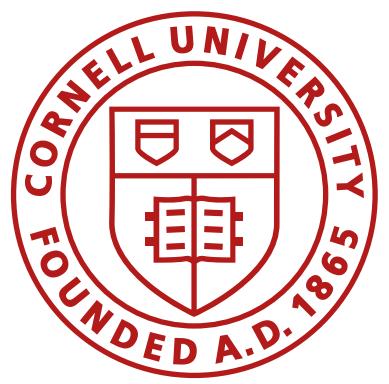


# Sensors II

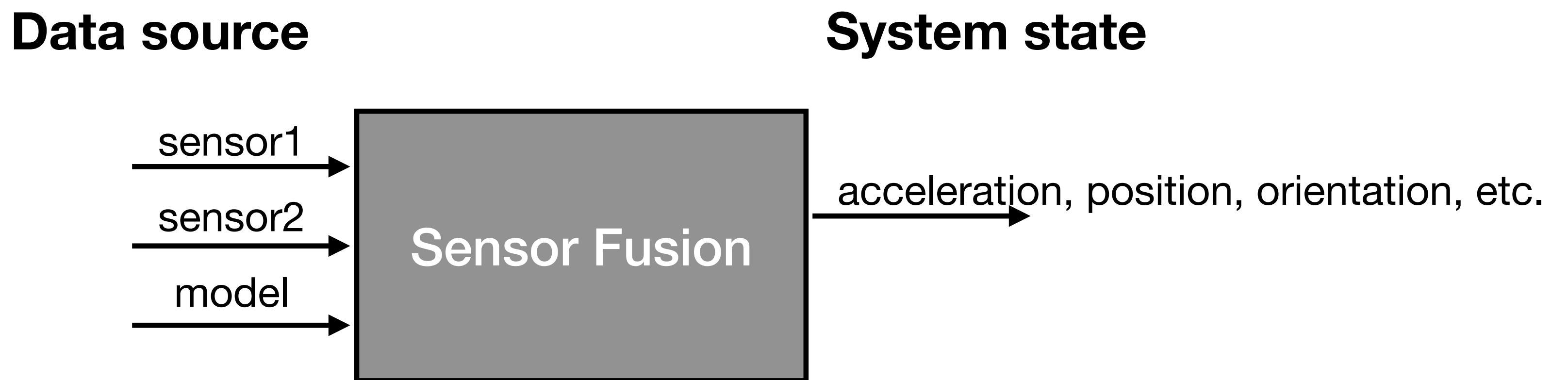
**Fast Robots, ECE4160/5160, MAE 4190/5190**

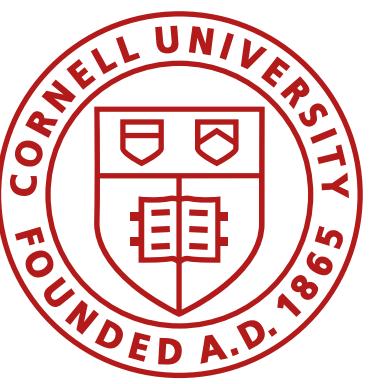
**E. Farrell Helbling, 1/29/25**



# Sensor Fusion

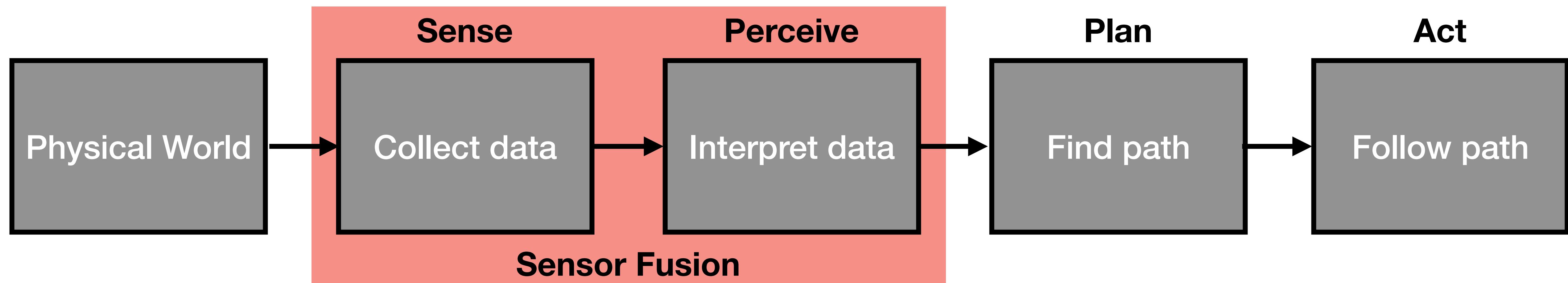
- Combine two or more data sources in a way that generates “better” understanding of the system
  - More consistent, accurate, and dependable signal over time



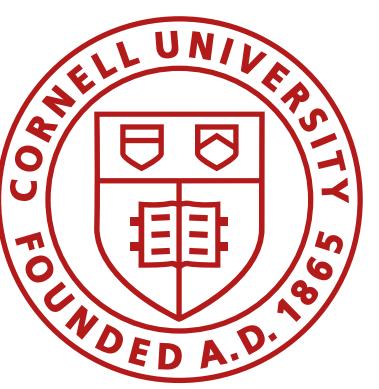


# Sensor Fusion

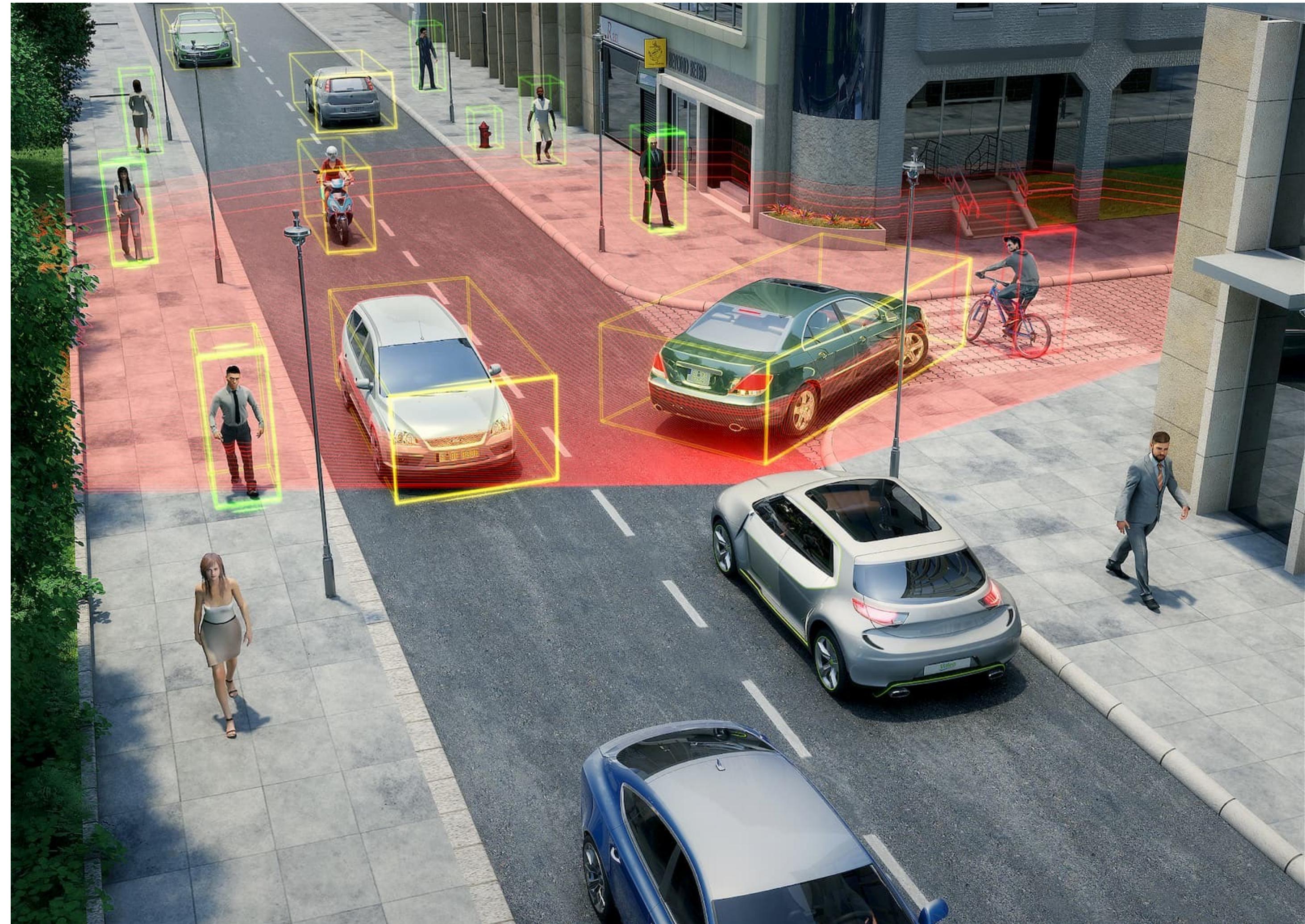
- Combine two or more data sources in a way that generates “better” understanding of the system
  - More consistent, accurate, and dependable signal over time



- Responsibilities:
  - Self-awareness (where am I? what am I doing? what is my current state?)
  - Situational awareness (detection/ tracking)



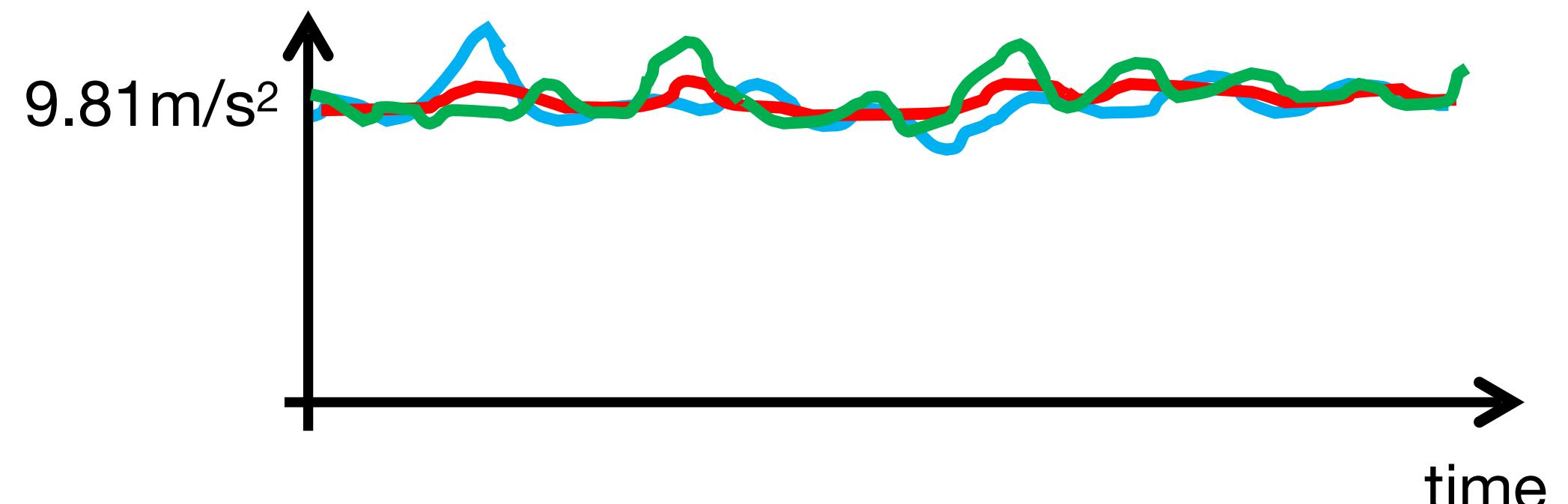
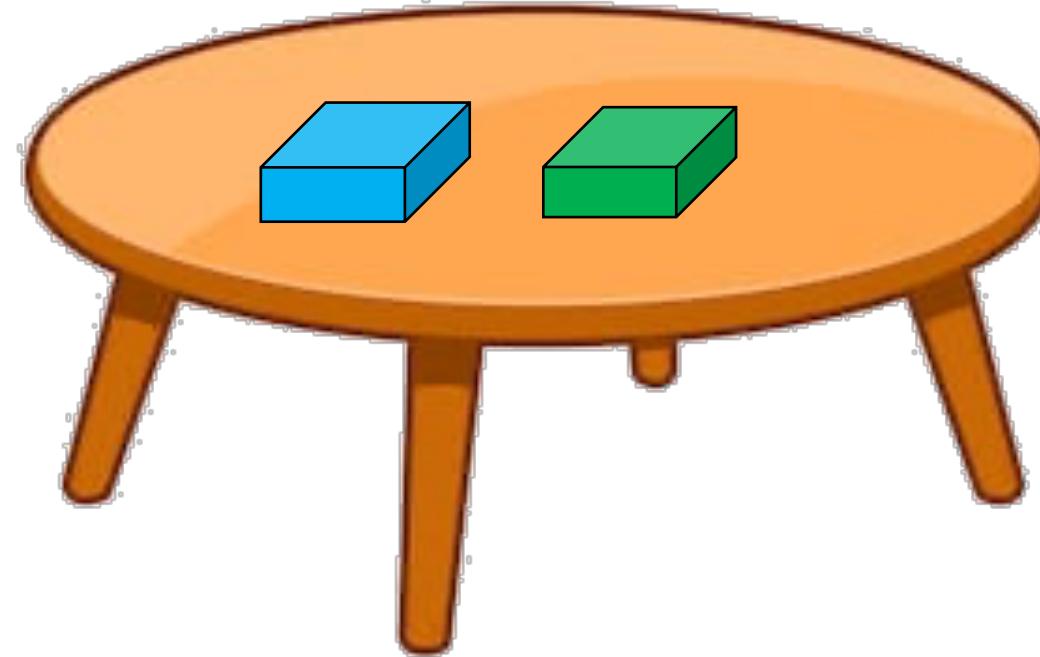
# Situational awareness



Valeo's LIDAR

# Sensor Fusion

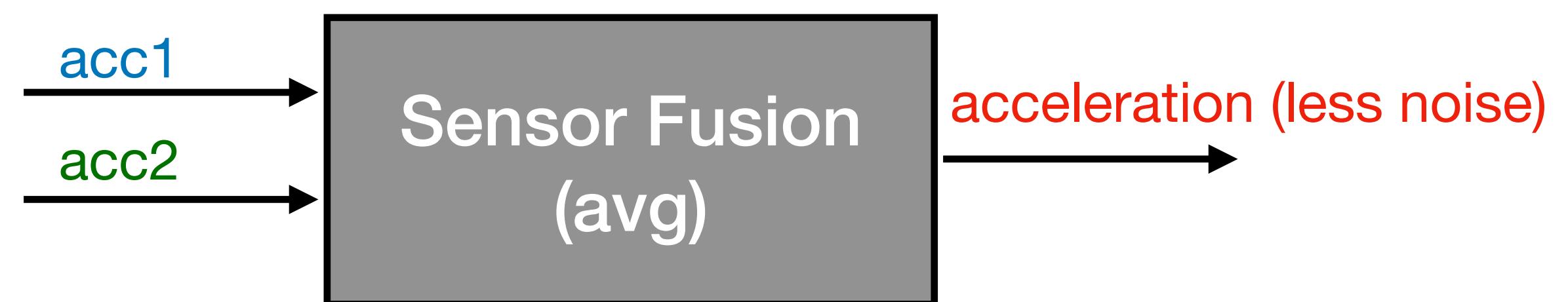
- Increase the quality of the data: less noise, uncertainty, deviation



- Adding sensors lowers noise:

- $n = 1/\sqrt{N}$

- Only true if the noise is not correlated!

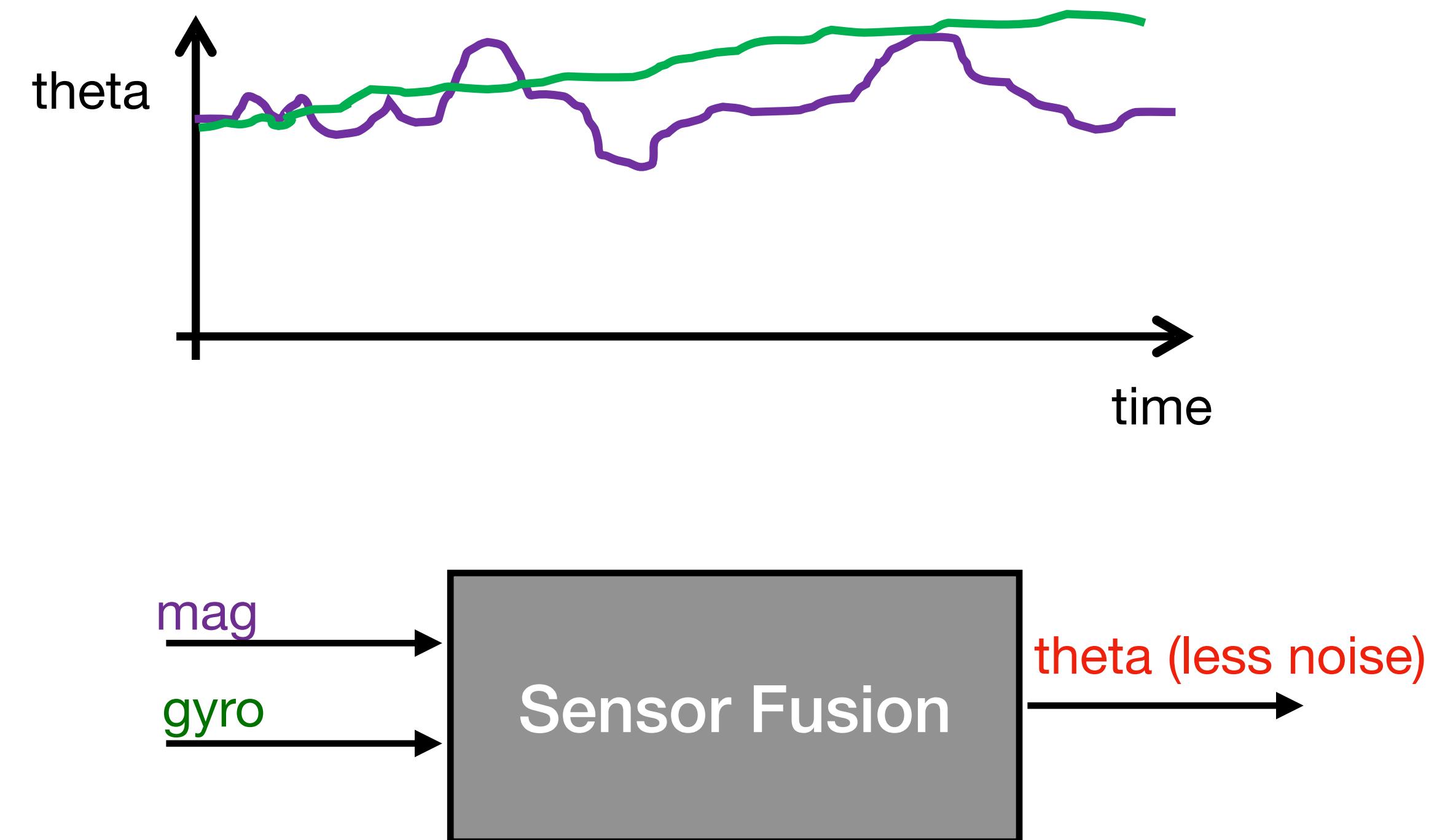


# Sensor Fusion

- Increase the quality of the data: less noise, uncertainty, deviation

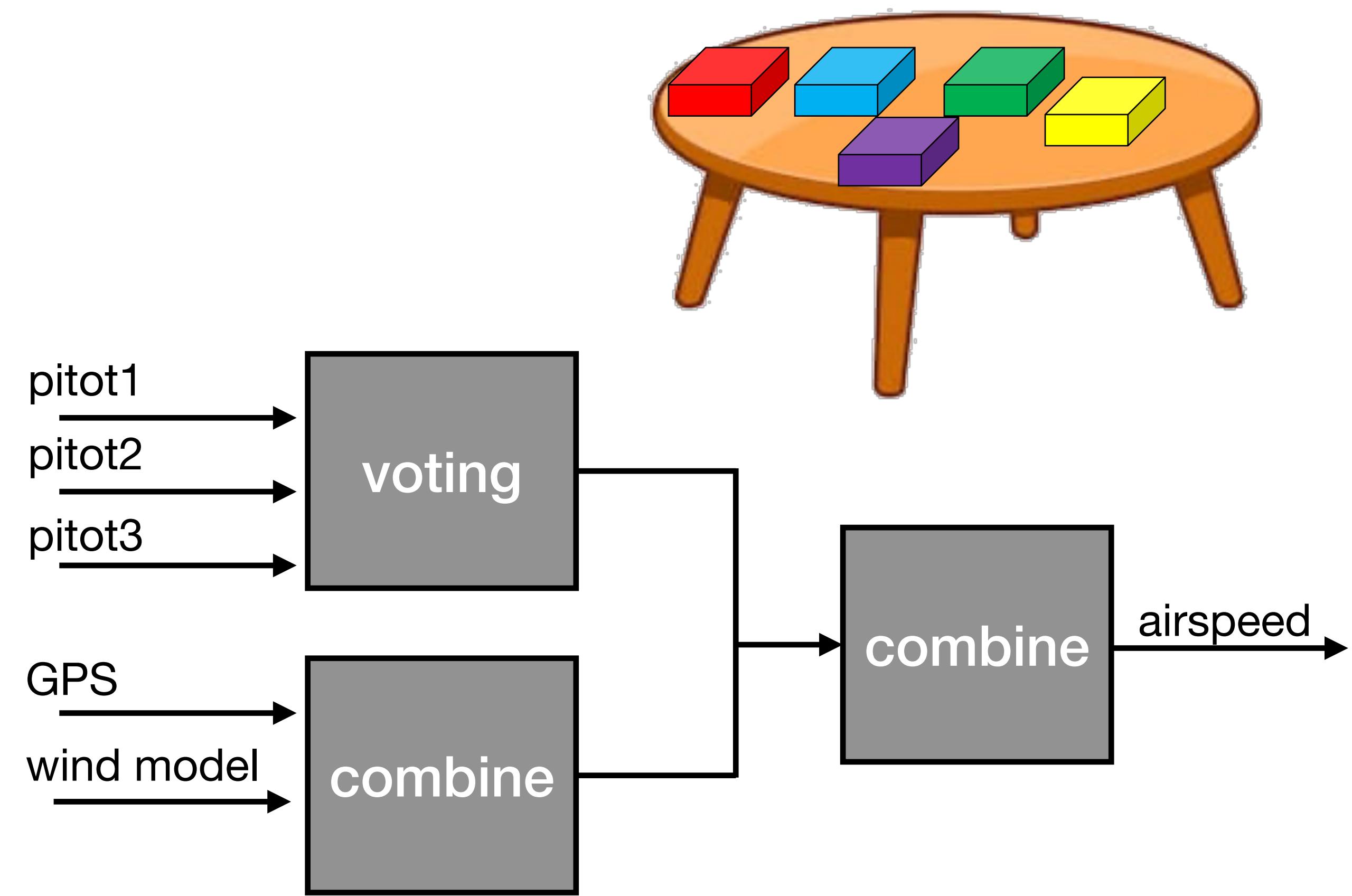


- Add a second mag?
- Move the sensor away from the field
- Low pass filter
- Fuse the mag data with the gyroscope data



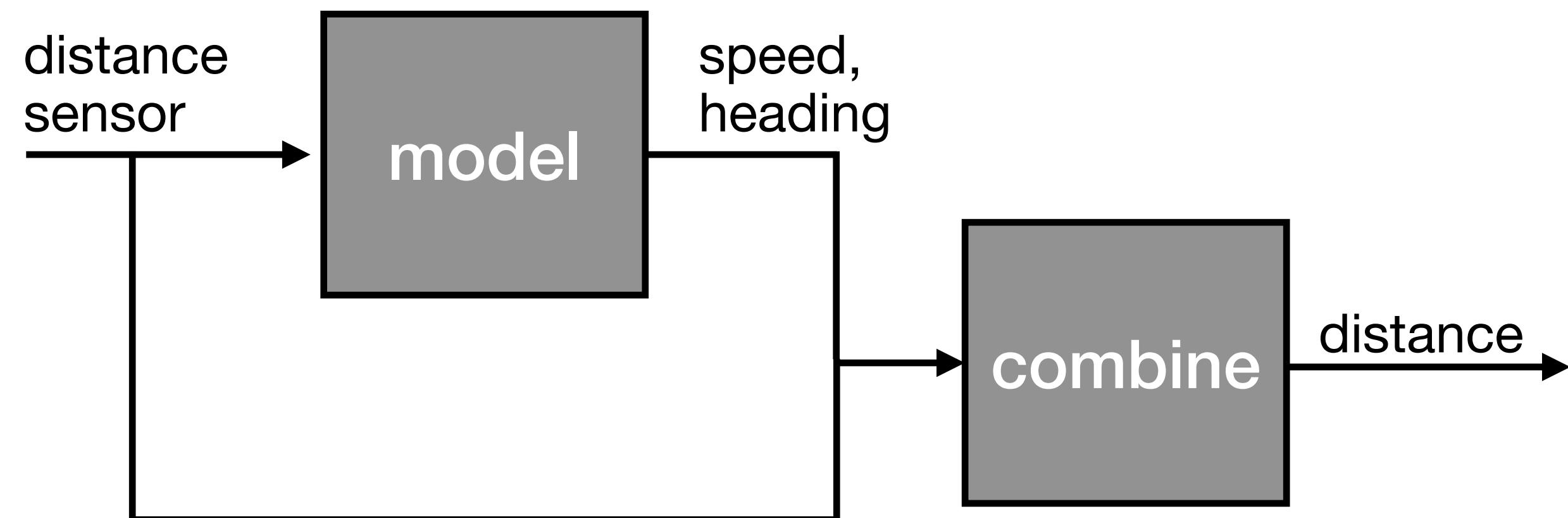
# Sensor Fusion

- Increase the quality of the data: less noise, uncertainty, deviation
- Increase data reliability



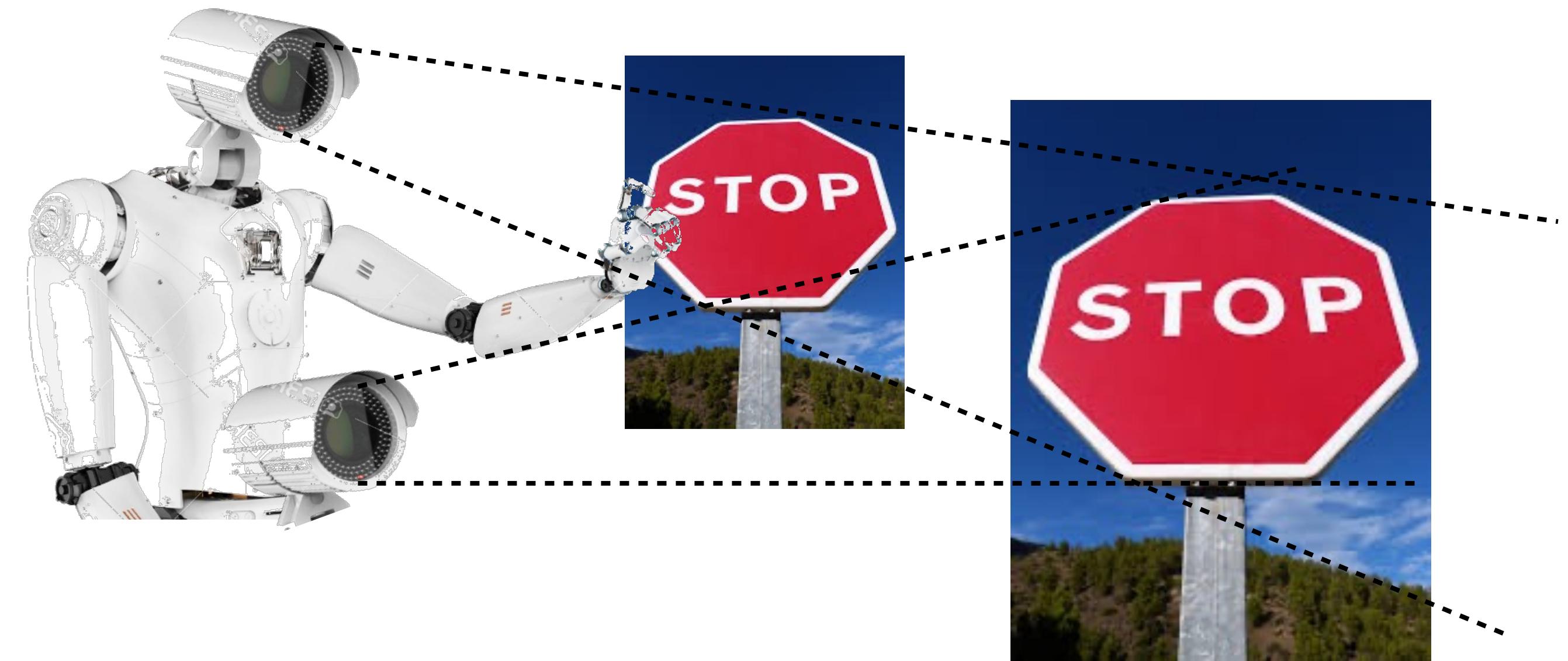
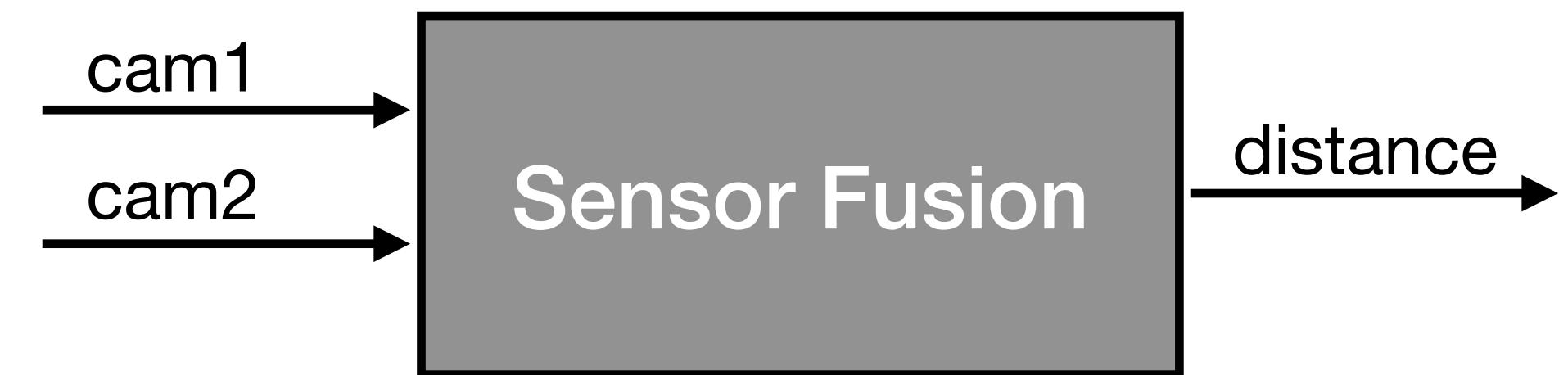
# Sensor Fusion

- Increase the quality of the data: less noise, uncertainty, deviation
- Increase data reliability



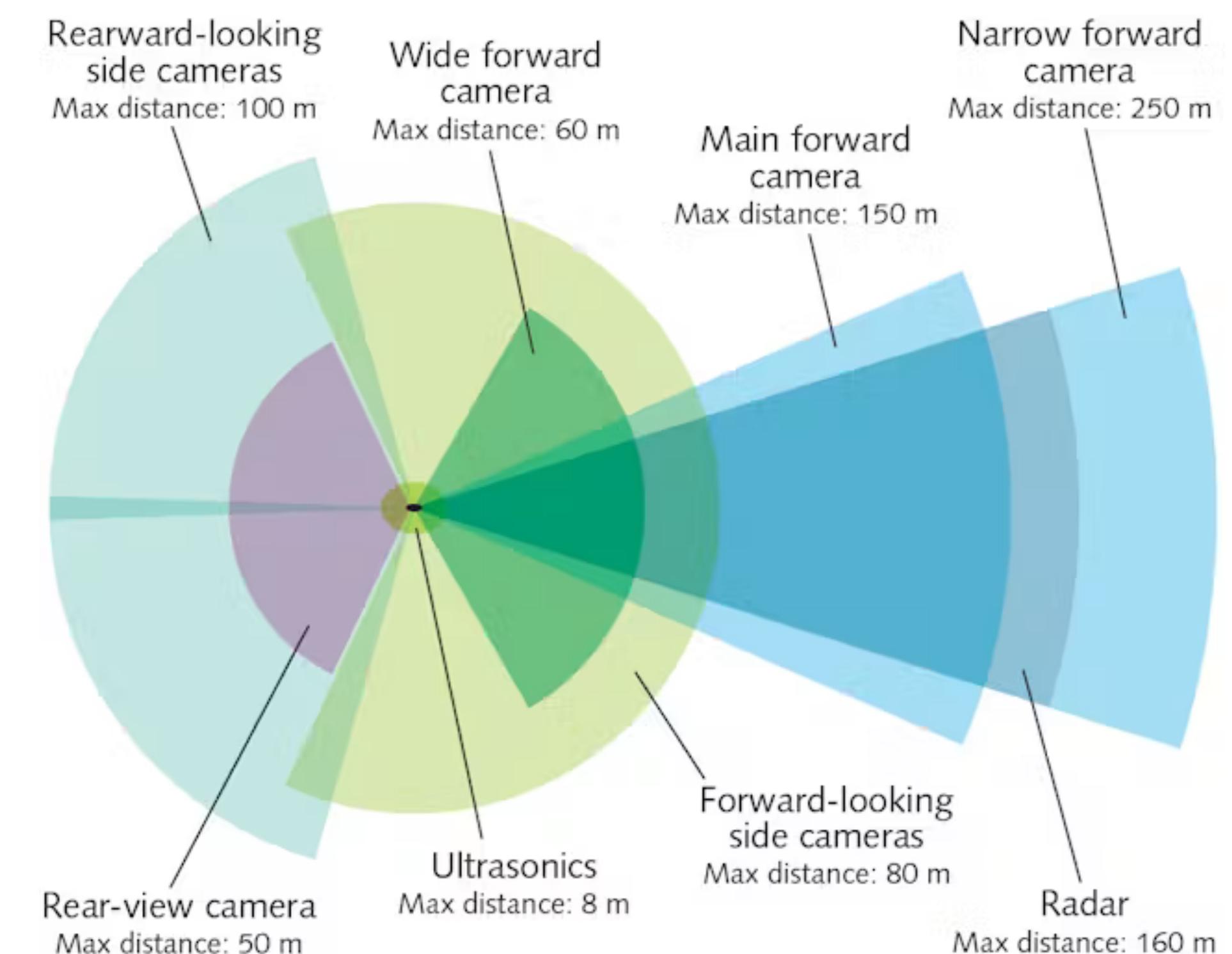
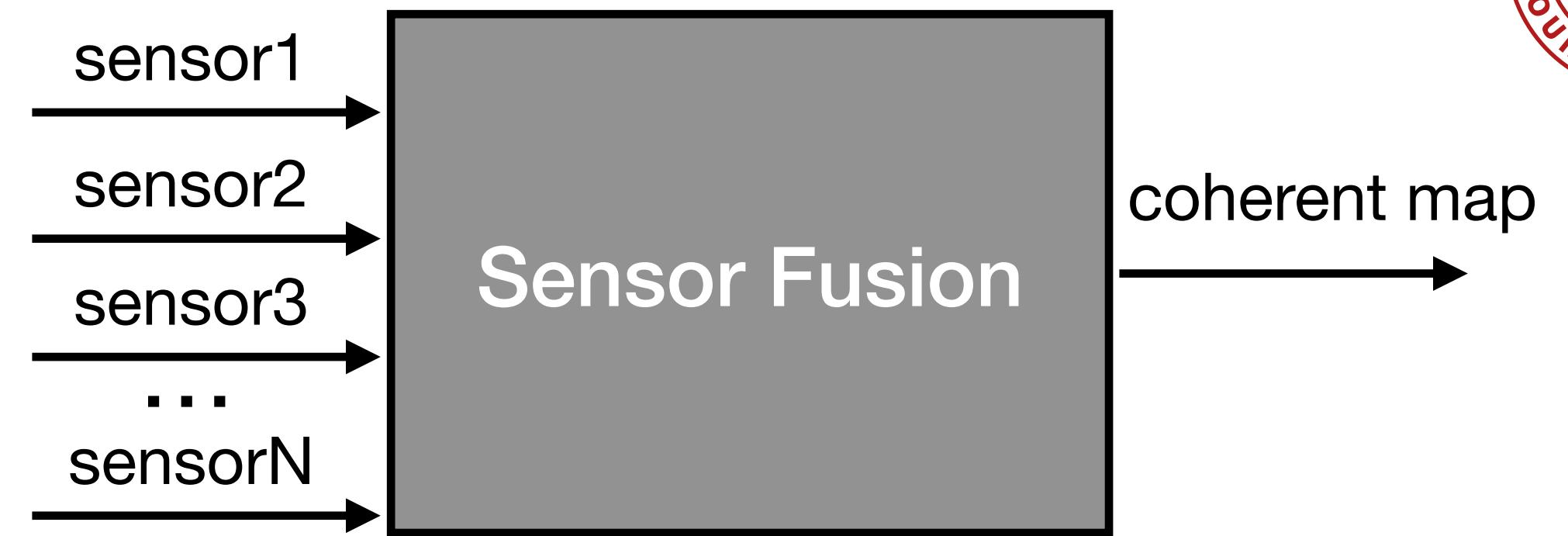
# Sensor Fusion

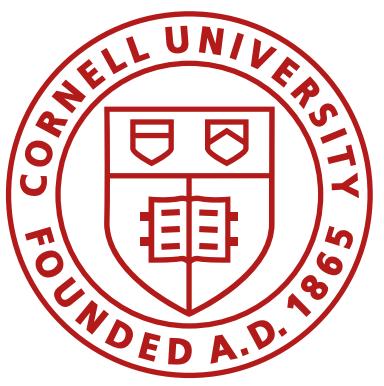
- Increase the quality of the data: less noise, uncertainty, deviation
- Increase data reliability
- Measure unmeasured states



# Sensor Fusion

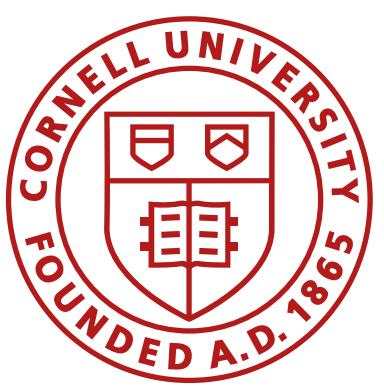
- Increase the quality of the data:
  - less noise, uncertainty, deviation
- Increase data reliability
- Measure unmeasured states
- Increase coverage area



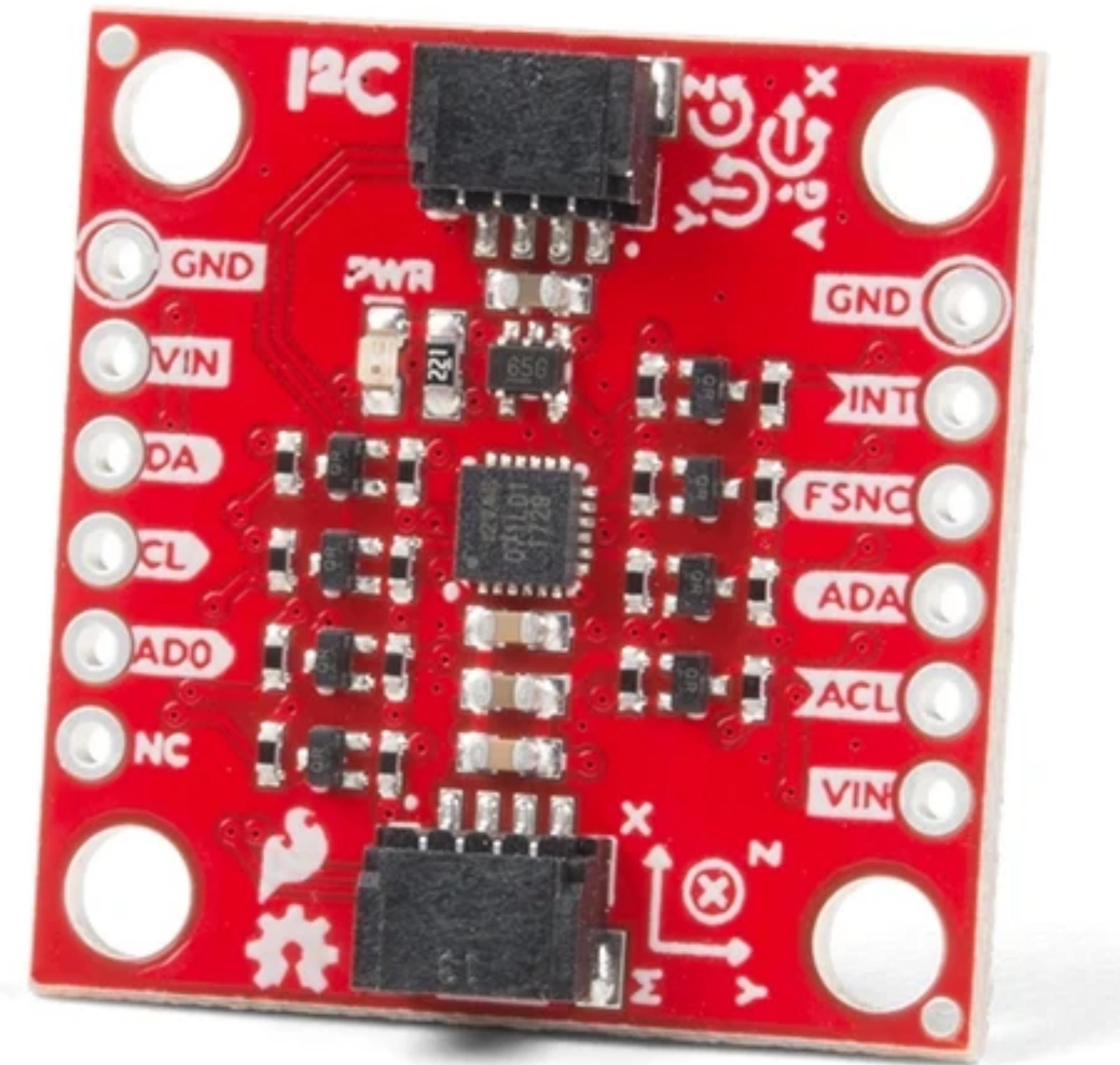


# Sources and references

- <http://www.cs.cmu.edu/~rasc/Download/AMRobots4.pdf>
- [https://www.ti.com/lit/ug/sbau305b/sbau305b.pdf?ts=1599417595209&ref\\_url=https%253A%252F%252Fwww.google.com%252F](https://www.ti.com/lit/ug/sbau305b/sbau305b.pdf?ts=1599417595209&ref_url=https%253A%252F%252Fwww.google.com%252F)
- <https://hmc.edu/lair/ARW/ARW-Lecture01-Odometry.pdf>
- Matlab Tech Talks on Sensor Fusion (<https://www.youtube.com/watch?v=6qV3YjFppuc>)
- Prof. Kirstin Petersen



# IMU



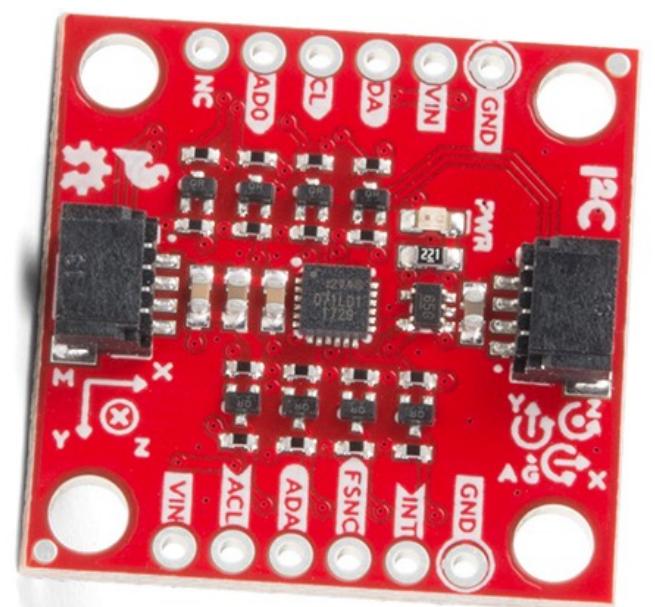
# IMU

- Data related to orientation, velocity, and gravity



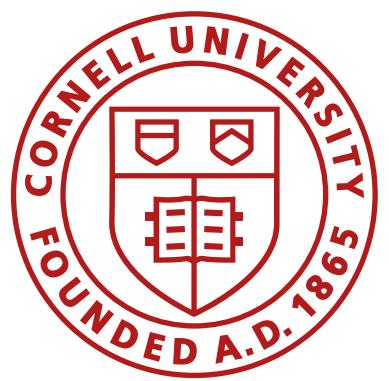
# IMU

- Accelerometer
    - Linear acceleration,  $a = \dot{v}$  [m/s<sup>2</sup>]
  - Gyroscope
    - Angular velocity,  $\omega = \frac{\Delta\theta}{\Delta t}$  [°/s]
  - Magnetometer
    - Magnetic field strength, [uT] or [Gauss]
  - NB: Gravity, magnetic fields, accelerations affect these sensors in many ways!
- ▶ Track orientation (position)
- ▶ Track orientation
- ▶ Get absolute orientation
- Dead reckoning



**ICM-20948**

- \$16
- Low power
- 9-axis



# IMU

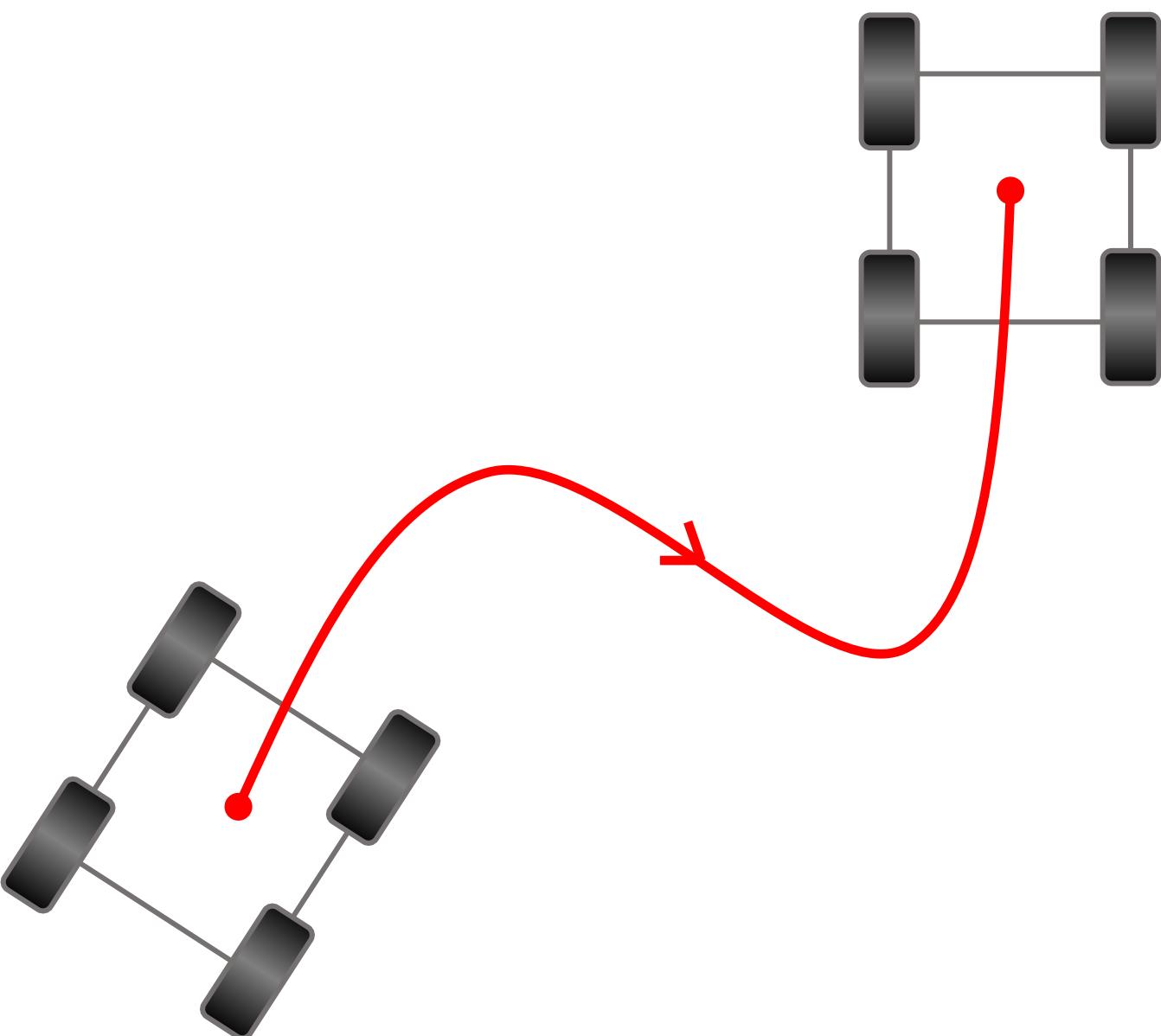
## Demo

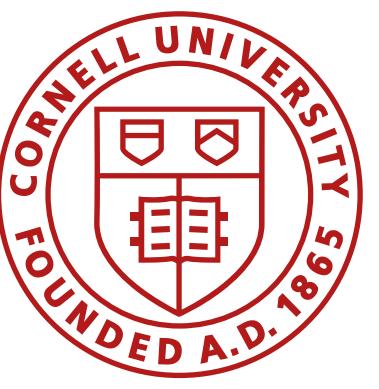
- Install Sparkfun 9DOF IMU – ICM 20948 library
- Follow the basics example

```
COM4
Initialization of the sensor returned: All is well.
Waiting for data
Scaled. Acc (mg) [ -00093.75, 00001.46, 01019.53 ], Gyr (DPS) [ -00000.96, 00001.80, -00002.67 ], Mag (uT) [ 00001.05, -00049.95, 00049.50 ], Tmp (C) [ 00024.35 ]
Scaled. Acc (mg) [ -00090.82, 00010.74, 01012.21 ], Gyr (DPS) [ 00001.40, 00000.82, 00001.05 ], Mag (uT) [ 00002.10, -00050.10, 00049.05 ], Tmp (C) [ 00024.16 ]
Scaled. Acc (mg) [ -00089.84, 00001.46, 01025.39 ], Gyr (DPS) [ 00001.19, 00000.60, 00002.05 ], Mag (uT) [ 00001.95, -00049.95, 00049.95 ], Tmp (C) [ 00024.16 ]
Scaled. Acc (mg) [ -00104.00, 00007.32, 01018.07 ], Gyr (DPS) [ -00001.53, 00001.66, -00002.59 ], Mag (uT) [ 00002.70, -00051.45, 00048.75 ], Tmp (C) [ 00024.07 ]
Scaled. Acc (mg) [ -00087.89, -00003.91, 01010.74 ], Gyr (DPS) [ -00000.18, 00001.04, 00001.18 ], Mag (uT) [ 00001.50, -00050.40, 00049.20 ], Tmp (C) [ 00024.16 ]
Scaled. Acc (mg) [ -00087.89, -00004.39, 01024.90 ], Gyr (DPS) [ 00003.80, -00001.62, -00000.11 ], Mag (uT) [ 00001.95, -00050.70, 00050.70 ], Tmp (C) [ 00024.26 ]
Scaled. Acc (mg) [ -00096.19, 00007.32, 01017.09 ], Gyr (DPS) [ 00000.19, 00002.37, -00002.16 ], Mag (uT) [ 00002.10, -00050.55, 00049.05 ], Tmp (C) [ 00024.35 ]
Scaled. Acc (mg) [ -00089.36, -00002.44, 01021.97 ], Gyr (DPS) [ 00000.73, -00000.73, 00004.83 ], Mag (uT) [ 00003.30, -00050.10, 00050.10 ], Tmp (C) [ 00024.40 ]
Scaled. Acc (mg) [ -00100.59, -00002.93, 01012.21 ], Gyr (DPS) [ 00001.35, 00000.65, 00001.63 ], Mag (uT) [ 00002.25, -00050.70, 00049.95 ], Tmp (C) [ 00024.07 ]
Scaled. Acc (mg) [ -00103.52, -00001.46, 01014.16 ], Gyr (DPS) [ -00000.80, 00001.38, -00004.44 ], Mag (uT) [ 00001.05, -00050.40, 00049.20 ], Tmp (C) [ 00024.35 ]
Scaled. Acc (mg) [ -00095.21, -00000.49, 01015.14 ], Gyr (DPS) [ 00000.66, -00000.41, 00001.28 ], Mag (uT) [ 00001.95, -00051.00, 00049.20 ], Tmp (C) [ 00024.45 ]
```

Autoscroll  Show timestamp

Newline 115200 baud Clear output





# Accelerometer

# Accelerometer

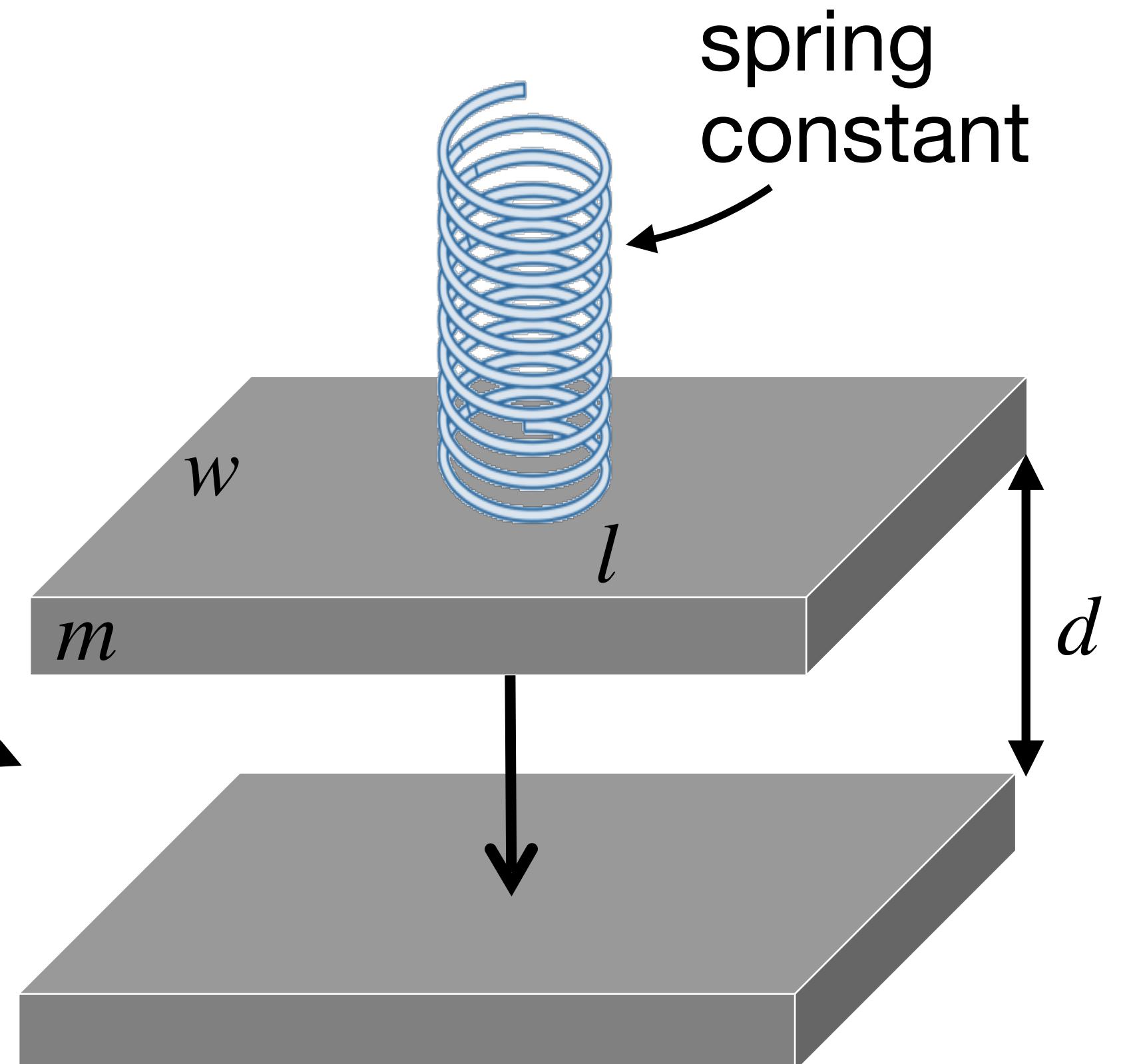
## Measure acceleration

$$C = \frac{\epsilon A}{d}$$

$$F = k\Delta d$$

$$F = ma$$

dielectric  
constant



# Accelerometer

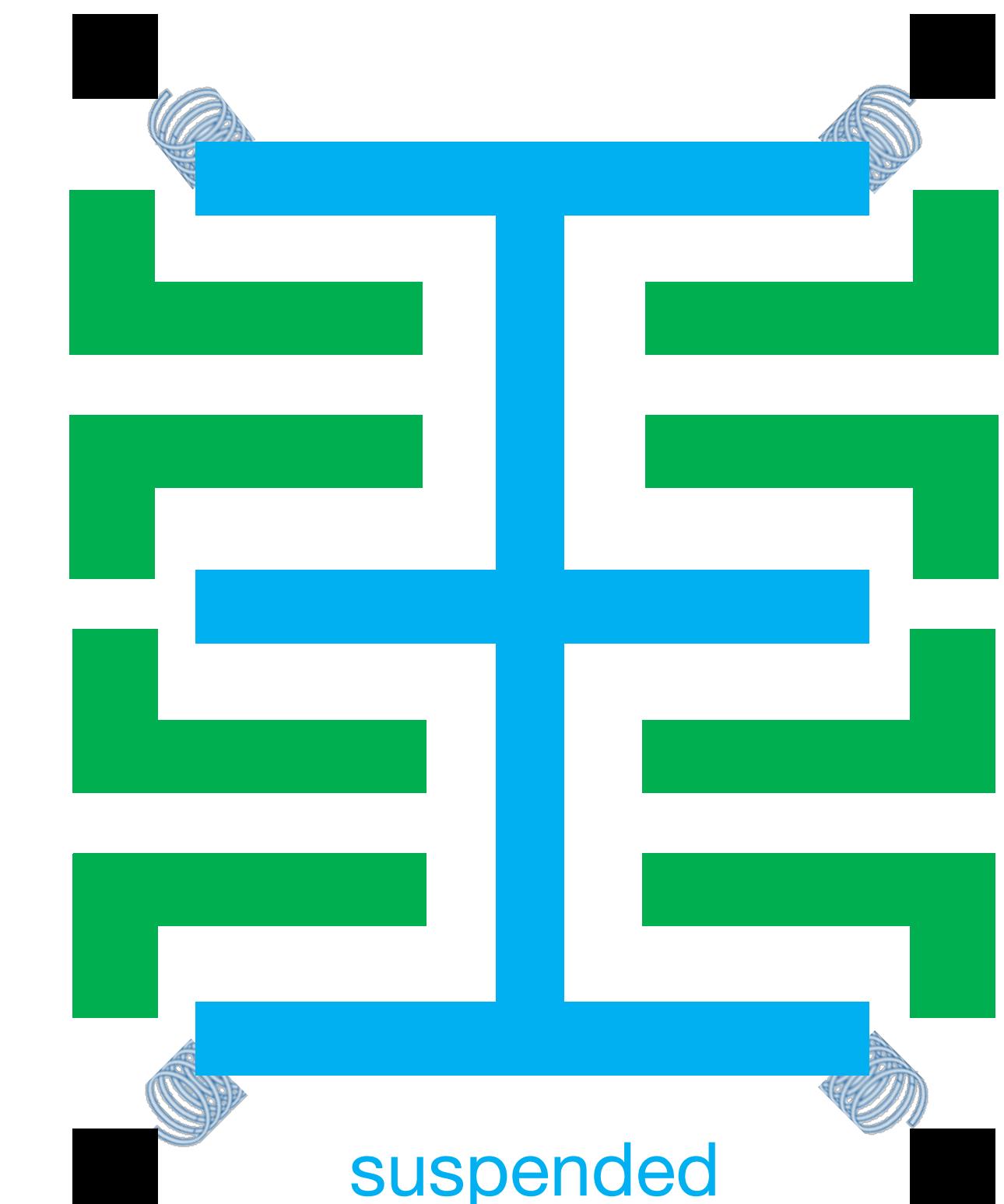
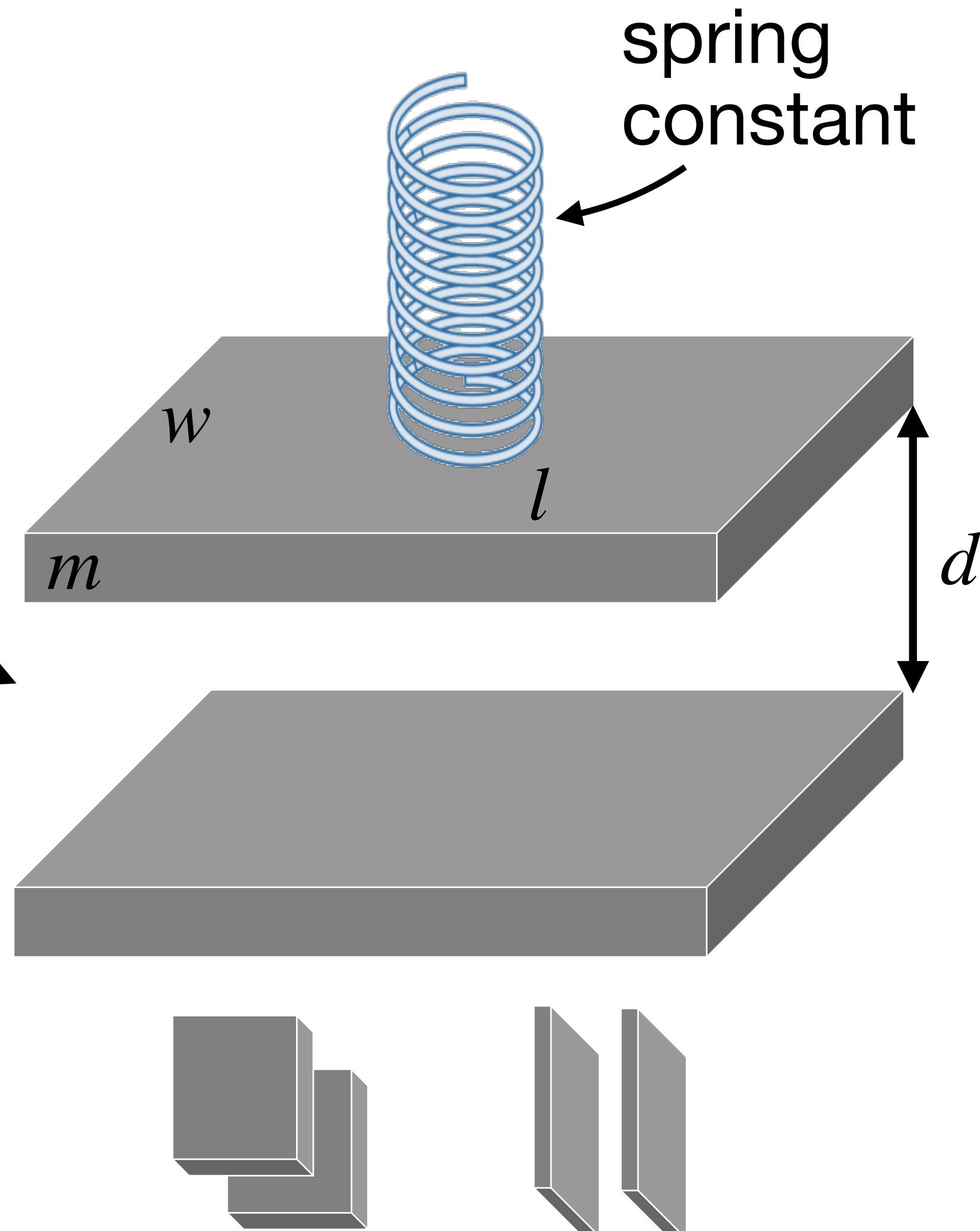
Measure acceleration in 3D

$$C = \frac{\epsilon A}{d}$$

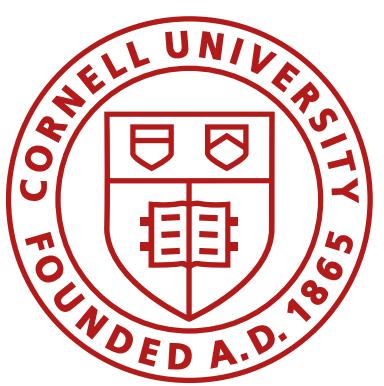
$$F = k\Delta d$$

$$F = ma$$

dielectric  
constant



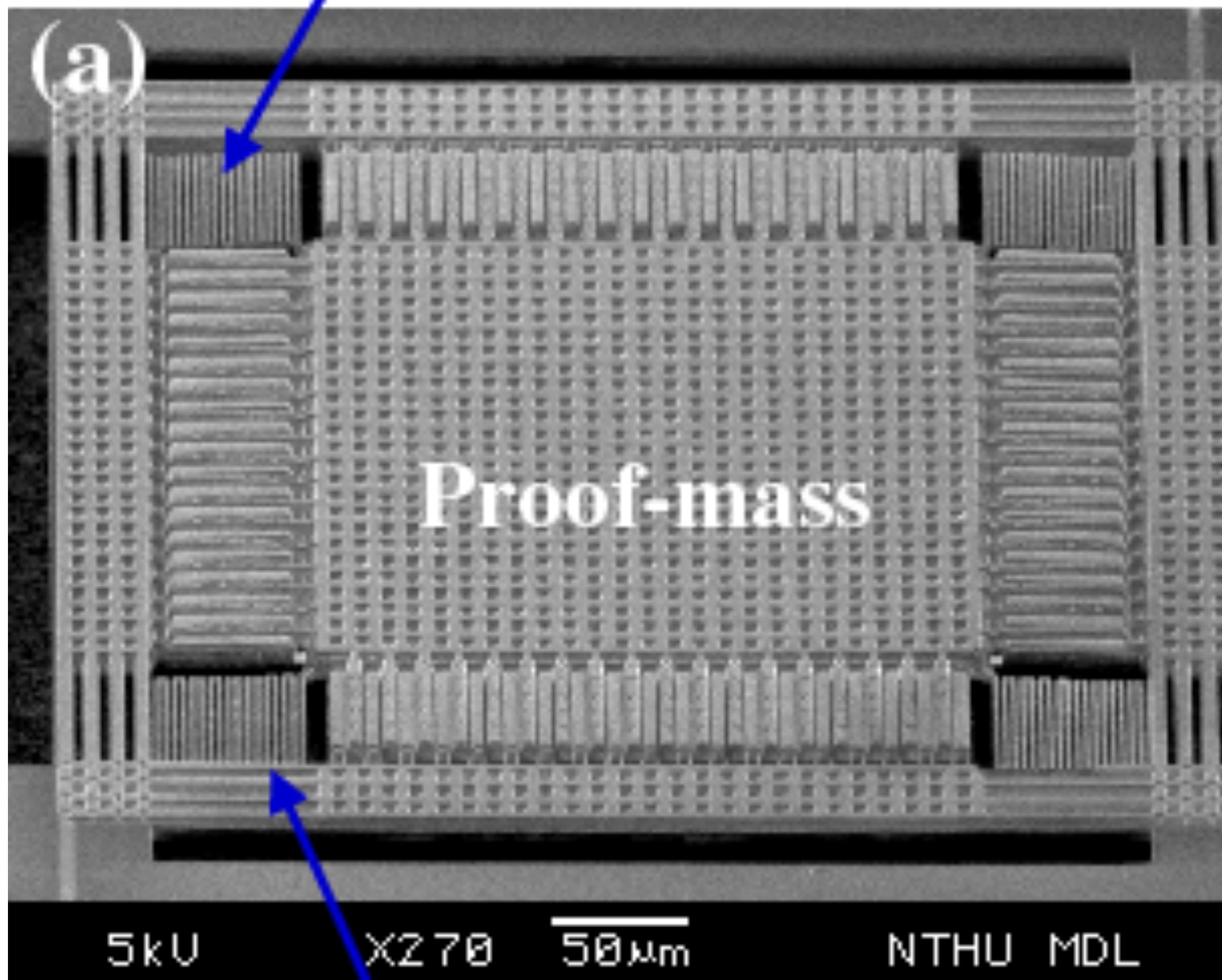
suspended  
conductive material  
conductive  
beams (fixed)



# Accelerometer

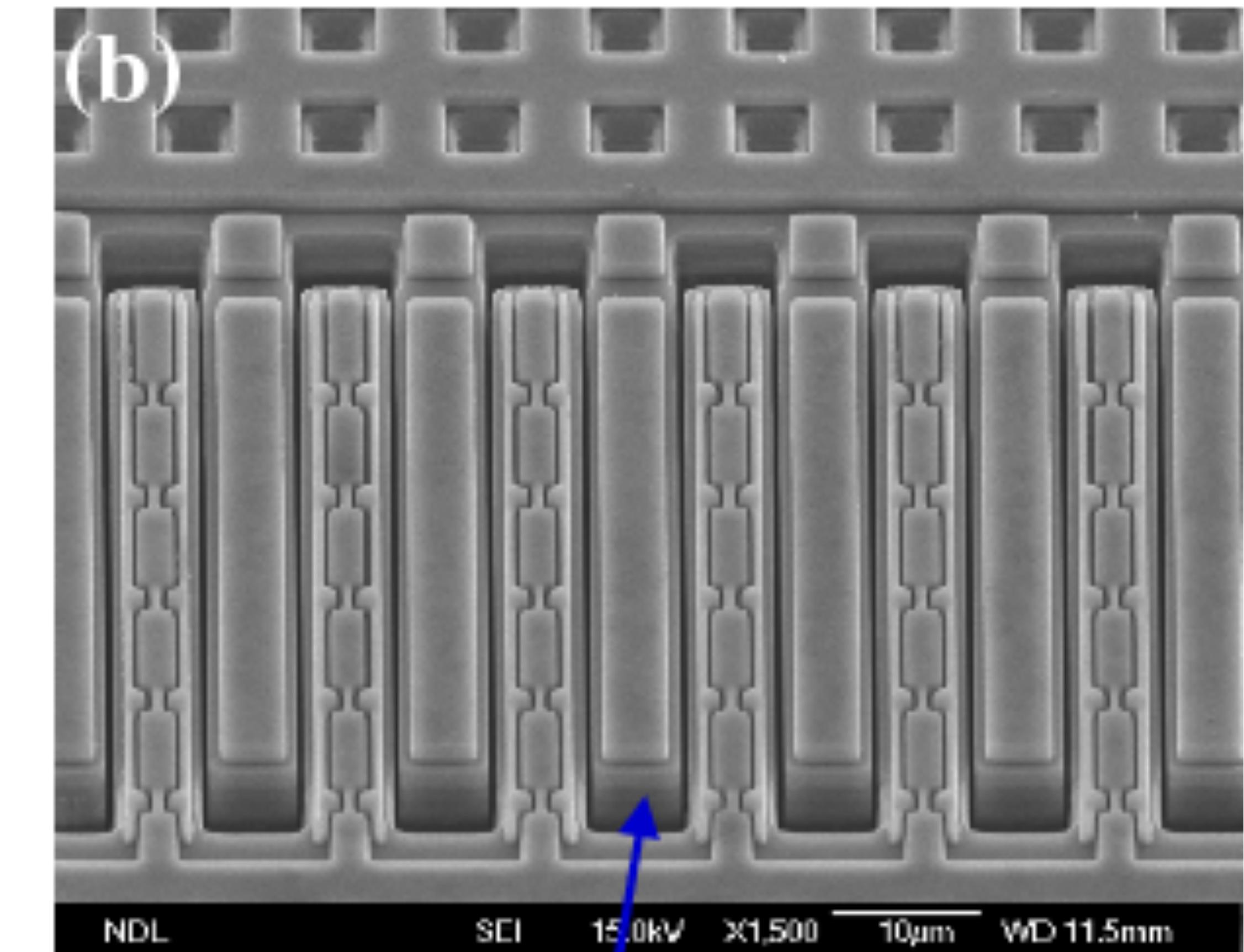
Measure acceleration in 3D

Spring

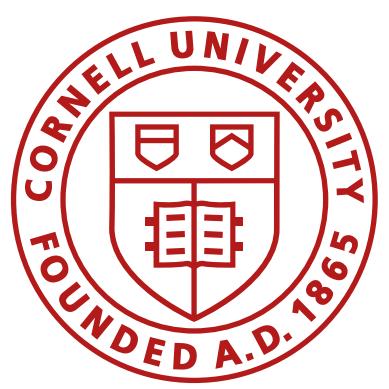


Supporting frame

Fang, 2011

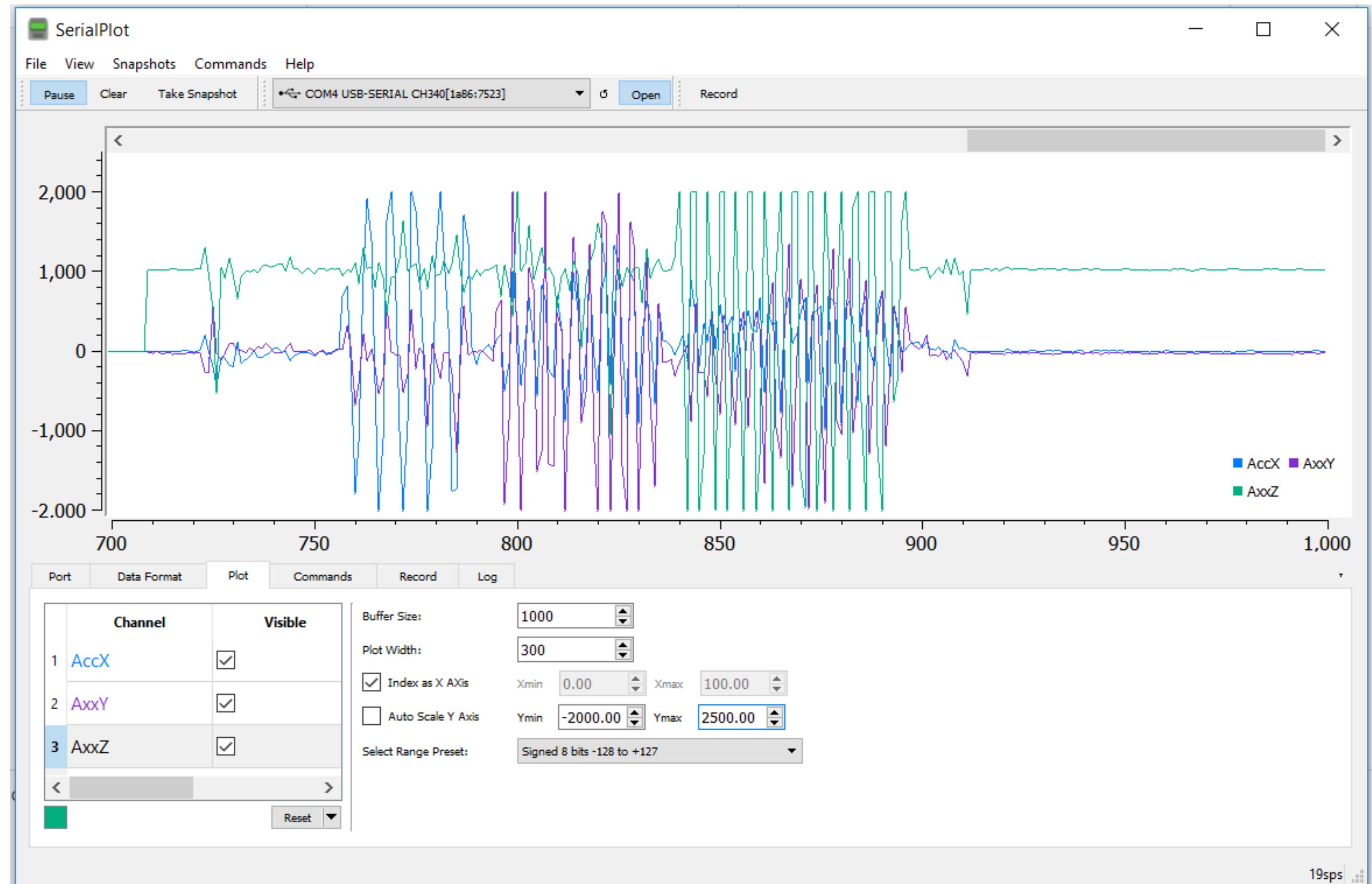


Sensing electrodes



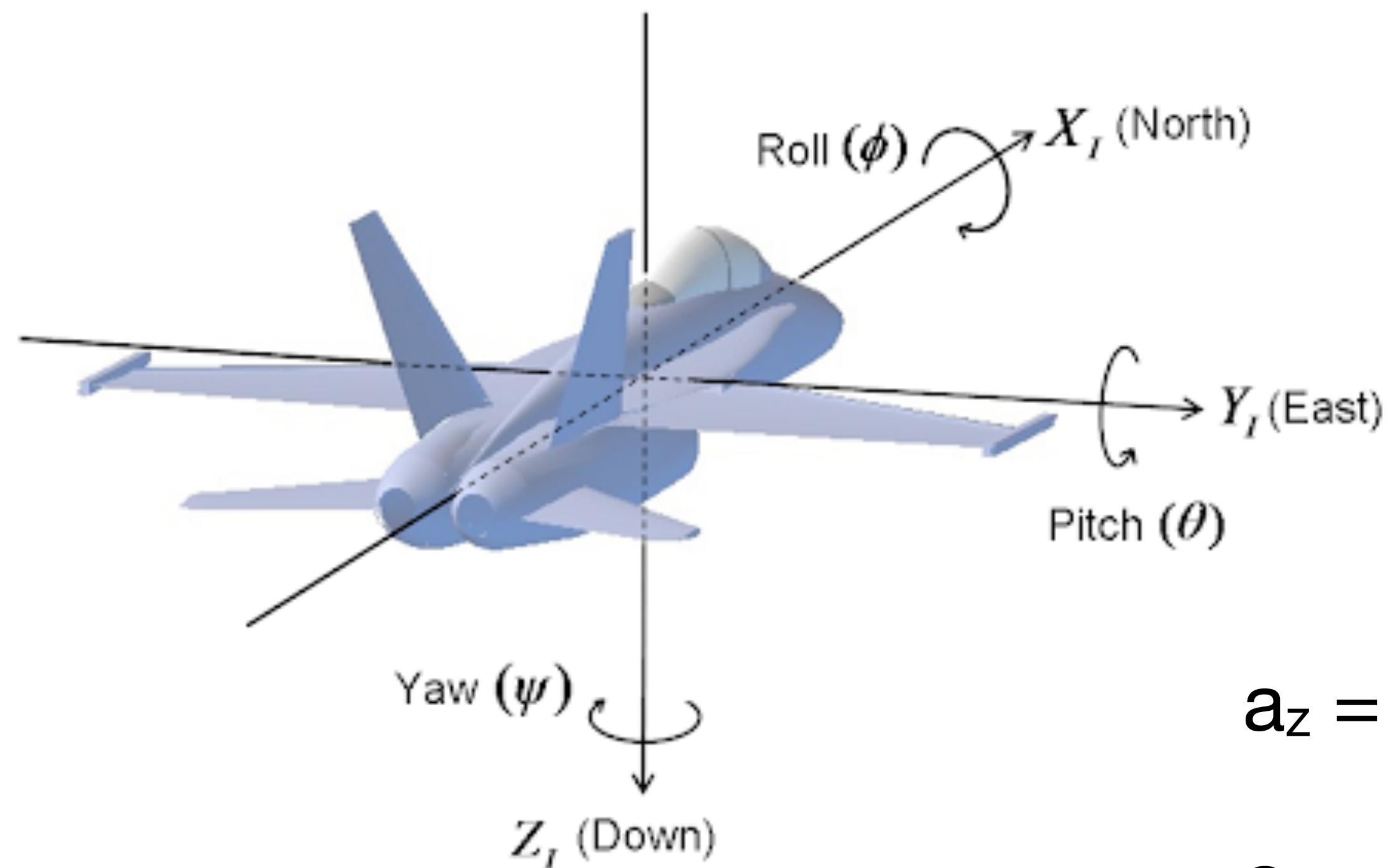
# Accelerometer

## Use Serial Monitor or Serial Plot



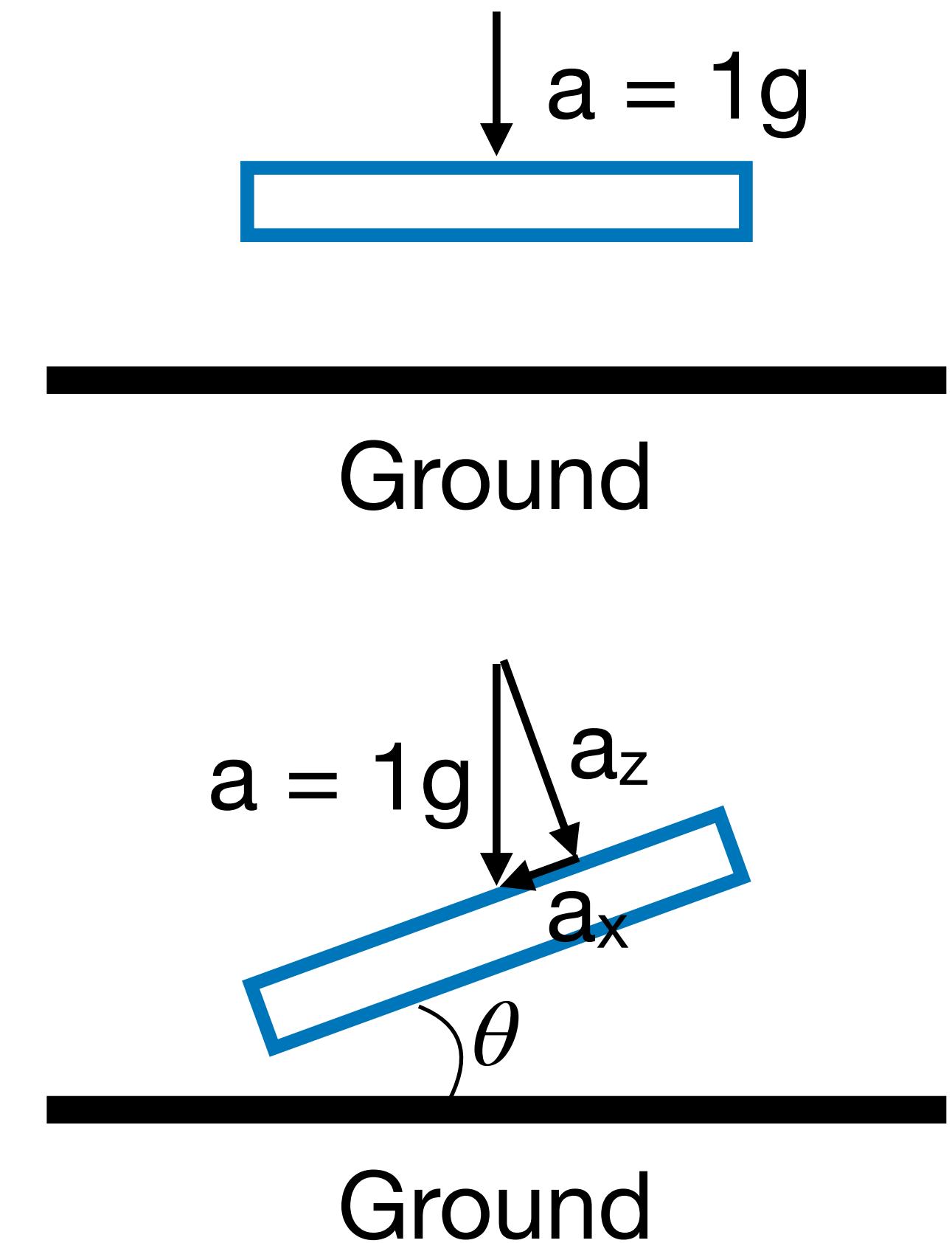
# Accelerometer

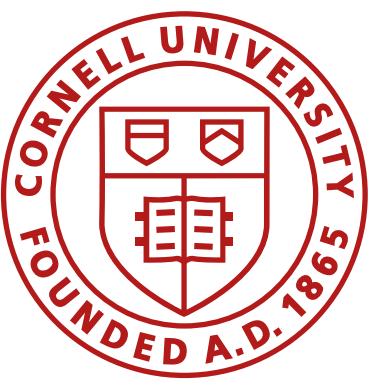
## Roll, Pitch, Yaw



$$a_z = 1g \cos(\theta)$$

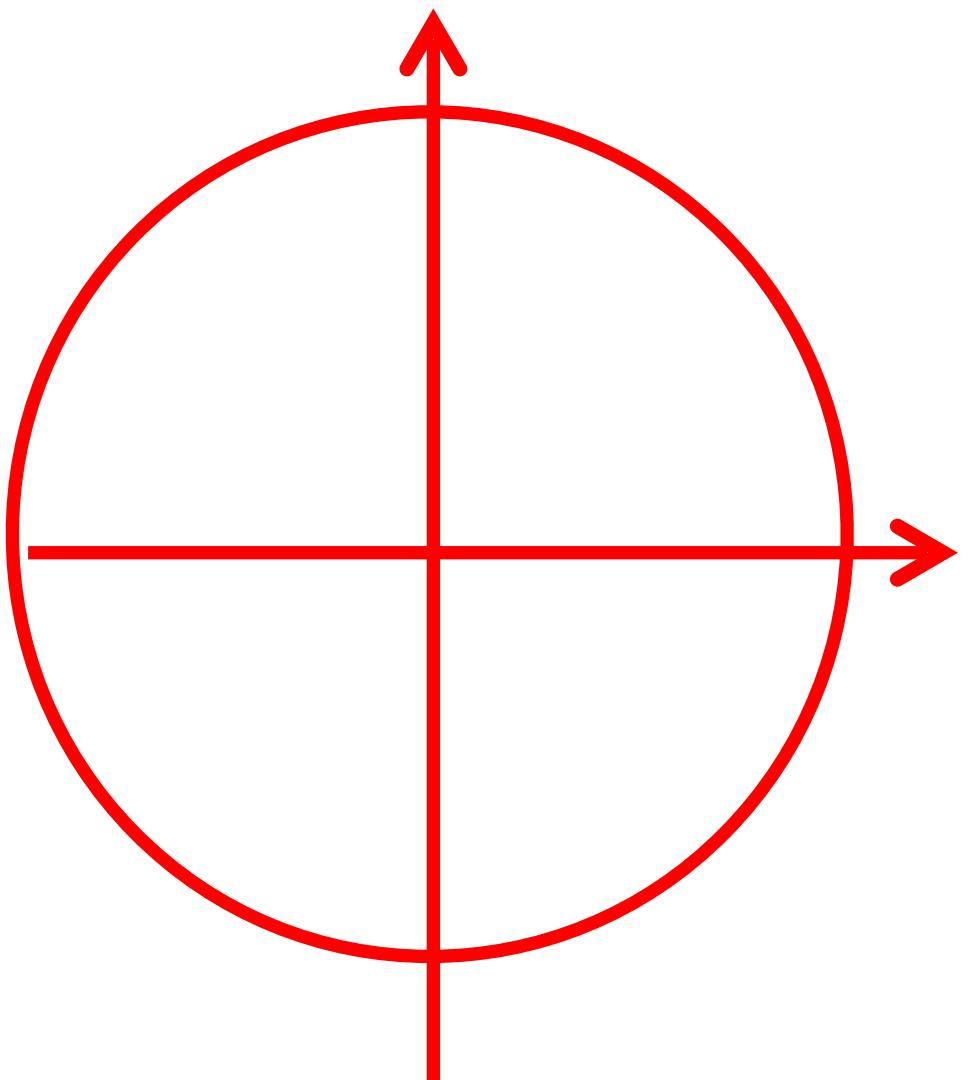
$$a_x = 1g \sin(\theta)$$





# atan vs. atan2

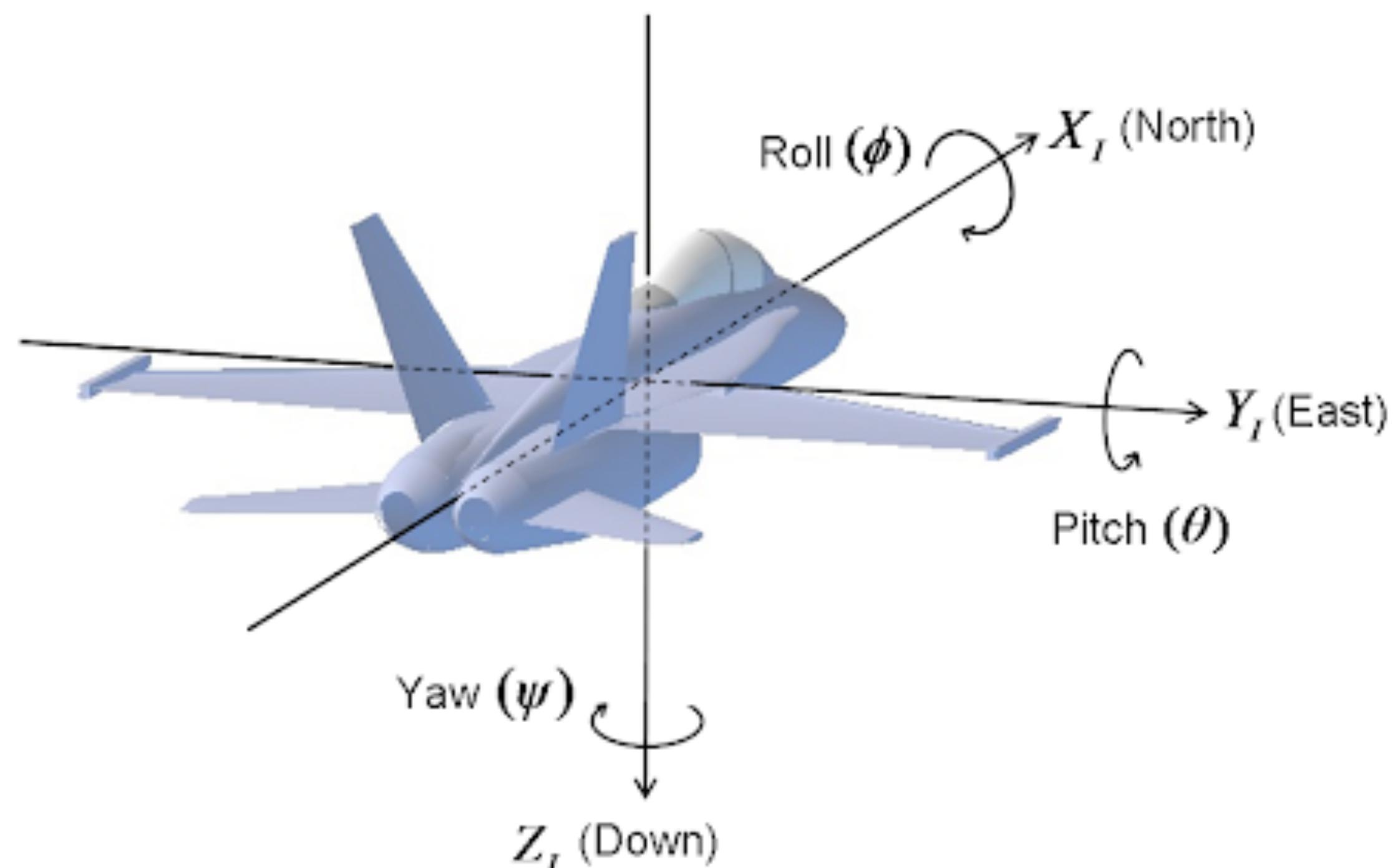
- $\text{atan}(a_x, a_z)$  returns  $[-\pi/2, \pi/2]$
- Instead use  $\text{atan2}(a_x, a_z)$  which returns  $[-\pi, \pi]$



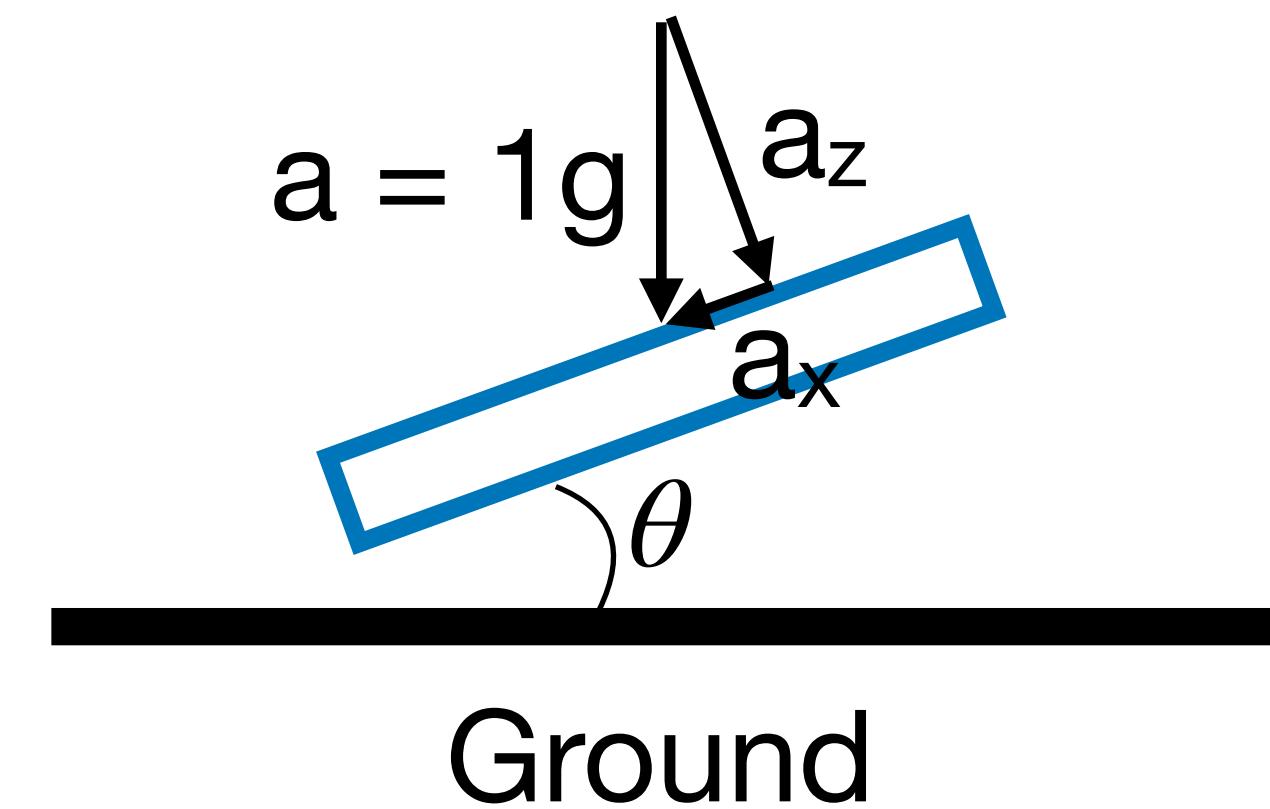
```
float atan2(float x, float y) {  
    if (x > 0.0)  
        return atan(y/x);  
    if (x < 0.0) {  
        if (y >= 0.0)  
            return (PI + atan(y/x));  
        else  
            return (-PI + atan(y/x));  
    }  
    if (y > 0.0) // x == 0  
        return PI_ON_TWO;  
    if (y < 0.0)  
        return -PI_ON_TWO;  
    return 0.0; // Should be undefined  
}
```

# Accelerometer

## Roll, Pitch, Yaw



**Can we estimate yaw?**



$$a_z = 1g \cos(\theta)$$

$$a_x = 1g \sin(\theta)$$

$$\theta = \text{atan2}(a_x, a_z)$$

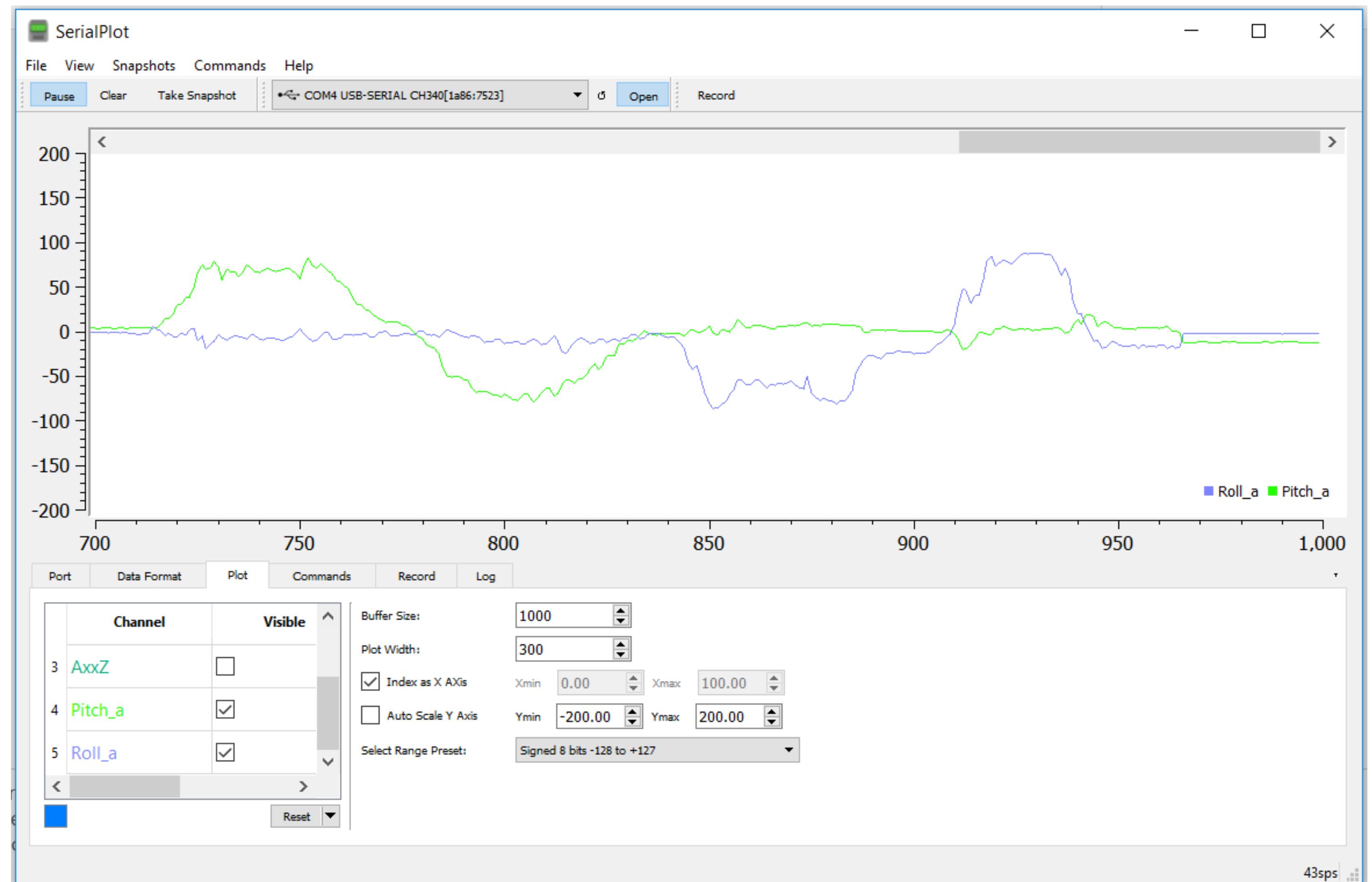
$$\phi = \text{atan2}(a_y, a_z)$$

# Accelerometer

## Pitch and Roll

$$\theta = \text{atan2}(a_x, a_z)$$

$$\phi = \text{atan2}(a_y, a_z)$$



# Accelerometer

## Roll and Pitch

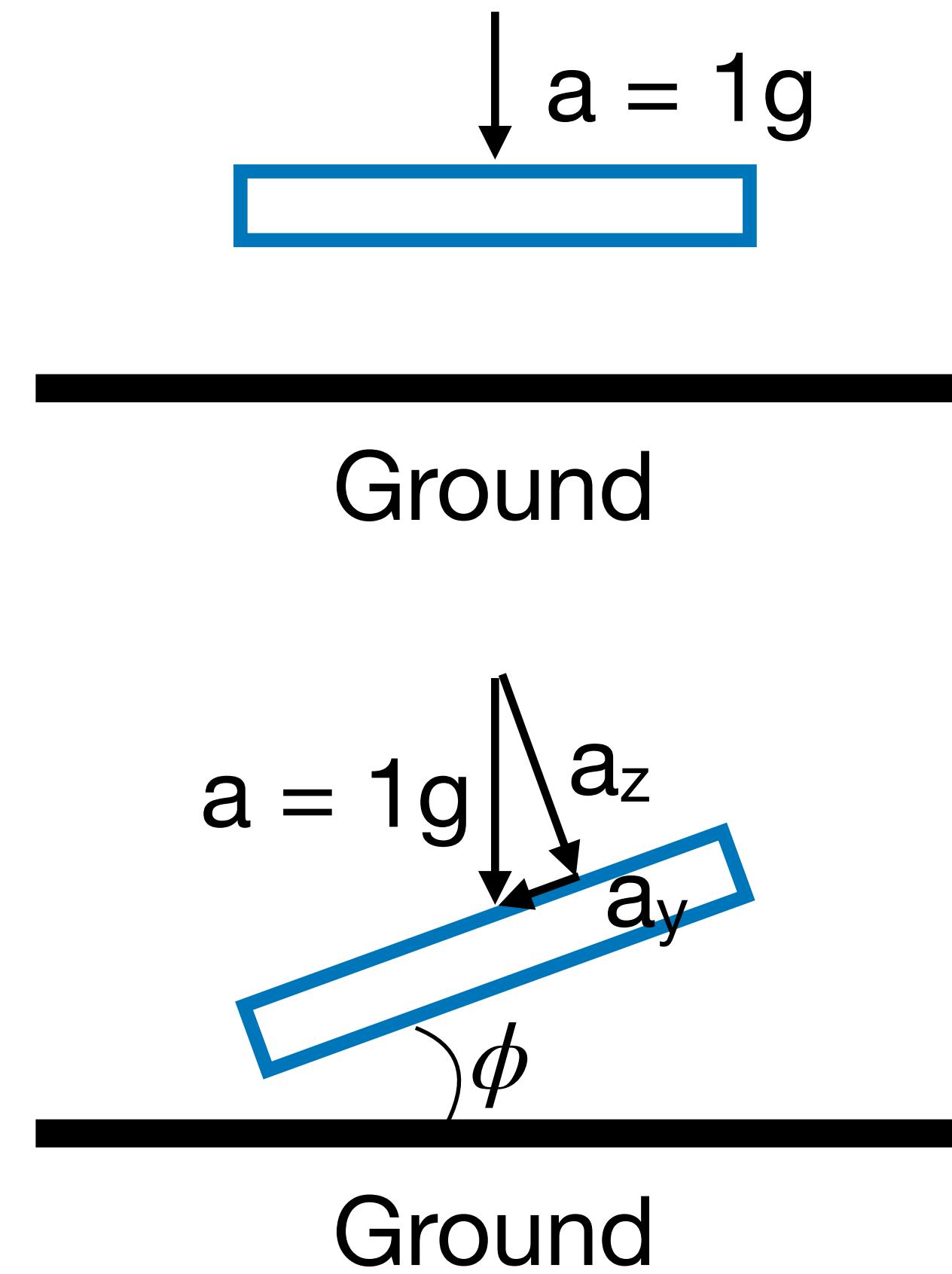
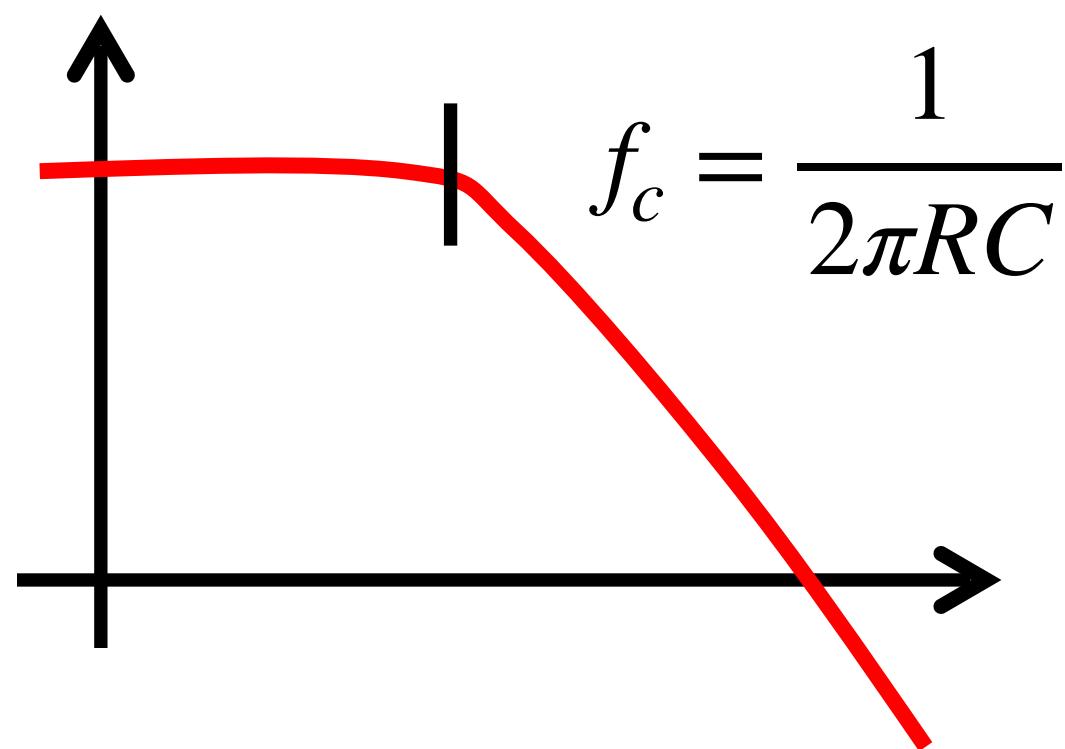
- Good (very accurate on average) vs. bad (noisy)
- Low pass filter

$$\theta_{\text{LPF}}[n] = \alpha * \theta_{\text{RAW}} + (1 - \alpha) * \theta_{\text{LPF}}[n-1]$$

$$\theta_{\text{LPF}}[n-1] = \theta_{\text{LPF}}[n]$$

- Think of your frequency like an RC low-pass filter:

$$\alpha = \frac{T}{T + RC}$$



# Accelerometer

## Dead Reckoning

- Use the accelerometer to do dead reckoning?

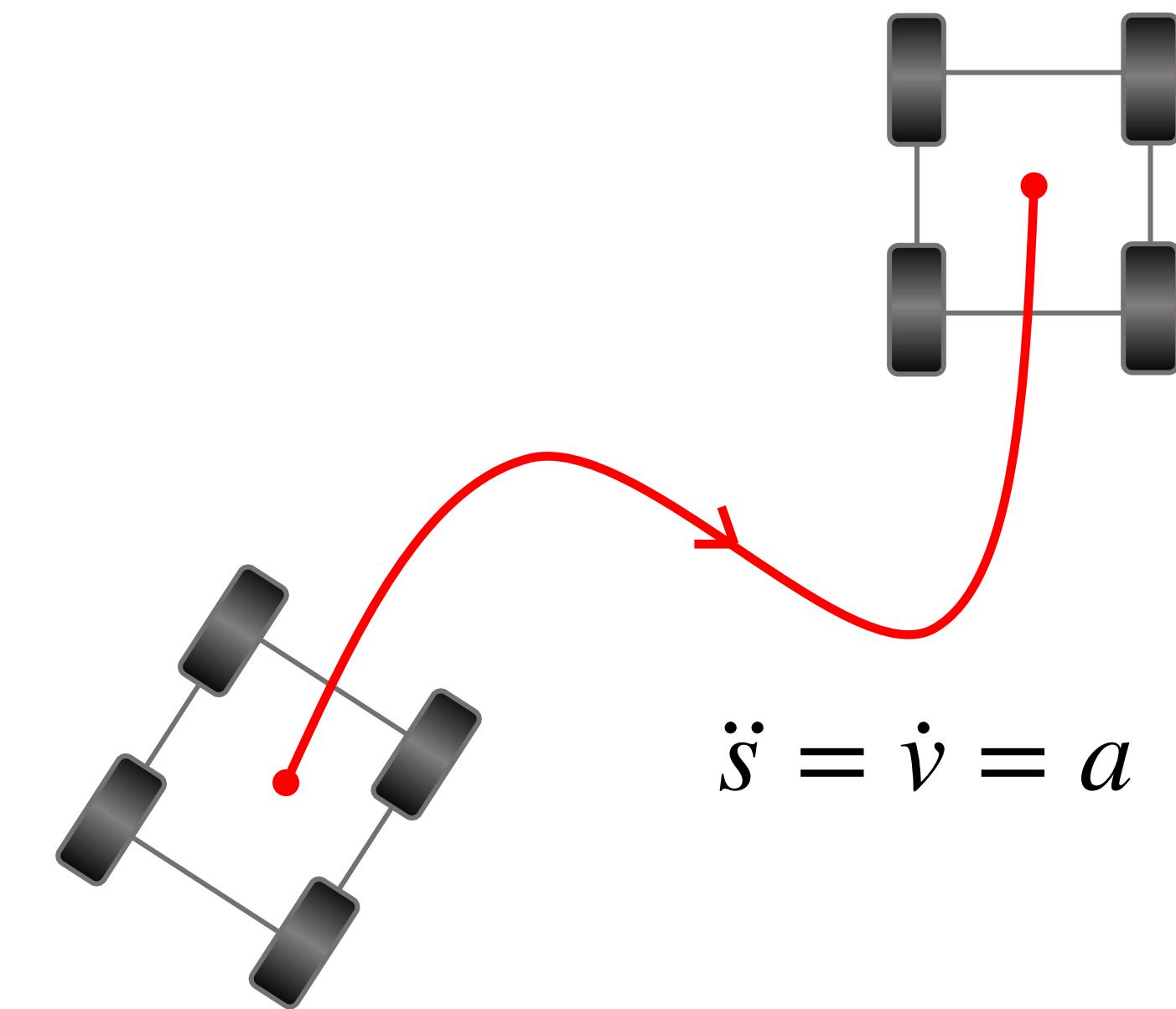
$$v = \int a$$

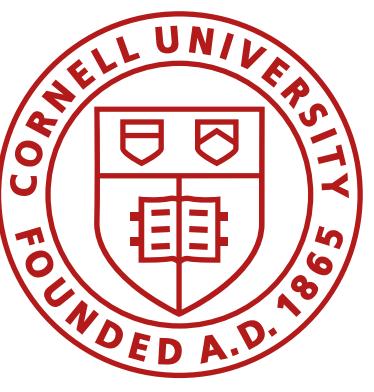
$$s = \int \int a$$

$$v[k+1] = v[k] + a[k] * dt$$

$$s[k+1] = s[k] + v[k] * dt$$

- If you do this at home, remember unit conversion!  
Accelerometer output is in mg ( $1g \sim 9.81 \text{ m/s}^2$ )





# Accelerometer

## Dead Reckoning

- Issue: Distinguishing sensor acceleration from gravity
  - Solution 1:** Calibrate the offset
  - Solution 2:** Low pass filter
  - Solution 3:** Minimum signal cut-off

Errors only accumulate, and they grow fast!

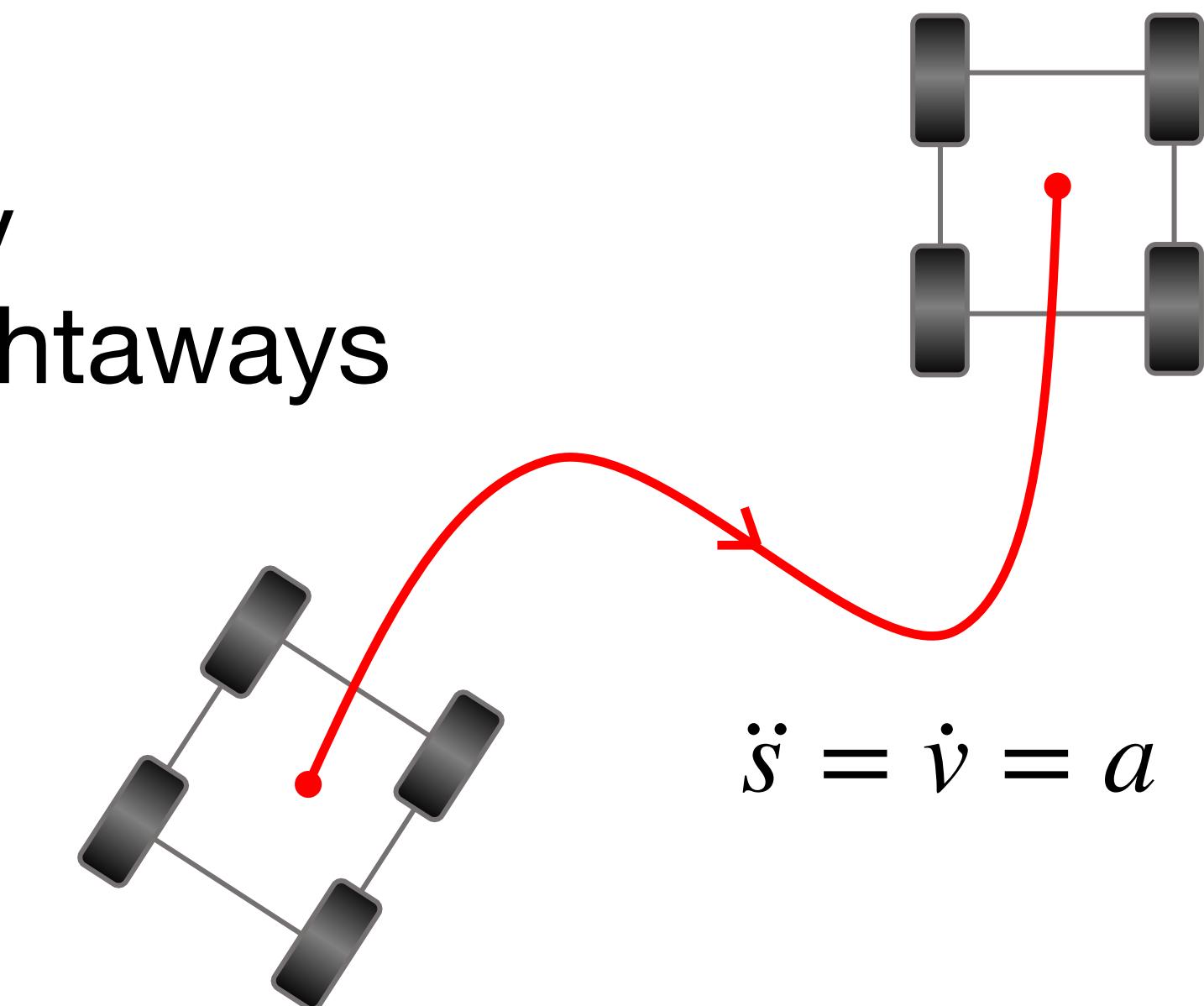
[www.chrobotics.com](http://www.chrobotics.com)

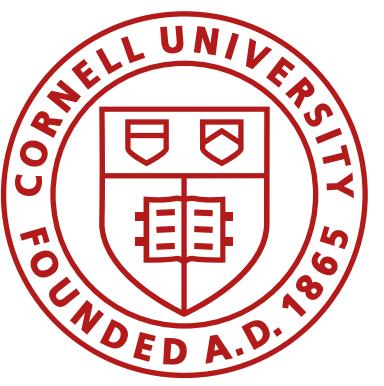
Angle Error (degrees)	Acceleration Error (m/s/s)	Velocity Error (m/s) at 10 seconds	Position Error (m) at 10 seconds	Position Error (m) at 1 minute	Position Error (m) at 10 minutes	Position Error (m) at 1 hour
0.1	0.017	0.17	1.7	61.2	6120	220 e 3
0.5	0.086	0.86	8.6	309.6	30960	1.1 e 6
1.0	0.17	1.7	17	612	61200	2.2 e 6
1.5	0.256	2.56	25.6	921.6	92160	3.3 e 6
2.0	0.342	3.42	34.2	1231.2	123120	4.4 e 6
3.0	0.513	5.13	51.3	1846.8	184680	6.6 e 6
5.0	0.854	8.54	85.4	3074.4	307440	11 e 6

Table 1 - A summary of velocity and position errors caused by attitude estimation error.

# Accelerometer Dead Reckoning

- Issue: Distinguishing sensor acceleration from gravity
  - **Solution 1:** Calibrate the offset
  - **Solution 2:** Low pass filter
  - **Solution 3:** Minimum signal cut-off
  - **Solution 4:** Stop periodically and zero the velocity
  - **Solution 5:** Use in combination with ToF on straightaways
  - **Solution 6:** Buy a more expensive IMU
  - etc...

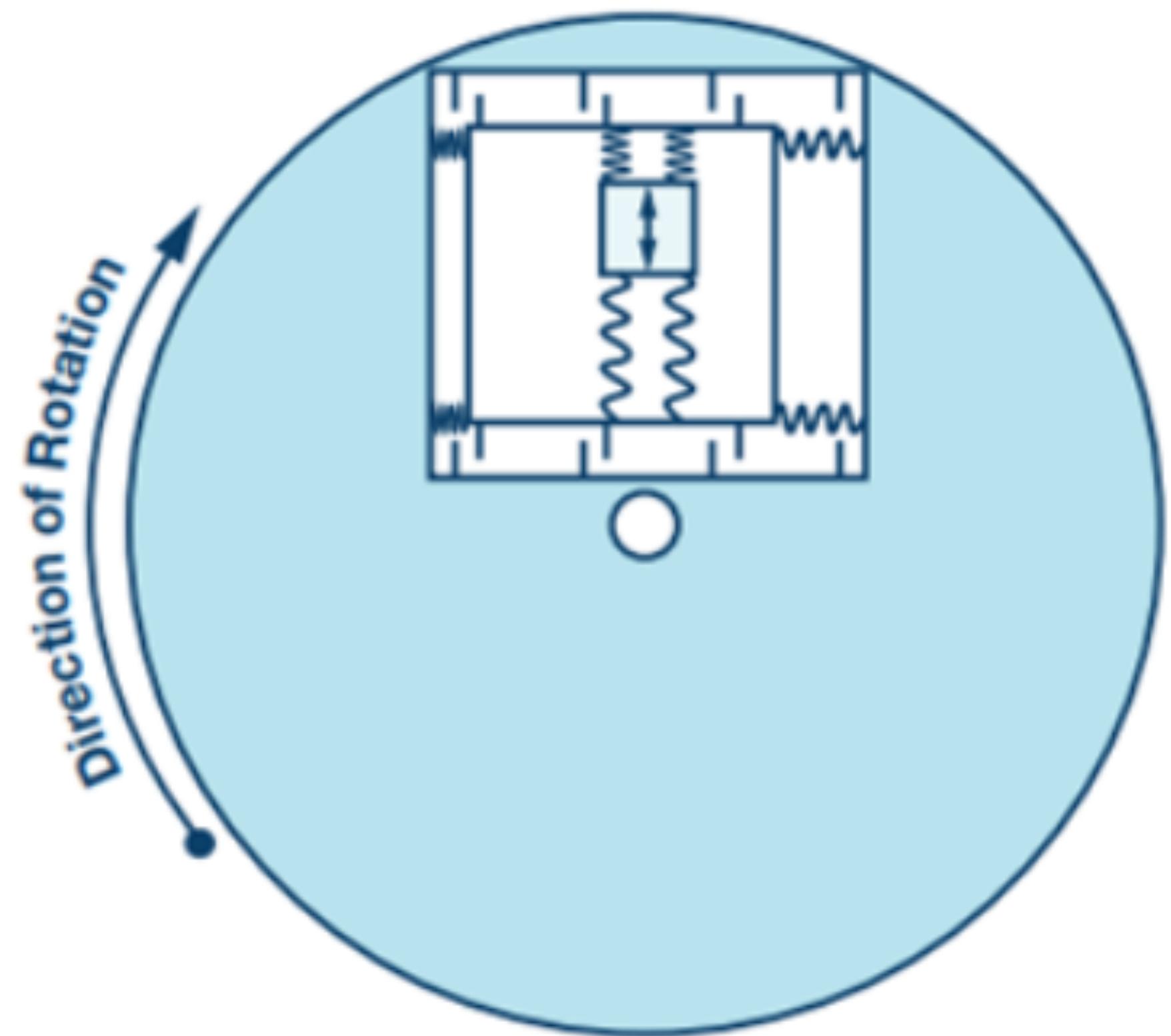
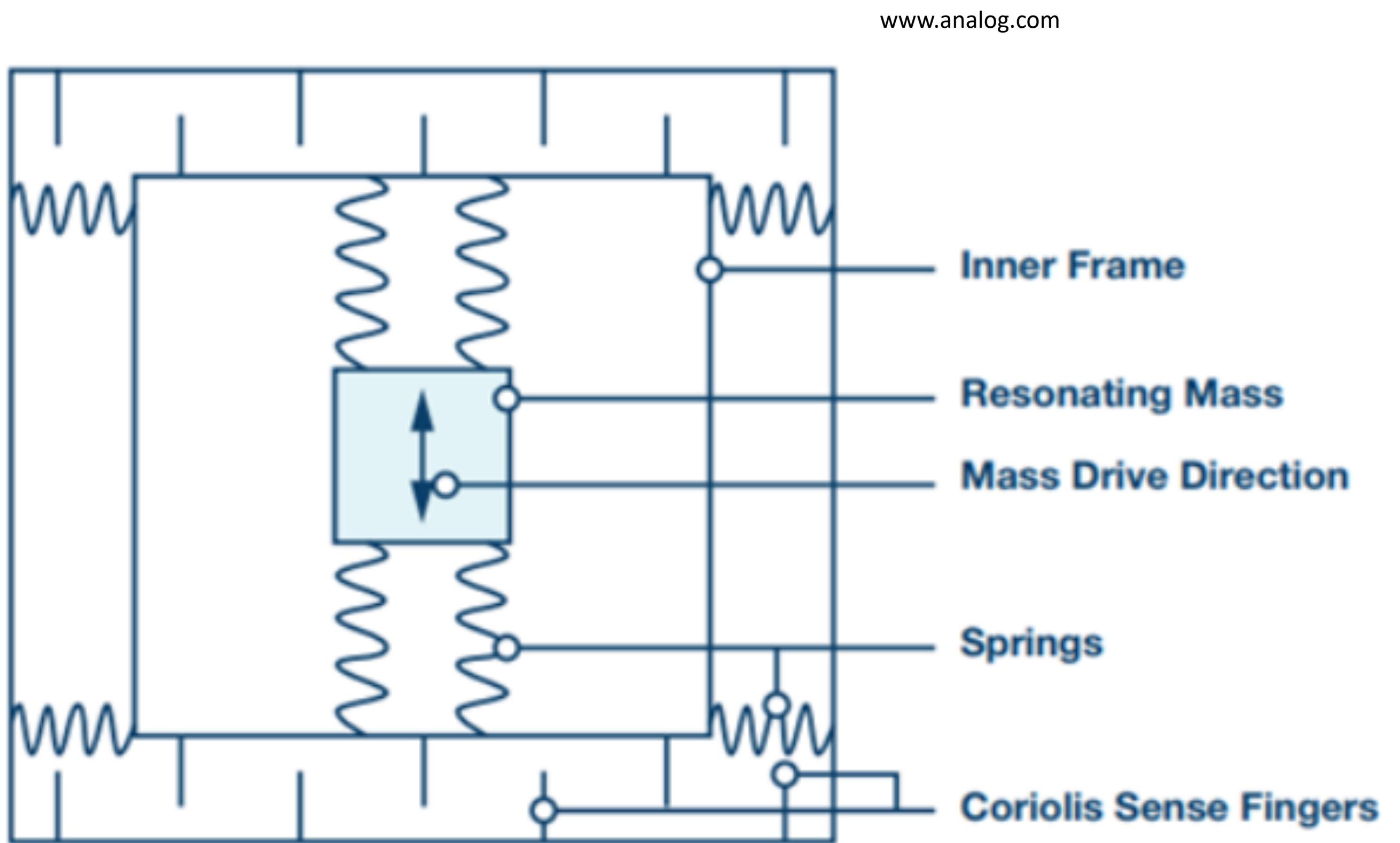


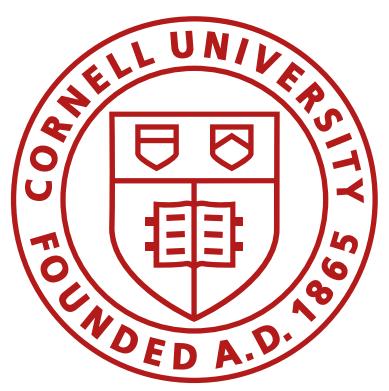


# Gyroscope

# Gyroscope

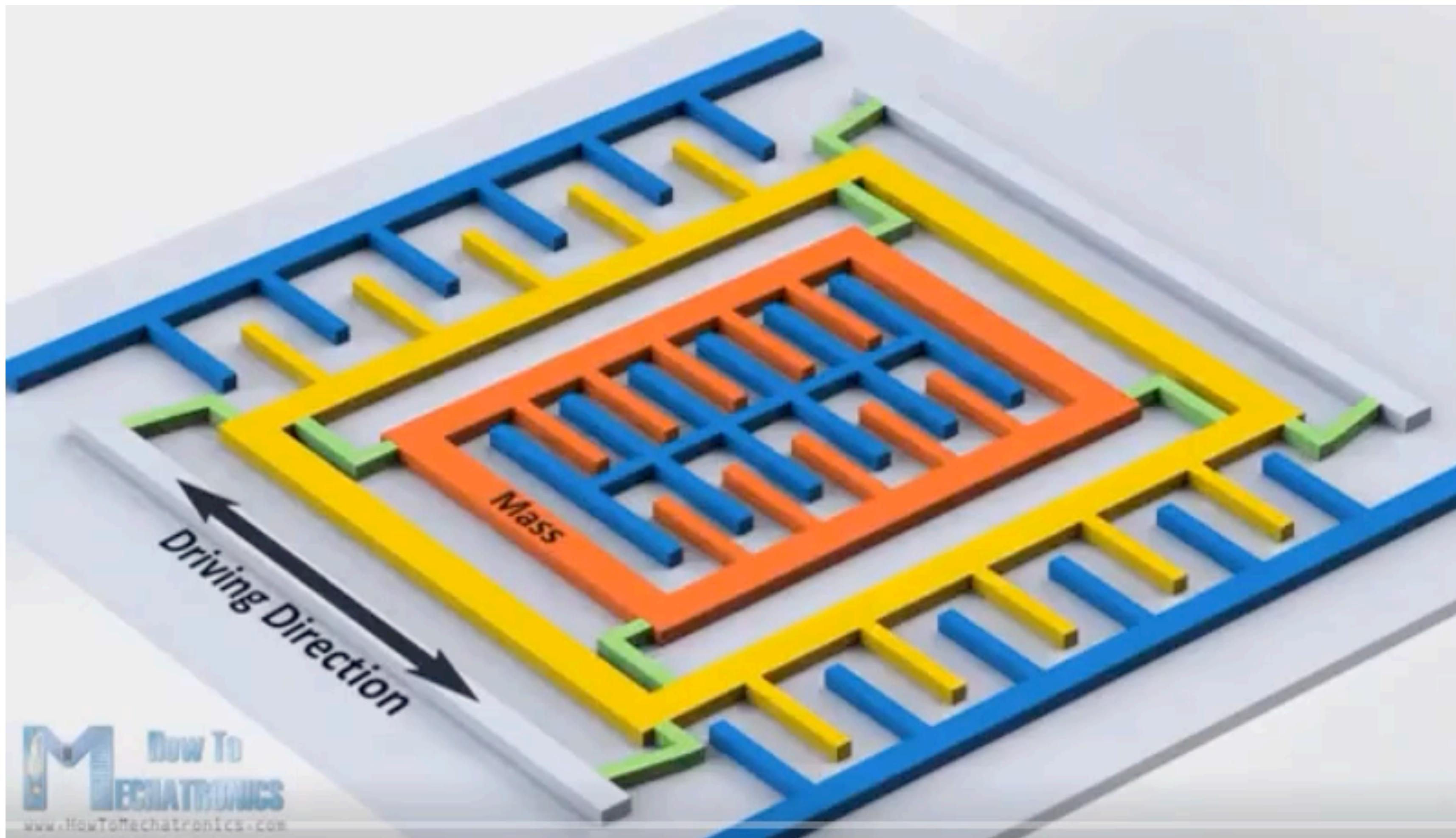
## Measure rate of angular change [°/s]

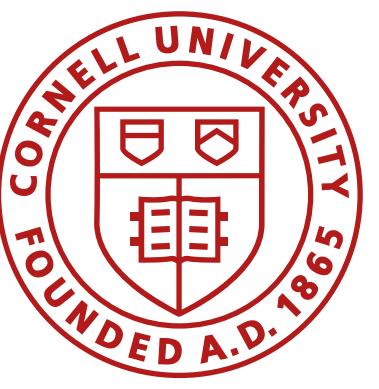




# Gyroscope

## Measure rate of angular change [°/s]



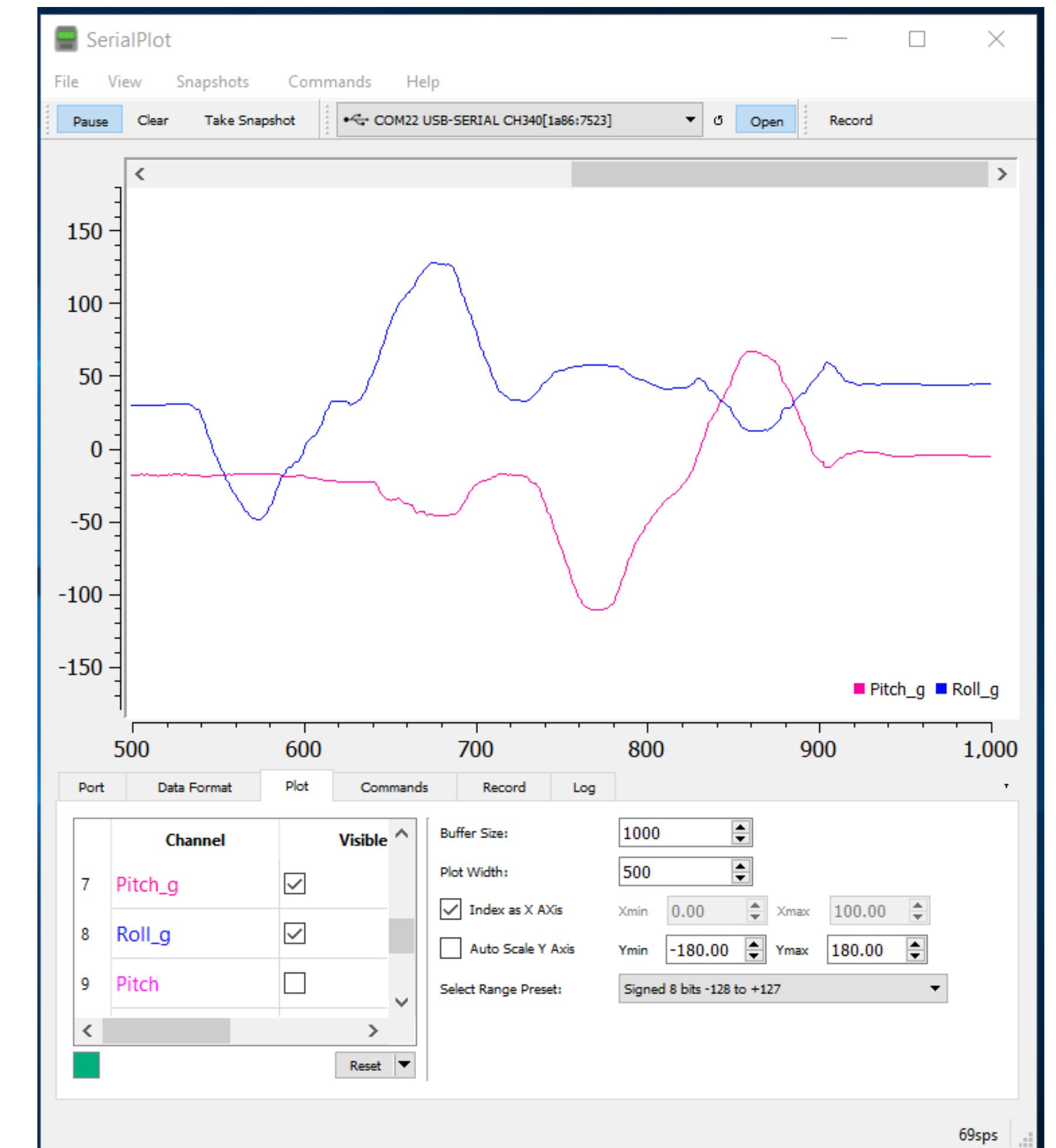


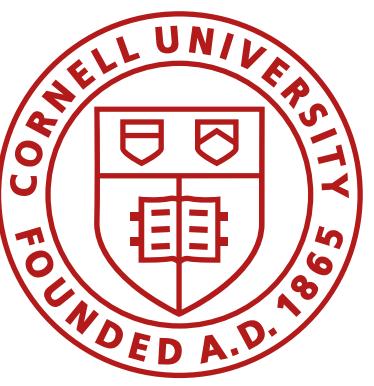
# Gyroscope

## Measure rate of angular change [°/s]

- How to measure angles?

- $\theta_g = \theta_g + \text{gyro\_reading} \times dt$
- Drift, but low noise





# Gyroscope

Measure rate of angular change [°/s]

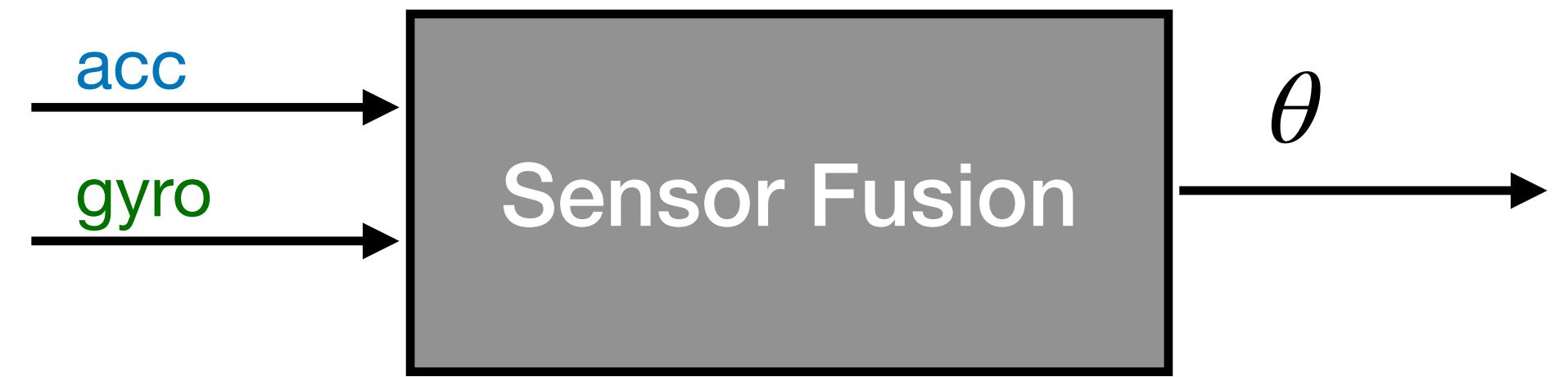
- How to measure angles?

- $\theta_g = \theta_g + \text{gyro\_reading} \times dt$

- Drift, but low noise

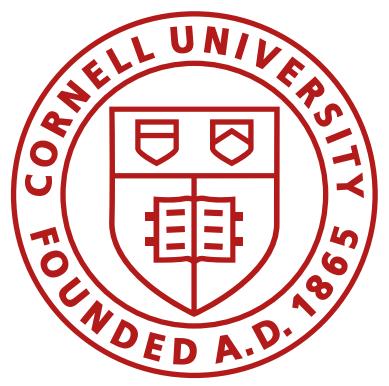
- Complementary filter:

- $\theta = (\theta + \theta_g)(1 - \alpha) + \theta_a \alpha$

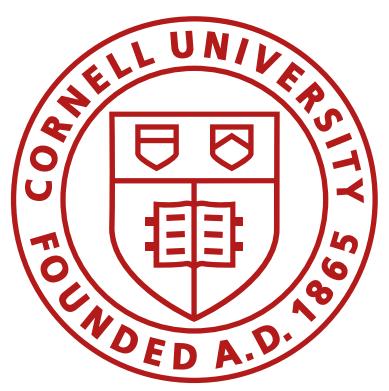


**Can we estimate yaw?**

Yes, but no complementary data from the accelerometer



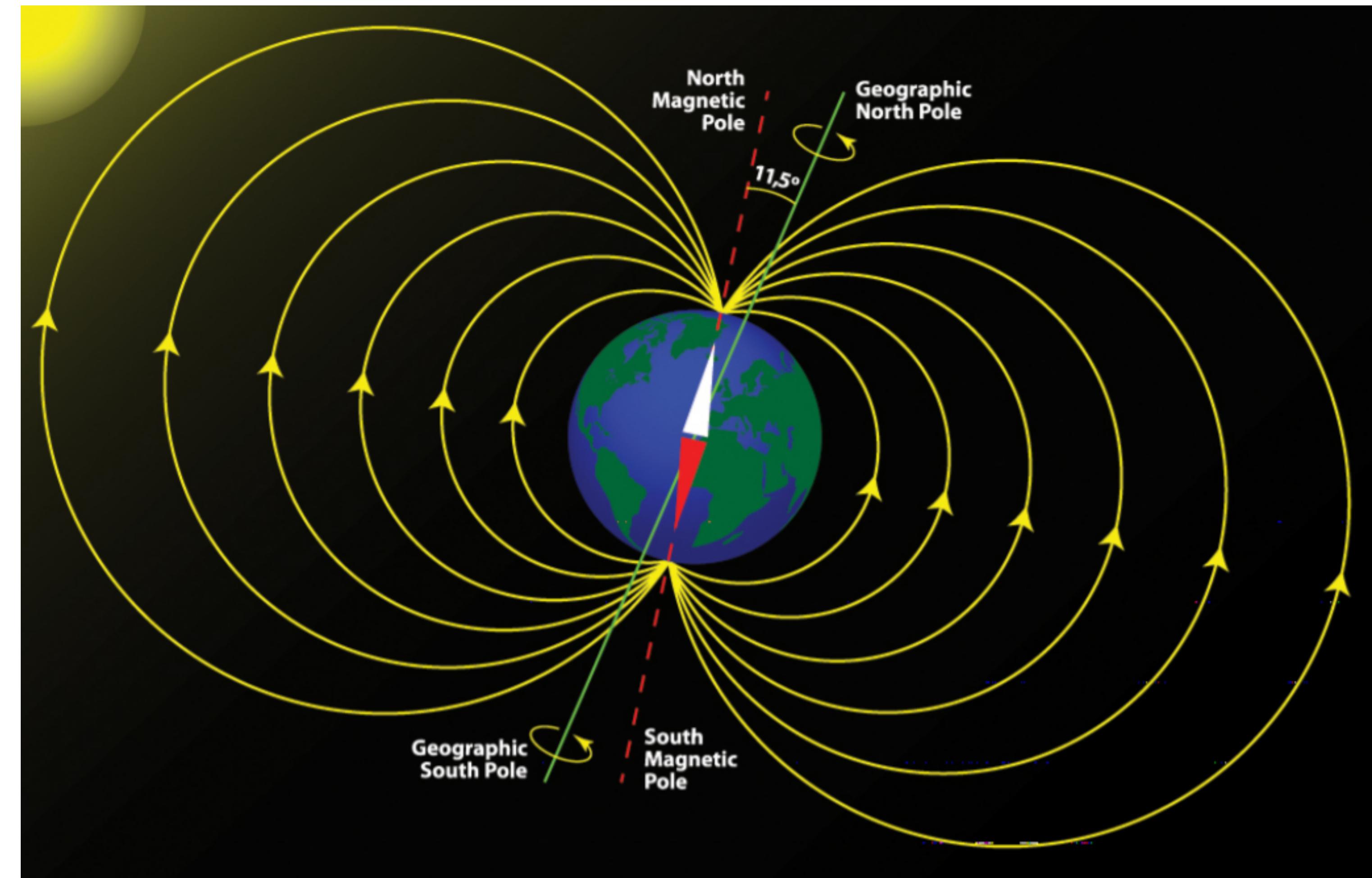
# Magnetometer

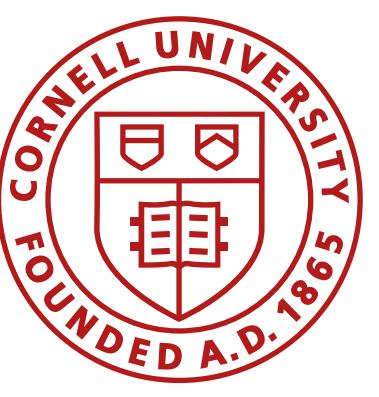


# Magnetometer

Measure Earth's magnetic field

- Depends on location, time

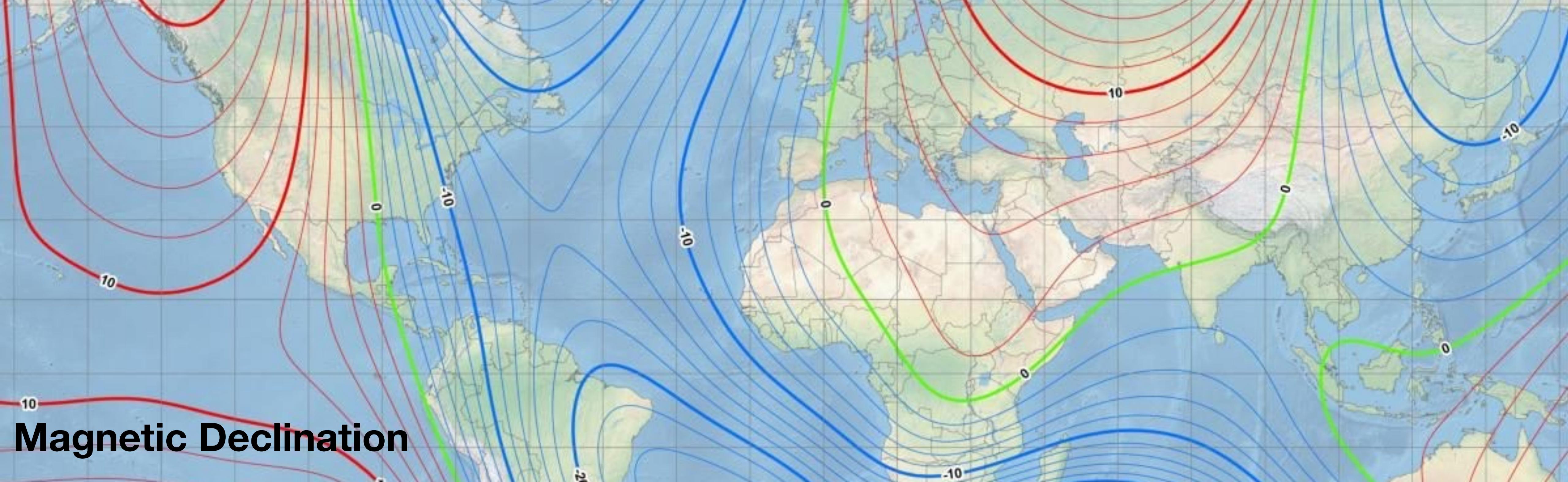
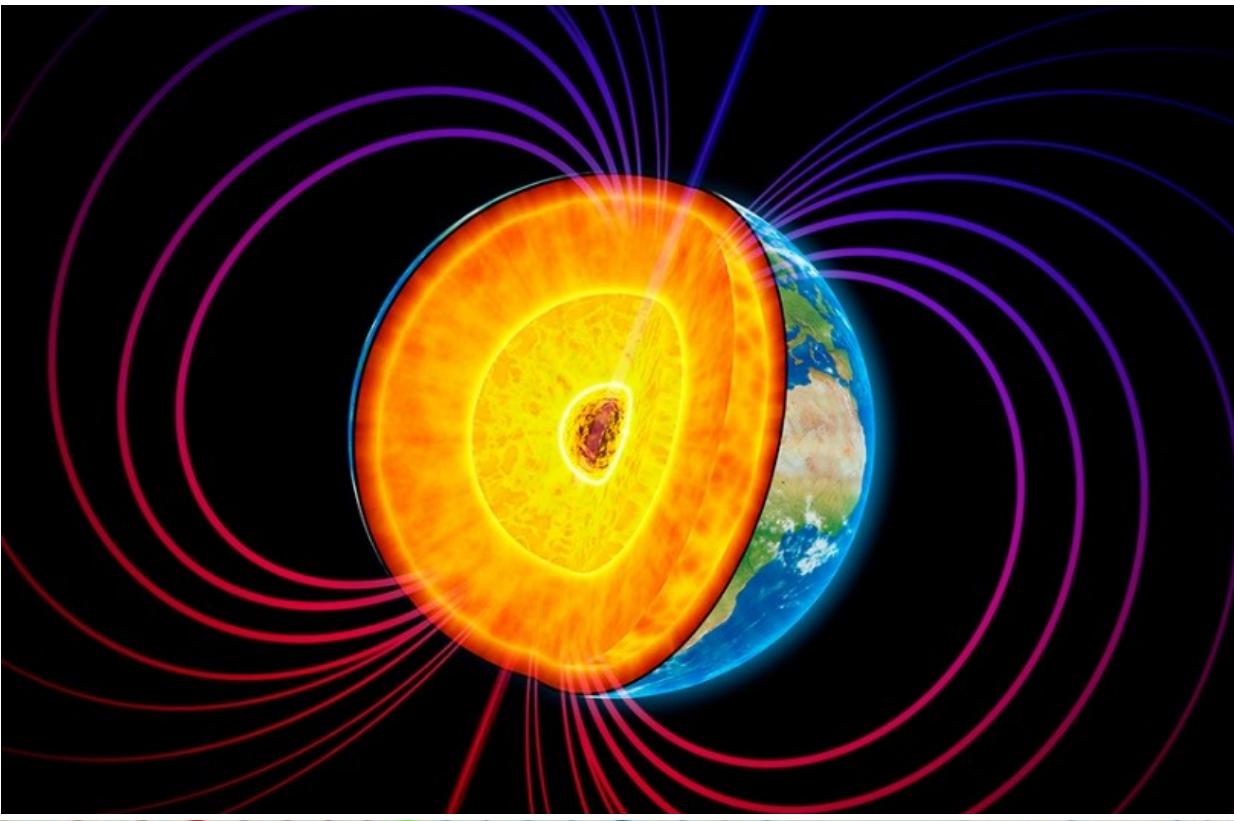




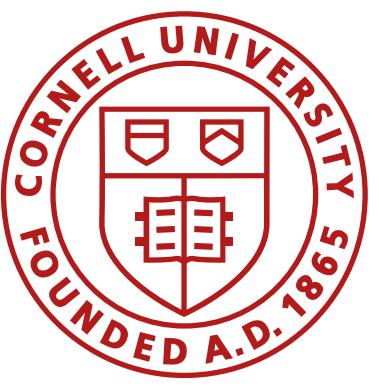
# Magnetometer

Measure Earth's magnetic field

- Depends on location, time
- Distortion due to metal objects or nearby EM fields



Magnetic Declination

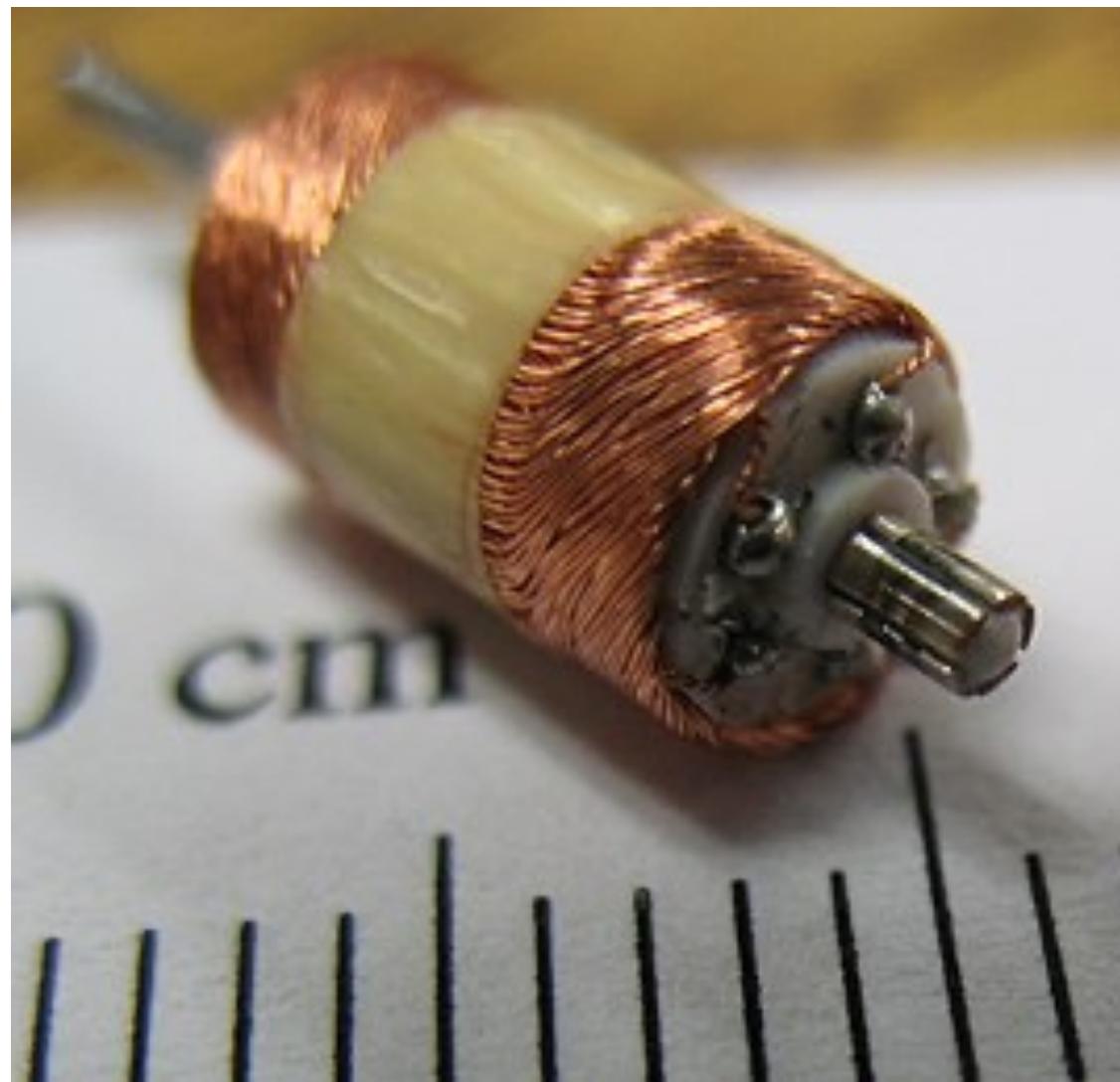


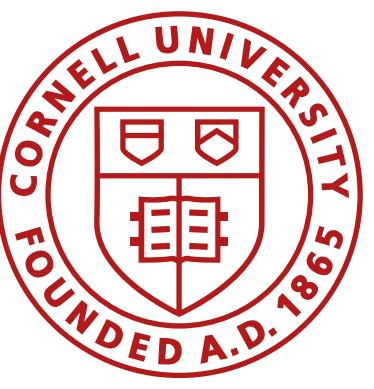
# Magnetometer

## Measure Earth's magnetic field

- Depends on location, time
- Distortion due to metal objects or nearby EM fields

### Examples



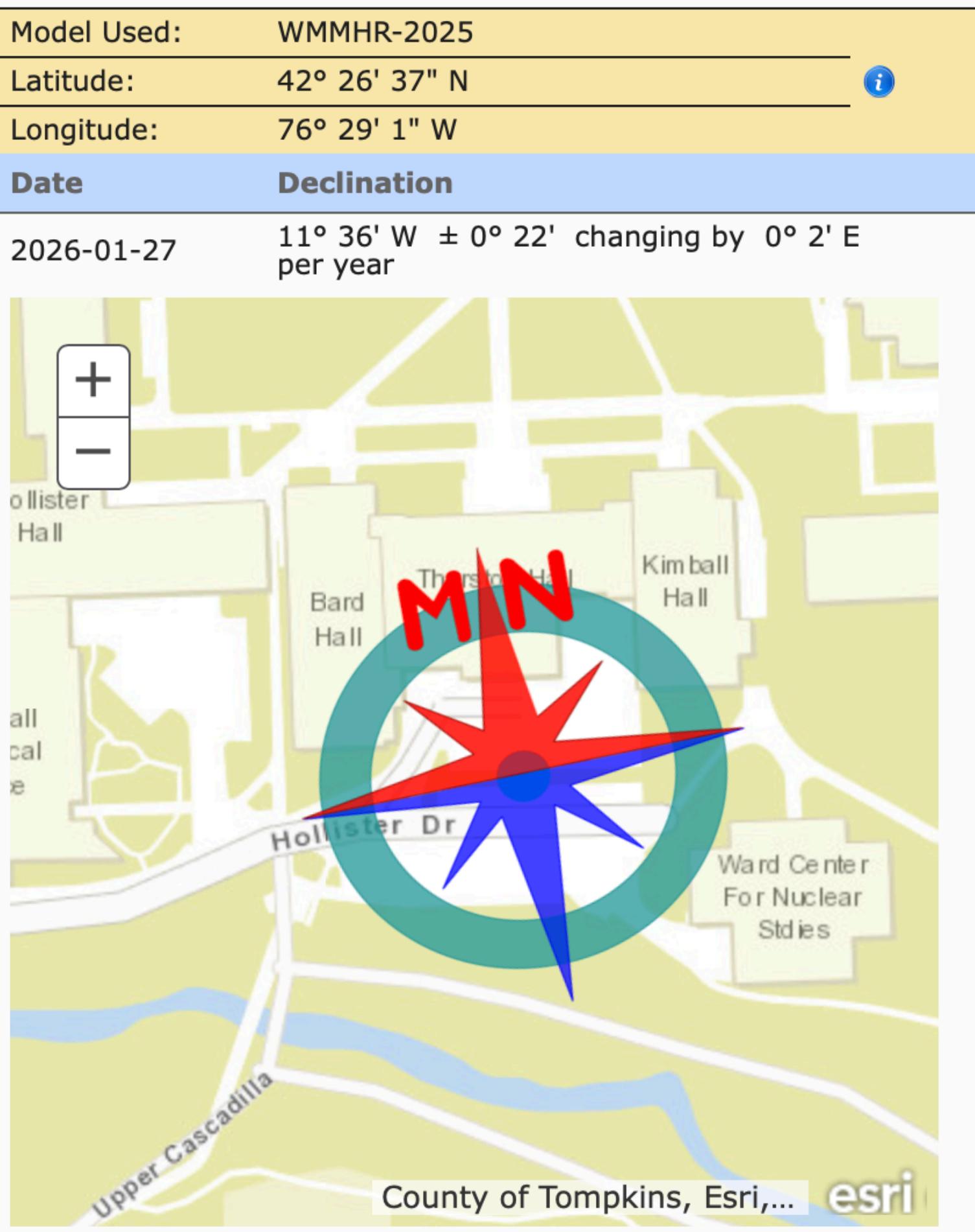


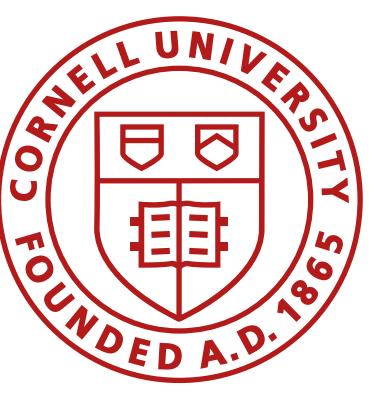
# Magnetometer

## Measure Earth's magnetic field

- Magnetic North is along the xmax-axis

Model Used:	WMMHR-2025						
Latitude:	42° 26' 37" N						
Longitude:	76° 29' 1" W						
Elevation:	0.0 km Mean Sea Level						
Date	Declination ( + E   - W )	Inclination ( + D   - U )	Horizontal Intensity	North Comp ( + N   - S )	East Comp ( + E   - W )	Vertical Comp ( + D   - U )	Total Field
2026-01-27	-11° 35' 31"	67° 31' 32"	19,930.1 nT	19,523.6 nT	-4,004.8 nT	48,176.1 nT	52,135.8 nT
Change/year	0° 2' 26"/yr	-0° 5' 57"/yr	41.6 nT/yr	43.6 nT/yr	5.5 nT/yr	-135.4 nT/yr	-109.2 nT/yr
Uncertainty	0° 22'	0° 11'	130 nT	135 nT	85 nT	134 nT	134 nT

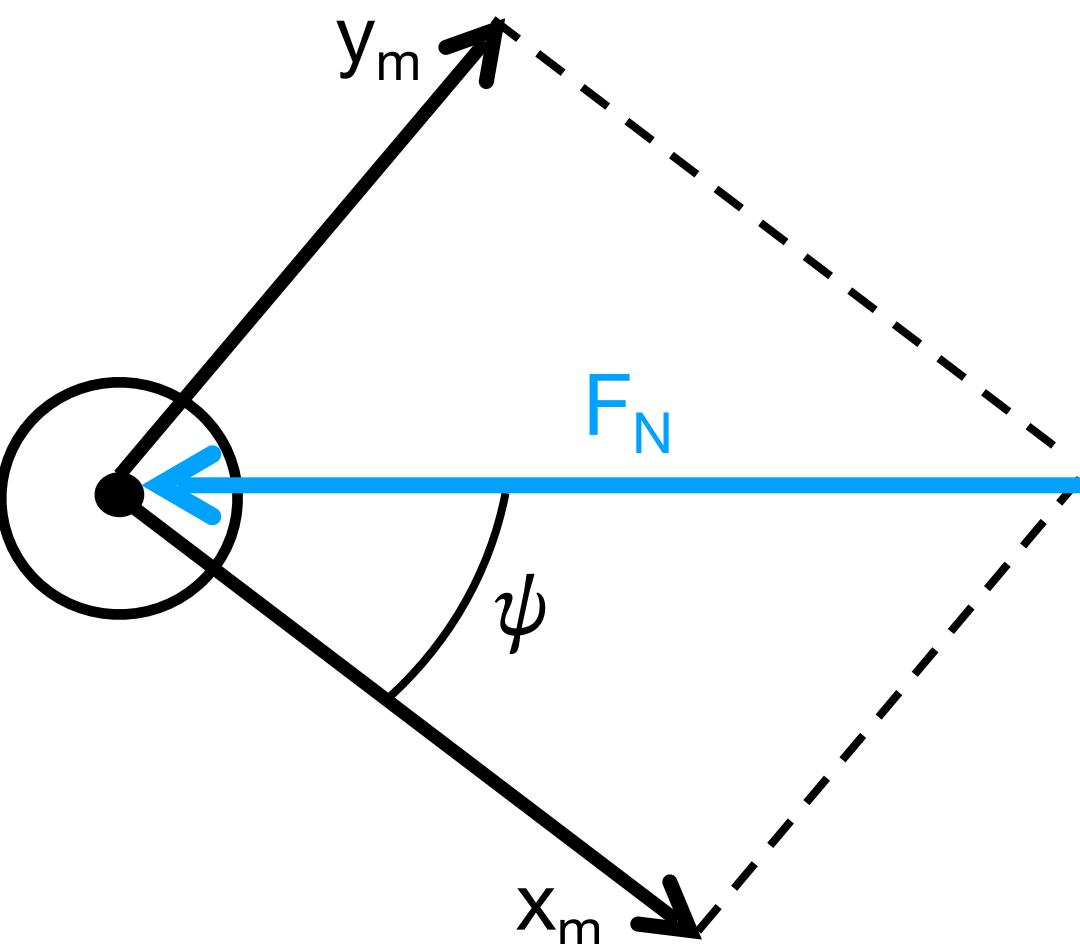
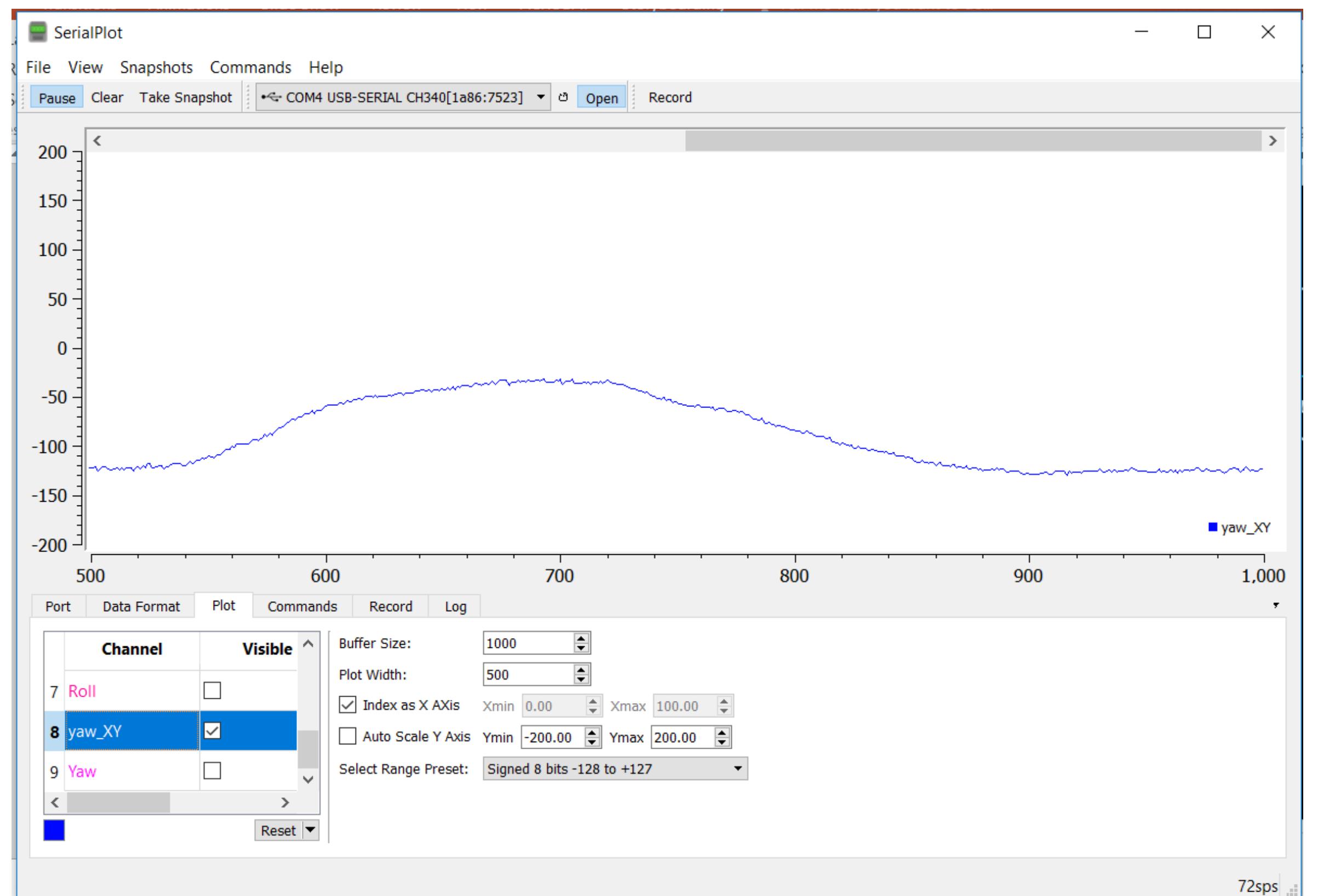




# Magnetometer

## Measure Earth's magnetic field

- $\psi = \text{atan}2(x_m, y_m)$



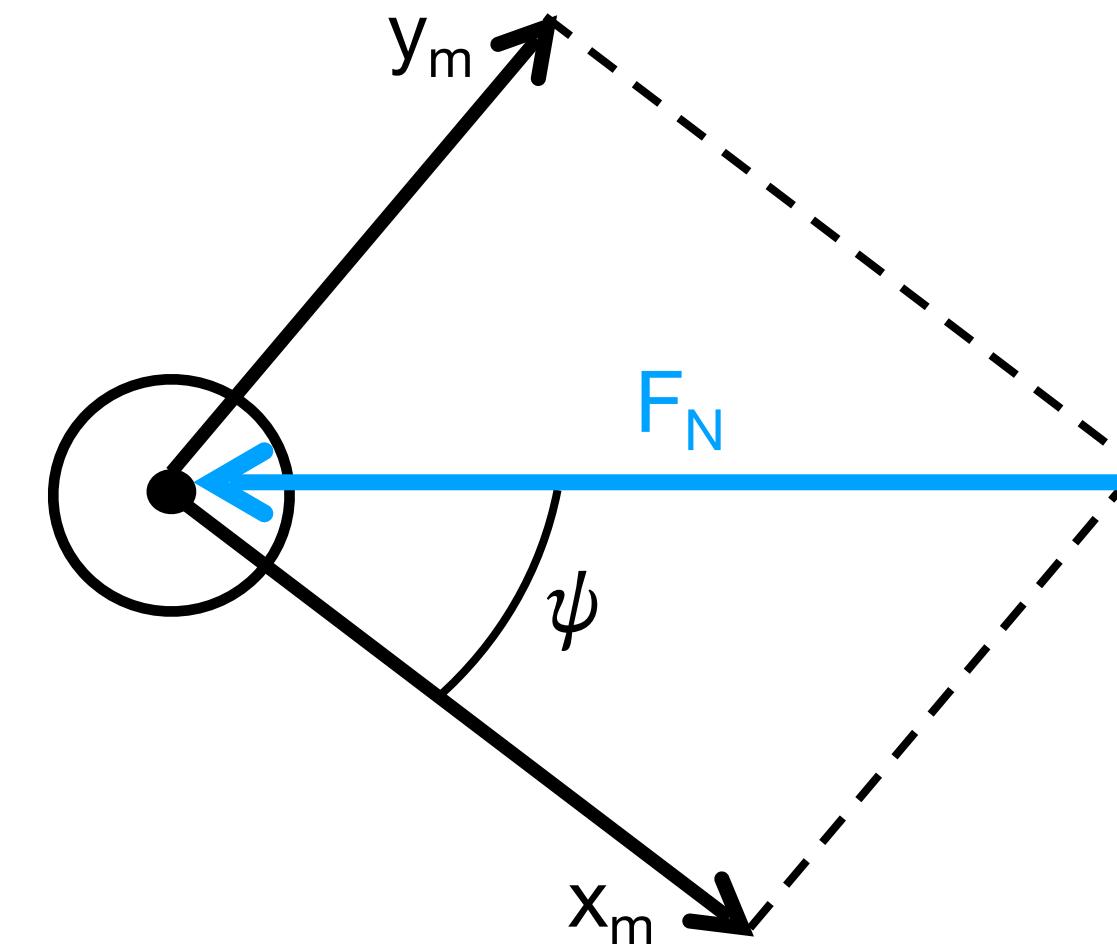
# Magnetometer

Measure Earth's magnetic field

- $\psi = \text{atan2}(x_m, y_m)$
- How to compensate for tilt? – Fuse data

$$\begin{bmatrix} x_m \\ y_m \\ z_m \end{bmatrix} = R_{x,\phi} R_{y,\theta} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = R_{x,\phi}^T R_{y,\theta}^T \begin{bmatrix} x_m \\ y_m \\ z_m \end{bmatrix} = \begin{bmatrix} c_\theta & 0 & c_\phi - s_\theta \\ s_\phi s_\theta & c_\phi & c_\theta s_\phi \\ c_\phi s_\theta & -s_\phi & c_\phi c_\theta \end{bmatrix} \begin{bmatrix} x_m \\ y_m \\ z_m \end{bmatrix}$$



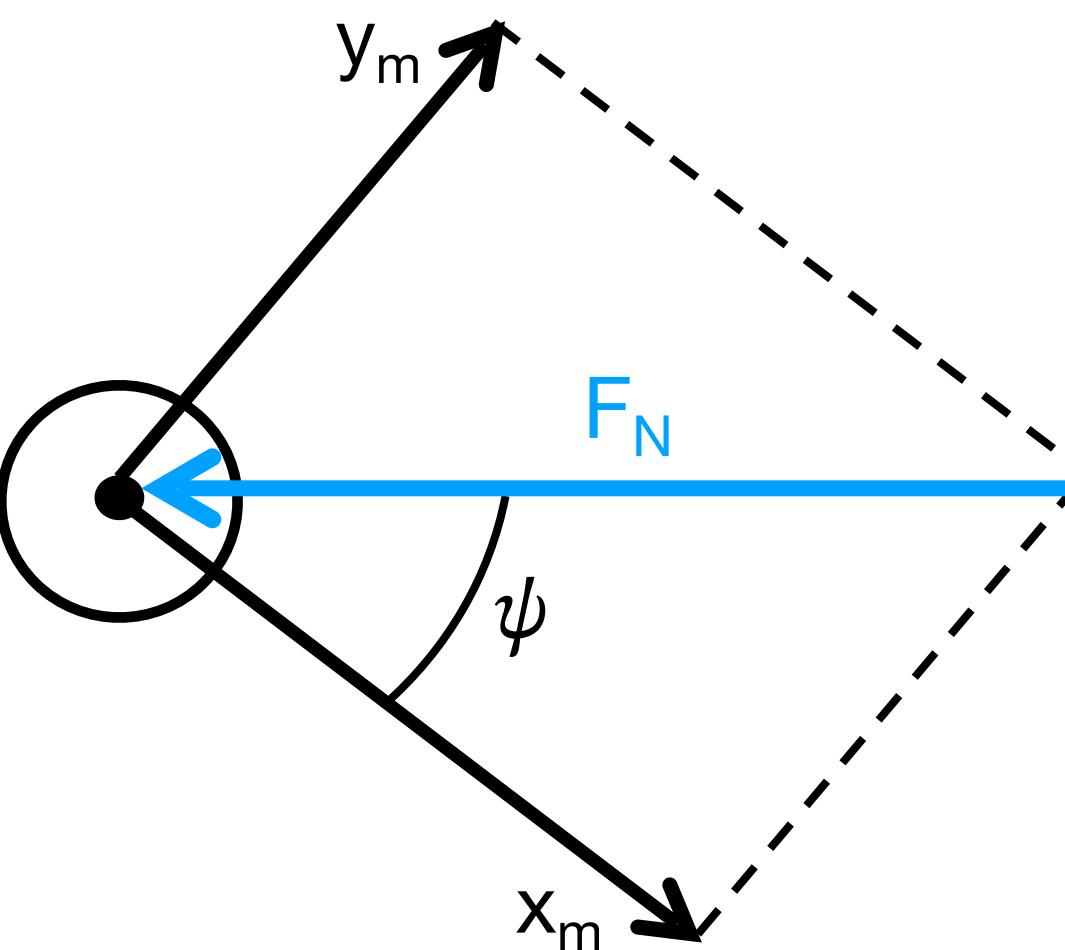
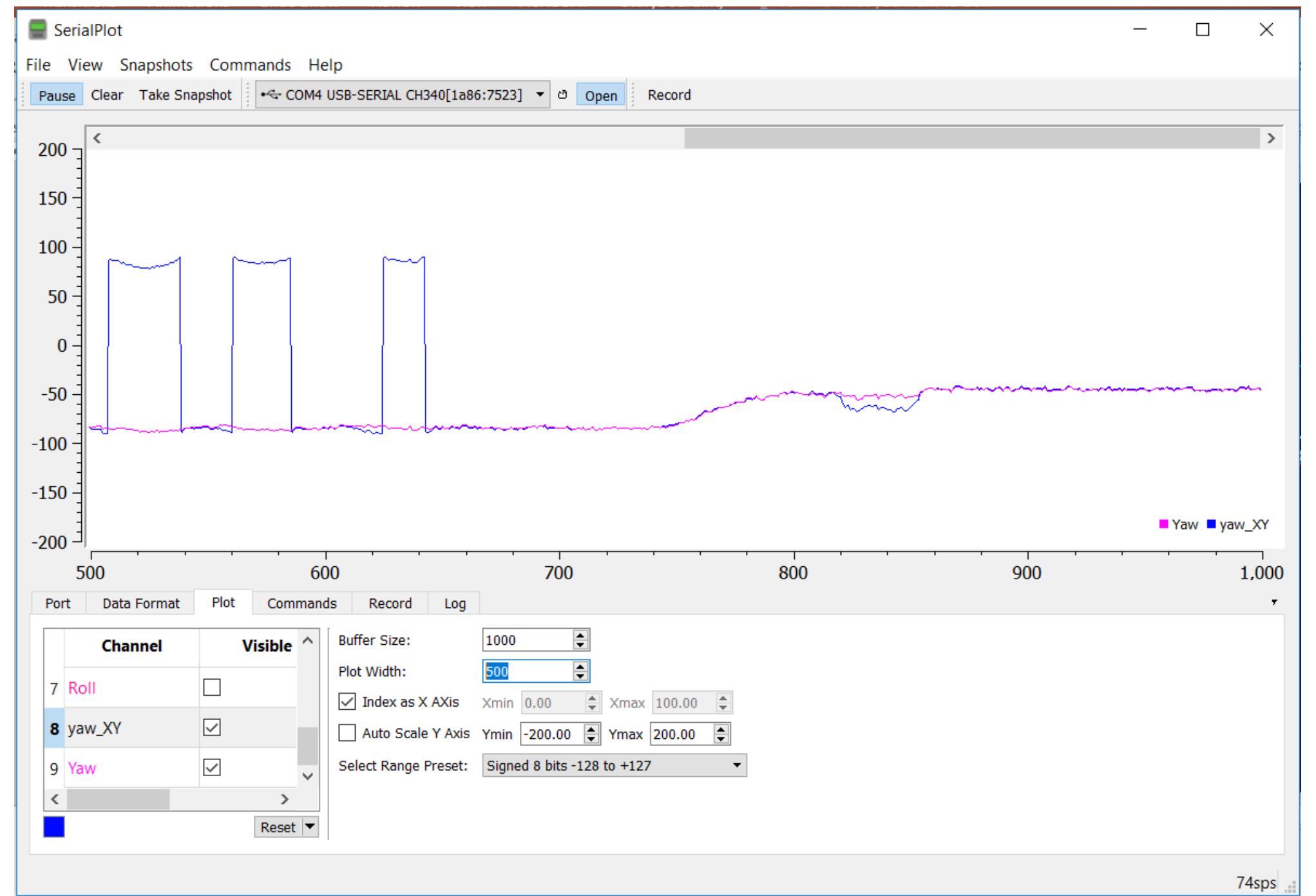
$$\begin{aligned} x &= y_m \cos(\phi) - z_m \sin(\phi) \\ y &= x_m \cos(\theta) + y_m \sin(\phi)\sin(\theta) \\ &\quad + z_m \cos(\phi)\sin(\theta) \end{aligned}$$

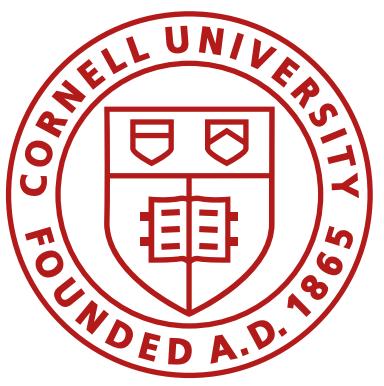
$$\psi = \text{atan2}(x, y)$$

# Magnetometer

## Measure Earth's magnetic field

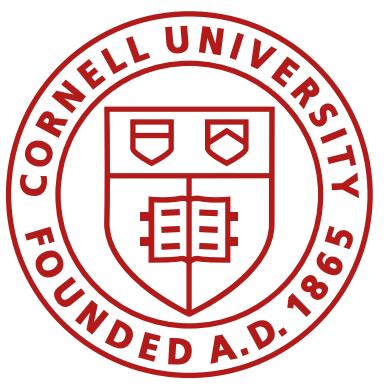
- $\psi = \text{atan}2(x_m, y_m)$
- How to compensate for tilt? – Fuse data





# Sources and references

- <http://www.chrobotics.com/library/accel-position-velocity>
- EE 267 Virtual Reality, by Gordon Wetzstein at Stanford University
- Analog.com
- <https://toptechboy.com/>
- Prof. Kirstin Petersen



# Class Action Items

- If you want to drop the class, please let me know **ASAP** and return your lab kits!
- **January 30th, midnight:** Make a GitHub repository and build your Github page
  - Include: name, photo, a small introduction, and the class number
  - Share **the page link** in the canvas assignment
- **February 3rd (8am)** for Lab 401, and **February 4th (8am)** for Labs 402 & 403:  
Lab 1A and Lab 1B write-ups are due!