1) Sea $A(x_1, x_2, ..., x_n)$ donde $A \sim N(\mu, \sigma)$

muestre que los estimadores de máxima verosimilitud son:

$$\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$\hat{\sigma}^2 = \frac{1}{2}\sum_{i=1}^{n}$$

sol:

$$\mathcal{L} = \prod_{i=1}^{n} \frac{e^{-\frac{1}{2}\left(\frac{x_i - \mu}{\sigma}\right)^2}}{\sqrt{2\pi\sigma^2}} = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\frac{(x_i-\mu)^2}{\sigma^2}}$$

Ahora

$$\ln(\mathcal{L}(x, \mu, \sigma)) = \ln\left(\prod_{i=1}^{n} \frac{e^{-\frac{1}{2}\frac{(x_i-\mu)^2}{\sigma^2}}}{\sqrt{2\pi\sigma^2}}\right] = \sum_{i=1}^{n} \ln\left[\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\frac{(x_i-\mu)^2}{\sigma^2}}\right]$$

$$= \sum_{i=1}^{n}\left[\ln\left(\frac{1}{\sqrt{2\pi\sigma^2}}\right) + \ln\left[e^{-\frac{1}{2}\frac{(x_i-\mu)^2}{\sigma^2}}\right]\right] = \sum_{i=1}^{n}\left[\ln\frac{1}{\sqrt{2\pi\sigma^2}} - \frac{1}{2\sigma^2}(x_i-\mu)^2\right]$$

derivar parcialmente respecto a μ

$$\frac{d}{d\mu}\ln[\mathcal{L}] = -\frac{1}{2\sigma}\sum_{i=1}^{n} 2(x_i - \mu) = 0$$

$$\sum_{i=1}^{n} x_i - \mu = 0 \rightarrow \sum_{i=1}^{n} x_i = n\mu$$

$$\therefore \mu = \frac{1}{n}\sum_{i=1}^{n} x_i$$

derivar respecto σ

$$\frac{d(\ln(L))}{d\sigma} = \sum_{i=1}^{n} \frac{d}{d\sigma}\left[\ln\frac{1}{\sqrt{2\pi\sigma^2}} - \frac{1}{2}\frac{(x_i-\mu)^2}{\sigma^2}\right]$$

$$= \sum_{i=1}^{n}\left[-\frac{d}{d\sigma}\left(\ln\sqrt{2\pi\sigma^2}\right) - \frac{(x_i-\mu)^2}{2}\frac{d}{d\sigma}\left(\frac{1}{\sigma^2}\right)\right]$$

$$= \sum_{i=1}^{n}\left[-\frac{\cancel{\sqrt{2\pi}}}{\sigma\cancel{\sqrt{2\pi}}} + \frac{(x_i-\mu)^2}{\sigma^3}\right] = 0$$

$$\sum_{i=1}^{n}\frac{1}{\sigma} = \sum_{i=1}^{n}\frac{(x_i-\mu)^2}{\sigma^3}$$

$$n\sigma^2 = \sum_{i=1}^{n}(x_i-\mu)^2$$

Por definición $\mu = \bar{x}$

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2$$