

SESA 6071

Spacecraft Propulsion

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Definitions

I_t	Total Impulse (Ns)	I_{sp}	Specific Impulse (s)
F	Rocket Thrust (N)	g_0	Standard Gravitational Accel (m/s^2)
\dot{m}	Propellant mass flow rate (kg/s)	m_p	Expelled propellant mass (kg)
c	Effective exhaust velocity (m/s)	η_T	Power Conversion Efficiency
P_{in}	Input Power (W)	m	Spacecraft or launch vehicle mass (kg)
α	Specific power plant mass (kg/W)	M_{pow}	Power plant mass (kg)
v_e	Exhaust velocity (m/s)	P_e	Exhaust pressure (Pa)
P_a	Atmospheric pressure (Pa)	A_e	Exhaust area (m^2)
c^*	Characteristic velocity (m/s)	P_c	Chamber pressure (Pa)
A_t	Throat area (m^2)	M	Mass fraction
M_0	Initial mass (kg)	M_P	Propellant mass (kg)
M_f	Fuel mass (kg)	ΔV	Change in velocity (m/s)
α	Angle of attack ($^\circ$ or rad)	δ	Gimbal angle ($^\circ$ or rad)
γ	Flight path angle ($^\circ$ or rad)	θ	Pitch angle ($^\circ$ or rad)
D	Drag (N)	c_p	Specific heat at a constant pressure (J/kgK)
c_v	Specific heat at a constant volume (J/kgK)	θ	Pitch angle ($^\circ$ or rad)
D	Drag (N)	c_p	Specific heat at a constant pressure (J/kgK)
C_F	Coefficient of thrust	h	Enthalpy (J/ mol)
k	Ratio or specific heats		

1. Lecture 1

1.1. What is Rocket Propulsion ?

Propulsion itself is the **act of changing the motion of a body**, typically by using newtons third law and it can be classified in various types of ways. A more colloquial way of defining rocket propulsion is as **mass drivers**, throwing out mass one way to yield an acceleration in the other.

1.2. Rocket Propulsion Family Tree

In **Figure 1** the rocket propulsion types are grouped by the energy source.

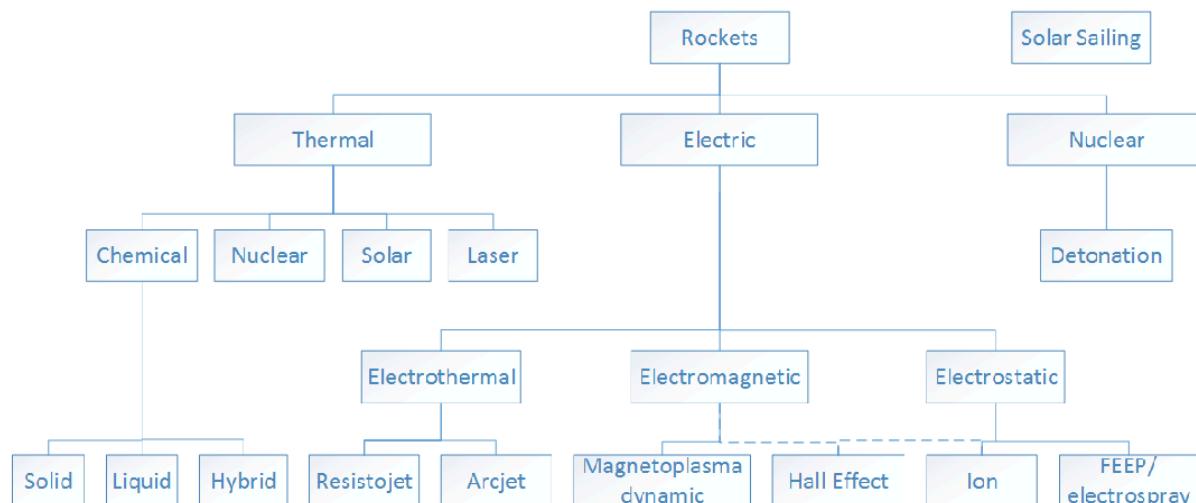


Figure 1: Flowchart of the rocket propulsion family tree

1.2.1. Chemical Rockets

These utilize either a chemical reaction or decomposition to generate energy. Gas is heated to between **700°C - 1300°C** and to speeds between **1.5 km/s - 4.5 km/s**. These require a **fuel and oxidizer** and come in the following types:

- **Solid:** Fuel and oxidizer mixed within into a solid grain which cannot stop burning once ignited. feature **high thrust with low performance**.
- **Liquid:** Burn a liquid fuel and oxidizer allowing for repeated firings and variable thrust. Feature **high performance and thrust with high complexity**.
- **Hybrid:** Have a liquid oxidizer but a solid fuel allowing for better performance than solid with lower complexity.

1.2.2. Electric Rockets

These use electrical energy to generate thrust without utilizing combustion. Typically have very high exhaust velocities ($\sim 60,000 \text{ m/s}$) and therefore **very high performance** at the costs of **high complexities and very low thrust**. The four distinct groups are:

- **Electrothermal:** Uses electrical energy to heat a propellant (Resistojet). Are **simple to build** at the cost of **low thrust**.
- **Electrostatic:** Uses electrical energy to accelerate ionized fuel across an electric fields. Feature **good performance** at the cost of **being expensive and low thrust**.
- **Electromagnetic:** Accelerates an ionized fuel using a magnetic field. Fall issue to **low efficiency unless power input is high**.
- **Hall Effect Thruster:** Uses a mixture of both electrostatic and electromagnetic propulsion methods to accelerate propellant. These are the most **commonly used**.

1.2.3. Nuclear Rockets

Broadly speaking there are two types of nuclear rockets, these are:

- **Nuclear Detonation:** Use the shockwave produced when nuclear bombs are detonated to produce thrust (Orion Drive). **High performance and thrust** but are **very dangerous and have limited testing**.
- **Nuclear Thermal:** Uses the heat energy produced during nuclear fission to heat a propellant (typically hydrogen) which is then exhausted. These have **high performance and thrust** but are **dangerous and have limited testing**.

1.2.4. Solar and Laser Rockets

These systems use large diameter telescopes to focus in a laser or solar radiation to heat up a propellant. These systems feature **high theoretical performance and moderate thrust** but are **very complex and lack any real testing**.

1.2.5. Solar Sails

These systems use no propellant at all and instead produce thrust through the momentum gained when a photon is incident on the sail. These systems feature **good performance with no fuel** but fall victim to **low thrust and engineering complexity**.

1.3. Rocket Propulsion Applications

Instead of grouping together rocket propulsion methods using the energy source, the rocket application can also be used, for example:

- **High Thrust/Maneuverability:** Typically have the cost of **low performance** and use **chemical or solid** propulsion methods.
- **High Performance:** Typically have the cost of **low thrust** and use **electrical** propulsion methods.
- **Balanced Thrust and Performance:** Typically the middle ground is **nuclear thermal**.

2. Lecture 2

2.1. Definitions and Fundamentals

To develop an empirical measure of performance we should first consider **Eq. 1**.

$$I_t = \int_0^t F \, dt \quad (1)$$

Where:

- I_t : Total Impulse (Ns)
- F : Thrust Force (N)
- t : Burn Duration (s)

Note that for **Eq. 1**, if F is constant then the equation simplified to $I_t = Ft$. A more useful measure of performance for rocket engines is shown in **Eq. 2**.

$$I_{sp} = \frac{\int_0^t F \, dt}{g_0 \int_0^t \dot{m} \, dt} = \frac{I_t}{g_0 \int_0^t \dot{m} \, dt} \quad (2)$$

Where:

- I_{sp} : Specific Impulse (s)
- g_0 : Standard Gravitational Accel (m/s^2) = $9.81 \, m/s^2$
- \dot{m} : Propellant mass flow rate (kg/s)

There is no concrete reason on why g_0 is present in this equation, however one common theory is that it allows I_{sp} to be in seconds instead of featuring a length unit which would eliminate any error in conversion from metric to imperial. If F and \dot{m} are both constant over the t then **Eq. 2** simplifies to **Eq. 3**.

$$I_{sp} = \frac{I_t}{g_0 m_p} \quad (3)$$

Where:

- m_p : Expelled propellant mass (kg) = $\dot{m}t$

Another useful parameter for defining engine performance is shown in **Eq. 4**.

$$c = \frac{F}{\dot{m}} \quad (4)$$

Where:

- c : Effective exhaust velocity (m/s)

The exhaust velocity is called as such as the **velocity profile of the exhaust is not uniform**, this is most seen in chemical rockets due to the **no slip condition** but is slightly seen in electrical rockets too. Rearranging all of the previous equations together yields a definition for I_{sp} in terms of c .

$$I_{sp} = \frac{c}{g_0} \quad (5)$$

Typical I_{sp} values for the rocket engine types defined in the previous lecture are shown in **Table 1**.

Rocket Engine Type	$I_{sp}(s)$	Thrust (N)	Efficiency	Propellant
Chemical bi-propellant	200 - 450	$\leq 10MN$	0.8	Liquid or Solid Propellents
Chemical mono-propellant	150 - 250	0.03 - 100	0.9	N_2H_4
Thermal Nuclear Fission	500 - 860	$\leq 10MN$	0.5	H_2
Resistojet - electrothermal	150 - 350	0.01 - 10	0.4	N_2H_4 , NH_3 , H_2
Ion Thruster - electrostatic	1500-8000	$10^{-5} - 0.5$	0.65	Xe
Hall Effect Thruster	1500-2000	$10^{-5} - 2$	0.55	Xe

Table 1: Typical values of I_{sp}

2.2. Maximum Chemical Performance

A typical chemical reaction used in chemical rockets is combustion shown in **Eq. 6**.



Combustion as shown in **Eq. 6** is an exothermic reaction as the energy of the reactants is more than the energy of the products, allowing for an excess of energy after the reaction. To estimate an effective upper limit to the energy released during combustion, the bond energies shown in **Table 2** can be used.

Chemical	Bond Energy (kJ/mol)
H_2	436
O_2	498
H_2O	428
	498.7

Table 2: Respective bond energies of reactants and products in combustion.

Note that there are two bond energies in **Table 2** due to the OH and the OH - H bonds. The maximum energy can be calculated and are shown in **Figure 2**.

Energy Per Kilogram Released During Combustion

$$M_{Water} := 18.01528 \frac{gm}{mol} \quad BE_{H2} := 436 \frac{kJ}{mol} \quad BE_{O2} := 498 \frac{kJ}{mol}$$

$$BE_{OH} := 428 \frac{kJ}{mol} \quad BE_{OHH} := 498.7 \frac{kJ}{mol}$$

$$E_{kg} := \frac{(BE_{OH} + BE_{OHH}) - \left(BE_{H2} + \frac{BE_{O2}}{2} \right)}{M_{Water}} = 13416388.75 \frac{J}{kg}$$

Maximum Chemical Performance

$$\eta := 0.9 \quad g_0 := 9.81 \frac{m}{s^2}$$

$$c := \sqrt{2 \eta \cdot E_{kg}} = 4914.21 \frac{m}{s} \quad I_{sp} := \frac{c}{g_0} = 500.94 \text{ s}$$

Figure 2: Calculations for maximum chemical rocket engine performance

Note that in this calculation, the bond energy of oxygen is halved as per [Eq. 6](#) and the equation for effective exhaust velocity comes from the kinetic energy equation and noting that $E_{kg} = Energy/mass$.

2.3. Comparative Electric Performance

To compare the efficiency of chemical propulsion to electric propulsion consider an electrostatic propulsion system shown in [Figure 3](#).

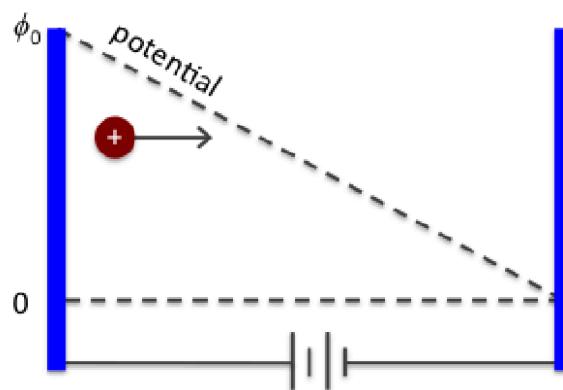


Figure 3: Basic principle of an electrostatic propulsion system.

A charged ion (assumed for these calculations to be a water ion) enters an electric field which causes it to be accelerated to the more negative (lower potential plate). By setting the electric potential energy gained by the ion equal to the kinetic energy ($\eta E_p = E_k$) then the I_{sp} can be calculated, shown in [Figure 4](#).

Voltage Required for an Isp of 500s Using Ionized Water

$$M_{Water} := 18.01528 \frac{gm}{mol} \quad \eta := 0.9 \quad g_0 := 9.81 \frac{m}{s^2} \quad N_a := 6.023 \cdot 10^{23} mol^{-1}$$

$$I_{sp} := 500 \text{ s} \quad q := 1.6 \cdot 10^{-19} C$$

$$V := \frac{\frac{1}{2} \frac{M_{Water}}{N_a} (I_{sp} \cdot g_0)^2}{\eta \cdot q} = 2.5 \text{ V}$$

Figure 4: Comparative electrical propulsion system voltage calculations.

As shown in **Figure 4** the voltage required to match the performance of a chemical system is very low and easily achievable, in reality electrostatic systems can achieve efficiencies in excess of 10,000s.

2.4. Nuclear Performance

To estimate the performance of a thermal nuclear rocket engine, Uranium-235 fission is considered, where the energy released in one fission event is immediately transferred to a water molecule, this calculation is shown in **Figure 5**.

Energy Transferred to One Water Molecule During One Nuclear Fission Event

$$E_{U235} := 180 \text{ MeV} = (2.88 \cdot 10^{-11}) J \quad M_{Water} := 18.01528 \frac{gm}{mol} \quad N_a := 6.023 \cdot 10^{23} mol^{-1}$$

$$m_{Water} := \frac{M_{Water}}{N_a} = 0 \text{ kg} \quad <- \text{ is } 2.99 \times 10^{-26} \text{ but is too small for mathcad to show}$$

$$E_{kg} := \frac{E_{U235}}{m_{Water}} = (9.63 \cdot 10^{14}) \frac{J}{kg}$$

Performance of a Nuclear Thermal System

$$\eta := 0.9 \quad g_0 := 9.81 \frac{m}{s^2}$$

$$c := \sqrt[2]{2 \eta \cdot E_{kg}} = 41631151.15 \frac{m}{s}$$

$$I_{sp} := \frac{c}{g_0} = (4.24 \cdot 10^6) \text{ s}$$

Figure 5: Maximum nuclear thermal propulsive system performance.

Note that this I_{sp} is a theoretical upper limit and in reality the true performance is much lower and is limited by material limits due to heat.

2.5. Definitions and Fundamentals Cont.

For propulsion systems, efficiency can be defined in terms of the fraction of source power that is converted to jet power, this efficiency is shown in **Eq. 7**.

$$\eta_T = \frac{\dot{m}c^2}{2P_{in}} \quad (7.1)$$

$$P_{in} = \frac{\dot{m}c^2}{2\eta_T} = \frac{Fc}{2\eta_T} \quad (7.2)$$

$$\frac{P_{in}}{m} = \frac{F}{m} \frac{c}{2\eta_T} = a \frac{c}{2\eta_T} \quad (7.3)$$

Where:

- η_T : Power conversion efficiency
- a : Acceleration (m/s^2)
- P_{in} : Input or Source power (W)
- m : Spacecraft mass (kg)

Note that for electrical systems P_{in} , the power must come from a source e.g., solar panel array. **Eq. 7** Also shows that **for a fixed specific power: ($\frac{P_{in}}{m}$) a high effective exhaust speed (c) means a low acceleration**. It is also useful to define a specific power plant mass as shown in **Eq. 8**.

$$\alpha = \frac{M_{pow}}{P_{in}} \quad (8)$$

Where:

- α : Specific power plant mass (kg/W)
- M_{pow} : Power plant mass (kg)

By manipulating equations **Eq. 8** and **Eq. 7**, as well as assuming that $\eta_T \approx 1$ and $M_{pow} \approx 0.1m$ then the acceleration can be written as **Eq. 9**.

$$a = \frac{0.2}{\alpha c} \quad \begin{cases} M_{pow} \approx 0.1m \\ \eta_T \approx 1 \end{cases} \quad (9)$$

Eq. 9 shows that a and c are inversely proportional from one another, meaning a high acceleration will typically mean a low effective exhaust velocity and vice versa. A showing how performance varies with acceleration is shown in **Figure 6**.

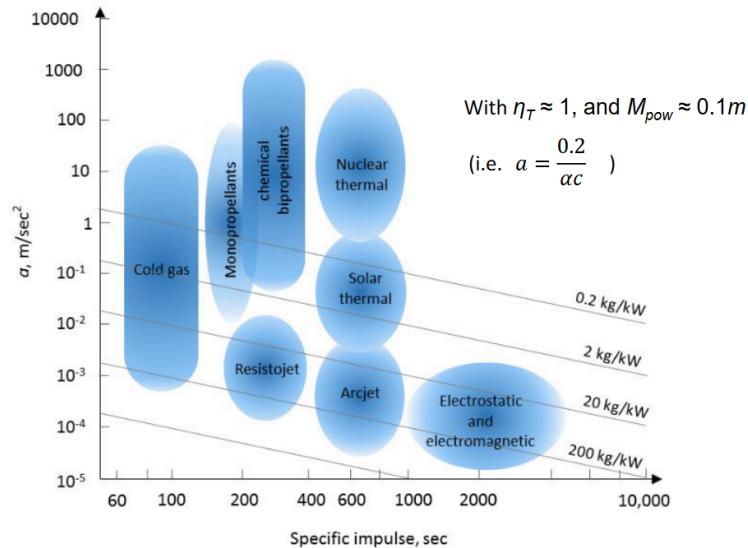


Figure 6: Variation of spacecraft acceleration against performance.

Note that for electrical propulsion systems shown in **Figure 6** a higher I_{sp} means a lower acceleration as $I_{sp} \propto c \propto \frac{1}{a}$. Different power sources have different values of α , for example:

- Nuclear Reactors $\Rightarrow 2\text{kg/kW}$
- Solar Panels $\Rightarrow 20\text{kg/kW}$
- RTGs $\Rightarrow 200\text{kg/kW}$

2.6. Thrust Fundamentals

By applying Newton's second law to a rocket nozzle, considering the difference in atmospheric and exhaust pressure as well as using the equations derived in the previous sections, **Eq. 10** can be derived.

$$F = \dot{m}v_e + (P_e - P_a)A_e \quad (10.1)$$

$$c = v_e + \frac{(P_e - P_a)A_e}{\dot{m}} \quad (10.2)$$

$$I_{sp} = \frac{1}{g_0} \left(v_e + \frac{(P_e - P_a)A_e}{\dot{m}} \right) \quad (10.3)$$

Where:

- | | |
|---|--|
| • v_e : Exhaust velocity (m/s) | • P_e : Exhaust Pressure (Pa) |
| • A_e : Exhaust Area (m^2) | • P_a : Atmospheric Pressure (Pa) |

One key thing to note about **Eq. 10** is that the thrust is made up of two parts, the first part being the **momentum thrust** accounting for the majority of the thrust (90-70%) and the second part is the **pressure thrust** (10-30%).

Crucially, as $P_a(h)$ then the I_{sp} and c vary with the height, typically being lower at lower altitudes and increasing up to reaching their maximums in the thinner sections of the atmosphere.

Another impartial performance parameter for chemical rockets which does not depend on the altitude is shown in **Eq. 11**.

$$c^* = \frac{P_c A_t}{\dot{m}} \quad (11)$$

Where:

- c^* : Characteristic velocity (m/s)
- P_c : Chamber pressure (Pa)
- A_t : Throat area (m^2)

Typical values of c^* are 1500 m/s for a solid rocket and 2500 for H_2/O_2 liquid bi-propelled rocket.

2.7. Tsiolkovsky Rocket Equation

One way to represent the quantity of propellant to the structure of the rocket is by using the **propellant mass fraction** shown in **Eq. 12**.

$$\mu = \frac{M_P}{M_0} \quad (12)$$

Where:

- μ : Mass fraction
- M_P : Propellant mass (kg)
- M_0 : Structure Mass (kg)

For a well designed rocket $\mu \approx 0.8 - 0.85$. The famous rocket equation is derived by starting with Newtons second law and considering the momentum of the fuel leaving the engine and integrating that equation, this yields .

$$\Delta V = c \ln \left(\frac{M_0}{M_f} \right) = I_{sp} g_0 \ln \left(\frac{M_0}{M_f} \right) \quad (13)$$

Where:

- ΔV : Change in velocity (m/s)
- M_f : Final mass (kg)

The ΔV and the M_0/M_f are plotted against one another in **Figure 7**. Note that for a single stage rocket $M_0/M_f \approx 20$ and the ΔV required to reach LEO is 9.5 km/s and so a single stage to rocket is on the boundary of being possible using a chemical bi-propellant rocket.

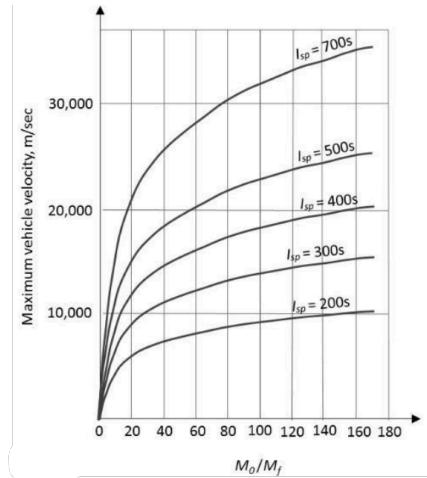


Figure 7: Plot of ΔV against M_0/M_f

3. Lecture 3

3.1. Rocket Staging

The typical ΔV s required for different manoeuvre are shown in **Table 3**.

Manoeuvre	Req ΔV (km/s)
Surface of Earth to LEO (inc drag and grav losses)	9.5
LEO to GEO (impulsive no plane change)	3.95
LEO to GEO (low thrust no plane change)	4.71
LEO to Lunar (impulsive)	3.9
LEO to Lunar (low thrust)	8
LEO to Mars (impulsive)	5.7
GEO station keeping	50 m/s /year
LEO station keeping	< 25 m/s /year

Table 3: Typical ΔV values for different manoeuvre

For a conventional chemical rocket, to reach LEO from the surface of the Earth, assuming an ideal mass ratio ($I_{sp} \approx 450s$, $\Delta V \approx 9.5km/s$, $M_o/M_f \approx 8.6$) then the mass fraction μ would have to be $\approx 90\%$, leaving 10% for the payload itself. This is mitigated through using **rocket staging**. Stages offer various benefits, the most prominent of which is the gain in ΔV when compared with one stage. The expression of the ΔV of a multistage rocket is shown in **Eq. 14**.

$$\Delta V_{\text{Total}} = \Delta V_{\text{Stage 1}} + \Delta V_{\text{Stage 2}} + \dots + \Delta V_{\text{Stage } n} \quad (14.1)$$

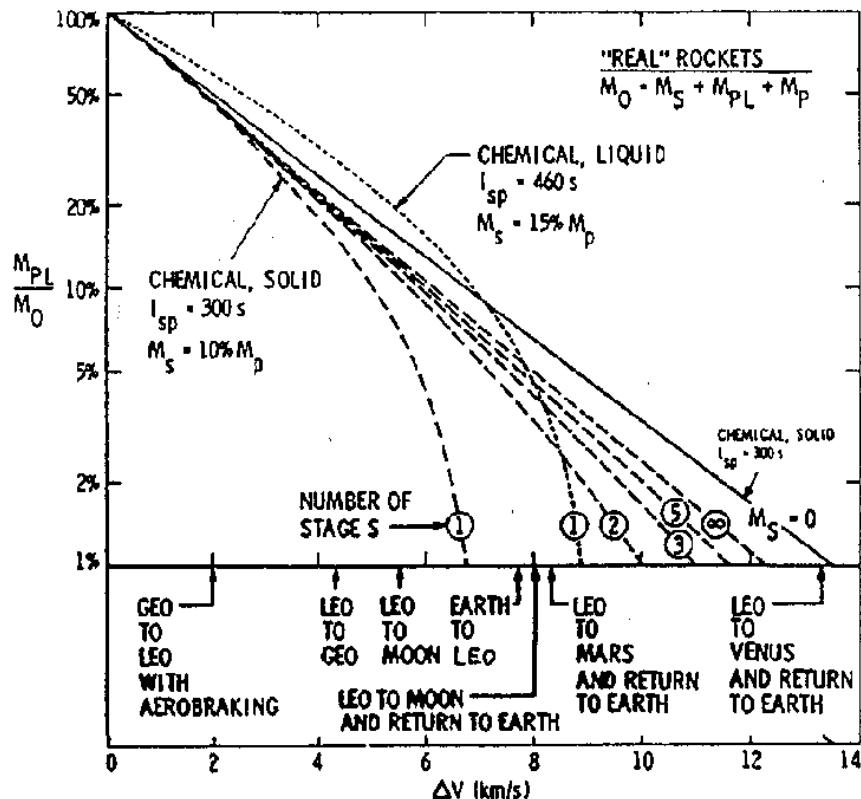
$$\Delta V_{\text{Total}} = I_{sp \text{ Stage 1}} g_0 \left(\frac{M_0 \text{ Stage 1}}{M_1 \text{ Stage 1}} \right) \quad (14.2)$$

$$+ I_{sp \text{ Stage 2}} g_0 \left(\frac{M_0 \text{ Stage 2}}{M_1 \text{ Stage 2}} \right) \quad (14.3)$$

$$+ \dots \quad (14.4)$$

$$+ I_{sp \text{ Stage } n} g_0 \left(\frac{M_0 \text{ Stage } n}{M_1 \text{ Stage } n} \right) \quad (14.5)$$

An image depicting the payload fraction against delta V is shown in **Figure 8**.

Figure 8: Plot depicting the effect of staging on the ΔV for a given payload fraction.

3.2. Launch Vehicle Dynamics

The key forces acting on a launch vehicle during launch are shown in **Figure 9**.

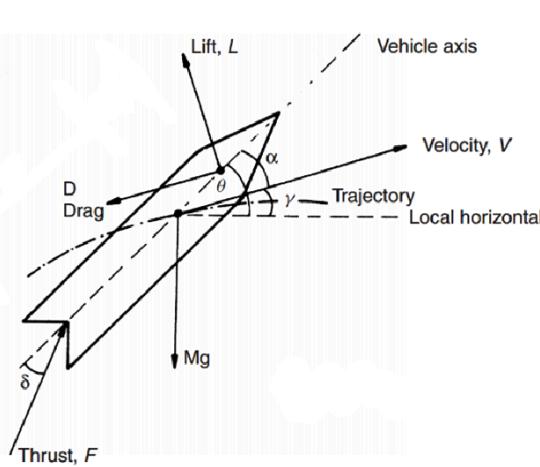


Figure 9: Plot illustrating the forces present on a launch vehicle.

Taking the forces shown in **Figure 9**, a differential expression can be generated for the motion of the craft, using Newton's second law, this is shown in

$$M \left(\frac{dV}{dt} \right) = F \cos(\alpha - \delta) - Mg \sin(\gamma) - D \quad (15)$$

Where:

- M : Total launch vehicle mass (kg)
- F : Thrust (N)
- δ : Gimbal angle ($^{\circ}$ or rad)
- θ : Pitch angle ($^{\circ}$ or rad) $\theta = \gamma + \alpha$
- V : Spacecraft velocity (m/s)
- α : Angle of attack ($^{\circ}$ or rad)
- γ : Flight path angle ($^{\circ}$ or rad)
- D : Drag (N)

Note that within **Eq. 15**, many of the terms depend on the time as well as on one another. These equations can be rearranged and manipulated to yield **Eq. 16** (assuming $V_0 \approx 0, \alpha \approx 0, \delta \approx 0$).

$$\Delta V = \Delta V_{\text{ideal}} - \Delta V_g - \Delta V_D \quad (16.1)$$

$$\Delta V_{\text{ideal}} = \bar{c} \ln \left(\frac{M_0}{M_f} \right) \quad (16.2)$$

$$\Delta V_g = \int_0^{t_b} g \sin(\gamma) dt \quad (16.3)$$

$$\Delta V_D = \int_0^{t_b} \frac{D/M_0}{1 - \mu t/t_p} dt \quad (16.4)$$

Note that for **Eq. 16**, $\Delta V_g \approx 1.1 \text{ km/s}$, $\Delta V_D \approx 0.2 \text{ km/s}$. Additionally a boost of 0.5 km/s can be gained by launching at the equator. Note that \bar{c} is an averaged effective exhaust velocity.

3.3. Converging Diverging Nozzle

START OF WEEK 2

All of the thermal rockets that were shown in **Figure 1** will most likely use a converging diverging nozzle (De-Laval nozzle) to accelerate the hot exhaust gas and increase the thrust of the engine. They effectively **convert the gases thermal energy to kinetic energy**. Note that when considering gaseous or liquid flow in this module, the following assumptions will be made:

- The fluid used are homogeneous.
- The species are gaseous.
- No heat transfer across the rocket walls (adiabatic assumption).
- No friction and all boundary layer effects effects negligible
- No shock waves or discontinuities in the nozzle
- Gas composition does not change in the nozzle (frozen flow) (not necessarily true but will assume for simplification that all reactions occur in the combustion chamber)

A plot of how the temperature, pressure, velocity and Mach number change over the nozzle is shown in **Figure 10**.

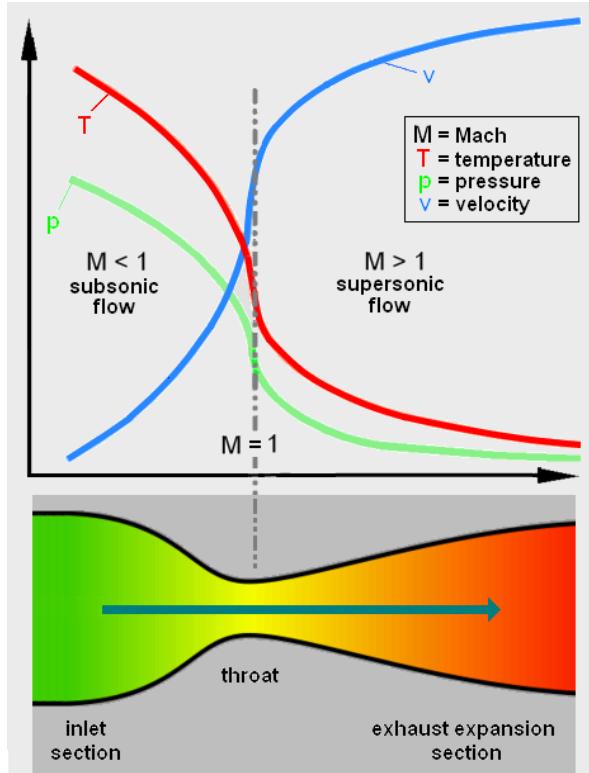


Figure 10: Plot of pressure, temperature, velocity and Mach number over a De-Laval nozzle.

3.4. Exit Velocity Equation

Utilizing the isentropic flow equations it is possible to derive equations for many of the nozzle and engine parameters that have been previously stated. To derive an expression for the **exit velocity** v_e from isentropic flow equations, we first start with the expression for stagnation enthalpy and apply the following criteria shown in **Eq. 17**.

$$h_0 = h_e + \frac{v_e^2}{2} \quad (17.1)$$

$$c_p T_0 = c_p T_e + \frac{v_e^2}{2} \quad \begin{cases} 1. \text{ Ideal gas} \\ 2. c_p \text{ is constant at a given } T \end{cases} \quad (17.2)$$

Where:

- h_0 : Stagnation enthalpy (J/mol).
- c_p : Specific heat at a constant pressure (J/mol).
- T_e : Temperature at nozzle exit (T).
- h_e : Enthalpy at nozzle exit (J/mol).
- T_0 : Stagnation temperature (T).

This equation can be further developed by **assuming isentropic flow** from the stagnation point to the exhaust point. This allows for the isentropic flow equations to apply, which are shown in **Eq. 18**.

$$\frac{T_0}{T_e} = \left(\frac{P_0}{P_e} \right)^{\frac{k-1}{k}} = \left(\frac{\rho_0}{\rho_e} \right)^{k-1} \quad (18)$$

Where:

- P_0 : Stagnation pressure (pa).
- ρ_0 : Stagnation density (kg/m³).
- k : Ratio of specific heats.
- P_e : Pressure at nozzle exit (pa).
- ρ_e : Density at nozzle exit (kg/m³).

Finally the last equation that is needed for a useful expression for v_e is the equation for the specific heat capacity at a constant pressure c_p , this is shown in **Eq. 19**.

$$c_p = \frac{R}{W} \frac{k}{k-1} \quad (19)$$

Where:

- R : Molar gas constant (J/(mol K)).
- W : Molecular weight (kg/mol).

Using **Eq. 19**, **Eq. 18** and **Eq. 17**, a useful expression for the exhaust velocity v_e can be derived, this is shown in **Eq. 20**.

$$v_e = \sqrt{\frac{R}{W} \frac{2k}{k-1} T_0 \left(1 - \left(\frac{P_e}{P_0} \right)^{\frac{k-1}{k}} \right)} \quad (20)$$

Where T_0, P_0 can be assumed to be the combustion conditions. Alternatively, **Eq. 20** can also be used to define the I_{sp} , shown in **Eq. 21** (assuming ideal expansion).

$$I_{sp} = \frac{1}{g_0} \sqrt{\frac{R}{W} \frac{2k}{k-1} T_0 \left(1 - \left(\frac{P_e}{P_0} \right)^{\frac{k-1}{k}} \right)} \quad (21)$$

To see what parameters effect the value of v_e and what need sto be maximized, various plots are shown in **Figure 11**.

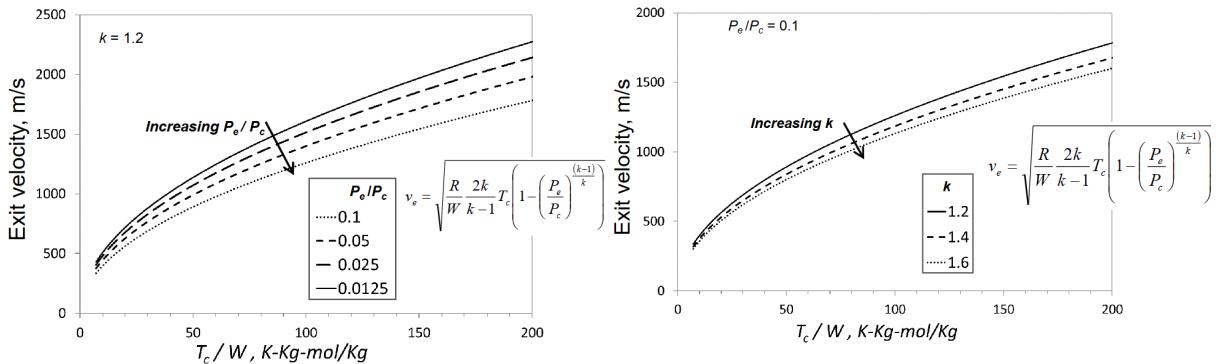


Figure 11: Plot of exit velocity for increasing P_e/P_c ratios [Left], Plot of exit velocity for increasing k ratios [Right]

From **Figure 11** it is clear to see that to maximize the value of v_e the following optimizations of parameters must occur:

- **Minimizing the molecular weight M of the reactants** will have a substantial effect on v_e .
- **Maximizing the combustion temperature T_c** will have a substantial effect on v_e .
- **Decreasing the ratio of P_e/P_c** will have a small impact on v_e .

- Decreasing the ratio of k will have a small impact on v_e .

3.5. Mass Flow Rate Equation

Assuming choked flow ($M_a @ \text{Throat} \approx 1$), the mass flow rate \dot{m} is given by the expression shown in **Eq. 22**.

$$\dot{m} = \rho_t A_t v_t \quad (22)$$

Where:

- ρ_t : Density at the throat (kg/m^3)
- A_t : Area of the throat (m^2)
- v_t : Velocity at the throat (m/s).

Ideally **Eq. 22** should be expressed in terms of chamber parameters. The first substitution that can be made is an expression for the velocity at the throat v_t using the speed of sound equation, this equation is shown in **Eq. 23**. **Eq. 17** can then be used to yield an expression for the stagnation/chamber pressure, shown again in **Eq. 23**.

$$v_t = a = \sqrt{\frac{kRT_t}{W}} \quad (23.1)$$

$$T_0 = T_t + \frac{v_t^2}{2c_p} = T_t + \frac{\left(M_a \sqrt{\frac{kRT_t}{W}}\right)^2}{2c_p} \quad (23.2)$$

Where:

- a : Speed of sound (m/s)
- M_a : Mach number

The next goal is to find expressions for the throat temperature and densities as this will then eliminate them from the equation. By using **Eq. 19**, **Eq. 18** and assuming that $M_t \approx 1$ **Eq. 24** can be derived for T_t as well as for ρ_t .

$$T_t = \frac{2T_c}{k+1} \quad \rho_t = \rho_c \left(\frac{2}{k+1} \right)^{\frac{1}{k-1}} \quad (24)$$

Finally, **Eq. 24** and **Eq. 23** can be substituted into **Eq. 22** to yield **Eq. 25**.

$$\dot{m} = \frac{A_t \rho_c k}{\sqrt{\frac{kRT_c}{W}}} \sqrt{\left(\frac{2}{k+1} \right)^{\frac{k+1}{k-1}}} \quad (25)$$

4. Lecture 4

4.1. Nozzle Expansion Ratio Equation

Momentum conservation can be applied between the exhaust and the throat to yield an expression including A_t and A_e , this expression is shown in **Eq. 26**.

$$\dot{m} = \rho_t A_t v_t = \rho_e A_e v_e \rightarrow \frac{A_t}{A_e} = \frac{\rho_e v_e}{\rho_t v_t} \quad (26)$$

Equations for v_e , v_t and ρ_e/ρ_t have already been defined and so by substituting **Eq. 23**, **Eq. 20** and **Eq. 18** into **Eq. 26** will yield **Eq. 27**.

$$\frac{A_t}{A_e} = \left(\frac{k+1}{2}\right)^{\frac{1}{k-1}} \left(\frac{P_e}{P_c}\right)^{\frac{1}{k}} \sqrt{\frac{k+1}{k-1} \left(1 - \frac{P_e}{P_c}\right)^{\frac{k-1}{k}}} \quad (27)$$

Note that for low altitude rockets $\frac{A_e}{A_t} \approx 3 - 25$ and for high altitude rockets $\frac{A_e}{A_t} \approx 40 - 200$.

4.2. Characteristic Velocity Equation

The characteristic velocity was first defined in **Eq. 11**. It can be rewritten in terms of the equations that have been previously defined to yield **Eq. 28**.

$$c^* = \frac{P_c A_t}{\dot{m}} = \frac{\sqrt{\frac{kRT_c}{W}}}{k \sqrt{\frac{2}{k+1}}^{\frac{k+1}{k-1}}} \quad (28)$$

Note that for a liquid oxygen, liquid hydrogen bipropellant rocket, $c^* \approx 2300m/s$ and for an ammonium perchlorate + polymer + Al solid rocket, $c^* \approx 1590m/s$.

4.3. Thrust Equation

Similarly to characteristic velocity, the thrust can be written in terms of the equations that have just been derived, mainly **Eq. 20** and **Eq. 25** to yield **Eq. 29**.

$$F = \dot{m} v_e + (P_e - P_a) A_e = A_t P_c \sqrt{\frac{2k^2}{k-1} \left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}} \left(1 - \left(\frac{P_e}{P_c}\right)^{\frac{k-1}{k}}\right)} + (P_e - P_a) A_e \quad (29)$$

4.4. Coefficient of Thrust Equation

A useful parameter when quantifying the performance of a nozzle is the coefficient of thrust C_F . The definition of C_F as well as the equation after substituting **Eq. 29** into it are shown in **Eq. 30**.

$$C_F = \frac{F}{P_c A_t} = \sqrt{\frac{2k^2}{k-1} \left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}} \left(1 - \left(\frac{P_e}{P_c}\right)^{\frac{k-1}{k}}\right)} + \frac{(P_e - P_a) A_e}{P_c A_t} \quad (30)$$

Values of $C_F \approx 0.8 - 1.9$ with a higher value meaning better thrust amplification. C_F is a peak when there is ideal expansion ($P_e = P_a$) at a constant P_a/P_c . Note that the equation

has a **momentum part** and a **pressure part** similar to the thrust itself. The behavior of the C_F against the area and pressure ratios is shown in **Figure 12**

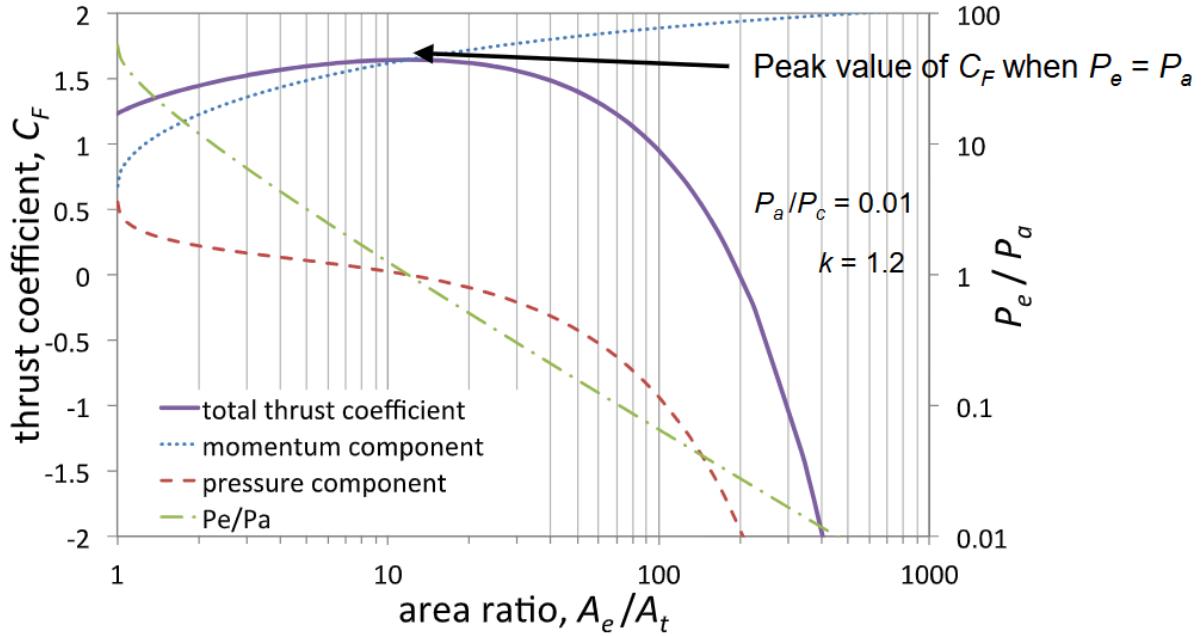


Figure 12: Plot of C_F against area and pressure ratios

Note that as the area ratio increases the momentum component increases but the pressure component decreases. This is interesting as the area ratio **does not appear in the momentum section of the equation**. In reality there is still a dependency as area ratio depends on pressure ratio which is present in the area ratio equation. Another plot is shown in **Figure 13**.

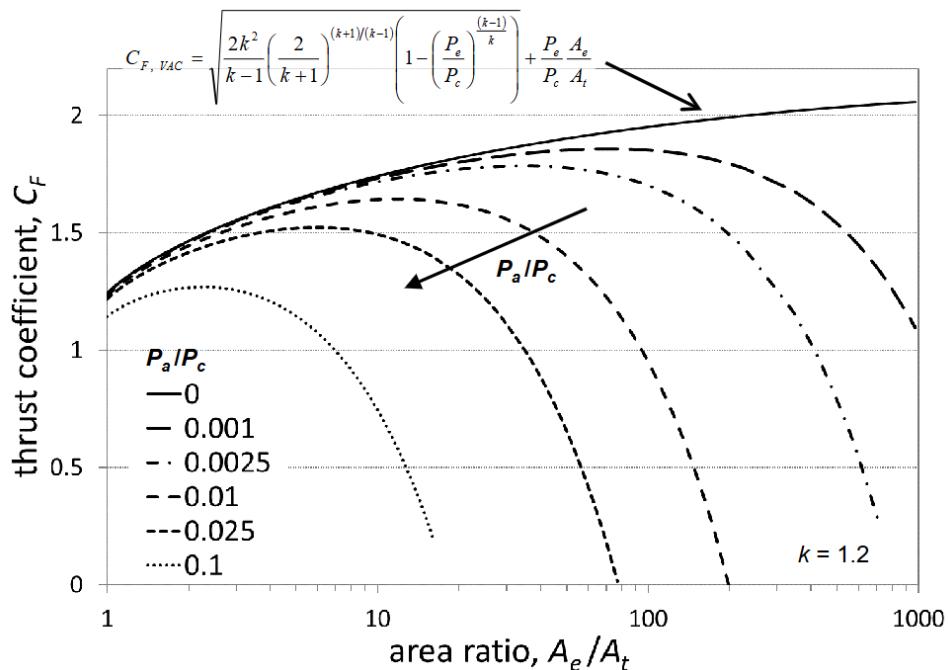


Figure 13: Plot of C_F against area ratio for varying pressure ratios

Note that in **Figure 13**, increasing the pressure ratio will decrease the thrust coefficient. The highest possible thrust coefficient is given when the pressure ratio is zero such as in a vacuum.

4.5. Summary of Equations

$$v_e(R, W, k, T_0, P_e, P_0) = \sqrt{\left(\frac{R}{W}\right) \frac{2k}{k-1} T_0 \left(1 - \left(\frac{P_e}{P_0}\right)^{\frac{k-1}{k}}\right)} \quad (31.1)$$

$$\dot{m}(A_t, \rho_c, k, R, T_c, W) = \frac{A_t \rho_c k}{\sqrt{\frac{kRT_c}{W}}} \sqrt{\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}} \quad (31.2)$$

$$\frac{A_t}{A_e}(k, P_e, P_c) = \left(\frac{k+1}{2}\right)^{\frac{1}{k-1}} \left(\frac{P_e}{P_c}\right)^{\frac{1}{k}} \sqrt{\frac{k+1}{k-1} \left(1 - \left(\frac{P_e}{P_c}\right)^{\frac{k-1}{k}}\right)} \quad (31.3)$$

$$c^*(T_c, k, R, W) = \frac{\sqrt{\frac{kRT_c}{W}}}{k \sqrt{\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}}} \quad (31.4)$$

$$F(A_t, P_c, k, P_e, A_e, P_a) = A_t P_c \sqrt{\frac{2k^2}{k-1} \left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}} \left(1 - \left(\frac{P_e}{P_c}\right)^{\frac{k-1}{k}}\right)} + (P_e - P_a) A_e \quad (31.5)$$

$$C_{F(k, P_e, P_a, A_e, P_c, A_t)} = \sqrt{\frac{2k^2}{k-1} \left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}} \left(1 - \left(\frac{P_e}{P_c}\right)^{\frac{k-1}{k}}\right)} + \frac{(P_e - P_a) A_e}{P_c A_t} \quad (31.6)$$

4.6. Equations Involving Mach Relations

Many of the previous equations can be represented in terms of mach number, namely **Eq. 18**, which are shown in **Eq. 32**.

$$T_0 = T \left(1 + \frac{1}{2}(k-1)M^2\right) \quad P_0 = P \left(1 + \frac{1}{2}(k-1)M^2\right)^{\frac{k}{k-1}} \quad \rho_0 = \rho \left(1 + \frac{1}{2}(k-1)M^2\right)^{\frac{1}{k-1}} \quad (32)$$

The Mach relations can be applied to **Eq. 27** to yield an expression for the area ratio in terms of Mach number shown in **Eq. 33**.

$$\frac{A_y}{A_x} = \frac{M_x}{M_y} \sqrt{\left(\frac{1 + \frac{1}{2}(k-1)M_y^2}{1 + \frac{1}{2}(k-1)M_x^2}\right)^{\frac{k+1}{k-1}}} \quad (33)$$

Eq. 33 shows that area ratio is directly proportional to the Mach ratio. Furthermore this equation is also proportional to coefficient of thrust as was previously stated, and this relation is also shown in **Figure 14**.

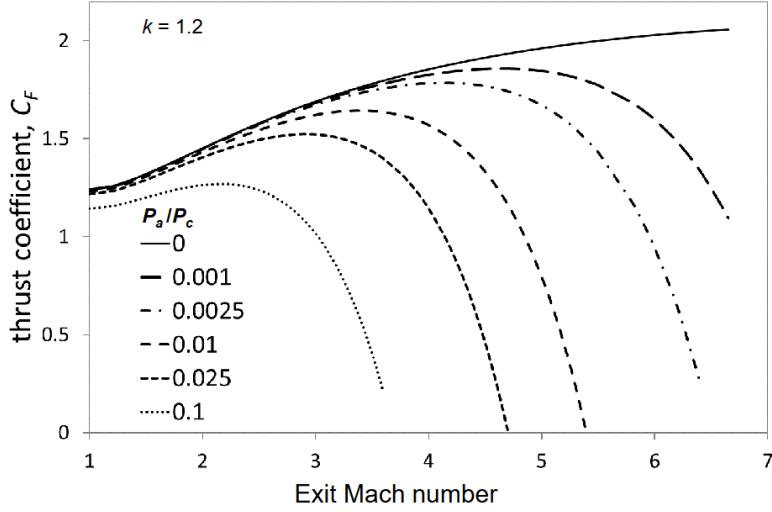


Figure 14: Plot of C_F against exit Mach number for varying pressure ratios

Figure 14 is effectively the same as **Figure 13** apart from altering the x-axis. A larger mach number will require a larger area ratio which will drive up the C_F as it depends on the pressure ratio which is proportional.

4.7. Coefficient of Thrust for Converging Nozzles

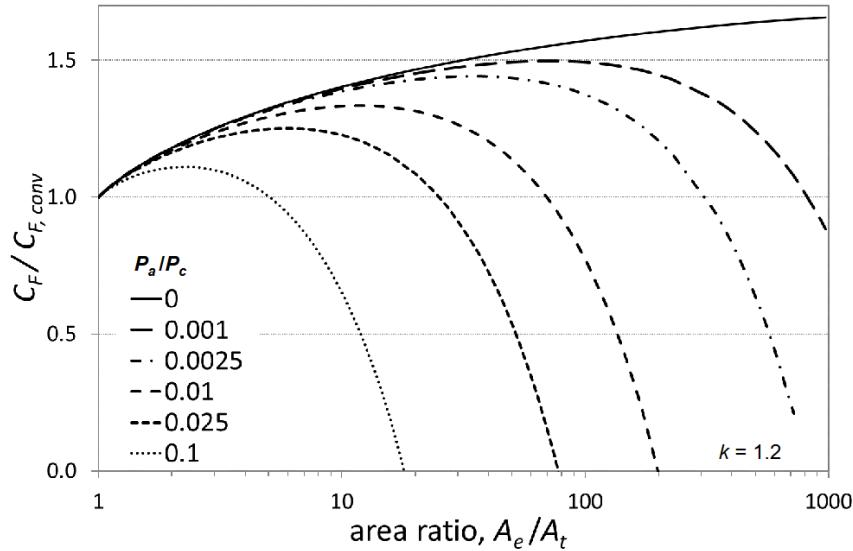
Figure 14 can be further edited to yield a neater plot. To get to this, consider the pressure equation in **Eq. 32** when there is no diverging nozzle. This would mean that $M_e = 1$ and **Eq. 32** can therefore be then written as **Eq. 34**.

$$\frac{P_c}{P_e} = \left(1 + \frac{1}{2}(k-1)M_e^2\right)^{\frac{k}{k-1}} \quad \text{If } M_e = 1 \rightarrow \quad \frac{P_e}{P_c} = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} \quad (34)$$

Eq. 34 can be substituted into **Eq. 30** to yield an equation for C_F for the converging section of the nozzle, this is shown in **Eq. 35**.

$$C_F \text{ Converging} = (k+1) \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} - \frac{P_a}{P_c} \quad (35)$$

Using **Eq. 35** a modified version of **Figure 14** can be plotted, this plot is shown in **Figure 15**. This plot now has a point where all lines roginiate, when the ratio of $C_F/C_F \text{ Converging} = 1$ and $A_e/A_t = 1$ when there is no diverging section at all.

Figure 15: Plot of $C_F / C_{F,conv}$ against exit area ratio for varying pressure ratios

4.8. Under, Ideal and Over Expanded Nozzles

Depending on the relationship between the exit pressure P_e and the ambient pressure P_a , there are three cases of nozzle exhaust flow, these are:

- **Under-expanded ($P_e > P_a$):**

- Typically occurs at **high altitudes** and happens when the **nozzle is too short**. Exhaust wasn't expanded enough and so expands out the back of the nozzle via expansion waves.
- C_F and thrust are **below maximum**.

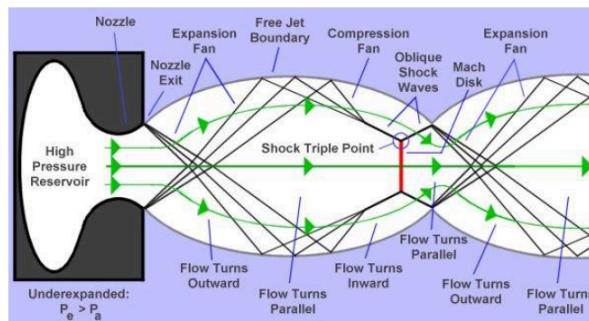


Figure 16: Under-expanded flow out of a nozzle

- **Ideally Expanded ($P_e \approx P_a$):**

- **Nozzle is perfect length** and exhaust exits in a perfect rectangular plume with no losses or shocks.
- C_F and thrust are **maximized**.
- $v_e = c$, exhaust velocity is equal to effective exhaust velocity.

- **Over-expanded ($P_e < P_a$):**

- Typically occurs at **low altitudes** and happens when the **nozzle is too long**. Exhaust is at a lower pressure than ambient causing shocks and possible flow separation within the nozzle.
- C_F and thrust are **below maximum**.

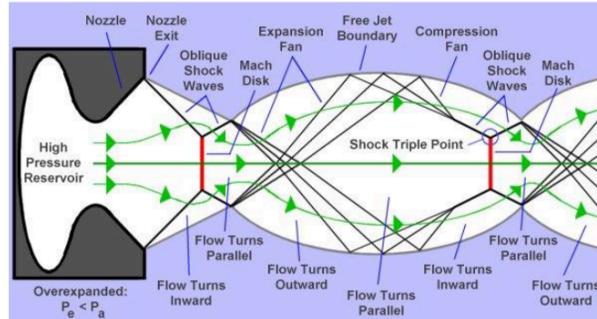
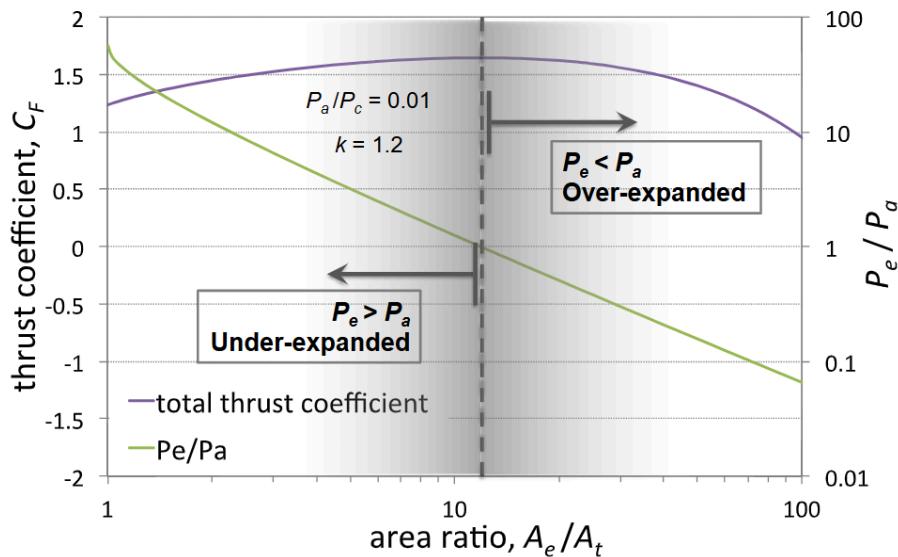


Figure 17: Over-expanded flow out of a nozzle

Plotting the behavior of the thrust coefficient against pressure ratio and the area ratio yields **Figure 18**. Note that the value of C_F is maximized when $P_e = P_a$ and $P_e/P_a = 1$.

Figure 18: Plot of C_F against pressure ratio and area ratio

4.8.1. Summerfield Criterion

The Summerfield criterion applies to heavily over-expanded nozzles and describes when the flow is likely to separate from inside of the nozzle and create shocks. The criterion is shown in **Eq. 36**.

$$P_e < (0.25 \text{ to } 0.4)P_a \quad (36)$$

Eq. 36 as well as the line of ideal expansion can be applied to **Figure 15** to produce **Figure 19**.

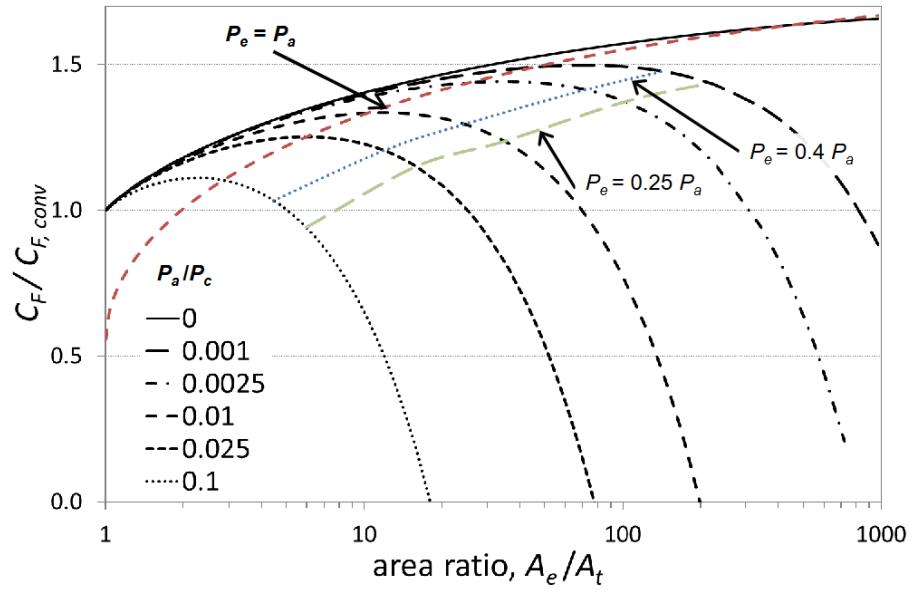


Figure 19: Plot of $C_F/C_{F, \text{conv}}$ against exit area ratio for varying pressure ratios with summerfield criterion and ideal expansion line.

On **Figure 19**, the red dotted line represents ideal expansion. **Below this line** sits **over-expanded flow**. **Above this line** sits **under-expanded flow**. **Below the yellow and blue lines** sits **super over-expansion** when the Summerfield criterion applies. Note that a typical rocket fired at sea level will undergo the following movements through this graph:

1. Initially **over-expanded** at sea level.
2. As the altitudes rises the rocket engine moves vertically upwards on the graph and the engine becomes less and less over-expanded until it is **ideally-expanded**.
3. As the rocket ascends further, the engine starts to become **under-expanded** and thrust and C_F start to decrease.

5. Lecture 5

5.1. Nozzle Designs and the Perfect Nozzle

Ideally a nozzle's expansion ratio A_e/A_t should increase as the rocket increases in altitude so that the flow is constantly ideally expanded. Some rockets achieve this using a skirt which drops down at higher altitudes to increase A_e/A_t . Some nozzle designs are shown in **Figure 20**

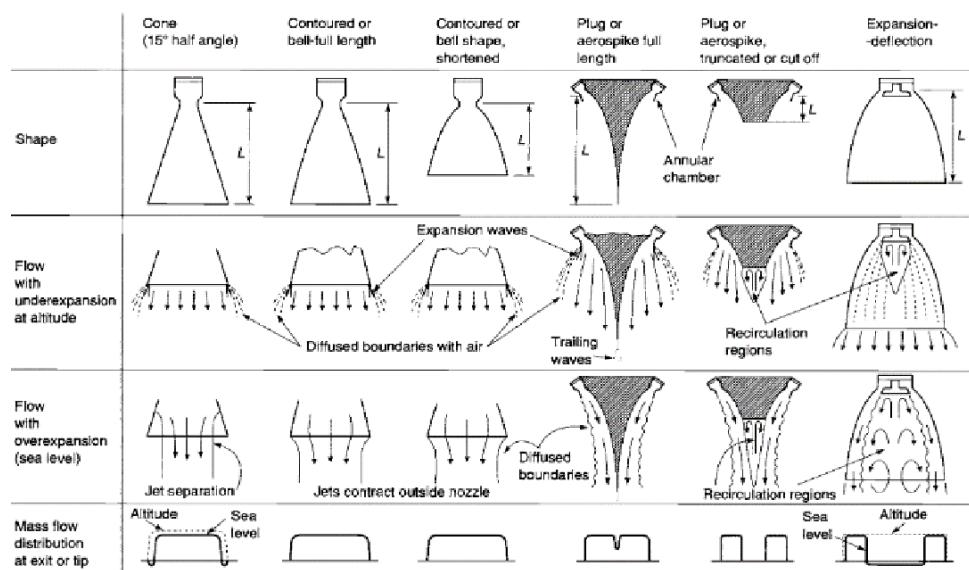


Figure 20: Various nozzle designs.

5.2. Conical Nozzles

Conical nozzles are a relatively simple nozzle design that is also easy to manufacture. There are various parameters that control the shape of a conical nozzle, these are depicted within **Figure 21**.

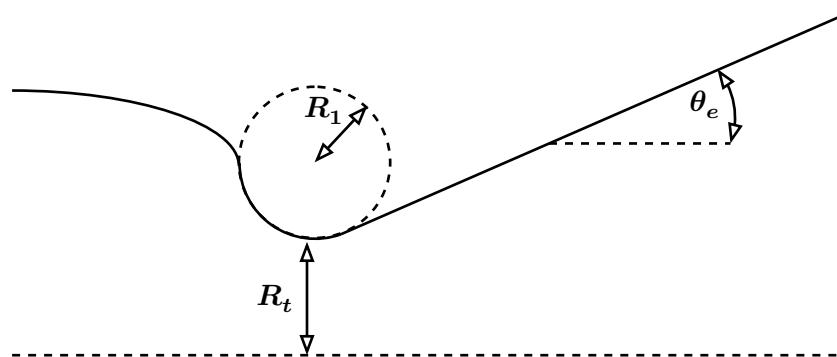


Figure 21: Definitions for a conical nozzle

Where:

- R_t : Throat radius (m).
- R_1 : Throat radius of curvature (m) $\approx 1.5 \times R_t$
- θ_e : Cone divergence half angle ($^{\circ}$ or Rad)

Ideally $\theta_e \approx 12^\circ - 18^\circ$ with:

- **Smaller angles** constituting a larger I_{sp} but higher mass and more complexity
- **Larger angles** constituting a lower I_{sp} but lower mass.

One issue with conical nozzles is that the flow does not all go directly straight out of the nozzle, it is instead directed outwards slightly at the edges. This introduces losses which are characterized by **Eq. 37** and are only applied to the **momentum term**. This is then applied to C_F .

$$\lambda = \frac{1}{2}(1 + \cos \theta_e) \quad (37.1)$$

$$C_{F(\lambda, k, P_e, P_a, A_e, P_c, A_t)} = \lambda \sqrt{\frac{2k^2}{k-1} \left(\frac{2}{k+1} \right)^{\frac{k+1}{k-1}} \left(1 - \left(\frac{P_e}{P_c} \right)^{\frac{k-1}{k}} \right)} + \frac{(P_e - P_a) A_e}{P_c A_t} \quad (37.2)$$

5.3. Bell (Rao) Nozzles

Bell nozzles have typically higher efficiency than conical by allowing the flow to quickly expand whilst it has high pressure and then slowly redirecting the flow to be as axial as possible by the end. An image showing the key dimensions for a bell nozzle are shown in.

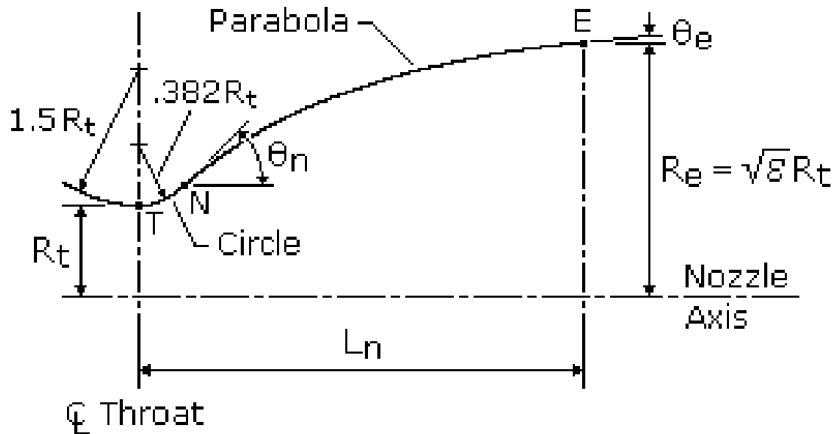


Figure 22: Bell nozzle dimensions.

Note that the bell curve will have a **point of inflection** along it. The coordinates of the inflection point are given by **Eq. 38** the following coordinates relative to the center of the throat (where R_t is measured from).

$$X_n = R_t \sin \theta_N \quad Y_n = R_t + R_1(1 - \cos \theta_e) \quad (38)$$

Note that θ_N here is the angle that the line at the inflection point makes with the horizontal datum (initial large divergence angle). If the initial diverging section of the nozzle is conical then this would be the cone angle for that portion. The equation for the parabolic low divergence angle section is shown in **Eq. 39**.

$$y' = Px' + Q + (Sx' + T)^{0.5} \quad (39.1)$$

$$P = \frac{y'_E \tan \theta_N + y'_E \tan \theta_E - 2x'_E \tan \theta_E \tan \theta_N}{2y'_E - x'_E \tan \theta_M - x'_E \tan \theta_E} \quad S = \frac{(y'_E - Px'_E)^2 (\tan \theta_N - P)}{x'_E \tan \theta_N - y'_E} \quad (39.2)$$

$$Q = \frac{S}{2(\tan \theta_N - P)} \quad T = Q^2 \quad (39.3)$$

Note that in **Eq. 39** any terms with a subscript of E are the coordinates and angles relating to the exit of the nozzle and the coordinates themselves are relative to the inflection point. Typically $\theta_E \approx 2^\circ - 8^\circ$. The length of a bell nozzle is compared to a 15° conical nozzle using **Eq. 40**.

$$L_{15} = \eta_{\text{Bell}} \frac{R_T(\sqrt{\varepsilon} - 1) + R_1 \left(\frac{1}{\cos \theta_e} - 1 \right)}{\tan \theta_e} \quad (40)$$

Where:

- L_{15} : Length of a 15° conical nozzle (m).
- ε : Expansion ratio $= A_e/A_t$
- θ_e : Divergence angle at exit ($^\circ$ or Rad)
- η_{Bell} : Percentage of full bell.

To obtain an values for θ_E, θ_N and ε for a given value of η_{Bell} then **Figure 23** can be used.

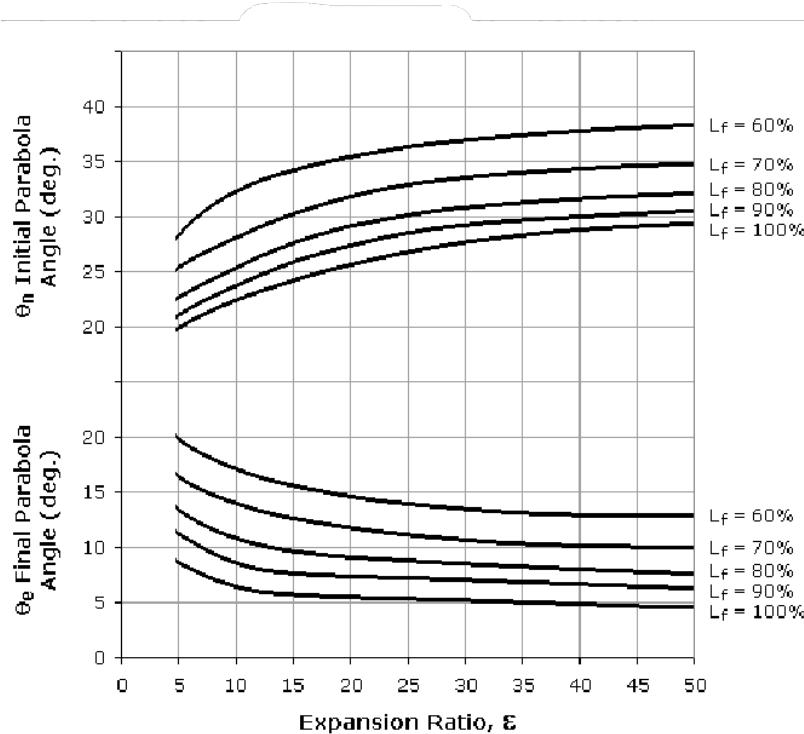


Figure 23: Bell curves for various values of η_{Bell} .

5.4. Aerospike Nozzles

Aerospike nozzles are a version of altitude compensating engine where the external air pressure changes the value of A_e Effective as it rises in altitude. They commonly feature many smaller combustion chambers which then have their exhausts directed onto a spike. The two main types of aerospike engines (shown in **Figure 20**) are:

- **Full Aerospike:** Feature a full length spike where there is no recirculation region, however the end of the spike is typically difficult to cool.
- **Truncated Aerospike:** The end of the spike is missing which allows for better cooling at penalty of lower performance.

Aerospikes can come in linear and annular forms. They are typically smaller than typical bell nozzles and can still vector thrust by controlling the thrust coming from individual combustion chambers. They haven't yet had much proven flight experience and lack any larger surface area examples.

5.5. Expansion Deflection Nozzle

Make use of a pintle at the center of the nozzle which redirects the flow along the walls of the nozzle (again shown in **Figure 20**). At low altitudes a large recirculation area caused by the high ambient pressure causes a smaller value of A_e . At higher altitudes the lower ambient pressure means the recirculation area is much smaller and the value of A_e is bigger. These engines haven't seen much use with one issue being keeping the pintle itself cool.

5.6. Intro to Liquid Propulsion

START OF WEEK 3

There are three main sub categories within liquid propulsion, these are shown in the bullet pointed list below. The relative performance of these liquid propulsion methods is shown in **Table 4:**

Type	$I_{sp}(s)$	$T_{max}(^{\circ}C)$	Thrust (N)	Propellants
Monopropellant	200 - 250	600 - 800	0.03 - 100	N_2H_4 , H_2O_2
Bipropellant	200 - 468	2500 - 4100	≤ 10 MN	N_2H_4 , H_2 , Kerosene, N_2O_4
Cold Gas	50 - 100	N/A	0.01 - 270	He, H_2 , Kr, N_2

Table 4: Typical liquid propellant parameters.

- **Bipropellant:** Mix together a liquid fuel and liquid oxidizer and combust them to produce thrust.
- **Monopropellant:** Flow a liquid fuel over a catalyst bed where it undergoes a exothermic decomposition reaction.
- **Cold Gas:** A gas is stored at pressure where it is released and flows through a nozzle to accelerate it.

From **Eq. 20**, a good rocket engine will maximize and minimize the following parameters:

- **Low molecular weight of combustion products W .** This is also why typically rocket engines operate fuel rich as the low molecular weight fuel dominates the reaction.
- **High combustion temperature T_c .**
- **High combustion pressure P_c ,** though there is a smaller gain from this parameter.
- **Low ratio of specific heats k ,** though there is a smaller gain from this parameter. Typically k sits at about one anyways.

6. Lecture 6

6.1. Performance of Liquid Thrusters

There are many engine parameters which can be calculated using the equations in [Eq. 31](#), however there are many parameters, mainly the combustion parameters (W and k) which cannot be calculated as easily. A **chemical reaction simulator** such as NASA's Glenn Chemical Equilibrium with Applications simulator can be used ([Press to go to link](#)). One useful parameter is the **oxidizer to fuel ratio**, shown in [Eq. 41](#).

$$r = \frac{\dot{m}_{\text{ox}}}{\dot{m}_{\text{fuel}}} \quad (41)$$

Typically, rocket engines **burn fuel rich** and not purely stoichiometric. This is to decrease W as lighter molecules make up more of the exhaust. An example of this is shown in [Figure 24](#).

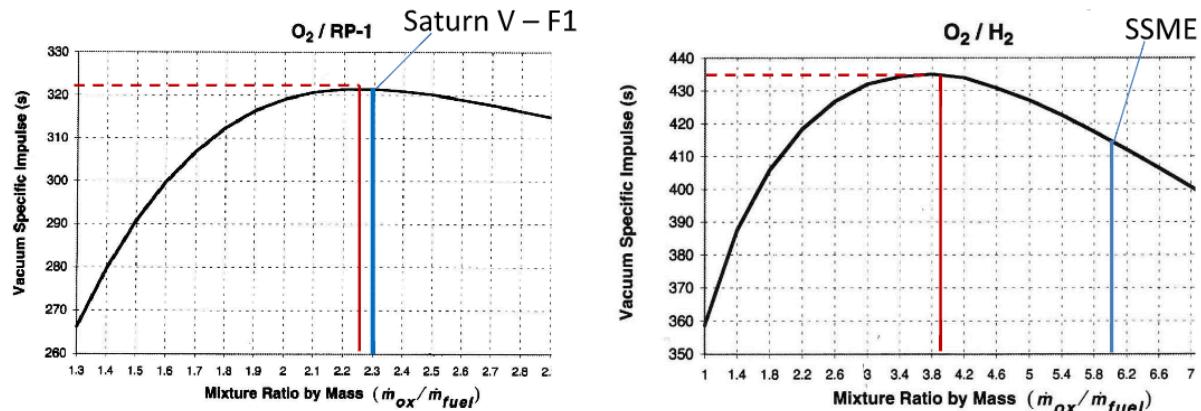


Figure 24: $\dot{m}_{\text{ox}}/\dot{m}_{\text{fuel}}$ against I_{sp} for different rocket engines.

Note that the stoichiometric ratio for r for the Saturn V F1 engines was 3.4 and for the Space Shuttle Main Engines (SSME) it was 8. Note that the Space shuttle couldn't reach r_{optimum} due to the low density of hydrogen and the lack of anymore space in the main fuel tanks for any more fuel.

6.2. Fuels

Name	Chemical Formula	Chemical Structure	Type	Toxicity	Corrosivity	Flammability	Hypergolic?	Molecular Weight (g/mol)	Density (kg/m ³)	Boiling and Freezing Temp (°C)
Fuels										
Liquid Hydrogen (LH ₂)	H ₂	H — H	Bi	None	None	High	No	2.016	70.85	Boil: -253 Freeze: -259
RP-1 (Rocket Propellant Group 1)	C _n H _{1.97n}		Bi	Moderate	None	High	No	170	810	Boil: 277 Freeze: -40
Hydrazine	N ₂ H ₄		Mono	High	Moderate	Spontaneous	N/A	32.05	1021	Boil: 113.5 Freeze: 2
Monomethylhydrazine (MMH)	CH ₃ NHNH ₂		Bi	High	High	High	Yes	46.07	880	Boil: 87.5 Freeze: -52.4
Unsymmetrical Dimethylhydrazine (UDMH)	(CH ₃) ₂ NNH ₂		Bi	High	High	High	Yes	60.1	793	Boil: 63 Freeze: -58
Methane	C ₂ H ₄		Bi	None	None	High	No	16.04	422.6	Boil: -162 Freeze: -183
Ethanol	C ₂ H ₅ OH		Bi	High	Moderate	High	No	46.07	789	Boil: 78.37 Freeze: -114
Pentaborane	B ₅ H ₉		Bi	Extremely	Extremely	Pyrophoric	Yes	63.13	618	Boil: 58.4 Freeze: -46.8

Name	Chemical Formula	Chemical Structure	Type	Toxicity	Corrosivity	Flammability	Hypergolic?	Molecular Weight (g/mol)	Density (kg/m ³)	Boiling and Freezing Temp (°C)
Oxidizers										
Liquid Oxygen (LOX)	O ₂	O = O	Bi	None	None	N/A	No	32	1141	Boil: -182 Freeze: -218
Dinitrogen Tetroxide	N ₂ O ₄		Bi	Extremely	High	N/A	Yes	92.01	1440	Boil: 21.2 Freeze: -11.2
Hydrogen Peroxide	H ₂ O ₂		Bi	High	High	N/A	No	34.014	1450	Boil: 150.2 Freeze: -0.43
Nitric Acid	HNO ₃		Bi	High	High	N/A	No	63.012	1510	Boil: 83 Freeze: -42
Chlorine Trifluoride	ClF ₃		Bi	Extreme	Extreme	N/A	Yes	92.45	1800	Boil: 11.75 Freeze: -76.34
Oxygen Difluoride	OF ₂		Bi	Extreme	Extreme	N/A	No	54	1880	Boil: -144.75 Freeze: -223.8
Fluorine	F ₂	F — F	Bi	Extreme	Extreme	N/A	Yes	38	1513	Boil: -188.1 Freeze: -219.6

Table 5: Data on common liquid fuels and oxidizers

Note that nitrogen tetroxide shown in **Table 5** is usually mixed with nitric oxide NO to increase the temperature range over which it is liquid, this mixture is called **Mixed Oxides of Nitrogen** (MON). To compare different fuel and oxidizer ratios, **Table 6** can be used.

Oxidizer	Fuel	Mass Mixture Ratio	ρ (g/cm ³)	c^* (m/s)	Sea Level I_{sp} (s)
O_2	Methane	3.20	0.81	1.84	296
	Hydrazine	0.74	1.06	1.87	301
	Hydrogen	3.40	0.26	2.43	386
	RP-1	2.24	1.01	1.77	300
	UDMH	1.39	0.96	1.84	295
F_2	Hydrazine	2.30	1.31	2.21	365
	Hydrogen	4.54	0.33	2.53	389
N_2O_4	Hydrazine	1.08	1.20	1.77	283
	RP-1	3.4	1.23		297
	MMH	1.65	1.16	1.59	278
H_2O_2	RP-1	7.0	1.29		297

Table 6: Performance parameters for different fuel and oxidizer combinations.

One new area of interest is **green hypergolic fuels** such as Dimethylthioformamide. These are safer and more environmentally friendly alternatives to traditional rocket fuels.

6.3. Overview of Monopropellant Thruster Systems

These systems generate thrust by flowing a propellant over/through a catalyst bed, where it endothermically decomposes, generating a hot, high velocity exhaust gas. The performance of such systems is shown in **Table 4** with a top level systems drawing of a thruster shown in **Figure 25**.

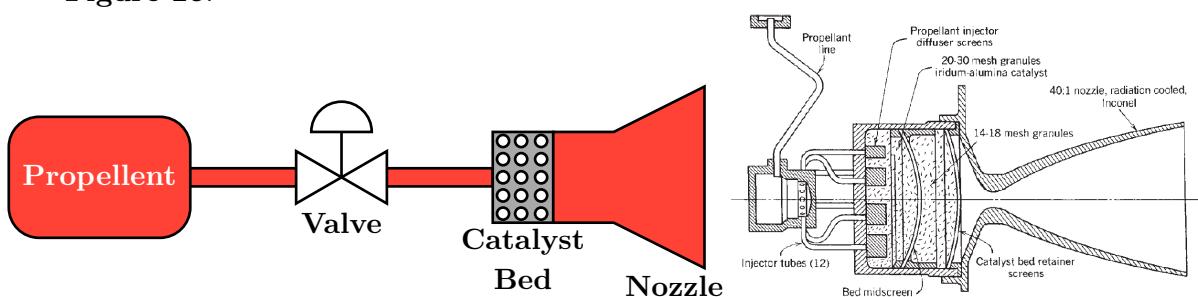


Figure 25: System engineering diagram of a monopropellant thruster system [Left], real schematic of a monopropellant thruster [Right]

6.3.1. Decomposition of Hydrazine

Hydrazine is the most common monopropellant fuel used in rockets. It decomposes over a catalyst bed following the reaction detailed in **Figure 26**.

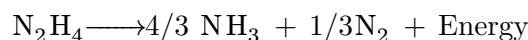


Figure 26: Decomposition reaction of hydrazine.

This exothermic decomposition reaction has a equilibrium temperature of 1650K and releases a large amount of energy ($\approx 111.7\text{ kJ/mol}$ of hydrazine). One issue is that at this temperature, the ammonia produced is unstable and itself decays via the reaction shown in **Figure 27**.

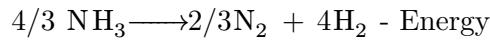


Figure 27: Thermal breakdown of ammonia.

This reaction is **slow and endothermic**, reducing the energy released in the decomposition reaction and subsequently reducing thrust. As this reaction is slow, the **dwell time** (the amount of time the fuel remains in the combustor), plays a large roll in the energy released. The full decomposition reaction for hydrazine can therefore be written as **Figure 28**.

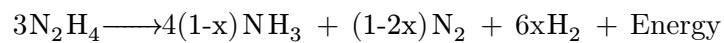


Figure 28: Full decomposition reaction for hydrazine.

Where x is the degree of ammonia dissociation. This parameter depends on the:

- Catalyst type
- Geometry
- Dwell time
- Size of chamber
- Chamber pressure

Note that the real schematic shown in the right of **Figure 25** has a very small combustor section, to ensure a low dwell time and thus increase thrust by reducing energy losses through the endothermic reaction of ammonia breakdown. Some key performance parameters for a hydrazine thruster are plotted in **Figure 29**.

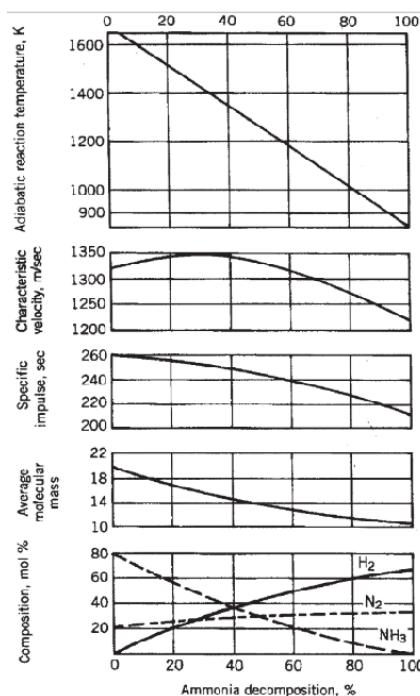


Figure 29: Performance parameters throughout a hydrazine decomposition reaction.

Note that the main parameter effecting the performance of the hydrazine thruster is the percentage of ammonia decomposition, meaning that dwell time is therefore one of the main contributing factors for engine performance.

6.3.2. Dangers of Hydrazine

Hydrazine is highly toxic, corrosive and flammable, meaning that working with this propellant is dangerous and costly. Some of the warning symbols related to hydrazine are shown in **Figure 30**.



Figure 30: Hazard symbols associated with hydrazine (1.flammable 2.corrosive 3.acutely toxic 4.serious health hazard 5.hazardous to the environment)

This provides motivation for the development of **green monopropellants** to decrease costs, allow for in-house refueling and ease of use.

6.3.3. Green Monopropellants: HAN

HAN or hydroxylammonium nitrate is an ionic liquid based compound which is solid at room temperature and often used in solid rocket motors. If mixed with water alcohol it becomes a monopropellant, the chemical formula as well as chemical structure are shown in **Figure 31**.

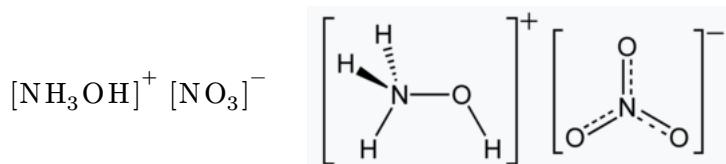


Figure 31: Chemical formula [Left] and chemical structure [right] of HAN.

6.3.4. Green Monopropellants: ADN

ADN or ammonium dinitramide is another ionic liquid based compound which is mixed with methanol, water and ammonia to yield a usable monopropellant. The chemical formula as well as the chemical structure are shown in **Figure 32**.

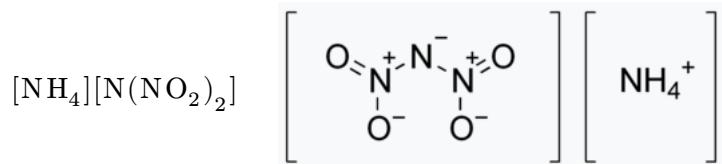


Figure 32: Chemical formula [Left] and chemical structure [right] of ADN.

6.3.5. Green Monopropellant: Hydrogen Peroxide

Hydrogen peroxide (H_2O_2) can also be used as a monopropellant where it decomposes over a catalyst bed into oxygen and water, however the efficiency of hydrogen peroxide as a monopropellant has a lower I_{sp} than the other monopropellant options.

6.3.6. Comparison of Green Monopropellants and Downsides

A table showing the performance of green monopropellents in comparison with traditional monopropellants (hydrazine) is shown in **Table 7**.

Name	Density g/m^3	Specific Impulse s	Temperature $^{\circ}C$	Density Specific Impulse $s \cdot g/m^3$
Hydrazine	1.01	230	1120	232
ADN-based LMP-103S	1.24	244	1600	302
HAN-based AF-M315E	1.46	248	1900	362
98% Hydrogen Peroxide	1.439	198	955	285

Table 7: Performance of hydrazine vs green monopropellants

Some of the main downsides or caveats of green monopropellants are:

- **High temperatures** which require more exotic materials or active cooling.
- **Pre-heating of thruster** up to high temperatures to reach equilibrium temperature of reaction.
- Exact compositions are **owned by companies**.

6.4. Propellant Feed Mechanisms

There exist two main propellant feed mechanisms, these are **pressure driven** and **pump driven**. A system level image for both methods is shown in **Figure 33**.

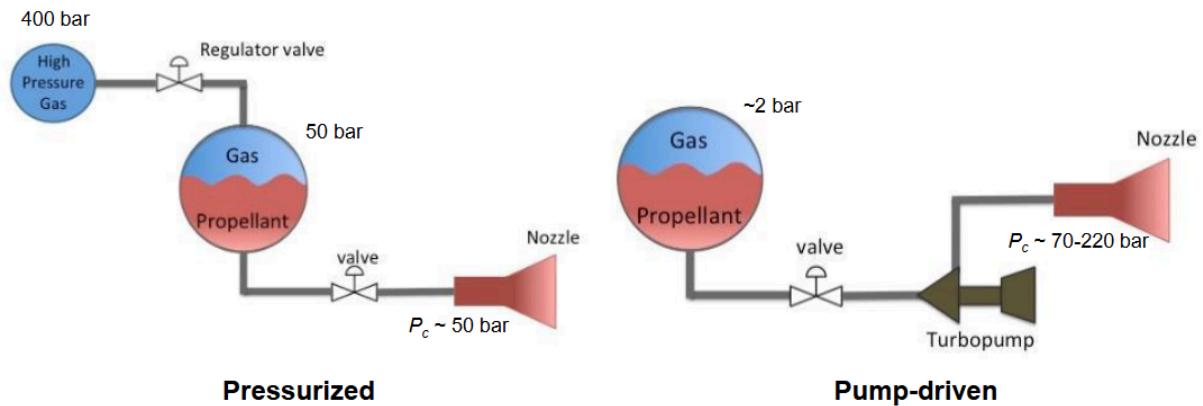


Figure 33: Pressure driven feed system [Left], pump driven feed system [Right]

For launch vehicles which require higher I_{sp} , higher thrust, higher pressures and higher flow rates a **pump driven** system is usually chosen. For spacecraft which require less weight, lower pressures and volume, a **pressure driven** system is chosen.

6.4.1. Pump Driven Propellant Systems: Turbopumps

Turbopumps pressurize the propellant before it enters the combustor, allowing for much higher pressures and flow rates. An example of a turbopump is shown in **Figure 34**.

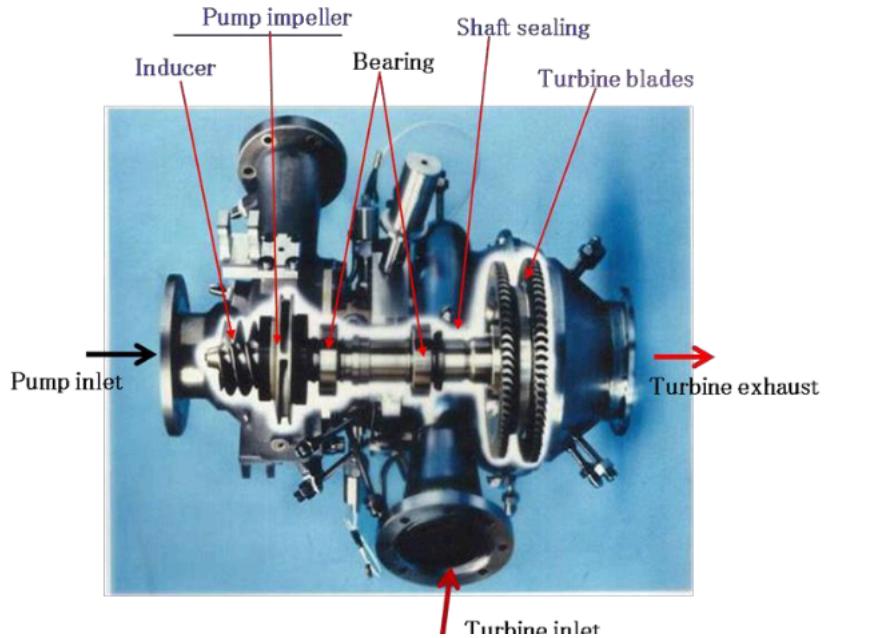


Figure 34: Spacecraft turbopump.

The working principle of a turbopump is that:

1. Propellant enters through the **propellant inlet**.
2. The **inducer** gradually increases the propellant pressure by a small bit, decreasing the chance of cavitation (like pre-stage fan in a jet engine).
3. **Impeller** increases the pressure and kinetic energy of the flow by rotating it axially out of the turbopump.
4. A **diffuser/volute** is placed at the exit of the impeller and converts any kinetic energy the flow has into static pressure.
5. This system is all driven by the **turbine** which is connected on the same shaft as the inducer and impeller. It takes hot energetic flow from the **combustor** and extracts work from it to drive the pump.

6.4.2. Pump Driven Propellant Systems: Open VS Closed Cycle

A pump driven propellant system can be categorized on whether the exhaust from the turbopumps is fed back into the main combustion chamber (**closed cycle**) or if it is dumped externally (**open cycle**). System level diagrams of these are shown in **Figure 35**.

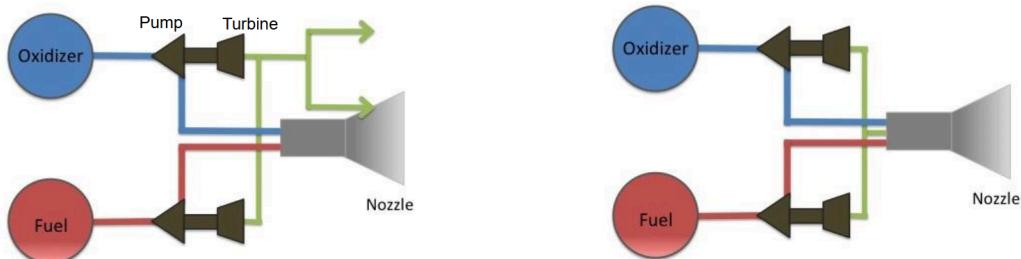


Figure 35: Open cycle propellant feed system [Left], closed cycle propellant feed system [Right].

While open cycle systems are simpler as the turbine's of the turbopump are operated at a lower relative pressure, the closed cycle systems offer higher efficiency (I_{sp}) at the cost of added complexity due to a much higher turbine pressure (turbine pressure must be close to or match combustion pressure.)

6.4.3. Open Cycle Pump Driven Propellant Systems: Gas Generator

In this system, a small amount of fuel and oxidizer are burnt in a **gas generator** (burnt **fuel rich**, keeping temperature low). The exhaust of the gas generator then powers the turbopumps. These systems are **simpler** and have a **lower mass** but have a **lower efficiency** (due to wasting fuel) than closed cycle systems by a few percent. An image of such a systems is shown in **Figure 36**.

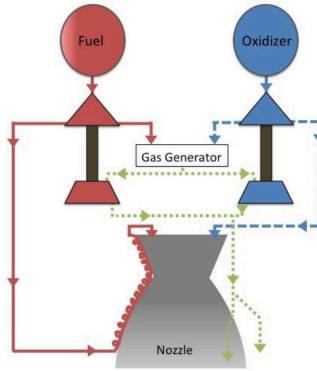


Figure 36: A system level image of a open cycle, gas generator, pump driven propulsion system.

6.4.4. Closed Cycle Pump Driven Propellant Systems: Staged Combustion

Similar to the gas generator system, some fuel and oxidizer are burnt in a **pre-combustor** which drives the turbines. The difference here, is that the exhaust is then piped into the main combustor. This system allows for **higher efficiencies** at the cost of **higher pre-combustor temperatures and pressures**. Note that the pre-combustor is less fuel rich than before, allowing for higher pressures. The pressure in the pre-combustor **must be higher** than the main combustor to avoid backflow, increasing complexity. An image of such a systems is shown in **Figure 37**.

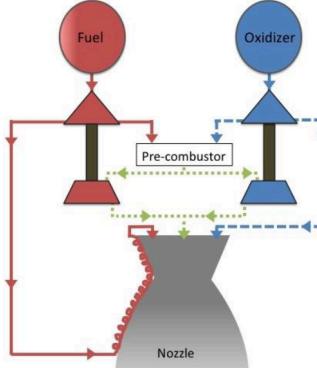


Figure 37: A system level image of a closed cycle, staged combustion, pump driven propulsion system.

6.4.5. Closed Cycle Pump Driven Propellant Systems: Expander Cycle

In this cycle, the turbines are driven by the fuel which is used to cool the nozzle. During the cooling process, the fuel is heated into a gas which is used to drive the turbines. Note that for this system, an initial starter or ignition is required in order to start the system. An image of such a systems is shown in **Figure 38**.

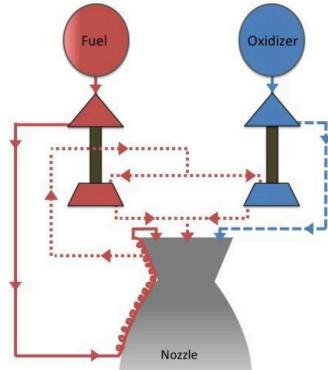


Figure 38: A system level image of a closed cycle, expander cycle, pump driven propulsion system.

6.4.6. Pump Driven Propellant Systems: Comparing cycles

The chart compares six rocket engines: Merlin, RD-180, F-1, Raptor, BE-4, and RS-25. The engines are shown in a row above the table. The table has a dark header and light gray rows for data.

	Merlin	RD-180	F-1	Raptor	BE-4	RS-25
Cycle	Open	Closed (LOX rich)	Open	Closed (Full Flow)	Closed (LOX rich)	Closed (Fuel Rich)
Fuel Type	RP-1	RP-1	RP-1	Methane	Methane	Hydrogen
Total Thrust	0.84 MN	3.83 MN	6.77 MN	2.00 MN	~2.40 MN	1.86 MN
Thrust : Weight	198 : 1	78 : 1	94 : 1	107 : 1	~80 : 1	73 : 1
Specific Impulse (ISP)	282 sl 311 vac	311 sl 338 vac	263 sl 304 vac	330 sl ~350 vac	~310 sl ~340 vac	366 sl 452 vac
Chamber Pressure	97 bar	257 bar	70 bar	270 bar	~135 bar	206 bar

Figure 39: Comparison of different pump driven rocket engines.