SESA 6071

Spacecraft Propulsion

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Definitions

$\boldsymbol{I_t}$	Total Impulse (Ns)	$\boldsymbol{I_{sp}}$	Specific Impulse (s)
$\boldsymbol{F}$	Rocket Thrust (N)	$\boldsymbol{g_0}$	Standard Gravitational Accel (m/s^2)
$\boldsymbol{\dot{m}}$	Propellent mass flow rate (kg/s)	$\boldsymbol{m_p}$	Expelled propellent mass (kg)
$\boldsymbol{c}$	Effective exhaust velocity (m/s)	$\boldsymbol{\eta_T}$	Power Conversion Efficiency
$\boldsymbol{P_{in}}$	Input Power (W)	$\boldsymbol{m}$	Spacecraft or launch vehicle mass (kg)
$\boldsymbol{\alpha}$	Specific power plant mass (kg/W)	$\boldsymbol{M_{pow}}$	Power plant mass (kg)
$\boldsymbol{v_e}$	Exhaust velocity (m/s)	$\boldsymbol{P_e}$	Exhaust pressure (Pa)
$\boldsymbol{P_a}$	Atmospheric pressure (Pa)	$\boldsymbol{A_e}$	Exhaust area (m^2)
$\boldsymbol{c^*}$	Characteristic velocity (m/s)	$\boldsymbol{P_c}$	Chamber pressure (Pa)
$\boldsymbol{A_t}$	Throat area (m^2)		

1. Lecture 1

1.1. What is Rocket Propulsion

Propulsion itself is the **act of changing the motion of a body**, typically by using newtons third law and it can be classified in various types of ways. A more colloquial way of defining rocket propulsion is as **mass drivers**, throwing out mass one way to yield an acceleration in the other.

1.2. Rocket Propulsion Family Tree

In **Figure 1** the rocket propulsion types are grouped by the energy source.

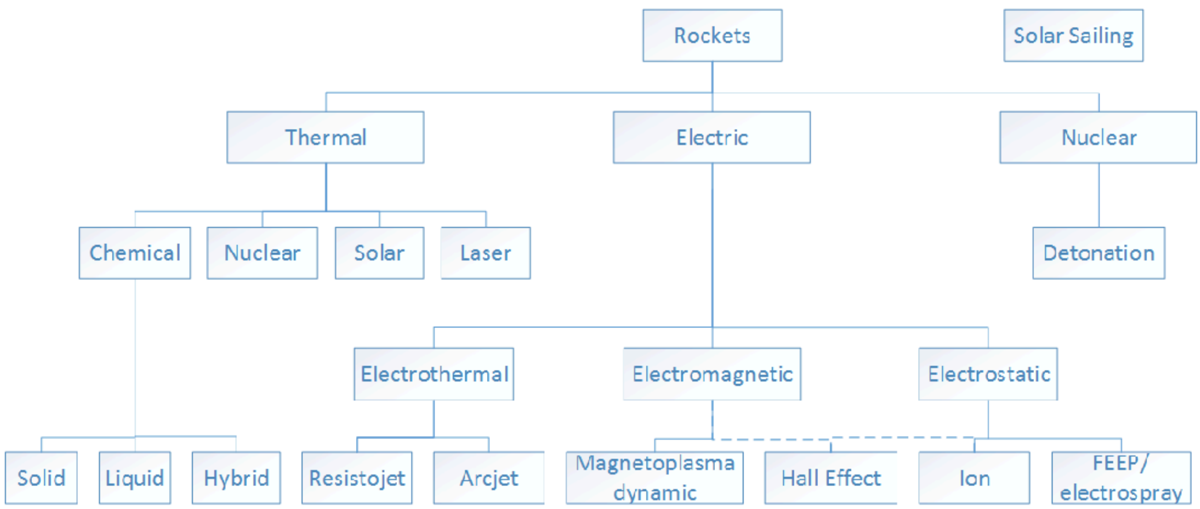

Figure 1: Flowchart of the rocket propulsion family tree

1.2.1. Chemical Rockets

These utilize either a chemical reaction or decomposition to generate energy. Gas is heated to between **700°C - 1300°C** and to speeds between **1.5 km/s - 4.5 km/s**. These require a **fuel and oxidizer** and come in the following types:

- **Solid:** Fuel and oxidizer mixed within into a solid grain which cannot stop burning once ignited. feature **high thrust with low performance**.
- **Liquid:** Burn a liquid fuel and oxidizer allowing for repeated firings and variable thrust. Feature **high performance and thrust with high complexity**.
- **Hybrid:** Have a liquid oxidizer but a solid fuel allowing for better performance than solid with lower complexity.

1.2.2. Electric Rockets

These use electrical energy to generate thrust without utilizing combustion. Typically have very high exhaust velocities ($\sim$ **$60,000$ m/s**) and therefore **very high performance** at the costs of **high complexities and very low thrust**. The four distinct groups are:

- **Electrothermal:** Uses electrical energy to heat a propellent (Resistojet). Are **simple to build** at the cost of **low thrust**.
- **Electrostatic:** Uses electrical energy to accelerate ionized fuel across an electric fields. Feature **good performance** at the cost of **being expensive and low thrust**.
- **Electromagnetic:** Accelerates an ionized fuel using a magnetic field. Fall issue to **low efficiency unless power input is high**.
- **Hall Effect Thruster:** Uses a mixture of both electrostatic and electromagnetic propulsion methods to accelerate propellent. These are the most **commonly used**.

1.2.3. Nuclear Rockets

Broadly speaking there are two types of nuclear rockets, these are:

- **Nuclear Detonation:** Use the shockwave produced when nuclear bombs are detonated to produce thrust (Orion Drive). **High performance and thrust** but are **very dangerous and have limited testing**.
- **Nuclear Thermal:** Uses the heat energy produced during nuclear fission to heat a propellent (typically hydrogen) which is then exhausted. These have **high performance and thrust** but are **dangerous and have limited testing**.

1.2.4. Solar and Laser Rockets

These systems use large diameter telescopes to focus in a laser or solar radiation to heat up a propellent. These systems feature **high theoretical performance and moderate thrust** but are **very complex and lack any real testing**.

1.2.5. Solar Sails

These systems use no propellent at all and instead produce thrust through the momentum gained when a photon is incident on the sail. These systems feature **good performance with no fuel** but fall victim to **low thrust and engineering complexity**.

1.3. Rocket Propulsion Applications

Instead of grouping together rocket propulsion methods using the energy source, the rocket application can also be used, for example:

- **High Thrust/Maneuverability:** Typically have the cost of **low performance** and use **chemical or solid** propulsion methods.
- **High Performance:** Typically have the cost of **low thrust** and use **electrical** propulsion methods.
- **Balanced Thrust and Performance:** Typically the middle ground is **nuclear thermal**.

2. Lecture 2

2.1. Definitions and Fundamentals

To develop a empirical measure of performance we should first consider **Eq. 1**.

$$I_t = \int_0^t F \ \ dt \tag{1}$$

Where:
- $\boldsymbol{I_t}$: Total Impulse (Ns)
- $\boldsymbol{F}$: Thrust Force (N)
- $\boldsymbol{t}$: Burn Duration (s)

Note that for **Eq. 1**, if F is constant then the equation simplified to $I_t = Ft$. A more useful measure of performance for rocket engines is shown in **Eq. 2**.

$$I_{sp} = \frac{\int_0^t F \ \ dt}{g_0 \int_0^t \dot{m} \ \ dt} = \frac{I_t}{g_0 \int_0^t \dot{m} \ \ dt} \tag{2}$$

Where:
- $\boldsymbol{I_{sp}}$: Specific Impulse (s)
- $\boldsymbol{g_0}$: Standard Gravitational Accel (m/s^2) = 9.81 m/s^2
- $\boldsymbol{\dot{m}}$: Propellent mass flow rate (kg/s)

There is no concrete reason on why g_0 is present in this equation, however one common theory is that it allows $I_{sp}$to be in seconds instead of featuring a length unit which would eliminate any error in conversion from metric to imperial. If F and $\dot{m}$ are both constant over the t then **Eq. 2** simplifies to **Eq. 3**.

$$I_{sp} = \frac{I_t}{g_0 m_p} \tag{3}$$

Where:
- $\boldsymbol{m_p}$: Expelled propellent mass (kg) = $\dot{m}t$

Another useful parameter for defining engine performance is shown in **Eq. 4**.

$$c = \frac{F}{\dot{m}} \tag{4}$$

Where:
- $\boldsymbol{c}$: Effective exhaust velocity (m/s)

The exhaust velocity is called as such as the **velocity profile of the exhaust is not uniform**, this is most seen in chemical rockets due to the **no slip condition** but is slightly seen in electrical rockets too. Rearranging all of the previous equations together yields a definition for I_{sp} in terms of c.

$$I_{sp} = \frac{c}{g_0} \tag{5}$$

Typical I_{sp} values for the rocket engine types defined in the previous lecture are shown in **Table 1**.

Rocket Engine Type	$I_{sp}(s)$	**Thrust (N)**	**Efficiency**	**Propellent**
Chemical bi-propellent	200 - 450	$\leq 10MN$	0.8	Liquid or Solid Propellents
Chemical mono-propellent	150 - 250	0.03 - 100	0.9	N_2H_4
Thermal Nuclear Fission	500 - 860	$\leq 10MN$	0.5	H_2
Resistojet - electrothermal	150 - 350	0.01 - 10	0.4	N_2H_4 , NH_3 , H_2
Ion Thruster - electrostatic	1500-8000	$10^{-5} - 0.5$	0.65	Xe
Hall Effect Thruster	1500-2000	$10^{-5} - 2$	0.55	Xe

Table 1: Typical values of I_{sp}

2.2. Maximum Chemical Performance

A typical chemical reaction used in chemical rockets is combustion shown in **Eq. 6**.

$$H_2 + 1/2\ O_2 \longrightarrow H_2O + \text{Energy} \tag{6}$$

Combustion as shown in **Eq. 6** nis an exothermic reaction as the energy of the reactants is more than the energy of the products, allowing for an excess of energy after the reaction. To estimate an effective upper limit to the energy released during combustion, the bond energies shown in **Table 2** can be used.

Chemical	**Bond Energy (kJ/mol)**
H_2	436
O_2	498
H_2O	428
	498.7

Table 2: Respective bond energies of reactants and products in combustion.

Note that there are two bond energies in **Table 2** due to the OH and the OH - H bonds. The maximum energy can be calculated and are shown in **Figure 2**.

Energy Per Kilogram Released During Combustion

$$M_{Water} := 18.01528\ \frac{gm}{mol} \qquad BE_{H2} := 436\ \frac{kJ}{mol} \qquad BE_{O2} := 498\ \frac{kJ}{mol}$$

$$BE_{OH} := 428\ \frac{kJ}{mol} \qquad BE_{OHH} := 498.7\ \frac{kJ}{mol}$$

$$E_{kg} := \frac{\left(BE_{OH} + BE_{OHH}\right) - \left(BE_{H2} + \frac{BE_{O2}}{2}\right)}{M_{Water}} = 13416388.75\ \frac{J}{kg}$$

Maximum Chemical Performance

$$\eta := 0.9 \qquad g_0 := 9.81\ \frac{m}{s^2}$$

$$c := \sqrt[2]{2\ \eta \cdot E_{kg}} = 4914.21\ \frac{m}{s} \qquad I_{sp} := \frac{c}{g_0} = 500.94\ s$$

Figure 2: Calculations for maximum chemical rocket engine performance

Note that in this calculation, the bond energy of oxygen is halved as per **Eq. 6** and the equation for effective exhaust velocity comes from the kinetic energy equation and noting that $E_{kg} = Energy/mass$.

2.3. Comparative Electric Performance

To compare the efficiency of chemical propulsion to electric propulsion consider an electrostatic propulsion system shown in **Figure 3**.

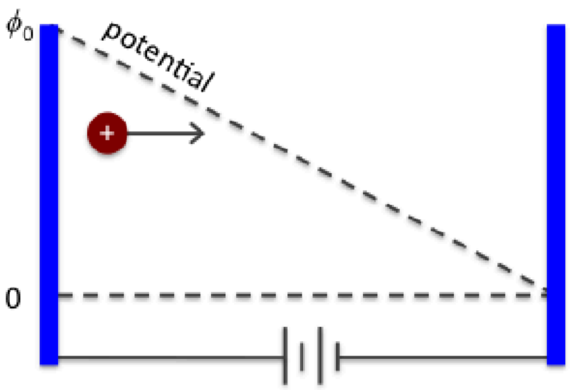

Figure 3: Basic principle of an electrostatic propulsion system.

A charged ion (assumed for these calculations to be a water ion) enters an electric field which causes it be accelerated to the more negative (lower potential plate). By setting the electric potential energy gained by the ion equal to the kinetic energy ($\eta E_p = E_k$) then the I_{sp} can be calculated, shown in **Figure 4**.

Voltage Required for an Isp of 500s Using Ionized Water

$M_{Water} := 18.01528\ \frac{gm}{mol} \qquad \eta := 0.9 \qquad g_0 := 9.81\ \frac{m}{s^2} \qquad N_a := 6.023 \cdot 10^{23}\ mol^{-1}$

$I_{sp} := 500\ s \qquad q := 1.6 \cdot 10^{-19}\ C$

$$V := \frac{\frac{1}{2}\frac{M_{Water}}{N_a}\left(I_{sp} \cdot g_0\right)^2}{\eta \cdot q} = 2.5\ V$$

Figure 4: Comparative electrical propulsion system voltage calculations.

As shown in **Figure 4** the voltage required to match the performance of a chemical system is very low and easily achievable, in reality electrostatic systems can achieve efficiencies in excess of 10,000s.

2.4. Nuclear Performance

To estimate the performance of a thermal nuclear rocket engine, Uranium-235 fission is considered, where the energy released in one fission event is immediately transferred to a water molecule, this calculation is shown in **Figure 5**.

Energy Transfered to One Water Molecule During One Nuclear Fission Event

$E_{U235} := 180\ MeV = \left(2.88 \cdot 10^{-11}\right)\ J \qquad M_{Water} := 18.01528\ \frac{gm}{mol} \qquad N_a := 6.023 \cdot 10^{23}\ mol^{-1}$

$m_{Water} := \frac{M_{Water}}{N_a} = 0\ kg$ <- is 2.99x10^-26 but is too small for mathcad to show

$E_{kg} := \frac{E_{U235}}{m_{Water}} = \left(9.63 \cdot 10^{14}\right)\ \frac{J}{kg}$

Performance of a Nuclear Thermal System

$\eta := 0.9 \qquad g_0 := 9.81\ \frac{m}{s^2}$

$c := \sqrt[2]{2\ \eta \cdot E_{kg}} = 41631151.15\ \frac{m}{s} \qquad I_{sp} := \frac{c}{g_0} = \left(4.24 \cdot 10^{6}\right)\ s$

Figure 5: Maximum nuclear thermal propulsive system performance.

Note that this I_{sp} is a theoretical upper limit and in reality the true performance is much lower and is limited by material limits due to heat.

2.5. Definitions and Fundamentals Cont.

For propulsion systems, efficiency can be defined in terms of the fraction of source power that is converted to jet power, this efficiency is shown in **Eq. 7**.

$$\eta_T = \frac{\dot{m}c^2}{2P_{in}} \tag{7.1}$$

$$P_{in} = \frac{\dot{m}c^2}{2\eta_T} = \frac{Fc}{2\eta_T} \tag{7.2}$$

$$\frac{P_{in}}{m} = \frac{F}{m}\frac{c}{2\eta_T} = a\frac{c}{2\eta_T} \tag{7.3}$$

Where:

- $\boldsymbol{\eta_T}$: Power conversion efficiency
- $\boldsymbol{a}$: Acceleration (m/s^2)
- $\boldsymbol{P_{in}}$: Input or Source power (W)
- $\boldsymbol{m}$: Spacecraft mass (kg)

Note that for electrical systems P_{in}, the power must come from a source e.g., solar panel array. **Eq. 7** Also shows that **for a fixed specific power:** $(\frac{P_{in}}{m})$ **a high effective exhaust speed** (c) **means a low acceleration.** It is also useful to define a specific power plant mass as shown in **Eq. 8**.

$$\alpha = \frac{M_{pow}}{P_{in}} \tag{8}$$

Where:

- $\boldsymbol{\alpha}$: Specific power plant mass (kg/W)
- $\boldsymbol{M_{pow}}$: Power plant mass (kg)

By manipulating equations **Eq. 8** and **Eq. 7**, as well as assuming that $\eta_T \approx 1$ and $M_{pow} \approx 0.1m$ then the acceleration can be written as **Eq. 9**.

$$a = \frac{0.2}{\alpha c} \quad \begin{cases} M_{pow} \approx 0.1m \\ \eta_T \approx 1 \end{cases} \tag{9}$$

Eq. 9 shows that a and c are inversely proportional from one another, meaning a high acceleration will typically mean a low effective exhaust velocity and vice versa. A showing how performance varies with acceleration is shown in **Figure 6**.

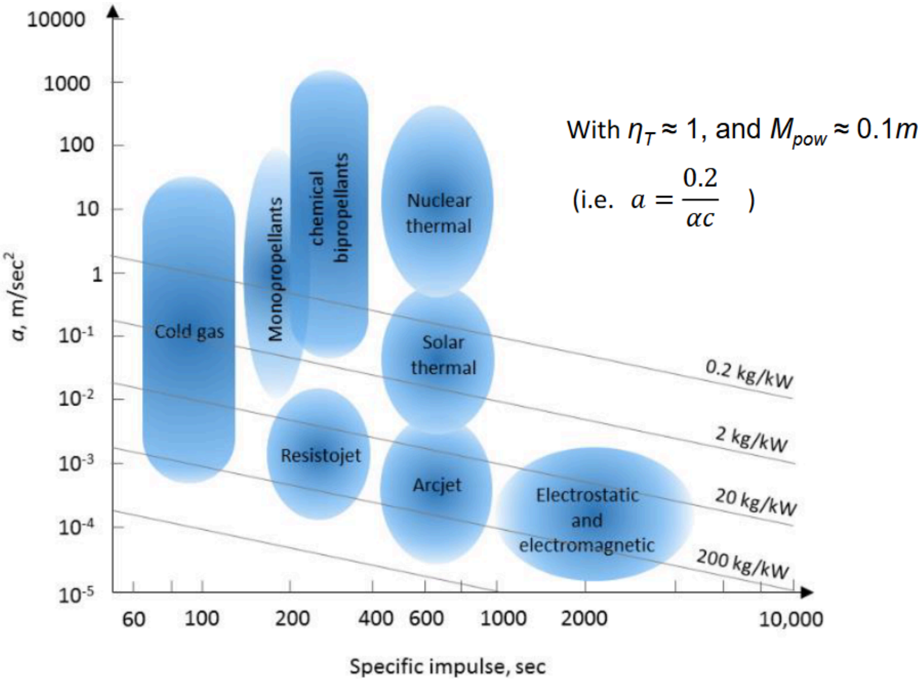

Figure 6: Variation of spacecraft acceleration against performance.

Note that for electrical propulsion systems shown in **Figure 6** a higher I_{sp} means a lower acceleration as $I_{sp} \propto c \propto \frac{1}{a}$. Different power sources have different values of α, for example:

- Nuclear Reactors $\Rightarrow 2kg/kW$
- Solar Panels $\Rightarrow 20kg/kW$
- RTGs $\Rightarrow 200kg/kW$

2.6. Thrust Fundamentals

By apply Newton's second law to a rocket nozzle, considering the difference the atmospheric and exhaust pressure as well as using the equations derived in the previous sections, **Eq. 10** can be derived.

$$F = \dot{m}v_e + (P_e - P_a)A_e \tag{10.1}$$

$$c = v_e + \frac{(P_e - P_a)A_e}{\dot{m}} \tag{10.2}$$

$$I_{sp} = \frac{1}{g_0}\left(v_e + \frac{(P_e - P_a)A_e}{\dot{m}}\right) \tag{10.3}$$

Where:

- $\boldsymbol{v_e}$: Exhaust velocity (m/s)
- $\boldsymbol{A_e}$: Exhaust Area (m^2)
- $\boldsymbol{P_e}$: Exhaust Pressure (Pa)
- $\boldsymbol{P_a}$: Atmospheric Pressure (Pa)

One key thing to note about **Eq. 10** is that the thrust is made up of two parts, the first part being the **momentum thrust** accounting for the majority of the thrust (90-70%) and the second part is the **pressure thrust** (10-30%).

Crucially, as $P_a(h)$ then the I_{sp} and c vary with the height, typically being lower at lower altitudes and increasing up an reaching their maximums in the thinner sections of the atmosphere.

Another impartial performance parameter for chemical rockets which does not depend on the altitude is shown in **Eq. 11**.

$$c^* = \frac{P_c A_t}{\dot{m}} \tag{11}$$

Where:

- $\boldsymbol{c^*}$: Characteristic velocity (m/s)
- $\boldsymbol{P_c}$: Chamber pressure (Pa)
- $\boldsymbol{A_t}$: Throat area (m^2)

Typical values of c^* are 1500 m/s for a solid rocket and 2500 for H_2/O_2 liquid bi-propelled rocket.