

SESA6085

Advanced Aerospace Engineering Management

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Definitions

$P(A)$	Probability of a general event A occurring.	N	Total number of equally likely possible outcomes in the sample space.
n	Number of favorable outcomes (ways in which event A occurs)	$P(AB)$	Probability of events A and B occurring.
$P(A + B)$	Probability of events A or B occurring.	$P(A B)$	Probability of event A given event B has already occurred
$P(A)$	Probability of event A given event B has already occurred	$P(\overline{A})$	Probability of event A not occurring.

1. Lecture 1

1.1. Probability Fundamentals, Rules and Notation

The most basic definition of the probability for a general event A occurring is **the ratio of the number of favorable outcomes n to the total number of equally likely possible outcomes N** , this is shown in a mathematical representation in **Eq. 1**.

$$P(A) = \frac{n}{N} \quad (1)$$

Where:

- $P(A)$: The probability of outcome A .
- N : Total number of equally likely possible outcomes in the sample space.
- n : Number of favorable outcomes (ways in which event A occurs)

Note that **Eq. 1** is only for events of equal probability, for example rolling a dice. Instead if N is **the number of experiments** then **Eq. 2** applies, implying that the larger the number of experiments the closer to **Eq. 1** the probability becomes.

$$P(A) = \lim_{N \rightarrow \infty} \left(\frac{n}{N} \right) \quad (2)$$

This module uses the following notation for the probability of combined events, these are:

- $P(A)$: Probability of event A occurring.
- $P(AB)$: Probability of events A and B occurring.
- $P(A + B)$: Probability of events A or B occurring.
- $P(A|B)$: Probability of event A given event B has already occurred.
- $P(\overline{A})$: Probability of event A not occurring (note that $P(A) = 1 - P(\overline{A})$).

1.2. Statical Independence

If two events are **statistically independent** (s-independent) from one another (meaning that the probability of one event occurring is completely separate from another event happening or not happening), then **Eq. 3** is true.

$$\left. \begin{aligned} P(A|B) &= P(A|\overline{B}) = P(A) \\ P(B|A) &= P(B|\overline{A}) = P(B) \end{aligned} \right\} \text{s-independent} \quad (3)$$

Furthermore, the joint probability of two s-independent events can be represented in the forms shown in **Eq. 4** with the further expressions derived from subbing in **Eq. 3**, **Eq. 4** is also known as the **product or series rule**.

$$P(AB) = P(A)P(B) \text{ } \} \text{s-independent} \quad (4)$$

1.3. Statistical Dependence

If two events are instead **statistically dependent** (s-dependent) from one another (the probability of on event happening or not happening **does** have an effect of the probability of another event), then the adjoint probability of these two events is shown in **Eq. 5**

$$\left. \begin{aligned} P(AB) &= P(A)P(B|A) \\ P(AB) &= P(A|B)P(B) \end{aligned} \right\} \text{s-dependent} \quad (5.1)$$

$$P(B|A) = \frac{P(AB)}{P(A)} \left\} \text{s-dependent and } P(A) \neq 0 \quad (5.2)$$

1.4. Probability Fundamentals, Rules and Notation Cont.

Generally speaking the probability of one event **or** another event occurring, whether they are s-dependant or s-independent is given by equation **Eq. 6**.

$$P(A + B) = P(A) + P(B) - P(AB) \quad (6.1)$$

$$P(A + B) = P(A) + P(B) - P(A)P(B) \left\} \text{s-independent} \quad (6.2)$$

Note that the $P(AB)$ in **Eq. 6** must be subtracted as it is counted twice in the first two terms.

1.5. Mutual Exclusivity

Two events can be said to be mutually exclusive if they **cannot occur at the same time as one another**. This means that the adjoint probability and or probability can be written in the form shown in **Eq. 7**.

$$\left. \begin{aligned} P(AB) &= 0 \\ P(A + B) &= P(A) + P(B) \end{aligned} \right\} \text{If A and B are Mutually Exclusive} \quad (7)$$

If instead there are **multiple mutually exclusive events** which together yield the probability of another event, then the probability of that event can be written in the form given by **Eq. 8**.

$$P(A) = \sum_i P(AB_i) = \sum_i P(A|B_i)P(B_i) \left\} \text{If A and all Bs are Mutually Exclusive} \quad (8)$$

1.6. Sequence Diagrams

Sequence diagrams act as an easy way of visualizing complex interactions and can be used to calculate overall probabilities, an example of a sequence diagram is shown in **Figure 1**.

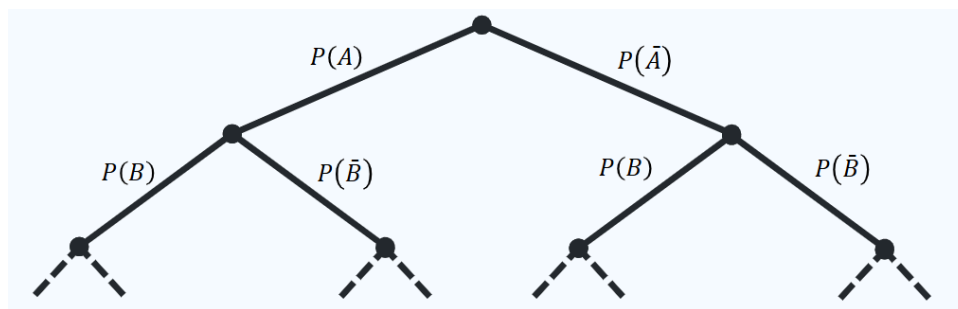


Figure 1: Example of a sequence diagram.

Probabilities down a leg are **and** probabilities and are therefore multiplied given that they are **statistically independent**. **Or** probabilities can be calculated by adding together subsequent probabilities.

1.7. Baye's Theorem

By rearranging **Eq. 5** a simple form of **Baye's theorem** which is shown in **Eq. 9**.

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} \quad \text{Given } P(B) \neq 0 \quad (9)$$

Eq. 9 can be further developed by substituting in **Eq. 8** which yields the **generalized Baye's theorem** shown **Eq. 10**.

$$P(A_j | B) = \frac{P(B|A_j)P(A_j)}{\sum_j P(B|A_j)P(A_j)} \quad (10)$$

Note that in **Eq. 10** A_j is the j th event effecting the event B . If the probability of event B depends on the probability of event A both happening and not happening then **Eq. 10** simplifies down to a form called the **binary partition** form, shown in **Eq. 11**.

$$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|\bar{A})P(\bar{A})} \quad (11)$$