SESA6085

Advanced Aerospace Engineering Management

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Contents

List of Figures

List of Tables

Definitions

$\boldsymbol{P(A)}$	Probability of a general event A occurring.	$\boldsymbol{N}$	Total number of equally likely possible outcomes in the sample space.
$\boldsymbol{n}$	Number of favorable outcomes (ways in which event A occurs)	$\boldsymbol{P(AB)}$	Probability of events $\boldsymbol{A}$ **and** $\boldsymbol{B}$ occurring.
$\boldsymbol{P(A+B)}$	Probability of events $\boldsymbol{A}$ **or** $\boldsymbol{B}$ occurring.	$\boldsymbol{P(A\vert B)}$	Probability of event $\boldsymbol{A}$ given event $\boldsymbol{B}$ has already occurred
$\boldsymbol{P(A)}$	Probability of event $\boldsymbol{A}$ given event $\boldsymbol{B}$ has already occurred	$\boldsymbol{P(\overline{A})}$	Probability of event $\boldsymbol{A}$ not occurring.
s-independent	statistically independent events	**s-dependent**	statistically dependent events
$\boldsymbol{f(t)}$	Probability Distribution Function (PDF)	$\boldsymbol{F(t)}$	Cumulative Distribution Function (CDF)
$\boldsymbol{R(t)}$	Reliability Function	$\boldsymbol{h(t)}$	Hazard Function

1. Lecture 1

1.1. Probability Fundamentals, Rules and Notation

The most basic definition of the probability for a general event A occurring **is the ratio of the number of favorable outcomes n to the total number of equally likely possible outcomes N**, this is shown in a mathematical representation in **Eq. 1**.

$$P(A) = \frac{n}{N} \tag{1}$$

Where:
- $\boldsymbol{P(A)}$: The probability of outcome A.
- $\boldsymbol{N}$: Total number of equally likely possible outcomes in the sample space.
- $\boldsymbol{n}$: Number of favorable outcomes (ways in which event A occurs)

Note that **Eq. 1** is only for events of equal probability, for example rolling a dice. Instead if **N is the number of experiments** then **Eq. 2** applies, implying that the larger the number of experiments the closer to **Eq. 1** the probability becomes.

$$P(A) = \lim_{N \to \infty} \left(\frac{n}{N}\right) \tag{2}$$

This module uses the following notation for the probability of combined events, these are:

- $\boldsymbol{P(A)}$: Probability of event $\boldsymbol{A}$ occurring.
- $\boldsymbol{P(AB)}$: Probability of events $\boldsymbol{A}$ **and** $\boldsymbol{B}$ occurring.
- $\boldsymbol{P(A+B)}$: Probability of events $\boldsymbol{A}$ **or** $\boldsymbol{B}$ occurring.
- $\boldsymbol{P(A|B)}$: Probability of event $\boldsymbol{A}$ given event $\boldsymbol{B}$ has already occurred.
- $\boldsymbol{P\left(\overline{A}\right)}$: Probability of event $\boldsymbol{A}$ **not** occurring (note that $P(A) = 1 - P\left(\overline{A}\right)$).

1.2. Statical Independence

If two events are **statistically independent** (s-independent) from one another (meaning that the probability of one event occurring is completely separate from another event happening or not happening), then **Eq. 3** is true.

$$\left.\begin{aligned} P(A|B) = P\left(A|\overline{B}\right) = P(A) \\ P(B|A) = P\left(B|\overline{A}\right) = P(B) \end{aligned}\right\} \text{s-independent} \tag{3}$$

Furthermore, the joint probability of two s-independent events can be represented in the forms shown in **Eq. 4** with the further expressions derived from subbing in **Eq. 3**, **Eq. 4** is also known as the **product or series rule**.

$$P(AB) = P(A)P(B)\} \text{ s-independent} \tag{4}$$

1.3. Statistical Dependence

If two events are instead **statistically dependent** (s-dependent) from one another (the probability of on event happening or not happening **does** have an effect of the probability of another event), then the adjoint probability of these two events is shown in **Eq. 5**

$$\left.\begin{array}{l} P(AB) = P(A)P(B|A) \\ P(AB) = P(A|B)P(B) \end{array}\right\} \text{s-dependent} \tag{5.1}$$

$$P(B|A) = \left.\frac{P(AB)}{P(A)}\right\} \text{s-dependent and } P(A) \neq 0 \tag{5.2}$$

1.4. Probability Fundamentals, Rules and Notation Cont.

Generally speaking the probability of one event **or** another event occurring , whether they are s-dependant or s-independent is given by equation **Eq. 6**.

$$P(A+B) = P(A) + P(B) - P(AB) \tag{6.1}$$

$$P(A+B) = P(A) + P(B) - P(A)P(B)\} \text{ s-independent} \tag{6.2}$$

Note that the $P(AB)$ in **Eq. 6** must be subtracted as it is counted twice in the first two terms.

1.5. Mutual Exclusivity

Two events can be said top be mutually exclusive if they **cannot occur at the same time as one another**. This means that the adjoint probability and or probability can be written in the form shown in **Eq. 7**.

$$\left.\begin{array}{l} P(AB) = 0 \\ P(A+B) = P(A) + P(B) \end{array}\right\} \text{If A and B are Mutually Exclusive} \tag{7}$$

If instead there are **multiple mutually exclusive events** which together yield the probability of another event, then then probability of that event can be written in the form given by **Eq. 8**.

$$P(A) = \textstyle\sum_i P(AB_i) = \sum_i P(A|B_i)P(B_i)\}\text{If A and all Bs are Mutually Exclusive} \tag{8}$$

1.6. Sequence Diagrams

Sequence diagrams act as an easy way of visualizing complex interactions and can be used to calculate overall probabilities, an example of a sequence diagram is shown in **Figure 1**.

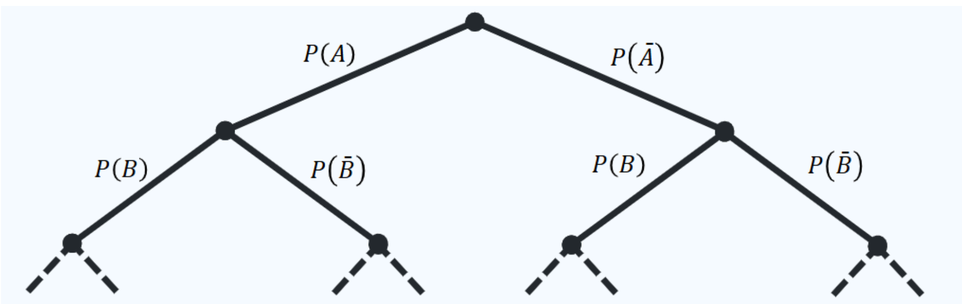

Figure 1: Example of a sequence diagram.

Probabilities down a leg are **and** probabilities and are therefore multiplied given that they are **statistically independent**. **Or** probabilities can be calculated by adding together subsequent probabilities.

1.7. Baye's Theorem

By rearranging **Eq. 5** a simple form of **Baye's theorem** which is shown in **Eq. 9**.

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} \qquad \text{Given } P(B) \neq 0 \tag{9}$$

Eq. 9 can be further developed by substituting in **Eq. 8** which yields the **generalized Baye's theorem** shown **Eq. 10**.

$$P\left(A_j \mid B\right) = \frac{P(B|A_i)P(A_i)}{\sum_j P\left(B|A_j\right)P\left(A_j\right)} \tag{10}$$

Note that in **Eq. 10** A_j is the jth event effecting the event B. If the probability of event B depends on the probability of event A both happening and not happening then **Eq. 10** simplifies down to a form called the **binary partition** form, shown in **Eq. 11**.

$$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P\left(B|\overline{A}\right)P\left(\overline{A}\right)} \tag{11}$$

2. Lecture 2

2.1. Frequency Histograms

A **frequency histogram** is a type of bar chart which is used to represent the distribution of data, on the **x axis** are bins of data and the **y axis** represents the frequency that occurs within that bin, an example of a frequency histogram is shown in **Figure 2**.

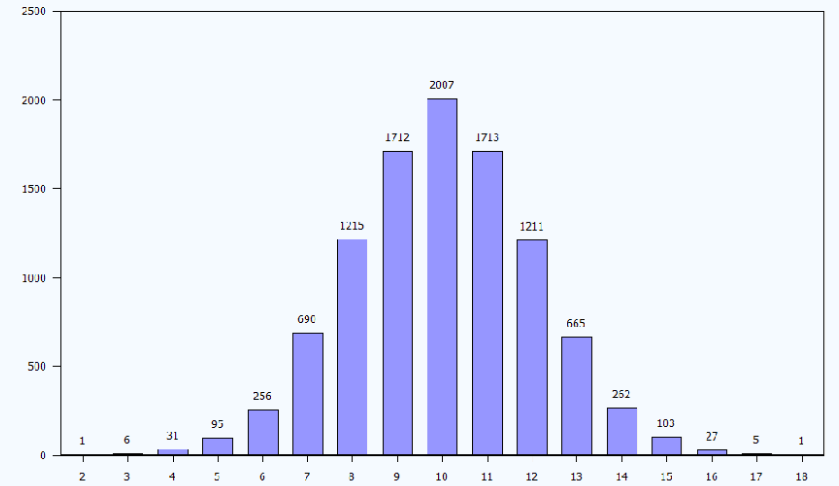

Figure 2: Example of a frequency histogram.

As we increase the number os samples taken, effectively decreasing the width of each bin, then the data will approach a smooth curve.

2.2. Definition of a PDF

As the frequency histogram bin width approaches an infinitesimal width, the histogram approaches a continuous curve known as the **Probability Density Function** (PDF). A PDF has one criteria in that the **area under the curve must be equal to 1**, the mathematical definition of a PDF is shown in **Eq. 12**.

$$\int_{-\infty}^{\infty} f(t)dt = 1 \tag{12}$$

PDFs can be used to find the probability that a certain value t is that value. In terms of reliability engineering its the **probability that a component fails** at the time t.

2.3. Definition of a CDF

A **Cumulative Distribution Function** (CDF) yields the probability that a given value will fall between the limits of $-\infty$ and t_1, its mathematical definition is shown in **Eq. 13**.

$$F(t) = \int_{-\infty}^{t_1} f(t)dt \tag{13}$$

2.4. Reliability Function

Reliability is the probability that a component will survive from a time $t = 0$ to a time $t = t_1$ and its mathematical definition is shown in **Eq. 14**, with a graph depicting the reliability function shown in **Figure 3**.

$$R(t) = 1 - F(t) = 1 - \int_{-\infty}^{t_1} f(t)dx \equiv \int_{x}^{\infty} f(t)dx \tag{14}$$

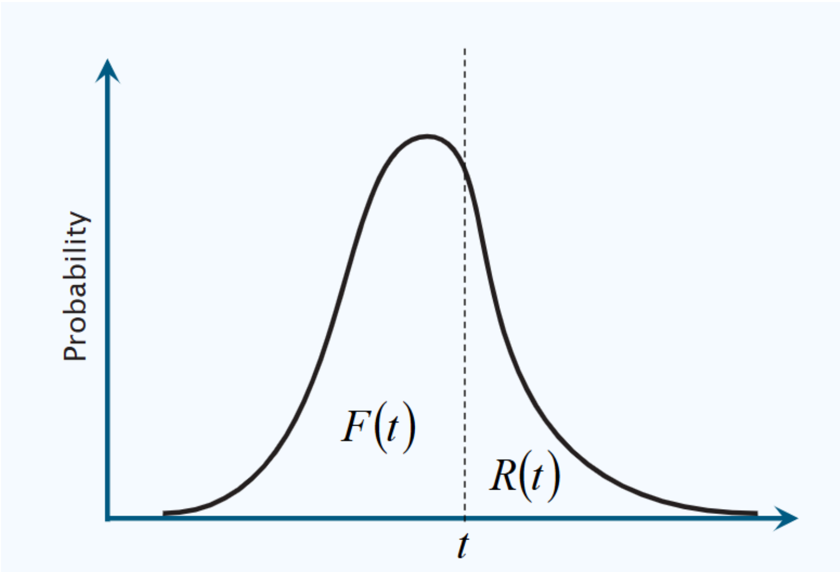

Figure 3: Graph of the reliability function and CDF on a PDF.

2.5. Hazard Function

Also known as the hazard rate the **hazard function** gives the probability of failure at a time t, given that there has not already been a failure. The mathematical definition for the hazard function is shown in **Eq. 15**.

$$h(t) = \frac{f(t)}{R(t)} = \frac{f(t)}{1 - R(t)} \tag{15}$$

The hazard function can be though of as a measure of the probability of eminent failure at t or the proneness of failure after t. Note that there does exist a cumulative hazard function which is not assessed as well as methods to rearrange between all of these functions.