SESM6047

Finite Element Analysis in Solid Mechanics

Author: Yusaf Sultan
Lecturer: Alex Dickinson
Word Count: 302

Contents

List of Figures

List of Tables

Definitions

$\boldsymbol{ABAB}$	Allahumabarik

0. Lecture 0

0.1. Matrix Terminology

A matrix is a structured way of organizing data in a rectangular array with rows and columns. For this module the notation of a matrix and reference to matrix terms is shown in **Eq. 1**.

$$[K]_{i\times j} = \begin{bmatrix} k_{11} & k_{12} & ... & k_{1j} \\ k_{21} & k_{22} & ... & k_{2j} \\ \vdots & \vdots & \ddots & \vdots \\ k_{i1} & k_{i2} & ... & k_{ij} \end{bmatrix}_{i\times j} \tag{1}$$

Where:

- $\boldsymbol{[K]_{i\times j}}$: The matrix $[K]$ (Note for this module a matrix will be defined by the [] notation).
- $\boldsymbol{i}$: The row index (indexed from 1).
- $\boldsymbol{j}$: The column index (indexed from 2).
- $\boldsymbol{i\times j}$: The shape of the matrix (i number of rows, j number of columns).
- $\boldsymbol{k_{ij}}$: The item in the ith row and the jth column.

A special type of commonly used matrix is a **diagonal matrix** shown in **Eq. 2**. This type of matrix only features terms that sit on the indexes where the row index is equal to the column index ($i = j$).

$$[K]_{i\times j} = \begin{bmatrix} k_{11} & 0 & ... & 0 \\ 0 & k_{22} & ... & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & ... & k_{ij} \end{bmatrix}_{i\times j} \tag{2}$$

0.2. Vector Terminology

A vector is a quantity that has a **magnitude** as well as a **direction**. A vector is like one column out of a matrix, the number of terms signifies the dimensions of a vector, the notation used in this module for a vector is shown in **Eq. 3**.

$$\{q\}_{i\times 1} = \begin{Bmatrix} q_1 \\ q_2 \\ \vdots \\ q_i \end{Bmatrix}_{i\times 1} \tag{3}$$

0.3. Matrix Addition and Subtraction

Matrixes with the same shape can be added or subtracted. Matrix addition or subtraction is a **commutative** property and the process to do so is shown in **Eq. 4**.

$$[A]_{i\times j} \pm [B]_{i\times j} \tag{4.1}$$

$$= \begin{bmatrix} a_{11} & a_{12} & ... & a_{1j} \\ a_{21} & a_{22} & ... & a_{2j} \\ \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & ... & a_{ij} \end{bmatrix}_{i\times j} \pm \begin{bmatrix} b_{11} & b_{12} & ... & b_{1j} \\ b_{21} & b_{22} & ... & b_{2j} \\ \vdots & \vdots & \ddots & \vdots \\ b_{i1} & b_{i2} & ... & b_{ij} \end{bmatrix}_{i\times j} = \begin{bmatrix} a_{11}\pm b_{11} & a_{12}\pm b_{12} & ... & a_{1j}\pm b_{1j} \\ a_{21}\pm b_{21} & a_{22}\pm b_{22} & ... & a_{2j}\pm b_{2j} \\ \vdots & \vdots & \ddots & \vdots \\ a_{i1}\pm b_{i1} & a_{i2}\pm b_{i2} & ... & a_{ij}\pm b_{ij} \end{bmatrix}_{i\times j} \tag{4.2}$$

$$[A]_{i\times j} \pm [B]_{i\times j} = [B]_{i\times j} \pm [A]_{i\times j} \tag{4.3}$$

0.4. Matrix Multiplication

0.4.1. Scalar Multiplication

A scalar can be multiplied with a matrix or vector of any shape, the scalar value is multiplied with every term within the matrix, as shown in **Eq. 5**.

$$k[A]_{i\times j} = k \begin{bmatrix} a_{11} & a_{12} & ... & a_{1j} \\ a_{21} & a_{22} & ... & a_{2j} \\ \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & ... & a_{ij} \end{bmatrix}_{i\times j} = \begin{bmatrix} ka_{11} & ka_{12} & ... & ka_{1j} \\ ka_{21} & ka_{22} & ... & ka_{2j} \\ \vdots & \vdots & \ddots & \vdots \\ ka_{i1} & ka_{i2} & ... & ka_{ij} \end{bmatrix}_{i\times j} \tag{5}$$

0.4.2. Matrix Multiplication

Any two matrices can be multiplied as long as **the number of columns of the first matrix is the same as the number of rows in the second matrix**. Given this criteria is satisfied the formula to then multiply two matrices together is shown in **Eq. 6**.

$$[A]_{i\times k}[B]_{k\times j} = \begin{bmatrix} a_{11} & a_{12} & ... & a_{1k} \\ a_{21} & a_{22} & ... & a_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & ... & a_{ik} \end{bmatrix}_{i\times k} \begin{bmatrix} b_{11} & b_{12} & ... & b_{1j} \\ b_{21} & b_{22} & ... & b_{2j} \\ \vdots & \vdots & \ddots & \vdots \\ b_{k1} & b_{k2} & ... & b_{kj} \end{bmatrix}_{k\times j} \tag{6.1}$$

$$= \begin{bmatrix} \sum_{r:1}^{k}(a_{1r}b_{r1}) & \sum_{r:1}^{k}(a_{1r}b_{r2}) & ... & \sum_{r:1}^{k}(a_{1r}b_{rj}) \\ \sum_{r:1}^{k}(a_{2r}b_{r1}) & \sum_{r:1}^{k}(a_{2r}b_{r2}) & ... & \sum_{r:1}^{k}(a_{2r}b_{rj}) \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{r:1}^{k}(a_{ir}b_{r1}) & \sum_{r:1}^{k}(a_{ir}b_{r2}) & ... & \sum_{r:1}^{k}(a_{ir}b_{rj}) \end{bmatrix}_{i\times j} \tag{6.2}$$

Note that matrix multiplication is **not commutable** ($[A]_{i\times k}[B]_{k\times j} \neq [B]_{k\times j}[A]_{i\times k}$).