

# Daily Integral

Easy Derivatives

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$$1. 05/02/2026: f(x) = 3e^{2x} + e^{\sqrt{x}} + \ln\left(\frac{\pi}{4x}\right), f'\left(\frac{\pi}{13}\right) = ?$$

After defining the derivative, it can be split utilizing the **sum rule** into a system of separate derivatives, this is shown in **Eq. 1**.

$$f'(x) = \frac{d}{dx}(f(x)) = \frac{d}{dx}\left(3e^{2x} + e^{\sqrt{x}} + \ln\left(\frac{\pi}{4x}\right)\right) = \frac{d}{dx}(3e^{2x}) + \frac{d}{dx}(e^{\sqrt{x}}) + \frac{d}{dx}\left(\ln\left(\frac{\pi}{4x}\right)\right) \quad (1)$$

Each of these sub derivatives ( $A$ ,  $B$  and  $C$ ) can be solved through utilizing the **chain rule**, this is shown for  $\frac{d}{dx}(3e^{2x})$  in **Eq. 2**.

$$A = 3e^{2x} \rightarrow \text{Let } u = 2x \therefore A = 3e^u \quad (2.1)$$

$$\frac{du}{dx} = 2 \quad \frac{dA}{du} = 3e^u \quad (2.2)$$

$$\frac{dA}{dx} = \frac{dA}{du} \cdot \frac{du}{dx} = 3e^u \cdot 2 = 6e^u = 6e^{2x} \quad (2.3)$$

The derivative for  $B$  ( $e^{\sqrt{x}}$ ) using the chain rule is shown in **Eq. 3**

$$B = e^{\sqrt{x}} \rightarrow \text{Let } u = \sqrt{x} \therefore B = e^u \quad (3.1)$$

$$\frac{du}{dx} = \frac{1}{2\sqrt{x}} \quad \frac{dB}{du} = e^u \quad (3.2)$$

$$\frac{dB}{dx} = \frac{dB}{du} \cdot \frac{du}{dx} = e^u \cdot \frac{1}{2\sqrt{x}} = \frac{e^u}{2\sqrt{x}} = \frac{e^{\sqrt{x}}}{2\sqrt{x}} \quad (3.3)$$

The derivative for  $C$  ( $\ln\left(\frac{\pi}{4x}\right)$ ) using the chain rule is shown in **Eq. 4**

$$C = \ln\left(\frac{\pi}{4x}\right) \rightarrow \text{Let } u = \frac{\pi}{4x} \therefore C = \ln(u) \quad (4.1)$$

$$\frac{du}{dx} = -\frac{\pi}{4x^2} \quad \frac{dC}{du} = \frac{1}{u} \quad (4.2)$$

$$\frac{dC}{dx} = \frac{dC}{du} \cdot \frac{du}{dx} = \frac{1}{u} \cdot -\frac{\pi}{4x^2} = -\frac{\pi}{4x^2u} = -\frac{1}{x} \quad (4.3)$$

The derivative of  $f(x)$  is therefore the sum of  $A'$ ,  $B'$  and  $C'$ , combining together equations **Eq. 2**, **Eq. 3** and **Eq. 4**. The derivative is shown in **Eq. 5**.

$$f'(x) = 6e^{2x} + \frac{e^{\sqrt{x}}}{2\sqrt{x}} - \frac{1}{x} \quad (5)$$

To solve the question,  $x = \frac{\pi}{13}$  is substituted into **Eq. 5**, the final result of which is shown in **Eq. 6**.

$$f'\left(\frac{\pi}{13}\right) = 6e^{\frac{2\pi}{13}} + \frac{e^{\sqrt{\frac{\pi}{13}}}}{2\sqrt{\frac{\pi}{13}}} - \frac{13}{\pi} \approx 7.25 \quad (2 \text{ d.p}) \quad (6)$$

**2. 06/02/2026:**  $f(x) = \sin x \cos x \sqrt{x}$ ,  $f'(\frac{\pi}{2}) = ?$ 

The derivative can be simplified utilizing the **double angle formula** and by then utilizing the **product rule**. The double angle formula used to simplify the expression is shown in **Eq. 7**.

$$\sin(2a) = 2 \sin a \cos a \quad \therefore \quad \sin a \cos a = \frac{1}{2} \sin(2a) \quad (7)$$

Applying **Eq. 7** to the derivative yields a function that is the product of two other functions, This is shown in **Eq. 8**.

$$f(x) = \sin x \cos x \sqrt{x} = \frac{1}{2} \sin(2x) \sqrt{x} \quad (8.1)$$

$$\text{Let } u = \frac{1}{2} \sin(2x) \quad v = \sqrt{x} \quad (8.2)$$

To differentiate  $u$ , the **chain rule** as the expression is a composite function, this is applied in **Eq. 9**.

$$u = \frac{1}{2} \sin(2x) \longrightarrow \text{Let } z = 2x \therefore u = \frac{1}{2} \sin(z) \quad (9.1)$$

$$\frac{dz}{dx} = 2 \quad \frac{du}{dz} = \frac{1}{2} \cos(z) \quad (9.2)$$

$$\frac{du}{dx} = \frac{du}{dz} \cdot \frac{dz}{dx} = \cos(z) = \cos(2x) \quad (9.3)$$

Now that an expression for  $\frac{du}{dx}$  has been obtained, the product rule can be applied, this is shown in **Eq. 10**.

$$\text{Let } u = \frac{1}{2} \sin(2x) \quad v = \sqrt{x} \quad (10.1)$$

$$\frac{du}{dx} = \cos(2x) \quad \frac{dv}{dx} = \frac{1}{2\sqrt{x}} \quad (10.2)$$

$$f'(x) = u \frac{dv}{dx} + \frac{du}{dx} v = \frac{\sin(2x)}{4\sqrt{x}} + \sqrt{x} \cos(2x) \quad (10.3)$$

Finally to obtain a solution to the question,  $x = \frac{\pi}{2}$  can be plugged into  $f'(x)$ , this is shown in **Eq. 11**.

$$f'\left(\frac{\pi}{2}\right) = \frac{\sin(2(\frac{\pi}{2}))}{4\sqrt{\frac{\pi}{2}}} + \sqrt{\frac{\pi}{2}} \cos\left(2\left(\frac{\pi}{2}\right)\right) = 0 + \sqrt{\frac{\pi}{2}} \approx -1.25 \quad (2 \text{ d.p}) \quad (11)$$

**3. 17/02/2026:**  $f(x) = x^9 + x^8 + x^7$ ,  $f^{(7)}\left(\frac{1}{2}\right) = ?$

This derivative can be solved by taking successive derivatives to obtain  $f^{(7)}$ , this is shown in **Eq. 12.**

$$f(x) = x^9 + x^8 + x^7 \quad (12.1)$$

$$f'(x) = 9x^8 + 8x^7 + 7x^6 \quad (12.2)$$

$$f''(x) = (9 \times 8)x^7 + (8 \times 7)x^6 + (7 \times 6)x^5 \quad (12.3)$$

$$f'''(x) = (9 \times 8 \times 7)x^6 + (8 \times 7 \times 6)x^5 + (7 \times 6 \times 5)x^4 \quad (12.4)$$

$$\dots \quad (12.5)$$

$$f^{(7)}(x) = \left( \prod_{k=3}^9 k \right) x^2 + \left( \prod_{k=2}^8 k \right) x + \left( \prod_{k=1}^7 k \right) = 181440x^2 + 40320x + 5040 \quad (12.6)$$

The value of  $x = \frac{1}{2}$  can then be inputted into **Eq. 12** to obtain a solution, this is shown in **Eq. 13.**

$$f^{(7)}\left(\frac{1}{2}\right) = 181440\left(\frac{1}{2}\right)^2 + 40320\left(\frac{1}{2}\right) + 5040 = 70560 \quad (13)$$

