Daily Integral

Medium Derivatives

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1. 05/02/2026: $f(x) = xe^{\sin x},\ f''(0) = ?$

This derivative can be solved by applying **product rule** as well as **chain rule**. Initially the u and v are assigned for the function, shown in **Eq. 1**.

$$\text{Let } u = x \quad v = e^{\sin(x)} \tag{1}$$

The chain rule can be applied to differentiate v, this is shown in **Eq. 2**.

$$v = e^{\sin(x)} \longrightarrow \text{Let } z = \sin(x) \therefore v = e^z \tag{2.1}$$

$$\frac{dz}{dx} = \cos(x) \quad \frac{dv}{dz} = e^z \tag{2.2}$$

$$\frac{dv}{dx} = \frac{dv}{dz} \cdot \frac{dz}{dx} = \cos(x)e^z = \cos(x)e^{\sin(x)} \tag{2.3}$$

Now that an expression for $\frac{dv}{dx}$ has been obtained, the product rule can be applied to obtain an expression for $f'(x)$, this is shown in **Eq. 3**.

$$\text{Let } u = x \quad v = e^{\sin(x)} \tag{3.1}$$

$$\frac{du}{dx} = 1 \quad \frac{dv}{dx} = \cos(x)e^{\sin(x)} \tag{3.2}$$

$$f'(x) = u\frac{dv}{dx} + \frac{du}{dx}v = x\cos(x)e^{\sin(x)} + e^{\sin(x)} = e^{\sin(x)}(x\cos(x) + 1) \tag{3.3}$$

The equation for $f'(x)$ is the product of two separate functions so product rule can be applied again, the u and v are assigned for the function, shown in **Eq. 4**.

$$\text{Let } u = e^{\sin(x)} \quad v = x\cos(x) + 1 \tag{4}$$

v itself is a function made up of the product of two sub functions, so the product rule is applied here to obtain $\frac{dv}{dx}$ shown in **Eq. 5** (as 1 differentiates to 0 it is omitted).

$$\text{Let } A = x \quad B = \cos(x) \tag{5.1}$$

$$\frac{dA}{dx} = 1 \quad \frac{dB}{dx} = -\sin(x) \tag{5.2}$$

$$\frac{dv}{dx} = A\frac{dB}{dx} + \frac{dA}{dx}B = -x\sin(x) + \cos(x) \tag{5.3}$$

Now that an expression for $\frac{dv}{dx}$ has been obtained, the product rule can be applied to obtain an expression for $f''(x)$, this is shown in **Eq. 6** (note that $\frac{du}{dx}$ is obtained via **Eq. 2**).

$$\text{Let } u = e^{\sin(x)} \quad v = x\cos(x) + 1 \tag{6.1}$$

$$\frac{du}{dx} = \cos(x)e^{\sin(x)} \quad \frac{dv}{dx} = \cos(x) - x\sin(x) \tag{6.2}$$

$$f'(x) = u\frac{dv}{dx} + \frac{du}{dx}v = [\cos(x) - x\sin(x)]e^{\sin(x)} + [x\cos(x) + 1]\cos(x)e^{\sin(x)} \tag{6.3}$$

To solve the question, $x = 0$ is substituted into $f''(0)$, which is shown in **Eq. 7**.

$$f''(0) = [1 - 0] \times 1 + [0 + 1] \times 1 \times 1 = 2 \tag{7}$$

2. 06/02/2026: $f(x) = x \arctan\left(\sqrt{x^2 + e^x}\right), \ f'(0) = ?$

This derivative can be solved utilizing the **product rule** and **chain rule**. Initially the u and v are assigned for the function, shown in **Eq. 8**.

$$\text{Let } u = x \quad v = \arctan\left(\sqrt{x^2 + e^x}\right) \tag{8}$$

The chain rule can be applied to differentiate v and obtain an expression for $\frac{dv}{dx}$, this is shown in **Eq. 9**.

$$v = \arctan\left(\sqrt{x^2 + e^x}\right) \longrightarrow \text{Let } z = \sqrt{x^2 + e^x} \therefore v = \arctan(z) \tag{9.1}$$

$$z = \sqrt{x^2 + e^x} \longrightarrow \text{Let } q = x^2 + e^x \therefore z = \sqrt{q} \tag{9.2}$$

$$\frac{dz}{dq} = \frac{1}{2\sqrt{q}} \quad \frac{dq}{dx} = 2x + e^x \quad \frac{dv}{dz} = \frac{1}{1 + z^2} \tag{9.3}$$

$$\frac{dv}{dx} = \frac{dv}{dz} \cdot \frac{dz}{dx} = \frac{dv}{dz} \cdot \frac{dz}{dq} \cdot \frac{dq}{dx} = \frac{2x + e^x}{2\sqrt{q}(1 + z^2)} = \frac{2x + e^x}{2\sqrt{x^2 + e^x}(1 + x^2 + e^x)} \tag{9.4}$$

Now that an expression for $\frac{dv}{dx}$ has been obtained, the product rule can be applied to obtain an expression for $f'(x)$, this is shown in **Eq. 10**.

$$\text{Let } u = x \quad v = \arctan\left(\sqrt{x^2 + e^x}\right) \tag{10.1}$$

$$\frac{du}{dx} = 1 \quad \frac{dv}{dx} = \frac{2x + e^x}{2\sqrt{x^2 + e^x}(1 + x^2 + e^x)} \tag{10.2}$$

$$f'(x) = u\frac{dv}{dx} + \frac{du}{dx}v = \frac{2x^2 + xe^x}{2\sqrt{x^2 + e^x}(1 + x^2 + e^x)} + \arctan\left(\sqrt{x^2 + e^x}\right) \tag{10.3}$$

To solve the question, $x = 0$ is substituted into $f'(0)$, which is shown in **Eq. 11**.

$$f'(0) = 0 + \arctan(1) = \frac{1}{4}\pi \approx 0.79 \quad \text{(2 d.p)} \tag{11}$$