Daily Integral

Easy Derivatives

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1. 05/02/2026: $f(x) = 3e^{2x} + e^{\sqrt{x}} + \ln\left(\frac{\pi}{4x}\right),\ f'\left(\frac{\pi}{13}\right) = ?$

After defining the derivative, it can be split utilizing the **sum rule** into a system of separate derivatives, this is shown in **Eq. 1**.

$$f'(x) = \frac{d}{dx}(f(x)) = \frac{d}{dx}\left(3e^{2x} + e^{\sqrt{x}} + \ln\left(\frac{\pi}{4x}\right)\right) = \frac{d}{dx}(3e^{2x}) + \frac{d}{dx}\left(e^{\sqrt{x}}\right) + \frac{d}{dx}\left(\ln\left(\frac{\pi}{4x}\right)\right) \tag{1}$$

Each of these sub derivatives (A, B and C) can be solved through utilizing the **chain rule**, this is shown for $\frac{d}{dx}(3e^{2x})$ in **Eq. 2**.

$$A = 3e^{2x} \longrightarrow \text{Let } u = 2x \therefore A = 3e^u \tag{2.1}$$

$$\frac{du}{dx} = 2 \quad \frac{dA}{du} = 3e^u \tag{2.2}$$

$$\frac{dA}{dx} = \frac{dA}{du} \cdot \frac{du}{dx} = 6e^u = 6e^{2x} \tag{2.3}$$

The derivative for B ($e^{\sqrt{x}}$) using the chain rule is shown in **Eq. 3**

$$B = e^{\sqrt{x}} \longrightarrow \text{Let } u = \sqrt{x} \therefore B = e^u \tag{3.1}$$

$$\frac{du}{dx} = \frac{1}{2\sqrt{x}} \quad \frac{dB}{du} = e^u \tag{3.2}$$

$$\frac{dB}{dx} = \frac{dB}{du} \cdot \frac{du}{dx} = \frac{e^u}{2\sqrt{x}} = \frac{e^{\sqrt{x}}}{2\sqrt{x}} \tag{3.3}$$

The derivative for C ($\ln\left(\frac{\pi}{4x}\right)$) using the chain rule is shown in **Eq. 4**

$$C = \ln\left(\frac{\pi}{4x}\right) \longrightarrow \text{Let } u = \frac{\pi}{4x} \therefore C = \ln(u) \tag{4.1}$$

$$\frac{du}{dx} = -\frac{\pi}{4x^2} \quad \frac{dC}{du} = \frac{1}{u} \tag{4.2}$$

$$\frac{dC}{dx} = \frac{dC}{du} \cdot \frac{du}{dx} = -\frac{\pi}{4x^2 u} = -\frac{1}{x} \tag{4.3}$$

The derivative of $f(x)$ is therefore the sum of A', B' and C', combining together equations **Eq. 2**, **Eq. 3** and **Eq. 4**. The derivative is shown in **Eq. 5**.

$$f'(x) = 6e^{2x} + \frac{e^{\sqrt{x}}}{2\sqrt{x}} - \frac{1}{x} \tag{5}$$

To solve the question, $x = \frac{\pi}{13}$ is substituted into **Eq. 5**, the final result of which is shown in **Eq. 6**.

$$f'\left(\frac{\pi}{13}\right) = 6e^{\frac{2\pi}{13}} + \frac{e^{\sqrt{\frac{\pi}{13}}}}{2\sqrt{\frac{\pi}{13}}} - \frac{13}{\pi} \approx 7.25 \ \ \text{(2 d.p)} \tag{6}$$

2. 06/02/2026: $f(x) = \sin x \cos x\sqrt{x},\ f'\left(\frac{\pi}{2}\right) = ?$

The derivative can be simplified utilizing the **double angle formula** and by then utilizing the **product rule**. The double angle formula used to simplify the expression is shown in **Eq. 7**.

$$\sin(2a) = 2\sin a\cos a \quad \therefore \quad \sin a\cos a = \frac{1}{2}\sin(2a) \tag{7}$$

Applying **Eq. 7** to the derivative yields a function that is the product of two other functions, This is shown in **Eq. 8**.

$$f(x) = \sin x\cos x\sqrt{x} = \frac{1}{2}\sin(2x)\sqrt{x} \tag{8.1}$$

$$\text{Let } u = \frac{1}{2}\sin(2x) \quad v = \sqrt{x} \tag{8.2}$$

To differentiate u, the **chain rule** as the expression is a composite function, this is applied in **Eq. 9**.

$$u = \frac{1}{2}\sin(2x) \longrightarrow \text{Let } z = 2x \therefore u = \frac{1}{2}\sin(z) \tag{9.1}$$

$$\frac{dz}{dx} = 2 \quad \frac{du}{dz} = \frac{1}{2}\cos(z) \tag{9.2}$$

$$\frac{du}{dx} = \frac{du}{dz}\cdot\frac{dz}{dx} = \cos(z) = \cos(2x) \tag{9.3}$$

Now that an expression for $\frac{du}{dx}$ has been obtained, the product rule can be applied, this is shown in **Eq. 10**.

$$\text{Let } u = \frac{1}{2}\sin(2x) \quad v = \sqrt{x} \tag{10.1}$$

$$\frac{du}{dx} = \cos(2x) \quad \frac{dv}{dx} = \frac{1}{2\sqrt{x}} \tag{10.2}$$

$$f'(x) = u\frac{dv}{dx} + \frac{du}{dx}v = \frac{\sin(2x)}{4\sqrt{x}} + \sqrt{x}\cos(2x) \tag{10.3}$$

Finally to obtain a solution to the question, $x = \frac{\pi}{2}$ can be plugged into $f'(x)$, this is shown in **Eq. 11**.

$$f'\left(\frac{\pi}{2}\right) = \frac{\sin(2(\frac{\pi}{2}))}{4\sqrt{\frac{\pi}{2}}} + \sqrt{\frac{\pi}{2}}\cos\left(2\left(\frac{\pi}{2}\right)\right) = 0 + \sqrt{\frac{\pi}{2}} \approx -1.25 \ \ (2\text{ d.p}) \tag{11}$$