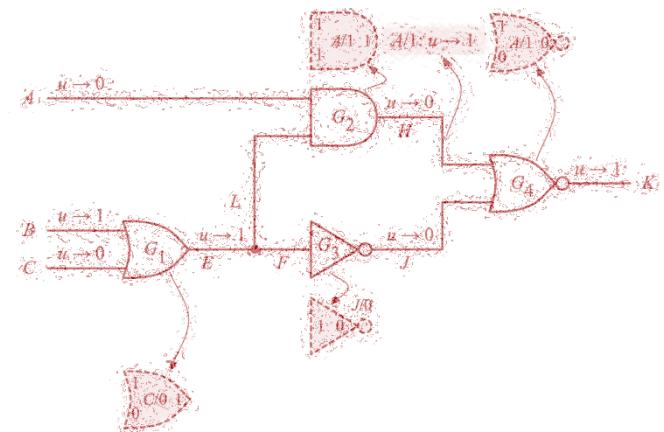


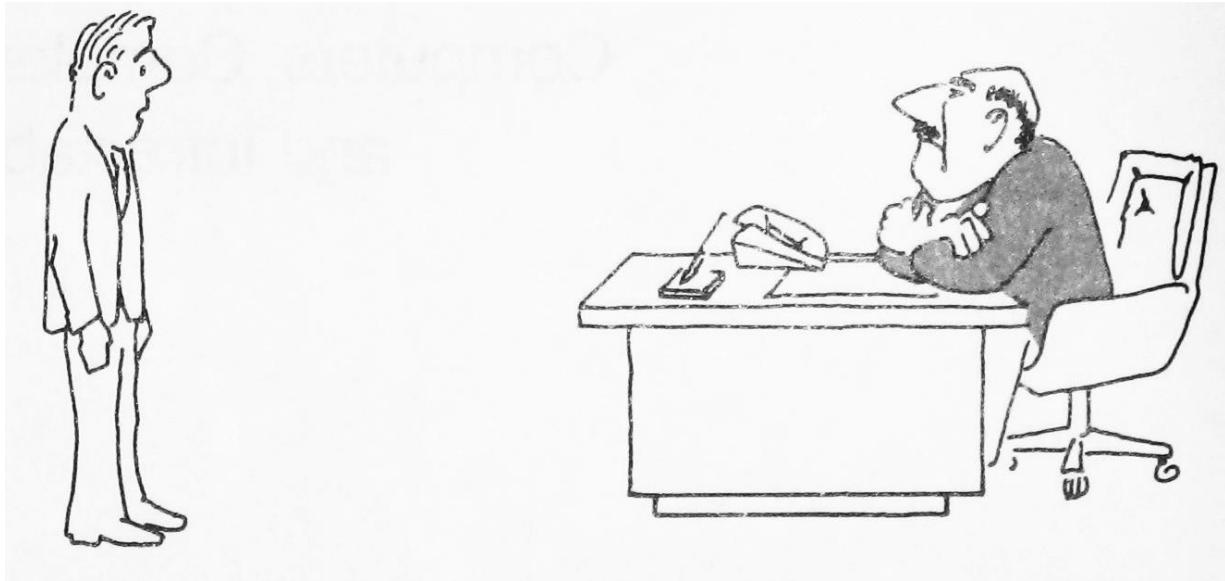
# Fault Simulation

- Introduction
- Fault simulation techniques
- Comparison of fault simulation
- Alternatives to fault simulation
  - ◆ Toggle coverage
  - ◆ Fault sampling (1974)
  - ◆ Critical path tracing (1979)
- Issues of fault simulation
- Concluding remarks



# Motivating Problem

- **Boss:** Why can't we speed up design sign-off?
- **You:** But the fault simulation is slow...
- **Boss:** We do not need exact fault coverage. Just give me an estimation and DO IT FAST!



**What Should You Do?**

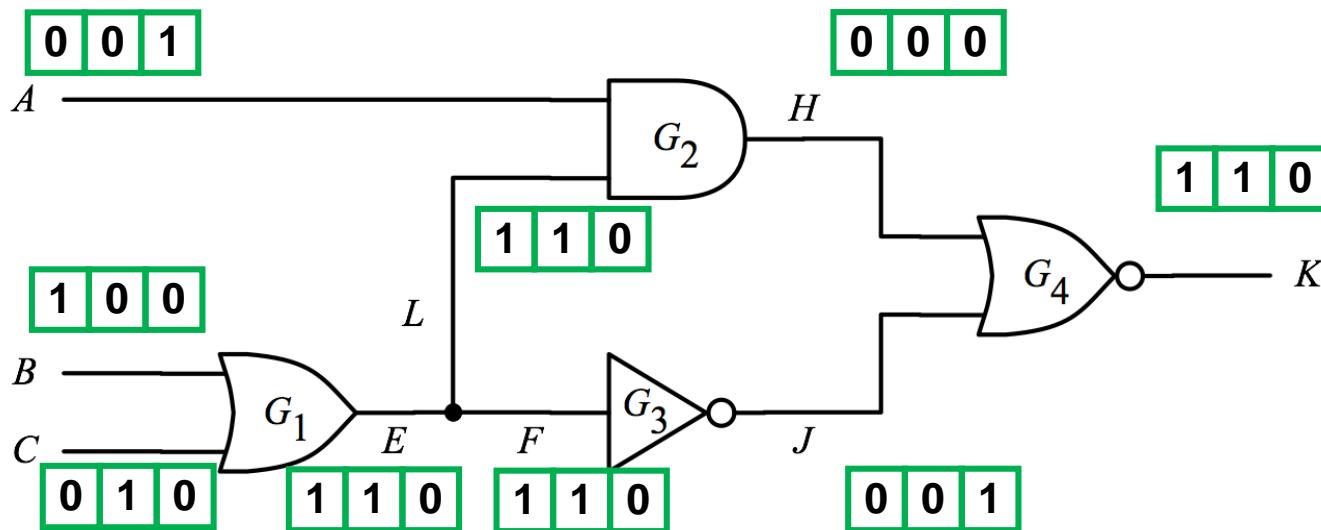
# Alternatives to Fault Simulation

- Q: Is exact fault simulation necessary?
  - ◆ Exact fault simulation is too expensive
  - ◆ Sometimes, we just want to know *approximate* fault coverage
    - \* with reasonable error
- Approximation is good for
  - ◆ DFT check in early design phase
  - ◆ Functional test pattern evaluation
- Approximation is NOT good for
  - ◆ ATPG
  - ◆ Diagnosis

# Toggle Coverage (DEF-1)

$$\text{toggle coverage} = \frac{\sum_{\text{all nodes } i} \# \text{ of different values in node } i}{2 \times \text{total } \# \text{ of nodes}}$$

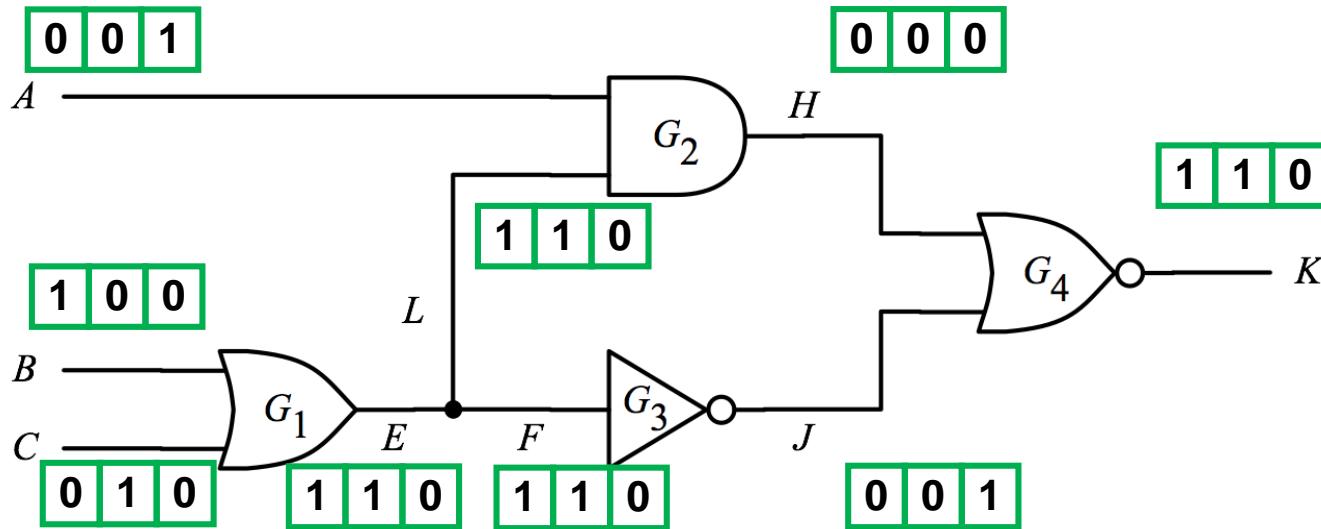
# = numbers



**Toggle Coverage = 17/18 = 94%**

# Toggle Coverage

- 😊 Advantage: Toggle coverage is easy to obtain
  - ♦ Logic simulation only, NO fault simulation
- 😢 Disadvantage: Toggle coverage is very optimistic
  - ♦ Fault activation only, NO fault propagation



**TC is Upper Bound of FC**

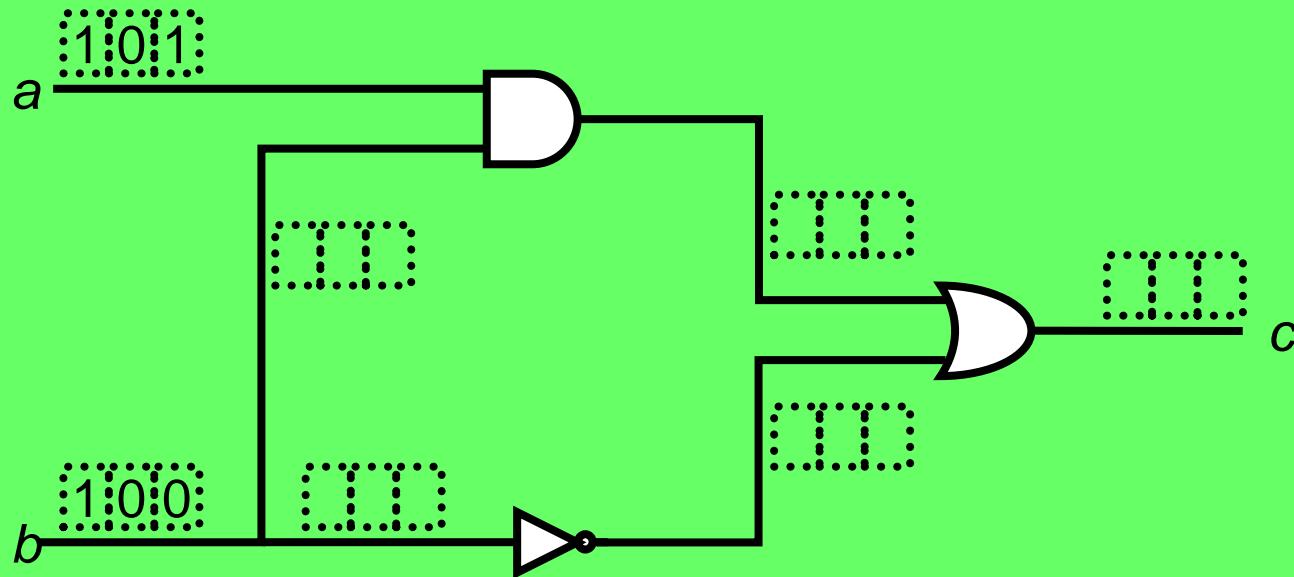
**Fault Coverage = 11/18 = 61% see 5.3 P.10**

**Toggle Coverage = 17/18 = 94%**

# Quiz

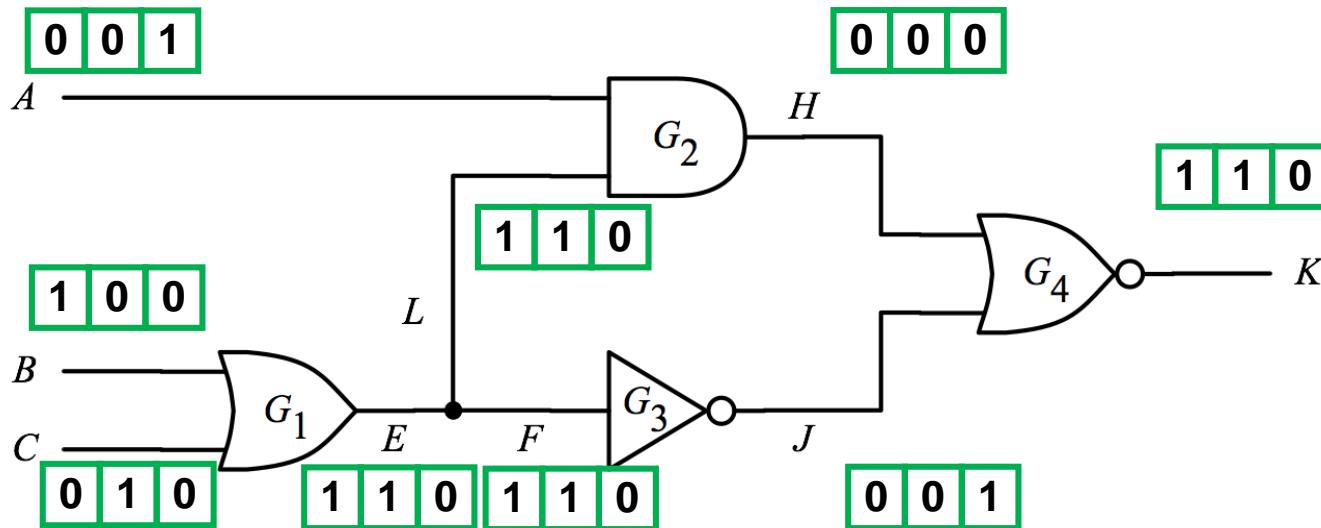
Q: Apply 3 patterns to this circuit of 7 nodes. Toggle coverage =?

A:



# Toggle Coverage (DEF-2)

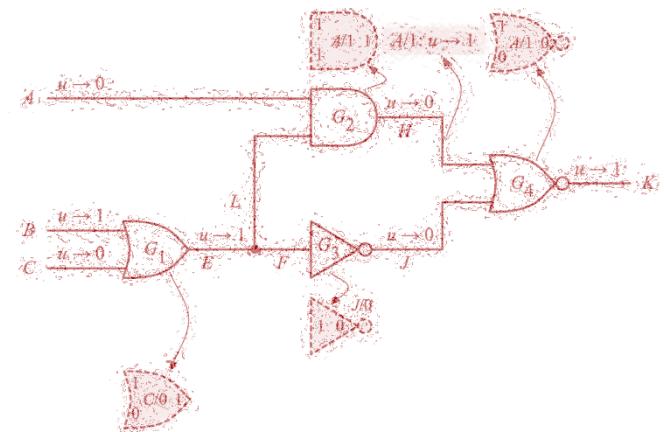
$$\text{toggle coverage} = \frac{\sum_{\text{all nodes } i} \# \text{ of different } \underline{\text{transitions}} \text{ in node } i}{2 \times \text{total } \# \text{ of nodes}}$$



**Toggle Coverage = 9/18 = 50%**  
DEF-2 more stringent

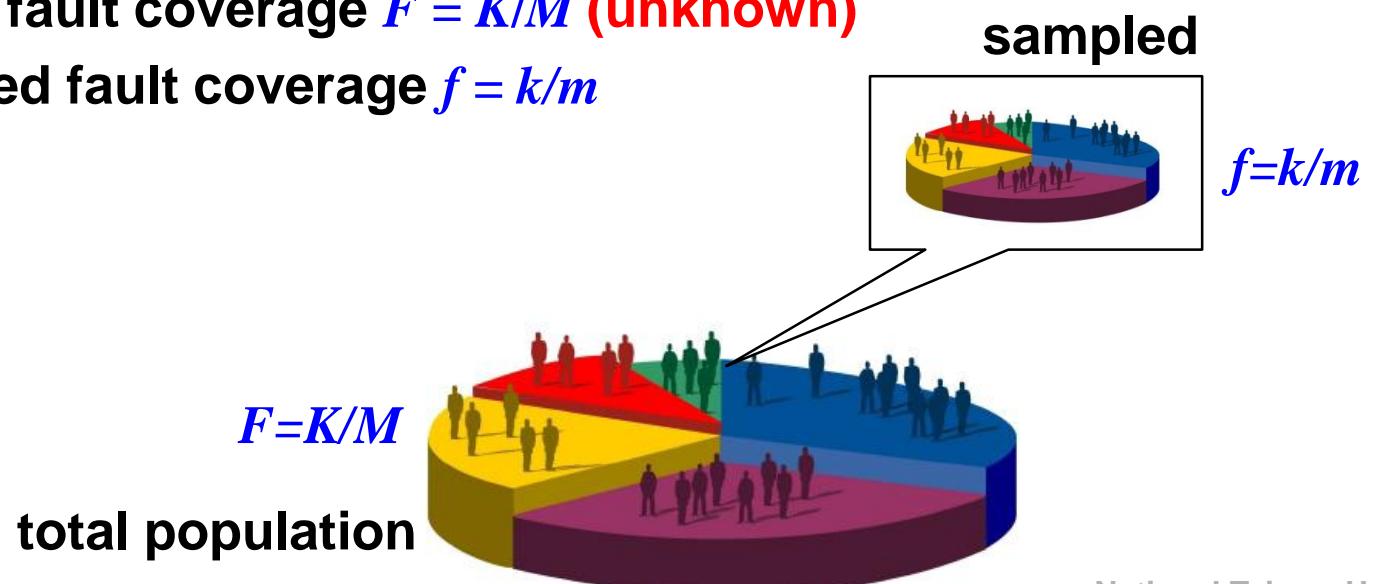
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# Fault Sampling [Butler 74]

- Idea: Actual fault simulation of whole circuit is too slow
  - ◆ Can we just sample a small portion of faults?
  - ◆ Like **Polling** before election
- Notation
  - ◆  $M$ : number of total faults
  - $m$ : number of sampled faults
  - $K$ : number of total faults detected (**unknown**)
  - $k$ : number of sampled faults detected
  - ◆ Actual fault coverage  $F = K/M$  (**unknown**)
  - ◆ Sampled fault coverage  $f = k/m$



# Fault Sampling (2)

- $P_k(M,m,K)$  = probability that a test detects  $k$  faults from a random sample size of  $m$ , given that it detects  $K$  faults from  $M$  total faults
  - ◆  $P_k$  is **hypergeometric distribution** (discrete valued)

$$P_k(m, M, K) = \frac{C_k^K C_{m-k}^{M-K}}{C_m^M}$$

- For large  $M$ ,
  - ◆  $P_k$  can be approximated by **normal distribution** (continuous valued)
  - ◆ with mean  $\mu_k$  and standard deviation  $\sigma_k$

$$\mu_k = m \frac{K}{M} = mF$$

$$\sigma_k^2 = m \frac{K}{M} \left(1 - \frac{K}{M}\right) \frac{M-m}{M-1} \cong mF(1-F)\left(1 - \frac{m}{M}\right)$$

See papers for detailed derivation

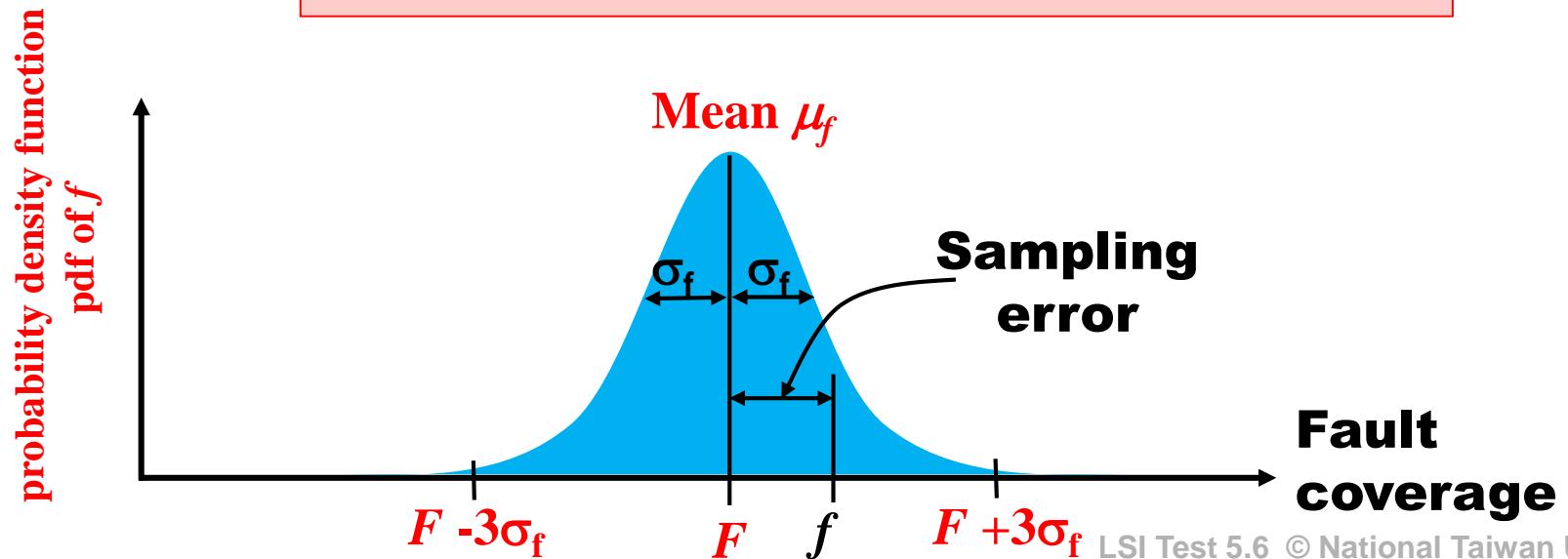
# Fault Sampling (3)

- For large  $M$ , sampled fault coverage  $f$  is random variable with normal distribution:

$$\mu_f = \mu_k / m = F$$

$$\sigma_f^2 = \sigma_k^2 / m^2 = F(1 - F)(1 - m/M) / m$$

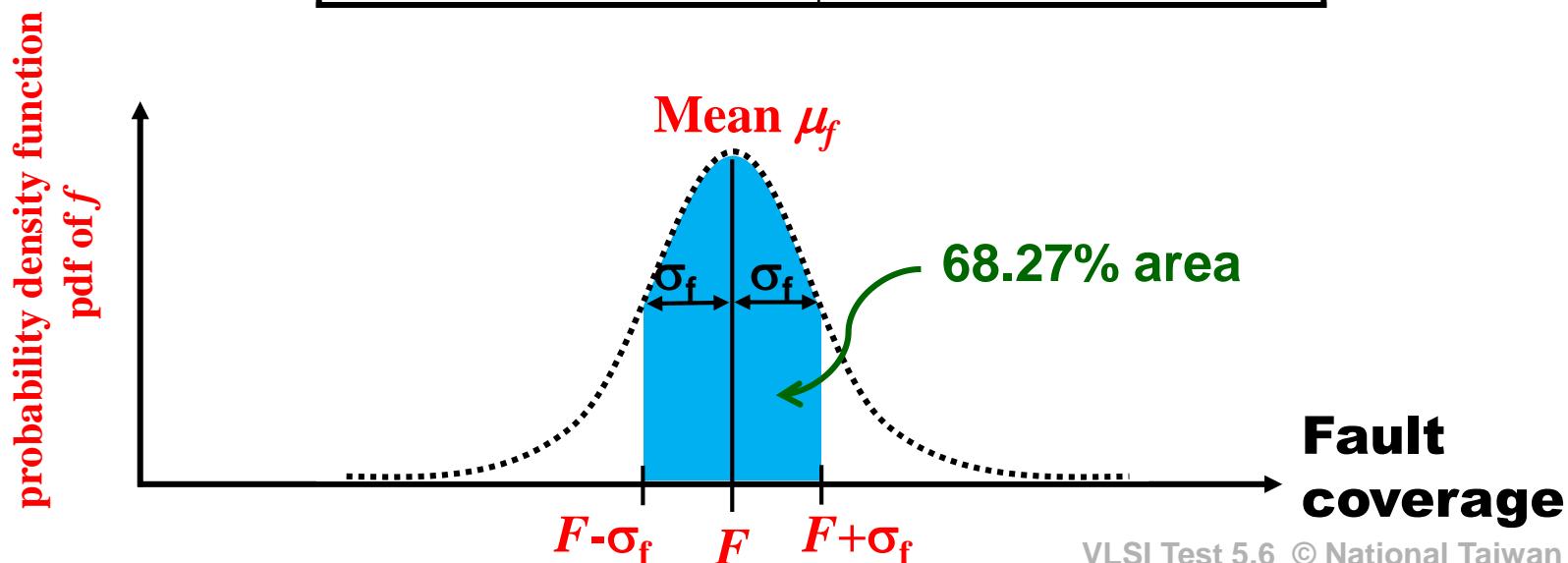
$$\begin{aligned}\sigma_f &= \sqrt{F(1 - F)(1 - m/M) / m} \\ &\cong \sqrt{F(1 - F) / m} \quad \text{if } M \gg m\end{aligned}$$



# Confidence Level

- **Confidence Level** = probability that estimated fault coverage  $f$  falls in between the *confidence interval*
- If we assume normal distribution:

Confidence interval	Confidence level
$\mu_f \pm \sigma_f$	68.27%
$\mu_f \pm 2\sigma_f$	95.44%
$\mu_f \pm 3\sigma_f$	99.73%
$\mu_f \pm 4\sigma_f$	99.99%



# Example

- $M=1,000,000$
- $m=10,000$  (1% of  $M$ )
- NOTE:  $F$  is unknown so we use  $\mu_f$

$$\begin{aligned}\sigma_f &= \sqrt{F(1-F)(1-m/M) / m} \\ &\cong \sqrt{F(1-F) / m}\end{aligned}$$

$\mu_f (=F)$	70%	80%	90%	95%
$\sigma_f$	0.5%	0.4%	0.3%	0.2%
68% CL	$70\% \pm 0.5\%$	$80\% \pm 0.4\%$	$90\% \pm 0.3\%$	$95\% \pm 0.2\%$
95% CL	$70\% \pm 1\%$	$80\% \pm 0.8\%$	$90\% \pm 0.6\%$	$95\% \pm 0.4\%$
99% CL	$70\% \pm 1.5\%$	$80\% \pm 1.2\%$	$90\% \pm 0.9\%$	$95\% \pm 0.6\%$

Accurate Prediction:  
High CL and Small CI

# Quiz

**Q:** Suppose FC=95% but we want to improve std  $\sigma_f$  to 0.1%.  
How many samples do we need,  $m=?$

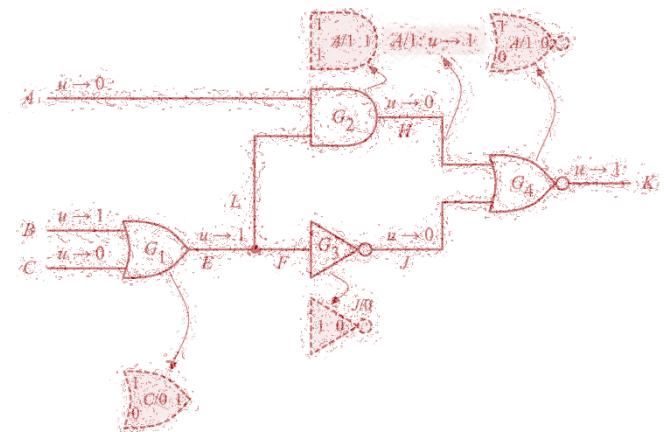
**A:**

$$\begin{aligned}\sigma_f &= \sqrt{F(1-F)(1-m/M) / m} \\ &\cong \sqrt{F(1-F) / m}\end{aligned}$$

$\mu_f (=F)$	70%	80%	90%	95%
$\sigma_f$	0.5%	0.4%	0.3%	0.2%
68% CI	70% $\pm$ 0.5%	80% $\pm$ 0.4%	90% $\pm$ 0.3%	95% $\pm$ 0.2%
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99% CI	70% $\pm$ 1.5%	80% $\pm$ 1.2%	90% $\pm$ 0.9%	95% $\pm$ 0.6%

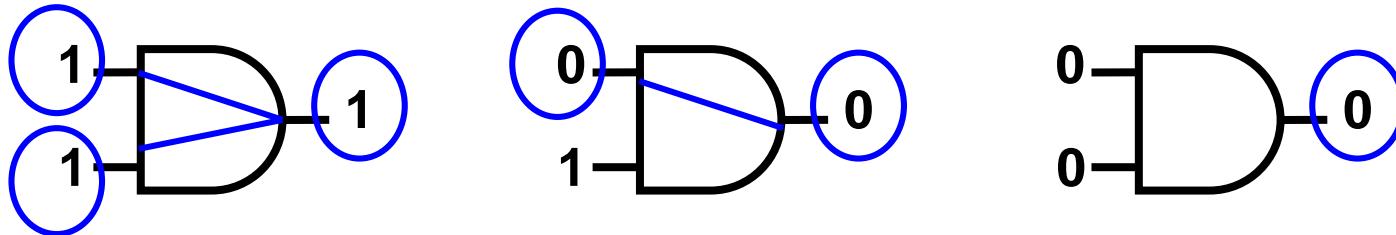
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# Critical Path Tracing (CPT) [Roth 79]

- $x$  is **Critical Signal**
  - ◆  $x$ 's value change causes some primary output values to change
- **Critical path**
  - ◆ All signals on this path that are critical signals
- Example: critical signals are circled

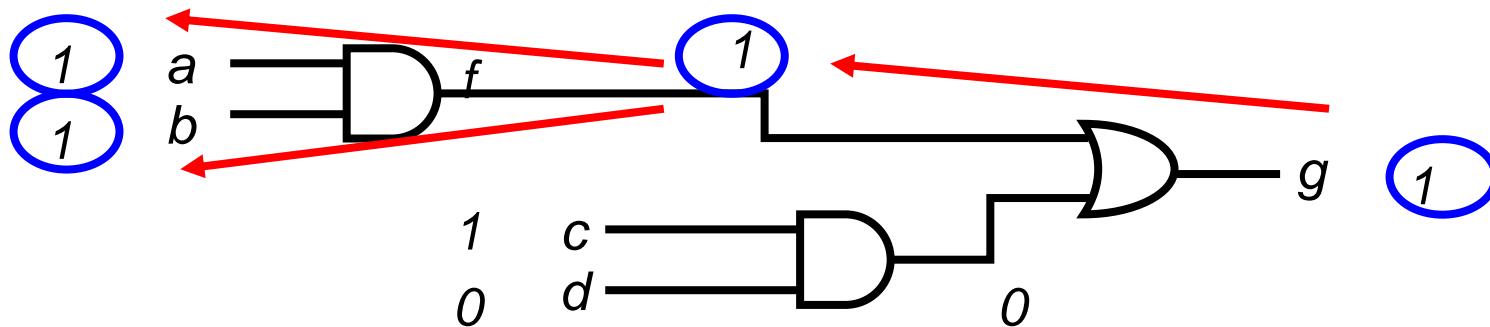


Note: In timing analysis, *critical path* has different definition

Critical path = Path with the longest path delay

# CPT Algorithm [Abramovici 84]

- **Critical Path Tracing:** Start from primary outputs to primary inputs
  - ◆ If gate output is critical, backtrace its critical gate input(s)
- Example: fanout-free cone (FFC)
  - ◆ Critical signals: *a, b, f, g*. Critical paths: *afg* and *bfg*



CPT ( $O$ ) /\* $O$  is a node\*/

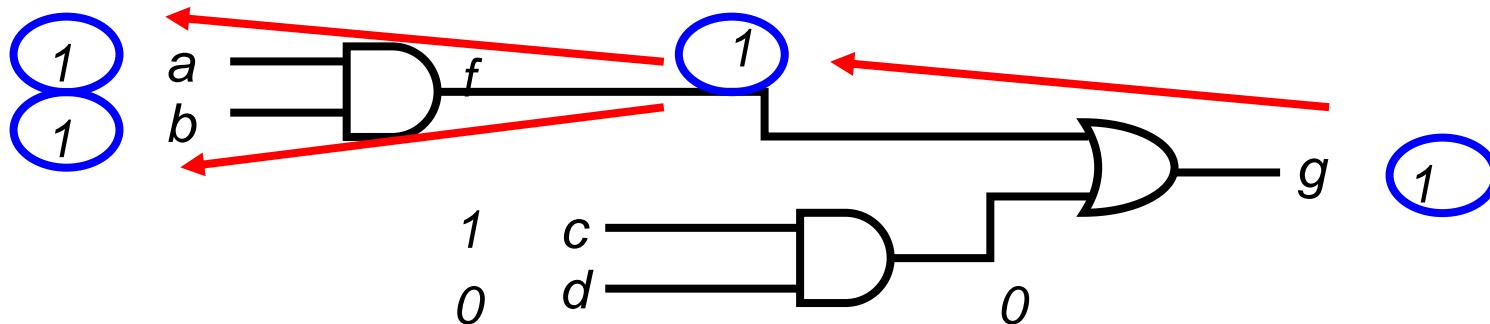
1. **foreach** gate input  $g$  of  $O$
2.   **if** ( $g$  is critical signal) **then**
3.       CPT ( $g$ ) ;
4.   **else return;**

# CPT Theorem

**THEOREM:**

If critical signal  $g$  has good value  $v$ , then  $g$  stuck-at  $v'$  fault is detected

- Example: Detected faults:  $a$  SA0,  $b$  SA0,  $f$  SA0,  $g$  SA0

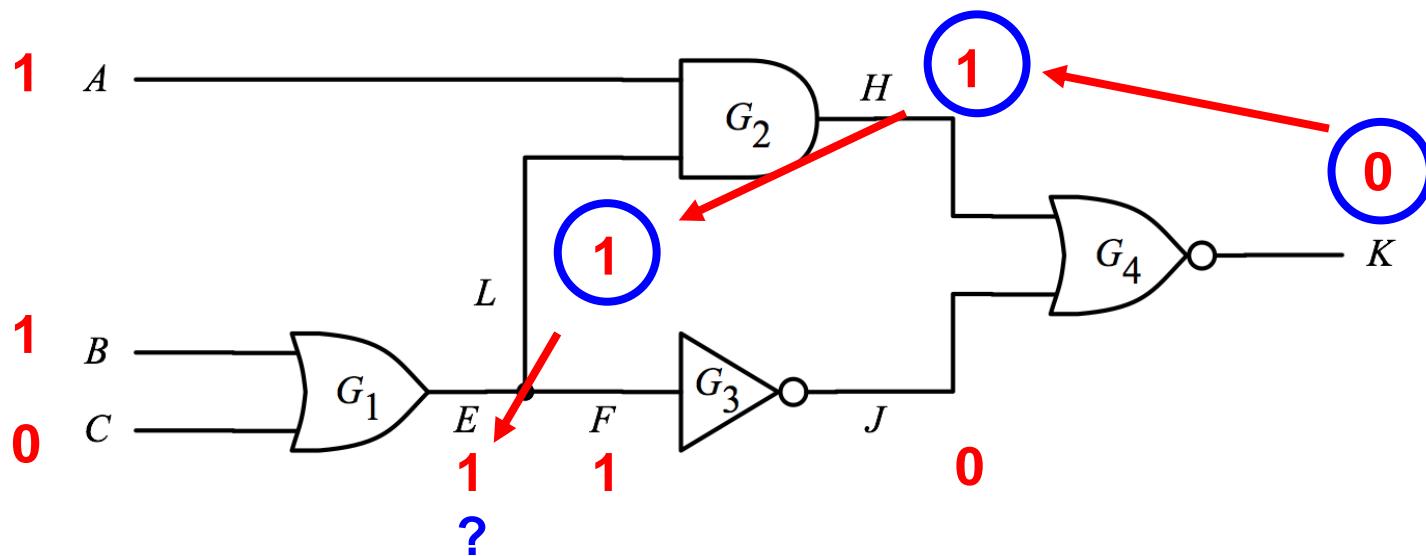


- CPT Advantages:
  - ◆ CPT can be done in linear time
  - ◆ Only logic simulation needed for FFC
  - ◆ Used in first commercial ATPG tool, LASER

**CPT Is Very Fast ... but**

# How about Fanout Stem?

- $L$  is critical;  $F$  is non-critical
- $E$  is non-critical

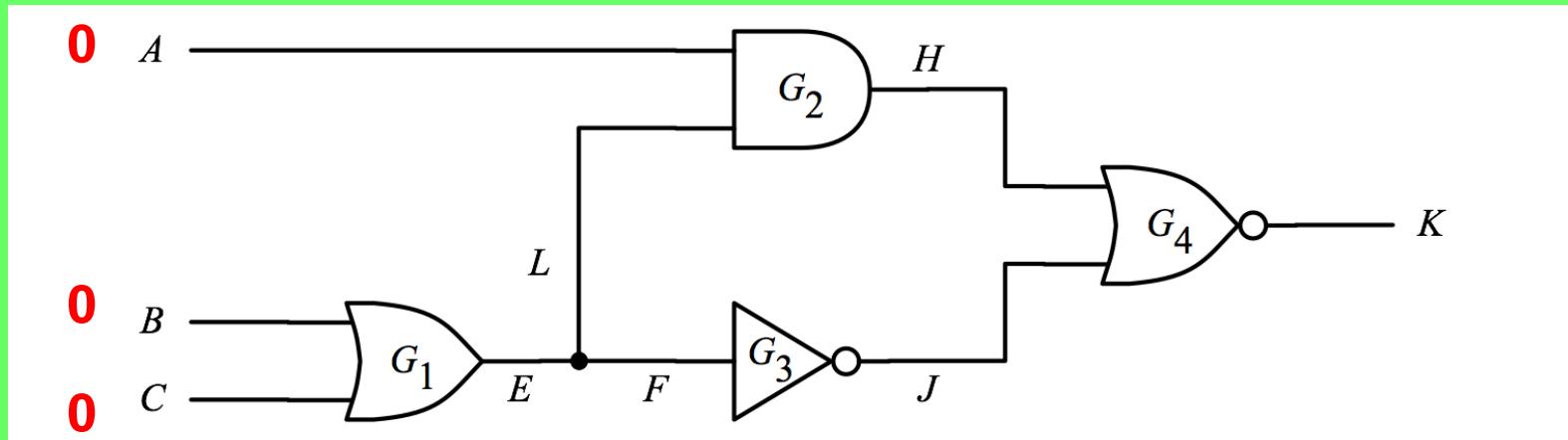


**Is This Always The Case?**  
Fanout stems are non-critical?

# Quiz

Q: Determine if fanout stem E is critical or not

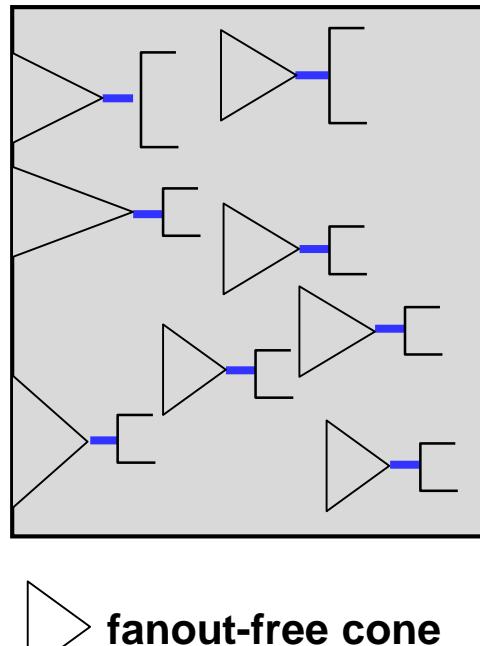
A:



CPT Does NOT Work Well  
when Fanout Reconverges

# So... What Should We Do?

- 1. Count fanout stems as non-critical
  - ◆ Fast but fault coverage pessimistic
- 2. Only fault simulate fanout stems, CPT in fanout-free cones
  - ◆ Slow but fault coverage accurate



# Summary

- Alternatives to fault simulation, fast but inaccurate

## ① Toggle coverage

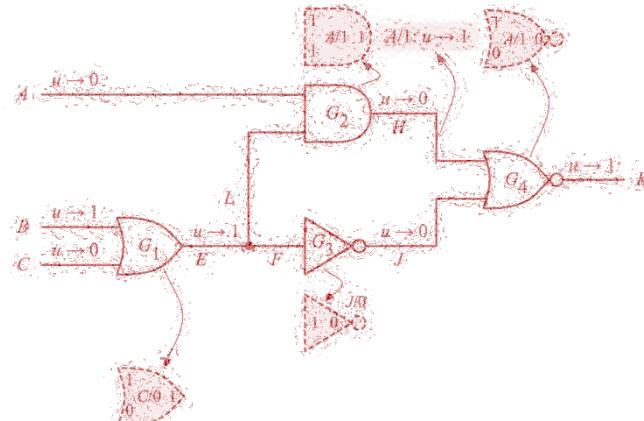
- ◆ Only logic simulation needed

## ② Fault Sampling

- ◆ Only fault simulate a small portion of faults
- ◆ Confidence interval and confidence level

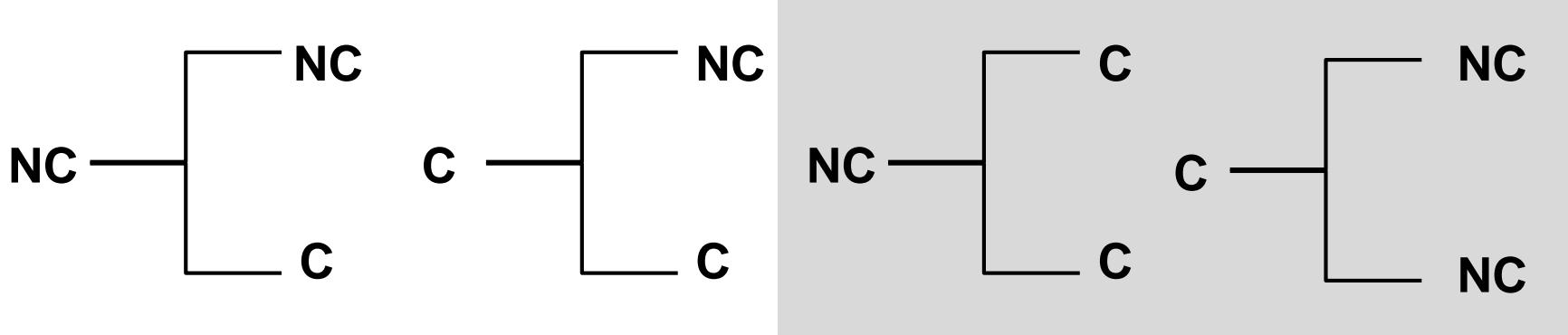
## ③ Critical path tracing

- ◆ Linear time for fanout-free cones
- ◆ Still need fault simulation when fanouts reconverge



# FFT

- Q1: For CPT, all four cases are possible.
  - ◆ We have shown first two cases
  - ◆ Find example for latter two cases?



- Q2: Toggle coverage  $\geq$  fault coverage. Can you prove it?
  - ◆ assume DEF-1 toggle coverage