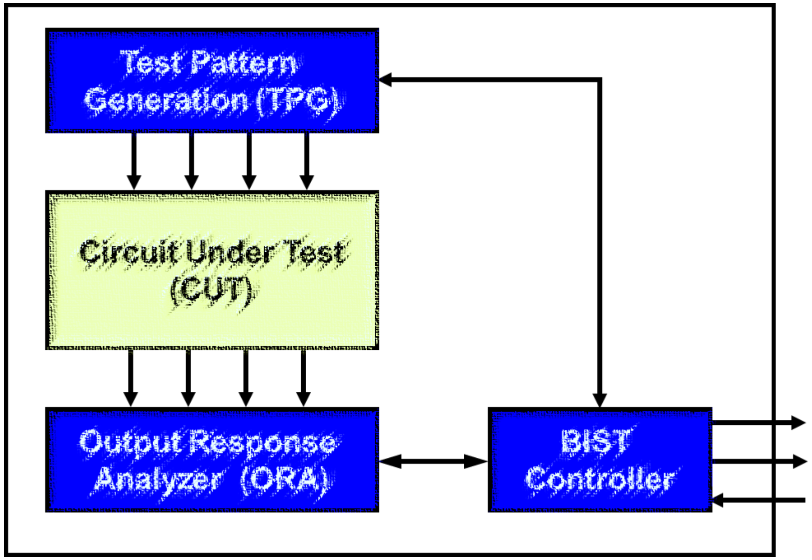
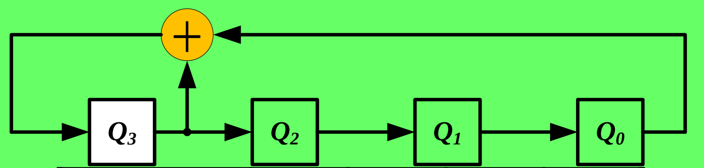
BIST Part1 - TPG

- Introduction
- Test Pattern Generation (TPG)
  - Deterministic: ROM, Algorithm, Counter
  - Pseudo Random
    - Linear Feedback Shift Register, LFSR (1977)
      - Two types of LFSR
      - Design of LFSR
        - ⇒ How to find seed?
        - ⇒ What polynomial?
        - ⇒ All-zero pattern?
        - ⇒ What is LFSR degree?
    - Cellular Automata, CA (1984)



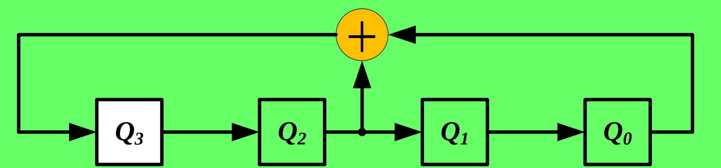
Quiz

Q: Given two LFSR of x^4+x^3+1, x^4+x^2+1. seed=1000. Fill in table. L_c=?



L_c=15

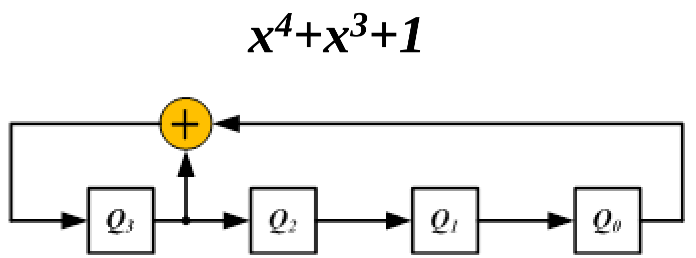
state	Q_3	Q_2	Q_1	Q_0
0	1	0	0	0
1	1	1	0	0
2	1	1	1	0
3	1	1	1	1
4	0	1	1	1
5	1	0	1	1
6	0	1	0	1
7	1	0	1	0
8	1	1	0	1
9	0	1	1	0
10	0	0	1	1
11	1	0	0	1
12	0	1	0	0
13	0	0	1	0
14	0	0	0	1
15 (=0)	1	0	0	0

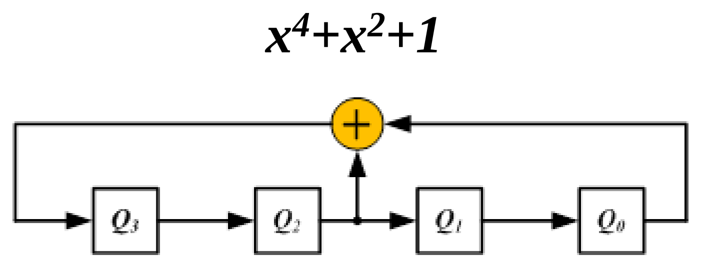


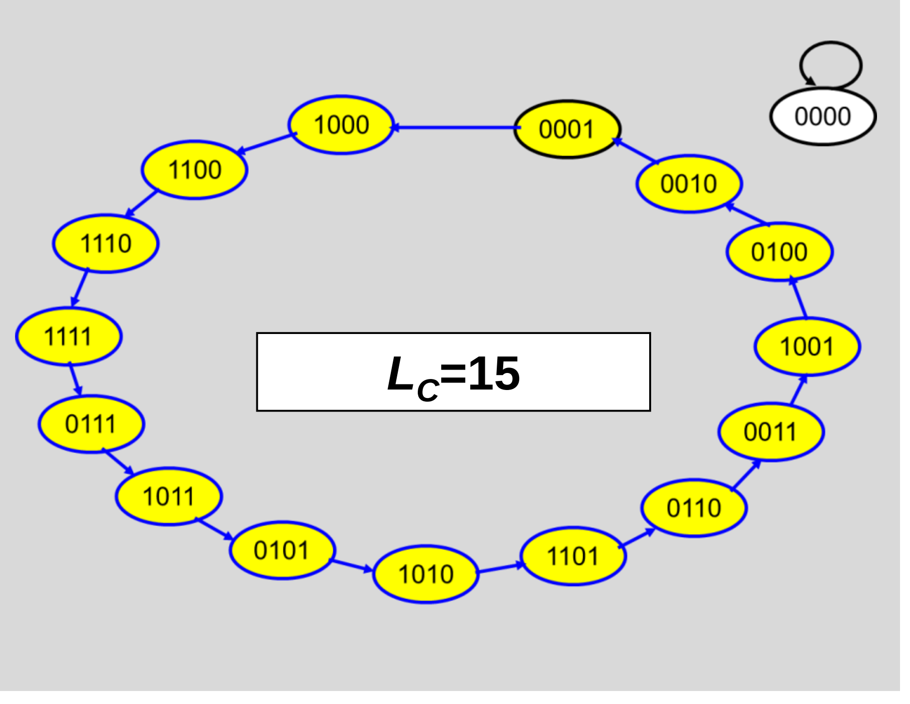
state	Q_3	Q_2	Q_1	Q_0
0	1	0	0	0
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				
15				

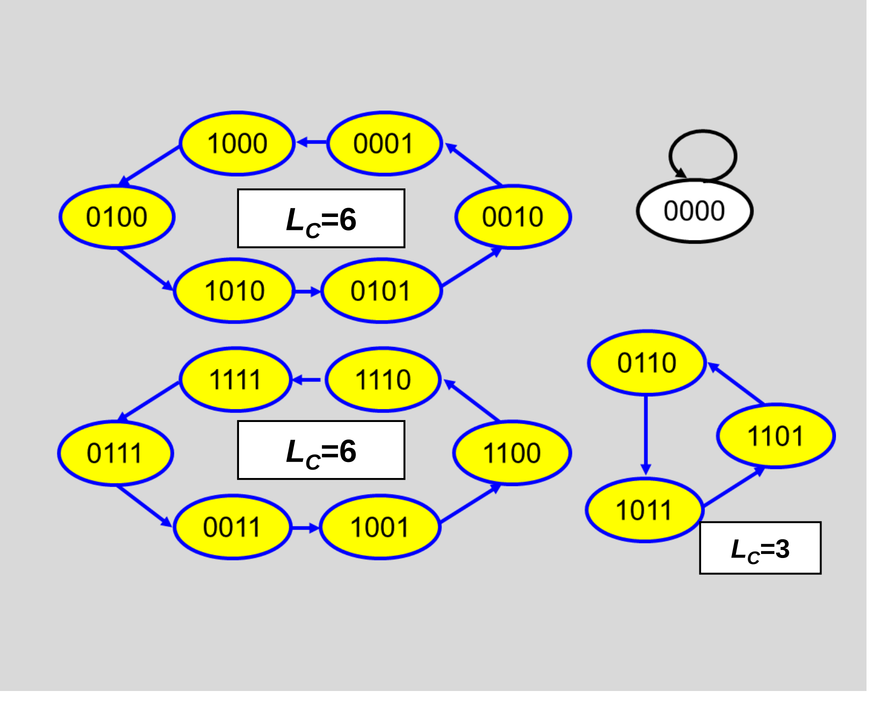
L_c=?

Polynomial and L_c









Larger L_c , Better TPG

Primitive Polynomial

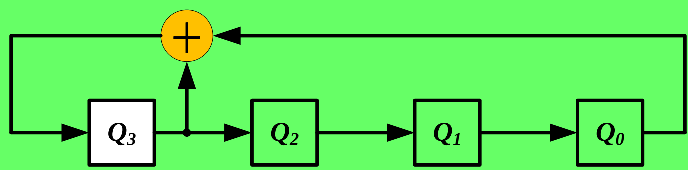
- *Primitive polynomial* (PP) generates *maximum-length sequence*
  - $L_c = 2^N-1$, N=degree of PP
- PP NOT unique for a given N
  - $1+x+x^4$, $1+x^3+x^4$ are both PP
- PP of degree N must satisfy:
  - PP is *irreducible*
  - PP has odd number of terms
  - PP divides $1+x^{2^N-1}$;N>3
  - more details see (BMS 87)
- PP can be found by *Seive Method*
  - Many available on line

N	primitive polynomial $f(X)$
1,2,3,4,6,7,15,22	$1 + X + X^N$
5,11,21,29	$1 + X^2 + X^N$
10,17,20,25,28,31	$1 + X^3 + X^N$
9	$1 + X^4 + X^N$
23	$1 + X^5 + X^N$
18	$1 + X^7 + X^N$
8	$1 + X^2 + X^3 + X^4 + X^N$
12	$1 + X + X^2 + X^4 + X^N$
13	$1 + X + X^3 + X^4 + X^N$
14,16	$1 + X^3 + X^4 + X^5 + X^N$
19,27	$1 + X + X^2 + X^5 + X^N$
24	$1 + X + X^2 + X^7 + X^N$
26	$1 + X + X^2 + X^6 + X^N$
30	$1 + X + X^2 + X^{23} + X^N$
32	$1 + X + X^2 + X^{22} + X^N$

PP Often Used to Design TPG

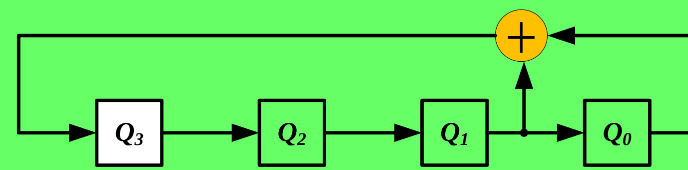
Quiz

Q: Compare two PP x^4+x^3+1, x^4+x+1. Fill in table. L_c=?



state	Q_3	Q_2	Q_1	Q_0
0	1	0	0	0
1	1	1	0	0
2	1	1	1	0
3	1	1	1	1
4	0	1	1	1
5	1	0	1	1
6	0	1	0	1
7	1	0	1	0
8	1	1	0	1
9	0	1	1	0
10	0	0	1	1
11	1	0	0	1
12	0	1	0	0
13	0	0	1	0
14	0	0	0	1
15 (=0)	1	0	0	0

L_c=15

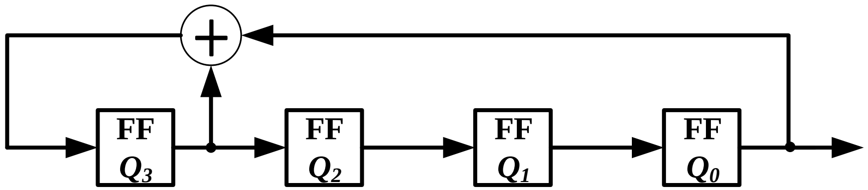


state	Q_3	Q_2	Q_1	Q_0
0	1	0	0	0
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				
15 (=0)				

L_c=?

m-Sequence

- A sequence generated by *primitive polynomial* LFSR is called
  - *Maximum-length sequence* (*m-sequence*)
  - *aka. pseudorandom sequence, pseudonoise sequence*
- m-sequence has period $L_c=2^N-1$
- Example: N=4
  - x^4+x^3+1
  - Period L_c= 15



m-sequence=000111101011001
(left bit first)

state	Q_3	Q_2	Q_1	Q_0
0	1	0	0	0
1	1	1	0	0
2	1	1	1	0
3	1	1	1	1
4	0	1	1	1
5	1	0	1	1
6	0	1	0	1
7	1	0	1	0
8	1	1	0	1
9	0	1	1	0
10	0	0	1	1
11	1	0	0	1
12	0	1	0	0
13	0	0	1	0
14	0	0	0	1

m-sequence

Properties of m-Sequence (1)

- 1. Period $L_c = 2^N - 1$
  - Example: x^4+x^3+1, N=4
  - 000111101011001
  - L_c=15
- 2. 2^{N-1} ones and $2^{N-1}-1$ zeros (almost equal probability)
  - Example:
  - 8 ones, 7 zeros
- 3. If a window of width N is slid along m-sequence, each of 2^N-1 non-zero N-tuple is seen exactly once in a period

0	0	0	1	1	1	1	0	1	0	1	1	0	0	1	0	0	0
0	0	0	1														
	0	0	1	1													
		0	1	1	1												
			1	1	1	1											
				1	1	1	0										
					1	1	0	1									
						1	0	1	0								
							0	1	0	1							
								1	0	1	1						
									0	1	1	0					
										1	1	0	0				
											1	0	0	1			
												0	0	1	0		
													0	1	0	0	
														1	0	0	0

Properties of m-Sequence (2)

- **4. m-sequence + its *cyclic shift* = another cyclic shift of itself**
  - ***cyclic-shift-and-add* property**
  - **This property is useful for *phase shifter* (see BIST-2 chapter)**
  - **Example: m-sequence of x^4+x^3+1, N=4**
    - **NOTE: mod-2 add = XOR**

	0	0	0	1	1	1	1	0	1	0	1	1	0	0	1	Original m-sequence
$\oplus$	1	0	0	0	1	1	1	1	0	1	0	1	1	0	0	Right cyclic-shift by 1 bit
	1	0	0	1	0	0	0	1	1	1	1	0	1	0	1	Right cyclic-shift by 4 bits

- **See *finite filed theory* or (BMS 87) for more properties**

m-sequence Has Nice Properties

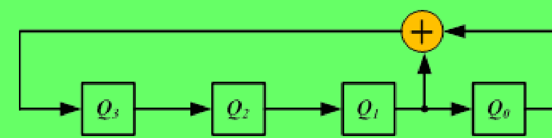
Quiz

Q: Given another m-sequence 000100110101111 generated by PP x^4+x+1.
Show that it also satisfies properties:
1) L_c=15
2) 8 ones and 7 zeros
3) it has cyclic-shift-add property

original + left-shift-by-2-bit = left-shift-by-?-bit

A:

	0	0	0	1	0	0	1	1	0	1	0	1	1	1	1
$\oplus$	0	1	0	0	1	1	0	1	0	1	1	1	1	0	0



	Q_3	Q_2	Q_1	Q_0
0	1	0	0	0
1	0	1	0	0
2	0	0	1	0
3	1	0	0	1
4	1	1	0	0
5	0	1	1	0
6	1	0	1	1
7	0	1	0	1
8	1	0	1	0
9	1	1	0	1
10	1	1	1	0
11	1	1	1	1
12	0	1	1	1
13	0	0	1	1
14	0	0	0	1

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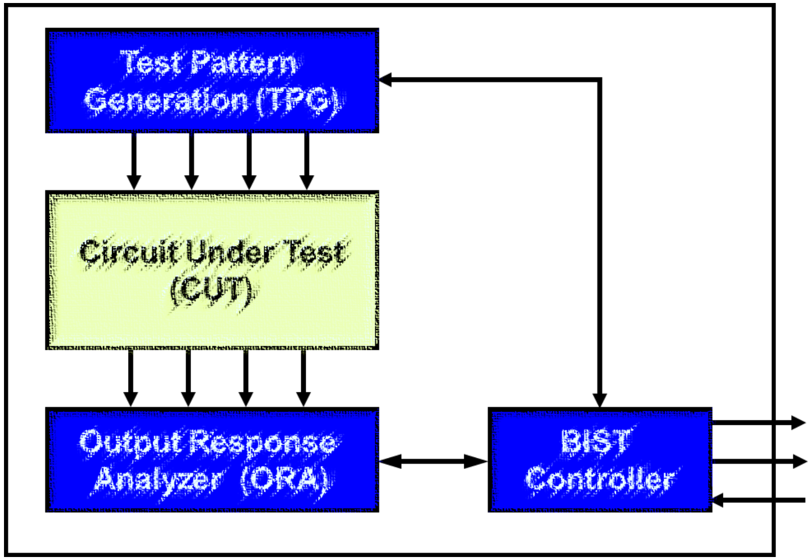
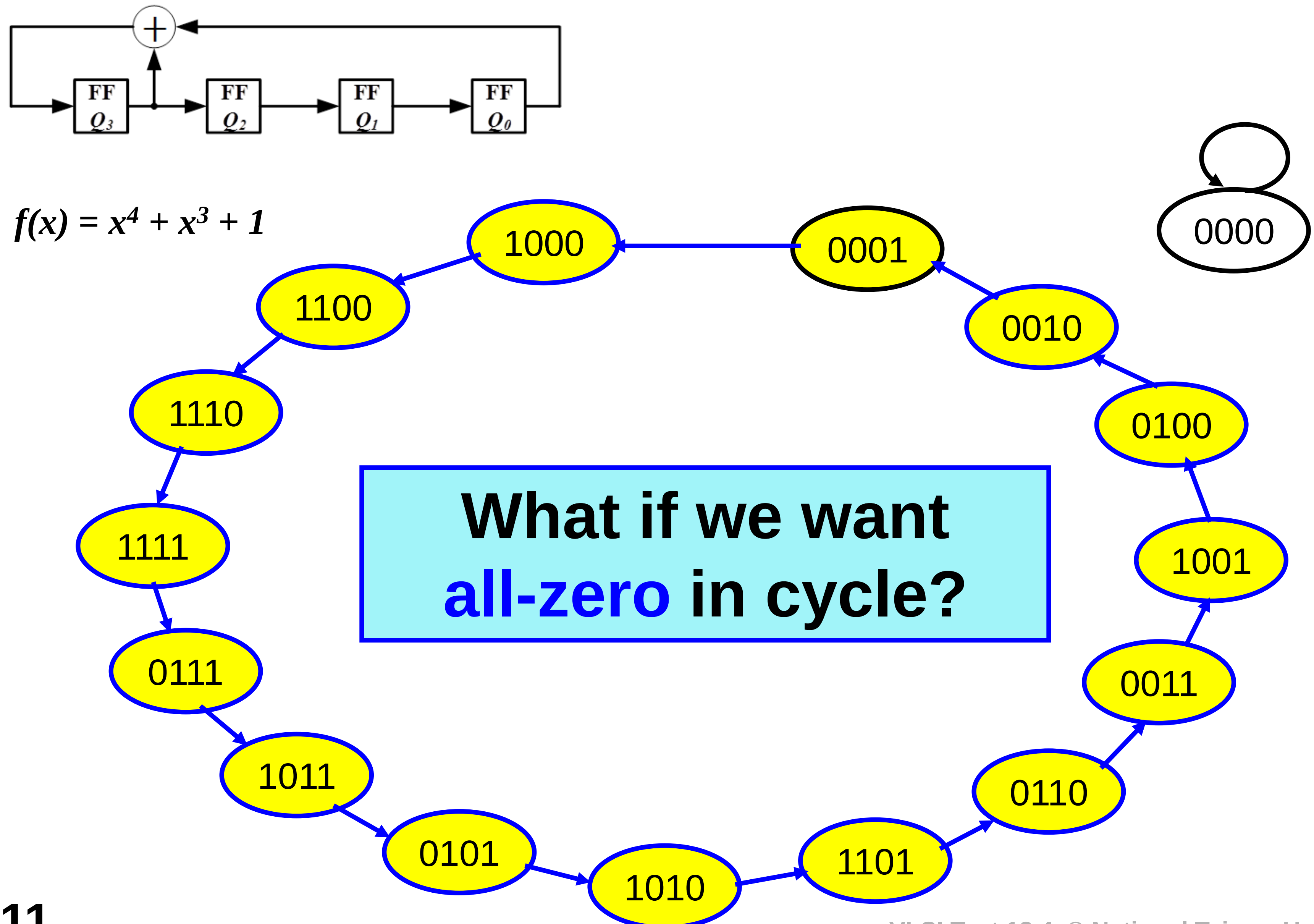


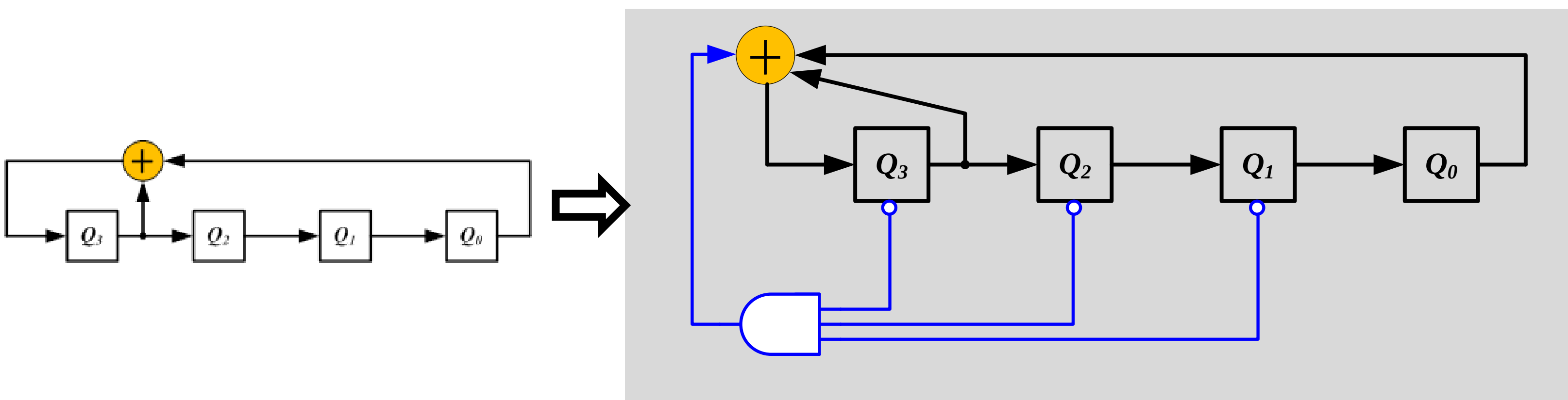
State Diagram



What if we want all-zero in cycle?

Exhaustive Feedback Shift Register

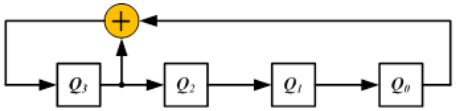
- **Exhaustive feedback Shift Register (aka. *De Bruijn Counter*)**
  - **2^N different states**
- **Example: *N*=4; 16 exhaustive states**
  - **0001→ <u>0000</u> →1000**

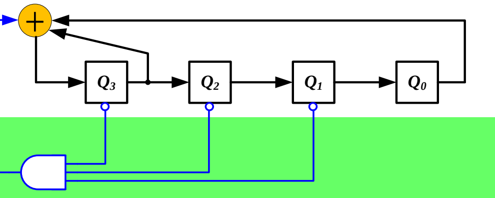


- **NOTE: This is NOT LFSR because AND is nonlinear**

DB Counter Generates All-zero Patterns

Quiz





Q: Show this DB counter generates all 16 states. Fill in table.

state	Q_3	Q_2	Q_1	Q_0
0	1	0	0	0
1	1	1	0	0
2	1	1	1	0
3	1	1	1	1
4	0	1	1	1
5	1	0	1	1
6	0	1	0	1
7	1	0	1	0
8	1	1	0	1
9	0	1	1	0
10	0	0	1	1
11	1	0	0	1
12	0	1	0	0
13	0	0	1	0
14	0	0	0	1
15 =0	1	0	0	0

state	Q_3	Q_2	Q_1	Q_0
0	1	0	0	0
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				
15				
16				

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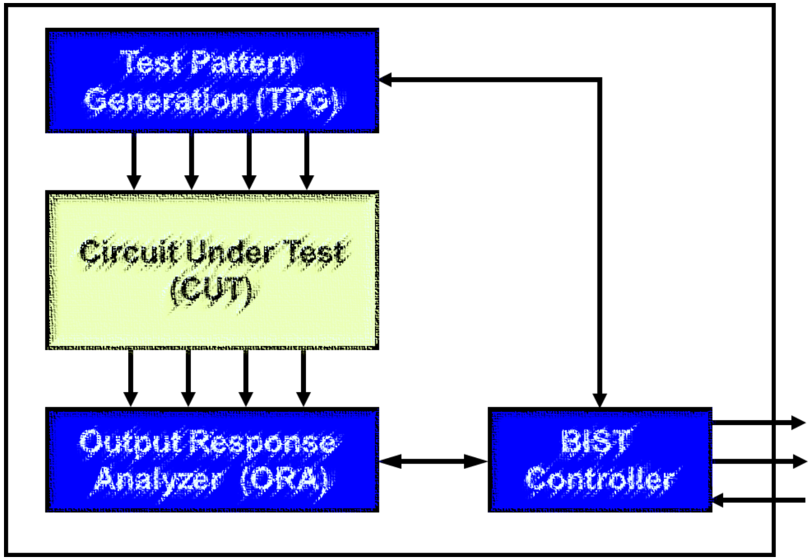


What is LFSR Degree? [Konemann 91] *not in exam

- Suppose N-*degree* LFSR, S care bits (*i.e.* specified bits)
  - N variables, S equations
  - What is minimum N to guarantee a solution (seed) can be found?
- Probability of not finding a solution

$$P(S,N)=\prod_{i=0}^{S-1}\frac{\left(2^N-1\right)-\left(2^i-1\right)}{\left(2^N-1\right)-1}\approx 1-e^{\left(-2^{(S-N)}\right)}$$

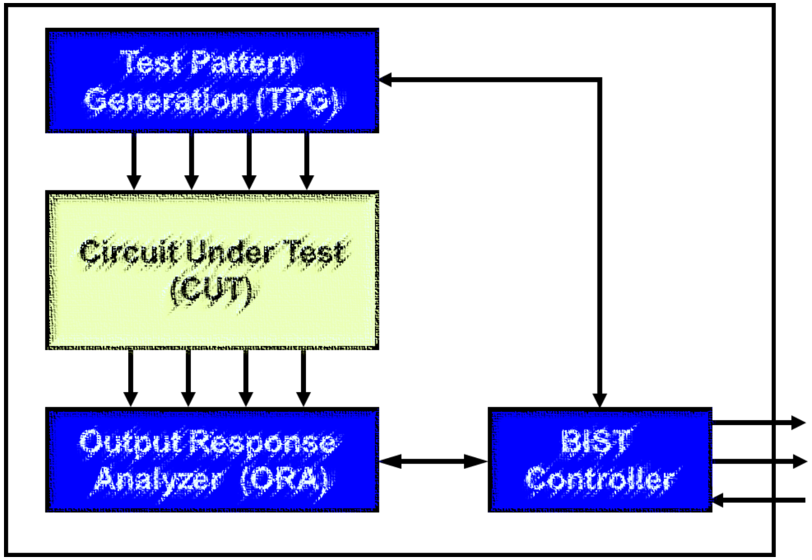
  - When $N \geq (S+20)$, probability not finding solution is 10^{-6}

- Example: scan chain length = 1,000. Assume 5% care bits
  - S = 1000x5%=50; N = 50+20=70
  - Primitive polynomial: $f(x)=x^{70}+x^{16}+x^{15}+x+1$

LFSR Degree $N \geq (S + 20)$

Summary

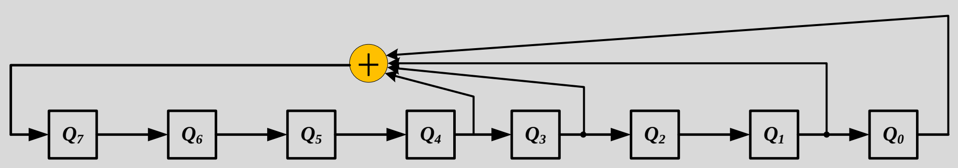
- What polynomial to use?
  - Primitive polynomials generates m-sequence
    - $L_c=2^N-1$
    - Almost half one and half zero
    - Cyclic-shift-and-add property
- How to generate all-zero pattern?
  - Exhaustive feedback shift register
- How many degree *N*?
  - $N \geq$ care-bits + 20



Pros and Cons of LFSR

- Advantages
  - ☺ Small area.
  - ☺ Easy to design: PP → structure
  - ☺ m-sequence has nice properties
- Disadvantages
  - ☹ Not enough randomness
  - ☹ Large LFSR has many tap points. too slow!

How to solve? see Cellular Automata



$$f(x) = x^8 + x^4 + x^3 + x^2 + 1$$

state	Q_0	Q_1	Q_2	Q_3
0	1	0	0	0
1	0	1	0	0
2	0	0	1	0
3	0	0	0	1
4	1	1	0	0
5	0	1	1	0
6	0	0	1	1
7	1	1	0	1
8	1	0	1	0
9	0	1	0	1
10	1	1	1	0
11	0	1	1	1
12	1	1	1	1
13	1	0	1	1
14	1	0	0	1

↔ 1 ↔ 1

LFSR Is Most Popular TPG

FFT

- Q: *De Bruijn Counter* generates 2^N sequence
  - N=4, L_c=16. 0001111010110010
  - Does it satisfy cyclic-shift-and-add property?

	0	0	0	1	1	1	1	0	1	0	1	1	0	0	1	0
⊕	0	0	0	0	1	1	1	1	0	1	0	1	1	0	0	1

?