

Test without Fault Model

- Introduction
- **Boolean Tests without Fault Model**
 - ◆ Toggle Test
 - ◆ Design Verification
 - ◆ Exhaustive Test
 - ◆ Pseudo Exhaustive Test (PET)
 - * Individual Output Verification
 - Dependence Matrix
 - Minimum IOV test
 - * Segment Verification
- Conclusions



Examples in this PPT are courtesy of McCluskey's lecture note at Stanford U.

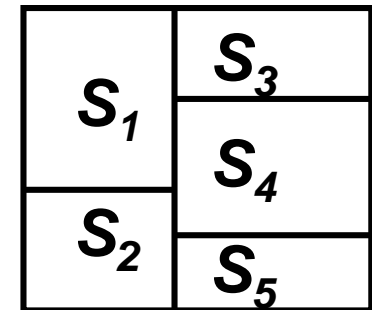
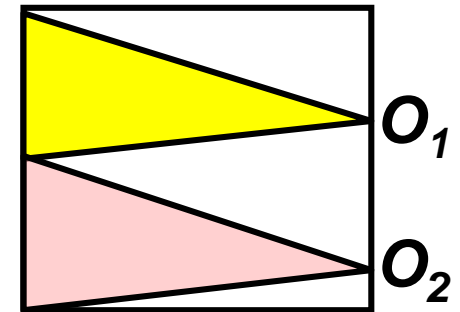
Exhaustive Test

- Apply all possible input combination
 - ◆ Combinational circuits with n inputs
 - * Test length = 2^n
- Advantages of Exhaustive Test
 - + No ATPG required
 - + No fault simulation
 - + Suitable for BIST (no storage needed)
 - + Very high fault coverage
- Disadvantages
 - Long test length

But... Really Need 2^n Patterns?

Pseudo Exhaustive Test (PET) [McCluskey 84]

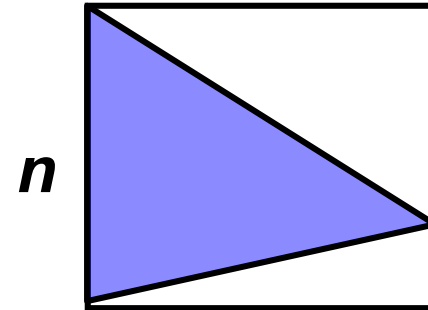
- Idea
 - ♦ Do not need exhaustive test for whole circuit
 - ♦ Test each circuit **partition** exhaustively
- Two categories:
 1. **Individual Output Verification (IOV)**
 - * Exhaustive test of each output
 - * This video
 2. **Segment Verification**
 - * Exhaustive test of each segment
 - * Next video



IOV Test, Circuit Classification

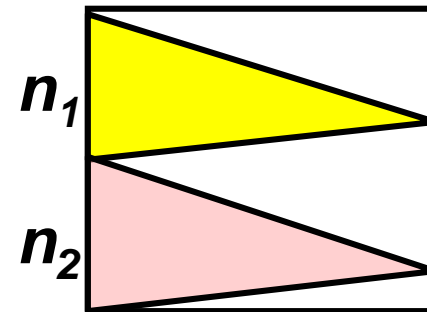
- **Full Dependence Circuit - FDC**

- ♦ Some Outputs Depend on All Inputs
- ♦ IOV **not effective**
 - * Exhaustive test length: 2^n
 - * IOV test length: 2^n



- **Partial Dependence Circuit - PDC**

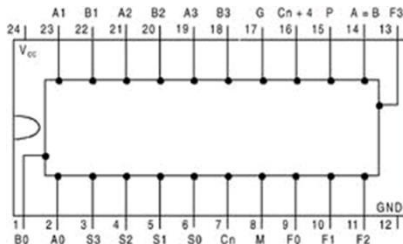
- ♦ No output depends on all inputs
- ♦ IOV effectively reduce test length
 - * $2^{n_1+n_2} \gg (2^{n_1} + 2^{n_2})$



IOV Test is Useful to PDC, not FDC

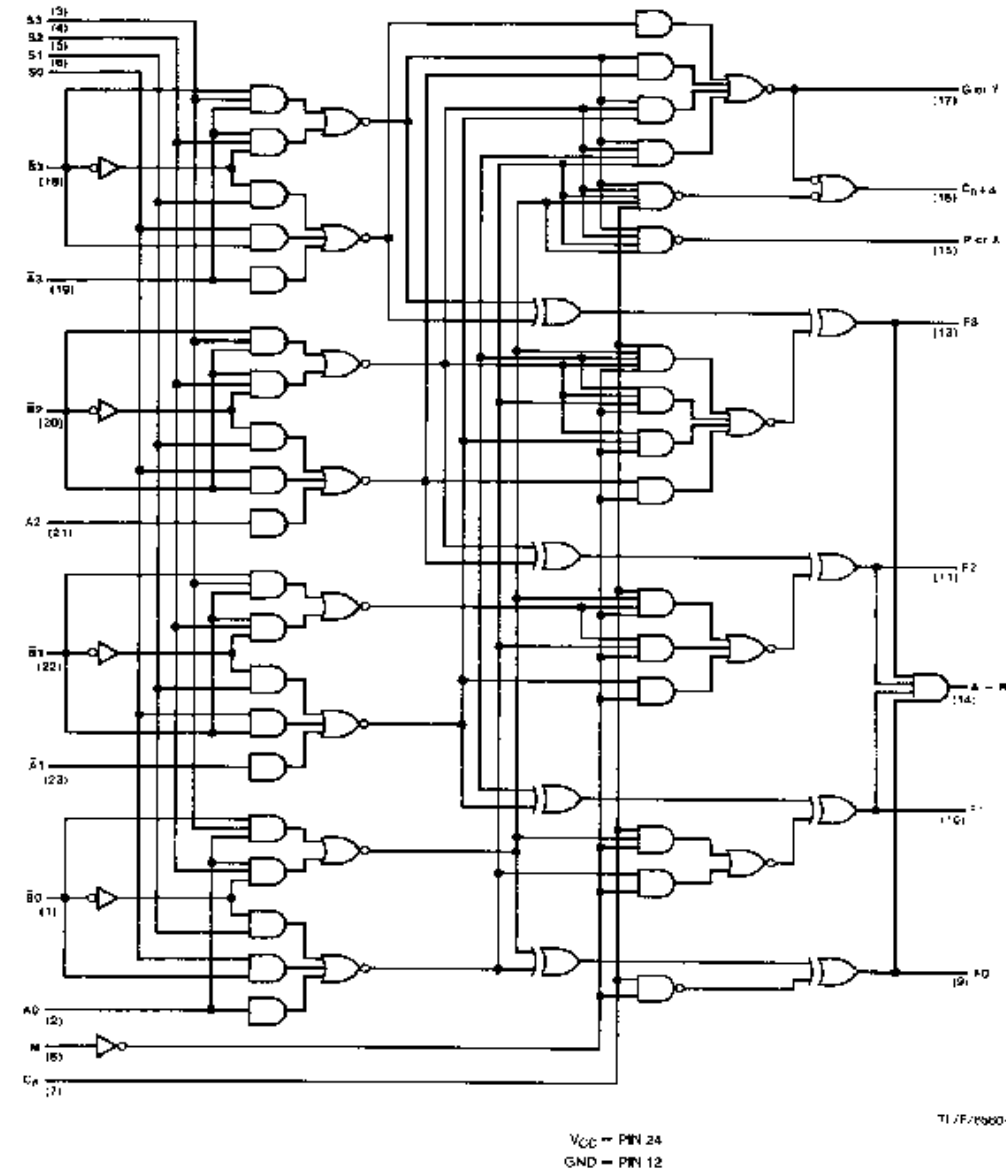
FDC Example - 74181 ALU

- 4-bit arithmetic logic unit (ALU)¹⁸¹
 - First ALU on single chip
 - Used as arithmetic/logic core in CPUs of many historically significant mini-computers
- 14 inputs, 8 outputs
 - Some outputs depend on **all inputs**
 - $IOV = 2^{14} = \text{exhaustive}$

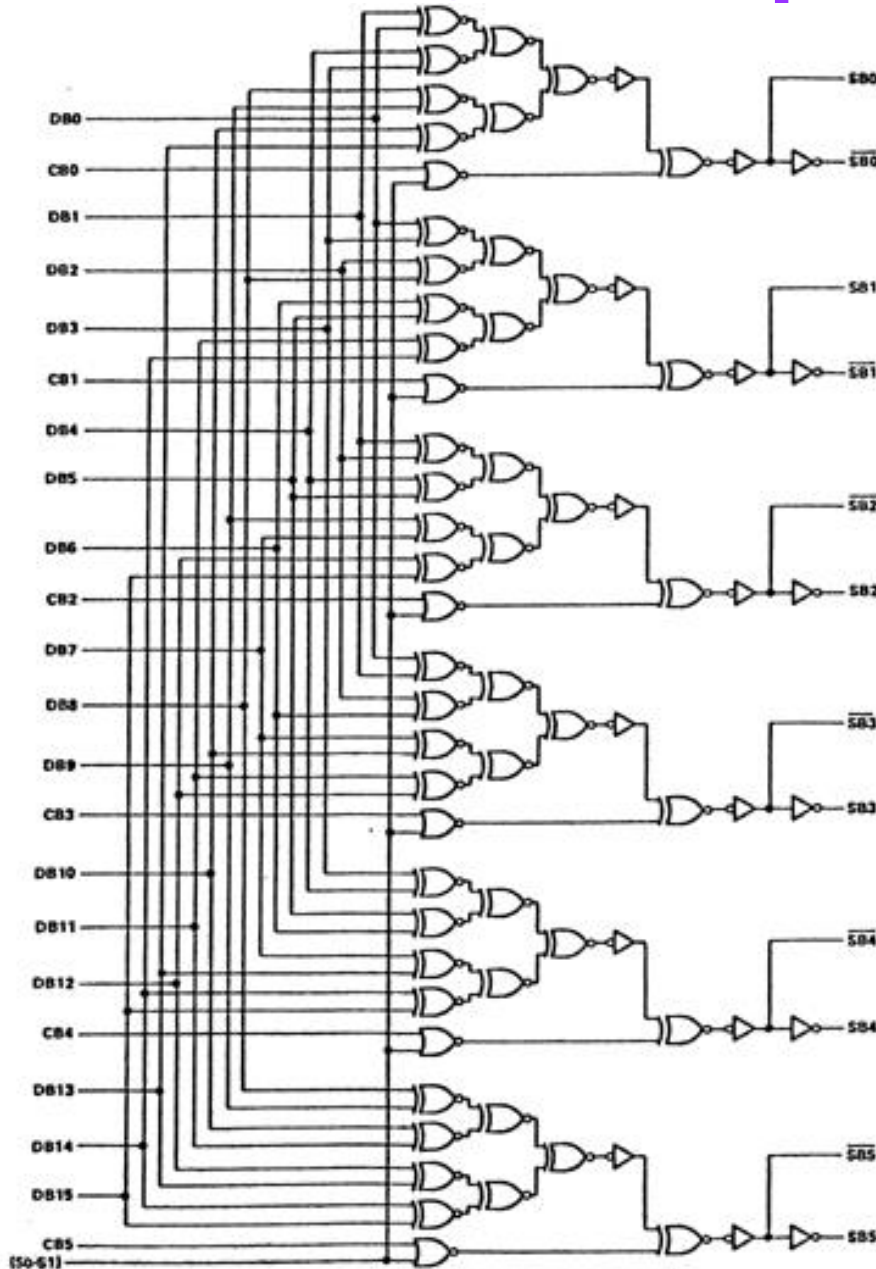


IOV not Useful to FDC

Logic Diagram



PDC Example – 74LS630



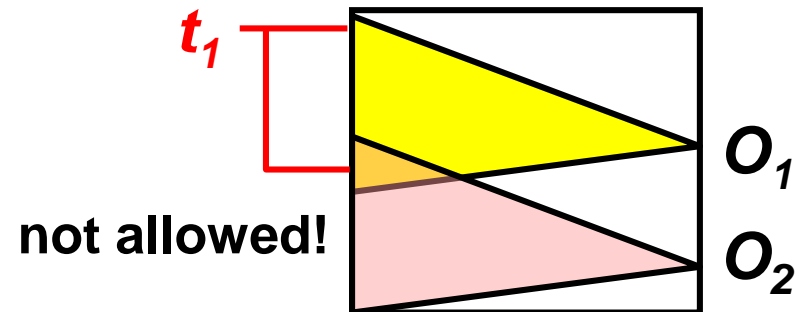
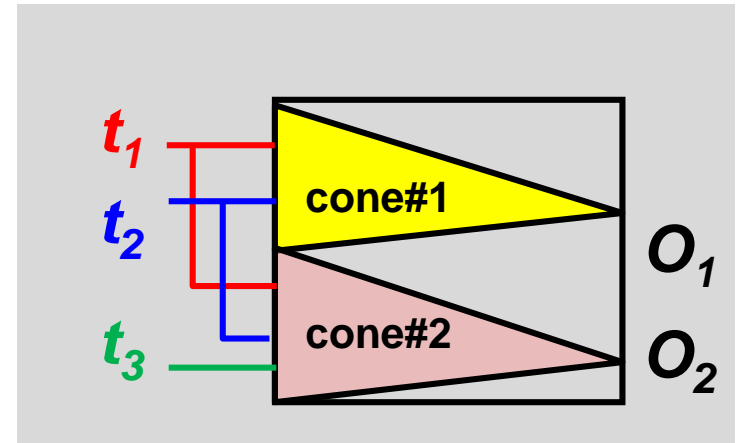
- 16-BIT parallel error detection and correction IC
- 24 inputs, 6 outputs
 - ♦ Each output depends on **10** inputs
 - ♦ Exhaustive test = 2^{24}
 - ♦ IOV test = 6×2^{10}



Can Do Better than 6×2^{10} ?

We Can Share Test Signals !

- Share **test signals** to test each output simultaneously can further reduce TL
- If cones do NOT overlap
 - ♦ Can arbitrarily share test signals with inputs of different output
 - ♦ $TL = 2^{n_1} + 2^{n_2} \rightarrow 2^{\max(n_1, n_2)}$
 - * e.g. $n_1=2, n_2=3$; $TL=2^3$
- If cones overlap
 - ♦ CANNOT share test signals with inputs of same output

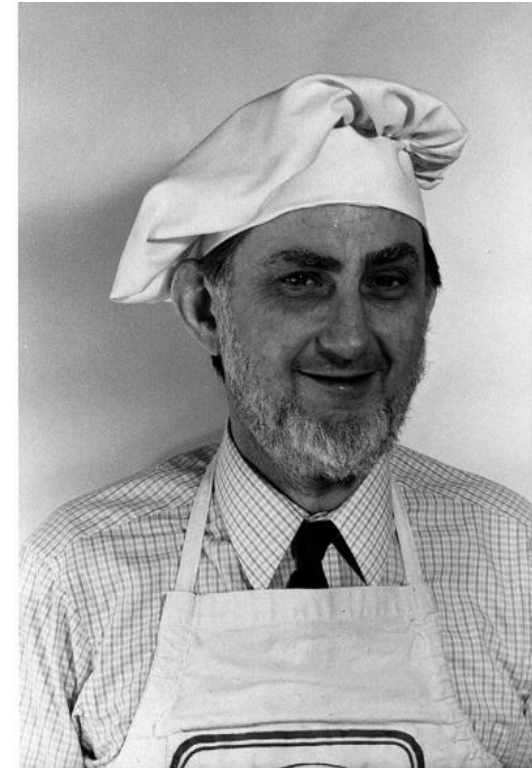


Cannot Share Inputs of Same Cone

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**How Many Test Signals
We Can Share?**



McCluskey and his
collection of hats

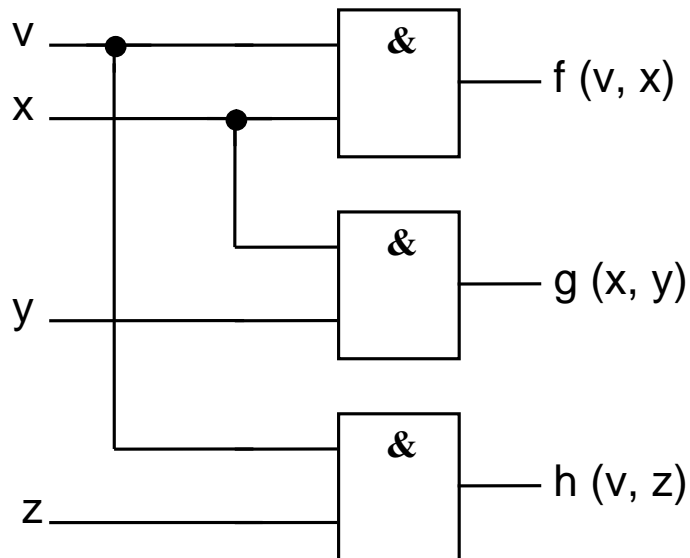
Dependence Matrix

- **Dependence matrix**

- ♦ Each row is an output; each column is an input
- ♦ $D_{ij} = 1$ iff output i depends on input j
- ♦ **Row weight** = sum of each row
- ♦ **Max Row Weight** = w = Max number of inputs to an output

- **Example**

- ♦ $w=2$



Outputs	Inputs				Row weight
	v	x	y	z	
$f(v, x)$	1	1	0	0	2
$g(x, y)$	0	1	1	0	2
$h(v, z)$	1	0	0	1	2

Max row weight = 2 = w

Partitioned Dependence Matrix

- **Partitioned dependence matrix**

- ◆ Each row of a partition has **at most one 1 entry**

- * Do not share test signals with inputs of same output

- ◆ **Number of partitions = p** = Number of test signals needed

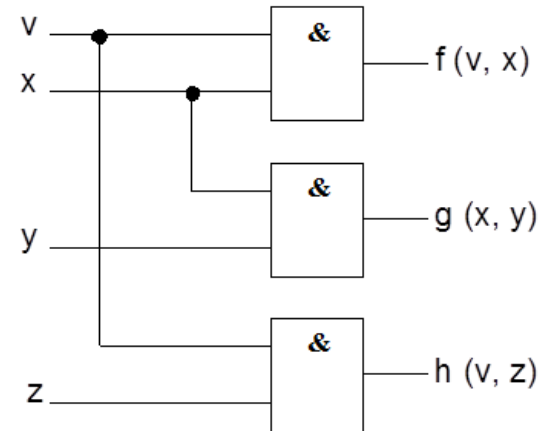
- **Example**

- ◆ $p=3$

Outputs	Inputs			
	v	x	y	z
$f(v, x)$	1	1	0	0
$g(x, y)$	0	1	1	0
$h(v, z)$	1	0	0	1

- ◆ $p=2$

Outputs	Inputs			
	v	y	x	z
$f(v, x)$	1	0	1	0
$g(x, y)$	0	1	1	0
$h(v, z)$	1	0	0	1

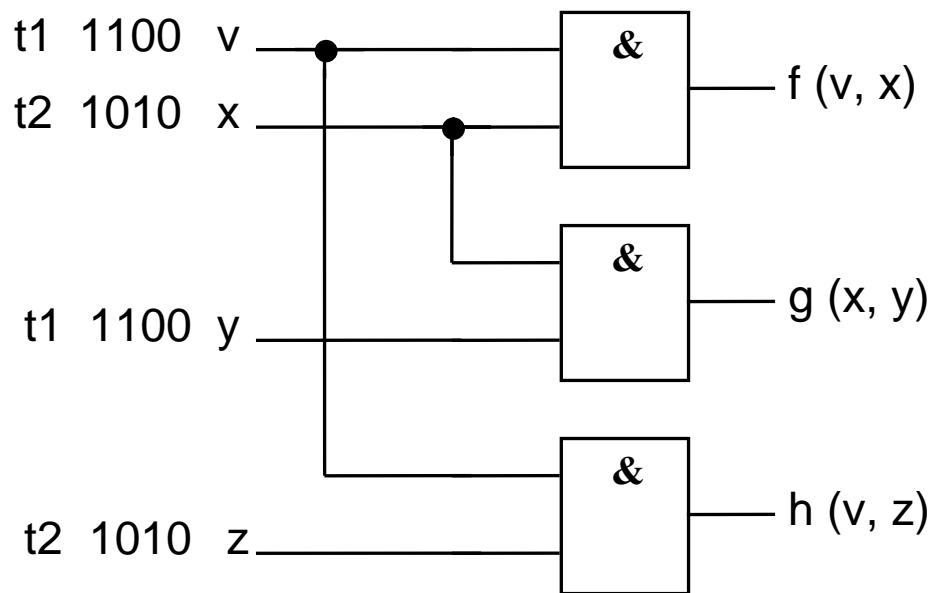


$p \geq w$, Why?

- PDM is not unique. Smaller p is preferred

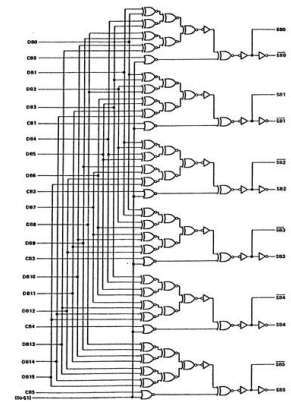
Minimum Test Length =?

- Example: $p=2$; $w=2$
 - ◆ Number of test signals required = $p = 2$
 - ◆ Shortest test length = $2^w = 4$
 - ◆ Each output is exhaustively tested



Outputs	Inputs			
	v	y	x	z
$f(v, x)$	1	0	1	0
$g(x, y)$	0	1	1	0
$h(v, z)$	1	0	0	1

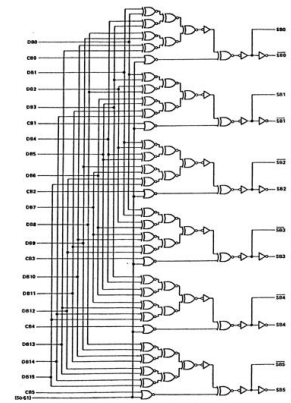
DM of 74LS630



- 74LS630
 - ◆ 24 inputs, 6 outputs, $w=10$

	DATA BITS																CHECK BITS							
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	0	1	2	3	4	5	S0.S1	rw
SB0	1	1	0	1	1	0	0	0	1	1	1	0	0	1	0	0	1	0	0	0	0	0	1	10
SB1	1	0	1	1	0	1	1	0	1	0	0	1	0	0	1	0	0	1	0	0	0	0	1	10
SB2	0	1	1	0	1	1	0	1	0	1	0	0	1	0	0	1	0	0	1	0	0	0	1	10
SB3	1	1	1	0	0	0	1	1	0	0	1	1	1	0	0	0	0	0	0	1	0	0	1	10
SB4	0	0	0	1	1	1	1	1	0	0	0	0	0	1	1	1	0	0	0	0	1	0	1	10
SB5	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0	0	1	1	10

PDM of 74LS630



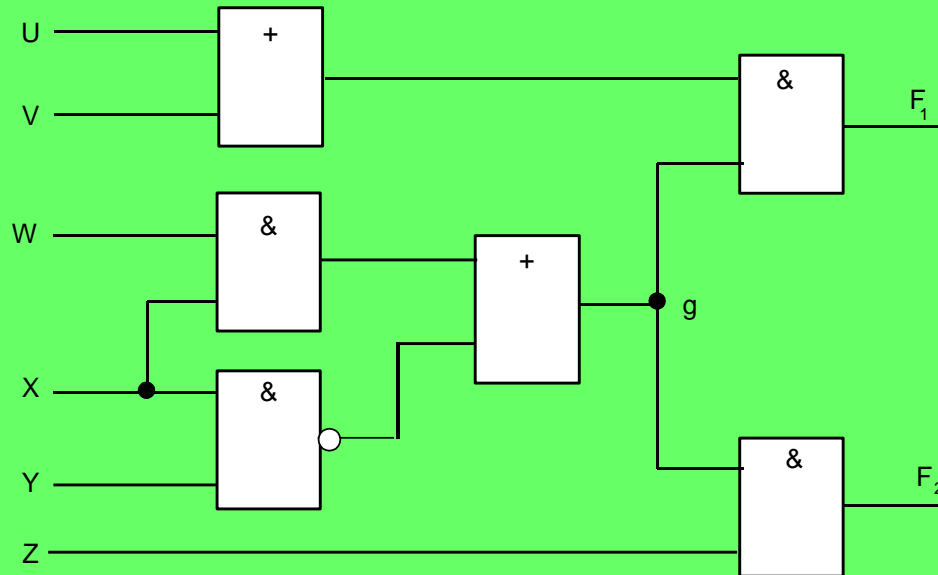
- $p=w=10$
- Minimum 10 test signals required
- Minimum test length = $6 \times 2^{10} \rightarrow 2^{10}$

	DATA BITS												CHECK BITS						S0					
	0	15	1	14	3	12	4	11	8	7	9	6	10	5	13	2	0	1	2	3	4	5	.S1	
SB0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	0	0	0	0	0	1
SB1	1	0	0	1	1	0	0	1	1	0	0	1	0	1	0	1	0	1	0	0	0	0	0	1
SB2	0	1	1	0	0	1	1	0	0	1	1	0	0	1	0	1	0	0	1	0	0	0	0	1
SB3	1	0	1	0	0	1	0	1	0	1	0	1	1	0	0	1	0	0	0	1	0	0	0	1
SB4	0	1	0	1	1	0	1	0	0	1	0	1	0	1	1	0	0	0	0	0	1	0	0	1
SB5	0	1	0	1	0	1	0	1	1	0	1	0	1	0	1	0	0	0	0	0	0	0	1	1

p = 10, w = 10

Quiz

Q: Find the partitioned dependence matrix for this circuit. $p=?$ $w=?$



	W	X	Y	Z	U	V	Row Weight
F1							
F2							

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**Now We Have p & w ,
What is Mini IOV Test?**

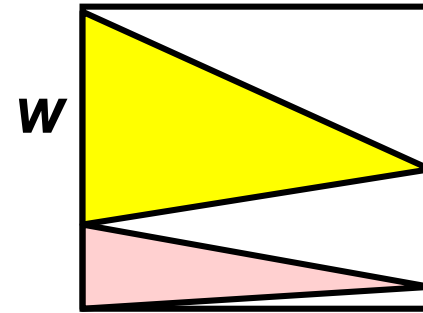


McCluskey and his
collection of hats

Minimum IOV Test Length

- Minimum IOV test length must $\geq 2^w$

- ♦ why?



- Three cases:

- ① $p = w$

- * Minimum IOV test length $= 2^w$ always possible

- ② $p = w + 1$

- * Minimum IOV test length $= 2^w$ always possible

- ③ $p > w + 1$

- * Minimum IOV test length $\geq 2^w$

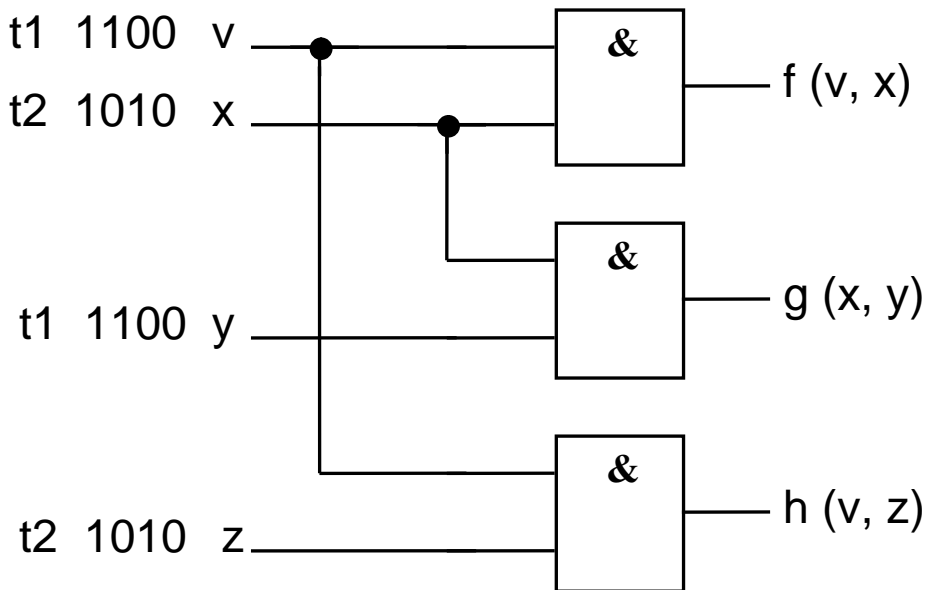
- * 2^w may or may not be achievable

Mini IOV TL Has 3 Cases

① Example: $p=w$

- Example: $p=2, w=2$

♦ Minimum IOV test length = $2^w = 2^2 = 4$



DP(N)				
Outputs	Inputs			
	v	y	x	z
$f(v, x)$	1	0	1	0
$g(x, y)$	0	1	1	0
$h(v, z)$	1	0	0	1

① Minimum IOV TL = 2^w

② Example: $p=w+1$

- Given w test signals t_1, t_2, \dots, t_w
 - XOR them to generate $t_{w+1} = t_1 \oplus t_2 \oplus \dots \oplus t_w$
- Example: $p = 3, w = 2$
 - $z = x \oplus y$
 - Any pair is exhaustive: $(x,y) (y,z) (x,z)$. This is *orthogonal array*

dependency matrix

	x	y	z
f1	1	1	0
f2	0	1	1
f3	1	0	1

IOV Test Patterns

x	y	$z=x \oplus y$
0	0	0
1	1	0
0	1	1
1	0	1

② Minimum IOV TL = 2^w

Orthogonal Array [Rao 1946]

- Entries are a finite **symbol set**, like {0,1}
- For every selection of p columns
 - ♦ All ordered **w -tuples** of symbols appear same number of times
 - ♦ w is called **strength** of orthogonal array
- Example: OA with symbol set {1,0}, strength $w=2$
 - ♦ (0,0)(1,0)(0,1)(1,1) appear **exactly once for every 2 columns**
 - ♦ OA is NOT unique

x	y	z
0	0	0
1	1	0
0	1	1
1	0	1

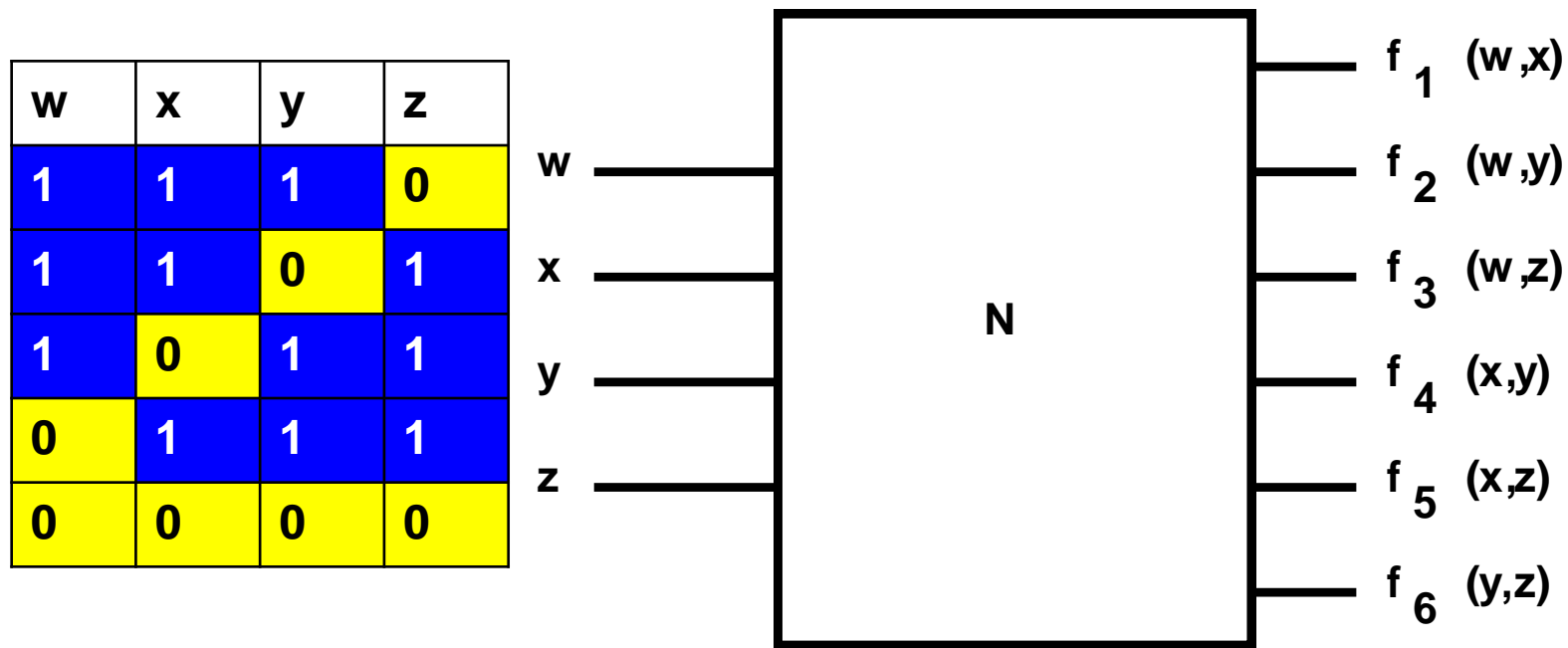
x	y	z
1	1	1
1	0	0
0	1	0
0	0	1

OA Provides IOV Test for $p=w+1$

③ Example: $p > w+1$

- Example: $p=4, w=2$

♦ Minimum IOV test length = row = $5 > 2^w$



How to Find IOV Test for Given $p > w+1$?

Universal IOV Test Table [McCluskey 84]

- This table provides general solution for $p > w+1$
- $U[i,j]$ represents p -column matrix with
 - ♦ all possible weight i and all possible weight j rows

w	U[i, j]		Range	Number of Tests
2	U[0, p-1]	U[1, p]	$p > 3$	$p+1$
3	U[1, p-1]		$p > 4$	$2p$
4	U[1, p-2]	U[2, p-1]	$p > 5$	$p(p+1)/2$
5	U[2, p-2]		$p > 6$	$p(p-1)$
6	U[2, p-3]	U[3, p-2]	$p > 8$	$B(p+1, 3)$
6	U[1, 4, 7]		$p = 8$	$p(p+1)/2$
7	U[3, p-3]		$p > 9$	$2 B(p, 3)$
7	U[0, 3, 6, 9]		$p = 9$	170
8	U[3, p-4]	U[4, p-3]	$p > 11$	$B(p+1, 4)$
8	U[0, 3, 6, 9]	U[1, 4, 7, 10]	$p = 10$	341
8	U[0, 4, 8]	U[3, 7, 9]	$p = 11$	496
9	U[4, p-4]		$p > 12$	$2 B(p, 4)$
9	U[1, 4, 7, 10]		$p = 11$	682
9	U[0, 4, 8, 12]		$p = 12$	992
10	U[4, p-5]	U[5, p-4]	$p > 14$	$B(p+1, 5)$
10	U[1, 4, 7, 10]	U[2, 5, 8, 11]	$p = 12$	1365
10	U[0, 4, 8, 12]	U[1, 5, 9, 13]	$p = 13$	2016
10	U[0, 5, 10]	U[4, 9, 14]	$p = 14$	3004

$B(i,j)$ is
binomial coefficient,
 i things taken j

Example: $p=4, w=2$

w	U[i, j]		Range	Number of Tests
2	U[0, p-1]	U[1, p]	$p > 3$	$p+1$
3	U[1, p-1]		$p > 4$	$2p$
4	U[1, p-2]	U[2, p-1]	$p > 5$	$p(p+1)/2$
5	U[2, p-2]		$p > 6$	$p(p-1)$
6	U[2, p-3]	U[3, p-2]	$p > 8$	$B(p+1, 3)$
6	U[1, 4, 7]		$p = 8$	$p(p+1)/2$
7	U[3, p-3]		$p > 9$	$2 B(p, 3)$
7	U[0, 3, 6, 9]		$p = 9$	170
8	U[3, p-4]	U[4, p-3]	$p > 11$	$B(p+1, 4)$
8	U[0, 3, 6, 9]	U[1, 4, 7, 10]	$p = 10$	341
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10	U[0, 4, 8, 12]	U[1, 5, 9, 13]	$p = 13$	2016
10	U[0, 5, 10]	U[4, 9, 14]	$p = 14$	3004

- Choice 1
- $U[0, p-1]=U[0,3]$

t_1	t_2	t_3	t_4
0	0	0	0
1	1	1	0
0	1	1	1
1	1	0	1
1	0	1	1

\leftarrow RW=0
 $\left\{ \right.$ RW=3

- Choice 2
- $U[1, p]=U[1,4]$

t_1	t_2	t_3	t_4
1	0	0	0
0	1	0	0
0	0	1	0
0	0	0	1
1	1	1	1

$\left\{ \right.$ RW=1
 \leftarrow RW=4

③ Example: $p > w + 1$

- Universal IOV table may **NOT be optimal** solution
 - ◆ Sometimes, we can do better than universal IOV
- Example: $p=5, w=3$
 - ◆ Minimum IOV TL = $8 = 2^w < 2p$

w	U[i, j]		Range	Number of Tests
2	U[0, p-1]	U[1, p]	$p > 3$	$p+1$
3	U[1, p-1]		$p > 4$	$2p$
4	U[1, p-2]	U[2, p-1]	$p > 5$	$p(p+1)/2$
5	U[2, p-2]		$p > 6$	$p(p-1)$
6	U[2, p-3]	U[3, p-2]	$p > 8$	$B(p+1, 3)$
6	U[1, 4, 7]		$p = 8$	$p(p+1)/2$

Dependence Matrix

	u	v	X	Y	z
$f_1(uvx)$	1	1	1	0	0
$f_2(uvy)$	1	1	0	1	0
$f_3(uxz)$	1	0	1	0	1
$f_4(uyz)$	1	0	0	1	1
$f_5(vxy)$	0	1	1	1	0
$f_6(vxz)$	0	1	1	0	1
$f_7(wyz)$	0	1	0	1	1
$f_8(xyz)$	0	0	1	1	1

Mini IOV Test

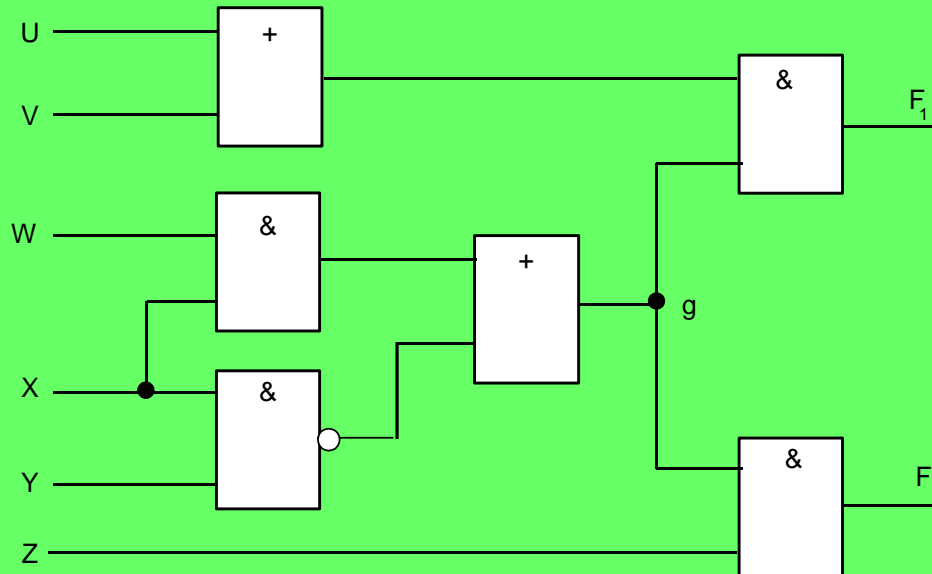
	u	v	X	Y	z
	1	0	0	0	0
	0	0	0	1	1
	0	0	1	0	1
	1	0	1	1	0
	1	1	0	0	1
	0	1	0	1	0
	0	1	1	0	0
	1	1	1	1	1

Test
Patterns

Quiz

Q1: (Cont'd) $p=5$ $w=5$. Which case is this?

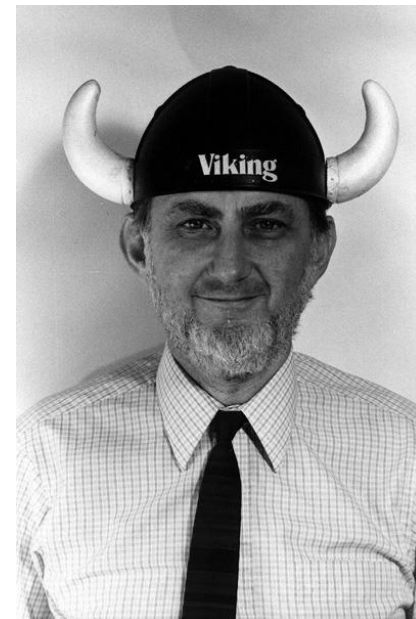
Q2: What is minimum IOV test length?



Can We Do Better ? See Next Video

Summary

- Individual Output Verification (IOV)
 - ◆ Test every output exhaustively
- Q1: How many test signals?
 - ◆ Partitioned dependence matrix
 - ◆ $w = \text{max row weight}$
 - ◆ $p = \text{partition}$
- Q2: What is minimum IOV TL?
 - ◆ $p = w$: $TL = 2^w$
 - ◆ $p = w + 1$: $TL = 2^w$
 - * Orthogonal Array
 - ◆ $p > w + 1$: $TL \geq 2^w$
 - * Universal IOV test table



McCluskey and his collection of hats

FFT

- Q: $p \geq w$, why?
- Three cases:
 - ♦ $p = w$
 - * Minimum IOV test length = 2^w
 - ♦ $p = w + 1$
 - * Minimum IOV test length = 2^w
 - ♦ $p > w + 1$
 - * Minimum IOV test length $\geq 2^w$

Orthogonal Array [Rao 1946]

- $N \times k$ array
- Entries are a finite **symbol set**, $\{1, 2, \dots, s\}$
- For every selection of t columns
 - ♦ All **t -tuples** of symbols (s^t) appear same number of times
 - ♦ t is called **strength** of orthogonal array
- Example: OA with symbol set $\{1, 0\}$, strength = 2
 - ♦ $N=4$, $k=3$, $s=2$, $t=2$
 - ♦ $(0,0)(1,0)(0,1)(1,1)$ appear exactly once for every 2 columns

1	1	1
0	0	1
1	0	0
0	1	0