

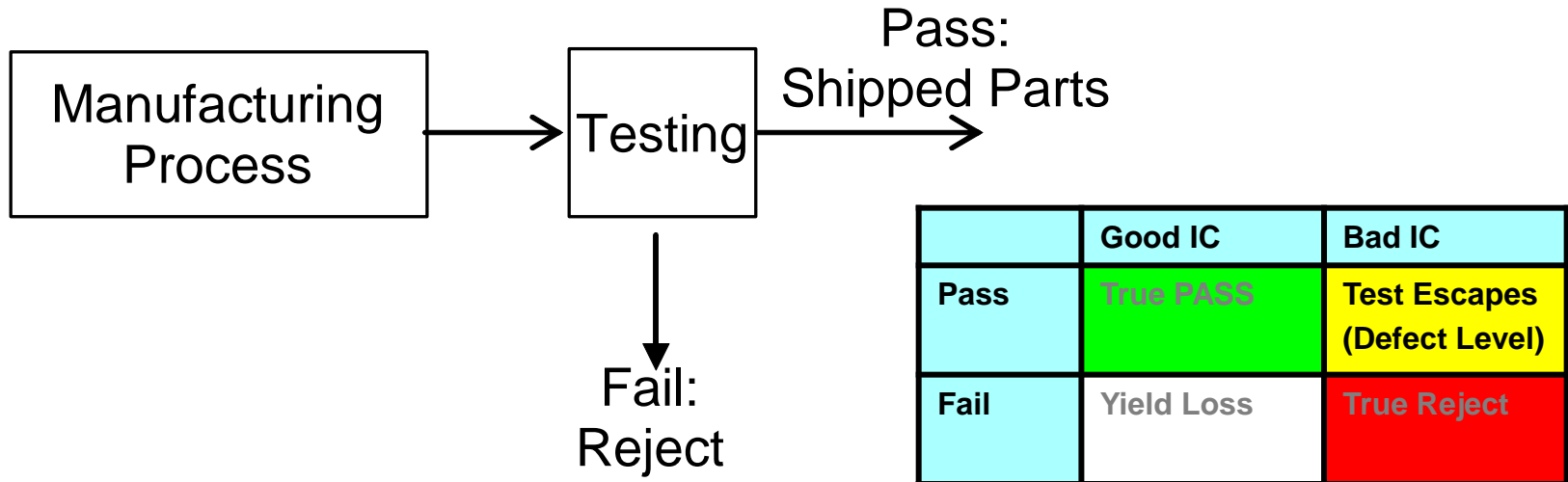
# Introduction

- What Is Testing
- Types of Testing
- Test Quality
  - ◆ Defect per Million (DPM)
  - ◆ Models to Predict DPM
  - ◆ Models to Predict Yield
- Test Economics
- Issues in Testing
- Conclusion



# How to Measure Test Quality?

- **Defect Level (DL)**
  - ♦ Fraction of bad IC passing the test (test escapes)
  - ♦ Measured by **Defective Parts per Million (DPM)**
    - \* < 200 DPM is acceptable for some IC
    - \* >1,000 DPM is very bad for IC



**Low DPM Means Good Test Quality**

# IC DPM → System DPM

- If you think DPM= 1,000 (0.1%) is good enough, consider this case
- A system has twenty IC (each 1000 DPM), what is system DPM?
  - ◆ No IC is defective for system to work correctly
    - \*  $\text{System DPM} = 1 - (1 - 0.1\%)^{20} = 2\% = 20,000 \text{ DPM} !$
  - ◆ If you sell a million system
    - \* You have to fix 20,000 of them!

**Low IC DPM is Very Important**

# Quiz

**Q: (Cont'd from last slide)**

**20 IC in a system.**

**Suppose we improve IC test quality to 200DPM**

**What is system DPM?**

**A:**

**IC DPM ↓ 5X, System DPM ↓ 5X**

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    - \* Brown & Williams (IBM, 1981), Binomial distribution
    - \* Agrawal (Bell Lab, 1982), , Poisson distribution
  - ♦ Models for Yield
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**Measure *Real DPM* Is difficult.  
Can we predict it?**



# Brown & Williams Model [Williams 81]

$$DL = 1 - Y^{(1-FC)}$$

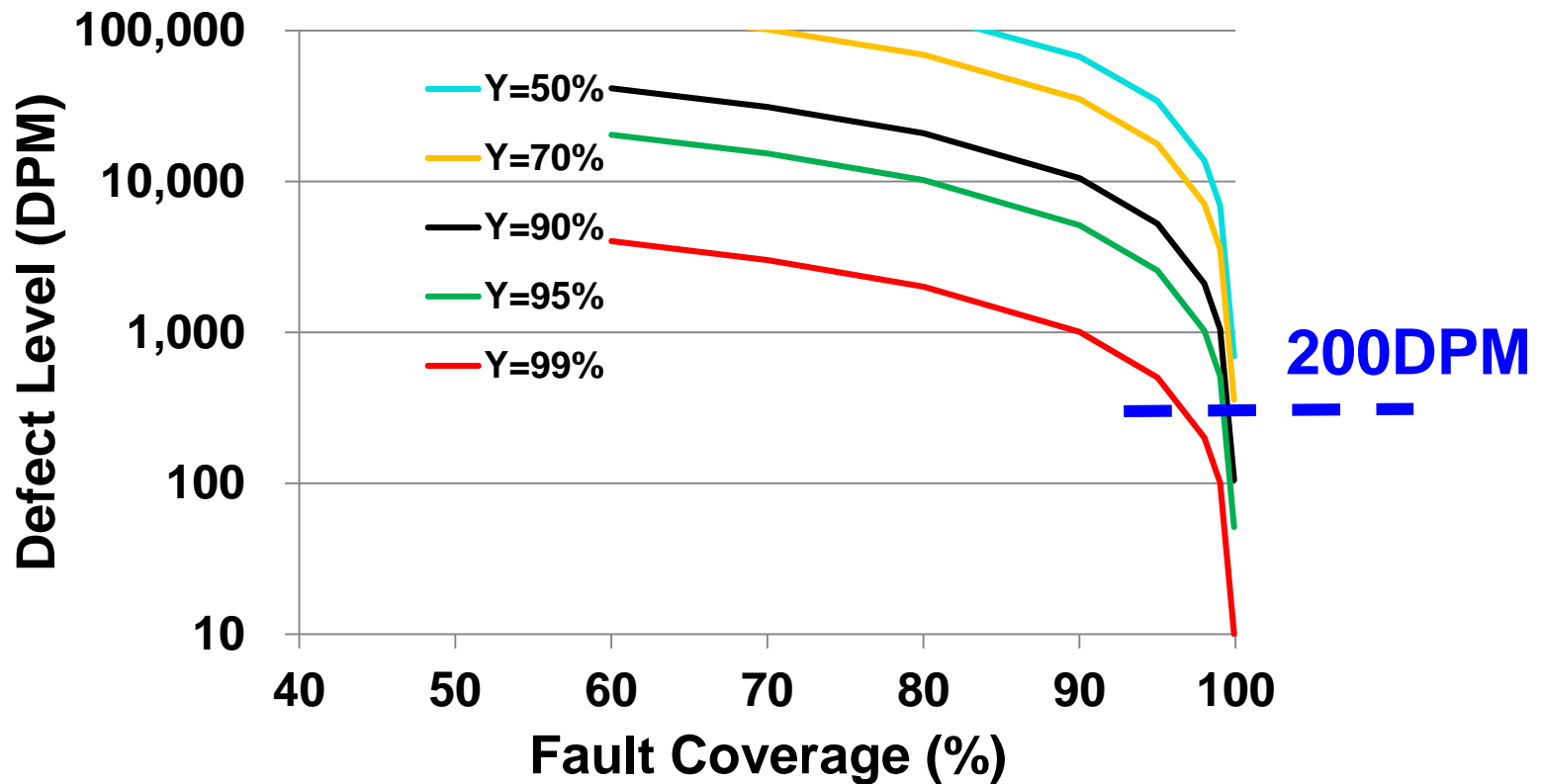
- ♦ ***Y=Yield***, fraction of total manufactured IC that are good
  - \* High Y means good **manufacturing quality**
  - \* **Unknown** parameter, estimated by fab. data or prediction

$$\text{yield, } Y = \frac{\text{number of good chips}}{\text{number of total chips}} \leq 100\%$$

- ♦ ***FC = Fault Coverage***, fraction of detected faults
  - \* High FC means good **test quality**
  - \* **Known** parameter, from *fault simulator* (see *fault simulation*)

$$\text{fault coverage, } FC = \frac{\text{number of detected faults}}{\text{number of total faults}} \leq 100\%$$

# DL v.s. FC



- Required DL = 200 DPM, then we need

Yield	50%	70%	90%	95%	99%
FC	99.97%	99.95%	99.8%	99.6%	98%

\*  $DL \approx (1-FC)(1-Y)$  when  $Y \approx 1$

poor yield  
good test



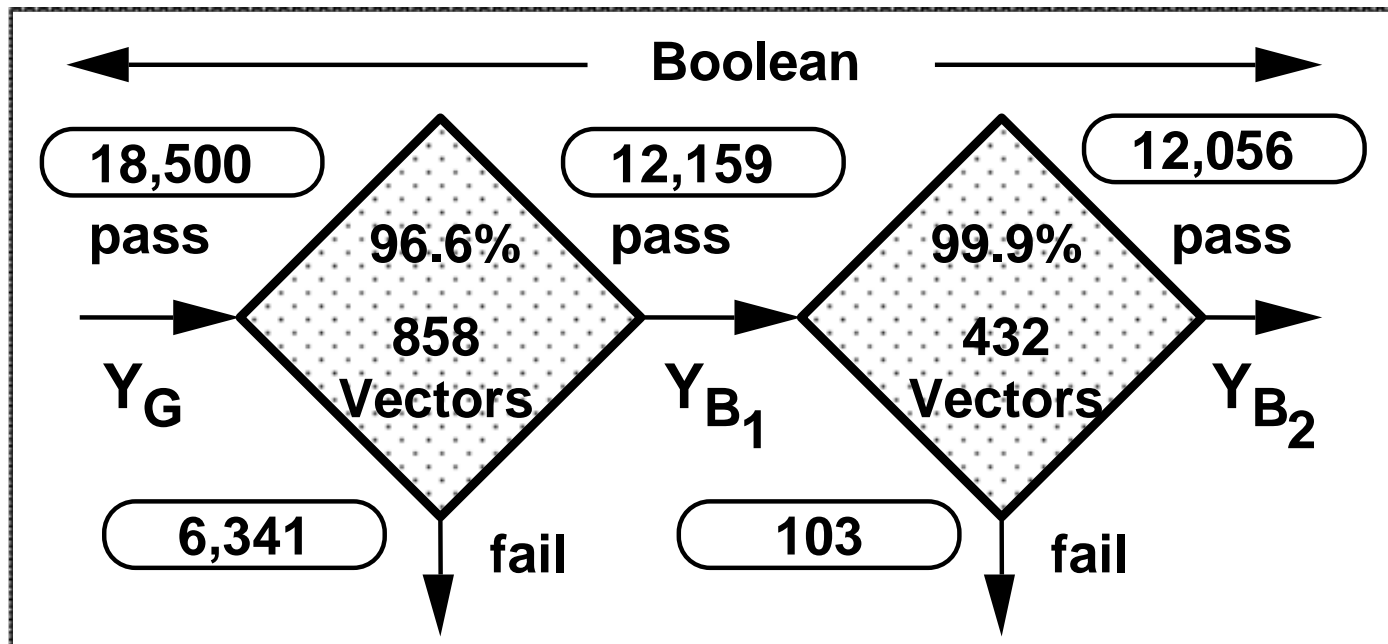
good yield  
poor test

# B&W Proof (simplified!)

- Assume each fault occurrence probability is  $q$  (uniform independent)
  - ♦ Total  $t$  faults in an IC
  - ♦  $d$  out of  $t$  faults detected by this test  $\rightarrow FC = d/t$
- Event A = an IC is good (no fault at all)
  - ♦  $\text{Prob}(A)$  = probability that an IC is good  $\rightarrow \text{Prob}(A) = (1-q)^t$
  - ♦  $Y$  = fraction of IC that are good  $\rightarrow Y = \text{Prob}(A)$
- Event B = an IC pass the test
  - ♦  $\text{Prob}(B)$  = an IC free of  $d$  faults  $\rightarrow \text{Prob}(B) = (1-q)^d$
- Event AB = an IC is good and pass test
  - ♦ Assume no overkill  $\rightarrow \text{Prob}(AB) = \text{Prob}(A)$
- Quality Level (QL) = fraction of good IC passing the test
  - ♦  $QL = \text{Prob}(A/B) = \text{Prob}(AB)/\text{Prob}(B) = (1-q)^{t-d} = (1-q)^{t(1-d/t)} = Y^{(1-FC)}$
- Defect Level (DL) = fraction of bad IC passing the test
  - ♦  $DL = 1-QL = 1-Y^{(1-FC)}$  QED

# Motorola 6802 Experiment [McCluskey 88]

- $Y = 12,056/18,500 = 65.16\%$
- test #1 FC = 96.6%
- Theoretical DL =  $1 - Y^{(1-FC)} = 14,454$  DPM
- Experimental DL = 8,471 DPM (103 over 12,159 IC passing test #1)



# Quiz

Q1: Use B&W model.

**Y=98%, FC=70%. DL=?**

$$DL = 1 - Y^{(1-FC)}$$

A:

Q2: Your boss is not happy about DL. If we add some design for testability (DFT) circuits to improves FC but harms yield a little.

**Y=97%, FC=99%. DL=?**

A:

**DFT Very Important for DL Improvement**

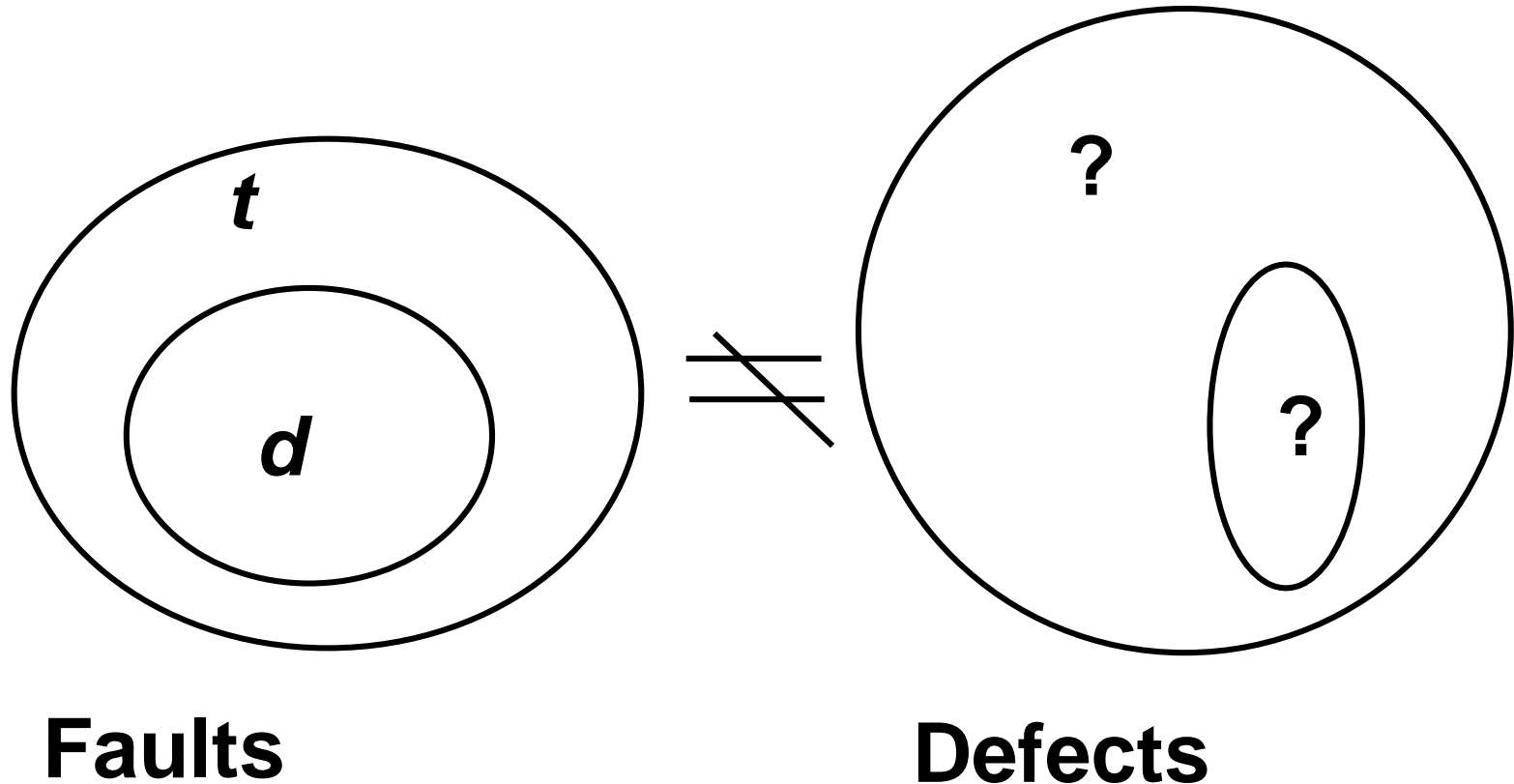
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  - ◆ Models for Yield
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# Why B&W Not Accurate?

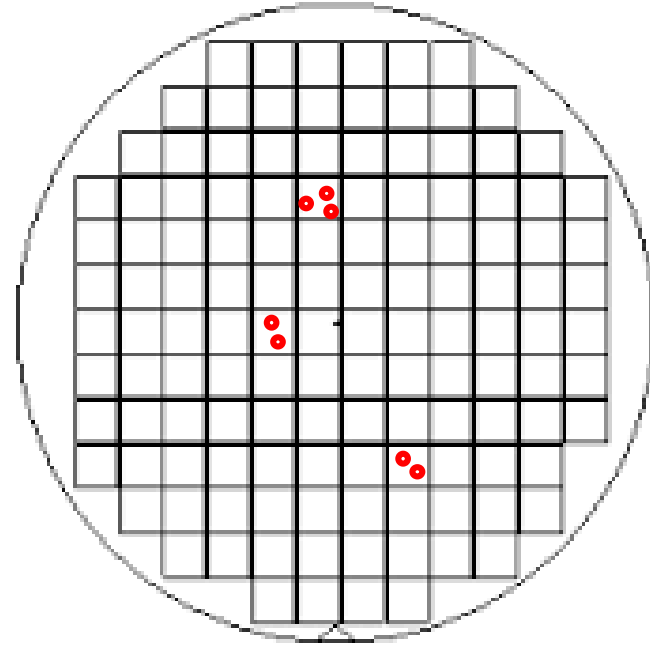
- 1. Fault coverage does not represent defect coverage
  - ♦ **100% FC  $\neq$  0 DPM** (see *fault modeling* chapter)
- 2. Defects are clustered
  - ♦ Agrawal used Poisson distribution



# Agrawal Model [Agrawal 82]

$$DL = \frac{(1 - FC)(1 - Y)e^{-(n-1)FC}}{Y + (1 - FC)(1 - Y)e^{-(n-1)FC}}$$

- Actual data show defects are **clustered**
  - ◆  $n$  = average num. of defects on a bad die
- We need two unknown parameters
  - ◆  $n$ : from experiment
  - ◆  $Y$ : from fab. or prediction
- Motorola experiment revisited
  - ◆ Experiment DL = **8,471** DPM
  - ◆ B&W DL = **14,454** DPM
  - ◆ Agrawal
    - \*  $n = 1$ , DL = **17,849** DPM
    - \*  $n = 2$ , DL = **6,869** DPM
    - \* experimental  $n$  between 1 and 2



7 defects  
3 bad die  
 $n = 7/3 = 2.3$

# Introduction

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  - ♦ Models for Yield
    - \* Simple
    - \* Critical Area
- Test Economy
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**Measure *Real Yield* Is too late.  
Can we predict it?**



# Yield Estimation (Simple)

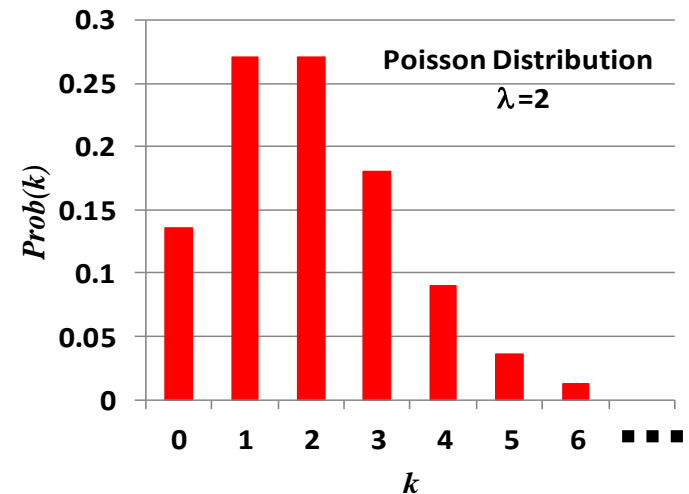
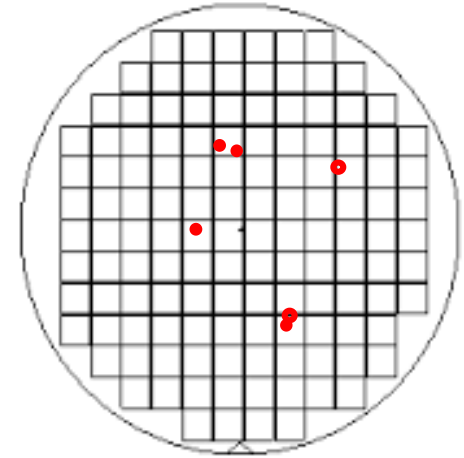
- Number of defects  $k$  on a die obeys **Poisson distribution** with mean  $\lambda$ 
  - ♦  $Prob(k)$  = prob exactly  $k$  defects on a die
  - ♦ A die is good when  $k=0$
  - ♦  $Yield = Prob(k=0)$

$$Prob(k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

$$Y = Prob(k=0) = e^{-\lambda}$$

- $A$  = die area,  $D$  = defect density
- $\lambda = AD$  = ave. number of defects per die

$$Y = e^{-AD} \quad \text{Larger area, lower yield}$$



# Quiz

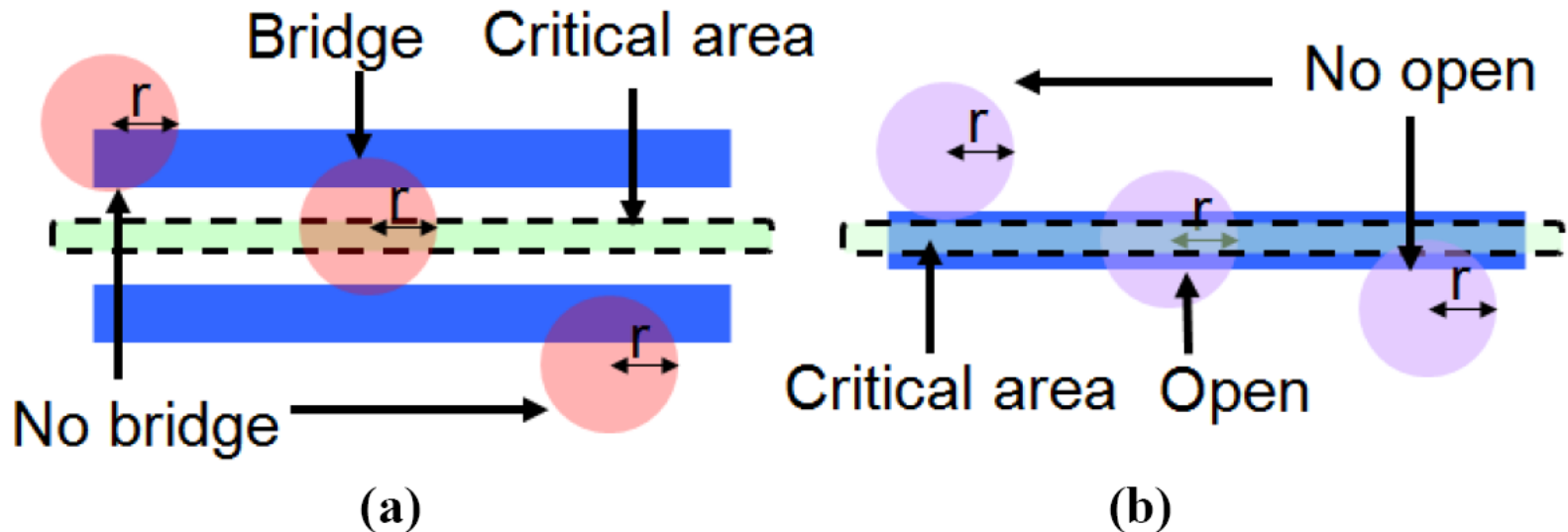
Q:  $A=0.8\text{cm}^2$ ,  $D=0.5/\text{cm}^2$ . What is yield?

$$Y = e^{-AD}$$

A:

# Critical Area , CA

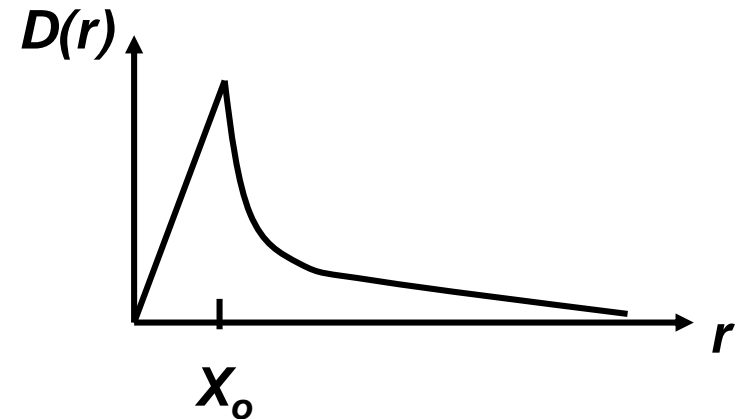
- Problem with  $Y^{AD}$  model
  - ♦ Some area NOT sensitive to defect. Use **total area is pessimistic**
- **Critical area,  $CA(r)$** 
  - ♦ Area where center of defect (radius  $r$ ) land to create a *fault*
- Example:
  - ♦ (a) bridging fault    (b) open fault



# Defect Size and Density Distribution, DSDD

- $D(r)$  = defect density per unit defect radius  $r$
- Typical DSDD curve:

$$D(r) = \begin{cases} D_0 \frac{2(p-1)r}{(p+1)X_o^2}, & 0 \leq r < X_o \\ D_0 \frac{2(p-1)X_o^{p-1}}{(p+1)r^p}, & r \geq X_o \end{cases}$$



- $D(r)$  is process dependent.
  - ♦ Example values:
  - ♦  $4.5 \times 10^{-6} \text{ cm} \leq X_o \leq 5.5 \times 10^{-6} \text{ cm}$
  - ♦  $2.0 \leq p \leq 3.0$
  - ♦  $0.25 / \text{cm}^2 \leq D_o \leq 0.6 / \text{cm}^2$

$$\text{Total Defect Density} = D_o = \int_0^{\infty} D(r) dr$$

# Yield Estimation (CA)

- Expected number of defects per die **landing in CA**

$$\lambda = \int_0^{\infty} CA(r) \times D(r) dr$$

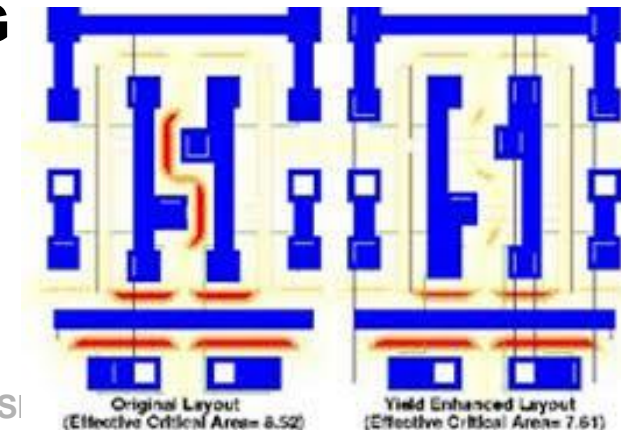
- Again, *Poisson distribution* with mean  $\lambda$ 
  - Yield = *Prob*( $k=0$ ), zero defect in CA

$$Y = e^{-\lambda} = e^{-\int_0^{\infty} CA(r) \times D(r) dr}$$

simple model

$$\Leftrightarrow Y = e^{-AD}$$

- Critical area analysis (CAA)** is useful tool to estimate yield
  - 1. modify design for yield improvement
  - 2. extract bridging or open faults for ATPG



# Summary

- Test quality measured by **Defect per Million (DPM)**
- Models for DPM
  - ♦ Brown & Williams :

$$DL = 1 - Y^{(1-FC)}$$

- ♦ Agrawal

$$DL = \frac{(1-FC)(1-Y)e^{-(n-1)FC}}{Y + (1-FC)(1-Y)e^{-(n-1)FC}}$$

- Models for Yield
  - ♦ Simple model

$$Y = e^{-AD}$$

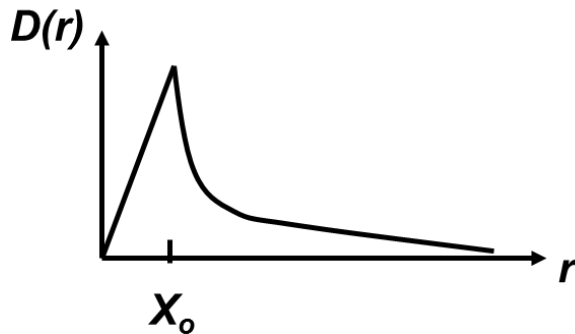
- ♦ Critical area

$$Y = e^{-\int_0^{\infty} CA(r) \times D(r) dr}$$



# Food for Thoughts, FFT

- Q: What is unit of  $D(r)$  ?
  - ♦  $D(r)$  = defect density per unit defect radius  $r$



$$\text{Total Defect Density} = D_0 = \int_0^{\infty} D(r) dr$$



# APPENDIX#1: Agrawal Proof (1)

- Number of defects  $k$  on a bad IC obeys **Poisson distribution** with mean  $n$   $p(k)$  = probability exactly  $k$  faults in a bad IC ( $k > 0$ )

$$p(k) = (1 - Y) \frac{(n - 1)^{k-1} e^{-(n-1)}}{(k - 1)!} \quad k = 1, 2, 3... \text{ (shifted Poisson)}$$

$$p(0) = Y$$

- Suppose total  $t$  defects,  $d$  is detected by test, so  $FC = d/t$
- $q_m(k)$  = prob. detecting exactly  $m$  defects when an IC has  $k$  defects

$$q_m(k) = \frac{\binom{k}{m} \binom{t-k}{d-m}}{\binom{t}{d}} \text{ hypergeometric distribution}$$

- $q_0(k)$  = prob. of passing a bad IC, having exactly  $k$  defects

$$q_0(k) = \frac{\binom{t-k}{d}}{\binom{t}{d}} \approx \left(1 - \frac{d}{t}\right)^k = (1 - FC)^k$$

# Agrawal Proof (2)

- **TE = Prob. bad IC passing the test**

$$\begin{aligned} TE &= \sum_{k=1}^t q_0(k) p(k) = \sum_{k=1}^t (1 - FC)^k (1 - Y) \frac{(n-1)^{k-1}}{(k-1)!} e^{-(n-1)} \\ &= (1 - Y) \sum_{k=1}^t (1 - FC) \frac{[(1 - FC)(n-1)]^{k-1}}{(k-1)!} e^{-(n-1)(1-FC)} e^{-(n-1)FC} \\ &= (1 - Y)(1 - FC) e^{-(n-1)FC} \sum_{k=1}^t \frac{[(1 - FC)(n-1)]^{k-1}}{(k-1)!} e^{-(n-1)(1-FC)} \\ &\approx (1 - Y)(1 - FC) e^{-(n-1)FC} \end{aligned}$$

$$DL = \frac{TE}{Y + TE} = \frac{(1 - FC)(1 - Y) e^{-(n-1)FC}}{Y + (1 - FC)(1 - Y) e^{-(n-1)FC}}$$

- **QED**

# APPENDIX #2: B&W Approximation

Taylor Expansion  $DL(Y) = \sum_{n=0}^{\infty} \frac{f^{(n)}(Y)}{n!} (Y - a)^n = f(1) + \frac{f'(1)}{1!} (Y - 1) + \frac{f''(1)}{2!} (Y - 1)^2 + \dots$

when  $Y \approx 1$ , we expand  $DL(Y)$  around  $a=1$

$$\begin{aligned} DL(Y) &= 1 - Y^{1-FC} \\ &= (1 - 1^{1-FC}) - (1 - FC) \cdot 1^{-FC} (Y - 1) - \frac{(-FC)(1 - FC)1^{-FC-1}}{2!} (Y - 1)^2 - \dots \\ &= 0 + (1 - FC)(1 - Y) + \frac{(1 - FC)FC}{2} (1 - Y)^2 + \dots \\ &\approx (1 - FC)(1 - Y) \end{aligned}$$

Example  $FC=98.5\%$ ,  $Y=95\%$

$$DL(Y) = 1 - Y^{1-FC} = 1 - 0.95^{0.015} = 769\text{DPM}$$

$$DL(Y) \approx (1 - FC)(1 - Y) = 750\text{DPM}$$

$$DL(Y) \approx (1 - FC)(1 - Y) + \frac{FC(1 - FC)}{2} (1 - Y)^2 = 768\text{DPM}$$

$$DL \approx (1-FC)(1-Y)$$