

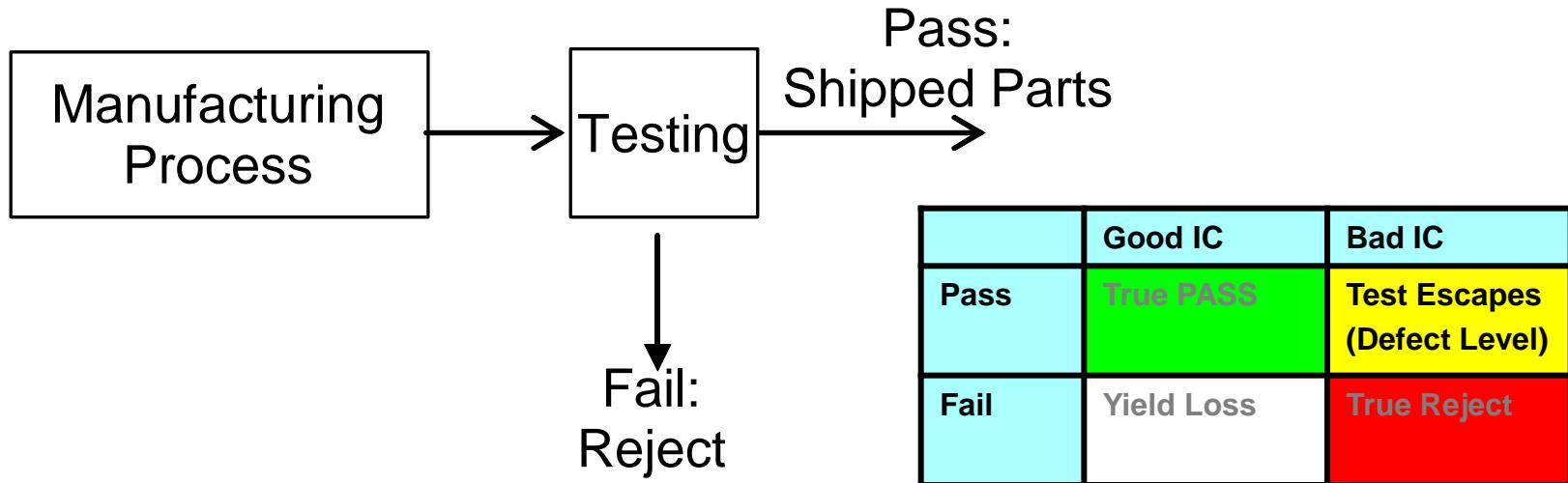
Introduction

- What Is Testing
- Types of Testing
- Test Quality
 - ◆ Defect per Million (DPM)
 - ◆ Models to Predict DPM
 - ◆ Models to Predict Yield
- Test Economics
- Issues in Testing
- Conclusion



How to Measure Test Quality?

- **Defect Level (DL)**
 - ◆ Fraction of bad IC passing the test (test escapes)
 - ◆ Measured by **Defective Parts per Million (DPM)**
 - * < 200 DPM is acceptable for some IC
 - * >1,000 DPM is very bad for IC



Low DPM Means Good Test Quality

IC DPM → System DPM

- If you think DPM= 1,000 (0.1%) is good enough, consider this case
- A system has twenty IC (each 1000 DPM), what is system DPM?
 - ◆ No IC is defective for system to work correctly
 - * $\text{System DPM} = 1 - (1 - 0.1\%)^{20} = 2\% = 20,000 \text{ DPM} !$
 - ◆ If you sell a million system
 - * You have to fix 20,000 of them!

Low IC DPM is Very Important

Quiz

Q: (Cont'd from last slide)

20 IC in a system.

Suppose we improve IC test quality to 200DPM

What is system DPM?

A:

IC DPM ↓ 5X, System DPM↓ 5X

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 - * Agrawal (Bell Lab, 1982), Poisson distribution
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**Measure *Real DPM* is difficult.
Can we predict it?**



Brown & Williams Model [Williams 81]

$$DL = 1 - Y^{(1-FC)}$$

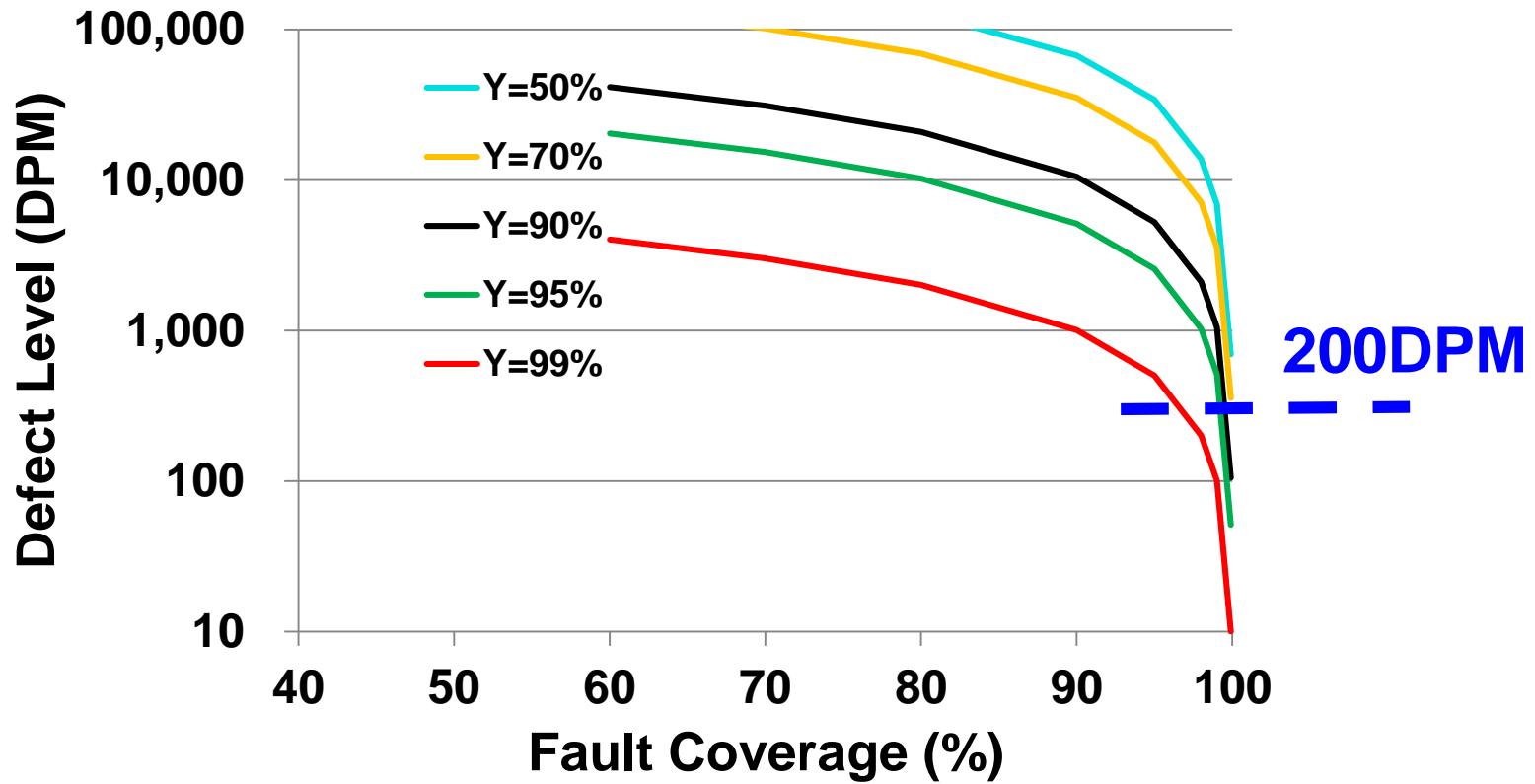
- ♦ **Y=Yield**, fraction of total manufactured IC that are good
 - * High Y means good **manufacturing quality**
 - * **Unknown** parameter, estimated by fab. data or prediction

$$\text{yield, } Y = \frac{\text{number of good chips}}{\text{number of total chips}} \leq 100\%$$

- ♦ **FC = Fault Coverage**, fraction of detected faults
 - * High FC means good **test quality**
 - * **Known** parameter, from **fault simulator** (see **fault simulation**)

$$\text{fault coverage, } FC = \frac{\text{number of detected faults}}{\text{number of total faults}} \leq 100\%$$

DL v.s. FC



- Required DL = **200 DPM**, then we need

Yield	50%	70%	90%	95%	99%
FC	99.97%	99.95%	99.8%	99.6%	98%

* $DL \approx (1-FC)(1-Y)$ when $Y \approx 1$

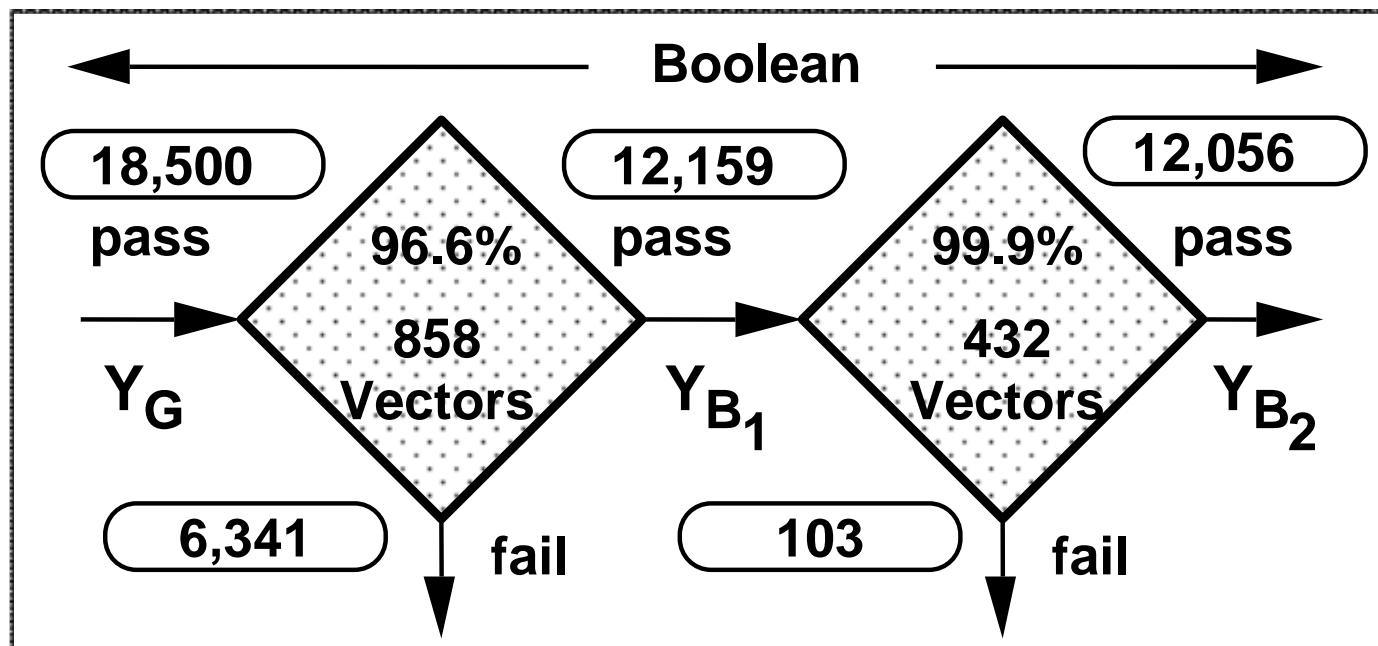
poor yield
good test good yield
poor test

B&W Proof (simplified!)

- Assume each fault occurrence probability is q (uniform independent)
 - ◆ Total t faults in an IC
 - ◆ d out of t faults detected by this test $\rightarrow FC = d/t$
- Event A = an IC is good (no fault at all)
 - ◆ Prob(A) = probability that an IC is good $\rightarrow Prob(A) = (1-q)^t$
 - ◆ Y = fraction of IC that are good $\rightarrow Y = Prob(A)$
- Event B = an IC pass the test
 - ◆ Prob(B) = an IC free of d faults $\rightarrow Prob(B) = (1-q)^d$
- Event AB = an IC is good and pass test
 - ◆ Assume no overkill $\rightarrow Prob(AB) = Prob(A)$
- Quality Level (QL) = fraction of good IC passing the test
 - ◆ $QL = Prob(A|B) = Prob(AB)/Prob(B) = (1-q)^{t-d} = (1-q)^{t(1-d/t)} = Y^{(1-FC)}$
- Defect Level (DL) = fraction of bad IC passing the test
 - ◆ $DL = 1-QL = 1-Y^{(1-FC)}$ QED

Motorola 6802 Experiment [McCluskey 88]

- $Y = 12,056/18,500 = 65.16\%$
- test #1 FC = 96.6%
- Theoretical DL = $1-Y^{(1-FC)} = 14,454$ DPM
- Experimental DL = 8,471 DPM (103 over 12,159 IC passing test #1)



Quiz

**Q1: Use B&W model.
Y=98%, FC=70%. DL=?**

$$DL = 1 - Y^{(1-FC)}$$

A:

**Q2: Your boss is not happy about DL. If we add some design for testability (DFT) circuits to improves FC but harms yield a little.
Y=97%, FC=99%. DL=?**

A:

DFT Very Important for DL Improvement

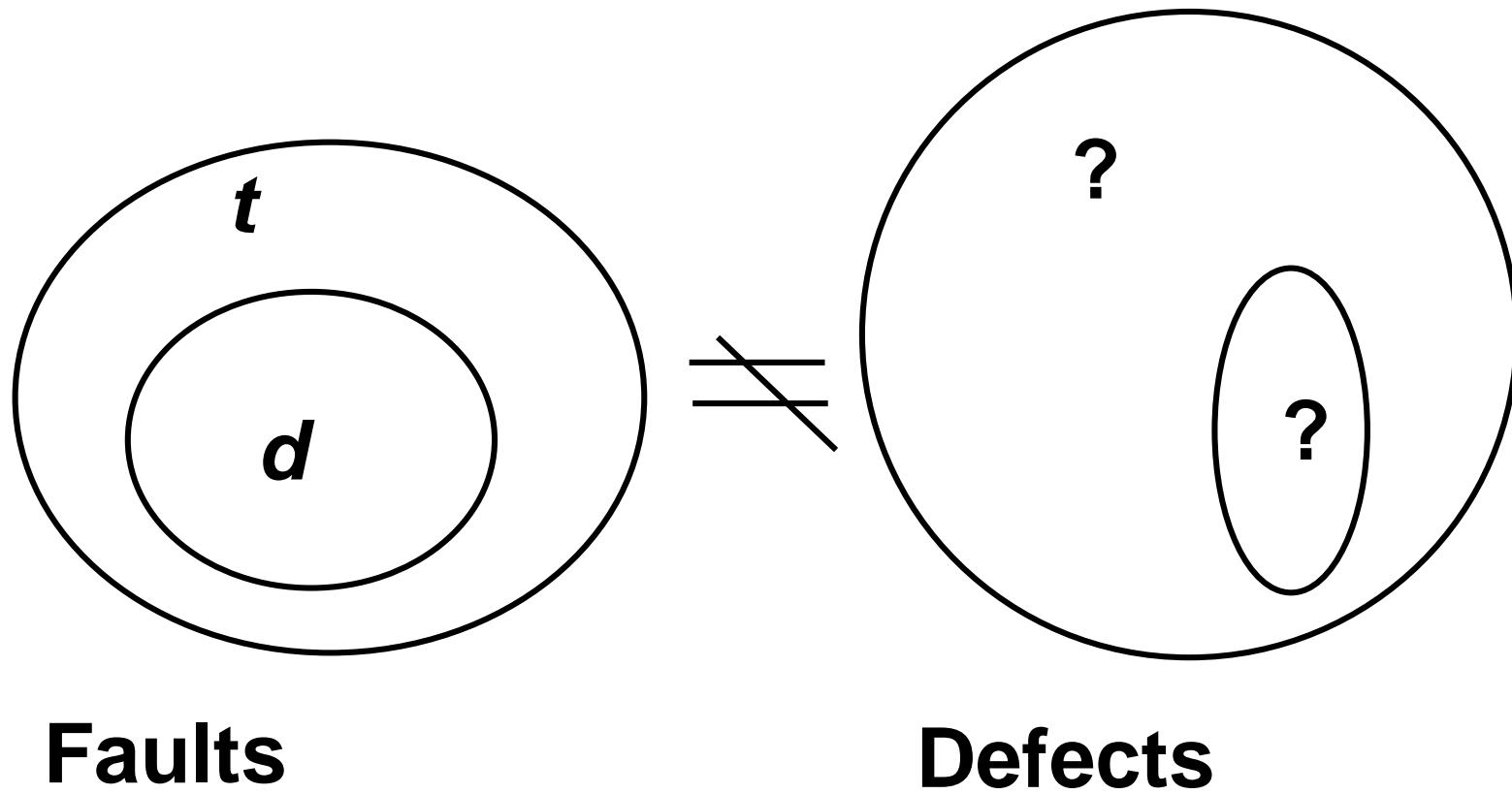
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Why B&W Not Accurate?

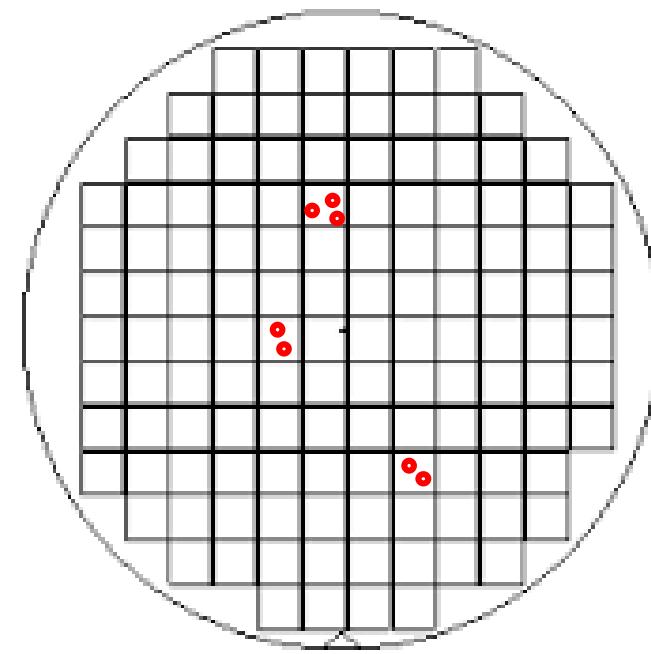
- 1. Fault coverage does not represent defect coverage
 - ◆ $100\% \text{ FC} \neq 0 \text{ DPM}$ (see *fault modeling* chapter)
- 2. Defects are clustered
 - ◆ Agrawal used Poisson distribution



Agrawal Model [Agrawal 82]

$$DL = \frac{(1 - FC)(1 - Y)e^{-(n-1)FC}}{Y + (1 - FC)(1 - Y)e^{-(n-1)FC}}$$

- Actual data show defects are **clustered**
 - n = average num. of defects on a bad die
- We need two unknown parameters
 - n : from experiment
 - Y : from fab. or prediction
- Motorola experiment revisited
 - Experiment DL = **8,471 DPM**
 - B&W DL = **14,454 DPM**
 - Agrawal
 - * $n = 1$, DL = **17,849 DPM**
 - * $n = 2$, DL = **6,869 DPM**
 - * experimental n between 1 and 2



7 defects
3 bad die
 $n=7/3=2.3$

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 - ◆ Models for Yield
 - * Simple
 - * Critical Area
- Test Economy
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**Measure *Real Yield* Is too late.
Can we predict it?**



Yield Estimation (Simple)

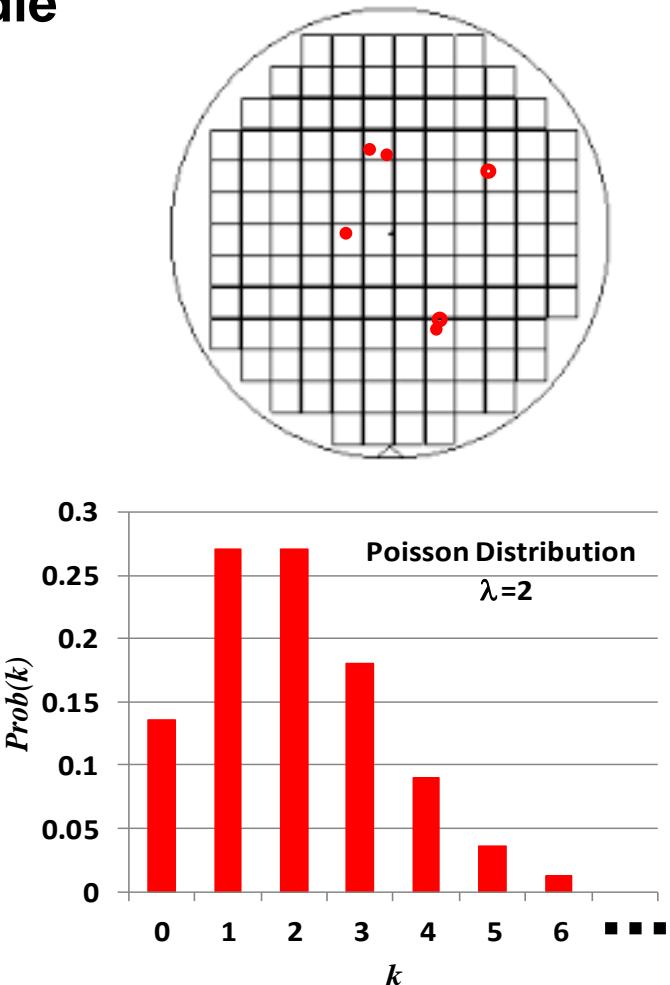
- Number of defects k on a die obeys **Poisson distribution** with mean λ
 - Prob(k) = prob exactly k defects on a die
 - A die is good when $k=0$
 - Yield = $Prob(k=0)$

$$Prob(k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

$$Y = Prob(k=0) = e^{-\lambda}$$

- A = die area, D = defect density
- $\lambda = AD$ = ave. number of defects per die

$$Y = e^{-AD} \quad \text{Larger area, lower yield}$$



Quiz

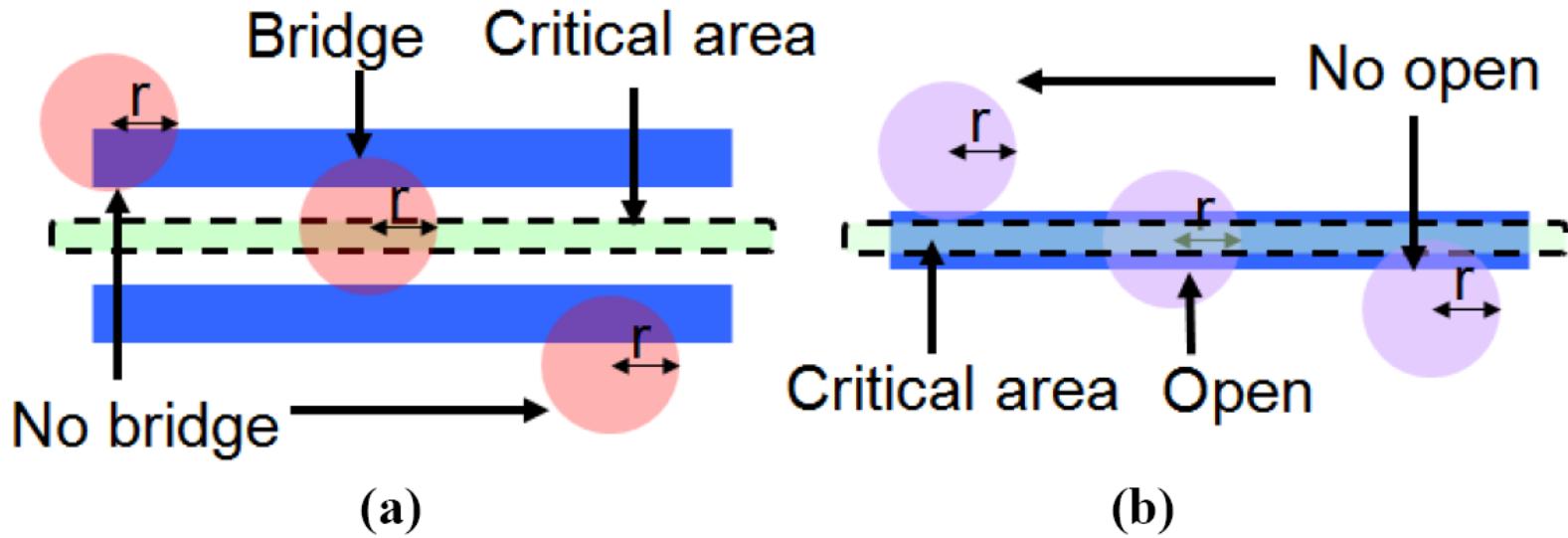
Q: A=0.8cm², D=0.5/cm². What is yield?

$$Y = e^{-AD}$$

A:

Critical Area , CA

- Problem with γ -AD model
 - ◆ Some area NOT sensitive to defect. Use total area is pessimistic
- **Critical area, $CA(r)$**
 - ◆ Area where center of defect (radius r) land to create a *fault*
- Example:
 - ◆ (a) bridging fault (b) open fault

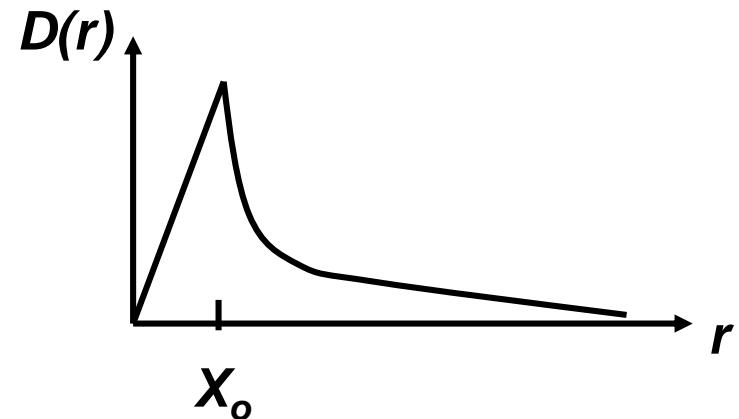


Defect Size and Density Distribution, DSDD

- $D(r)$ = defect density per unit defect radius r
- Typical DSDD curve:

$$D(r) = \begin{cases} D_0 \frac{2(p-1)r}{(p+1)X_o^2}, & 0 \leq r < X_o \\ D_0 \frac{2(p-1)X_o^{p-1}}{(p+1)r^p}, & r \geq X_o \end{cases}$$

- $D(r)$ is process dependent.
 - ◆ Example values:
 - ◆ $4.5 \times 10^{-6} \text{ cm} \leq X_o \leq 5.5 \times 10^{-6} \text{ cm}$
 - ◆ $2.0 \leq p \leq 3.0$
 - ◆ $0.25 / \text{cm}^2 \leq D_o \leq 0.6 / \text{cm}^2$



$$\text{Total Defect Density} = D_o = \int_0^\infty D(r) dr$$

Yield Estimation (CA)

- Expected number of defects per die landing in CA

$$\lambda = \int_0^{\infty} CA(r) \times D(r) dr$$

- Again, *Poisson distribution* with mean λ

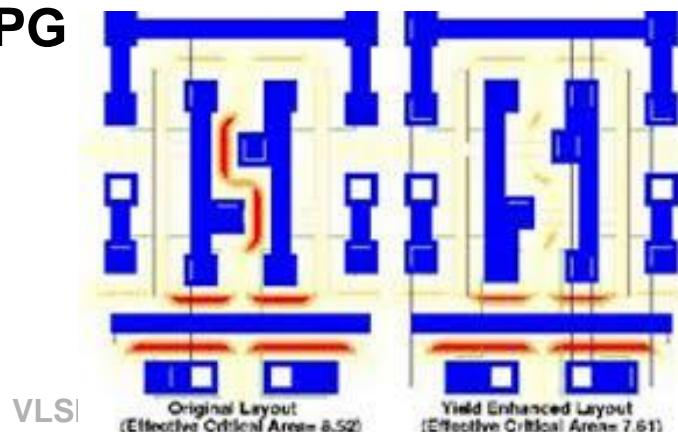
- ◆ Yield = $Prob(k=0)$, zero defect in CA

$$Y = e^{-\lambda} = e^{-\int_0^{\infty} CA(r) \times D(r) dr}$$

simple model

$$\Leftrightarrow Y = e^{-AD}$$

- *Critical area analysis (CAA)* is useful tool to estimate yield
 - ◆ 1. modify design for yield improvement
 - ◆ 2. extract bridging or open faults for ATPG



Summary

- Test quality measured by **Defect per Million (DPM)**
- Models for DPM
 - ◆ Brown & Williams :

$$DL = 1 - Y^{(1-FC)}$$

- ◆ Agrawal

$$DL = \frac{(1-FC)(1-Y)e^{-(n-1)FC}}{Y + (1-FC)(1-Y)e^{-(n-1)FC}}$$

- Models for Yield
 - ◆ Simple model

$$Y = e^{-AD}$$

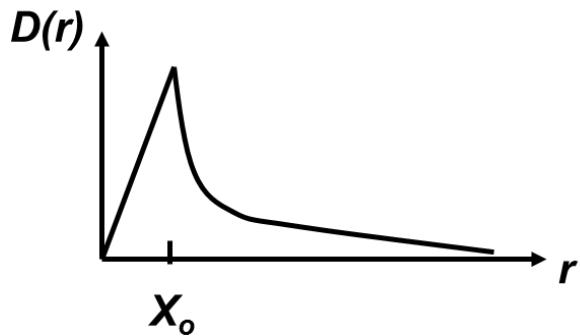
- ◆ Critical area

$$Y = e^{-\int_0^{\infty} CA(r) \times D(r) dr}$$



Food for Thoughts, FFT

- Q: What is unit of $D(r)$?
 - ◆ $D(r)$ = defect density per unit defect radius r



$$\text{Total Defect Density} = D_0 = \int_0^{\infty} D(r) dr$$



APPENDIX#1: Agrawal Proof (1)

- Number of defects k on a bad IC obeys **Poisson distribution** with mean $n p(k)$ = probability exactly k faults in a bad IC ($k > 0$)

$$p(k) = (1 - Y) \frac{(n-1)^{k-1} e^{-(n-1)}}{(k-1)!} \quad k = 1, 2, 3, \dots \text{ (shifted Poisson)}$$

$$p(0) = Y$$

- Suppose total t defects, d is detected by test, so $FC = d/t$
- $q_m(k)$ = prob. detecting exactly m defects when an IC has k defects

$$q_m(k) = \frac{\binom{k}{m} \binom{t-k}{d-m}}{\binom{t}{d}} \quad \text{hypergeometric distribution}$$

- $q_0(k)$ = prob. of passing a bad IC, having exactly k defects

$$q_0(k) = \frac{\binom{t-k}{d}}{\binom{t}{d}} \approx \left(1 - \frac{d}{t}\right)^k = (1 - FC)^k$$

Agrawal Proof (2)

- **TE = Prob. bad IC passing the test**

$$\begin{aligned} TE &= \sum_{k=1}^t q_0(k) p(k) = \sum_{k=1}^t (1-FC)^k (1-Y) \frac{(n-1)^{k-1}}{(k-1)!} e^{-(n-1)} \\ &= (1-Y) \sum_{k=1}^t (1-FC) \frac{[(1-FC)(n-1)]^{k-1}}{(k-1)!} e^{-(n-1)(1-FC)} e^{-(n-1)FC} \\ &= (1-Y)(1-FC)e^{-(n-1)FC} \sum_{k=1}^t \frac{[(1-FC)(n-1)]^{k-1}}{(k-1)!} e^{-(n-1)(1-FC)} \\ &\approx (1-Y)(1-FC)e^{-(n-1)FC} \end{aligned}$$

$$DL = \frac{TE}{Y + TE} = \frac{(1-FC)(1-Y)e^{-(n-1)FC}}{Y + (1-FC)(1-Y)e^{-(n-1)FC}}$$

- **QED**

APPDENDIX #2: B&W Approximation

Tylor Exapnsion $DL(Y) = \sum_{n=0}^{\infty} \frac{f^{(n)}(Y)}{n!} (Y - a)^n = f(1) + \frac{f'(1)}{1!}(Y - 1) + \frac{f''(1)}{2!}(Y - 1)^2 + \dots$

when $Y \approx 1$, we expand $DL(Y)$ around $a=1$

$$\begin{aligned} DL(Y) &= 1 - Y^{1-FC} \\ &= (1 - 1^{1-FC}) - (1 - FC) \cdot 1^{-FC} (Y - 1) - \frac{(-FC)(1 - FC)1^{-FC-1}}{2!} (Y - 1)^2 - \dots \\ &= 0 + (1 - FC)(1 - Y) + \frac{(1 - FC)FC}{2} (1 - Y)^2 + \dots \\ &\approx (1 - FC)(1 - Y) \end{aligned}$$

Example FC=98.5%, Y=95%

$$DL(Y) = 1 - Y^{1-FC} = 1 - 0.95^{0.015} = 769\text{DPM}$$

$$DL(Y) \approx (1 - FC)(1 - Y) = 750\text{DPM}$$

$$DL(Y) \approx (1 - FC)(1 - Y) + \frac{FC(1 - FC)}{2} (1 - Y)^2 = 768\text{DPM}$$

DL \approx (1-FC)(1-Y)