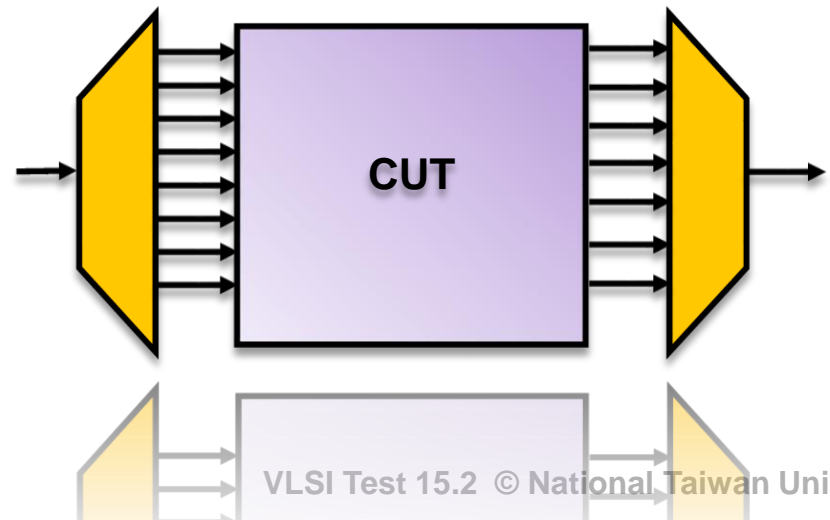


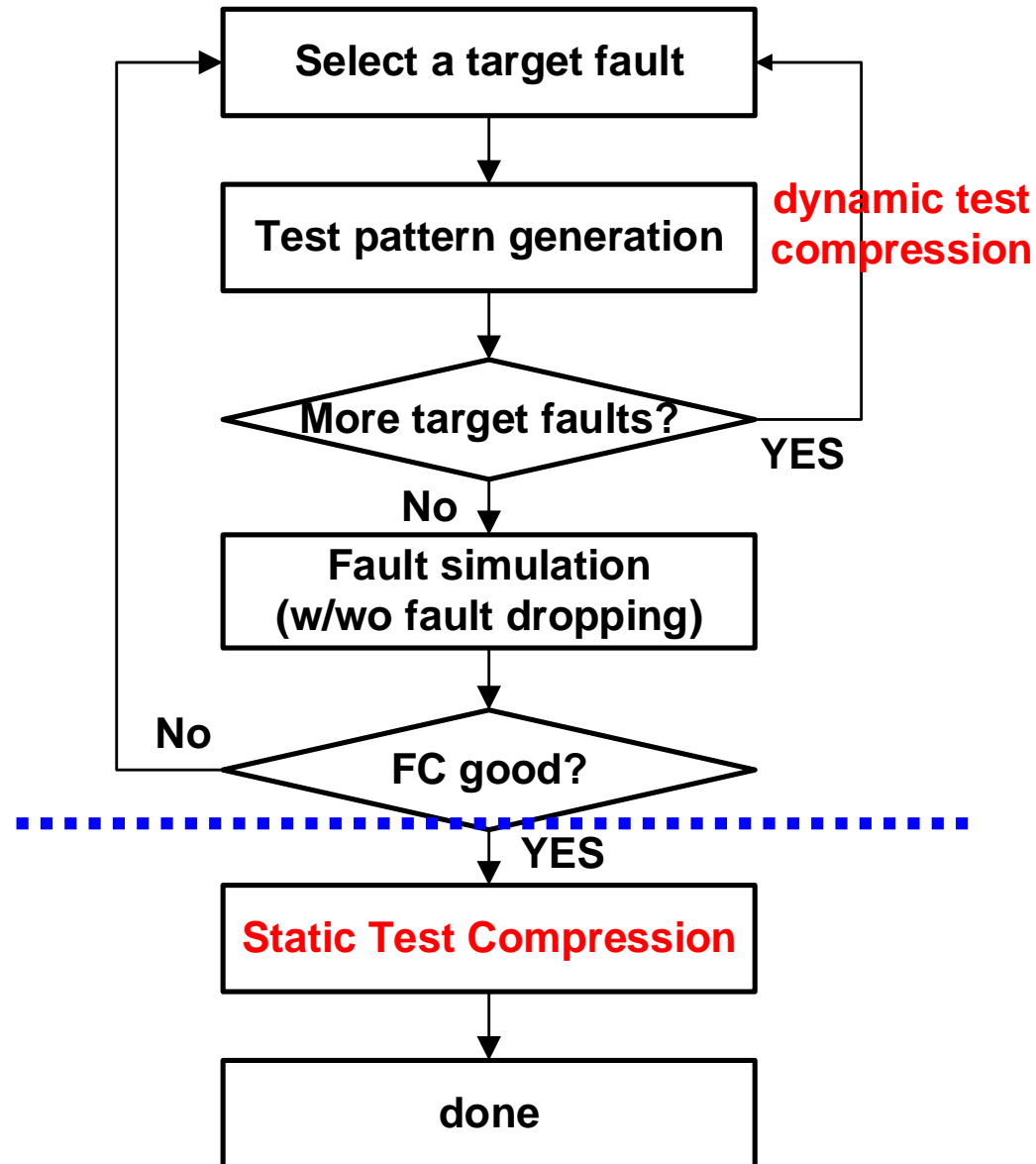
# Test Compression

- Introduction
- Software Techniques
  - ♦ Dynamic Test Compression (DTC)
  - ♦ Static Test Compression (STC)
- Hardware Techniques
  - ♦ Test Stimulus Compression
  - ♦ Test Response Compression
  - ♦ Industry Practices
- Conclusion



# Review: STC vs. DTC

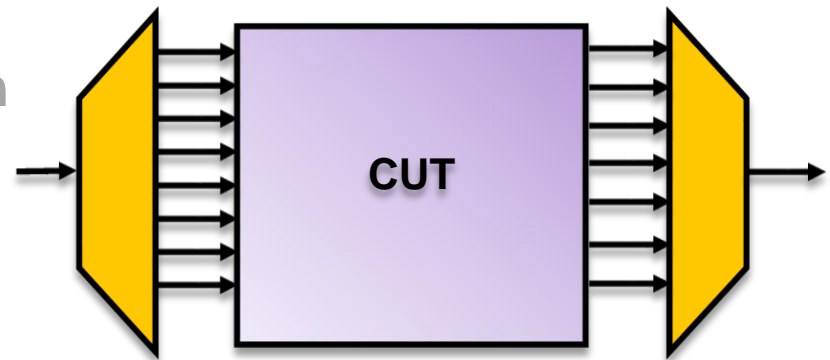
- **Dynamic test compression**
  - ♦ performed **during** TPG
  - ♦ more CPU time
  - ♦ more effective
- **Static test compression**
  - ♦ performed **after** TPG
  - ♦ less CPU time
  - ♦ less effective



# Test Compression

- Introduction
- Software Techniques
  - ♦ Dynamic Test Compression
  - ♦ Static Test Compression
    - \* With fault dictionary
    - \* Without fault dictionary
      - Compatibility graph (X-unfilled)
      - Reverse order fault simulation (X-filled)
- Hardware Techniques
  - ♦ Test Stimulus Compression
  - ♦ Test Response Compression
  - ♦ Industry Practices
- Conclusion

} 3 cases



# STC with Fault Dictionary

- Suppose we have a fault dictionary (**without fault dropping**)
- **Covering table**
  - ♦ Each row is a test pattern, each column is a fault
- Goal: Find minimum test set (select fewest test patterns)
  - ♦ Detect all faults
- Finding minimum test set is **minimum set covering problem**
  - ♦ NP-hard, but don't give up...

	$f_1$	$f_2$	$f_3$	$f_4$	$f_5$
$t_1$				X	
$t_2$		X	X		X
$t_3$		X		X	X
$t_4$	X			X	

Each row is a pattern  
Each column is a fault  
X= detection  
(not don't care!)

# Quine-McCluskey Method [McCluskey 56]

- First EDA algorithm for 2-level logic synthesis
- Fault that is detected only once is **essential fault**
  - ♦ Must select **essential patterns** that detect essential faults
- Example:
  - ♦  $f_1, f_3$  are essential faults;  $t_2, t_4$  are essential patterns
  - ♦ Test set selected =  $\{t_2, t_3, t_4\}$  or  $\{t_1, t_2, t_4\}$ , minimum test length = 3

	$f_1$	$f_2$	$f_3$	$f_4$	$f_5$
$t_1$		X		X	
$t_2$			X		X
$t_3$		X		X	X
$t_4$	X			X	

Each row is a pattern  
Each column is a fault  
X= detection

**What if No Essential Faults?**

# Quine-McCluskey Method (cont'd)

- 1. Remove redundant **equivalent row**, keep one row is enough
  - Row  $t_1$  is equal to row  $t_2$  because they have X in same columns
- 2. Remove **dominated row**
  - Row  $t_3$  dominates row  $t_4$  because
    - (1) row  $t_3$  has X in all columns where row  $t_4$  has X, and
    - (2) row  $t_3$  has at least one X where row  $t_4$  does not have X

	$f_1$	$f_2$	$f_3$	$f_4$	$f_5$	
$t_1$	X	X	X		X	
$t_2$	X	X	X		X	equivalent row
$t_3$		X	X	X	X	
$t_4$			X	X	X	dominated row
$t_5$	X			X	X	

# Quine-McCluskey Method (cont'd)

- 3. Remove **dominating column**
  - Column  $f_5$  dominates column  $f_4$  ( $f_3, f_2, f_1$  also) because
    - (1) column  $f_5$  has X in all rows where column  $f_4$  has X, and
    - (2) column  $f_5$  has at least one X where column  $f_4$  does not have X
- 4. **Secondary essential**
  - After steps 1~3,  $t_3$  is now secondary essential pattern
- Minimum test set  $\{t_1, t_3\}$  or  $\{t_3, t_5\}$ , minimum test length =2

dominating column

	$f_1$	$f_2$	$f_3$	$f_4$	$f_5$
$t_1$	X	X	X		X
$t_3$		X	X	X	X
$t_5$	X				X

✓

(cont'd from last page)

# Quiz

Q1: Which are essential faults?

Q2: Which are dominated rows? Dominating columns?

Q3: What is minimum test length?

	$f_1$	$f_2$	$f_3$	$f_4$	$f_5$
$t_1$			X	X	X
$t_2$		X	X		X
$t_3$			X		
$t_4$	X				
$t_5$		X		X	X

**QM Solves Many Cases in Polynomial Time**  
but not all...



# FFT

- Mini-set covering is NP-hard
  - ♦ Q1: Show an special case where no rule of QM can be applied
  - ♦ Q2: What are you going to do with it?

	$f_1$	$f_2$	$f_3$	$f_4$
$t_1$	X	X		
$t_2$		X	X	
$t_3$			X	X
$t_4$	X			X

# Alternative Solution, 01-ILP

- Model STC as **01-Integer Linear Programming** problem

$$\begin{aligned} \text{Objective: } & \min \sum_i t_i \\ \text{s.t. } & \sum_i d_{i,j} \times t_i \geq 1, \quad \text{foreach fault } j \end{aligned}$$

- ♦  $t_i=1$  ,if test  $i$  is selected;  $t_i=0$  otherwise
- ♦  $d_{i,j}=1$  if test pattern  $t_i$  detects fault  $f_j$

- Example:  $t_1=0, t_2=t_3=t_4=1$

	$f_1$	$f_2$	$f_3$	$f_4$	$f_5$
$t_1$		X			
$t_2$			X		X
$t_3$		X		X	X
$t_4$	X			X	

$$\begin{aligned} \min \quad & t_1 + t_2 + t_3 + t_4 \\ \text{s.t.} \quad & \end{aligned}$$

$$t_4 \geq 1$$

$$t_1 + t_3 \geq 1$$

$$t_2 \geq 1$$

$$t_3 + t_4 \geq 1$$

$$t_2 + t_3 \geq 1$$

# It is Well-solved... What Is Wrong?

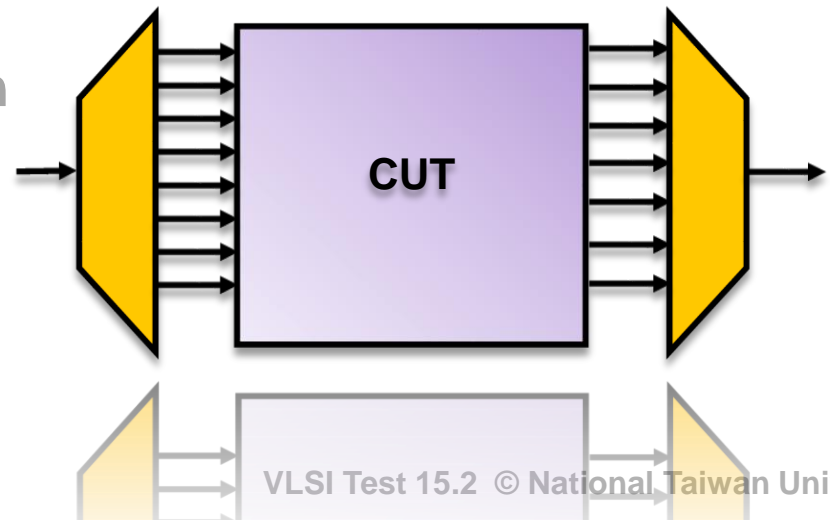
- Q: What is practical issue of STC with dictionary?
  - ◆ A: Complete fault dictionary is very large, very slow

	$f_1$	$f_2$	$f_3$	$f_4$	$f_5$
$t_1$		X	X		
$t_2$	X	X	X		X
$t_3$			X		X
$t_4$	X			X	

**Need STC without Dictionary**

# Test Compression

- Introduction
- Software Techniques
  - ♦ Dynamic Test Compression
  - ♦ Static Test Compression
    - \* With fault dictionary
    - \* Without fault dictionary
      - Compatibility graph (X-unfilled)
      - Reverse order fault simulation (X-filled)
- Hardware Techniques
  - ♦ Test Stimulus Compression
  - ♦ Test Response Compression
  - ♦ Industry Practices
- Conclusion



# STC w/o Dictionary (X-unfilled)

- Suppose that we do NOT have dictionary
  - ♦ but we have don't care bits in test cubes
- Two test cubes are **compatible** iff no conflict in specified bits
- Compatible test cubes can be **merged** into one test cube
- Example
  - ♦  $t_0$  and  $t_1$  are compatible, merged to  $0xx10$
  - ♦ Feasible solution:
    - \* **4 patterns:**  $\{t_0+t_1+t_2, t_3+t_6, t_4+t_5, t_7\}$
    - \* Any better solution?

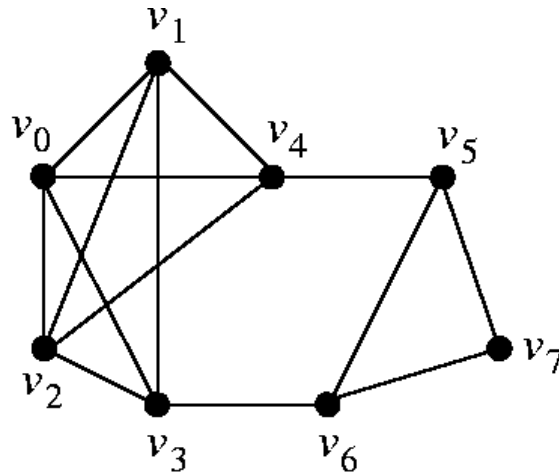
$t_0$	0xx10
$t_1$	0xx1x
$t_2$	0x01x
$t_3$	01xx0
$t_4$	x0xx0
$t_5$	1xxxx
$t_6$	x1x00
$t_7$	11xx0

x = don't cares

**Any Algorithm  
to Solve This Problem?**

# Compatibility Graph

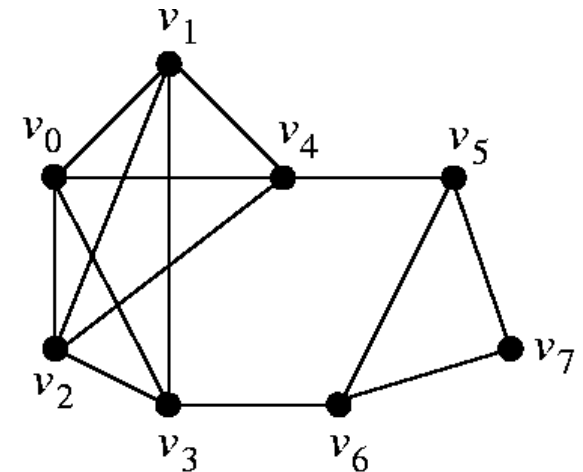
- **Compatibility graph**  $G(V, E)$ 
  - ♦ Vertex  $v_i$  represents a test cube  $t_i$
  - ♦ Edge  $e_{ij}$  between  $v_i$  and  $v_j$  means two test cubes are **compatible**
  - ♦ **Adjacent vertices** can be merged into one
- **Example**
  - ♦  $t_0$  and  $t_1$  are adjacent, can be merged



$t_0$	0xx10
$t_1$	0xx1x
$t_2$	0x01x
$t_3$	01xx0
$t_4$	x0xx0
$t_5$	1xxxx
$t_6$	x1x00
$t_7$	11xx0

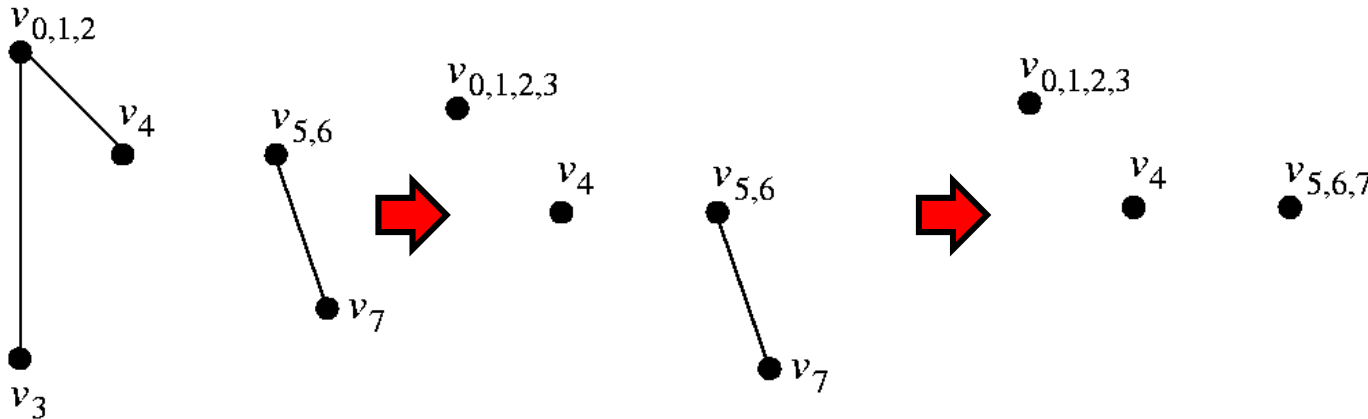
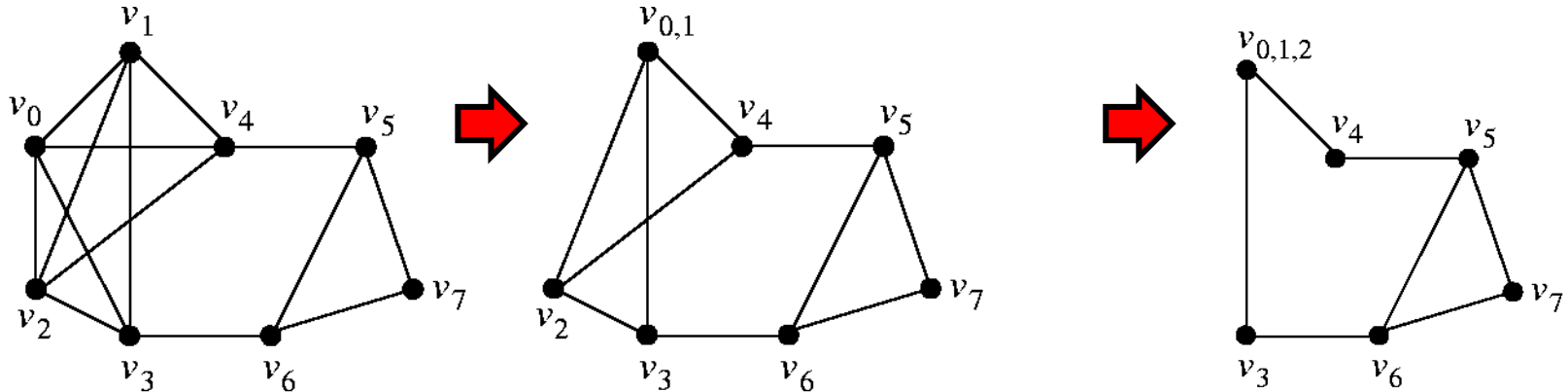
# Clique

- **Clique** is a subset of vertices such that
  - ♦ Each pair of vertices are connected
  - ♦ Clique is a *complete subgraph*
- **Minimum clique partition problem**
  - ♦ Partition graph into minimum number of cliques
- Example:
  - ♦  $\{v_0, v_1, v_2, v_3\}$   $\{v_5, v_6, v_7\}$  are cliques
  - ♦ minimum clique partition
    - \*  $\{v_0, v_1, v_2, v_3\} \{v_4\} \{v_5, v_6, v_7\}$
    - \* 3 partitions
- MCP is NP-hard problem
  - ♦ Can be solved by Tseng-Siewiorek
    - \* Greedy algorithm, does NOT guaranteed optimal solution



# Tseng-Siewiorek Idea

- Select two **adjacent vertices** of maximum **common neighbors**
- Merge two vertices into a **supervertex**
  - ♦ e.g., merge  $v_0 v_1 \Rightarrow v_{0,1}$  supervertices
- Iteratively merge vertices until no more edge



$t_{0,1,2,3}$	01010
$t_4$	x0xx0
$t_{5,6,7}$	11xx00



# Tseng-Siewiorek Algorithm (1)

```

 $k \leftarrow 0;$ 
 $G_c^k(V_c^k, E_c^k) \leftarrow G_c(V_c, E_c);$ 
while ( $E_c^k \neq \emptyset$ ) {
    find  $(v_i, v_j) \in E_c^k$  with largest set of common neighbors;
     $N \leftarrow$  set of common neighbors of  $v_i$  and  $v_j$ ;
     $s \leftarrow i \cup j$ ;
     $V_c^{k+1} \leftarrow V_c^k \cup \{v_s\} \setminus \{v_i, v_j\};$   $\setminus$  means remove
     $E_c^{k+1} \leftarrow \emptyset;$ 
    for each  $(v_m, v_n) \in E_c^k$  build new edges
        if ( $v_m \neq v_i \wedge v_m \neq v_j \wedge v_n \neq v_i \wedge v_n \neq v_j$ )
             $E_c^{k+1} \leftarrow E_c^{k+1} \cup \{(v_m, v_n)\};$ 
        for each  $v_n \in N$ 
             $E_c^{k+1} \leftarrow E_c^{k+1} \cup \{(v_n, v_s)\};$ 
     $k \leftarrow k + 1;$ 
}

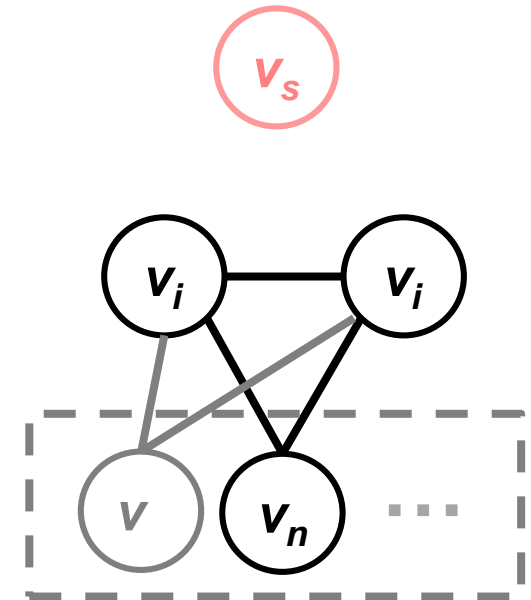
```

$G_c^k$  = compatibility graph in  $K_{th}$  iteration

$V_c^k$  = set of vertices in  $G_c^k$

$E_c^k$  = set of edges in  $G_c^k$

$v_s$  = supervertex of  $v_i$  and  $v_j$



$N = \{\text{common neighbors}\}$

# Tseng-Siewiorek Algorithm (2)

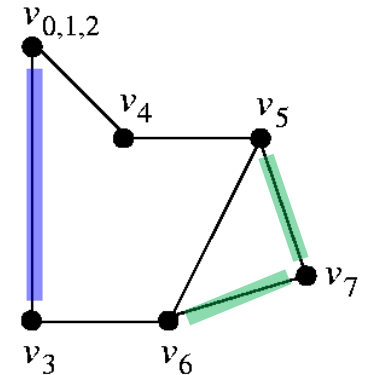
```

 $k \leftarrow 0;$ 
 $G_c^k(V_c^k, E_c^k) \leftarrow G_c(V_c, E_c);$ 
while ( $E_c^k \neq \emptyset$ ) {
    find  $(v_i, v_j) \in E_c^k$  with largest set of common neighbors;
     $N \leftarrow$  set of common neighbors of  $v_i$  and  $v_j$ ;
     $s \leftarrow i \cup j$ ;
     $V_c^{k+1} \leftarrow V_c^k \cup \{v_s\} \setminus \{v_i, v_j\};$ 
     $E_c^{k+1} \leftarrow \emptyset;$ 
    for each  $(v_m, v_n) \in E_c^k$ 
        if ( $v_m \neq v_i \wedge v_m \neq v_j \wedge v_n \neq v_i \wedge v_n \neq v_j$ )
             $E_c^{k+1} \leftarrow E_c^{k+1} \cup \{(v_m, v_n)\};$ 
    for each  $v_n \in N$ 
         $E_c^{k+1} \leftarrow E_c^{k+1} \cup \{(v_n, v_s)\};$ 
     $k \leftarrow k + 1;$ 
}

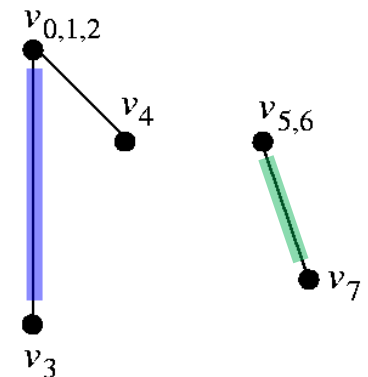
```

keep edges touch  
neither  $v_i$  nor  $v_j$   
e.g. edge ( $v_{012}$ ,  $v_3$ )

new edge between  $N$  and  $v_s$   
e.g. edge ( $v_5$ ,  $v_7$ )



$v_i, v_j =$   
 $v_5, v_6$



# Quiz

**Q1: Draw compatibility graph**

**Q2: What is minimum test length using T-S algorithm?**

$t_0$	0xx10
$t_1$	x1x10
$t_2$	0x11x
$t_3$	00x11

# FFT

- Q: What is practical problem with this method?
  - ◆ Practically, there are many don't cares in test cubes
  - ◆ X-unfilled test generation is slower and length is longer
    - \* than X-filled test generation

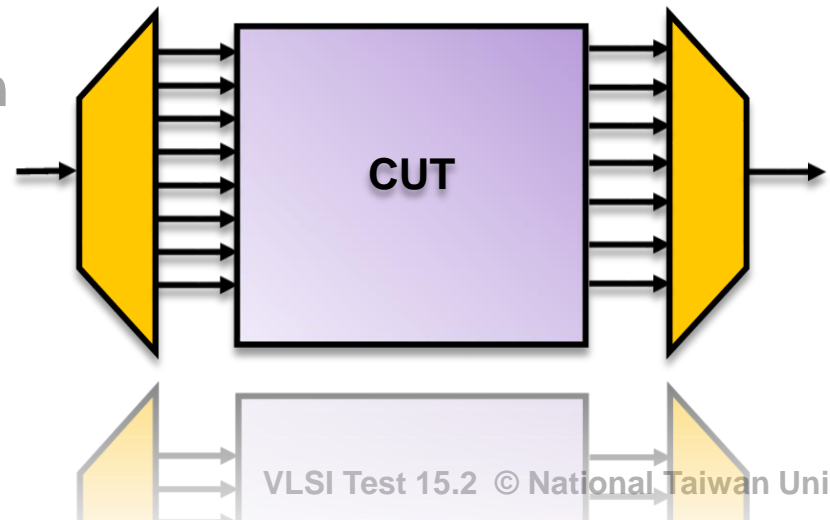
$t_0$	0xx10xxxxxx1
$t_1$	0xx1xxxxxxxxxx
$t_2$	0x01xxx1xxxx
$t_3$	01xx0xxxxxxxx
$t_4$	X0xx0xxxxx0x
$t_5$	1xxxxxxxxxxxx
$t_6$	x1x00xxxx1xx
$t_7$	11xx0xxxxxx0

$t_0$	011101100011
$t_1$	010111000000
$t_2$	010011010101

**Need STC with X-filled**

# Test Compression

- Introduction
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  - ♦ Dynamic Test Compression
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    - \* With fault dictionary
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  - ♦ Test Stimulus Compression
  - ♦ Test Response Compression
  - ♦ Industry Practices
- Conclusion



# STC with X-filled (1)

- **Reverse-order fault simulation**

- ♦ Fault simulate X-filled patterns in reverse order of ATPG

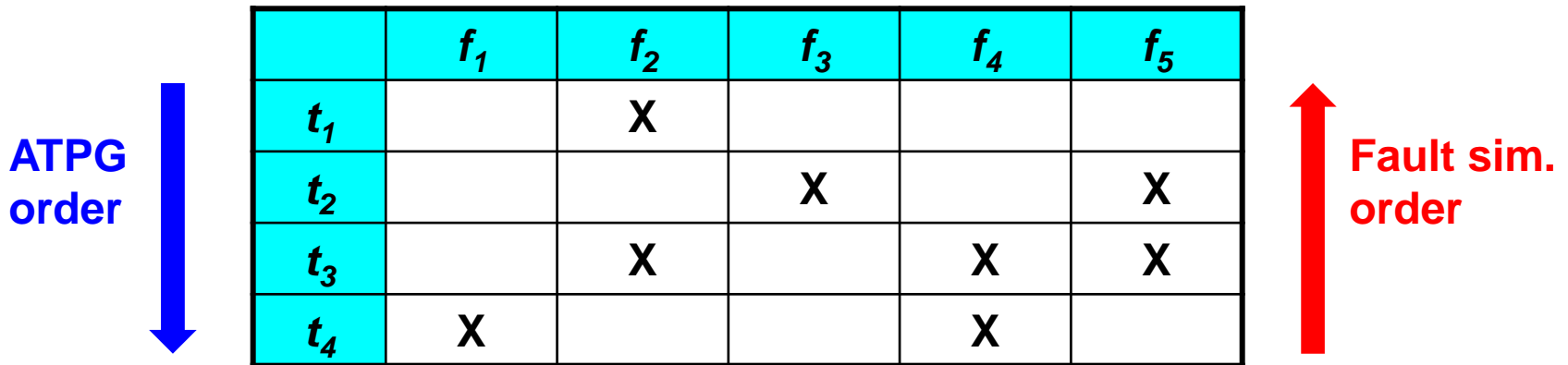
- \* Delete redundant test patterns

- ♦ Example:

- \* First simulate  $t_4$ , and then  $t_3, t_2, t_1$

- \* Delete  $t_1$ . Choose test set  $\{t_4, t_3, t_2\}$

- ♦ Advantage: **Simple, no dictionary needed. Most popular STC**



The diagram illustrates the relationship between ATPG order and fault simulation order. A blue arrow labeled 'ATPG order' points downwards, indicating the sequence of test patterns generated by ATPG. A red arrow labeled 'Fault sim. order' points upwards, indicating the sequence of test patterns simulated for faults. The matrix shows that faults are simulated in reverse order of their generation.

	$f_1$	$f_2$	$f_3$	$f_4$	$f_5$
$t_1$		X			
$t_2$			X		X
$t_3$		X		X	X
$t_4$	X			X	

- Q: Why ATPG generated redundant  $t_1$  at beginning?

# STC with X-filled (2)

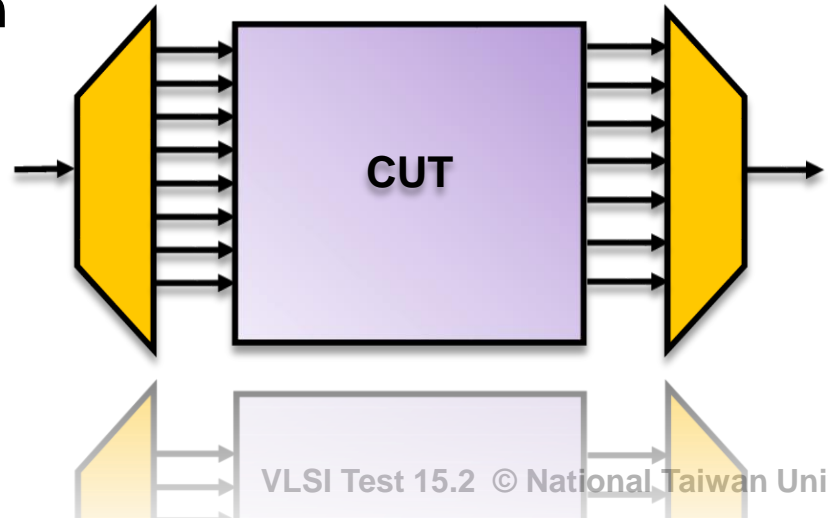
- **Random-order fault simulation**
  - ♦ Fault simulate test patterns in random order
  - ♦ Example:
    - \* First simulate  $t_4$ , and then  $t_2$ ,  $t_1$ ,  $t_3$
    - \* Choose test set  $\{t_4, t_1, t_2\}$

	$f_1$	$f_2$	$f_3$	$f_4$	$f_5$
$t_1$		X			
$t_2$			X		X
$t_3$		X		X	X
$t_4$	X			X	

**Too Many Orders to Try**

# Summary

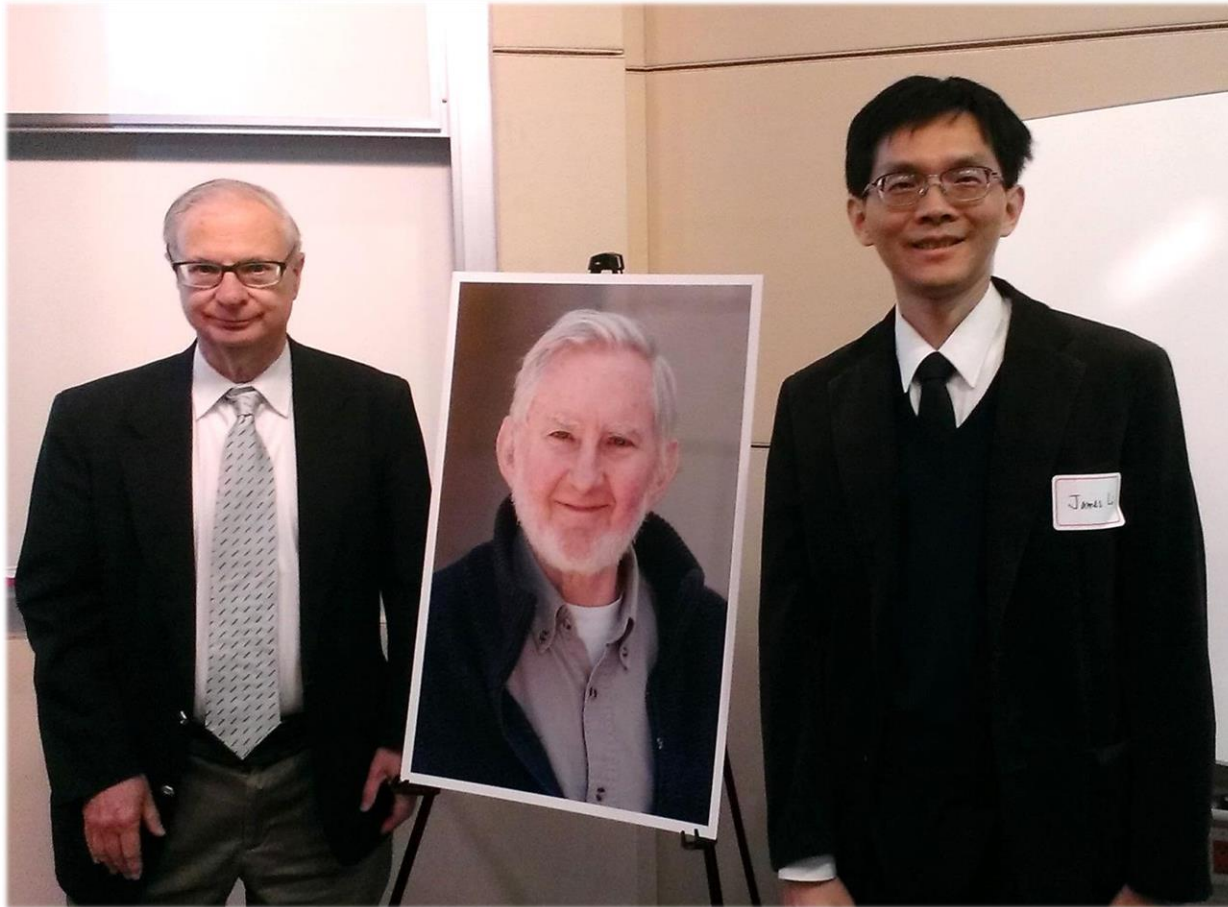
- Static test compression
  - ◆ With fault dictionary
    - \* Minimum set-covering problem
      - Quine-McCluskey or 01-ILP
    - \* Too large dictionary
  - ◆ Without fault dictionary
    - \* Compatibility graph (X-unfilled)
      - T-S Algorithm
    - \* Reverse order fault simulation (X-filled)
      - Most popular solution





# Three Authors Together

- Prof. Siewiorek, Prof. McCluskey, Prof. James Li
- 2016 Stanford University



# FFT1

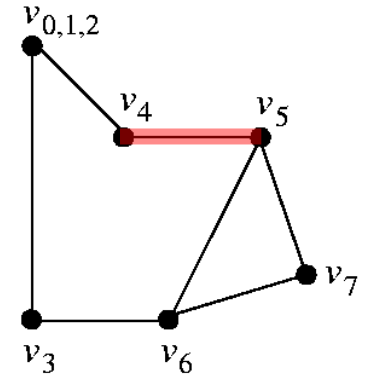
```

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     $N \leftarrow$  set of common neighbors of  $v_i$  and  $v_j$ ;
     $s \leftarrow i \cup j$ ;
     $V_c^{k+1} \leftarrow V_c^k \cup \{v_s\} \setminus \{v_i, v_j\};$ 
     $E_c^{k+1} \leftarrow \emptyset;$ 
    for each  $(v_m, v_n) \in E_c^k$ 
        if ( $v_m \neq v_i \wedge v_m \neq v_j \wedge v_n \neq v_i \wedge v_n \neq v_j$ )
             $E_c^{k+1} \leftarrow E_c^{k+1} \cup \{(v_m, v_n)\};$ 
    for each  $v_n \in N$ 
         $E_c^{k+1} \leftarrow E_c^{k+1} \cup \{(v_n, v_s)\};$ 
     $k \leftarrow k + 1;$ 
}

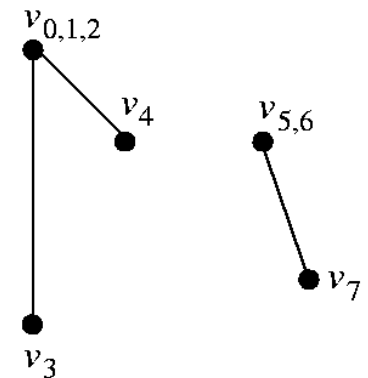
```

keep edges touch  
neither  $v_i$  nor  $v_j$   
e.g. edge ( $v_{012}$ ,  $v_3$ )

new edges between  $N$  and  $v_s$   
e.g. edge ( $v_5$ ,  $v_7$ )



$v_i, v_j = v_5, v_6$



**Why Edge ( $v_4, v_5$ ) Removed?**

# FFT2

- MCP is NP-hard
- Show an example when TS-algorithm fails to find an optimal solution

