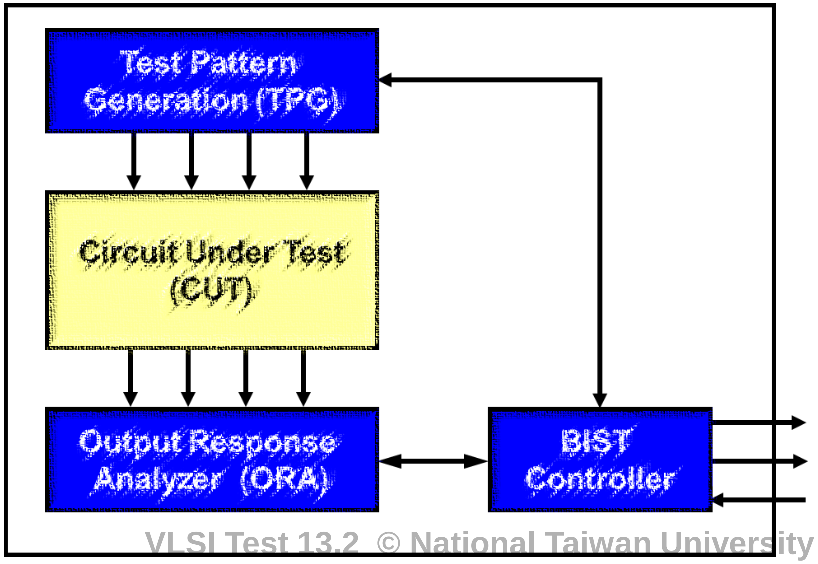
# BIST Part1 - TPG

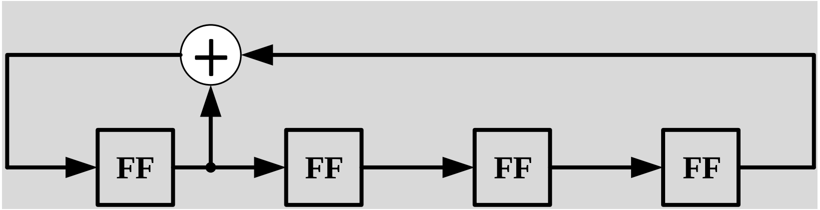
- Introduction
- Test Pattern Generation (TPG)
  - Deterministic: ROM Algorithm Counter
  - **Pseudo Random**
    - ***Linear Feedback Shift Register, LFSR* (1977)**
      - **Two types of LFSR**
      - Design of LFSR
    - Cellular Automata, CA (1984)



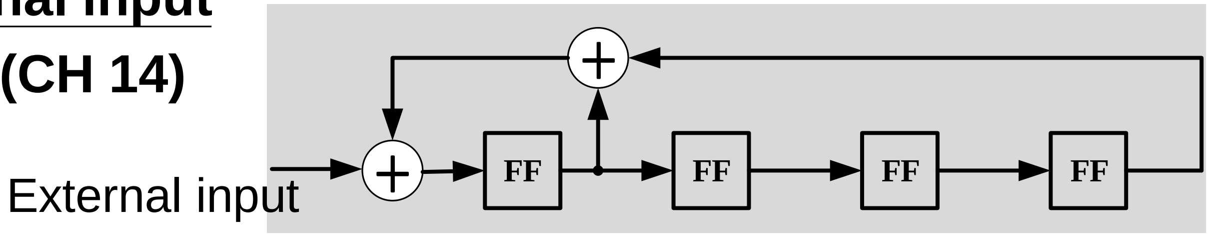
# Linear Feedback Shift Register [Frohwerk 77]

- LFSR consist of unit delays (flip-flops, FF) and feedback (XOR )
- Two applications of LFSR:

1. LFSR without external input
   - Used for TPG
   - aka *Autonomous LFSR*




2. LFSR with external input
   - Used for ORA (CH 14)

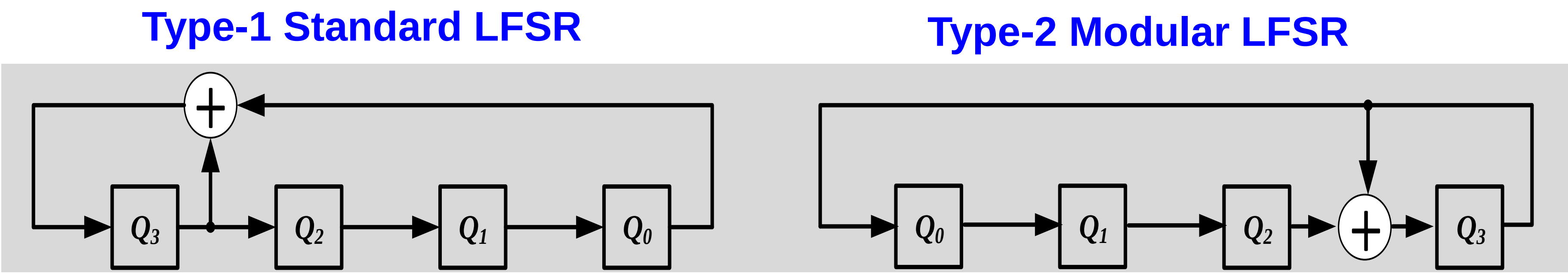



⊕ =XOR

- *"Linear"* because XOR is *mod-2 addition*

# Two Types of Autonomous LFSR

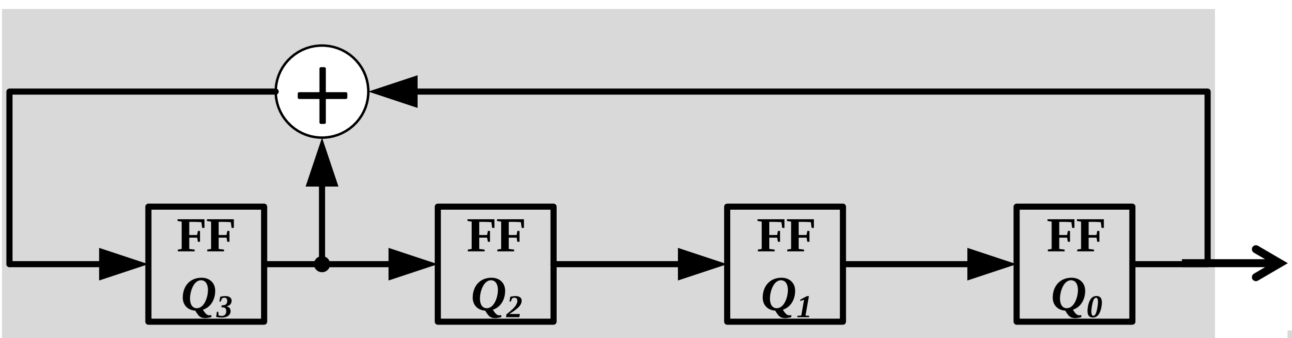
- **Autonomous LFSR is *Modular Counter***
  - **Very small area, generate *pseudo random* outputs**
- **Two structures:**
  - ***Type 1: Standard Form* (aka *external XOR*) LFSR**
  - ***Type 2: Modular Form* ( aka *internal XOR*) LFSR**




**LFSR is Good for TPG**

# Type-1, Standard Form LFSR

- Three ways to describe LFSR
  1. ***Next state equation***: $Q_3^+ = Q_3 \oplus Q_0$
     - $Q_3^+$ means next state of FF $Q_3$
  2. ***Recurrence equation***: $q_m = q_{m-1} \oplus q_{m-4}$
  3. ***Characteristic polynomial***: $f(x) = x^4 + x^3 + 1$
     - Most popular. will see why 13.4




1 0 0 0

output bit stream
($q_0$ is first)

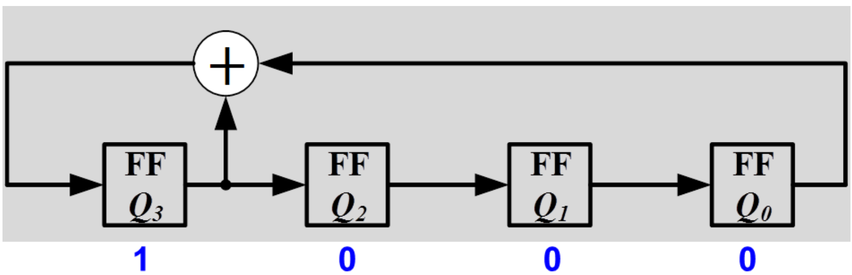
| $q_0$ | $q_1$ | $q_2$ | $q_3$ | $q_4$ | $q_5$ | $q_6$ | $q_7$ |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 |

$q_4 = q_3 \oplus q_0$
$q_5 = q_4 \oplus q_1$
...
$q_m = q_{m-1} \oplus q_{m-4}$

- NOTE: Modular-2 arithmetic: $\oplus$ + - are same
  - $1\oplus1=0$; $0\oplus1=1$; $1\oplus0=1$; $0\oplus0=0$

# State Sequence of $x^4+x^3+1$ LFSR

- *Seed* = Initial state of LFSR
  - must be non-zero
- Total $2^4$-1 = 15 distinct states
  - All-zero state not included
- Periodical.
  - Cycle length $L_c$= 15

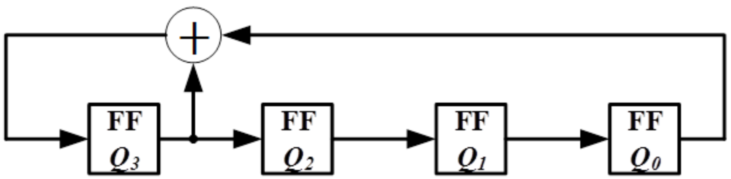



Back to seed after 15 cycles →

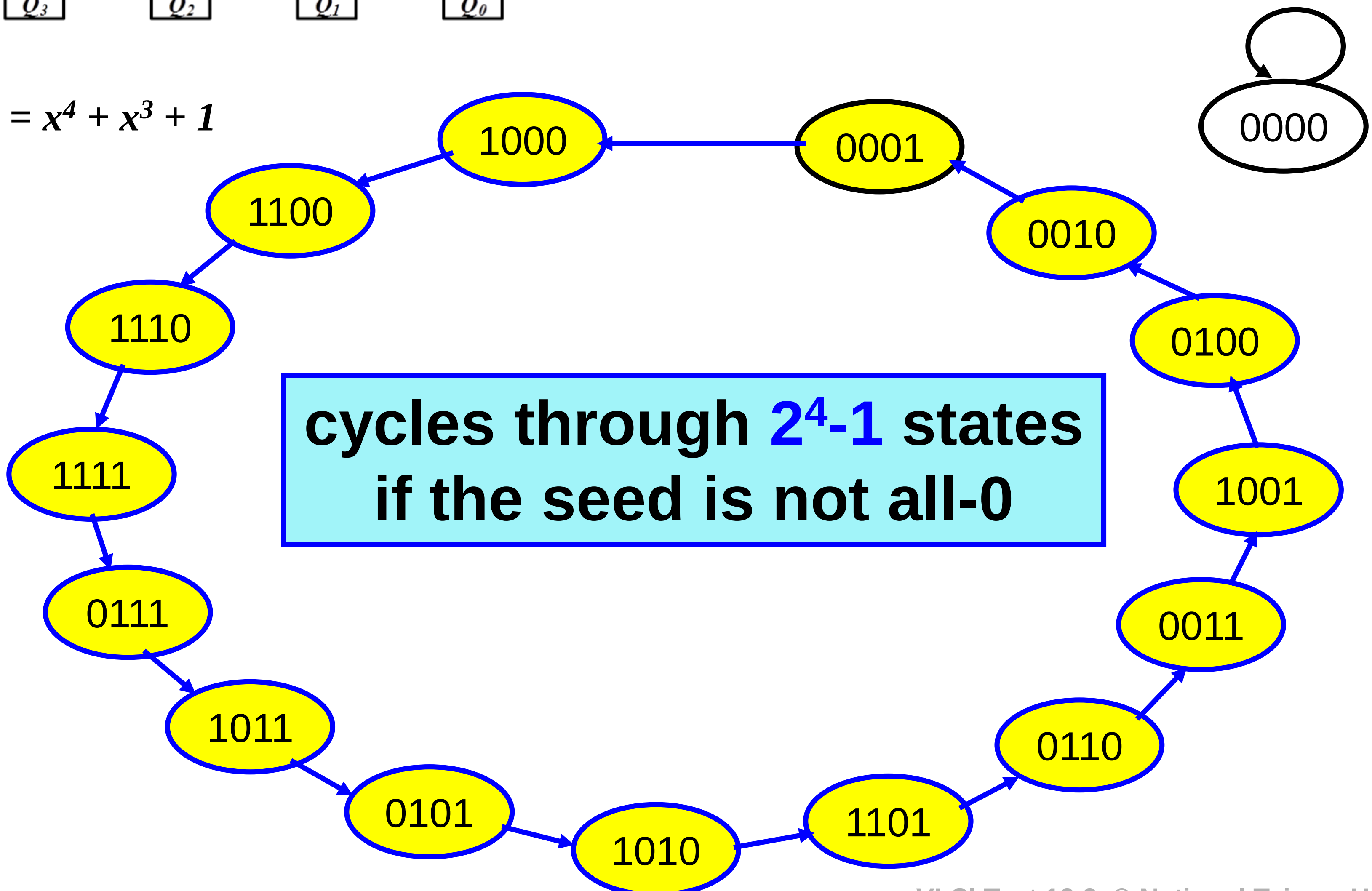
| state | $Q_3$ | $Q_2$ | $Q_1$ | $Q_0$ |
|---|---|---|---|---|
| 0 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 |
| 2 | 1 | 1 | 1 | 0 |
| 3 | 1 | 1 | 1 | 1 |
| 4 | 0 | 1 | 1 | 1 |
| 5 | 1 | 0 | 1 | 1 |
| 6 | 0 | 1 | 0 | 1 |
| 7 | 1 | 0 | 1 | 0 |
| 8 | 1 | 1 | 0 | 1 |
| 9 | 0 | 1 | 1 | 0 |
| 10 | 0 | 0 | 1 | 1 |
| 11 | 1 | 0 | 0 | 1 |
| 12 | 0 | 1 | 0 | 0 |
| 13 | 0 | 0 | 1 | 0 |
| 14 | 0 | 0 | 0 | 1 |
| 15 (=0) | 1 | 0 | 0 | 0 |

seed

# State Diagram

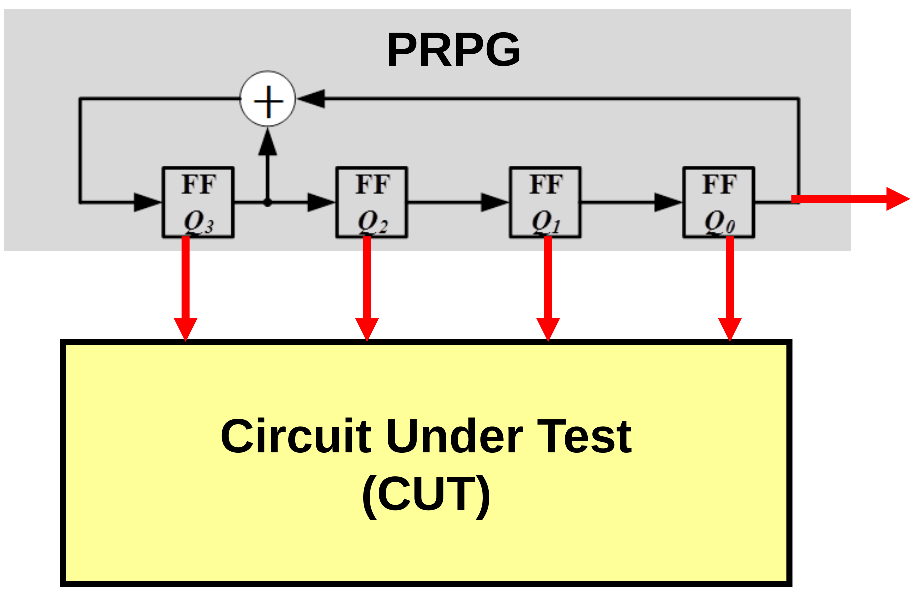



$f(x) = x^4 + x^3 + 1$



# Pseudo Random Pattern Generator (PRPG)

- ***Pseudo random.*** **NOT truly random**
- **Serial PRPG:** $q_0, q_1, q_2, q_3, \ldots$
  - **Periodical.** $L_c$**= 15**
- **Parallel PRPG:** $(Q_3\ Q_2\ Q_1\ Q_0)$
  - **Each output shifted by one cycle**
  - ***Phase difference*** **=1**




| state | $Q_3$ | $Q_2$ | $Q_1$ | $Q_0$ | |
|---|---|---|---|---|---|
| 0 | 1 | 0 | 0 | 0 | $q_0$ |
| 1 | 1 | 1 | 0 | 0 | $q_1$ |
| 2 | 1 | 1 | 1 | 0 | $q_2$ |
| 3 | 1 | 1 | 1 | 1 | $q_3$ |
| 4 | 0 | 1 | 1 | 1 | $q_4$ |
| 5 | 1 | 0 | 1 | 1 | $q_5$ |
| 6 | 0 | 1 | 0 | 1 | $q_6$ |
| 7 | 1 | 0 | 1 | 0 | $q_7$ |
| 8 | 1 | 1 | 0 | 1 | $q_8$ |
| 9 | 0 | 1 | 1 | 0 | $q_9$ |
| 10 | 0 | 0 | 1 | 1 | $q_{10}$ |
| 11 | 1 | 0 | 0 | 1 | $q_{11}$ |
| 12 | 0 | 1 | 0 | 0 | $q_{12}$ |
| 13 | 0 | 0 | 1 | 0 | $q_{13}$ |
| 14 | 0 | 0 | 0 | 1 | $q_{14}$ |
| 15 (=0) | 1 | 0 | 0 | 0 | $q_{15}$ |

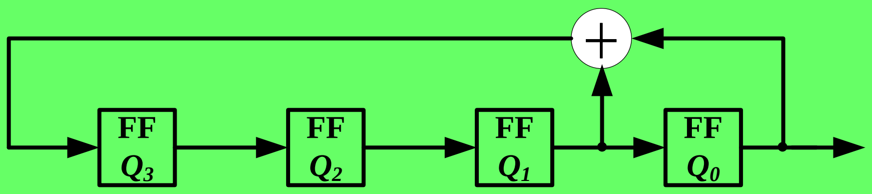
$\leftrightarrow$ $\leftrightarrow$ $\leftrightarrow$

**Phase diff. =1**

# Quiz

**Q: Given this new LFSR, show the state sequences with seed [1 0 0 0]**

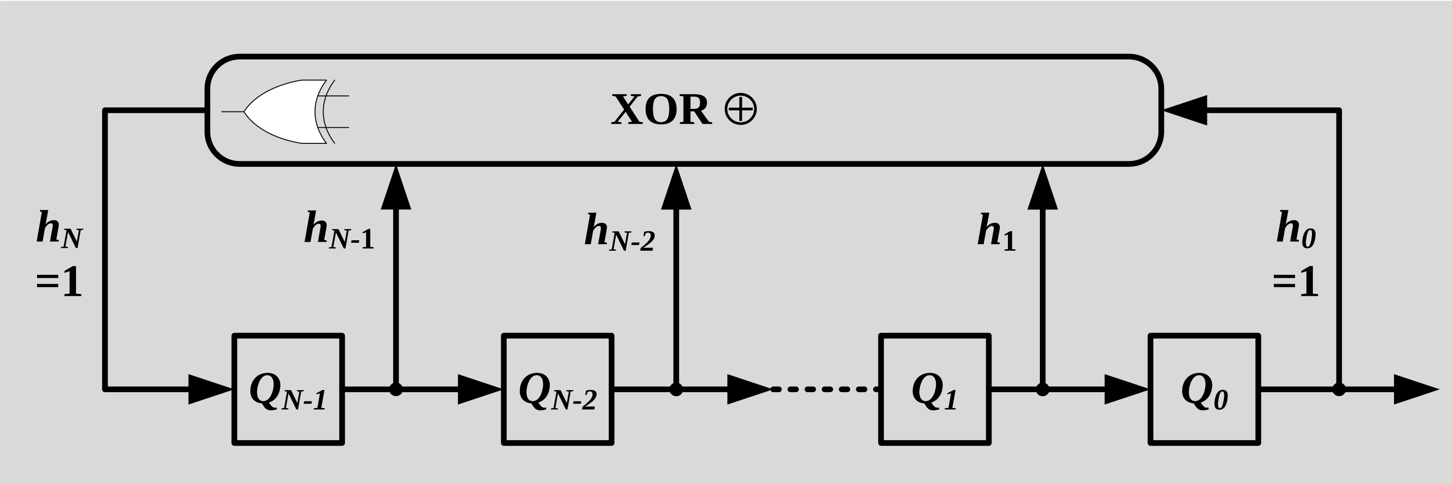
**Q2: What is cycle length $L_c$=?**



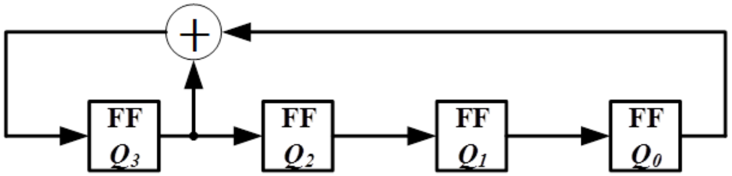

| state | $Q_3$ | $Q_2$ | $Q_1$ | $Q_0$ |
|---|---|---|---|---|
| 0 | 1 | 0 | 0 | 0 |
| 1 | | | | |
| 2 | | | | |
| 3 | | | | |
| 4 | | | | |
| 5 | | | | |
| 6 | | | | |
| 7 | | | | |
| 8 | | | | |
| 9 | | | | |
| 10 | | | | |
| 11 | | | | |
| 12 | | | | |
| 13 | | | | |
| 14 | | | | |
| 15 | | | | |

# *N*-degree Standard Form LFSR

- ***N = LFSR degree*** = number of FF = characteristic polynomial degree




$h_i$ = 1 if feedback exists
$h_i$ = 0 if no feedback
$h_N$ =1, $h_0$ =1




$f(x) = x^4 + x^3 + 1$

- ***Characteristic polynomial***: (details see appendix)

$$f(x) = \sum_{i=0}^{N} h_i x^i$$

***f(x)* Can Be Written from LFSR Structure**

# Matrix Representation (Type-1 LFSR)

Identity matrix

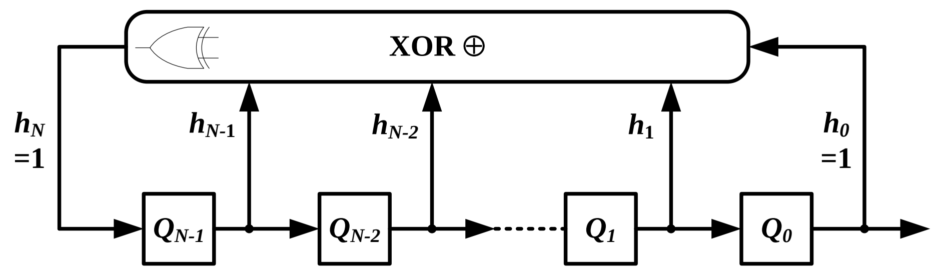
$$\begin{bmatrix} Q_0^+ \\ Q_1^+ \\ \cdot \\ \cdot \\ \cdot \\ Q_{N-3}^+ \\ Q_{N-2}^+ \\ Q_{N-1}^+ \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 & 0 \\ 0 & 0 & 0 & \dots & 0 & 1 \\ h_0 & h_1 & h_2 & \dots & h_{N-2} & h_{N-1} \end{bmatrix} \begin{bmatrix} Q_0 \\ Q_1 \\ \cdot \\ \cdot \\ \cdot \\ Q_{N-3} \\ Q_{N-2} \\ Q_{N-1} \end{bmatrix}$$

$Q^+ = T\,Q$ (mod-2)

$Q$ = current state

$Q^+$ = next state

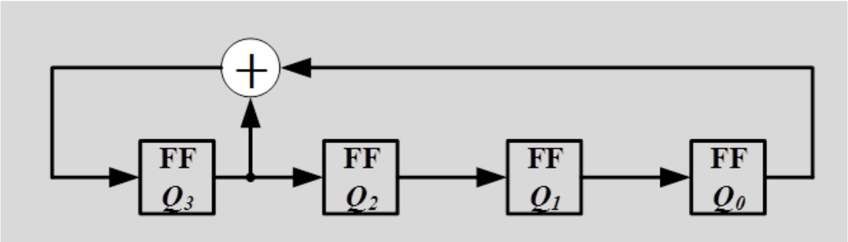
$T$ = ***companion matrix***




$Q_{N-1}^+ = \Sigma\, h_i Qi$

$Q_0^+ = Q_1$

# Example




| state | $Q_3$ | $Q_2$ | $Q_1$ | $Q_0$ |
|---|---|---|---|---|
| 0 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 |
| 2 | 1 | 1 | 1 | 0 |
| 3 | 1 | 1 | 1 | 1 |

$$T = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

$$T^2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

mod-2 arithmetic:
1x1=1 1x0=0 0x0=0
1+1=0 1+0=1 0+0=0

$$Q_{seed} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

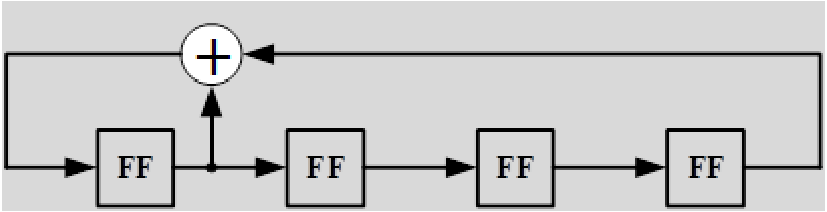
$$TQ_{seed} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

$$T^2Q_{seed} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$T^3Q_{seed} == T^2TQ_{seed} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

**Matrix Multiplication Same as Simulation**

# Linear Relationship between States




- $0_{th}$ *cycle: LFSR state* = $Q_{seed}$
- $1_{st}$ cycle: $TQ_{seed}$
- $2_{nd}$ cycle: $TTQ_{seed} = T^2Q_{seed}$
- $4_{th}$ cycle: $T^4Q_{seed} = (T^2)^2Q_{seed}$
- $16_{th}$ cycle: $T^{16}Q_{seed} = (T^8)^2Q_{seed} = (((T^2)^2)^2)^2Q_{seed}$
  - only 4 matrix squares needed

- After $L_c$ cycles, LFSR returns to initial state
  - $T^{Lc}Q_{seed} = Q_{seed}$
  - In this case $L_c$=15. $T^{15} = I$, so $T^{16} = T$

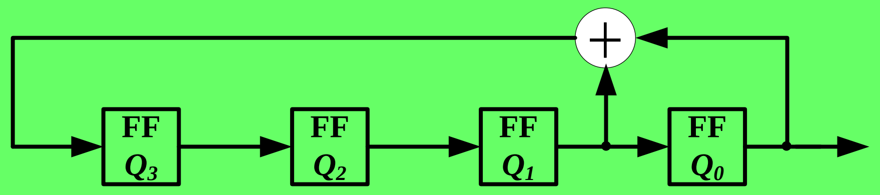
**Matrix Faster Than Simulation**

# Quiz

**Q1: Given this LFSR, What is its characteristic polynomial?**

**Q2: What is its companion matrix *T*?**

**Q3: Use *T* to derive state after three cycles, starting from seed [1 0 0 0]**




| state | $Q_3$ | $Q_2$ | $Q_1$ | $Q_0$ |
|---|---|---|---|---|
| 0 | 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 |
| 2 | 0 | 0 | 1 | 0 |
| 3 | 1 | 0 | 0 | 1 |

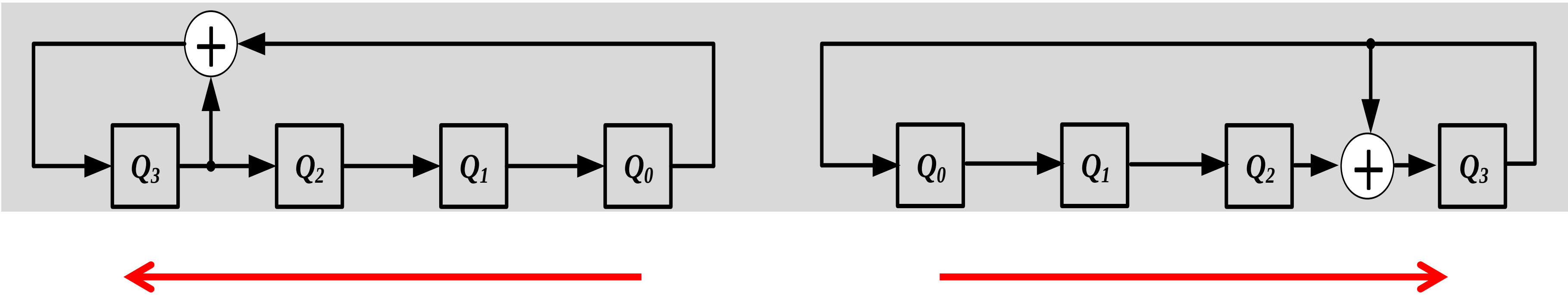
**ANS**

# LFSR Types

- *Type 1: Standard Form* (aka *external XOR*) LFSR
- ***Type 2: Modular Form ( aka internal XOR) LFSR***
  - XOR gates are internal to LFSR
  - as opposed to standard form LFSR, in which XOR are external

Type-1 Standard LFSR

Type-2 Modular LFSR



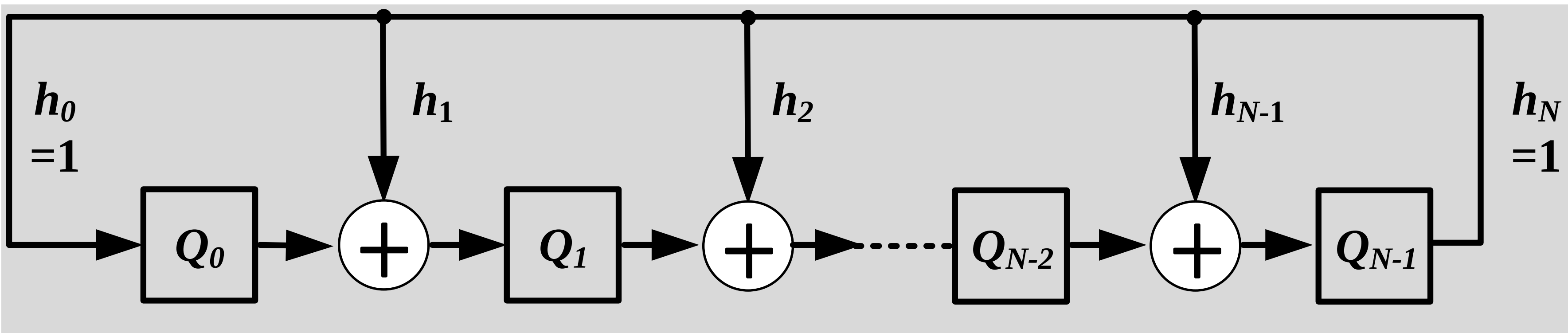

**NOTE: Order of FF Different**



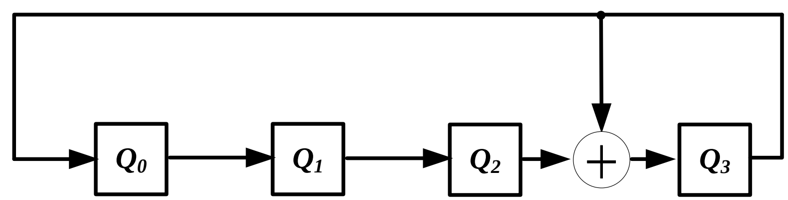
 

# *N*-degree Modular Form LFSR

- ***N* = *LFSR degree*** **= number of FF = characteristic polynomial degree**




**$h_i$ = 1 if feedback exists**
**$h_i$ = 0 if no feedback**
**$h_N$ =1, $h_0$ =1**




$f(x) = x^4 + x^3 + 1$

- ***Characteristic polynomial***: $f(x) = \sum_{i=0}^{N} h_i x^i$

## *f(x)* Same as Type-1 LFSR

**(remember correct order of FF !)**

# Matrix Representation (Type-2 LFSR)

$$\begin{bmatrix} Q_0^+ \\ Q_1^+ \\ \cdot \\ \cdot \\ \cdot \\ Q_{N-3}^+ \\ Q_{N-2}^+ \\ Q_{N-1}^+ \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & h_0 \\ 1 & 0 & 0 & \cdots & 0 & h_1 \\ \vdots & 1 & \vdots & & \vdots & h_2 \\ 0 & 0 & 1 & \cdots & 0 & \vdots \\ 0 & 0 & 0 & \cdots & 0 & h_{N-2} \\ 0 & 0 & 0 & \cdots & 1 & h_{N-1} \end{bmatrix} \begin{bmatrix} Q_0 \\ Q_1 \\ \cdot \\ \cdot \\ \cdot \\ Q_{N-3} \\ Q_{N-2} \\ Q_{N-1} \end{bmatrix}$$

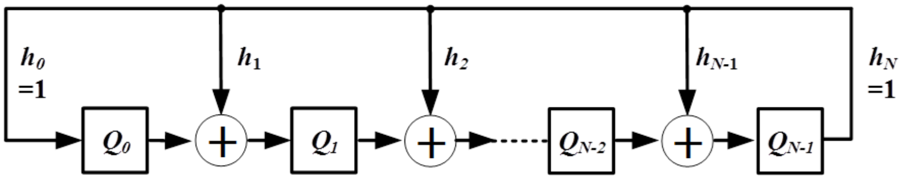
Identity matrix

$Q^+ = T\,Q$

$Q$ = current state

$Q^+$ = next state

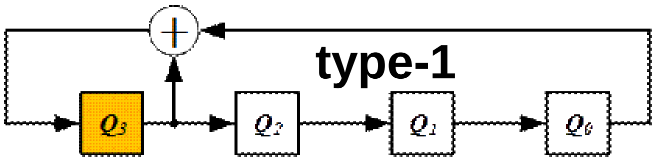
$T$ = Companion matrix

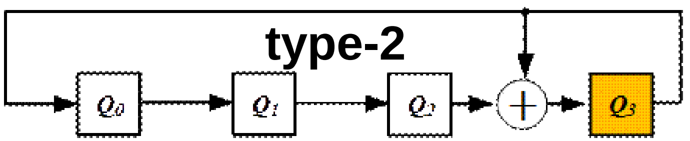



$$Q_1^+ = Q_0 + h_1 Q_{N-1}$$

# Quiz

Q: Given two types LFSR of $x^4+x^3+1$, same seed 1000, fill in table.



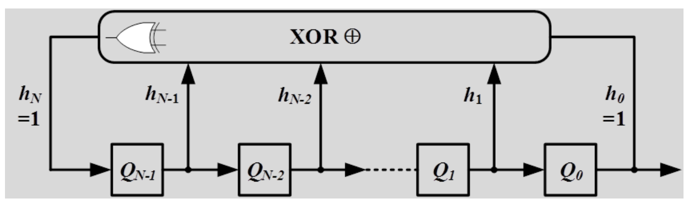
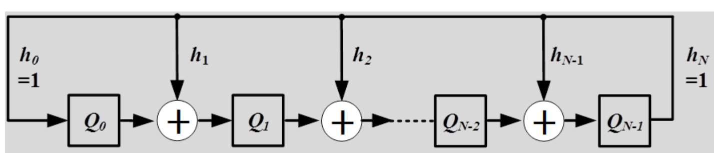




| state | $Q_3$ | $Q_2$ | $Q_1$ | $Q_0$ |
|---|---|---|---|---|
| 0 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 |
| 2 | 1 | 1 | 1 | 0 |
| 3 | 1 | 1 | 1 | 1 |
| 4 | 0 | 1 | 1 | 1 |
| 5 | 1 | 0 | 1 | 1 |
| 6 | 0 | 1 | 0 | 1 |
| 7 | 1 | 0 | 1 | 0 |
| 8 | 1 | 1 | 0 | 1 |
| 9 | 0 | 1 | 1 | 0 |
| 10 | 0 | 0 | 1 | 1 |
| 11 | 1 | 0 | 0 | 1 |
| 12 | 0 | 1 | 0 | 0 |
| 13 | 0 | 0 | 1 | 0 |
| 14 | 0 | 0 | 0 | 1 |
| 15 (=0) | 1 | 0 | 0 | 0 |

| state | $Q_0$ | $Q_1$ | $Q_2$ | $Q_3$ |
|---|---|---|---|---|
| 0 | 1 | 0 | 0 | 0 |
| 1 | | | | |
| 2 | | | | |
| 3 | | | | |
| 4 | | | | |
| 5 | | | | |
| 6 | | | | |
| 7 | | | | |
| 8 | | | | |
| 9 | | | | |
| 10 | | | | |
| 11 | | | | |
| 12 | | | | |
| 13 | | | | |
| 14 | | | | |
| 15 (=0) | | | | |

# Summary – LFSR

- **Two types** of LFSR. **Simple** structure, good for **TPG**
- LFSR generate **pseudo random test patterns.** Repeat every $L_c$ cycles
- Linear relationship between states: $Q^+ = TQ$
- **Characteristic polynomial** is often used to describe LFSR

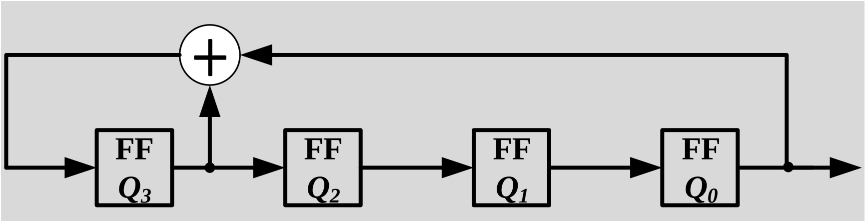
| | Structure | Matrix |
|---|---|---|
| **type1**<br>standard-form | <br>index right to left | $\begin{bmatrix} Q_0^+ \\ Q_1^+ \\ \vdots \\ Q_{N-3}^+ \\ Q_{N-2}^+ \\ Q_{N-1}^+ \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & 0 & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ h_0 & h_1 & h_2 & \cdots & h_{N-2} & h_{N-1} \end{bmatrix} \begin{bmatrix} Q_0 \\ Q_1 \\ \vdots \\ Q_{N-3} \\ Q_{N-2} \\ Q_{N-1} \end{bmatrix}$ |
| **type2**<br>modular-form | <br>index left to right | $\begin{bmatrix} Q_0^+ \\ Q_1^+ \\ \vdots \\ Q_{N-3}^+ \\ Q_{N-2}^+ \\ Q_{N-1}^+ \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & h_0 \\ 1 & 0 & 0 & \cdots & 0 & h_1 \\ \vdots & 1 & \vdots & & \vdots & h_2 \\ 0 & 0 & 1 & \cdots & 0 & \vdots \\ 0 & 0 & 0 & \cdots & 0 & h_{N-2} \\ 0 & 0 & 0 & \cdots & 1 & h_{N-1} \end{bmatrix} \begin{bmatrix} Q_0 \\ Q_1 \\ \vdots \\ Q_{N-3} \\ Q_{N-2} \\ Q_{N-1} \end{bmatrix}$ |
| **Char. Poly** | $f(x) = \sum_{i=0}^{N} h_i x^i$ | $f(\lambda) = \det(T - \lambda I)$ |

# FFT 1

- **Characteristic polynomial can also be derived from matrix *T***

$$\boxed{f(\lambda) = \det(T - \lambda I)}$$

- **Q1: Given this type-1 LFSR, Please verify that** $\det(T-\lambda I) = \lambda^4 - \lambda^3 - 1$
- **Q2: From structure, we know =** $f(x) = x^4 + x^3 + 1$
  - **Not the same, why?**

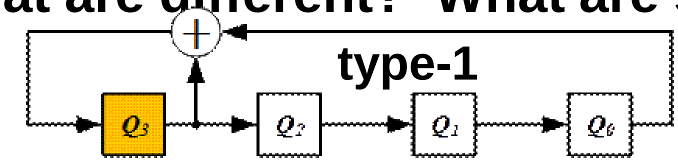


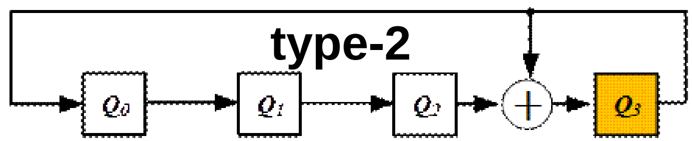

$$T = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

$$\det(T - \lambda I) = \begin{vmatrix} -\lambda & 1 & 0 & 0 \\ 0 & -\lambda & 1 & 0 \\ 0 & 0 & -\lambda & 1 \\ 1 & 0 & 0 & 1-\lambda \end{vmatrix} = ?$$

# FFT 2

- **Q: Given two types LFSR of same $x^4+x^3+1$, same seed 1000.**
- **What are different? What are same?**

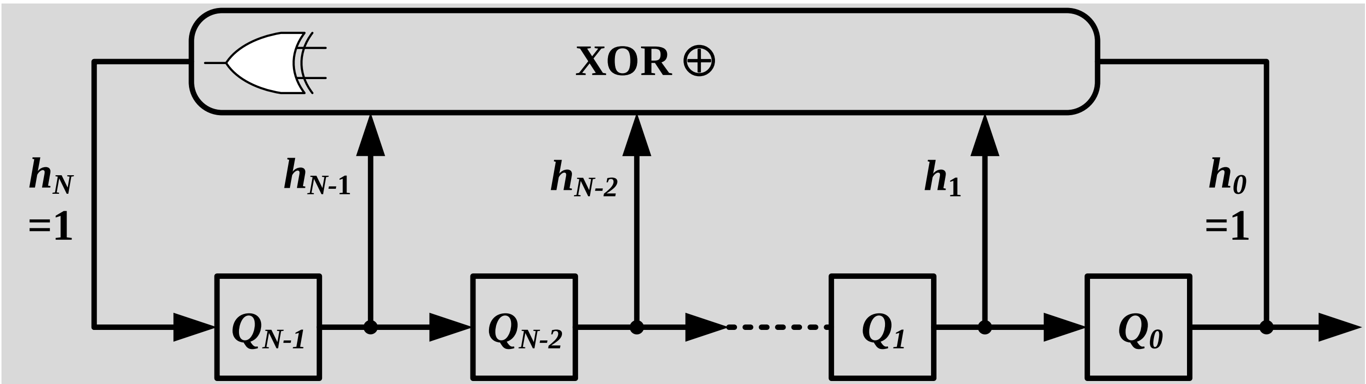



| state | $Q_3$ | $Q_2$ | $Q_1$ | $Q_0$ |
|---|---|---|---|---|
| 0 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 |
| 2 | 1 | 1 | 1 | 0 |
| 3 | 1 | 1 | 1 | 1 |
| 4 | 0 | 1 | 1 | 1 |
| 5 | 1 | 0 | 1 | 1 |
| 6 | 0 | 1 | 0 | 1 |
| 7 | 1 | 0 | 1 | 0 |
| 8 | 1 | 1 | 0 | 1 |
| 9 | 0 | 1 | 1 | 0 |
| 10 | 0 | 0 | 1 | 1 |
| 11 | 1 | 0 | 0 | 1 |
| 12 | 0 | 1 | 0 | 0 |
| 13 | 0 | 0 | 1 | 0 |
| 14 | 0 | 0 | 0 | 1 |
| 15 (=0) | 1 | 0 | 0 | 0 |

| state | $Q_0$ | $Q_1$ | $Q_2$ | $Q_3$ |
|---|---|---|---|---|
| 0 | 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 |
| 2 | 0 | 0 | 1 | 0 |
| 3 | 0 | 0 | 0 | 1 |
| 4 | 1 | 0 | 0 | 1 |
| 5 | 1 | 1 | 0 | 1 |
| 6 | 1 | 1 | 1 | 1 |
| 7 | 1 | 1 | 1 | 0 |
| 8 | 0 | 1 | 1 | 1 |
| 9 | 1 | 0 | 1 | 0 |
| 10 | 0 | 1 | 0 | 1 |
| 11 | 1 | 0 | 1 | 1 |
| 12 | 1 | 1 | 0 | 0 |
| 13 | 0 | 1 | 1 | 0 |
| 14 | 0 | 0 | 1 | 1 |
| 15 (=0) | 1 | 0 | 0 | 0 |

# APPENDIX: Characteristic Polynomial (1)




output bit stream: $q_0,\ q_1,\ q_2,\ \ldots\ (q_0$ first)

recurrence equation: $$q_m = \sum_{i=1}^{N} h_i q_{m-i}$$

generating function: $$G(x) = q_0 x^0 + q_1 x^1 + q_2 x^2 + \ldots = \sum_{m=0}^{\infty} q_m x^m$$

$$= \sum_{m=0}^{\infty}\left[ \left(\sum_{i=1}^{N} h_i q_{m-i}\right) \cdot x^m \right] = \sum_{i=1}^{N} h_i x^i \cdot \left[\sum_{m=0}^{\infty} q_{m-i} x^{m-i}\right]$$

$$= \sum_{i=1}^{N} h_i x^i \cdot \left[q_{-i} x^{-i} + \ldots + q_{-1} x^{-1} + \sum_{m=0}^{\infty} q_m x^m\right]$$

# APPENDIX: Characteristic Polynomial (2)

$$G(x) = \sum_{i=1}^{N} h_i x^i \cdot \left[ q_{-i} x^{-i} + \ldots + q_{-1} x^{-1} + G(x) \right]$$

$$G(x) = \frac{\sum_{i=1}^{N} h_i x^i \cdot [q_{-i} x^{-i} + \ldots + q_{-1} x^{-1}]}{1 - \sum_{i=1}^{N} h_i x^i}$$

**LFSR seed.** $q_{-N}, \ldots, q_{-1}$

**LFSR structure**

when seed $q_{-N} \ldots q_{-1}$=[1000...0]

$$G(x) = \frac{1}{1 - \sum_{i=1}^{N} h_i x^i} = \frac{1}{f(x)}$$

characteristic polynomial: $f(x) = 1 - \sum_{i=1}^{N} h_i x^i = \sum_{i=0}^{N} h_i x^i$