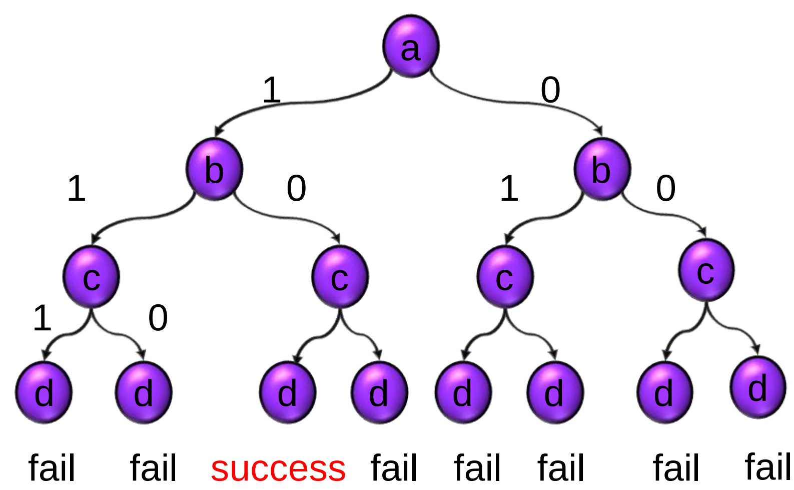
Combinational ATPG

- Introduction
- Deterministic Test Pattern Generation
  - Boolean difference*
  - Path sensitization**
  - D-Algorithm (1965)**
  - PODEM (1981)**
  - FAN(1985) **
  - SAT-based (1992)*
- Acceleration Techniques
- Concluding Remarks

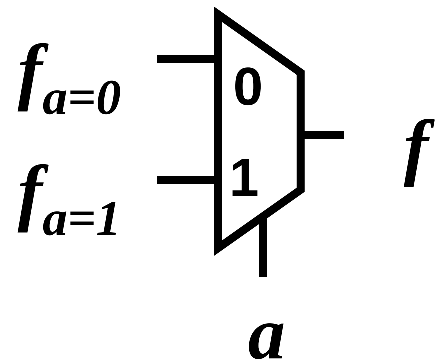
*Boolean-based methods
**path-based methods



Decomposition

- Consider a circuit that realizes the function $f(a, b, c, \ldots)$
- Fixed a to one:
  - *positive cofactor of f with respect to a*

$$f_{a=1} = f(a=1,\ b, c, \ldots)$$



- Fixed a to zero:
  - *negative cofactor of f with respect to a*

$$f_{a=0} = f(a=0,\ b, c, \ldots)$$

- ***Shannon's Expansion*** w.r.t. input a [Shannon 1948]

$$\boxed{f = a\ f_{a=1} + a'\ f_{a=0}}$$

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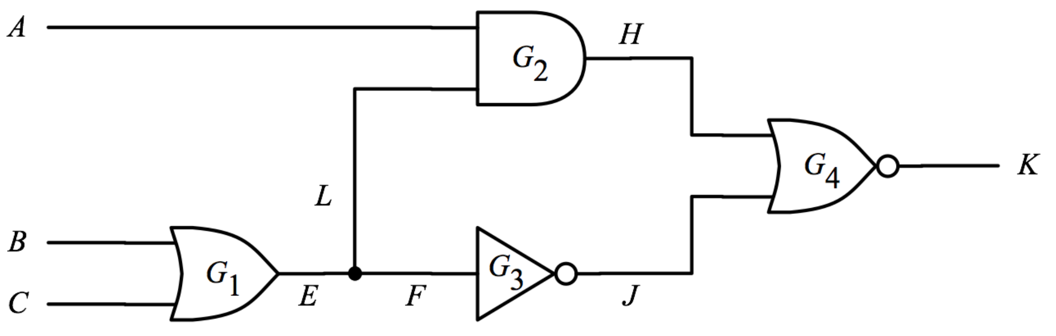
Quiz

Q: Shannon's Expansion for output *K* w.r.t. input B
A:



Boolean Difference (1)

- To detect *a* stuck-at zero fault
  - Good output (f) and faulty output ($f_{a=0}$) are different: $f \oplus f_{a=0} = 1$
  - After Shannon Expansion: $$\begin{aligned}&[af_{a=1} + a' f_{a=0}] \oplus f_{a=0} \\ &= a[f_{a=1} \oplus f_{a=0}] + a'[f_{a=0} \oplus f_{a=0}] \\ &= a[f_{a=1} \oplus f_{a=0}] + 0 \\ &= 1\end{aligned}$$
  - Thus: $\boxed{a[f_{a=0} \oplus f_{a=1}] = 1}$
- To detect *a* stuck-at one fault
  - Good output (f) and faulty output ($f_{a=0}$) are different: $f \oplus f_{a=1} = 1$
  - After Shannon Expansion: $[af_{a=1} + a' f_{a=0}] \oplus f_{a=1} = 1$
  - Thus: $\boxed{a'[f_{a=0} \oplus f_{a=1}] = 1}$
- ***Boolean difference* of *f* w.r.t. *a***

$$\boxed{\frac{df}{da} = [f_{a=0} \oplus f_{a=1}]}$$

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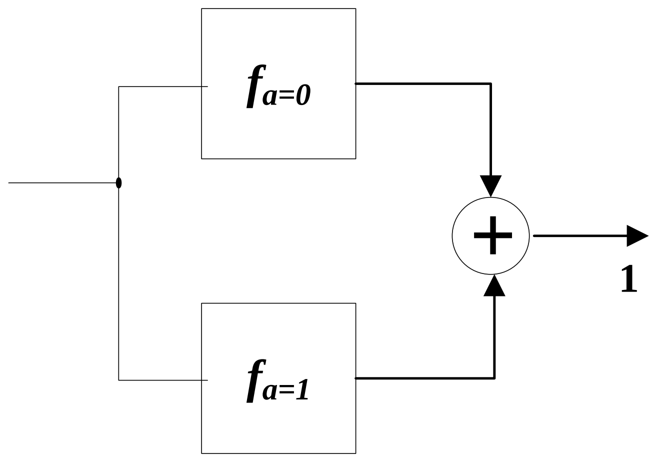
Boolean Difference (2)

- **Boolean Difference = 1 means** ***$f_{a=0}$ and $f_{a=1}$ are different***
  - **which means: *a* is sensitized to output *f***

$$\boxed{\frac{df}{da} = [f_{a=0} \oplus f_{a=1}] = 1}$$



- **Example: AND gate, when b=1, *f* = A**

$$f = ab$$

$$\frac{df}{da} = [f_{a=0} \oplus f_{a=1}] = 0 \oplus b = b$$

- **Example: OR gate, when b=0, *f* = A**

$$f = a + b$$

$$\frac{df}{da} = [f_{a=0} \oplus f_{a=1}] = b \oplus 1 = b'$$

BD=1: sensitization condition

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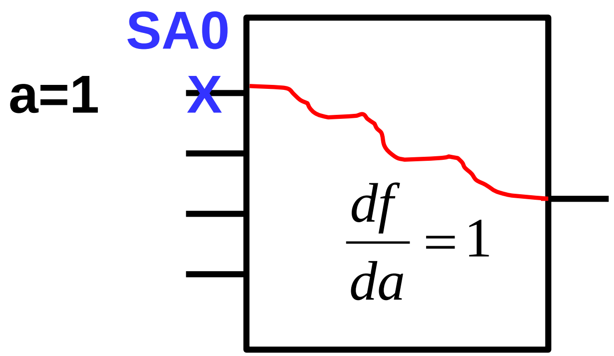
Boolean Difference (3)

- **All patterns to detect *a* stuck-at-0 fault**
  - ***a* = 1 : *fault excitation***
  - **BD = 1 : *sensitization* (aka *fault effect propagation*)**

$$a\frac{df}{da}=1$$



- **All patterns to detect *a* stuck-at-1 fault**
  - ***a* = 0 : fault excitation**
  - **BD = 1 : sensitization**

$$a'\frac{df}{da}=1$$

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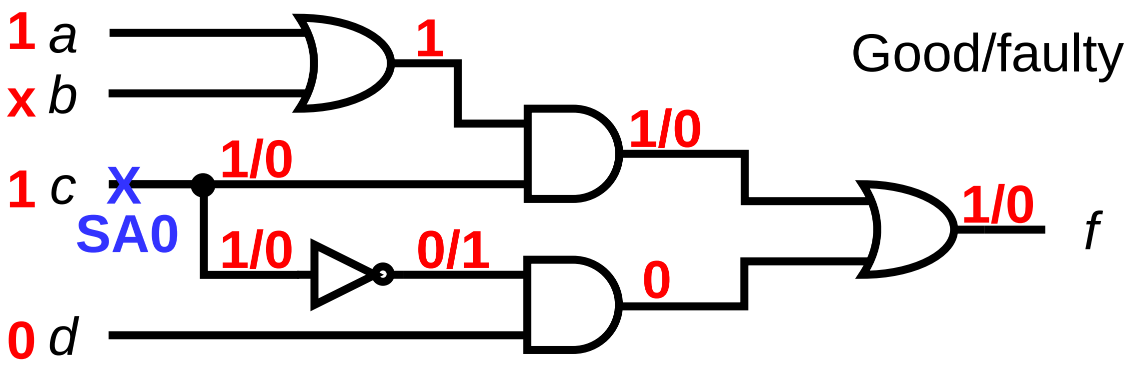
Test Generation Example



- ***f = (a + b)c + c'd***
- **Set of all tests for *c stuck-at-0* is** $\boxed{c\dfrac{df}{dc}=1}$

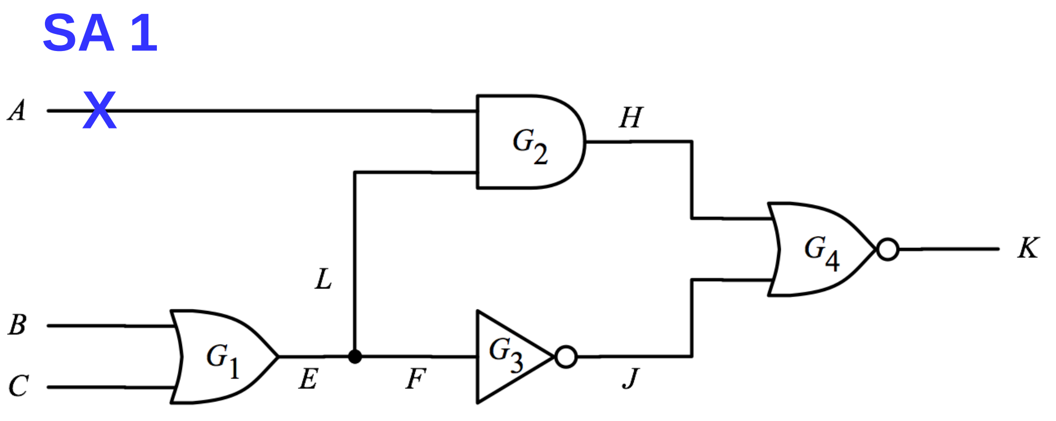
$$\frac{df}{dc} = f(a,b,0,d) \oplus f(a,b,1,d) = d \oplus (a+b) = ad'+bd'+a'b'd$$

$$c\frac{df}{dc} = acd'+bcd'+a'b'cd$$

- **Set of all tests = {1x10, x110, 0011}   (x = don't care inputs)**
- **One *fully specified test pattern:* *e.g.* 1110**
- **One *partially specified test pattern (aka. test cube):* 1x10**

Quiz

(Cont'd) We already known K=A'B'C+A'B
Q1: Boolean difference dK/dA=?
Q2: Use BD to find all test patterns for A stuck-at one fault.

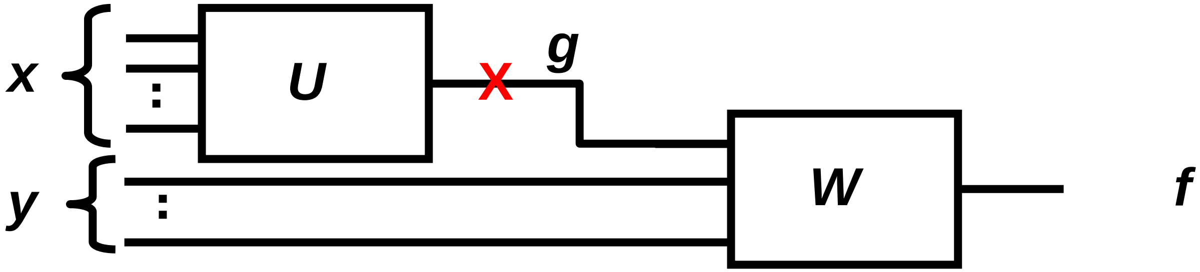


Internal Faults

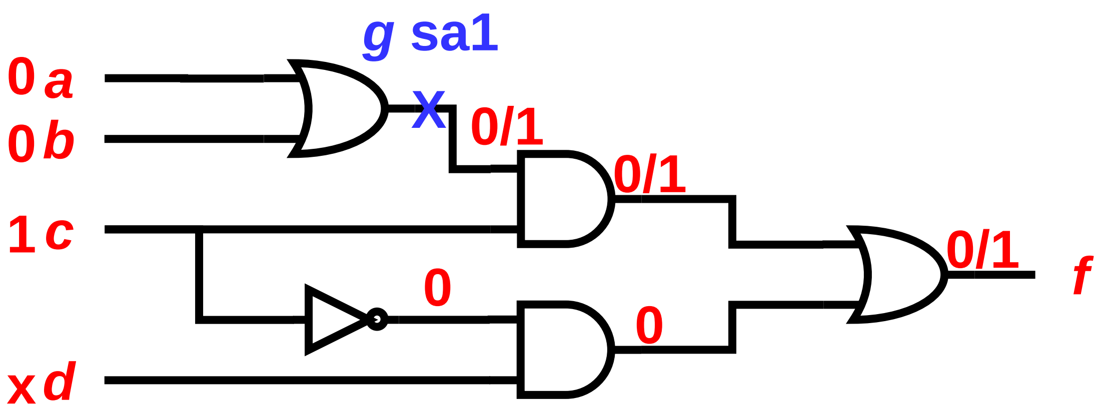
- **Same approach also used for stuck-at faults internal faults**
- **Let g be internal signal of Boolean function f**

$$f(x_1, x_2, ..., y_1, y_2, ...) = W(g, y_1, y_2, ...)$$

- **Test sets for faults**
  - ***g* SA0:** $\boxed{g\frac{dW}{dg}=1}$
  - ***g* SA1:** $\boxed{g'\frac{dW}{dg}=1}$



Example



$$f = (a+b)c + c'd \qquad g = a+b$$

$$W = gc + c'd$$

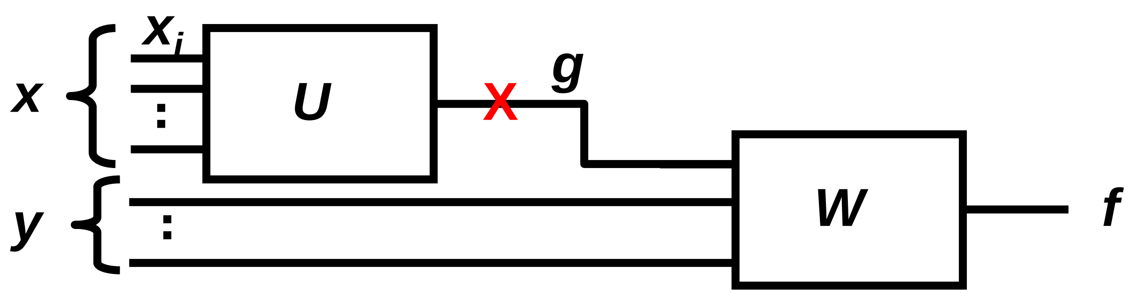
$$\frac{dW}{dg} = W_{g=0} \oplus W_{g=1} = c'd \oplus (c + c'd) = c$$

- **Tests for *g* sa0**
  (*a* + *b*)*c* = *ac* + *bc*
- **Tests for *g* sa1**
  (*a* + *b*)'*c* = *a*'*b*'*c*

Chain Rule

- **Chain rule:**

$$\frac{df}{dx_i} = \frac{df}{dg}\frac{dg}{dx_i}$$



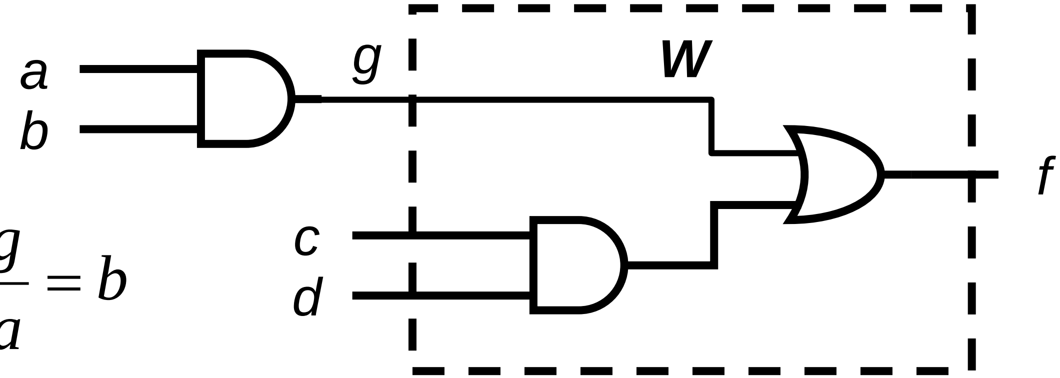
- **Example**

$$g = ab$$

$$W = g + cd$$

$$\frac{dW}{dg} = 1 \oplus cd = c' + d' \qquad \frac{dg}{da} = b$$

$$\frac{df}{da} = \frac{dW}{dg}\frac{dg}{da} = (c' + d')b$$



Problem with Boolean Difference

- **Boolean expression not always available**
  - **Especially in synthesized circuits**
- **Automatic algorithm is difficult**
- **Generating *ALL* test patterns for a fault is waste of time**
  - **Only need *ONE* test pattern for a fault**

BD is Not Useful in Practice

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Summary

- **Deterministic Test Pattern Generation**
  - **BD=1 means sensitization condition**

    $$\frac{df}{da} = [\, f_{a=0} \oplus f_{a=1} \,] = 1$$

  - **Test patterns to detect *a* stuck-at-0 fault**

    $$a\,\frac{df}{da} = 1$$

  - **Test patterns to detect *g* stuck-at-0 fault**

    $$g\,\frac{dW}{dg} = 1$$

  - **Chain rule**

    $$\frac{df}{dx_i} = \frac{df}{dg}\,\frac{dg}{dx_i}$$

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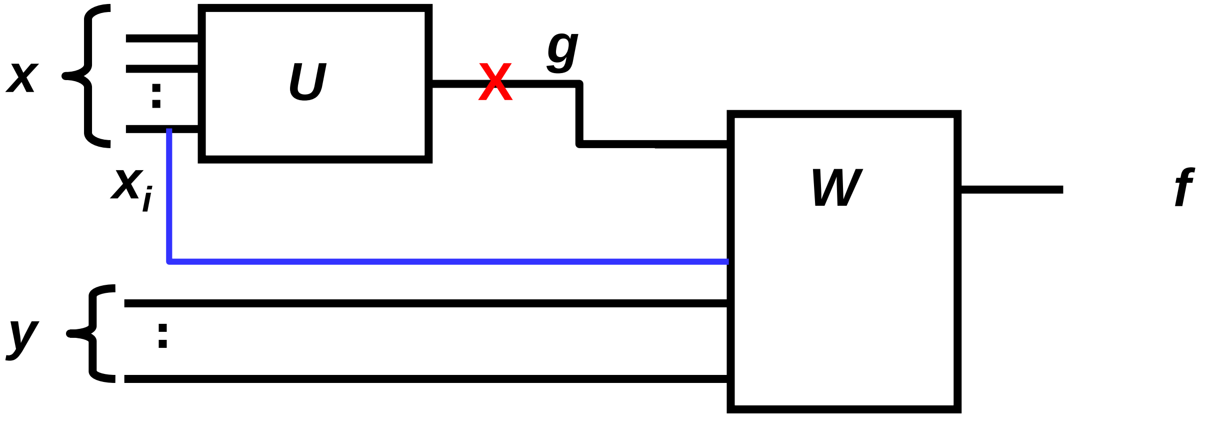
FFT1

- Q: Does it still apply if fanout exists?
  - U and W share some inputs x_i
- Test sets for faults
  - *g* sa0: $\boxed{g \frac{dW}{dg} = 1}$
  - *g* sa1: $\boxed{g' \frac{dW}{dg} = 1}$



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FFT2

- Q: Does it still apply if fanout exists?
  - U and W share some inputs x_i

- Chain rule:

$$\frac{df}{dx_i} = \frac{df}{dg}\frac{dg}{dx_i}$$