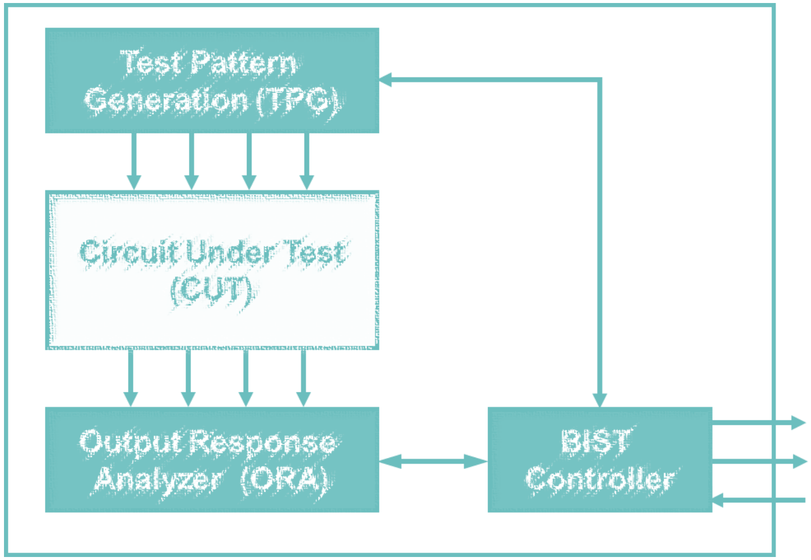
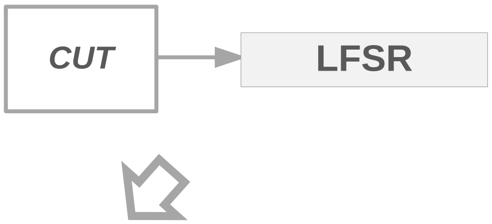
BIST Part 2

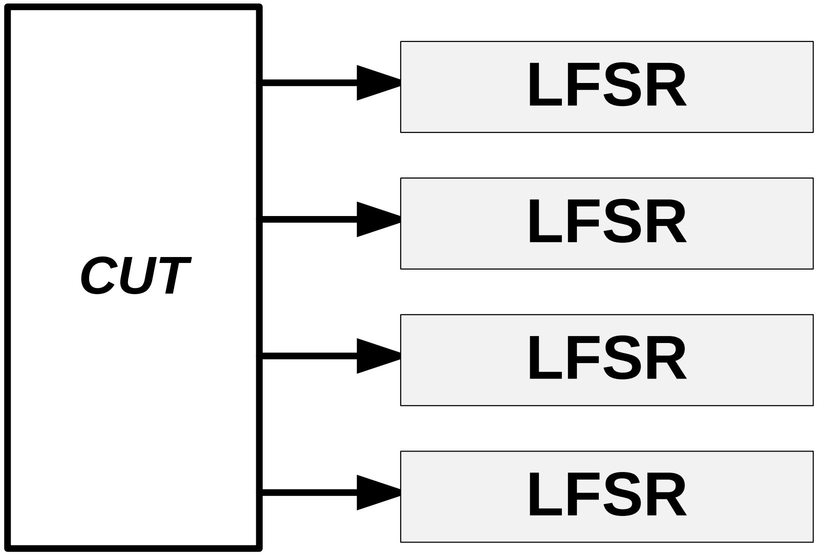
- <u>**Output Response Analysis**</u>
  - Simple ORA
  - <u>**LFSR-based ORA**</u>
    - Serial : compress one bit at a time
    - <u>**Parallel : compress multiple bits at a time**</u>
      - <u>**MISR**</u>
      - **Aliasing**
      - **Masking**
- BIST Architecture
- Issues with BIST
- Conclusions



Parallel ORA

- Serial ORA only compress one CUT output
- How about multiple CUT outputs?
  - One LFSR for each CUT output?
  - Too much hardware overhead!

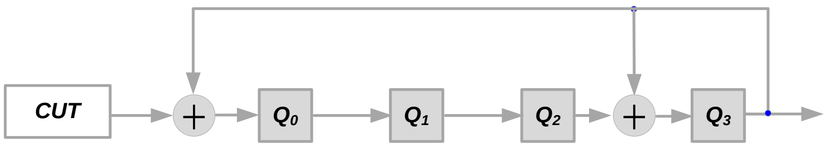


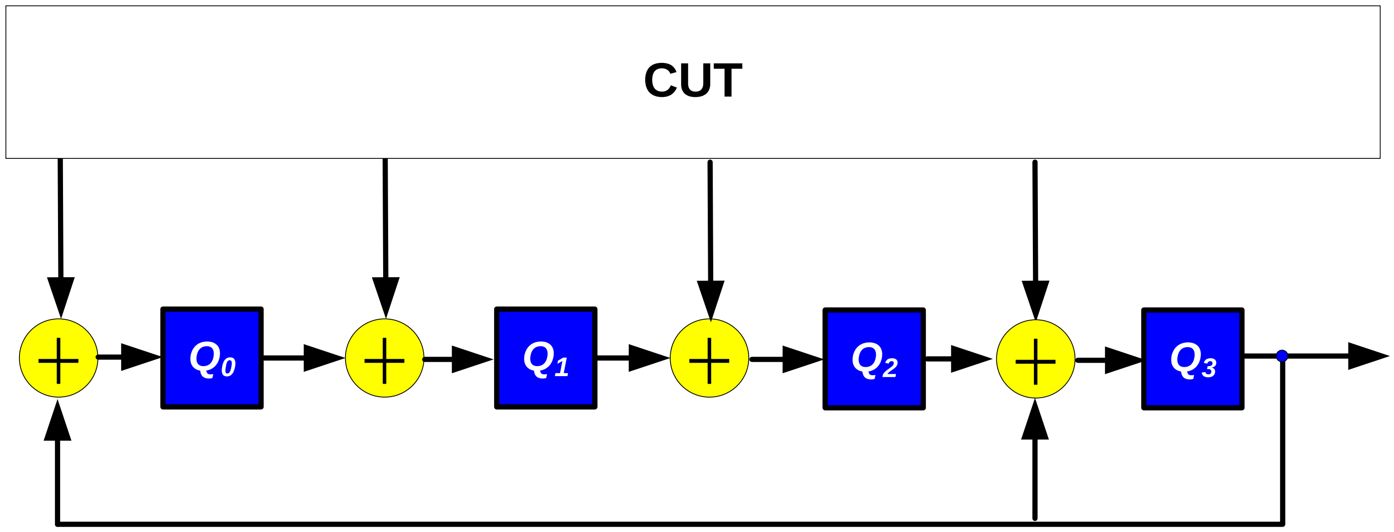


Any Better Idea?

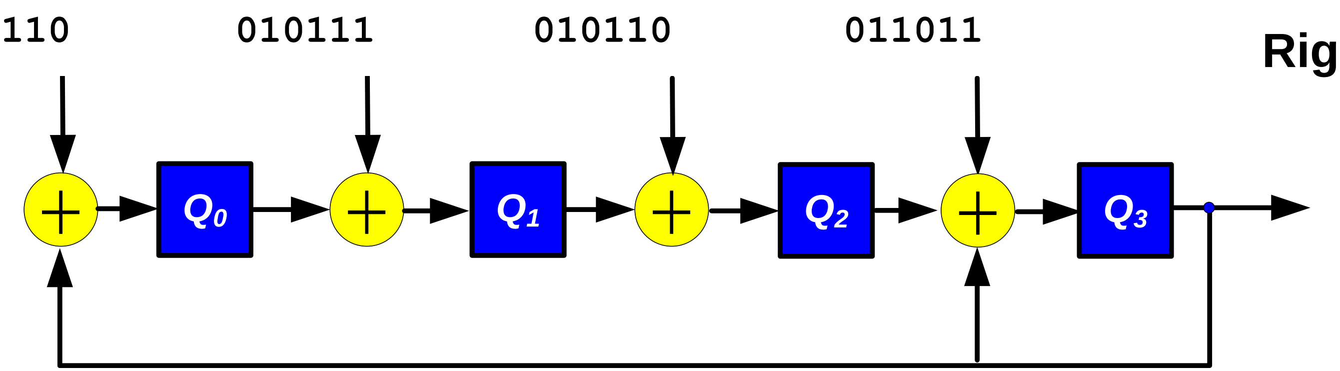
Multiple Input Signature Register, MISR

- *MISR* has similar structure to LFSR, except
  - Parallel inputs feed XOR between stages
- MISR characteristic polynomial same as LFSR
- Example: MISR with 4 parallel inputs
  - Modified from type-2 LFSR
  - $f(x) = x^4+x^3+1$





What is MISR Signature?

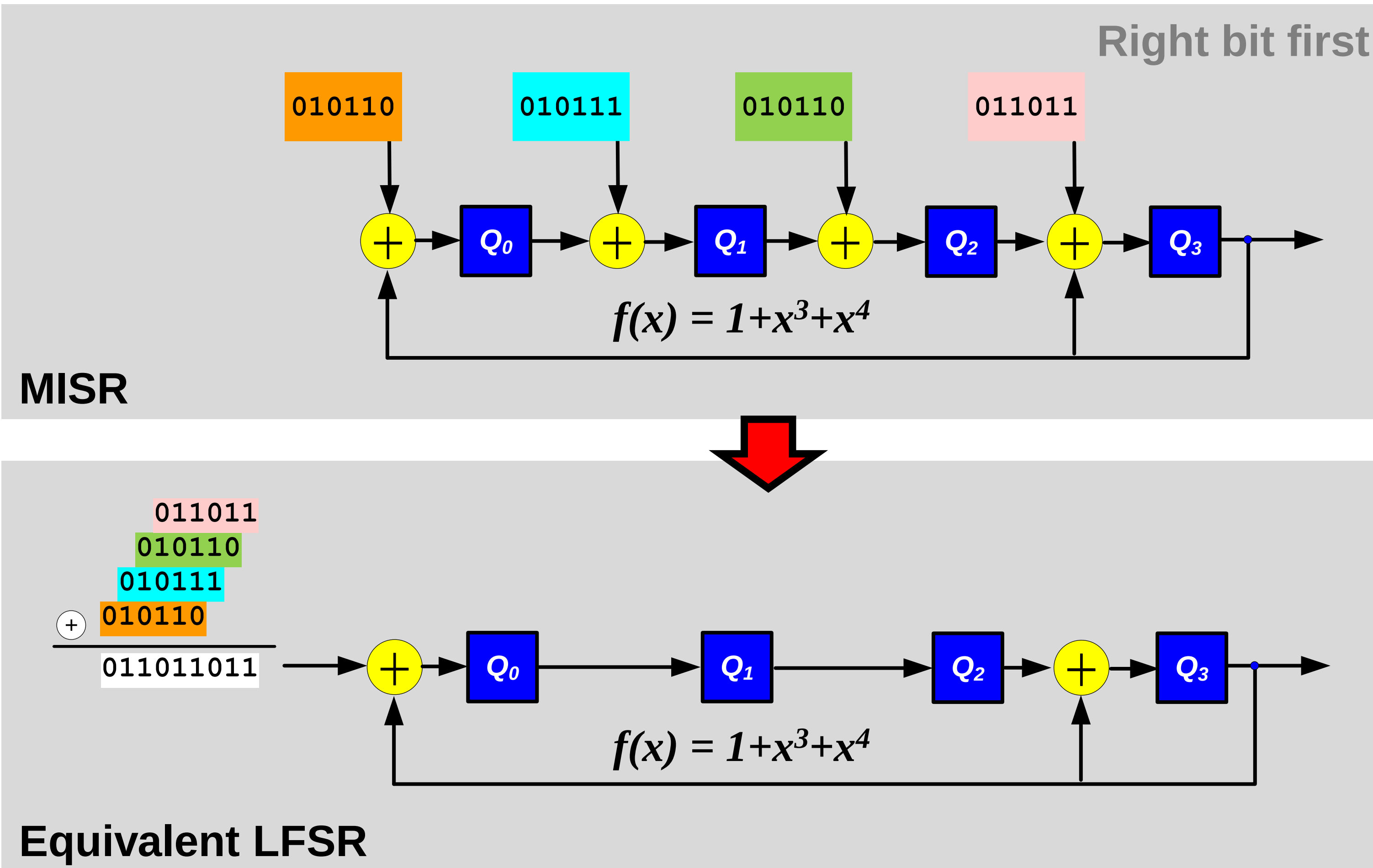


cycle	Q_0	Q_1	Q_2	Q_3	
0	0	0	0	0	initial state
1	0	1	0	1	
2	0	1	0	0	
3	1	1	0	0	
4	0	1	1	1	
5	0	1	0	1	
6	1	0	1	1	signature

Too Slow!

Equivalent LFSR

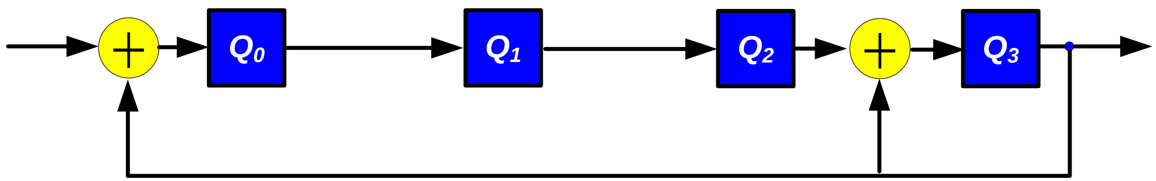
- Change MISR to ***equivalent LFSR***
  - Just phase shift and add many input bit streams



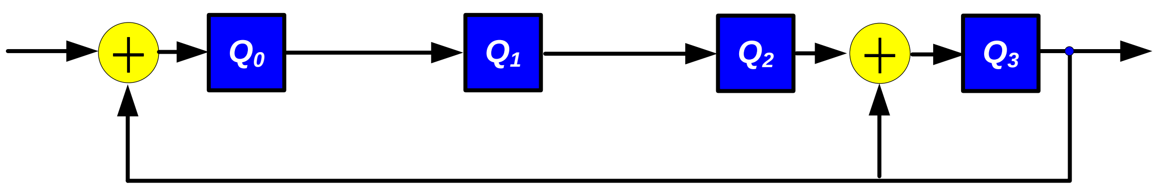
Signature Analysis Using CRC

assume type-2 LFSR

```
      011011
     010110
    010111
 ⊕ 010110
 ------------
   011011011
```



$$
\begin{array}{r}
(x+x^2+x^4+x^5)\,x^3 \\
(x+x^3+x^4)\,x^2 \\
(x+x^3+x^4+x^5)\,x \\
\oplus\ (x+x^3+x^4) \\
\hline
x+x^2+x^4+x^5+x^7+x^8
\end{array}
$$



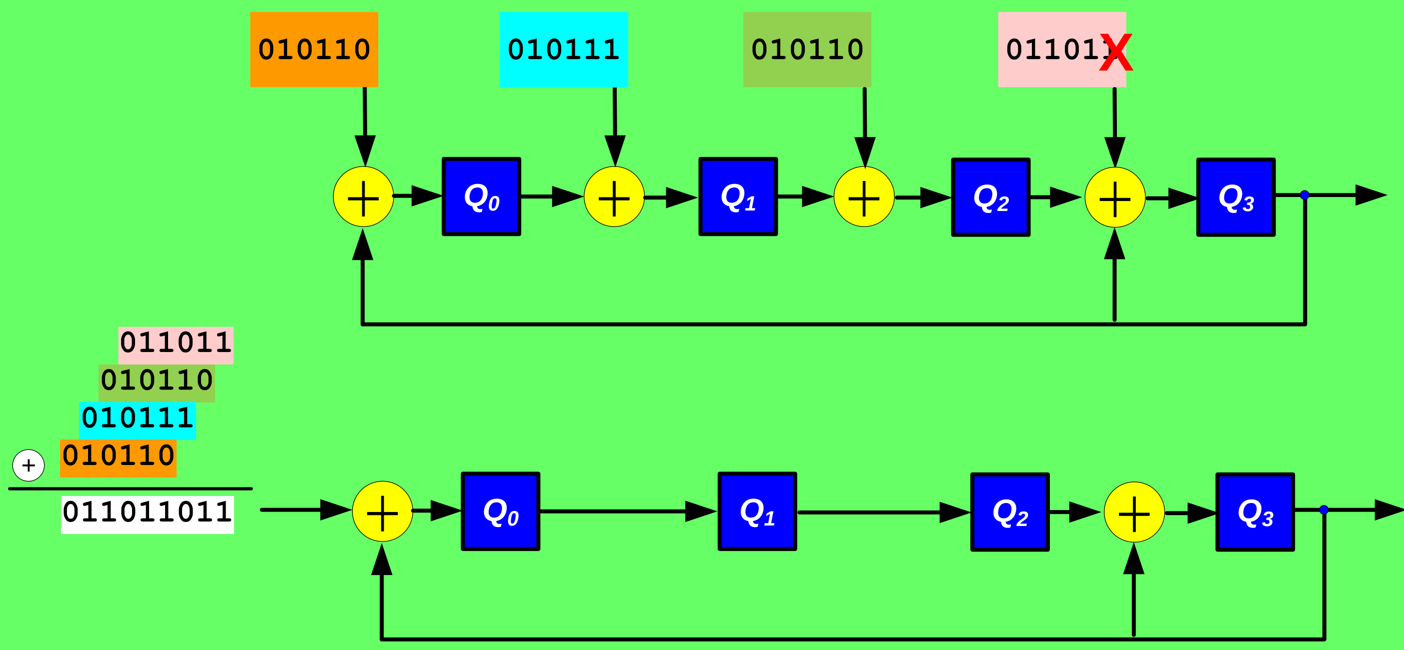
$$(x+x^2+x^4+x^5+x^7+x^8) \div (1+x^3+x^4) = 1+x+x^4 \ \ldots\ldots\ 1+x^2+x^3$$

$$M(x) \quad \div \quad f(x) \quad = \quad Q(x) \quad \ldots\ldots \quad R(x)$$

Quiz

Q: Given same example. The first bit of rightmost '1' is flipped to '0'

1) Find equivalent LFSR and associated input bit stream
2) What is MISR signature using CRC analysis?



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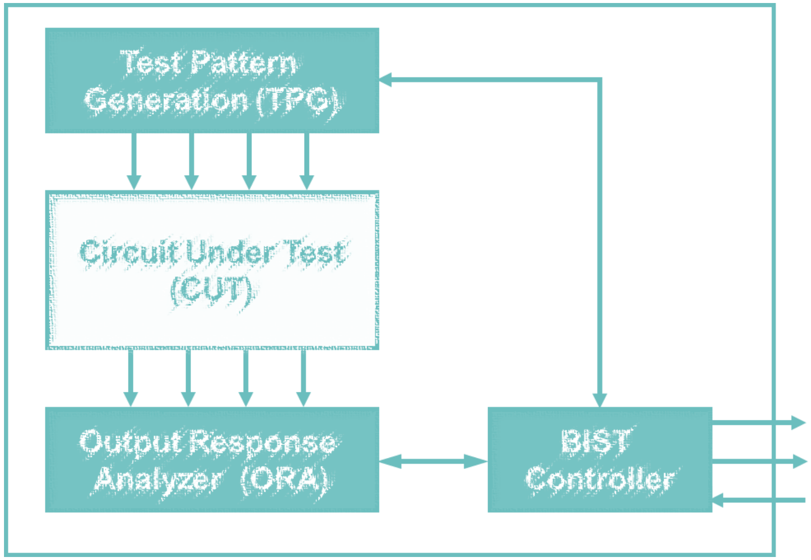
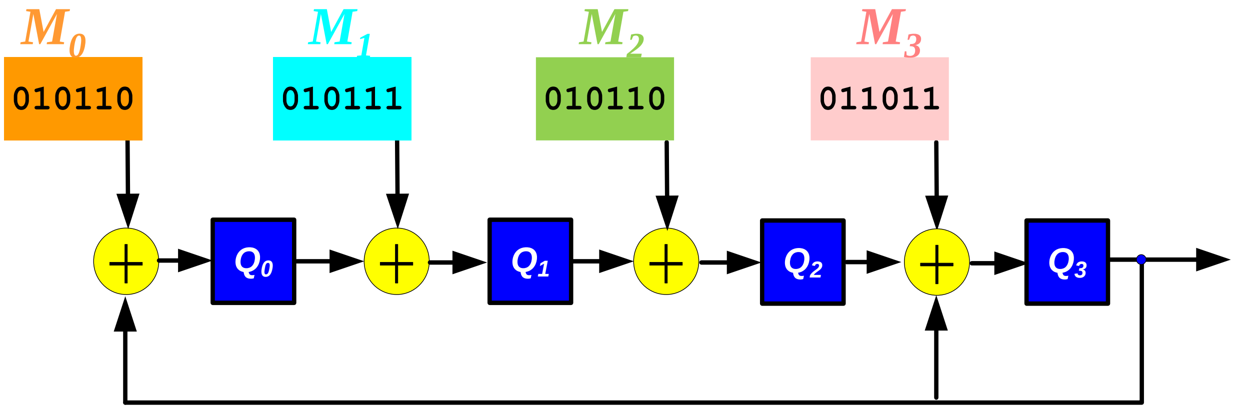


Good Signature (1/2)

- **Assume**
  - $f(x)$ = **N-degree characteristic polynomial, which is primitive**
  - **Totally N MISR inputs:** $n=0,1,2,\ldots,N-1$ **(from left to right)**
  - **Totally K cycles of MISR input bit stream:** $k=0,1,2,..,K-1$
  - $M_n(x)$ = **MISR n_{th} input polynomial**
- **Example**
  - $N=4, K=6, f(x) = x^4+x^3+1$
  - $M_0(x)=x+x^3+x^4$, $M_1(x)=x+x^3+x^4+x^5$, $M_2(x)=x+x^3+x^4$, $M_3(x)=x+x^2+x^4+x^5$

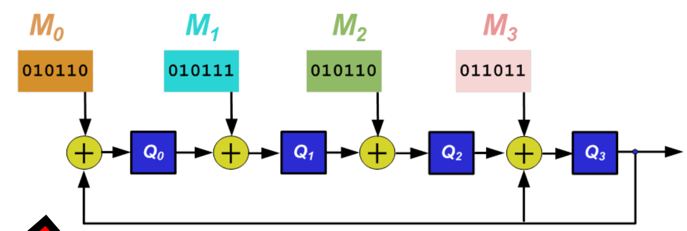


Good Signature (2/2)

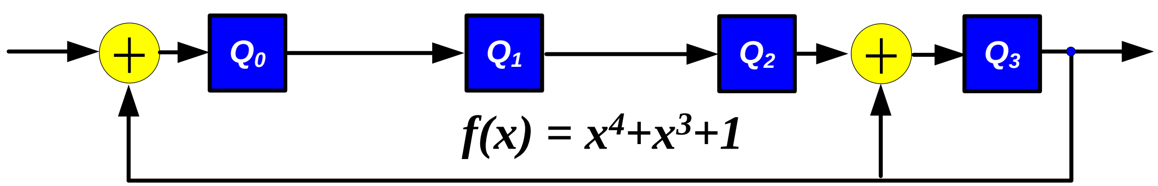
- $M_{good}(x)$ = good input bit stream; $R_{good}(x)$= good signature

$$M_{good}(x) = M_0(x) + x^1 M_1(x) + x^2 M_2(x) \ldots + x^{N-1} M_{N-1}(x)$$
$$R_{good}(x) = M_{good}(x) \bmod f(x)$$

- Example:



$(x+x^2+x^4+x^5)\,x^3$
$(x+x^3+x^4)\,x^2$
$(x+x^3+x^4+x^5)\,x$
$\oplus\ (x+x^3+x^4)$
$x + x^2 + x^4 + x^5 + x^7 + x^8$



$$M_{good}(x)=M_0(x)+xM_1(x)+x^2 M_2(x)+ x^3M_3(x)+x^4M_4(x)=x+x^2+x^4+x^5+x^7+x^8$$
$$R_{good}(x)=M_{good}(x) \bmod f(x) = 1+x^2+x^3$$

When Error Occurs

- $M_{faulty}(x) = M_{good}(x) + M_{error}(x)$

$$M_{faulty}(x) = M_0(x) + x^1 M_1(x) + x^2 M_2(x) \ldots + x^{N-1} M_{N-1}(x) + M_{error}(x)$$
$$R_{faulty}(x) = M_{faulty}(x) \bmod f(x)$$

01101~~1~~ X
010110
010111
⊕ 010110

$M_{good}(x)$ 011011011
⊕ $M_{error}(x)$ 000000001

$M_{faulty}(x)$ 011011010 → ***N*-degree LFSR**

$$M_{faulty}(x) = x + x^2 + x^4 + x^5 + x^7 + x^8 + x^8 = x + x^2 + x^4 + x^5 + x^7$$
$R_{faulty}(x) = M_{faulty}(x) \bmod f(x) = 1 + x$ **No aliasing.**

PAL=?

- Same analysis as LFSR
  - $M_{faulty}(x) = M_{good}(x) + M_{error}(x)$
  - Bit stream length $m = K+N-1$

$$PAL = \frac{2^{m-N}-1}{2^m-1} = \frac{2^{K-1}-1}{2^{N+K-1}-1} \approx 2^{-N}$$

- Example
  - N=4, K=6
  - m=9
  - PAL≅2^{-4}

```
         01101X
        010110
       010111
 ⊕    010110
---------------------------
   Mgood(x) 011011011
 ⊕ Merror(x) 000000001
---------------------------
 Mfaulty(x) 011011010  →  N-degree LFSR
```

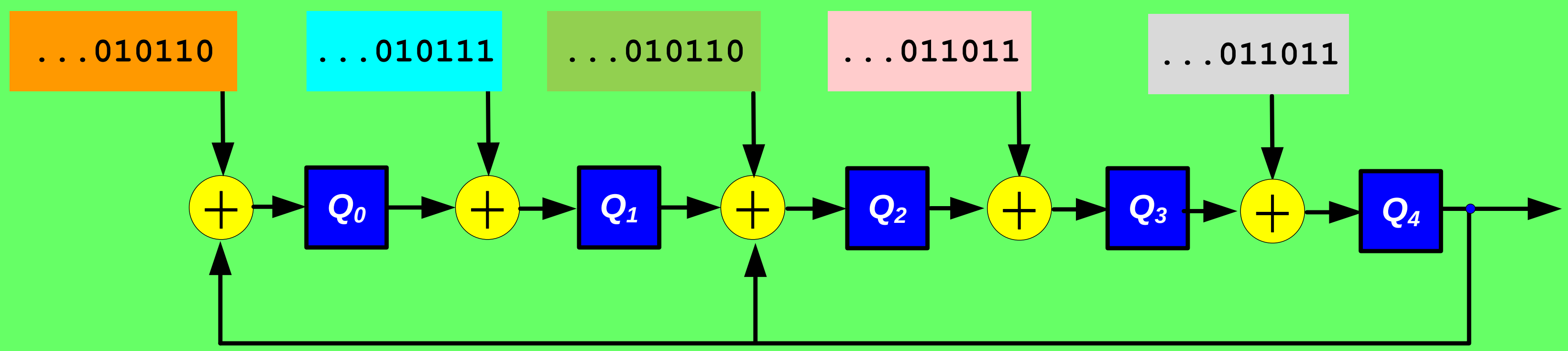
$\underbrace{011011010}_{K+N\text{-1 bits}}$

PAL=2^{-N}. Same as LFSR!

Quiz

Q: We have 5 inputs, each 100 bits long. 5-degree MISR (PP=$1+x^2+x^5$)
What is PAL of this MISR?

A:



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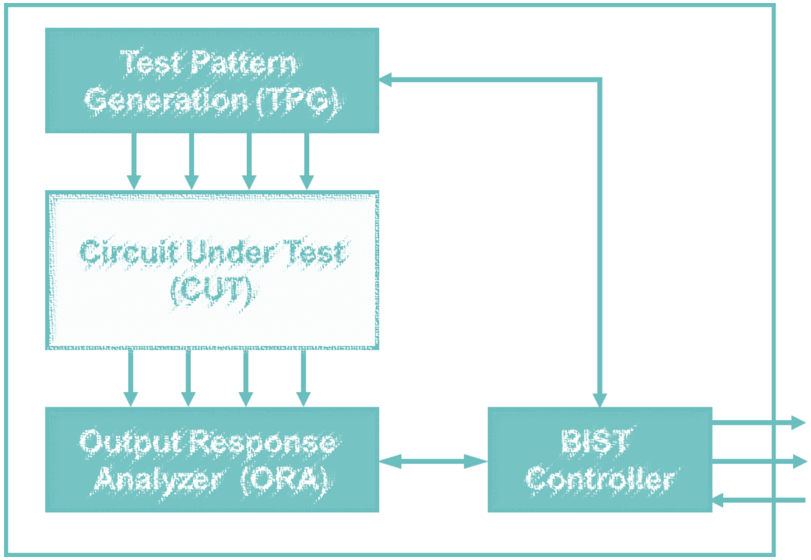


What If Two Errors?

- **Two cases:**

different columns

```
          011011
         01011X
        0101X1
+      010110
------------------
Mgood(x)   011011011
+ Merror(x) 000001010
------------------
Mfaulty(x) 011010001
```

This is modeled by PAL

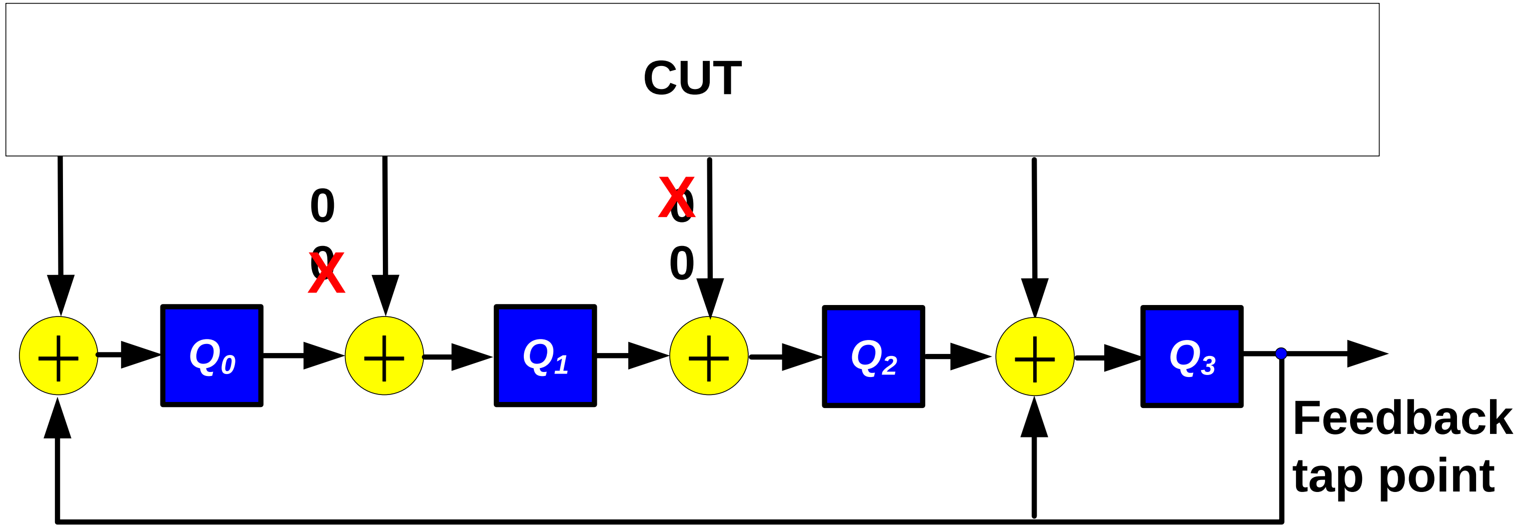
same column

```
          011011
         010X10
        0101X1
+      010110
------------------
Mgood(x)   011011011
+ Merror(x) 000000000
------------------
Mfaulty(x) 011011011
```

Error output → Good signature
But this is NOT aliasing
What is wrong?

Masking

- ***Masking*** **(aka. *error cancellation*)**
  - **One error bit cancels another error bits**
  - **Before reaching MISR feedback tap points**



Masking ≠ Aliasing

More Careful Analysis

- Actually, error can happen anywhere in the middle
  - Error bits can get cancelled before summation

$$M_{faulty}(x) = M_0(x) + M_{error0}(x) + x^1 M_1(x) + M_{error1}(x) + x^2 M_2(x) + M_{error2}(x) + \ldots$$

```
                       011011
Merror0(x)             000000
                      010110
Merror1(x)            000100
                     010111
Merror2(x)           000010
                    010110
+ Merror3(x)        000000
---------------------------------
Mfaulty(x)          011011011
```

Probability of Masking (Bardell 87)

- **Probability of masking =Probability of even ones in same column**

$$
\begin{array}{lr}
M_{error0}(x) & 000000 \\
M_{error1}(x) & 000100 \\
M_{error2}(x) & 000010 \\
\oplus \quad M_{error3}(x) & 000000 \\
\hline
M_{error}(x) & 000000000
\end{array}
$$

- **Assume all errors are equally likely, then**

$$
\boxed{\Pr(masking) \approx 2^{1-N-K} << 2^{-N} = PAL}
$$

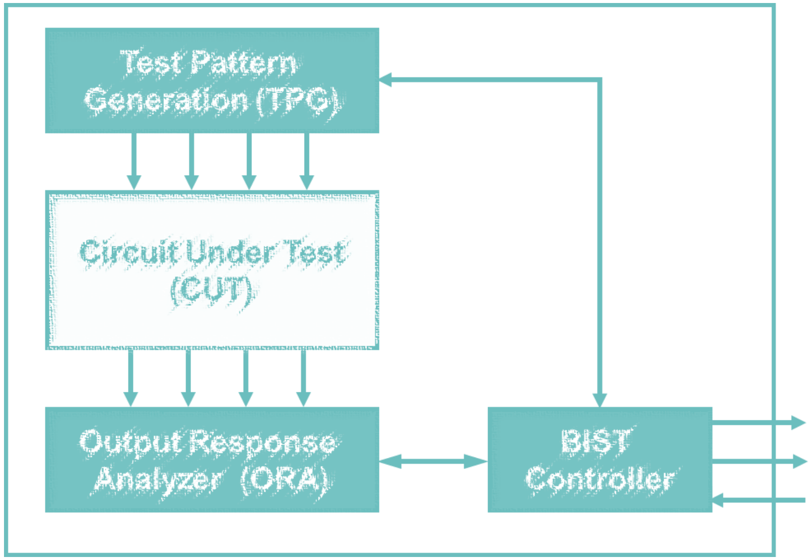
- **Conclusion: Prob. of masking is much smaller than PAL**

Aliasing and Masking

	Aliasing	Masking
Reason of happening	Error bits propagate through feedback path and cancel out with errors in later cycles	Error bit shifted down the MISR and cancelled by another error bit, before reaching feedback tap points
Probability	2^{-N}	2^{1-N-K} Smaller
Happens in	Both LFSR and MISR	Only MISR
Polynomial	Dependent	Independent

Summary

- MISR (Multiple input signature register)
  - Similar to LFSR but multiple inputs
  - Most popular parallel ORA
    - Small area, low PAL
- How to analyze MISR?
  - Convert to equivalent LFSR
- Aliasing
  - PAL = 2^{-N}. same as LFSR
- Masking
  - Prob. much smaller than PAL



FFT

- Q: Why is aliasing PAL polynomial-dependent?
  - What if non-primitive polynomial

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