Question (E): Explain why a negative natural borrowing limit results in linear consumption functions with this utility function while the VFI consumption functions had curvature at the low end of assets.

Answer: When the borrowing limit is set at its natural level (a negative value that still guarantees repayment), households can freely borrow against future income, so the borrowing constraint never binds. With a CRRA utility function and constant interest rate, the Euler equation holds everywhere, implying smooth proportional consumption growth. Because households can fully smooth their consumption, the relationship between cash-on-hand and consumption becomes almost perfectly linear: each extra dollar of wealth increases consumption by roughly one dollar.

By contrast, in the Value Function Iteration (VFI) case with a tight borrowing constraint, (typically zero borrowing) low-asset households cannot borrow to offset low income. The Euler condition breaks down at the lower bound, forcing them to consume all available resources. This creates curvature in the consumption function at the low end of assets, as the policy transitions from constrained (steep) to unconstrained (flatter) behavior.

Question (F): Explain why having the unemployment state induces curvature at the low end of cash-on-hand for this utility function

Answer: Adding an unemployment state adds income uncertainty. With CRRA preferences, this risk induces precautionary saving (households consume less today to protect against the possibility of zero income tomorrow). Because marginal utility is convex, the expected marginal utility of future consumption rises under risk, lowering optimal current consumption, especially for low-wealth agents. The result is that the consumption function bends and it becomes flatter at the low end of cash-on-hand, where the precautionary motive is strongest.

In contrast to the linear case without risk, unemployment introduces curvature because households can no longer perfectly smooth consumption, instead, they build buffer savings to insure themselves against potential income loss.