Texas Tech University
ECO 6363—Consumption & Investment Dynamics
Ongoing Coding Problems—Part 2

Due Date: Thursday October 30, 2025 at 12pm CT
Format: Matlab code (.m file)
Submission via: Github

Endogenous Labor Supply

Consider the problem below of an infinitely-lived household expressed by the Bellman equation (V) below and given constraints,

$$V(a,w) = \max_{c>0, n>0, a'>x} \{u(c,n) + \beta \mathbb{E} V(a',w')\}$$

where,

$$u(c) = \log\left[c - \Omega \frac{n^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}\right]$$

Note that x is the natural borrowing limit, which the bank assumes is derived from an income stream where the lowest possible w is earned every period. The period budget constraint of the household is given by,

$$a' = (1+r)(a + wn - c)$$

The stochastic process for income w' is autoregressive of order 1 [AR(1)], specifically,

$$w' = (1-\rho)\bar{w} + \rho w + \varepsilon', \qquad \varepsilon' \sim N(0,\sigma^2),\ 0 < \sigma < \infty,\ 0 \leq \rho < 1, \bar{w} > 0.$$

Calibrate [yearly] the parameters given above as,

$$\beta = 0.96, \quad \gamma = 1, \quad r = 0.04, \quad \rho = 0.9, \quad \sigma = 0.15, \quad \bar{w} = 2.5, \quad \varphi = 2$$

Calibrate Ω such that in steady state, the steady state level of hours is $40/168$. Complete the following:

(a) Numerically solve for $V(a,w)$, $a'(a,w)$ and $c(a,w)$ for a tolerance of 10^{-9} on the norm of the difference between consecutive iterations on V with a policy function iteration step included.

(b) Graph the converged value function in (a,w) space for all w.

(c) Generate a series of 1000 simulated income innovations for the given normal distribution and simulate the model for 1000 periods. Discard the first 500 simulations. Create a *tiledlayout* of simulated w, a', n and c.

(d) Calculate the standard deviation of simulated n. Explain what you would qualitatively expect would occur to the standard deviation of n in the following cases.

(a) The borrowing constraint were zero.

(b) The relative risk aversion parameter doubled.

(c) The Frisch labor supply elasticity doubled.

(d) Real wage volatility doubled.

Hints:

- The labor supply choice of the household must be considered.
- Explain the novel elements in determining the natural borrowing limit.

Variable Capital Utilization

Consider the problem of an infinitely lived firm expressed by the Bellman equation (V) below and given constraints,

$$V(k,w) = \max_{n>0,u>0,k'>0,inv'} \{f(uk,n) - inv' - wn + \beta \mathbb{E}V(k',w')\}$$

where,

$$f(uk,n) = (uk)^{\alpha} n^{1-\alpha}, \quad 0 < \alpha < 1$$
$$k' = [1-\delta(u)]k + inv'$$
$$\delta(u) = \delta_0 + \phi_1(u-1) + \frac{\phi_2}{2}(u-1)^2, \quad \delta_0 > 0, \phi_1 = \frac{1}{\beta} - (1-\delta_0), \phi_2 \in \mathbb{R}$$

The stochastic process for income w' is as in the previous section with identical calibratins of he autoregressive parameter and the process standard deviation. Calibrate [yearly] the remaining parameters given above as,

$$\beta = 0.96, \quad \alpha = 0.4, \quad \delta_0 = 0.1, \quad \phi_2 = 0.2.$$

Complete the following:

(a) Numerically solve for $V(k,w)$, $k'(k,w)$, $u(k,w)$, $n(k,w)$ and $inv'(k,w)$ for a tolerance of 10^{-9} on the norm of the difference between consecutive iterations on V.

(b) Graph the converged value function in (k,w) space for all w.

(c) Generate a series of 1000 simulated income innovations for the given normal distribution and simulate the model for 1000 periods. Discard the first 500 simulations. Create a *tiledlayout* of simulated w, k', n, inv', and u.

(d) Calculate the standard deviation of simulated n, u, and inv'. Explain what you would qualitatively expect would occur to the standard deviation of each in the following cases.

   (a) ϕ_2 doubled.

(b) The real interest rate doubled.

Hints:

- The utilization choice should be handled as the labor supply choice is handled in the previous section.

The assignment is graded as follows:

- 50% of the grade for returning a working code to me. That means no bugs and an output that is consistent with the underlying theory that is being numerically tested.
- 50% for in code commented derivations of the required analytical components and explanations of novel elements relative to part 1 of the coding assignment.