Texas Tech University
ECO 6363 — Consumption & Investment Dynamics
Problem Set 3

Due Date: November 4, 2025 at 12pm CT

Format: Commented Matlab Code (.m file)
Submission via: Github (1 .m file)

Consider the problem below of an infinitely-lived household expressed by the Bellman equation (V) below and given constraints,

$$V(a, y) = \max_{c,a'} \{u(c) + \beta \mathbb{E} V(a', y')\}$$

where,

$$u(c) = \begin{cases} \log c & \text{if} \quad \gamma = 1 \\ \frac{c^{1-\gamma}}{1-\gamma} & \text{if} \quad \gamma \neq 1 \end{cases}$$

The period budget constraint of the household is given by,

$$c + \frac{a'}{1+r} = a + \exp(y)$$

The stochastic process for income y is autoregressive of order 1 [AR(1)], specifically,

$$y' = \rho y + \varepsilon', \qquad \varepsilon' \sim N(0, \sigma_\varepsilon^2),\ 0 < \sigma_\varepsilon < \infty,\ 0 < \rho < 1.$$

which we will continue to discretize with the Tauchen or Rouwenhorst methods. Additionally, one of the income states is $y = 1$ or $exp(y = 1) = 0$. (unemployment/catastrophe). This state is reached from any employment state with probability $p_u = 0.05$. Emerging from unemployment occurs to the lowest income state only, at the probability of $p_e = 0.75$. So the Markov chain that provides transition probabilities between employment states must be augmented with the transition probabilities from the state of unemployment to all states, and the transition probabilities from employment states to the state of unemployment, in a probability theory compatible way. Note the effect that the introduction of the unemployment state has on the natural borrowing limit. Calibrate [yearly] the remaining parameters given above as,

$$\beta = 0.96, \quad \gamma = 3.5, \quad r = 0.04, \quad \rho = 0.92, \quad \sigma_\varepsilon = 0.05$$

We want to find an accurate numerical approximation of the consumption function. Specify cash-on-hand ($w(a, y)$) as,

$$w(a, y) = a + \exp(y)$$

We have one state variable being w and we are going to find an accurate approximation of the consumption function using the method of endogenous gridpoints, as opposed to Value Function Iteration. I will provide a sketch of the method below. As this is a very fast algorithm, feel free to make use of large grids.

The Method of Endogenous Gridpoints

A sketch of the algorithm is as follows. Given initial grids for a (large number of gridpoints) and y (small number of gridpoints), specify an initial guess of the policy function $c(w) = w$, the level that would correspond to the final period consumption choice in a finite time model. Note that by the Euler Equation, we have,

$$c = (u')^{-1} \left[\beta(1+r)\mathbb{E}u'(c')\right]$$

The algorithm proceeds as follows:

1. Set a convergence tolerance on $max||c^i - c^{i-1}||$.
2. Discretize a and y and calculate w.
3. Set $c^1 = w$.
4. Calculate $c^{i+1}(w^{i+1}) = (u')^{-1} \left[\beta(1+r)\mathbb{E}u'(c^i(w))\right]$. (Interpolation required. Use a methodology that preserves curvature.)
5. Back-out $w^{i+1} = \frac{a}{1+r} + c^{i+1}$, from the period budget constraint.
6. Check convergence. If converged, stop. If not converged, go to the next i and repeat from step 4.

Assignment Question

The assignment is graded as follows:

- 50% of the grade for returning a working code to me. That means no bugs and an output that is consistent with the underlying theory that is being numerically tested.
- 50% for explanations requested below presented in a LaTeX-typeset document.

(a) Solve for the consumption policy function using the method of endogenous gridpoints.

(b) Graph the consumption policy function on the cash-on-hand grid based on the assets and income states grid, for each income state.

(c) Use the `xcorr` function to calculate a correlogram between the simulated income and consumption series to 4 lags.

(d) Use the `corrplot` function to plot your correlogram.

(e) Explain why a negative natural borrowing limit results in linear consumption functions with this utility function while the VFI consumption functions had curvature at the low end of assets.

(f) Explain why having the unemployment state induces curvature at the low end of cash-on-hand for this utility function.