Texas Tech University
ECO 6363 — Consumption & Investment Dynamics
Replication Assignment

Due Date: December 2, 2025 at 11:59pm CT

Format: Commented Matlab Code (.m file)
Submission via: Github (1 .m file)

The firm solves the following Bellman equation:

$$
\begin{aligned}
V(k, A, \varepsilon) &= \max_{z} \ \left[V^R(k, A, \varepsilon), V^{NR}(k, A, \varepsilon))\right] \\
V^R(k, A, \varepsilon) &= A\varepsilon\lambda k - F + \beta \mathbb{E} V(1, A', \varepsilon') \\
V^{NR}(k, A, \varepsilon) &= A\varepsilon k + \beta \mathbb{E} V\left((1-\delta)k, A', \varepsilon'\right)
\end{aligned}
$$

where $\beta = 0.9,\ \lambda = 0.75,\ F = 0.2,\ \delta = 0.1$ are parameters. Aggregate productivity A is exogenous and follows a two-state Markov process with $A_H = 1.25$ and $A_L = 0.75$ and $P(A' = A_j | A = A_j) = 0.9 \,\forall\, j \in \{H, L\}$. The idiosyncratic productivity ε is exogenous and iid which follows a uniform distribution specifically, $\varepsilon \sim U(0.4, 1.6)$.

Set an equally spaced grid of 20 gridpoints for ε. The capital stock k evolves according to:

$$
k' = \begin{cases} (1-\delta)k & \text{if } z = 0 \\ 1 & \text{if } z = 1 \end{cases}
$$

The choice variable z is an indicator variable taking the value 1 if capital replacement occurs and 0 if the firm does not replace its capital in the period under consideration. This implies that the set of possible values of k is given by,

$$
k \in \{1, (1-\delta), (1-\delta)^2, (1-\delta)^3, \dots\}
$$

The Matlab function *cumprod* is helpful for discretizing this kind of grid.

Perform the following tasks and answer any questions asked by leaving comments in your .m file:

1. Initialize the following objects:
   (a) The value and revenue functions are functions of k, A, ε. On a grid, three dimensional functions become three dimensional arrays. Create a three dimensional array with any point giving the revenue level, the product $A\varepsilon k$.
   (b) Create a three dimensional first guess of the value function V.
   (c) Choose a tolerance level $\mu > 0$.
   (d) Pre-allocate the objects $V^R,\ V^{NR},\ z$, which should all be three dimensional.
2. Write the value function iteration loop. Each iteration of the loop will include several steps:

(a) Given a previous iteration i of V called V_i, calculate the value of replacement V_{i+1}^R and no replacement V_{i+1}^{NR}. You will have to make use of the stochastic process that governs the evolution of exogenous variables to do this.

(b) Update the value function V_{i+1} by choosing the maximum at each point between V_{i+1}^R and V_{i+1}^{NR}. Determine the policy function z_{i+1} by assigning the value 1 if replacement is chosen and zero if no replacement is chosen.

(c) Check if the tolerance is satisfied, if so, stop, if not, update the value function guess and repeat. The tolerance condition is $\max\max\max|V_i - V_{i+1}| < \mu$.

3. Examine the policy function. There should be a cut-off level of capital below which capital is always replaced. Reduce the size of the k dimension to 1 plus the cut-off level and repeat questions 1. and 2.

4. Plot the policy function using a *spy* plot. Comment on the important features of the policy function.

5. Plot the hazard function of capital replacement for the two values of A. Comment on the important features of the hazard function. Essentially, replicate Figure 1 in Cooper, Haltiwanger, & Power (1999)("the paper").

6. Simulate a time series for the evolution of one firm in the model using the policy function and the stochastic processes for the exogenous variables. Plot sample paths for something like 40 periods with the output of the firm and the capital stock of the firm. Comment on the behavior of firms in the model.

7. Assume that A is fixed. Replicate Figure 3 in the paper.

8. Now let A follow the Markov process specified earlier. Replicate Figure 4 in the paper.

9. Comment on the implications for firm investment behavior that you see as important from your replications of Figures 3 and 4.