Computational Physics Homework Report

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1. Logistic Map Bifurcation Diagram

1.1 Functions Description

f(r,n,x_0):

This function performs the logistic map for an initial value x_0 for n times with this formula:

$$x_{n+1} = 4rx_n(1 - x_n)$$

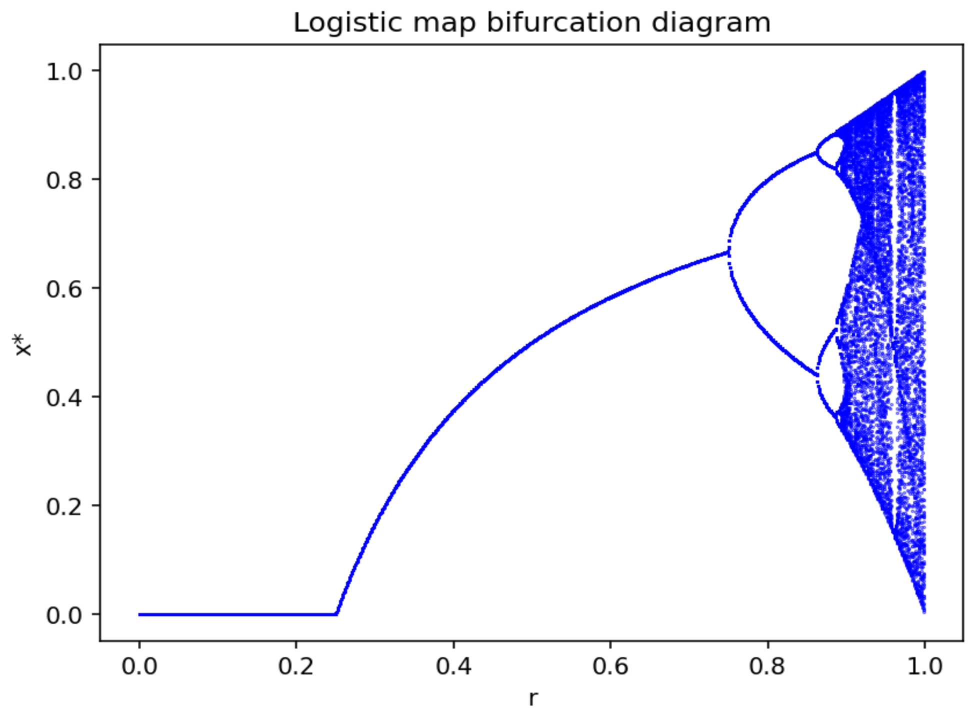
Then returns the last hundred answers as an array.

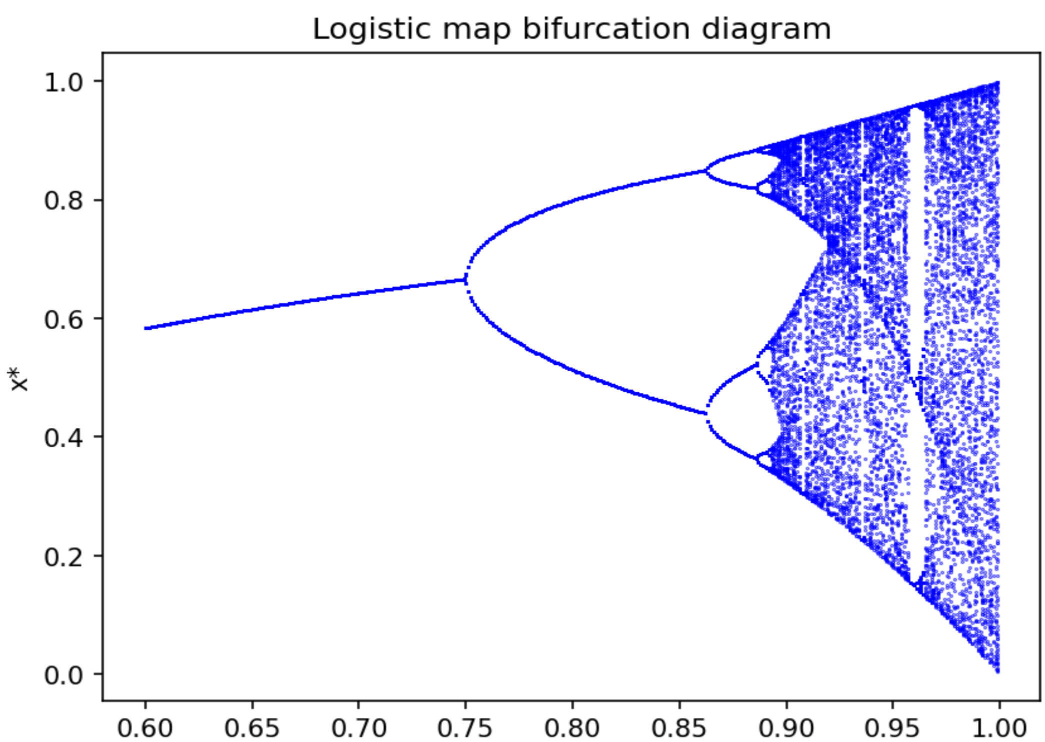
1.2 Main Code

Firstly, an array including desired r values is created using np.arange. Then a two dimensional array is created to store the value of f(r,n,x_0) for each r, and a loop starts to calculate this value for required times. lastly, a graph is plotted to show the result of f for each r.

1.3 Results

X_0=0.2 and n=1000





2. Bifurcations Place and Feigenbaum Constants

2.1 Functions Description:

f(r,n,x_0):

Same as the previous part.

2.2 Main Code:

The first part of the code is the same as the previous section, unless it rounds the results of f(r,n,x_0) up to three decimal places. Now each value that exists in f for a specific r is found using np.unique and sorted in different_val array. The length of this array tells us how many cycles we have for that specific value of r.

2.3 Results

For each r, the number of cycles and the values of x* in each is printed. We can see that after the first bifurcation, the number of cycles is equal to 2, then 4, 8 and 16. After that, the number of cycles would increase drastically. In fact these values are not cycles anymore and the chaotic behavior has started. In the table below, the values of r corresponding to each bifurcation are listed.

n	2	4	8	16	chaos
r	0.7500	0.8625	0.8865	0.8915	0.8925

Now, in order to find the first Feigenbaum constant we need this value:

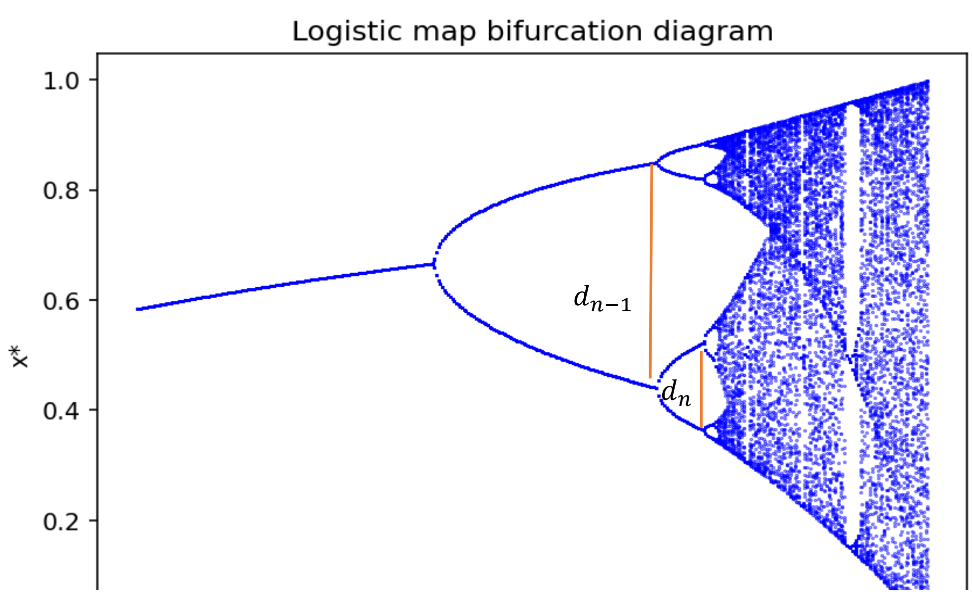
$$\delta = \frac{r_{n-1} - r_{n-2}}{r_n - r_{n-1}}$$

Using the first three r's we have:

$$\delta = \frac{0.8625 - 0.75}{0.8865 - 0.8625} = 4.6875$$

Which is pretty close to the expected value, 4.6692 …

The second constant is defined as the ratio between the width of a tine and the width of one of its two subtines. I used the distance between the last values of x* before the next bifurcations happen.



$$\alpha = \frac{d_{n-1}}{d_n} = \frac{0.85 - 0.44}{0.363 - 0.524} = -2.534$$

Which is close to its real value, -2.502…