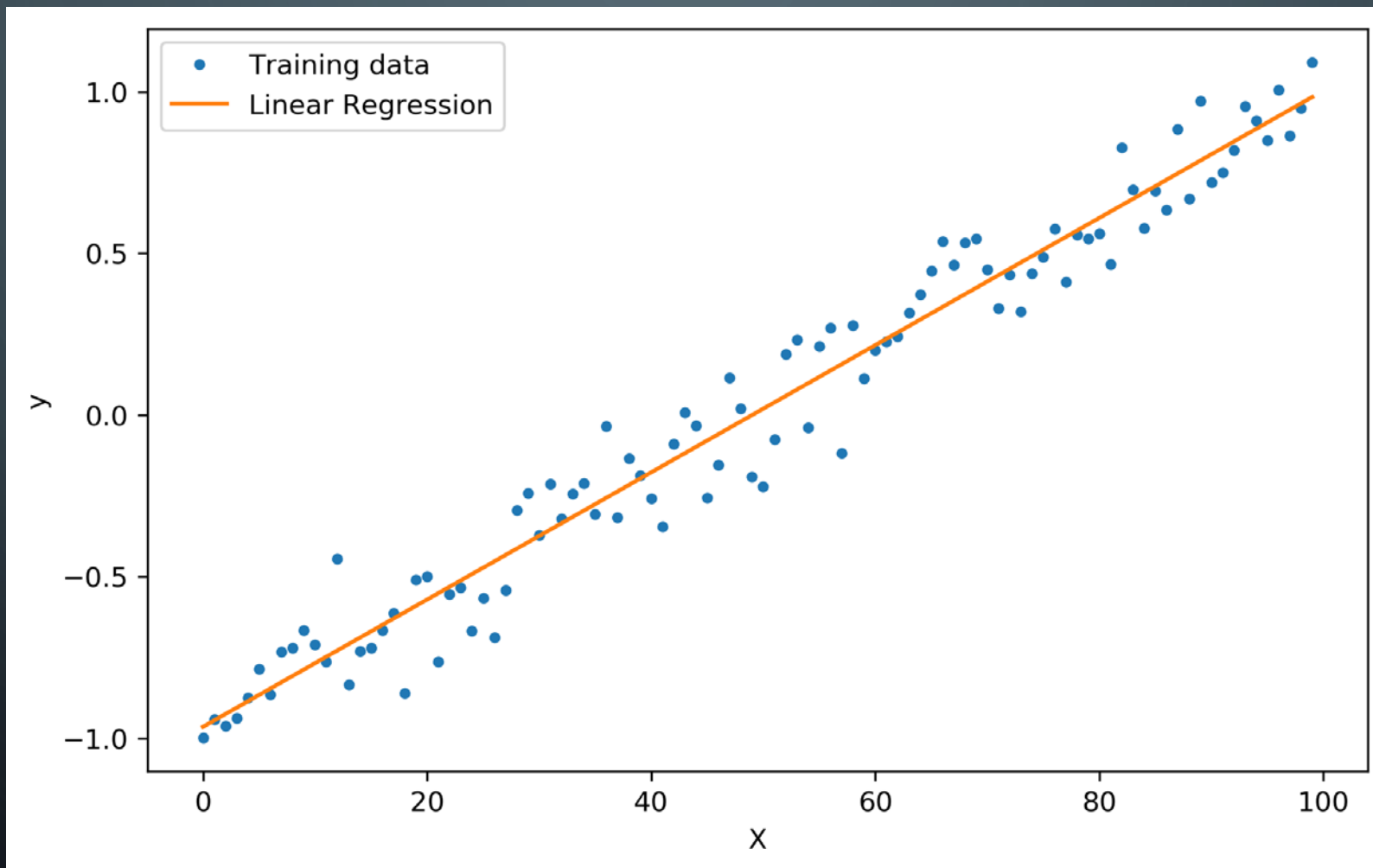




LINEAR REGRESSION

MOHAMMAD GHODDOSI

REGRESSION



LINE EQUATION

- Standard form

$$ax + by = c$$

- Point–Slope Form

$$y - y_1 = m(x - x_1)$$

- Intercept Form

$$\frac{x}{a} + \frac{y}{b} = 1$$

- Slope–Intercept Form

$$y = mx + b$$

LINE EQUATION

- Standard form (3 variables)

$$ax + by = c$$

- Point-Slope Form (3 variables)

$$y - y_1 = m(x - x_1)$$

- Intercept Form (2 variables)

$$\frac{x}{a} + \frac{y}{b} = 1$$

- Slope-Intercept Form (2 variables)

$$y = mx + b$$

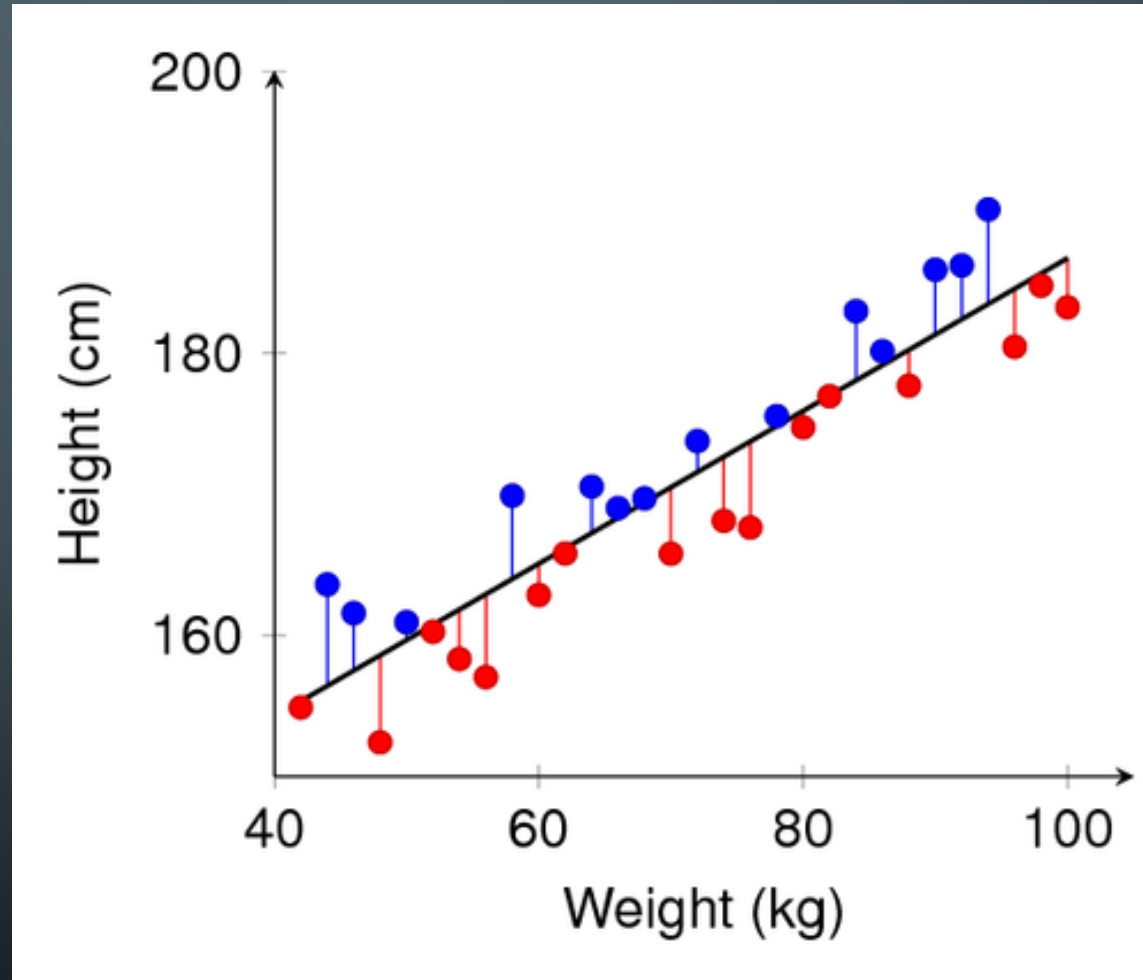
LINE EQUATION

- If we have more than one feature, use all:
- We want to predict y using x_1, x_2, x_3, x_4
 - $y = w_1x_1 + w_2x_2 + w_3x_3 + w_4x_4 + b$
- If we have n features:
 - $y = \sum_{i=1}^n w_i x_i + b$

The Jupyter logo is centered in the image. It consists of two orange, curved, crescent-like shapes that form a circle around the word "jupyter". There are four white circles of varying sizes positioned around the orange shapes: one at the top-left, one at the top-right, one at the bottom-left, and one at the bottom-right. The word "jupyter" is written in a white, lowercase, sans-serif font, centered within the orange shapes.

jupyter

MSE COST FUNCTION



MSE COST FUNCTION

- We need a performance measure (Tom Mitchell)
- We can use idea of euclidean distance

- 2d : $d(P, Q) = \sqrt{(P_x - Q_x)^2 + (P_y - Q_y)^2}$

- Md: $d(P, Q) = \sqrt{(P_1 - Q_1)^2 + (P_2 - Q_2)^2 + \dots (P_M - Q_M)^2}$

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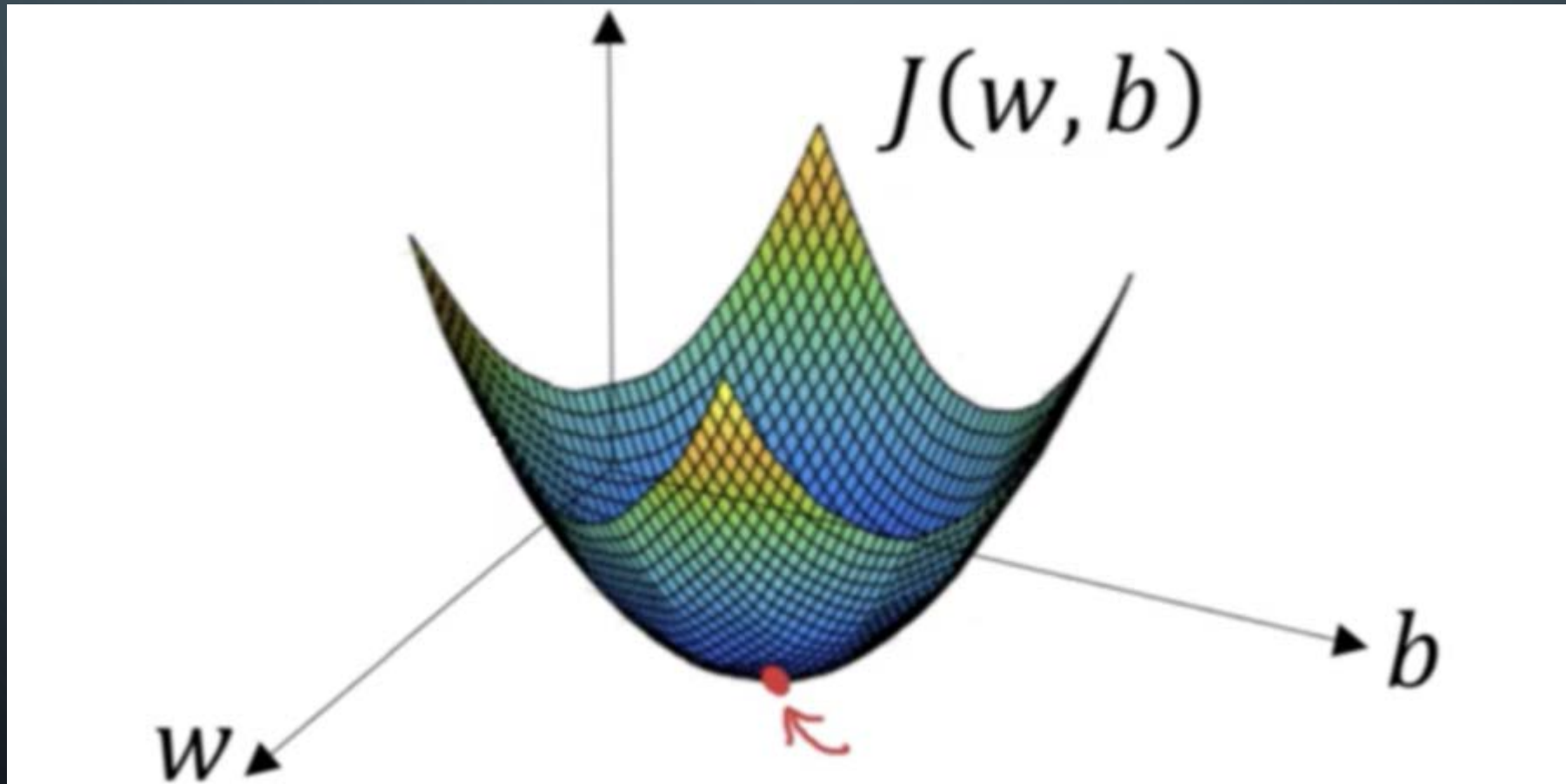
MSE COST FUNCTION

$$MSE = \frac{1}{n} \sum_{i=1}^n (Y_i - \hat{Y}_i)^2$$

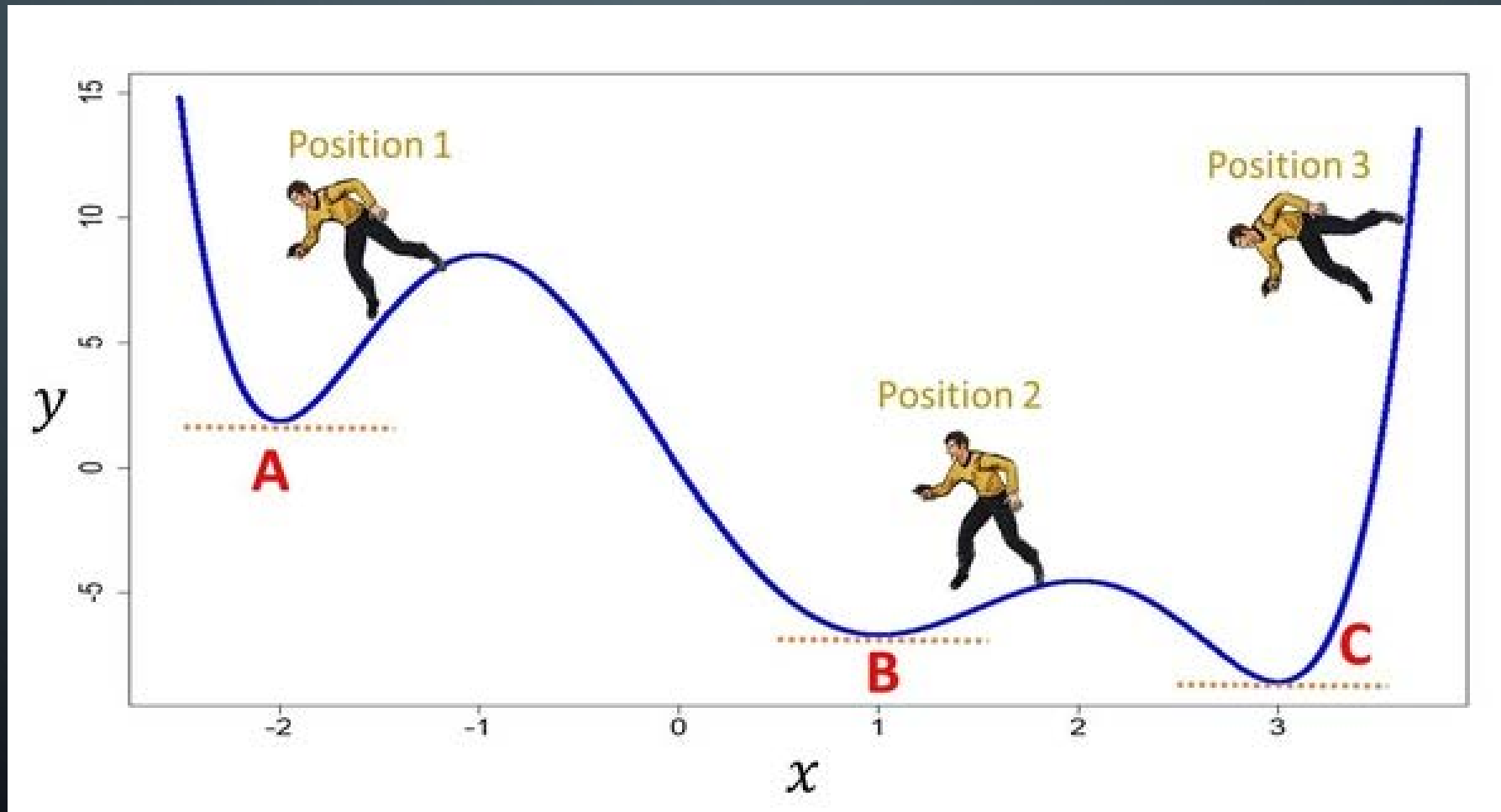
$$MSE = \frac{1}{2n} \sum_{i=1}^n (Y_i - \hat{Y}_i)^2$$

$$J(\theta) = \frac{1}{n} \sum_{i=1}^n (h_{\theta}(x^{(i)}) - y^{(i)})^2$$

OPTIMIZATION



OPTIMIZATION AND DERIVATIVES (2D)



OPTIMIZATION AND DERIVATIVES (2D)

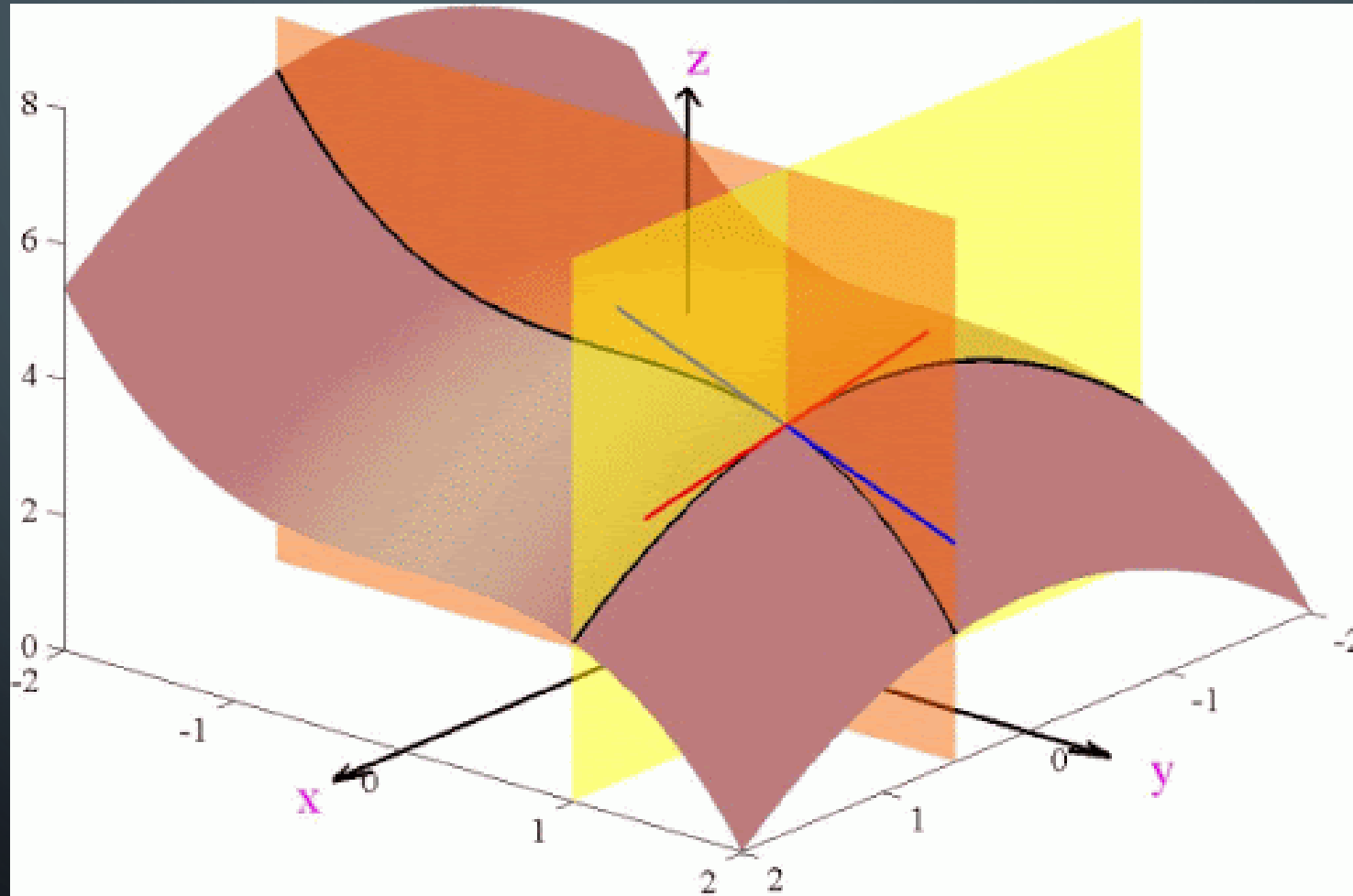
$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$\lim_{\Delta x \rightarrow 0} \Delta x * f'(x) = \lim_{\Delta x \rightarrow 0} f(x + \Delta x) - f(x)$$

$$\Delta x * f'(x_0) \approx f(x_0 + \Delta x) - f(x_0)$$

$$\begin{cases} \Delta f_{x_0} < 0 & \Delta x * f'(x_0) < 0 \\ \Delta f_{x_0} > 0 & \Delta x * f'(x_0) > 0 \end{cases}$$

OPTIMIZATION AND DERIVATIVES (ND)



OPTIMIZATION AND DERIVATIVES (ND)

$$\nabla f = \frac{\partial f}{\partial x_1} \quad \frac{\partial f}{\partial x_2} \quad \dots \quad \frac{\partial f}{\partial x_n}$$

$$f(\vec{X}_0 + \vec{u}) \approx f(\vec{X}_0) + \nabla f(\vec{X}_0) \cdot \vec{u}$$

$$\|f(\vec{X}_0 + \vec{u}) - f(\vec{X}_0)\| \approx \|\nabla f(\vec{X}_0) \cdot \vec{u}\|$$

$$\|f(\vec{X}_0 + \vec{u}) - f(\vec{X}_0)\| \approx \|\nabla f(\vec{X}_0)\| * \|\vec{u}\| * \cos(\theta_{X_0,u})$$

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WHY MSE + GD WORKS?

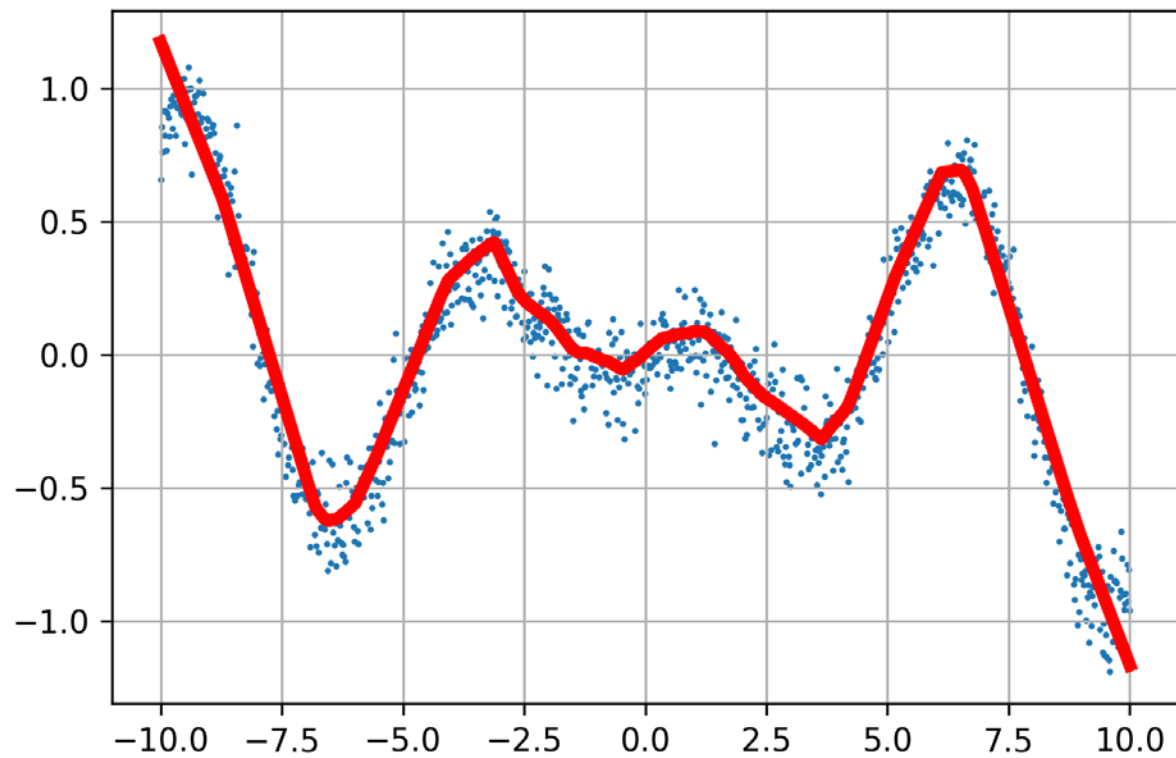
$$J(\theta) = \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2$$

Repeat until convergence {

$$\theta_j \leftarrow \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta)$$

}

NON-LINEAR REGRESSION



NON-LINEAR REGRESSION

- Use different degree of polynomial
 - $y = w_0 + w_1x + w_2x^2$
 - $y = w_0 + w_1x + w_2x^2 + w_3x^3$
 - $y = w_0 + w_1x + w_2x^2 + w_3x^3 + w_4x^4$
 - ...
- If we have more than one feature, use all possible terms:
 - $y = w_0 + w_1x + w_2y + w_3x^2 + w_4y^2 + w_5xy$
 - $y = w_0 + w_1x + w_2y + w_3x^2 + w_4y^2 + w_5xy + w_6y^3 + w_7x^3 + w_8y^2x + w_9x^2y$

VECTOR DOT PRODUCT

$$\vec{a} \cdot \vec{b} = [a_1 \quad a_2 \quad \dots \quad a_n] \cdot [b_1 \quad b_2 \quad \dots \quad b_n]$$

$$= a_1 b_1 + a_2 b_2 + \dots + a_n b_n$$

$$= \sum_{i=1}^n a_i b_i$$

MATRIX DOT PRODUCT

$$X.Y = \begin{bmatrix} X_{11} & X_{12} & \cdots & X_{1k} \\ X_{21} & X_{22} & \cdots & X_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ X_{n1} & X_{n2} & \cdots & X_{nk} \end{bmatrix} \cdot \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1m} \\ Y_{21} & Y_{22} & \cdots & Y_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{k1} & Y_{k2} & \cdots & Y_{km} \end{bmatrix}$$

MATRIX DOT PRODUCT

The diagram illustrates the matrix dot product operation. On the left, a column of input vectors $\vec{x}_1, \vec{x}_2, \dots, \vec{x}_n$ is shown with green arrows pointing to a matrix X . The matrix X is represented as a grid of elements X_{ij} where i ranges from 1 to n and j ranges from 1 to k . The first row is $[X_{11} \ X_{12} \ \dots \ X_{1k}]$, the second row is $[X_{21} \ X_{22} \ \dots \ X_{2k}]$, and the last row is $[X_{n1} \ X_{n2} \ \dots \ X_{nk}]$. To the right of matrix X is a dot operator \cdot , followed by a column of output vectors $\vec{y}_1, \vec{y}_2, \dots, \vec{y}_m$. Each output vector \vec{y}_j is shown with a red arrow pointing down to a column of elements Y_{ij} in a matrix Y . The first column of Y is $[Y_{11} \ Y_{21} \ \vdots \ Y_{k1}]$, the second column is $[Y_{12} \ Y_{22} \ \vdots \ Y_{k2}]$, and the last column is $[Y_{1m} \ Y_{2m} \ \vdots \ Y_{km}]$. Ellipses \dots are used between the second and last columns of Y to indicate intermediate columns.

$$\begin{matrix} \vec{x}_1 \\ \vec{x}_2 \\ \vdots \\ \vec{x}_n \end{matrix} \begin{matrix} \longrightarrow \\ \longrightarrow \\ \longrightarrow \\ \longrightarrow \end{matrix} \begin{bmatrix} X_{11} & X_{12} & \dots & X_{1k} \\ X_{21} & X_{22} & \dots & X_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ X_{n1} & X_{n2} & \dots & X_{nk} \end{bmatrix} \cdot \begin{bmatrix} Y_{11} & Y_{12} & \dots & Y_{1m} \\ Y_{21} & Y_{22} & \dots & Y_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{k1} & Y_{k2} & \dots & Y_{km} \end{bmatrix}$$

$\vec{y}_1 \quad \vec{y}_2 \quad \dots \quad \vec{y}_m$

MATRIX DOT PRODUCT

$$\begin{bmatrix} \overrightarrow{x_1} \cdot \overrightarrow{y_1} & \overrightarrow{x_1} \cdot \overrightarrow{y_2} & \cdots & \overrightarrow{x_1} \cdot \overrightarrow{y_m} \\ \overrightarrow{x_2} \cdot \overrightarrow{y_1} & \overrightarrow{x_2} \cdot \overrightarrow{y_2} & \cdots & \overrightarrow{x_2} \cdot \overrightarrow{y_m} \\ \vdots & \vdots & \ddots & \vdots \\ \overrightarrow{x_n} \cdot \overrightarrow{y_1} & \overrightarrow{x_n} \cdot \overrightarrow{y_2} & \cdots & \overrightarrow{x_n} \cdot \overrightarrow{y_m} \end{bmatrix}$$

LINEAR REGRESSION AS VECTOR MULTIPLICATION

$$y = w_1x_1 + w_2x_2 + w_3x_3 + w_4x_4 + b$$

$$y = w_1x_1 + w_2x_2 + w_3x_3 + w_4x_4 + w_5$$

$$y = \langle w_1 \ w_2 \ w_3 \ w_4 \ w_5 \rangle \cdot \langle x_1 \ x_2 \ x_3 \ x_4 \ 1 \rangle$$

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix} = \begin{bmatrix} x_{1,1} & \cdots & x_{M,1} & 1 \\ x_{1,2} & \cdots & x_{M,2} & 1 \\ \vdots & \ddots & \vdots & \vdots \\ x_{1,N} & \cdots & x_{M,N} & 1 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_M \end{bmatrix}$$

LINEAR REGRESSION AS VECTOR MULTIPLICATION

$$H = X.W^T$$

$$J = \frac{1}{2N} (Y - H)^T . (Y - H)$$

$$\nabla J = \frac{1}{N} (Y - H)^T . X$$

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