Evaluation Measures for Classification

Confusion Matrix

	Predicted Class	
Actual Class	Positive	Negative
Positive	TP: True Positive	FN: False Negative
Negative	FP: False Positive	TN: True Negative

Performance measures of classification models can be easily extracted from the confusion matrix. Here are the formulas for evaluation measures for binary classification problems.

$$\text{Accuracy} = \frac{\text{TP} + \text{TN}}{\text{TP} + \text{FN} + \text{FP} + \text{TN}} \qquad \text{Error rate} = \frac{\text{FP} + \text{FN}}{\text{TP} + \text{FN} + \text{FP} + \text{TN}}$$

$$\text{Precision} = \frac{\text{TP}}{\text{TP} + \text{FP}} \qquad \text{Sensitivity} = \frac{\text{TP}}{\text{TP} + \text{FN}} \qquad \text{Specificity} = \frac{\text{TN}}{\text{TN} + \text{FP}}$$

Note: Sensitivity is also called as **TP rate**, **hit rate** or **recall**.
FP / (TN+FP) = (1 – Specificity) is called **FP rate**, **false alarm rate**.
F Score is the harmonic average of precision and recall.
F Score = 2 (Precision x Recall) / (Precision + Recall)

Say, you are working on a project that users log on to their accounts by voice. For such a problem, *false positive* will denote wrongly logging on an impostor, and *false negative* will denote refusing a valid user. Please note that the two type of errors are not equally bad. For this problem false positive is much worse. True positive rate, TP-rate, also known as hit rate, measures what proportion of valid users we authenticate. False positive rate, FP-rate, also known as false alarm rate, is the proportion of impostors we wrongly accept.

Here is the confusion matrix for classification problems with three classes.

		Predicted Class		
		A	B	C
Actual Class	A	True A	Error AB	Error AC
	B	Error BA	True B	Error BC
	C	Error CA	Error CB	True C

We will read this three class confusion matrix as follows:
True X: Actual class is X and predicted as X.
Error XY: Actual class is X and predicted as Y.

When we extend the confusion matrix for two classes to a confusion matrix for multiple classes, we need to interpret TP, TN, FP and FN for multi classes.

TP means that the actual class is positive and predicted as positive. Let's extend this understanding for three classes. For instance, True Positive for class A will be denoted as "True A", interpreted as "actual class is A and predicted as A" and placed in the first row, first column of the confusion matrix. Similarly, True Positive of each class will be placed on the corresponding diagonal.

The total number of FP instances for a class is the sum of values in the corresponding column, excluding the TP of that class. For instance, FP of class A is the sum of Error BA and Error CA.

The total number of FN instances for a class is the sum of values in the corresponding row, excluding the TP of that class. For instance, FN of class A is the sum of Error AB and Error AC.

The total number of TN instances for a class is the sum of values in all rows and columns, excluding that class's row and column. For instance, TN of class A is the sum of True B, Error BC, Error CB and True C.

Here are the corresponding sensitivity and specificity formulas for three classes.

$$\text{Sensitivity(A)} = \frac{\text{True A}}{\text{True A} + \text{Error AB} + \text{Error AC}}$$

$$\text{Sensitivity(B)} = \frac{\text{True B}}{\text{True B} + \text{Error BA} + \text{Error BC}}$$

$$\text{Sensitivity(C)} = \frac{\text{True C}}{\text{True C} + \text{Error CA} + \text{Error CB}}$$

$$\text{Specificity(A)} = \frac{\text{True B} + \text{Error BC} + \text{Error CB} + \text{True C}}{\text{True B} + \text{Error BC} + \text{Error CB} + \text{True C} + \text{Error BA} + \text{Error CA}}$$

$$\text{Specificity(B)} = \frac{\text{True A} + \text{Error AC} + \text{Error CA} + \text{True C}}{\text{True A} + \text{Error AC} + \text{Error CA} + \text{True C} + \text{Error AB} + \text{Error CB}}$$

$$\text{Specificity(C)} = \frac{\text{True A} + \text{Error AB} + \text{Error BA} + \text{True B}}{\text{True A} + \text{Error AB} + \text{Error BA} + \text{True B} + \text{Error BC} + \text{Error AC}}$$

Please note that these definitions and formulas can easily be extended to more than three classes.