Classical Dynamics Project

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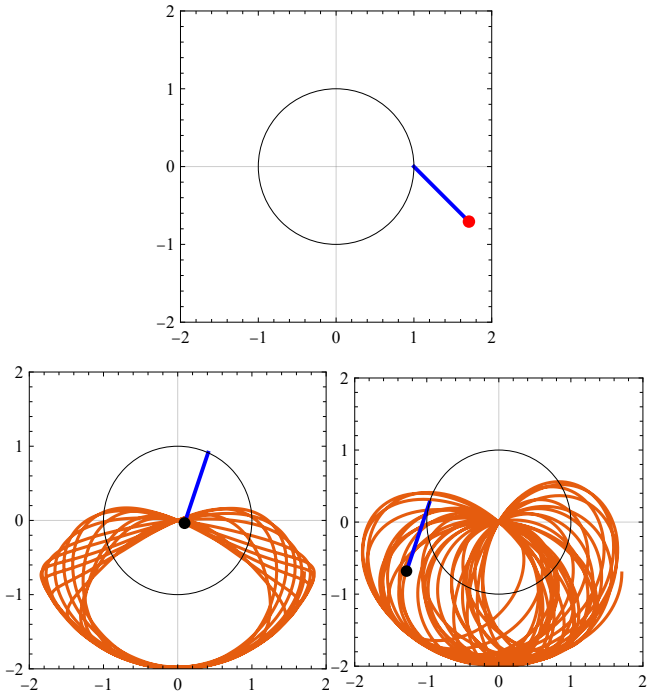
In this project we trying to simulate trajectory of pendulum which is connected to the disk and disk has an angular momentum ω.
we use Lagrangian mechanics and Euler-Lagrange equation to analysis this problem.



In[1]:=

```
g := 9.8
l := 1(*Pendulum arm length*)
r := 1(*radius of disk*)
w := -2(*Angular momentum of disk*)
tmax := 50(*time range*)
```

$$\begin{cases} x[t] = & l\text{Sin}[\theta] + r\text{Cos}[\phi] \\ y[t] = & r\text{Sin}[\phi] - l\text{Cos}[\theta] \end{cases}$$

$$\phi = \omega t$$

$$\rightarrow \begin{cases} x'[t] = & l\theta' \text{ Cos}[\theta] - r\omega\text{Sin}[\omega t] \\ y'[t] = & r\omega\text{Cos}[\omega t] + l\theta' \text{ Sin}[\theta] \end{cases}$$

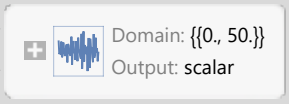
$$\rightarrow T = \frac{1}{2} m \left(x'^2 + y'^2\right) = \frac{1}{2} m\left[l^2 \theta'^2 + a^2 \omega^2 + 2\, r\omega l\theta' + \text{Sin}[\theta - \omega t]\right]$$

$$U = mgy = mg[r\text{Sin}[\phi] - l\text{Cos}[\theta]]$$

$$L = T - U = \frac{1}{2} m\left[l^2 \theta'^2 + a^2 \omega^2 + 2\, r\omega l\theta' + \text{Sin}[\theta - \omega t]\right] - mg[r\text{Sin}[\phi] - l\text{Cos}[\theta]]$$

$$\frac{\partial L}{\partial \theta} - \frac{d}{dt}\frac{\partial L}{\partial \theta'} = 0$$

In[6]:=
```
solv =
 Flatten[NDSolve[{θ''[t] == -g/l * Sin[θ[t]] + r * w^2/l Cos[θ[t] - w * t], θ[0] == π/4, θ'[0] == 0},
   {θ[t]}, {t, 0, tmax}]]
θ[t_] := Evaluate[θ[t] /. solv]
```

Out[6]=
```
{θ[t] → InterpolatingFunction[  [t]}
```


In[8]:=
```
y1[t_] := r * Sin[w * t]
x1[t_] := r * Cos[w * t]
x2[t_] := x1[t] + l * Sin[θ[t]]
y2[t_] := y1[t] - l * Cos[θ[t]]
```

In[ ]:=
```
Animate[Show[ParametricPlot[{x2[t1], y2[t1]}, {t1, 0, t},
    Background → White, PlotTheme → "Scientific", PlotRange → {{-2, 2}, {-2, 2}}],
  Graphics[{Thick, Blue, Line[{{x1[t], y1[t]}, {x2[t], y2[t]}}]}],
  Graphics[{Circle[{0, 0}, r]}],
  Graphics[{PointSize[0.04], Black, Point[{x2[t], y2[t]}]}]],
 {t, 10^-10, tmax}, AnimationRunning → False]
```

Out[ ]=