# Classical Dynamics Project

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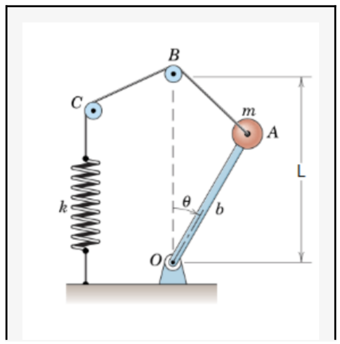
May 2022




$$l = cd + bc + ab$$

$$const = l - bc = cd + ab$$

$$ab = \sqrt{(l - y[t])^2 + (x[t])^2}$$

$$cd = const - \sqrt{(l - y[t])^2 + (x[t])^2}$$

$$x[t] = b\mathrm{Sin}[\theta[t]]$$

$$y[t] = b\mathrm{Cos}[\theta[t]]$$

$$T = \frac{1}{2} m \left((x'[t])^2 + (y'[t])^2\right)$$

$$U = mgy[t] + \frac{1}{2} k\,(cd)^2 = mgy[t] + \frac{1}{2} k\,(const - ab)^2$$

$$L = T - U = -b\,g\,m\,\mathrm{Cos}[\theta[t]] - \frac{1}{2} k \left(-bc + l - \sqrt{(l - b\,\mathrm{Cos}[\theta[t]])^2 + b^2\,\mathrm{Sin}[\theta[t]]^2}\right)^2 + \frac{1}{2} m \left(b^2\,\mathrm{Cos}[\theta[t]]^2\,\theta'[t]^2 + b^2\,\mathrm{Sin}[\theta[t]]^2\,\theta'[t]^2\right)$$

$$\frac{\partial L}{\partial \theta} - \frac{d}{dt}\frac{\partial L}{\partial \theta'} = 0$$

$$b\,g\,m\,\mathrm{Sin}[\theta[t]] + \frac{k\left(2\,b^2\,\mathrm{Cos}[\theta[t]]\,\mathrm{Sin}[\theta[t]] + 2\,b\,(l - b\,\mathrm{Cos}[\theta[t]])\,\mathrm{Sin}[\theta[t]]\right)\left(-bc + l - \sqrt{(l - b\,\mathrm{Cos}[\theta[t]])^2 + b^2\,\mathrm{Sin}[\theta[t]]^2}\right)}{2\sqrt{(l - b\,\mathrm{Cos}[\theta[t]])^2 + b^2\,\mathrm{Sin}[\theta[t]]^2}} - \frac{1}{2} m \left(2\,b^2\,\mathrm{Cos}[\theta[t]]^2\,\theta''[t] + 2\,b^2\,\mathrm{Sin}[\theta[t]]^2\,\theta''[t]\right) = 0$$

In[1]:=

```
g := 9.8
b := 1
l := 3
bc := 0.5
const := l - bc
k := 50
m := 2
ab := Sqrt[(l - y[t])^2 + (x[t])^2]
x[t_] := b * Sin[θ[t]]
y[t_] := b * Cos[θ[t]]
T := 1/2 * m * ((x'[t])^2 + (y'[t])^2)
u := m * g * y[t] + 1/2 * k * (const - ab)^2
L = T - u
```

Out[13]=

$$-19.6\,\mathrm{Cos}[\theta[t]] - 25\left(2.5 - \sqrt{(3 - \mathrm{Cos}[\theta[t]])^2 + \mathrm{Sin}[\theta[t]]^2}\right)^2 + \mathrm{Cos}[\theta[t]]^2\,\theta'[t]^2 + \mathrm{Sin}[\theta[t]]^2\,\theta'[t]^2$$

In[14]:=

```
eq = ∂_θ[t] L - D[∂_θ'[t] L, t]
```

Out[14]=

$$19.6\,\mathrm{Sin}[\theta[t]] + \left(25\,(2\,(3 - \mathrm{Cos}[\theta[t]])\,\mathrm{Sin}[\theta[t]] + 2\,\mathrm{Cos}[\theta[t]]\,\mathrm{Sin}[\theta[t]])\left(2.5 - \sqrt{(3 - \mathrm{Cos}[\theta[t]])^2 + \mathrm{Sin}[\theta[t]]^2}\right)\right) \Big/ \left(\sqrt{(3 - \mathrm{Cos}[\theta[t]])^2 + \mathrm{Sin}[\theta[t]]^2}\right) - 2\,\mathrm{Cos}[\theta[t]]^2\,\theta''[t] - 2\,\mathrm{Sin}[\theta[t]]^2\,\theta''[t]$$

In[15]:=

```
sol = θ[t] /. Flatten[NDSolve[{eq == 0, θ[0] == π/3, θ'[0] == 0}, θ[t], {t, 0, 10}]]
```

Out[15]=

```
InterpolatingFunction[ Domain: {{0., 10.}} Output: scalar ][t]
```

In[16]:=

```
θ[t_] = sol
```

Out[16]=

```
InterpolatingFunction[ Domain: {{0., 10.}} Output: scalar ][t]
```

In[17]:=
```
x11[t_] = x[t]
y11[t_] = y[t]
```

Out[17]=
```
Sin[InterpolatingFunction[ Domain: {{0., 10.}} Output: scalar ][t]]
```

Out[18]=
```
Cos[InterpolatingFunction[ Domain: {{0., 10.}} Output: scalar ][t]]
```

In[ ]:=
```
Quit
```

In[19]:=
```
Manipulate[
 Show[ParametricPlot[{x11[t1], y11[t1]}, {t1, 0, t}, PlotTheme → "Scientific",
   PlotRange → {{-2, 2}, {-2, 2}}], Graphics[{Line[{{0, 0}, {x11[t], y11[t]}}]}],
  Graphics[{PointSize[0.05], Green, Point[{x11[t], y11[t]}]}]], {t, 10^-10, 10}]
```

Out[19]=