

Machine Learning

Assignment 3

Fatemeh Nadi 810101285

December 18, 2022

Problem 1

A.

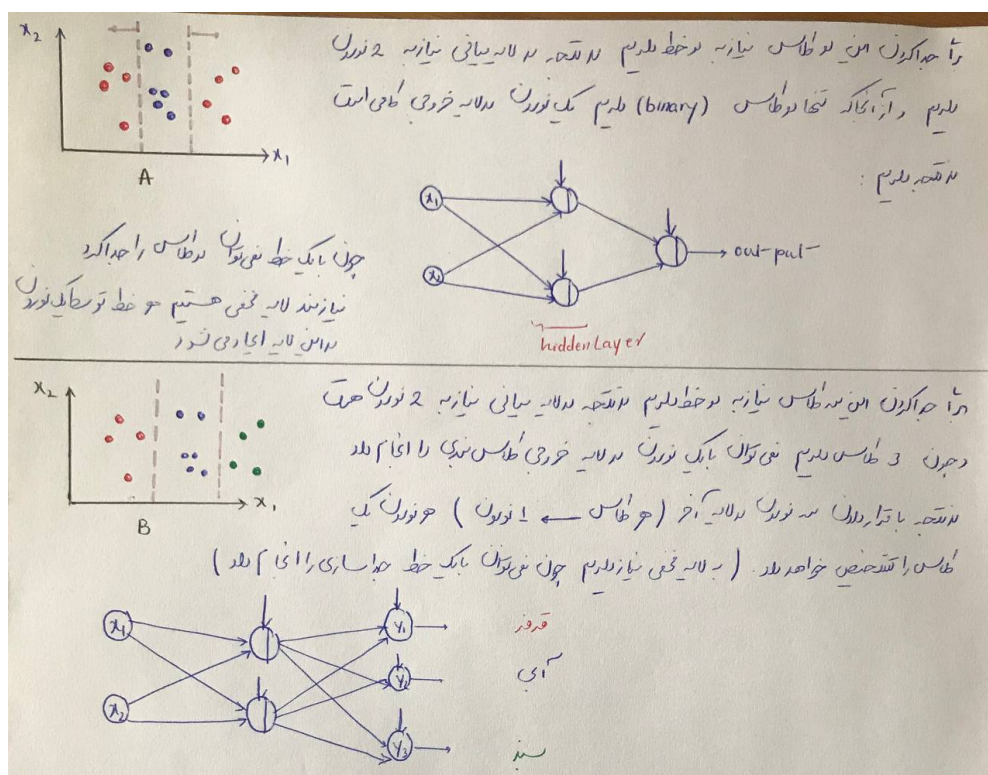


Figure 1

B.

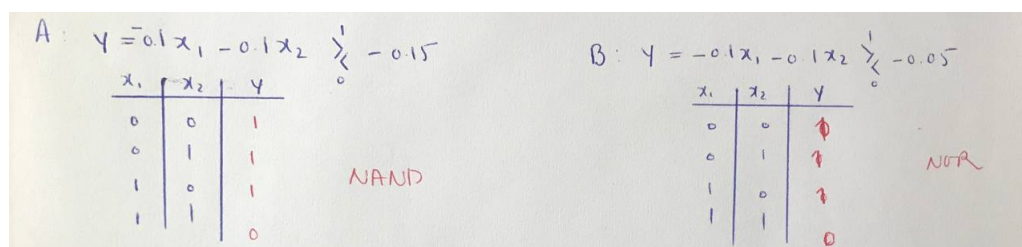


Figure 2

C.

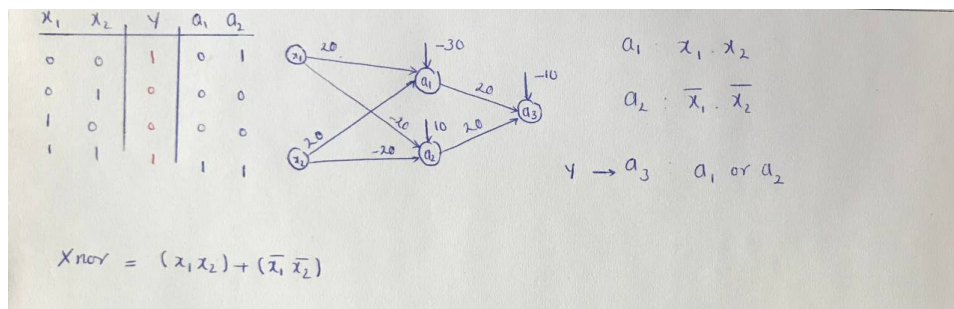


Figure 3

Problem 2

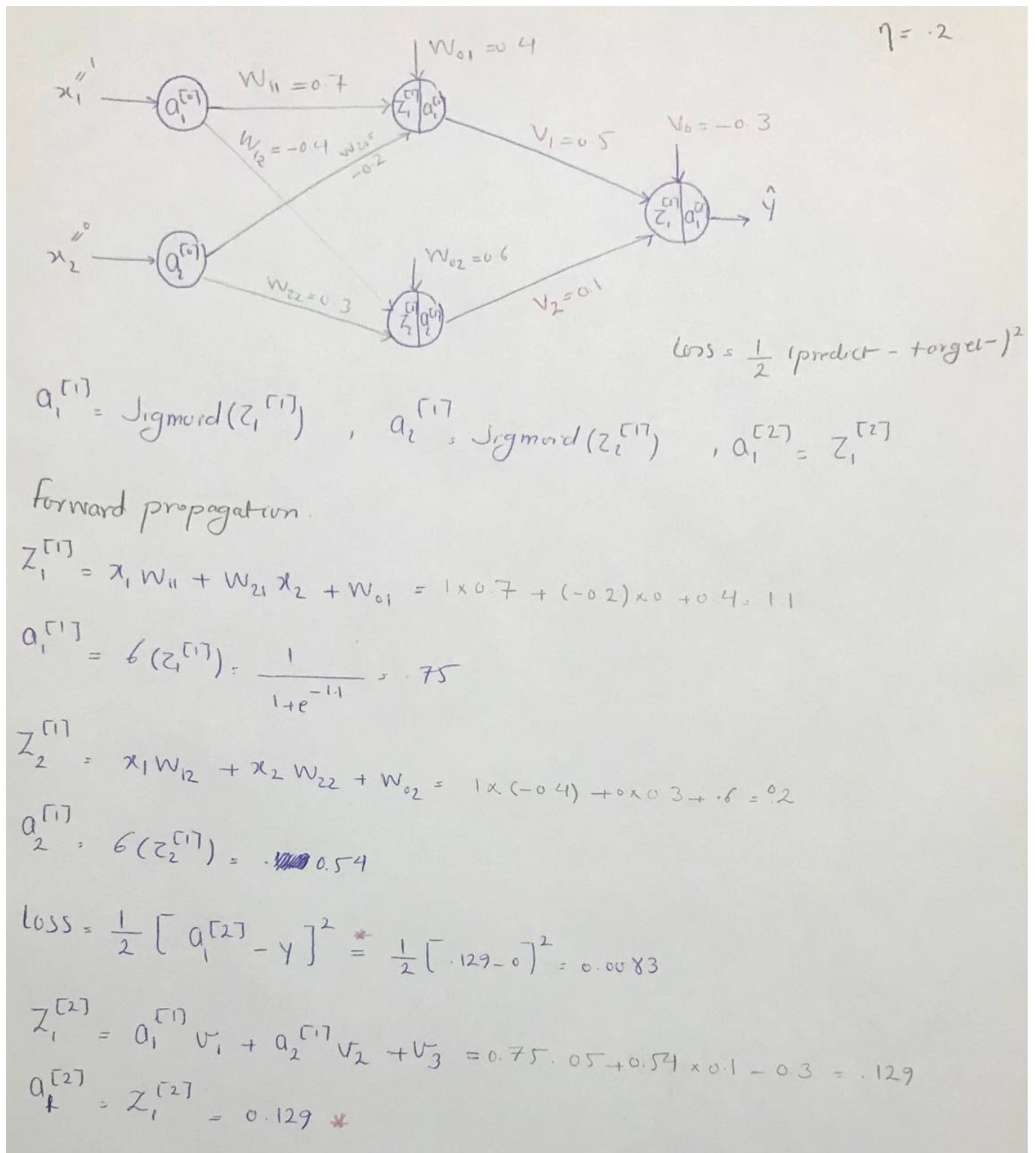


Figure 4

backward propagation $\text{cost} = L$

لايه لايه (نورتي) لايه

$$E_1 = \frac{dL}{dz_1^{(2)}} = \frac{dL}{da_1^{(2)}} \cdot \frac{da_1^{(2)}}{dz_1^{(2)}} = (a_1^{(2)} - y) \cdot 1 = 0.129$$

$$\frac{1}{2} \times 2 \cdot (a_1^{(2)} - y)$$

$$\frac{dL}{dv_2} = \frac{dL}{dz_1^{(2)}} \cdot \frac{dz_1^{(2)}}{dv_2} = 0.129 \times 0.54 = 0.06966$$

$$\rightarrow v_2 = v_2 - \eta \frac{dL}{dv_2} = 0.1 - 0.2 \times 0.06966 = 0.086068$$

$$\frac{dL}{dv_1} = \frac{dL}{dz_1^{(2)}} \cdot \frac{dz_1^{(2)}}{dv_1} = 0.129 \times 0.75 = 0.09675$$

$$\rightarrow v_1 = v_1 - \eta \frac{dL}{dv_1} = 0.5 - 0.2 \times 0.09675 = 0.48065$$

$$\frac{dL}{dv_0} = \frac{dL}{dz_1^{(2)}} \cdot \frac{dz_1^{(2)}}{dv_0} = 0.129 \rightarrow v_0 = v_0 - \eta \frac{dL}{dv_0} = 0.129 \times 0.2 = 0.3$$

$$= -0.3258$$

$$E_2 = \frac{dL}{dz_2^{(1)}} = \frac{dL}{da_2^{(1)}} \cdot \frac{da_2^{(1)}}{dz_2^{(1)}} = E_1 \cdot v_2 \cdot a_2^{(1)} (1 - a_2^{(1)})$$

لايه لايه (نورتي) لايه

$$= 0.129 \times 0.1 \times 0.54 (1 - 0.54) = 0.0032$$

$$\frac{dL}{dz_2^{(1)}} = \frac{dL}{dz_1^{(2)}} \cdot \frac{dz_1^{(2)}}{da_2^{(1)}} = 0.129 \times 0.1 \times 0.54 (1 - 0.54) = 0.0032$$

$$\frac{dL}{dw_{22}} = \frac{dL}{dz_2^{(1)}} \cdot \frac{dz_2^{(1)}}{dw_{22}} = 0.0032 \times 0 = 0 \rightarrow w_{22} = w_{22} - \eta \frac{dL}{dw_{22}} = 0.3$$

$$\frac{dL}{dw_{12}} = \frac{dL}{dz_2^{(1)}} \cdot \frac{dz_2^{(1)}}{dw_{12}} = 0.0032 \times 1 = 0.0032$$

$$\rightarrow w_{12} = w_{12} - \eta \frac{dL}{dw_{12}} = -0.4 - 0.2 \times 0.0032 = -0.4006$$

$$\frac{dL}{dw_{02}} = \frac{dL}{dz_2^{(1)}} \cdot \frac{dz_2^{(1)}}{dw_{02}} = 0.0032 \rightarrow w_{02} = w_{02} - \eta \frac{dL}{dw_{02}} = 0.6 - 0.2 \times 0.0032 = 0.59936$$

Figure 5

$$E_3 = \frac{dL}{dz_1^{[1]}} = \frac{dL}{da_1^{[1]}} \cdot \frac{da_1^{[1]}}{dz_1^{[1]}} = 0.129 \times 0.5 \times 0.75 \times 0.25 = 0.012$$

$$\downarrow$$

$$\frac{dL}{dz_1^{[2]}} = \frac{dL}{dz_1^{[1]}} \cdot \frac{dz_1^{[1]}}{da_1^{[1]}} = \frac{dL}{dz_1^{[1]}} \cdot a_1^{[1]} (1 - a_1^{[1]})$$

$$\frac{dL}{dz_1^{[2]}} = \frac{dL}{dz_1^{[1]}} \cdot \frac{dz_1^{[1]}}{dw_{11}} = 0.012 \times 1 = 0.012$$

$$\frac{dL}{dw_{11}} = \frac{dL}{dz_1^{[2]}} \cdot \frac{dz_1^{[2]}}{dw_{11}} = 0.012 \times 1 = 0.012$$

$$\rightarrow w_{11} = w_{11} - \eta \frac{dL}{dw_{11}} = 0.7 - 0.2 \times 0.012 = 0.697$$

$$\frac{dL}{dw_{21}} = \frac{dL}{dz_1^{[2]}} \cdot \frac{dz_1^{[2]}}{dw_{21}} = 0 \rightarrow w_{21} = w_{21} - \eta \frac{dL}{dw_{21}} = -0.2$$

$$\frac{dL}{dw_{01}} = \frac{dL}{dz_1^{[2]}} \cdot \frac{dz_1^{[2]}}{dw_{01}} = 0.012 \rightarrow w_{01} = w_{01} - \eta \frac{dL}{dw_{01}} = 0.4 - 0.2 \times 0.012 = 0.3975$$

Figure 6

Problem 3

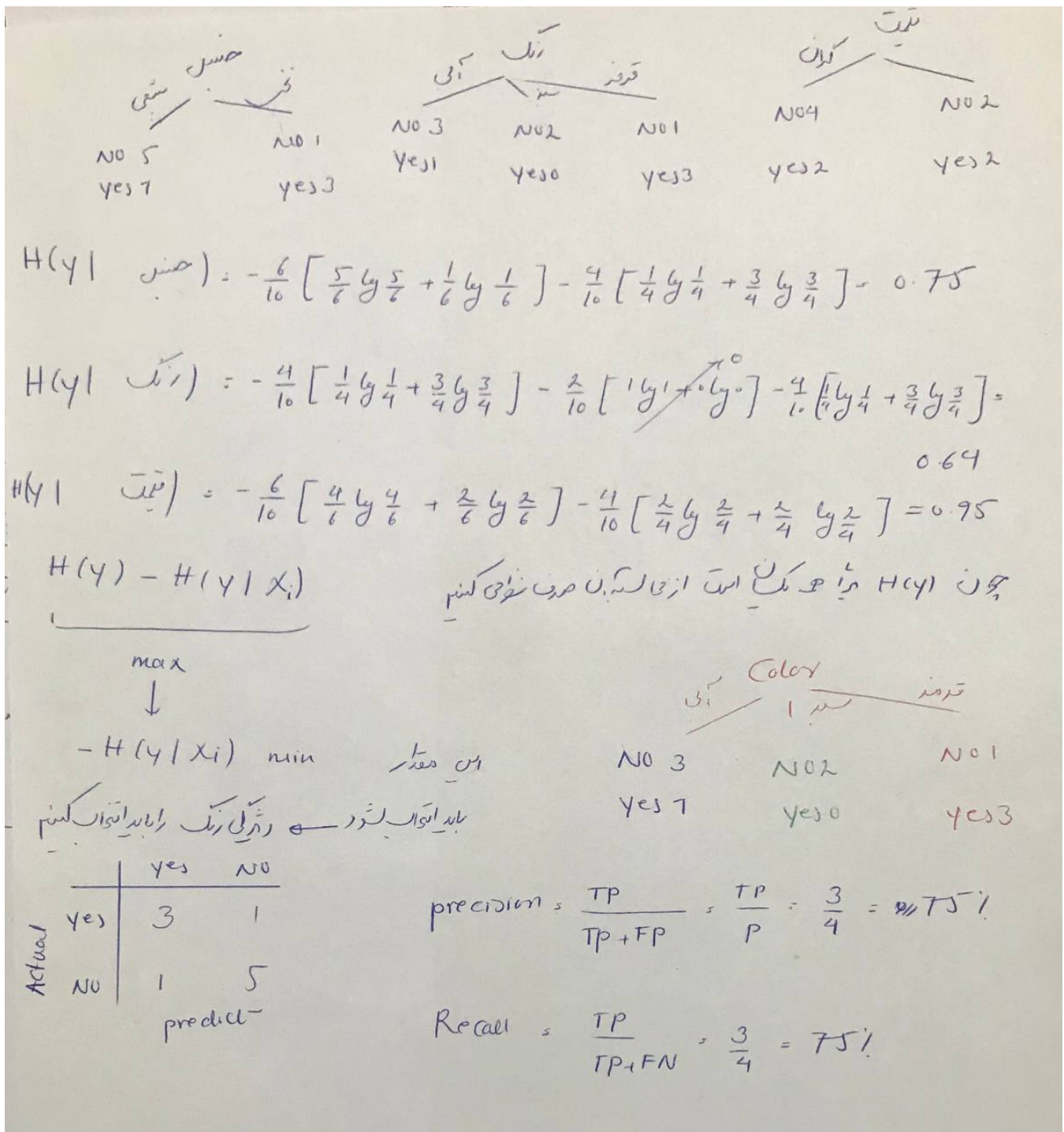


Figure 7

Problem 4

A.

B.

$$K=1 : P_n(x) = \frac{1}{2^n} \sum_{j=0}^{K=0} \binom{n}{j} = \frac{1}{2^n} = P_1$$

$$K \geq 2 : P_n(x) = \frac{1}{2^n} \sum_{j=0}^{K-1/2} \binom{n}{j} = P_\infty$$

با توجه به قضیه بیل می توان گفت $P_1 \leq P_\infty$
 \downarrow
 $K > 1$

چون هر دو طرف از توزیع کلاسیک محاسبه شدند بنابراین خطای $1NN$ کمتر از مابقی K های کمتر ^{positive}

Figure 9

C.

$$\begin{aligned}
 \lim_{n \rightarrow \infty} P_n(e) &= 0 \\
 \lim_{n \rightarrow \infty} P_n(e) &= \frac{1}{2^n} \sum_{j=0}^{K-1/2} \binom{n}{j} = \frac{1}{2^n} \sum_{j=0}^{K-1/2} \frac{n!}{(n-j)! j!} \approx \frac{1}{2^n} \sum_{j=0}^{K-1/2} \frac{n!}{n!} \\
 &= \lim_{n \rightarrow \infty} \frac{K-1}{2^{n-1}} = 0 \quad K=O(n)
 \end{aligned}$$

Figure 10

Problem 6

Please see this file: "Q6.ipynb"

Code and explanation are provided.

Problem 7

Please see this file: "Q7.ipynb"

Code and explanation are provided.

Problem 8

Please see this file: "Q8.ipynb"
Code and explanation are provided.