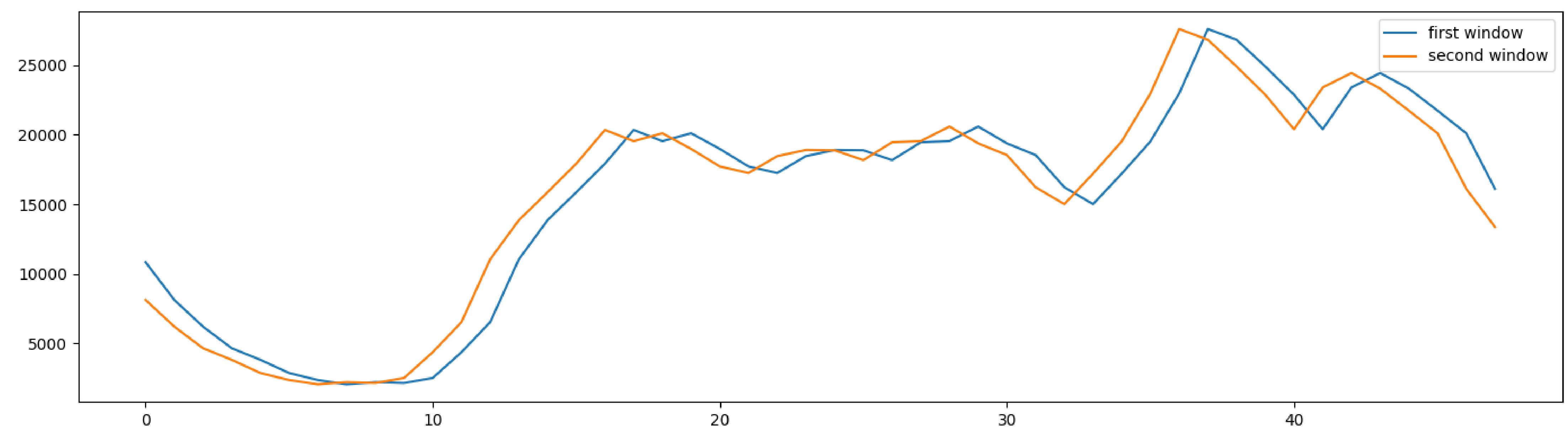
A Time-Dependent Estimator

Spotting the Problem

The sequence-based estimator we built learns from all the training data

This means it will learn from both these series, for example:

In [3]:
```
plt.figure(figsize=figsize)
plt.plot(wdata.iloc[0], label='first window')
plt.plot(wdata.iloc[1], label='second window')
plt.legend()
plt.tight_layout()
```



Spotting the Problem

Let us consider the first two window applications

- In the first window, the observations are x_0, x_1 and so on
- In the second window, the observations are x_1, x_2 and so on

x_0 is number of taxis as 00:00, x_1 at 00:30, and so on

- Hence, the first observation in the first window corresponds to 00:00
- ...But in the second window corresponds to 00:30

Our estimator learns a distribution for the observations:

- Moving the window forward changes "who is who"
- We learn the distribution of x_0 (and its correlations) multiple times!

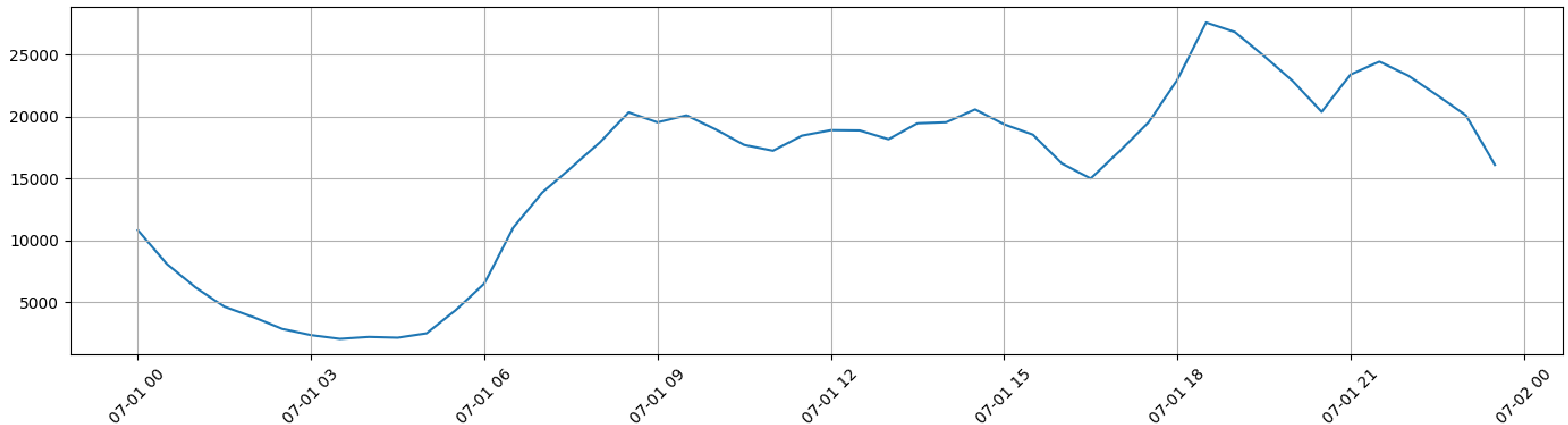
The learning problem is still well defined, but also very complex

This is the reason for (most of) the noise in the alarm signal

Rewind a Little

Remember why we introduced the sequence based estimator?

```
In [4]: util.plot_series(data.iloc[:48], figsize=figsize)
```

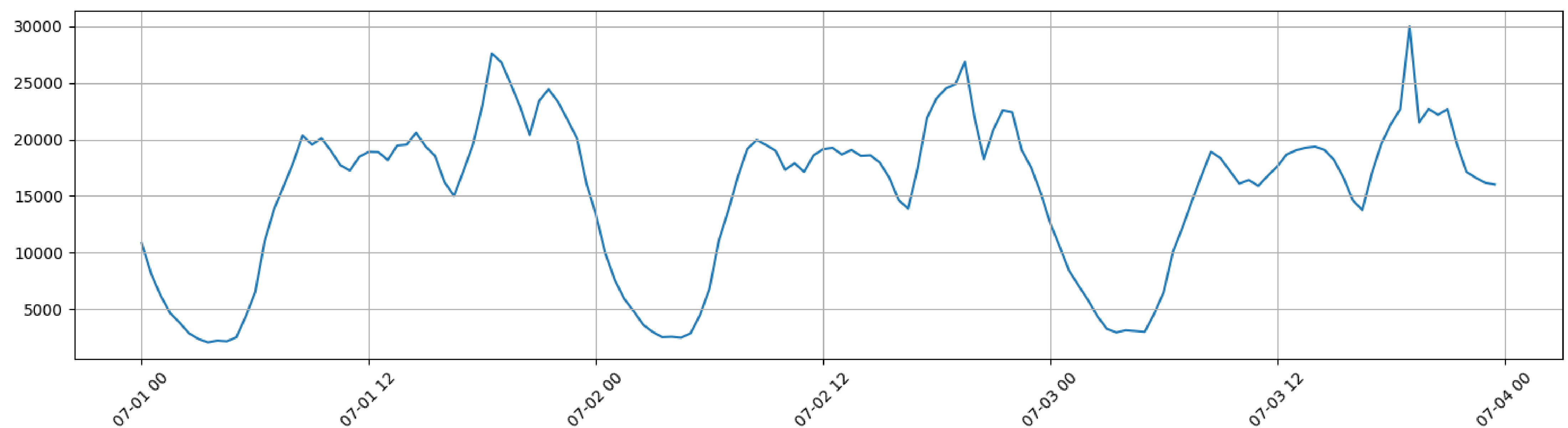


- We wanted to take advantage of correlation between nearby points

...Then Forward Again

But there is more! Let's look just a little bit further

```
In [5]: util.plot_series(data.iloc[:3*48], figsize=figsize)
```

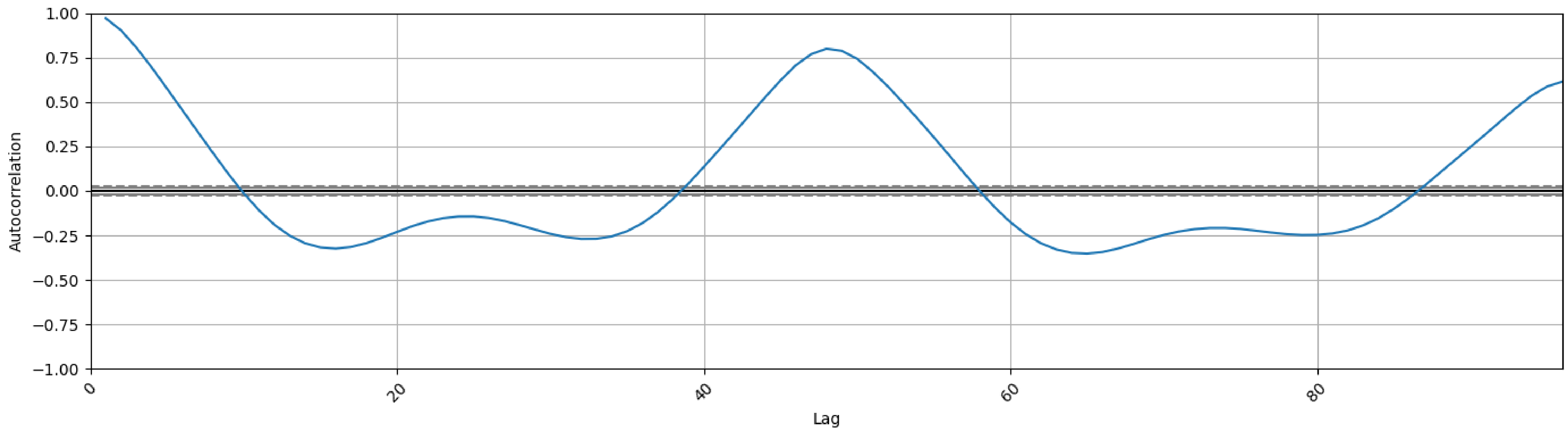


- There is recurring pattern!
- I.e. the series is approximately periodic

Determine the Period

This is even clearer in the autocorrelation plot

In [6]:
```
util.plot_autocorrelation(data, max_lag=96, figsize=figsize)
```



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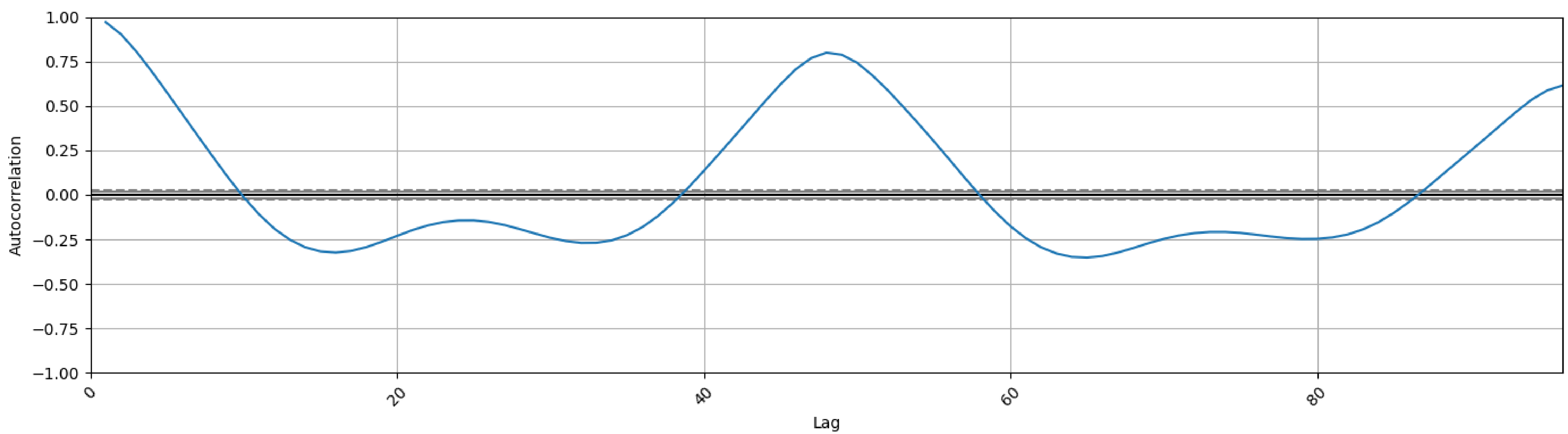


- There is strong peak at 48 time steps (a time step is 30 minutes)
- This is consistent with a period of 24 hours

Reevaluate

Let's recap our situation

Our sequence-based estimator

- ...Is solving a uselessly complicated problem
- ...And it's not using all the available knowledge

These are both very serious drawbacks

In any problem:

- Never introduce complications unless they are worth it
- Never willingly throw away information

Can we do something to tackle both problems?

Time as an Additional Input

One way to look at that

...Is that the distribution depends on the time of the day

- Therefore, we should consider the number of taxi calls x
- ...And the time of the day t together

Let us extract (from the index) the time information information:

In [11]:
```
dayhour = (data.index.hour + data.index.minute / 60)
```

We can then add it as a separate column to the data:

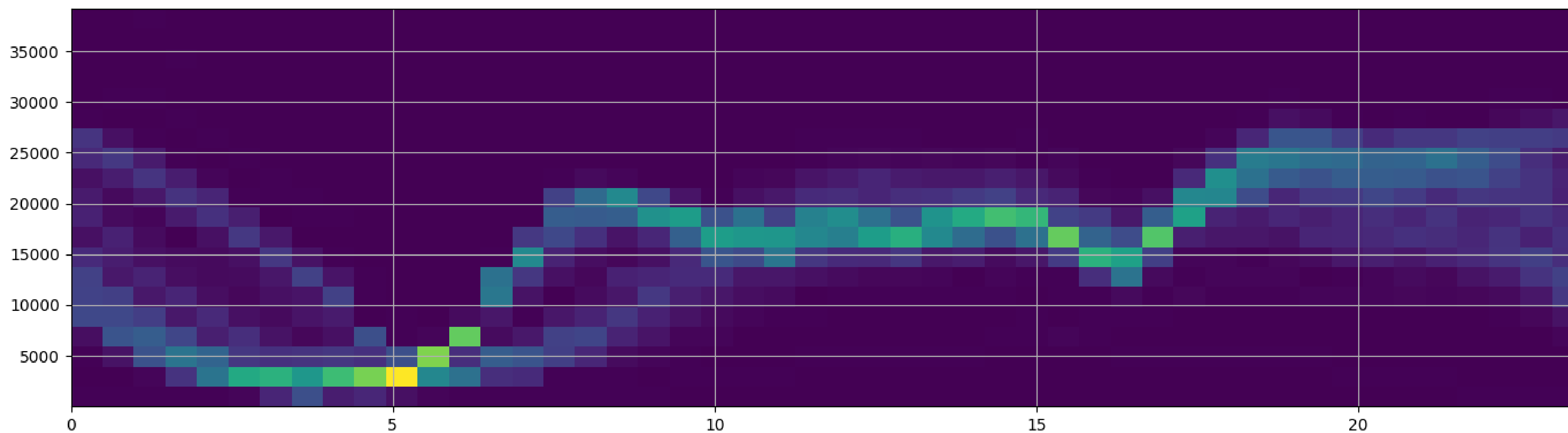
In [12]:
```
data2 = data.copy()
data2['dayhour'] = dayhour
```

Multivariate Distribution

Let us examine the resulting multivariate distribution

We can use a 2D histogram:

In [13]:
```
util.plot_histogram2d(data2['dayhour'], data2['value'], bins=(48, 20), figsize=figsize)
```



- x = time, y = value, color = frequency of occurrence

Anomaly Detection with Controlled Variables

If we feed this information to KDE

...We learn an estimator for the joint PDF:

$$f(t, x)$$

...Which is not exactly what we were looking for

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Assume we flag an anomaly when $f(t, x) \leq \theta$

- This may happen when x (the number of cars) takes an unlikely value
- ...Or when t (the time) does

Except that the time is completely predictable

- Any different in its estimated density is only due to sampling choices
- In practice, it's a controlled variable

Anomaly Detection with Controlled Variables

What we really care about is the conditional density, i.e.

$$f(x \mid t)$$

- I.e. the density value of the observed value of x
- Assuming that the time t is known

Our true anomaly detection conditions should then be:

$$f(x \mid t) \leq \varepsilon$$

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How to handle the conditioning variable?

Anomaly Detection with Controlled Variables

There's more than one way to do it

...The one we'll see starts with the definition of conditional probability:

$$f(t, x) = f(x \mid t) f(t)$$

Meaning that we can detect anomalies by evaluating:

$$\frac{f(t, x)}{f(t)} \leq \varepsilon$$

In order to pull this off, we need

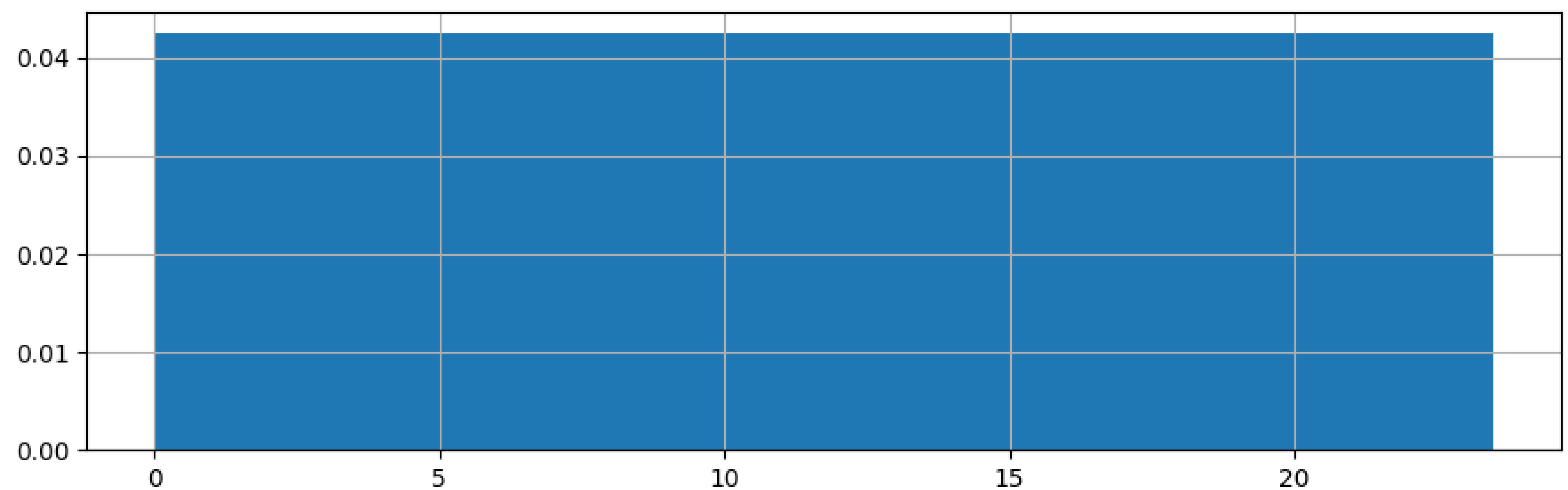
- An estimator for $f(t, x)$, which we already have
- An estimator for $f(t)$, which we can easily obtain (e.g. using KDE again)

...But in our specific case, things are even simpler

Time Distribution

In particular, the distribution of time values is uniform:

In [14]:
```
util.plot_histogram(data2['dayhour'], bins=48)
```



Our Time-Dependent Estimator

In non-degenerate cases, our condition can always be rewritten:

$$\frac{f(t,x)}{f(t)} \leq \varepsilon \quad \longrightarrow \quad f(t,x) \leq \varepsilon f(t)$$

- But since $f(t)$ is constant this is equivalent to checking the joint probability
- ...With a modified threshold

$$f(t,x) \leq \varepsilon'$$

- The threshold ε' now represents $\varepsilon f(t)$
- ...But since we still need to choose it value, it make little difference to us

Hence, for this problem we can use $f(t,x)$ for anomaly detection

Choosing a Bandwidth

We now need to pick a threshold

- We can use grid search and cross-validation again
- ...But this time we need to make sure to normalize the data

In [16]:
```
scaler = MinMaxScaler()
data2_n_tr = data2[data2.index < train_end].copy()
data2_n_tr[:] = scaler.fit_transform(data2_n_tr)
data2_n = data2.copy()
data2_n[:] = scaler.transform(data2)
```

This is due to a low-level technical detail:

- scikit-learn uses a very efficient KDE implementation
- ...But it requires using the same bandwidth for all input dimensions

Normalization makes this drawback less impactful

Choosing a Bandwidth

We can then optimize the bandwidth as usual

We'll use cross-validation, since we have vector input

In [17]:
```
from sklearn.model_selection import GridSearchCV
params = {'bandwidth': np.linspace(0.001, 0.01, 10)}
opt = GridSearchCV(KernelDensity(kernel='gaussian'), params, cv=5)
opt.fit(data2_n_tr);
opt.best_params_
```

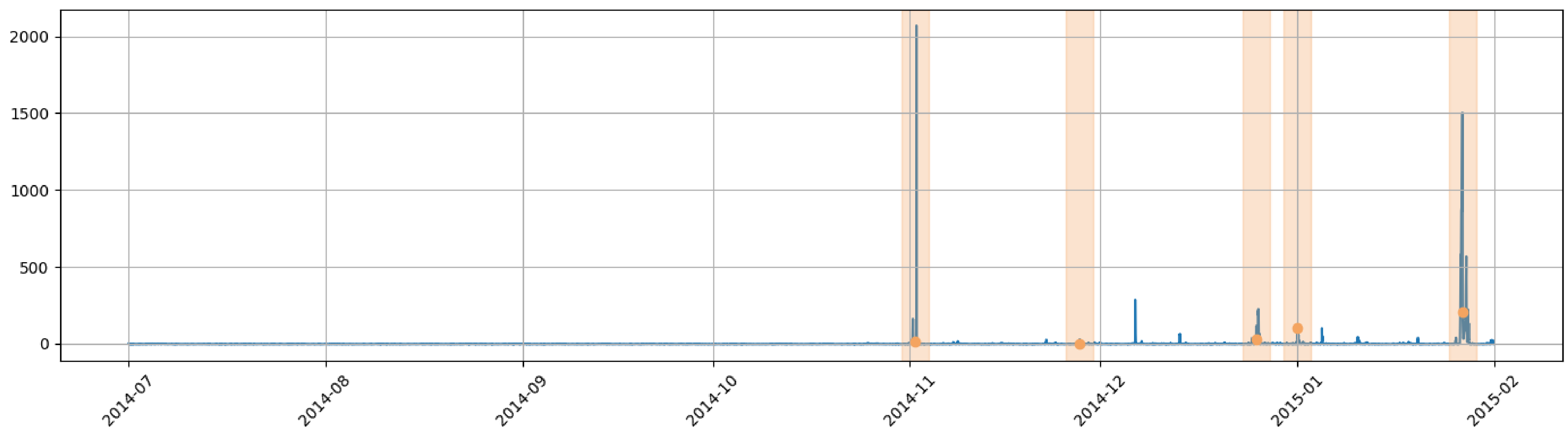
Out[17]:
```
{'bandwidth': 0.006}
```

- As another small advantage of normalization
- ...Choosing the grid search range becomes a bit easier

Alarm Signal

Let us obtain the alarm signal

In [18]:
```
ldens2 = opt.score_samples(data2_n)
signal2 = pd.Series(index=data2.index, data=-ldens2)
util.plot_series(signal2, labels=labels, windows=windows, figsize=figsize)
```



Threshold Optimization

Now, let us optimize our threshold:

In [19]:
```
signal2_opt = signal2[signal2.index < val_end]
labels_opt = labels[labels < val_end]
windows_opt = windows[windows['end'] < val_end]
thr2_range = np.linspace(10, 100, 100)
best_thr2, best_cost2 = util.opt_thr(signal2_opt, labels_opt, windows_opt, cmodel, thr2_rang
print(f'Best threshold: {best_thr2:.3f}, corresponding cost: {best_cost2:.3f}')
```

```
Best threshold: 27.273, corresponding cost: 9.000
```

On the whole dataset:

In [20]:
```
c2tst = cmodel.cost(signal2, labels, windows, best_thr2)
print(f'Cost on the whole dataset {c2tst}')
```

```
Cost on the whole dataset 18
```

- It was 45 for the first approach and 30 for the second

There is a second period in the data!
Can you guess which one?