MATHS **Max.Marks:80**

SECTION-1

(SINGLE CORRECT CHOICE TYPE)

Section-I (Single Correct Answer Type, Total Marks: 24) contains 8 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONLY ONE is correct. For each question you will be awarded 3 marks if you darken ONLY the bubble corresponding to the correct answer and zero marks if no bubble is darkened. In all other cases, minus one (-1) mark will be awarded.

41. Let P be the inflection point of the curve $y = x^3 + 3ax^2 + 4a^3 \quad (a > 0)$

Let $y = f(x)$ be the equation of the locus of P as the value of $'a'$ changes. The equation to the tangent to the curve $y = f(x)$ at $x = -1$ is

A) $18x + y + 12 = 0$ B) $x + 18y + 12 = 0$

C) $12x + y + 18 = 0$ D) $x - 18y + 12 = 0$

42. Number of points of inflection for the function $f(x) = \left(\frac{\pi}{3}\right)^{x^3-8}$ is

A) 0 B) 2 C) 1 D) 4

43. In the xy-plane, the curve $y = x^3$ intersects with the line $y = 3x + a$ in three points A, B and C. If the point D is given by $(a, 4a)$, the maximum value of the product $DA.DB.DC$ is

A) $\frac{16}{\sqrt{3}}$ B) $\frac{160}{3\sqrt{3}}$ C) $\frac{160\sqrt{10}}{3\sqrt{3}}$ D) $\frac{16\sqrt{10}}{\sqrt{3}}$

44. Three real numbers b, c and d are given. Then the set of all values of $'a'$ such that the tangents drawn to the curve $y = \sin x$ at the points $(a, \sin a), (b, \sin b), (c, \sin c), (d, \sin d)$ form a rectangle is

A) $\{n\pi : n \in \mathbb{Z}\}$
B) $\left\{\frac{n\pi}{2} : n \in \mathbb{Z}\right\}$
C) $\left\{(2n+1)\frac{\pi}{2} : n \in \mathbb{Z}\right\}$
D) $\{(2n+1)\pi : n \in \mathbb{Z}\}$

45. Let the functions $f(x) - x$ and $f(x) - x^3$ are monotonically increasing functions on R. Then the function $f(x) - \frac{\sqrt{3}}{2}x^2$ is

A) increasing on $R^+ \cup \{0\}$ only
B) decreasing on R
C) increasing on R
D) decreasing on $R^- \cup \{0\}$ only

46. In the parabola $x^2 = 4ay, a > 0$, consider a chord whose length is double that of the latusrectum. The angle at which the chord is inclined to the positive x-axis so that its mid point is at a minimum height from x-axis is given by

A) $\frac{\pi}{3}$
B) $\frac{\pi}{4}$
C) $\frac{\pi}{6}$
D) π

47. Let $f(x)=\frac{x^2+ax+b}{x^2+1}$ for some real numbers $a\&b$. If $|f(x)|\le 2, \forall x$, then the locus of the point (a,b) describes a region. Maximum distance between any two points in the region is

A) $2\sqrt{3}$ B) $4\sqrt{3}$ C) $\sqrt{3}$ D) $6\sqrt{3}$

48. Which of the following statements is false about any differentiable monotonic function $f:\mathbb{R}\to\mathbb{R}$ that satisfies the equation $f(xf(y)+x)=2f(x)+xy-1,\ \forall x,y\in\mathbb{R}$?

A) $y=f(x)$ has constant derivative in $\mathbb{R}$

B) There are only two functions which satisfy the above conditions

C) $y=f(x)$ is surjective

D) $y=f(x)$ has one point of inflection

SECTION-2

(MORE THAN ONE TYPE)

Section - II (Multiple Correct Answers Type, Total Marks: 16) contains 4 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONE or MORE may be correct. For each question you will be awarded 4 marks if you darken ALL the bubble(s) corresponding to the correct answer(s) ONLY and zero marks otherwise. There are no negative marks in this section.

49. P(x) is a monic polynomial with real coefficients satisfying the equation $P\left(\sqrt{2}x\right)=P\left(x+\sqrt{1-x^2}\right)$ for all $|x|\le 1$. Given that P(0)=0, which of the following conclusion(s) hold?

A) $P'(0)=0$ B) P(x) represents an even function

C) $x^2+y^2=2$ whenever $P(x)=P(y)$ D) $P(1)=0$

50. Let $f(x)$ be a function defined on $(0,\infty)$ as $f(x)=x+\dfrac{1}{x}$. Let $h(x)$ is a function defined for all $x\in(0,1)$ as $h(x)=\dfrac{x^4}{(1-x)^6}$. Suppose $g(x)=f(h(x))$ for all $x\in(0,1)$. Then which of the following statements is/are false?

A) It is not possible to find some $a\in(0,1)$ such that $g(x)$ strictly decreasing in $(0,a)$ and strictly increasing in $(a,1)$

B) If the option A is false, then there exists $'a'$ such that $\dfrac{1}{2}<a<1$

C) $f(x)$ is decreasing in its domain

D) $h(x)$ is increasing in its domain

51. If $f(x)=3x^4-4(a+7)x^3+6(7a+10)x^2-120ax+2$ defined on the interval $[4,\infty)$, then which of the following statements is/are true about f(x) and 'a'? (We write $f(x)\uparrow$ if f(x) increases and $f(x)\downarrow$ if f(x) decreases)

A) when $a>5$, $f(x)\uparrow$ in $(5,a)$ and $f(x)\downarrow$ in (a,∞)

B) when $a\in(2,4]$, then $f(x)\uparrow$ for $(5,\infty)$ and $f(x)\downarrow$ for $(4,5)$

C) when $a<3$, then $f(x)\downarrow$ in $(5,\infty)$ and $f(x)\downarrow$ for $(4,5)$

D) when $a<1$, $f(x)\uparrow$ for $(5,\infty)$ and $f(x)\downarrow$ for $(4,5)$

52. Suppose that $f(x)$ is a differentiable function such that $f'(x) \geq 3$ for all real numbers and $f(0) = -4$. Then which of the following cannot be a value for for $f(3)$?

A) 8 B) 2 C) 4 D) 6

SECTION-3

[INTEGER TYPE]

Section-III (Integer Answer Type, Total Marks: 24) contains 6 questions. The answer to each of the questions is a single-digit integer, ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the ORS. For each question you will be awarded 4 marks if you darken ONLY the bubble corresponding to the correct answer and zero marks otherwise. There are no negative marks in this section.

53. Number of non-zero integer values for K, so that the equation $x^4 - 4x^3 + 2x^2 + 4x + K = 0$ has four distinct real roots is

54. Smallest positive integer value for K so that the function $f(x) = \sin^3 x + K \sin^2 x$, fails to have exactly one minimum and exactly one maximum in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ is

55. If K is the minimum value of the expression $|x-a| + |x-15| + |x-a-15|$ for $x \in [a,15]$ and $a \in (0,15)$. Then $\frac{K}{3} =$

56. Given that a function $f(x)$ satisfies the relation for $f\left(\frac{x-3}{x+1}\right) + f\left(\frac{3+x}{1-x}\right) = x$ all real numbers x such that $|x| \neq 1$. Number of points on y=f(x) tangent at which is parallel to the line $7x - 2y = 15$ is

57. The parametric equations of the function $y = f(x)$ are given by $x = t^5 + 5t^3 - 20t + 4$, $y = 6t^3 - 21t^2 + 12t + 5$ then the point of minimum for the curve occurs at $t =$ ___

58. Consider the real valued functions f, g, h given by $g(x) = (f(x)+1)x + 4 = [h(x)]^2$ where $f(x) = x^3 - f''(1)x^2 - [f''(1) + f'(2)]x + 3f'(2)$. If K is minimum value of $h(x)$, then $4K =$

SECTION-4

[Matrix Matching Type]

Section-IV (Matrix-Match Type, Total Marks: 16) contains 2 questions. Each question has four statements (A, B, C and D) given in Column I and five statements (p, q, r, s and t) in Column II. Any given statement in Column I can have correct matching with ONE or MORE statement(s) given in Column II. For example, if for a given question, statement B matches with the statements given in q and r, then for the particular question, against statement B, darken the bubbles corresponding to q and r in the ORS. For each question you will be awarded 2 marks for each row in which you have darkened ALL the bubble(s) corresponding to the correct answer(s) ONLY and zero marks otherwise. Thus, each question in this section carries a maximum of 8 marks. There are no negative marks in this section.

59.

	Column I (Function)		Column II
A)	$f(x) = x^2 - \frac{1}{2}\ln\lvert x\rvert$	P)	strictly increasing in $\left(0, \frac{\pi}{4}\right)$
B)	$f(x) = 2x + e^{-x}$	Q)	strictly decreasing in $\left(-2\pi, \frac{-3\pi}{2}\right)$
C)	$f(x) = \cos x + e^{x}$	R)	strictly increasing in (e, ∞)
D)	$f(x) = e^{\cos x} - e^{-\cos x}$	S)	strictly decreasing in $(-\infty, e)$

60.

	Column I		Column II
A)	The tangent at any point P on the curve $x = at^3$, $y = at^4$ meets X, Y axes respectively at A and B so that $\frac{\text{PA}}{\text{PB}}$ is m:n, then $n + m$ is equal to (m and n are co-prime)	P)	0
B)	If the area of the triangle formed by normal at the point(1,0) on the curve $x = e^{\sin y}$ with co-ordinate axes is Δ then the value of 2Δ is	Q)	1
C)	If the angle between the curves $x^2 y = 1$ and $y = e^{2(1-x)}$ at the point (1,1) is θ , then $\tan\theta$ is equal to	R)	7
D)	The length of the subtangent at any point on the curve $y = be^{x/3}$ is equal to	S)	3