$$\frac{1}{2}2m\left(\frac{v}{2}\right)^2+\frac{1}{2}\ k\left(\frac{mg}{k}\right)^2 = 2mg\left(\frac{mg}{k}\right)$$

Solving we get g v = $\sqrt{\frac{6\,m\,g^2}{k}}$

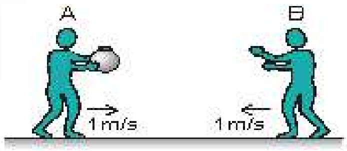
38 Ans 2

Sol. They can avoid the collision when separation between them starts.

Frist A throws ball towards B. Applying conservation of momentum on 'A + ball' system

$$80.1 = 70\ V_A + 10\ (5 + V_A)$$

Where V_A is speed of A towards B after throwing the ball.



$$V_A = \frac{3}{8}\ \text{m/s}$$

B catches the ball and throws towards A. Let V_B is speed of B towards A after the throw. Therefore

$$70.1 - 10\frac{43}{8}. = 70.\ V_B + 10\ (5 + V_B)$$

$$\frac{130}{8} - 50 = 80\ V_B$$

$$-\frac{370}{640} = V_B \qquad Þ\ V_B = -\frac{1}{2}$$

i.e. B is going towards right with speed more than that of A (they are separating).

39.

Ans. (A) p,q (B) p,q (C) q,r (D) q,r

Sol. In all cases speed of balls after collision will be same. In case of elastic collision speed of both balls after collision will be u, otherwise it will be less than u.

MATHS

41 . [$z=1, \cos\alpha + i\sin\alpha, \cos\alpha - i\sin\alpha$. Are the roots & area of triangle, is maximum when $\alpha = 120^{o}$]

42. [End points of a diameter of I circle are $\left(5/3, 0\right)$ & $(3,0)$ end points of a diameter of II circle are. $\left(4/3, 0\right)$ $(0,0)$]

43. $\sum PA^2 = \sum_{i=1}^{6} |z - z_i|^2 = \sum_{i=1}^{6} |z|^2 + |z_1|^2 - \left(2\overline{z}_1 + z_1\overline{z}\right) = 30$

$z_i, i = 1,2,3,4,5,6$ are the roots of $z^6 = 1$ & $|z| = 2$

44. $z^3 = -w^5 \Rightarrow |z|^3 = |w|^5 \Rightarrow |z|^6 = |w|^{10}$ …..1

$z^2 = \frac{1}{\overline{w}^4} \Rightarrow |z|^2 = \frac{1}{|w|^4} \Rightarrow |z|^6 = \frac{1}{|w|^{12}}$ …..2

From 1 & 2 $\Rightarrow |w| = 1 \,\&\, |z| = 1$

$\Rightarrow w\overline{w} = 1 \,\&\, z\overline{z} = 1$

& $z^6 = w^{10} \,\&\, z^6 w^{12} \Rightarrow z^6 = \frac{1}{\overline{w}^{12}} = w^{10}$

$\Rightarrow (w\overline{w})^{10}\overline{w}^2 = 1$

$\Rightarrow \overline{w}^2 = 1 \Rightarrow w = 1 \text{ or } -1$

when $\overline{w} = 1 \text{ or } -1$

$z^3 + 1 \,\&\, z^2 = 1$

$\Rightarrow z = \pm 1$

45. $z_1 + (\sqrt{3} + i)t - i = 0$ & arg $(z_1) = \pi/4$

$\Rightarrow \text{Re}(z_1) = \frac{\sqrt{3}}{2}\left(\sqrt{3} + 1\right)$

$z_2 + \frac{(\sqrt{3} + i)}{\sqrt{3}}\lambda - i = 0 \,\&\, \arg(z_2) = -3\pi/4$

$\Rightarrow \text{Re}(z_2) = -\frac{(\sqrt{3} + 1)}{2}$

$\therefore$ Area triangle ABC = $\frac{1}{2} \times 1 \times \left(\frac{\sqrt{3} + 1}{2}\right)(\sqrt{3} + 1) = \frac{\sqrt{3} + 2}{2}$

46. $\arg\left(\frac{-z_1}{z-z_1}\right)=\pi/2 \Rightarrow \frac{-z_1}{z_1-z}+\frac{\bar{z}_1}{\bar{z}_1-\bar{z}}=0 \to 1$

& B, O, P are collinear

$\Rightarrow \arg\left(\frac{z_2}{z_1}\right)=\pi \Rightarrow \frac{z_2}{z}=\frac{\bar{z}_2}{\bar{z}} \to 2$

From 1 and 2 $z=\frac{2|z_1|^2 z_2}{z_1\bar{z}_2+\bar{z}_1 z_2}$

$\Rightarrow |z-z_1|=\frac{z_i i.\operatorname{Im}(\bar{z}_1 z_2)}{\operatorname{Re}(\bar{z}_1 z_2)}$

47. Suppose $|\omega|=1$ & $\omega^{n+1}-\omega^n-1=0 \Rightarrow |\omega^n(\omega-1)|=1$ $|\omega-1|=1$& ω_1,ω_2 are the points of intersection of $|z|=1\&|z-1|=1$

$\omega_1=e^{i\pi/3},\omega_2=e^{-\pi i/3}$.

$\omega-1=e^{\pm\frac{2\pi i}{3}}=\omega_2 \Rightarrow 1=\omega^n(\omega-1)=w^{n+2}$

$$\Rightarrow \cos\left(\frac{n+2}{3}\right)\pi \pm i\sin\left(\frac{n+2}{3}\right)\pi \Rightarrow \left(\frac{n+2}{3}\right)\pi=2k\pi,(k\in I)$$

48. $\arg(z_1)+\arg(z_2)+...+\arg(z_n)=\pi\pm 2m\pi$

$\pi/n\left[3+5+7+...+(2n+1)\right]=\lambda\pm 2m\pi$

$\pi/n\left[n/2(6+2(n-1)\right]=\lambda\pm 2m\pi$

$3+n-1=1\pm 2m$

$\Rightarrow n=2m\pm 1]$

49. $\left[\frac{x^{2n}+1}{x^{2n-1}+x}=\frac{\cos n\theta}{\cos(n-1)\theta}\right]$

51. $a+b+c=0\ \&\ 1/a+1/b+1/c=0 \Rightarrow ab+bc+ca=0$

$\Rightarrow a^2+b^2+c^2=(a+b+c)^2$

$\Rightarrow a^3+b^3+c^3=3abc$

$a+b=-c \Rightarrow a^2+b^2-c^2=-2ab$

$a^4+b^4+c^4=2(a^2b^2+b^2c^2+c^2a^2)$

52. $|z_3| = \dfrac{|z_2||z_1 - z_4|}{|\overline{z}_1 z_4 - 1|} \le 1 \Rightarrow |z_1|^2 + |z_4|^2 - |z_1|^2|z_4|^2 - 1 \le 0$

$\Rightarrow \left(|z_1|^2 - 1\right)\left(|z_4|^2 - 1\right) \ge 0$

$\Rightarrow |z_4| \le 1$

53. [$\min\{|az_2 + (1-a)z_3 - z_1|\}$ = height , from z_1 to the line, joining z_2, z_3

$\therefore \dfrac{bc}{h} = \dfrac{bca}{2\Delta} = 2R = 6$]

57. $(x-1)^3 = (-2)^3$

$\Rightarrow \alpha - 1 = -2$

$\beta - 1 = -2w$

$\gamma - 1 = -2w^2$

$\dfrac{\alpha-1}{\beta-1} + \dfrac{\beta-1}{\gamma-1} + \dfrac{\gamma-1}{\alpha-1} = 3\omega^2$

But $3 + (3k+2) = 15$ not possible

58. $\alpha = a + ib$

$\gamma = c + id$

We need to minimize $\sqrt{a^2+b^2} + \sqrt{c^2+d^2}$

$f(i) \Rightarrow$ b+d+1=0

& $f(i) =$ pured

$\Rightarrow \sqrt{a^2+b^2} =$ ___