Sri Chaitanya IIT Academy, India

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A right Choice for the Real Aspirant

ICON CENTRAL OFFICE, MADHAPUR-HYD

Sec: Sr. IPLCO **Date: 05-09-15**

Time: 9:00 AM to 12:00 Noon **RPTM-5** **Max.Marks: 360**

KEY SHEET

PHYSICS			MATHS			CHEMISTRY	
Q.NO	ANSWER		Q.NO	ANSWER		Q.NO	ANSWER
1	2		31	3		61	4
2	1		32	1		62	2
3	1		33	4		63	1
4	2		34	3		64	4
5	3		35	3		65	4
6	2		36	1		66	4
7	1		37	4		67	2
8	1		38	1		68	4
9	3		39	1		69	3
10	1		40	3		70	2
11	2		41	2		71	3
12	3		42	1		72	4
13	2		43	4		73	1
14	2		44	1		74	2
15	2		45	4		75	3
16	2		46	4		76	3
17	4		47	2		77	3
18	1		48	3		78	3
19	3		49	3		79	4
20	3		50	1		80	4
21	2		51	4		81	4
22	3		52	4		82	4
23	3		53	1		83	3
24	4		54	3		84	3
25	2		55	2		85	2
26	2		56	3		86	2
27	1		57	1		87	2
28	4		58	3		88	2
29	2		59	2		89	4
30	1		60	2		90	2

MATHS

31. Let x = cos t, $t \in [0, \pi]$

So, we have $\sqrt{1-x^2} = \sin t$

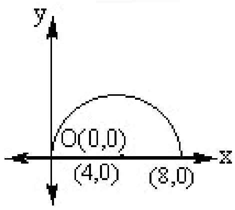
Then inequality becomes, sin t + cos t $\geq a$

The maximum value of f(t) = sin t + cos t on the interval [0, π] is $\sqrt{2}$.

Hence the range of a is set of all real numbers not exceeding $\sqrt{2}$.

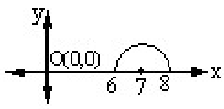
32. We have $f(x) = \sqrt{16-(x-4)^2} - \sqrt{1-(x-7)^2}$ Now consider $y = \sqrt{16-(x-4)^2}$

$\Rightarrow (x-4)^2 + y^2 = 16, y > 0$ is a semi circle with centre (4,0) and radius = 4.



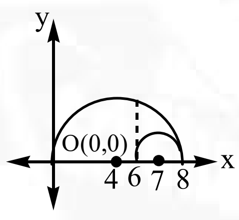
|||1y $y = \sqrt{1-(x-7)^2}$

$\Rightarrow (x-7)^2 + y^2 = 1, y > 0$



is a semi circle with centre (7,0) and radius = 1

Now on combining the 2 figures, we have



Now $f(x)]_{max}$ = maximum vertical distance between the 2 curves which occurs when x = 6.

$\therefore f(x)]_{max} = \sqrt{16-(6-4)^2} - 0 = \sqrt{12}$

33. Conceptual

34. Period of f(x) = |sin2x| + |cos2x| is π/4

but f(x) = ln ([|sin2x| + |cos2x|])

Max. value of |sin2x| + |cos2x| = $\sqrt{2}$

f(x) = ln ([$\sqrt{2}$]) = ln (1) = 0

$\Rightarrow$ it is periodic function but fundamental period not defined.

f(x) is many one and into function

35. "$x > 0$"; the given inequality is :

(i) $[x] < 2^x$ when $x \ge 1$

which has solution $[1, \infty)$

(ii) $2\ell nx < 2^x$ when $x \in (0,1)$

which has solution (0, 1)

Note: $2\ell n\, x < 0 < 2^x$

Solution set : $(0, \infty)$.

36. Let $g(x) = f(x + T/2) - f(x)$

then $g(k) = f(k + T/2) - f(k)$ ….. (1)

and $g(k + T/2) = f(k + T) - f(k + T/2)$

$= f(k) - f(k + T/2)$

$= -g(k)$

37. The equation $|2ax - 3| + |ax + 1| + |5 - ax|$......

$|2ax - 3| + |ax + 1| + |5 - ax| \ge |2ax - 3 + (-ax - 1) + 5 - ax| \ge 1$ so no solution for $\frac{1}{2}$

38. Put x = 2 $\Rightarrow$ a f(1) + bf(-2) = 5 $\Rightarrow$ a + 5b = 5

Put x = -1 $\Rightarrow$ a f(-2) + bf(1) = 3 $\Rightarrow$ 5a + b = 3

$\Rightarrow a = \frac{5}{12}, b = \frac{11}{12} \Rightarrow b - a = \frac{1}{2}$

39. $\frac{5^m + 3}{40} = \frac{1}{10}(5 + 5^2 + 5^3 + ... + 5^{m.1} + 2) \Rightarrow \lambda = \frac{1}{5}, \frac{7}{10}$

40. $-x^3 + x = -x \Rightarrow x = \pm\sqrt{2}$

41. Given $\left(1 + \frac{1}{n}\right)^{n + x_n} = e$ taking log

$$(n+x_n)l\,\text{n}\left(1+\frac{1}{n}\right)=1 \Rightarrow n+x_n=\frac{1}{l\text{n}\left(1+\frac{1}{n}\right)} \Rightarrow x_n=\frac{1}{\ln\left(1+\frac{1}{n}\right)}-n \quad ....(1)$$

Let $\frac{n+1}{n}=u \Rightarrow nu=n+1 \Rightarrow n=\frac{1}{u-1}$

$$x_n=\underset{u\to 1}{Lim}\frac{1}{l\,\text{n}\,u}-\frac{1}{u-1}=\underset{u\to 1}{Lim}\frac{(u-1)-l\,\text{n}\,u}{(u-1)l\,\text{n}\,u}=\underset{u\to 1}{Lim}\frac{1-\frac{1}{u}}{\frac{u-1}{u}+lnu}=\underset{u\to 1}{Lim}\frac{\frac{1}{u^2}}{\frac{1}{u^2}+\frac{1}{u}}=\frac{1}{2}$$

42. $L=\underset{n\to\infty}{Lim}\left(\frac{1}{2n+1}+\frac{1}{2n+2}+\frac{1}{2n+3}+......+\frac{1}{4n}\right)-\underset{n\to\infty}{Lim}\left(\frac{1}{2n+2}+\frac{1}{2n+4}+\frac{1}{2n+6}+.....+\frac{1}{4n}\right)$

$$=\underset{n\to\infty}{Lim}\left[\frac{1}{n}\sum_{r=1}^{2n}\frac{n}{2n+r}-\frac{1}{n}\sum_{r=1}^{n}\frac{n}{2n+2r}\right]=\underset{n\to\infty}{Lim}\left[\frac{1}{n}\sum_{r=1}^{2n}\frac{1}{2+\frac{r}{n}}-\frac{1}{n}\sum_{r=1}^{n}\frac{1}{2+2\left(\frac{r}{n}\right)}\right]$$

$$=\int_0^2\frac{1}{2+x}dx-\int_0^1\frac{1}{2+2x}dx=[l\text{n}(2+x)]_0^2-\frac{1}{2}[l\,\text{n}(1+x)]_0^1$$

$$=l\,\text{n}\,4-l\,\text{n}\,2-\frac{1}{2}l\,\text{n}\,2=\left(2-\frac{3}{2}\right)l\,\text{n}\,2$$

$$=\frac{1}{2}l\,n2=\frac{A}{B}l\,n\,C$$

$\therefore$ Least sum A + B + C = 1 + 2 + 2 = 5

43. $\lim\limits_{x\to\infty}\left(\frac{|x|}{|x|+2}\right)^{-x}=\lim\limits_{x\to-\infty}\left(\frac{2-x-2}{2-x}\right)^{x}=\lim\limits_{x\to-\infty}\left(1-\frac{2}{2-x}\right)^{x}$

$$x\to-\infty \Rightarrow |x|=-x \qquad x=-\frac{1}{y},\, y\to 0$$

$$\underset{y\to 0}{Lt}\left(1-\frac{2}{2+\frac{1}{y}}\right)^{\frac{1}{y}}=\underset{y\to 0}{Lt}\left(1-\frac{y}{2y+1}\right)^{\frac{1}{y}},1\text{from }\underset{=e^{r\to 0}}{Lt}\frac{1}{y}\left(1-\frac{y}{2y+1}-1\right)\underset{=e^{r\to 0}}{Lt}\left(\frac{1}{2y+1}\right)_{=e^1}$$

44. $\ln\frac{a}{b}=\lim\limits_{n\to\infty}\frac{1}{n}\ln\left(\frac{(2n+1)(2n+2).....(2n+n)}{(n+1)(n+2)....(n+n)}\right)$

$$=\underset{n\to\infty}{Lim}\frac{1}{n}\sum_{r=1}^{n}\ln\left(\frac{2+\frac{r}{n}}{1+\frac{r}{n}}\right)=\int_0^1\ln\left(\frac{2+x}{1+x}\right)dx=\ln\frac{27}{16}$$

$a=27, b=16$

45. $f(x)=\frac{x^2}{2}+\cos x+1, x\geq 0$

$=-\frac{x^2}{2}+\cos x+1, x<0$ Clearly it is increasing and Hence a Bijection.

46.

47. $L=\underset{x\to 0}{Lt}\frac{a-\sqrt{a^2-x^2}\;\frac{-x^2}{4}}{x^4}=\underset{x\to 0}{Lt}\frac{\frac{1}{\sqrt{a^2-x^2}}-\frac{1}{2}}{4x^2}$, if exist, then a = 2 and L = $\frac{1}{64}$

48. $\underset{x\to\frac{\pi}{2}}{Lt}\left(2^{\frac{1}{\cos^2 x}}+3^{\frac{1}{\cos^2 x}}+4^{\frac{1}{\cos^2 x}}+5^{\frac{1}{\cos^2 x}}+6^{\frac{1}{\cos^2 x}}\right)^{2\cos^2 x}$

$\underset{x\to\frac{\pi}{2}}{Lt}\ 36\left(\left(\frac{1}{3}\right)^{\sec^2 x}+\left(\frac{1}{2}\right)^{\sec^2 x}+\left(\frac{2}{3}\right)^{\sec^2 x}\left(\frac{5}{6}\right)^{\sec^2 x}+1\right)^{2\cos^2 x}=36$

49. $-2\leq x-2\leq 5\Rightarrow 0\leq x\leq 7\Rightarrow \alpha=0,\beta=7$

$1\leq f(x-2)\leq 12\Rightarrow -3\geq -3f(x-2)\geq -36$

$\Rightarrow -32\leq g(x)\leq 1$

$\Rightarrow \gamma=-32,\delta=1$

50. $\frac{1}{n^2+n+1}>\frac{1}{n^2+n+2}>.....>\frac{1}{n^2+n+n}$

$\therefore \frac{1+2+3+.....+n}{n^2+n+n}<\sum_{r=1}^{n}\frac{r}{n^2+n+r}<\frac{1+2+3+.........+n}{n^2+n+1}$

$\frac{1}{2}<\lim_{n\to\infty}\sum_{r=1}^{n}\frac{r}{n^2+n+r}<\frac{1}{2}$

$\therefore \lim_{n\to\infty}\sum_{r=1}^{n}\frac{r}{n^2+n+r}=\frac{1}{2}$

51. Conceptual

52. $\cos(\sin x)\geq 0$ true for all $x\in R;\log_x\{x\}\geq 0\ for\ x\in(0,1)$

53. Clearly $f'(x)=3\ e^{3x^2-3x+2}(x^2-1)\geq 0\Rightarrow 'f'$ is increasing f(x) is one-one range of $f=(0,e^4]\neq(0,e^5]\Rightarrow 'f'$ is into

54. g(-x) = - g(x) & f(-x) = f(x) $\Rightarrow$ x^2f(-x) – 2f$\left(\frac{-1}{x}\right)$ =g(-x) $\Rightarrow$ g(x) = 0

$\therefore$ $x^2f(x) - 2f\left(\frac{1}{x}\right) = 0 \Rightarrow \frac{1}{x^2} f\left(\frac{1}{x}\right) - 2f(x) = 0 \Rightarrow f(x) = 0 \forall x \in R - \{0\}$

55. $\lim\limits_{x\to\infty} \frac{(2^{x^n})^{\frac{1}{e^x}} - (3^{x^n})^{\frac{1}{e^x}}}{x^n} = \lim\limits_{x\to\infty} \frac{(3)^{\frac{x^n}{e^x}}\left(\left(\frac{2}{3}\right)^{\frac{x^n}{e^x}} - 1\right)}{x^n}$ Now, $\lim\limits_{x\to\infty} \frac{x^n}{e^x} = \lim\limits_{x\to\infty} \frac{n!}{e^x} = 0$

Hence, $L = \lim\limits_{x\to\infty}(3)^{\frac{x^n}{e^x}} \lim\limits_{x\to\infty} \frac{\left(\left(\frac{2}{3}\right)^{\frac{x^n}{e^x}} - 1\right)}{\frac{x^n}{e^x}} \lim\limits_{x\to\infty} \frac{1}{e^x}$

=1×log (2/3) × 0 = 0

56. Now $\lim\limits_{x\to a_m^-} (A_1A_2...A_n) = (-1)^{n-m+1}$ and $\lim\limits_{x\to a_m^+} (A_1A_2...A_n) = (-1)^{n-m}$ Hence, $\lim\limits_{x\to a_m} (A_1A_2...A_n)$ does not exist

57. $h(x) = \begin{cases} e^x, & x > 0 \\ 0, & x = 0 \\ -e^{-x}, & x < 0 \end{cases}$ there fore h(x) is odd

58. $\frac{1 + \sqrt{(r^2-1)(r^2+2r+1-1)}}{r(r+1)} = \frac{1}{r}\cdot\frac{1}{r+1} + \sqrt{1 - \frac{1}{r^2}}\sqrt{1 - \frac{1}{(r+1)^2}}$

$K = \sum\limits_{r=2}^{n} \cos^{-1}\left(\frac{1 + \sqrt{(r-1)r(r+1)(r+2)}}{r(r+1)}\right) = \sum\limits_{r=2}^{n} \cos^{-1}\left(\frac{1}{r+1}\right) - \cos^{-1}\left(\frac{1}{r}\right)$

$\underset{n\to\infty}{Lt} K = \frac{\pi}{6}$

59.

60. As f(x) is one-one function, so f(x^3 + 14x^2 + 13x -5) = f(1-x^2+x^3)

$\Rightarrow x^2 + 14x^2 + 13x - 5 = 1x^2 + x^3 \Rightarrow 15x^2 + 13x - 6 = 0$

$\Rightarrow 15x^2 + 18x - 5x - 6 = 0 \Rightarrow (3x-1)(5x+6) = 0$

$\Rightarrow x = \frac{1}{3}, \frac{-6}{5}\, but\, x \in [-1,1]$

Hence $x = \frac{1}{3} only.]$