Sri Chaitanya IIT Academy, India

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A right Choice for the Real Aspirant

ICON CENTRAL OFFICE, MADHAPUR-HYD

Sec: Sr.IPLCO	JEE–ADVANCE	Date: 16-08-15
Time: 3 Hours	2012-P1-Model	Max Marks: 210

PAPER-I
KEY & SOLUTIONS

PHYSICS

1	**D**	2	**B**	3	**A**	4	**B**	5	**D**	6	**C**
7	**C**	8	**A**	9	**B**	10	**D**	11	**ABCD**	12	**AB**
13	**BD**	14	**ACD**	15	**AB**	16	**1**	17	**1**	18	**6**
19	**7**	20	**4**								

CHEMISTRY

21	**B**	22	**A**	23	**C**	24	**A**	25	**A**	26	**C**
27	**D**	28	**C**	29	**D**	30	**C**	31	**BD**	32	**ABCD**
33	**BD**	34	**BCD**	35	**BCD**	36	**6**	37	**5**	38	**4**
39	**4**	40	**2**								

MATHS

41	**D**	42	**B**	43	**D**	44	**B**	45	**C**	46	**B**
47	**B**	48	**C**	49	**C**	50	**D**	51	**A,B,C,D**	52	**A,C**
53	**B,C**	54	**A,B,C**	55	**A,B,C,D**	56	**8**	57	**0**	58	**1**
59	**4**	60	**2**								

MATHS

41.

Sol:- Let mid-point of chords be $(t^2, 2t)$ As it lies inside the given ellipse

$\therefore t^4 + 8t^2 - 1 < 0$

$t^2 \in (0, \sqrt{17} - 4)$

Equation of chord with mid-point $(t^2, 2t)$ for given ellipse is

$t^2 x + (2t) \times 2y = t^4 + 8t^2$

$\Rightarrow (a+1) = t^2 + 8$

$t^2 = a - 7$

$\therefore t^2 + 7 \in (0+7, \sqrt{17} - 4 + 7)$

42.

Sol:- Circle 'C' is $x^2 + y^2 + \lambda y - 16 = 0,$ it intersects the circle $x^2 + y^2 - 6x - 10y + 9 = 0$, orthogonally then $\lambda = \frac{7}{5}.$

Now $|r_1 - r_2| < C_1C_2 < r_1 + r_2$

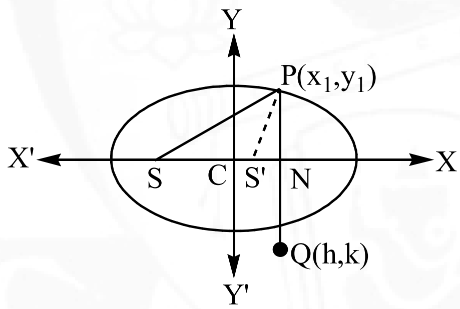
43. a – b > b

44. Eqn of chord $is\ \frac{x}{2\sqrt{6}} + \frac{y}{2} = 1$

45. Using reflection property. Area $= d\sqrt{a^2 - b^2}$. Where of d is $\perp^r$ dist from (0,0) to the chord

46.

Sol:-



$a^2 = 25$ and $b^2 = 16$

$\Rightarrow e = \sqrt{1 - \frac{16}{25}}$

$= \frac{3}{5}$

Let point Q be (h, k), where k < 0. Given that $k = SP = a + ex_1,$ where $P(x_1, y_1)$ lies on the ellipse

$\Rightarrow |k| = a + eh (as\ x_1 = h)$

$\Rightarrow -y = a + ex$

$\Rightarrow 3x + 5y + 25 = 0$

47. Write chord eqn. Homogenise & coeff of $x^2 + coeff$ of y^2 **= 0**

48. Conceptual

49. Ordinate of the point of intersection of the line $\frac{x}{a} - \frac{y}{b} = m$ and the hyperbola is given by

$\left(\frac{x}{a}-\frac{y}{b}\right)\left(\frac{x}{a}-\frac{y}{b}+\frac{2y}{b}\right)=1$ i.e. $m\left(m+\frac{2y}{b}\right)=1$ i.e. $y=\frac{b(1-m^2)}{2m}$

Similarly ordinate of the point of intersection of the line $\frac{x}{a}+\frac{y}{b}=m$ and the hyperbola is given by

$y=\frac{b(m^2-1)}{2m}$

$\therefore$ Sum of the ordinates is 0.

50. Equation of tangent is $hx+ky=h^2+k^2$ -------1

Also tangent at P is $\frac{x}{t}+yt=2c$ -------2

$\because$ (1) and (2) are identical eliminate t

Multiple correct answer type:

51. Conceptual

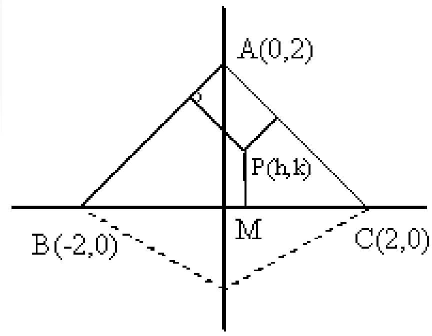
52.

Sol:- PM = K

Equation $AB \equiv -x+y=2$

Equation $AC \equiv x+y=2$

According



$$\left(\frac{2-h-k}{\sqrt{2}}\right)\left(\frac{2+h+k}{\sqrt{2}}\right)=2k^2$$

$$\Rightarrow h^2+3k^2+4k=4$$

$$\Rightarrow h^2+3\left(k^2+\frac{4}{3}k+\frac{4}{9}\right)=4+\frac{4}{3}$$

$$\Rightarrow h^2+3\left(k+\frac{2}{3}\right)^2=\frac{16}{3}$$

$$\Rightarrow \frac{h^2}{16/3}+\frac{\left(k+\frac{2}{3}\right)^2}{16/9}=1$$

$$\Rightarrow \text{ellipse with } e=\sqrt{\frac{2}{3}} \text{ and } D=\left(0,-\frac{2}{3}\right)$$

53.

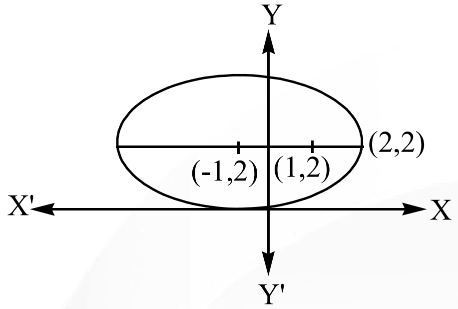
Sol:- $y^2 = 32x$ Let equation of tangent $y = mx + \frac{8}{m}$ $\frac{64}{m^2}=\frac{8}{9}m^2-\frac{8}{9}$ $m = \pm 3$, $y = \pm (3x + 8/3)$.

54. Sol:- The given equation is $\left(x-\frac{1}{13}\right)^2+\left(y-\frac{2}{13}\right)^2=\frac{1}{a^2}\left(\frac{5x+12y-1}{13}\right)^2$

It represents ellipse if $\frac{1}{a^2}<1 \Rightarrow a^2>1 \Rightarrow a>1$

$4x^2+8x+9y^2-36y=-4$

$\Rightarrow 4(x^2+2x+1)+9(y^2-4y+4)=36$



$\Rightarrow \frac{(x+1)^2}{9}+\frac{(y-2)^2}{4}=1$

Hence, (-1, 2) is focus and (1,2) lies on the major axis. Then required minimum distance is 1.

Equation of normal at $P(\theta)$ is 5 sec θ x $(-4\operatorname{cosec}\theta y=25-16,$ and it passes through $P(0,\alpha)$

$\therefore\ \alpha=\frac{-9}{4\operatorname{cosec}\theta}$

$\Rightarrow \alpha=\frac{-9}{4}\sin\theta$

$\Rightarrow |\alpha|<\frac{9}{4}$

$\frac{2b^2}{a}=\frac{2a}{3}\Rightarrow 3b^2=a^2$

$\Rightarrow$ from $b^2=a^2(1-e^2), 1=3(1-e^2)\Rightarrow e=\sqrt{2/3}$

55. Common normal must pass through (5,0). Not possible

Integer type:-

56.

Sol:- Let $P\left(ct_1,\frac{c}{t_1}\right)$ and $Q\left(ct_2,\frac{c}{t_2}\right)$ be any two point on $xy=c^2$. Then, tangents at P and Q are

$x+yt_1^2=2ct_1$ .....(i)

and $x+yt_2^2=2ct_2$ .....(ii)

The foot of the ordinate of P is (ct_1, 0) and it lies on Eq. (ii), then

$ct_1+0=2ct_2$

$\therefore$ $t_1=2t_2$

Then, from Eqs. (iii) and (iv),

$h=\frac{2c.2t_2.t_2}{2t_2+t_2}$

and $k=\frac{2c}{2t_2+t_2}$

$\therefore$ $h = \frac{4c}{3} t_2$

and $k = \frac{2c}{3t_2}$

$\therefore$ $h.k = \frac{4c}{3} t_2 \times \frac{2c}{3t_2}$

$\therefore$ $hk = \frac{8}{9} c^2$

$\therefore$ Locus of (h,k) is $xy = \frac{8}{9} c^2$,

$\therefore$ $\lambda = \frac{8}{9}$

Then $729\lambda = 648$

57. $\lambda \in (0,2) - \{1\}$

58. The equation of any tangent PQ to the ellipse $x^2/a^2 + y^2/b^2 = 1$ is $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$

This tangent cuts the ellipse $x^2/c^2 + y^2/d^2 = 1$ at the points P and Q.
Let the tangent at P and Q intersect at the point R(h,k).
Then PQ becomes the chord of contact w.r.t to the point R for the ellipse $x^2/c^2 + y^2/d^2 = 1$, i.e., the equation of PQ is

$\frac{hx}{c^2} + \frac{ky}{d^2} = 1$ .... (2) The equations (1) and (2) represent the same straight line. Therefore, $\frac{(\cos\theta)/a}{h/c^2} = \frac{(\sin\theta)/b}{k/d^2} = 1 \Rightarrow \cos\theta = \frac{ah}{c^2}$ and $\sin\theta = \frac{bh}{d^2}$ Squaring and adding, we get

$\frac{a^2h^2}{c^4} + \frac{b^2k^2}{d^4} = 1$ .... (3)

Which is the locus of the point R(h,k). If R(h, k) is the point of intersection of the two perpendicular tangents, then the locus of R should be the director circle of the ellipse

$x^2/c^2 + y^2/d^2 = 1, i.e., .x^2 + y^2 = c^2 + d^2$ $i.e., \frac{x^2}{c^2+d^2} + \frac{y^2}{c^2+d^2} = 1$

The equations (3) and (4) represent the same locus. Therefore,

$\frac{a^2}{c^4} = \frac{1}{c^2+d^2}$ and $\frac{b^2}{b^4} = \frac{1}{c^2+d^2}$ $\Rightarrow \frac{a^2}{c^2} + \frac{b^2}{d^2} = 1$

59. Foci of the ellipse $= (\pm\sqrt{7}, 0)$ there fore $r = \sqrt{7+9} = 4$

60. $2x + y = 1$ is tangent $\Rightarrow c^2 = a^2m^2 - b^2 \Rightarrow 1 = a^2 4 - b^2 \Rightarrow 1 = 4a^2 - a^2(e^2 - 1)$

$\Rightarrow 1 = 5a^2 - a^2e^2$ ------------ (1) also $2x + y = 1$ passes through $\left(\frac{a}{e}, 0\right) \Rightarrow 2\frac{a}{e} = 1$

$\Rightarrow a = \frac{e}{2}$ $\Rightarrow 1 = 5\frac{e^2}{4} - \frac{e^2}{4}e^2$ $e^4 - 5e^2 + 4 = 0 \Rightarrow e = 2$