Sri Chaitanya IIT Academy, India

A.P, TELANGANA, KARNATAKA, TAMILNADU, MAHARASHTRA, DELHI, RANCHI

A right Choice for the Real Aspirant

ICON CENTRAL OFFICE, MADHAPUR-HYD

Sec: Sr. IPLCO **Date: 07-11-15**

Time: 9:00 AM to 12:00 Noon **RPTM-11** **Max.Marks: 360**

KEY SHEET

CHEMISTRY		PHYSICS		MATHS	
Q.NO	ANSWER	Q.NO	ANSWER	Q.NO	ANSWER
1	1	31	3	61	3
2	1	32	1	62	2
3	1	33	2	63	4
4	1	34	1	64	4
5	1	35	2	65	3
6	3	36	2	66	3
7	3	37	3	67	3
8	4	38	1	68	3
9	3	39	1	69	4
10	1	40	3	70	2
11	2	41	2	71	3
12	1	42	4	72	3
13	4	43	1	73	2
14	1	44	4	74	3
15	3	45	2	75	4
16	4	46	1	76	2
17	3	47	3	77	2
18	1	48	3	78	1
19	4	49	2	79	3
20	3	50	3	80	2
21	4	51	2	81	4
22	4	52	2	82	4
23	4	53	4	83	3
24	2	54	4	84	3
25	2	55	2	85	4
26	2	56	1	86	3
27	4	57	1	87	3
28	4	58	3	88	2
29	4	59	2	89	4
30	2	60	2	90	3

MATHS

61. $f(x)=\int_0^x (x-t)\,dt+\int_x^1 (t-x)\,dt=-\left.\frac{(x-t)^2}{2}\right|_0^x+\left.\frac{(t-x)^2}{2}\right|_x^1=\frac{x^2}{2}+\frac{(1-x)^2}{2}=x^2-x+\frac{1}{2}$

$\int_0^1 f(x)\,dx=\int_0^1\left(x^2-x+\frac{1}{2}\right)dx=\frac{1}{3}$

62. $I=\int_{f(a)}^{f(b)} 2x\left(b-f^{-1}(x)\right)dx$

$f^{-1}(x)=t, x=f(t), dx=f'(t)\,dt\rightarrow I=\int_a^b (b-t)\left(2f(t)f'(t)\right)dt$

Integrating by parts,

$I=(b-t)f^2(t)\Big|_a^b+\int_a^b f^2(t)\,dt=-(b-a)f^2(a)+\int_a^b f^2(x)\,dx=\int_a^b\left(f^2(x)-f^2(a)\right)dx$

63. $f(x)=x^2+\int_0^x e^{-(x-t)}f(t)\,dt\rightarrow(1)$

$f'(x)=2x+f(x)-e^{-x}\int_0^x e^t f(t)\,dt\rightarrow(2)$

$(1)+(2)\rightarrow f'(x)+x^2+2x=\frac{x^3}{3}+x^2+c$ But $f(0)=0, c=0$

$f(1)=\frac{4}{3}$

64. $I=\int_0^{\frac{\pi}{4}}\frac{\sin x+\cos x}{4-(\sin x-\cos x)^2}=\frac{1}{2}.\frac{1}{2}\ln\left(\frac{2+\sin x-\cos x}{2-\sin x+\cos x}\right)\Bigg|_0^{\frac{\pi}{4}}=-\frac{1}{4}\ln\left(\frac{1}{3}\right)=\frac{1}{4}\ln 3$

65. $x=e^t, \int\frac{(1+t)^2 e^t\ dt}{1+(1+e^t)t+e^t t^2}=\log(1+te)_0^1=\log(1+e)$

66. $\tan\frac{x}{2}=t\rightarrow I=2\int_0^\infty\frac{1+t^2}{(t^2+9)^2}dt=$

$t=3\tan\theta\rightarrow I=\frac{2}{27}\int_0^{\frac{\pi}{2}}\left(\cos^2\theta+9\sin^2\theta\right)=\frac{2\times10}{27}\int_0^{\frac{\pi}{2}}\cos^2\theta\,d\theta=\frac{2\times10}{27}.\frac{\pi}{4}=\frac{10\pi}{54}=\frac{5\pi}{27}$

67. $t=\frac{1}{4}\int_{-1}^1\frac{(x+2)\,dx}{\left((x+1)^2+\frac{1}{2}\right)^2}$

Substitute $x+1=\frac{1}{\sqrt{2}}\tan\theta$

$=\frac{1}{2\sqrt{2}}\tan^{-1}\left(2\sqrt{2}\right)+\frac{1}{3}$

Since $\tan\theta=2\sqrt{2}$

$\Rightarrow\cos 2\theta=-\frac{7}{9}$ *and* $\sin 2\theta=\frac{4\sqrt{2}}{9}$

68. $$I=\int_0^{\frac{\pi}{2}}\frac{2\sin\frac{\pi}{6}dx}{\cos\left(x-\frac{\pi}{6}\right)\cos\left(x-\frac{\pi}{3}\right)}=2\int_0^{\frac{\pi}{2}}\frac{\sin\left(\left(x-\frac{\pi}{6}\right)-\left(x-\frac{\pi}{3}\right)\right)dx}{\cos\left(x-\frac{\pi}{6}\right)\cos\left(x-\frac{\pi}{3}\right)}=2\int_0^{\frac{\pi}{2}}\tan\left(x-\frac{\pi}{6}\right)\tan\left(x-\frac{\pi}{3}\right)$$

$$=2\ln\frac{\cos\left(x-\frac{\pi}{3}\right)}{\cos\left(x-\frac{\pi}{6}\right)}\Bigg|_0^{\frac{\pi}{2}}=4\ln\left(\frac{\cos\frac{\pi}{6}}{\cos\frac{\pi}{3}}\right)=4\ln\sqrt{3}=2\ln 3$$

69. $$I=\int_0^{\pi}\frac{xdx}{a^2\cos^2x+b^2\sin^2x},x\to\pi-x$$

$$I=\int_0^{\pi}\frac{(\pi-x)dx}{a^2\cos^2x+b^2\sin^2x}\Rightarrow 2I=\int_0^{\pi}\frac{dx}{a^2\cos^2x+b^2\sin^2x}=2\pi\int_0^{\frac{\pi}{2}}\frac{dx}{a^2\cos^2x+b^2\sin^2x}$$

$$I=\pi\int_0^{\frac{\pi}{2}}\frac{\sec^2xdx}{a^2+b^2\tan^2x}=\frac{\pi}{ab}\tan^{-1}\left(\frac{b}{a}\tan\theta\right)_0^{\frac{\pi}{2}}=\frac{\pi^2}{2ab}$$

70. $$t=\tan\frac{x}{2}\to I=2\int_0^1\frac{\left(3-t^2\right)dt}{\left(3+t^2\right)^2}$$

$$t=\sqrt{3}\tan\theta\to I=\frac{2}{\sqrt{3}}\int_0^{\frac{\pi}{6}}\cos 2\theta d\theta=\frac{2}{\sqrt{3}}\frac{\sin 2\theta}{2}\Bigg|_0^{\frac{\pi}{6}}=\frac{1}{2}$$

71. $$I=\int_0^{\frac{\pi}{4}}\frac{dx}{\left(\sin^2x+\cos^2x\right)^2-3\sin^2x\cos^2x}=\int_0^{\frac{\pi}{4}}\frac{4dx}{4-3\sin^2 2x},x=\tan 2x$$

$$2\int_0^{\infty}\frac{du}{u^2+4}=\tan^{-1}\frac{u}{2}\Bigg|_0^{\infty}=\frac{\pi}{2}$$

72. $$f'(x)=A(x+1)\left(x-\frac{1}{3}\right)=\frac{A}{3}\left(3x^2+2x-1\right)$$

$$f(x)=\frac{A}{3}\left(x^3+x^2-x\right)+B\Rightarrow 0=f(-2)\to B=\frac{2A}{3}$$

$$\therefore f(x)=\frac{A}{3}\left(x^3+x^2-x+2\right)$$

$$\frac{14}{3}=\int_{-1}^{1}f(x)dx=\frac{A}{3}\int_{-1}^{1}\left(x^3+x^2-x+2\right)dx=\frac{2A}{3}\int_0^1\left(x^2+2\right)dx=\frac{14A}{9}\to A=3$$

$$f(x)=x^3+x^2-x+2,f(1)=3$$

73. $f(x)=(1+a)\sin x$ where $a=\int_0^{\frac{\pi}{2}}f(t)\cos tdt=(1+a)\int_0^{\frac{\pi}{2}}\sin t\cos tdt=\frac{1+a}{2}\to a=1,g(x)=2\sin x$

$$\int_0^{\frac{\pi}{2}}f(x)dx=2\int_0^{\frac{\pi}{2}}\sin dx=2$$

74. $$f'(x)=f(x)\to f(x)=Ae^x$$

$f(0)=1 \to f(x)=e^x, g(x)=x^2-e^x$

$I=\int_0^1 e^x\left(x^2-e^x\right)dx=\int_0^1\left(x^2e^x-e^{2x}\right)dx=\left(x^2-2x+2\right)e^x-\left.\frac{e^{2x}}{2}\right|_0^1=e-2-\frac{e^2}{2}+\frac{1}{2}=e-\frac{e^2}{2}-\frac{3}{2}$

75. $5I_4+3I_2=\int_0^1 \tan^{-1}x.\left(5x^4+3x^2\right)dx=\left(x^5+x^3\right)\tan^{-1}x\Big|_0^1-\int_0^1\frac{x^5+x^3}{1+x^2}=\frac{\pi}{2}-\int_0^1 x^3dx=\frac{\pi}{2}-\frac{1}{4}$

76. $\dfrac{\sin\left(x+\frac{\pi}{4}\right)}{\cos x(1-\sin x)}=\frac{1}{\sqrt{2}}\sec^3 x(\sin x+\cos x)(1+\sin x)$c

$=\frac{1}{\sqrt{2}}\left(\sec^2 x+\sec x\tan^2 x\right)+\frac{1}{\sqrt{2}}(\sec x+1)\sec x\tan x$

$I=\left.\frac{e^{\sec x}.\tan x}{\sqrt{2}}\right|_0^{\frac{\pi}{4}}+\frac{1}{\sqrt{2}}\int_0^{\frac{\pi}{4}}e^{\sec x}(\sec+1)\sec x\tan x dx=\frac{1}{\sqrt{2}}e^{\sqrt{2}}+e^{\sqrt{2}}-\frac{e}{\sqrt{2}}=\left(1+\frac{1}{\sqrt{2}}\right)e^{\sqrt{2}}-\frac{e}{\sqrt{2}}$

77. Replacing $f(x)$ by x and x by $g(x)$, $x=\int_0^{g(x)}\frac{dt}{\sqrt{1+t^3}}$

Differentiating, $I=\frac{g'}{\sqrt{1+g^3}}\to(g')^2=1+g^3$

Differentiating again

$2g'g''=3g^2g'\to 2g''=3g^2$

78. $S_n<\lim_{n\to\infty}S_n=\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\frac{1}{1+\frac{k}{n}+\left(\frac{k}{n}\right)^2}=\frac{2}{\sqrt{3}}\tan^{-1}\left.\frac{2x+1}{\sqrt{3}}\right|_0^1=\frac{\pi}{3\sqrt{3}}$

79. $f'(x)=x\left(e^x-1\right)(x-1)(x-2)^3(x-3)^5$, $f(x)\to\infty$ as $x\to\infty$ Hence $x=3$, minimum, $x=2$, maximum , $x=1$, minimum $x\left(e^x-1\right)=x^2\left(1+\frac{x}{2}...\right)$

$\therefore x=0$ is an inflectional point

80. $L=\lim_{x\to\infty}\dfrac{\int_0^x e^{t^2}dt}{\frac{e^{x^2}}{1+x}}=\lim_{x\to\infty}\dfrac{1}{\frac{1}{\frac{2x}{1+x}}-\frac{1}{(1+x)^2}}=\frac{1}{2}$

81. $L=\lim_{n\to\infty}\dfrac{\left(\frac{1}{n}\sum_{r=1}^{n}\sqrt{\frac{r}{n}}\right)\left(\frac{1}{n}\sum_{r=1}^{n}\sqrt{\frac{n}{r}}\right)}{\frac{1}{n}\sum_{r=1}^{n}\frac{r}{n}}=\dfrac{\int_0^1\sqrt{x}dx.\int_0^1\frac{1}{\sqrt{x}}dx}{\int_0^1 xdx}=\frac{2}{3}.2.2=\frac{8}{3}$

82. $f(x)=\int_0^x e^{x-t}dt+\int_x^4 e^{t-x}dt=-e^{x-t}\Big|_0^x+e^{t-x}\Big|_x^4=e^x+e^{4-x}-2$

By symmetry, the minimum value of $f(x)$ is $f(2)=2\left(e^2-1\right)$

83. $\left[\tan^{-1}x\right]=\begin{cases}0, 0\le x\le\tan 1\\1, x>\tan 1\end{cases};\left[\cot^{-1}x\right]=\begin{cases}0, 0\le x\le\cot 1\\1, x>\cot 1\end{cases};$

The given expression becomes

$$\int_{\tan 1}^{2} 1dx + \int_{0}^{\cot 1} 1dx = 2 - \tan 1 + \cot 1 = 2 + \frac{2}{\tan 2} = 2(1 + \cot 2)$$

84. $\int_0^1 \sin^{-1}\left(2x\sqrt{1-x^2}\right)dx$; $x = \sin\theta$ f

$$= \int_0^{\frac{\pi}{2}} \sin^{-1}\sin 2\theta\cos\theta d\theta = \int_0^{\frac{\pi}{2}} 2\theta\cos\theta d\theta + \int_0^{\frac{\pi}{2}} (\pi - 2\theta)\cos\theta d\theta = \frac{\pi}{2\sqrt{2}} + \frac{2}{\sqrt{2}} - 2 - \frac{\pi}{2\sqrt{2} + \frac{2}{\sqrt{2}}} = 2\left(\sqrt{2}-1\right)$$

85. $\int_0^4 \left(x^2 - 2x + 1\right) f(x)dx = 3 - 4 + 1 = 0$

$\int_0^4 (x-1)^2 f(x)dx = 0$ If $f(x)$ is positive or negative in $[0,4]$, the integral can not be zero

$\therefore f(x)$ changes its sign

$\therefore f(x)$ has at least one root.

86. Let $g(x) = f(x) - x$, $g(0) = g(1) = 0$

$$\int_0^1 \left(f'(x)\right)^2 dx = \int_0^1 \left(1 + g'(x)\right)^2 dx \geq \int_0^1 \left(1 + 2g'(x)\right)dx = 1$$

87. $\int_0^1 \tan^{-1}\left(1 - x + x^2\right)dx = \int_0^1 \frac{\pi}{2}\cot^{-1}\left(1 - x + x^2\right)dx = \frac{\pi}{2} - 2\left[x\tan^{-1}x\Big|_0^1 - \int_0^1 \frac{x}{1+x^2}dx\right] = \ln 2$

88. $I = \int_{-\pi}^{\pi} \frac{2x\sin xdx}{1+\cos^2 x}$ dropping odd term

$I = 4\int_0^{\pi} \frac{x\sin xdx}{1+\cos^2 x}, x \to \pi - x$

$2I = 4\pi\int_0^{\pi} \frac{x\sin xdx}{1+\cos^2 x} = 8\pi\left(0 + \frac{\pi}{4}\right) = 2\pi^2 \to I = \pi^2$

89. $I_1 = \int_0^{\frac{\pi}{4}} f(\cos 2x)\cos xdx + \int_{\frac{\pi}{4}}^{\frac{\pi}{4}} f(\cos 2x)\cos xdx +$

Setting $x = \frac{\pi}{4} - t$ in the first integral and $x = \frac{\pi}{4} + t$ in the second interal,

$$I_1 = \int_0^{\frac{\pi}{4}} f(\sin 2t)\cos\left(\frac{\pi}{4} - t\right)dt + \int_0^{\frac{\pi}{4}} f(\sin 2t)\cos\left(\frac{\pi}{4} + t\right)dt = \sqrt{2}\int_0^{\frac{\pi}{4}} f(\sin 2t)\cos tdt = \sqrt{2}I_2$$

90. $\int \frac{\cos 9x + \cos 6x}{2\cos 5x - 1}dx = \int \frac{2\cos\frac{15x}{2} + \cos\frac{3x}{2}}{2\left(\cos^2\frac{5x}{2} - 1\right) - 1}dx$

By using $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

$\int 2\cos\frac{5x}{2}\cos\frac{3x}{2}dx = \int\left(\cos 4x + cosx\right)dx = \frac{1}{4}\sin 4x + \sin x + c$

$A + B = \frac{1}{4} + 1 = \frac{5}{4}$