Sri Chaitanya IIT Academy, India

A.P, TELANGANA, KARNATAKA, TAMILNADU, MAHARASHTRA, DELHI, RANCHI

A right Choice for the Real Aspirant

ICON CENTRAL OFFICE, MADHAPUR-HYD

Sec: Sr.IPLCO **Dt: 27-09-15**

Time: 09:00 AM to 12:00 Noon **RPTA-8** **Max.Marks: 180**

PAPER-1

KEY & SOLUTIONS

PHYSICS

1	**BD**	2	**BC**	3	**BCD**	4	**BD**	5	**CD**	6	**BC**
7	**ABCD**	8	**ABCD**	9	**BCD**	10	**CD**	11	**3**	12	**5**
13	**5**	14	**2**	15	**1**	16	**2**	17	**4**	18	**3**
19	**8**	20	**4**								

CHEMISTRY

21	**ABCD**	22	**AB**	23	**ABCD**	24	**AB**	25	**D**	26	**ABC**
27	**AB**	28	**AD**	29	**ACD**	30	**ABCD**	31	**1**	32	**1**
33	**3**	34	**6**	35	**4**	36	**0**	37	**6**	38	**2**
39	**2**	40	**6**								

MATHS

41	**CD**	42	**ABC**	43	**AC**	44	**CD**	45	**ACD**	46	**ACD**
47	**ACD**	48	**ABC**	49	**ABCD**	50	**ABD**	51	**4**	52	**7**
53	**1**	54	**2**	55	**8**	56	**6**	57	**2**	58	**1**
59	**5**	60	**9**								

SOLUTIONS
PHYSICS

1. In the frame of elevator

$$mg + ma - kx = m\frac{d^2x}{dt^2}$$

$$\Rightarrow \frac{d^2x}{dt^2} = -\frac{k}{m}\left[x - \frac{m(g+a)}{k}\right]$$

Or $\frac{d^2x}{dt^2} = -\frac{k}{m}\left[x - \frac{m(g+pt+a)}{k}\right]$

Three is a term involving t on R.H.S this does not represents S.H.M unless p = 0

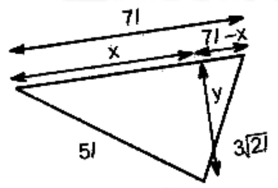
Differentiating again w.r.t time

$$\frac{d^3x}{dt^3} = -\frac{k}{m}\left[\frac{dx}{dt} - \frac{mp}{k}\right] \text{ or } \frac{d^2v}{dt^2} = -\frac{k}{m}\left[v - \frac{mp}{k}\right]$$

Thus the velocity of the block will vary simple harmonically.

2. $\Rightarrow y^2 = (5l)^2 - x^2$

$$= \left(3\sqrt{2}l\right)^2 - (7l - x)^2$$



$$\Rightarrow x = 4l \Rightarrow y = 3l$$

$$\Rightarrow T = 2\pi\sqrt{\frac{3l}{g}}$$

3. Acceleration $a = -\frac{1}{m}\frac{dU(x)}{dx}$

$$= \frac{-1}{2} \times 15(2x-2)$$

$$= -16(x-1)\, m/s^2$$

The particle executes S.H.M

$\omega^2 = 16$

$\Rightarrow t = \frac{2\pi}{\omega} = \frac{2\pi}{4} = \frac{\pi}{2} s$

At $x = 1m, F(x) = 0 (i.e)$ corresponds to equilibrium position

$\omega A = 2m/ \Rightarrow A = 0.5m$

The particle describes oscillatory motion from $x_1 = 0.5m \text{ to } x_2 = 1.5m$

4. The time period will change only when the additional of friction F. Hence the horizontal acceleration is always in the direction of F and its magnitude is $\frac{F}{m}$. The magnitude and the direction of F can thus be obtained from the magnitude and direction of the acceleration.

5. When a constant force is superimposed on a system undergoing S.H.M along the line of S.H.M the time period does not change. The mean position changes as this is the position where the net force on the particle is zero

6. $Y_1 = 6\left(\frac{\sqrt{3}}{2}\cos 3\pi t + \frac{1}{2}\sin 3\pi t\right)$

$= 6\sin\left(3\pi t + \frac{\pi}{2}\right)$

$\therefore A_1 = 6$

$V_1 = \left(\frac{dY_1}{dt}\right)_{\max} = (6)(3\pi) = 18\pi$

$Y_2 = 6\sin\left(6\pi t + \frac{\pi}{6}\right)$

$\therefore A_2 = 6$

And $V_2 = (6)(6\pi) = 36\pi$

$\therefore \frac{A_1}{A_2} = 1$

And $\frac{V_1}{V_2} = \frac{1}{2}$

7. $y = A\sin\frac{2\pi}{T}t$

$y\left(t = \frac{T}{8}\right) = A\sin\frac{\pi}{4} = \frac{4}{\sqrt{2}} (c) \text{ and } (d)$

$\frac{dy}{dt} = \frac{2\pi}{T}A\cos\frac{2\pi}{T}t$

$\Rightarrow \frac{dy}{dt}\left(t = \frac{T}{8}\right) = \frac{2\pi}{T}A\cos\frac{\pi}{t} = \frac{2\pi}{T}\frac{A}{\sqrt{2}} = \frac{v_{\max}}{\sqrt{2}}$

8. Poiseuile's equation can be written as

$$Q=\frac{p_1-p_2}{\left(\frac{8\eta L}{\pi R^4}\right)}=\frac{\Delta p}{X}$$

Where $x=\frac{8\eta L}{\pi R^4}$

This equation can be compared with the ohm's law equation $i=\frac{\Delta V}{R}$. Problem of series and parallel combination of pipes can be solved in the similar manner as done in case of an electrical circuit

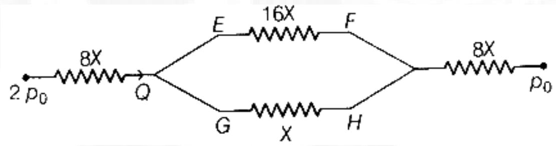
$$X=\frac{8\eta L}{\pi R^4}$$

$$\Rightarrow X\propto\frac{L}{R^4}$$

$$X_{AB}:X_{CD}:X_{EF}:X_{GH}$$

$$=\frac{\frac{1}{2}}{\left(\frac{1}{2}\right)^4}:\frac{\frac{1}{2}}{\left(\frac{1}{2}\right)^4}:\frac{1}{\left(\frac{1}{2}\right)^4}:\frac{1}{(1)^4}=8:8:16:1$$

So, the equivalent electrical circuit can be drawn as under



As, the current is distributed in the inverse ratio of the resistances in parallel. The Q will be distributed in the inverse ratio of X.

Thus, volume flow rate through EF will be $\frac{Q}{17}$ and that from GH will be $\frac{16}{17}Q$.

$$X_{net}=8X+\frac{(16X)(X)}{16X+X}+8X=\frac{288}{17}X$$

$$Q=\frac{\Delta p}{X_{net}}\qquad\left(same\ as,\ i=\frac{\Delta V}{R}\right)$$

$$Q=\frac{2p_0-p_0}{\frac{288}{17}X}=\frac{17p_0}{288X}$$

Let p_E be the pressure at E. Then

$$2p_0-p_E=Q(8X)\qquad(\Delta V=iR)$$

$$2p_0-p_E=\frac{17p_0}{288X}.8X=\frac{17p_0}{36}$$

$$\Rightarrow p_E = 2p_0 - \frac{17p_0}{36} = 1.53\,p_0$$

Let p_F be the pressure at F. Then $p_E - p_F = \frac{Q}{17}(16X)$

$$p_F = p_E - \frac{Q}{17}(16X) = p_E - \frac{16}{17}X\left(\frac{17p_0}{288X}\right)$$

$$= p_E - \frac{p_0}{18} = 1.53p_0 - 0.056p_0$$

$$\therefore p_F = 1.47\,p_0$$

9. Viscous force $= mg\sin\theta$ $\quad(\because F_{net} = 0)$

$$\eta a^2\frac{v}{t} = a^3\rho g\sin\theta \Rightarrow \eta = \frac{ta\rho g\sin\theta}{v}$$

10.

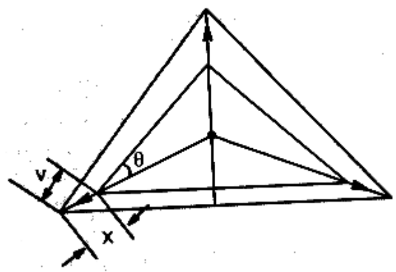
11. If x be the magnitude of the displacement of blocks

$y = x\cos\theta$

$\Rightarrow$ change in the length of each of the spring

$= 2y = 2x\cos\theta$

Net force $\vec{F}$ on each of the ball will have magnitude $4kx\cos\theta\cos\theta$



$\vec{F} = -\left(4k\cos^2\theta\right)\vec{x}$

Hence v (oscillation frequency)

$= \frac{1}{2\pi}\sqrt{\frac{4k\cos^2\theta}{m}}$

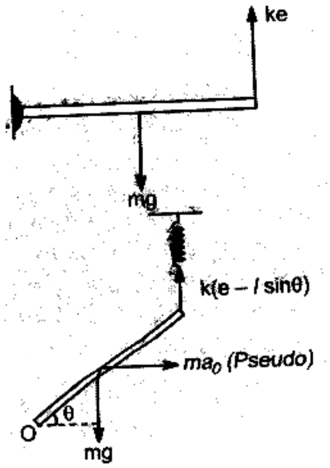
$= \frac{1}{2\pi}\sqrt{\frac{3k}{m}}\left(\theta = 30^\circ\right)$

12. In equilibrium, taking moments of the forces about O,

$mg\left(\ell/2\right) = kel$

$e = \left(mg/2k\right)$

$e \Rightarrow$ initial extension in the spring



During small oscillations, let the angular displacement be θ. In the reference of car, Restoring torque about O,

$$I_0\alpha = -\left[ma_0\left(\frac{l}{2}\sin gq\right)+mg\left(\frac{l}{2}\cos\theta\right)-k\left(e-l\sin\theta\right)l\cos\theta\right]$$

$$=-\left[ma_0\frac{l}{2}\theta+mg\frac{l}{2}\cos\theta-kel\cos\theta+kl^2\right]\therefore \sin\theta=\theta$$

$$\alpha=\frac{\left[ma_0\frac{l}{2}+kl^2\right]\theta}{ml^2/3}=\left(\frac{3a_0}{2l}+\frac{3k}{m}\right)\theta=\omega^2\theta$$

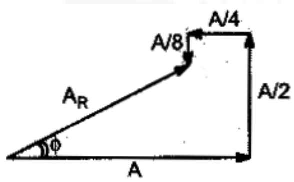
$$\Rightarrow T=\frac{2\pi}{\sqrt{\frac{3a_0}{2\ell}+\frac{3k}{m}}}$$

$$=2\pi\sqrt{\frac{2lm}{3ma_0+6kl}}$$ substituring values

$$T=\frac{2\pi}{\sqrt{3}\times 5}$$

13. From the vector diagram we can see that the resultant amplitude is

$$A_R=\sqrt{\left(A-\frac{A}{4}\right)^2+\left(\frac{A}{2}-\frac{A}{8}\right)^2}$$



$$=A\sqrt{\left(\frac{3}{4}\right)^2+\left(\frac{3}{8}\right)^2}$$

$$= \frac{3\sqrt{5}}{8} A$$

And $\tan\phi = \frac{(A/4)-(A/8)}{A-(A/4)} = \frac{1}{2}$

14. (a) Let A be the base area of the cone. In floating condition the cone is in equilibrium, i.e

Buoyant force = weight

$$\frac{1}{3}\left[A\left(\frac{h}{H}\right)^2\right]h\rho_w g = \left(\frac{1}{3}AH\right)\rho_w s g$$

Or

$$h = H(s)^{1/3}$$

For H =4m,

$$s = \frac{27}{64}$$

(b) when the cone is further, pushed by x, the restoring force is

$$F = A\left(\frac{h}{H}\right)^2 \rho_w g x = m\frac{d^2x}{dt^2}$$

$$= \frac{1}{3}AH\rho_w s\frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} + \left(\frac{4g}{H}\right)x = 0$$

Or

$$\omega = \sqrt{\frac{4g}{H}} = \frac{2\pi}{T}$$

Thus

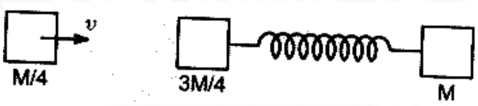
$$T = 2\pi\sqrt{\frac{H}{4g}}$$

On substituging numerical values;

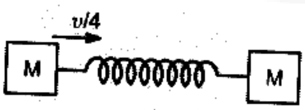
H = 4m; g = 10 m/s^2

We get T = 1.98 s

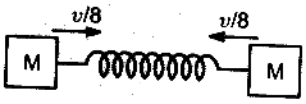
15.



After collision



In CM frame $v_{CM} = \frac{v}{8}$



Maximum compression of spring is given by

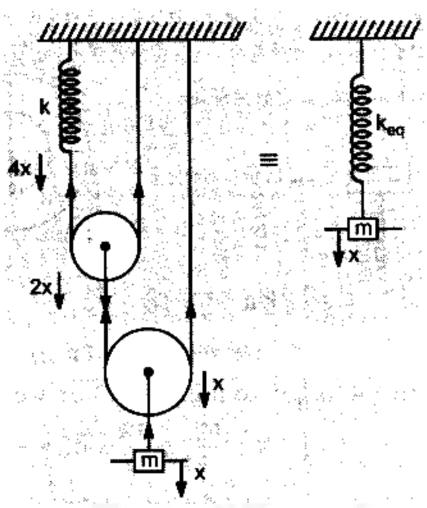
$$\frac{1}{2}kx_{\max}^2 = \frac{1}{2}\left(\frac{M}{2}\right)\left(\frac{v}{4}\right)^2$$

Or $x_{\max} = \frac{v}{4}\sqrt{\frac{M}{2k}}$

This is amplitude of oscillation and time period is given by $T = 2\pi\sqrt{\frac{\mu}{k}}$

$$= 2\pi\sqrt{\frac{M}{2k}}$$

16.



For same displacement two systems shown in figure are identical. Let block is displaced by x then spring will stretch by 4x. Thus force on block will be 16 kx.

Hence $16kx = k_{eq}x$

And $\omega = \sqrt{\frac{16k}{m}} = \frac{2\pi}{T} = 2\pi v$

$$v = \frac{1}{2\pi}\sqrt{\frac{16k}{m}}$$

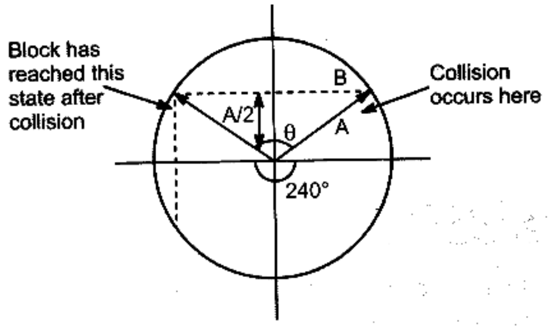
17. Time period $= \frac{2\pi}{\omega}$

Velocity $= \omega\sqrt{A^2 - x^2}$

$$= \frac{2\pi}{T}\sqrt{A^2 - x^2}$$

$$\cos\theta = \frac{A/2}{A} = 60^\circ$$

$\theta = \frac{\pi}{3}$



Particle has terned through $\frac{2\pi}{3}$. Thus, time t is $\frac{2\pi}{3} = \frac{2\pi}{T} \times t$

$t = \frac{T}{3}$

18. If P is the pressure difference between two ends of a capillary of length l and radius r and η be the coefficient of viscosity of liquid flowing through the capillary, then volume of liquid flowing through it per sec is

$V = \frac{\pi P r^4}{8\eta l}$ ……(1)

Given that : $V = 40 ml/\sec = 40 c.c./\sec$ *and* $P = \rho g h$ (as pressure difference = h cm of water)

$\therefore\ 40 = \frac{\pi \rho g h r^4}{8\eta l}$ …….(2)

Now, another capillary of same length and radius r/2 is connected is series with the first capillary hence, volume of liquid flowing per sec will be same through each capillary (= volume of liquid flowing/sec through combination) but pressure across the combination will be to sum of pressures across each tube i.e.,

$P_1 + P_2 = \rho g h$ ……….(3)

($\because$ combination is connected across the same pressure level)

And $V = \frac{\pi P_1 r^4}{8\eta l} = \frac{\pi P_2 (r/2)^4}{8\eta l}$ ………..(4)

Or $\frac{P_1}{P_2} = \frac{1}{16}$ ………..(5)

From (3) and (5), $P_1 = \frac{\rho gh}{17}$ *and* $P_2 = \frac{16\rho gh}{17}$

From equation (4), $V = \frac{\pi \rho gh r^4}{17 \times 8\eta l} = \frac{1}{17}\left(\frac{\pi \rho ghr^4}{8\eta l}\right) = \frac{40}{17}\, cc/\sec$

19. Given that $h = 2\times10^{-2}\, m, t = 1\ hr = 3600\sec, \rho = 1.8\times10^3\ kg/m^3, \sigma = 1\times10^3\ kg/m^3,$
$\eta = 10^{-2}\ Poise = 10^{-2}\ gm/cm/\sec = 10^{-3}\ kgm/m/\sec$

Terminal velocity of particles $v_T = \frac{2}{9}\frac{r^2(\rho-\sigma)g}{\eta}$

The particles remaining in suspension one hour later, will be those whose v_T is just

$\le \frac{h}{t} = \frac{2\times10^{-2}}{3600} = \frac{1}{18}\times10^{-4}\ m/s$.

Out of these particles, diameter will be largest for those particles whose terminal velocity is just equal to $\frac{1}{18}\times10^{-4}\ m/s$.

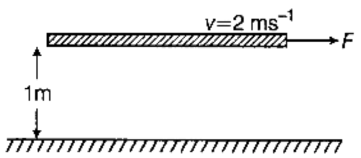
Hence $\frac{1}{18}\times10^{-4} = \frac{2}{9}\times\frac{r^2\times[1.8-1]\times10^3}{10^{-3}}\times10$

$\frac{1}{18}\times10^{-4} = \frac{2}{9}\times0.8\times10^7\times r^2$

$\therefore r^2 = \frac{9\times10^{-4}}{18\times2\times8\times10^6}$ *or* $4r^2 = \frac{1}{8}\times10^{-10}$ or $D = \frac{10^{-5}}{\sqrt{8}}m$

20. (4) Velocity gradient $= \frac{\Delta v}{\Delta y} = \frac{0.5-0}{1.25\times10^{-2}} = 40\, ms^{-1}\, m^{-1}$

$F = 2\left[\eta A\frac{\Delta v}{\Delta y}\right] \Rightarrow 1 = 2\times\eta\times0.5\times40$



$\Rightarrow \eta = \frac{1}{40}kg - sm^{-2}$

Or $\eta = 0.25\times10^{-1} = 25\times10^{-3}\ kgm^{-2}$

$\Rightarrow \beta = 5$