MATHS

31. The range of the parameter 'a' for which there exists a real number 'x' satisfying $\sqrt{1-x^2} \geq (a-x)$ is

1) $\left[-\sqrt{2}, \sqrt{2}\right]$ 2) $\left[0, \sqrt{2}\right]$ 3) $(-\infty, \sqrt{2}]$ 4) $(-\infty, -\sqrt{2}]$

32. If the largest positive value of the function defined as $f(x) = \sqrt{8x - x^2} - \sqrt{14x - x^2 - 48}$, is $m\sqrt{n}$ where $m, n \in N,\ (m \neq 1)$ then the value of (m + n) is

1) 5 2) 6 3) 7 4) 9

33. Which of the following pairs of functions have the same graph

1) $f(x) = \dfrac{|\sec x| + |\text{cosec } x|}{|\sec x||\text{cosec } x|}; g(x) = |\sin x| + |\cos x|$

2) $f(x) = \lim\limits_{n\to\infty} \dfrac{x^{2n}-1}{x^{2n}+1}; g(x) = \text{sgn}(1-|x|)$ (sgn x *denotes signum function*)

3) $f(x) = \sec^{-1} x; g(x) = e^{\ln(\sec^{-1} x)}$

4) $\forall x \in (2,3),\ f(x) = \cos^{-1}\sqrt{3-x},\ g(x) = \cot^{-1}\sqrt{\dfrac{3-x}{x-2}}$

34. $f: R \to [-1, \infty)$ and $f(x) = \ln([|\sin 2x| + |\cos 2x|])$ (where [.] is greatest integer function) then which of the following is not correct

1) $R^- \cap$ Range of f is null set

2) f(x) is periodic but fundamental period is not defined

3) f(x) is invertible in $\left[0, \frac{\pi}{4}\right]$

4) f(x) is into function

35. The solution set of $\ln|x| - |\ln x| + [x] < 2^x$, where $[.]$ denotes greatest integer, is

1) $[1, \infty)$ 2) All integers 3) $(0, \infty)$ 4) $[2, \infty)$

36. Let $R \to R$ be continuous and periodic with period T > 0. Then which of the following is always true

1) $f(x_0 + T/2) = f(x_0)$ for some $x_0 \in [k, k + T/2], K \in R$

2) $f(x_0 + T/2) = f(x_0)$ for some $x_0 \in (k, k + T/4), K \in R$

3) $f(x_0 + T/2) = f(x_0)$ for some $x_0 \in (k, k + T/3), K \in R$

4) $f(x_0 + T/2) = f(x_0)$ for some $x_0 \in (k, k + T/6), K \in R$

37. The equation $|2ax-3|+|ax+1|+|5-ax|=\frac{1}{2}$ possesses

1) Infinite number of real solutions for some $a \in R$

2) Finite number of real solutions for some $a \in R$

3) No real solution for some $a \in R$

4) No real solution for all $a \in R$

38. The function f : R→R satisfies the condition a f(x – 1) + b f(-x) = 2|x| + 1. If f(-2) = 5 and f(1) = 1, then (b – a) is

1) $\frac{1}{2}$ 2) $\frac{1}{4}$ 3) $\frac{3}{4}$ 4) $\frac{7}{4}$

39. If $\frac{5^m+3}{40}-\left[\frac{5^m+3}{40}\right]=\lambda \ (m \in N, m \geq 3)$ and [.] denote the G.I.F., then λ takes

1) two values 2) one value

3) infinite values 4) four values

40. $Let\, f(x) = -x^3 + x$ and $x \in (-\infty, -1] \cup [1, \infty)$ then the number of solutions of $f(x) = f^{-1}(x)\, is$

1) 0 2) 1 3) 2 4) 3

41. Let x_n be defined as $\left(1+\frac{1}{n}\right)^{n+x_n} = e$, then $\underset{n\to\infty}{Lim}\, x_n$ equals

1) 1 2) $\frac{1}{2}$ 3) $\frac{1}{e}$ 4) 0

42. The value of $\underset{n\to\infty}{Lim}\left(\frac{1}{2n+1}+\frac{1}{2n+3}+\frac{1}{2n+5}+.....+\frac{1}{4n-1}\right)$ is $\frac{a}{b}\ln c$ where a, b, c $\in$ N. Then the least value of a + b + c is

1) 5 2) 3 3) 1 4) 4

43. If $f(x) = \left(\frac{|x|}{|x|+2}\right)^{-x}$ then

1) $\lim_{x\to-\infty} f(x) = e^3$

2) $\lim_{x\to-\infty} f(x) = 0$

3) $\lim_{x\to 1} f(x) = \frac{1}{3}$

4) $\lim_{x\to\infty} f(x) = e^2$

44. If a,b are two positive co-prime integers such that $\lim_{n\to\infty}\left(\frac{^{3n}C_n}{^{2n}C_n}\right)^{\frac{1}{n}}=\frac{a}{b}$ then

1) a = 27 2) b = 13 3) a + b = 5 4) 2a = 3b

45. Let $f:R\to R$ be defined by $f(x)=\frac{x|x|}{2}+\cos x+1$ then $f(x)$ is.

1) One –one only 2) onto only

3) Neither one-one nor onto 4) bijection

46. $f:R\to R, f(x^2+x+3)+2f(x^2-3x+5)=6x^2-10x+17, \forall x\in R$ Then

1) $f(x)$ is an even function 2) $f(x)=0$ has a root in (0,1)

3) $f(x)$ is an odd function 4) $f(x)$ is invertible

47. Let $L=\underset{x\to 0}{Lt}\frac{a-\sqrt{a^2-x^2}-\frac{x^2}{4}}{x^4}, a\ge 0$. If L is finite, then $\frac{1}{8L}$ = is

1) 4 2) 8 3) 16 4) 32

48. $\underset{x\to\frac{\pi}{2}}{\mathrm{Lt}}\left(2^{\frac{1}{\cos^2 x}}+3^{\frac{1}{\cos^2 x}}+4^{\frac{1}{\cos^2 x}}+5^{\frac{1}{\cos^2 x}}+6^{\frac{1}{\cos^2 x}}\right)^{2\cos^2 x}$ is

1) 1 2) 6 3) 36 4) $\frac{1}{36}$

49. Suppose the domain of the function y = f(x) is $-2 \le x \le 5$ and the range is $1 \le y \le 12$.

Let g(x) = 4 – 3f (x – 2). If the domain of g(x) is $[\alpha,\beta]$ and the range of g(x) is $[\gamma,\delta]$, then which of the following relations(s) hold good?

1) $\alpha+\beta+\gamma+15=0$
2) $3\alpha+4\beta+\gamma+4\delta=12$
3) $\alpha+\beta+\gamma+\delta+24=0$
4) $5\beta+\gamma+\delta=14$

50. If $\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\frac{r}{n^2+n+r}=\frac{k}{6}$, then k is

1) 3
2) 4
3) 6
4) 2

51. X and Y are two sets and $f: X \to Y$. If $\{f(c)=y; c\subset X, y\subset Y\}$ and $\{f^{-1}(d)=x; d\subset Y, x\subset X\}$, then the true statement is

1) $f(f^{-1}(b))=b$
2) $f^{-1}(f(a))=a$
3) $f(f^{-1}(b))=b,\ b\subset y$
4) $f^{-1}(f(a))=a, a\subset x$

52. The domain of $f(x)=\sqrt{\cos(\sin x)}+\sqrt{\log_x\{x\}}$ where {.} is fractional part of x, is

1) $[1,\pi)$
2) $(0,2\pi)-[1,\pi)$
3) $\left(0,\frac{\pi}{2}\right)-\{1\}$
4) (0,1)

53. $f:(-\infty,-1]\to(0,e^5]$ defined by $f(x)=e^{x^3-3x+2}$ is

1) one-one and into
2) one-one and onto
3) many-one and into
4) many-one and onto

54. If f(x) is an even function and satisfies the relation $x^2f(x)-2f\left(\frac{1}{x}\right)=g(x)$, where g(x) is an odd function, then f(5) equals to

1) $\frac{50}{73}$
2) $\frac{49}{75}$
3) 0
4) $\frac{52}{37}$

55. The value of $\lim\limits_{x\to\infty}\dfrac{(2^{x^n})^{\frac{1}{e^x}}-(3^{x^n})^{\frac{1}{e^x}}}{x^n}$ (where $n\in N$) is

1) $\log n\left(\dfrac{2}{3}\right)$ 2) 0 3) $n\log n\left(\dfrac{2}{3}\right)$ 4) not defined

56. $A_i=\dfrac{x-a_i}{|x-a_i|}, i=1,2,\ldots,n$ and $a_1<a_2<a_3<\ldots<a_n$. If $a_m<a_1$(m < n) then the value of $\lim\limits_{x\to a_m}(A_1A_2\ldots A_n)$

1) is always 1
2) is always -1
3) does not exist
4) is $(-1)^{n-m}$

57. Let $f(x)=e^{\left\{e^{|x|}\operatorname{sgn}x\right\}}$ and $g(x)=e^{\left[e^{|x|}\operatorname{sgn}x\right]}, x\in R,$ where {.} and [.] denote the fractional and integral part functions, respectively. Also, $h(x)=\log(f(x))+\log(g(x))$. Then for real $x, h(x)$ is

1) an odd function
2) an even function
3) neither an odd nor an even function
4) both odd and even function

58. If $\underset{x\to\infty}{Lt}\sum_{r=2}^{n}\cos^{-1}\left(\frac{1+\sqrt{(r-1)(r)(r+1)(r+2)}}{r(r+1)}\right)=\frac{5\pi}{10\alpha}$ then α is

1) 1 2) 4 3) 3 4) 6

59. Let $f:R\to[\alpha,\infty), f(x)=x^2+3ax+b, g(x)=\sin^{-1}\frac{x}{4},(\alpha\in R)$ then which of the following is true

1) The number of possible integral values of 'a' for which $f(x)$ is many to one in [-3,5] is 4

2) If a = -1 and $gof(x)$ is defined for $x\in[-1,1]$ then number of possible integral values of 'b' can be 3.

3) If $a=2,\alpha=-8$ then the value of 'b' for which $f(x)$ is surjective is 2

4) If $a=1,b=2$ then exact number of integers in the range of $fog(x)$ is 3

60. Let $f:A\to B$ be an onto function defined as $f(x)=\frac{\sin^{-1}x+\tan^{-1}x}{\cos^{-1}x+\cot^{-1}x}$ then, the number of solutions of the equation $f(x^3+14x^2+13x-5)=f(1-x^2+x^3)$ is

1) 0 2) 1 3) 2 4) 3