Sri Chaitanya IIT Academy, India

A.P, TELANGANA, KARNATAKA, TAMILNADU, MAHARASHTRA, DELHI, RANCHI

A right Choice for the Real Aspirant

ICON CENTRAL OFFICE, MADHAPUR-HYD

Sec: Sr. IPLCO **Date: 14-11-15**

Time: 9:00 AM to 12:00 Noon **RPTM-12** **Max.Marks: 360**

KEY SHEET

PHYSICS		MATHS		CHEMISTRY	
Q.NO	ANSWER	Q.NO	ANSWER	Q.NO	ANSWER
1	2	31	1	61	2
2	3	32	2	62	4
3	1	33	4	63	4
4	1	34	1	64	2
5	2	35	3	65	1
6	1	36	1	66	4
7	3	37	1	67	1
8	2	38	3	68	4
9	1	39	2	69	2
10	4	40	4	70	3
11	1	41	2	71	2
12	3	42	4	72	4
13	2	43	3	73	2
14	1	44	2	74	3
15	3	45	2	75	3
16	3	46	2	76	3
17	2	47	1	77	1
18	3	48	2	78	2
19	2	49	4	79	3
20	1	50	2	80	3
21	3	51	4	81	2
22	4	52	4	82	3
23	1	53	3	83	1
24	4	54	4	84	4
25	1	55	1	85	3
26	2	56	3	86	4
27	3	57	3	87	2
28	1	58	1	88	4
29	1	59	2	89	3
30	1	60	3	90	1

MATHS

31. $$\frac{x\,dy - y\,dx}{x^2} = \left(x^2 + y^2\right) x dx$$

$$\frac{x\,dy - y\,dx}{x^2 + y^2} = x^3\,dx$$

$$\int \frac{d\left(\frac{y}{x}\right)}{1 + \left(y/x\right)^2} = \int x^3\,dx$$

$$\tan^{-1} y/x = \frac{x^4}{4} + c$$

32. $$x^2 y^2 dx + e^x y dx - e^x dy = 0$$

$$x^2\,dx + \frac{y\,d\left(e^x\right) - e^x.d\left(y\right)}{y^2} = 0$$

$$x^2 dx + d\left(\frac{e^x}{y}\right) = 0$$

$$\frac{x^3}{3} + \frac{e^x}{y} = c \Rightarrow x^3 y + 3e^x = 3cy$$

33. $$xdy + ydx + xy\left(xdy - ydx\right) = 0$$

$$\frac{d\left(xy\right)}{xy} + x\,dy - y\,dx = 0$$

$$\frac{d\left(xy\right)}{\left(xy\right)^2} + \frac{x^2}{xy}\left(\frac{xdy - ydx}{x^2}\right) = 0$$

$$\frac{d\left(xy\right)}{\left(xy\right)^2} + \frac{d\left(y/x\right)}{\left(y/x\right)} = c$$

$$\frac{-1}{xy} + \log_e\left(y/x\right) = c$$

34. $$\frac{1}{2}\frac{d\left(x^2 + y^2\right)}{\sqrt{a^2 - \left(x^2 + y^2\right)}} = \frac{xdy - ydx}{\sqrt{x^2 + y^2}}$$

$$\frac{1}{2}.\frac{d\left(x^2+y^2\right)}{\sqrt{x^2+y^2}\sqrt{a^2-\left(\sqrt{x^2+y^2}\right)}}=\frac{xdy-ydx}{x^2+y^2}$$

$$=\int\frac{d\left(\sqrt{x^2+y^2}\right)}{\sqrt{a^2-\left(\sqrt{x^2+y^2}\right)^2}}=\int\frac{d\left(y/x\right)}{1+\left(y/x\right)^2}$$

$$=\sin^{-1}\left(\frac{\sqrt{x^2+a^2}}{a}\right)=\tan^{-1}\left(y/x\right)+c$$

35. $$\frac{dx}{x}-\frac{dy}{y}+\frac{x^2dy-y^2dx}{\left(x-y\right)^2}=0$$

$$\frac{dx}{x}-\frac{dy}{y}+\frac{\frac{dy}{y^2}-\frac{dx}{x^2}}{\left(\frac{1}{y}-\frac{1}{x}\right)^2}=0$$

$$\int\frac{dx}{x}-\int\frac{dy}{y}+\int\frac{\frac{dy}{y^2}-\frac{dx}{x^2}}{\left(\frac{1}{y}-\frac{1}{x}\right)^2}dx$$

$$\log_e x-\log_e y+\frac{1}{\left(\frac{1}{y}-\frac{1}{x}\right)}=c$$

$$\log_e\left(x/y\right)+\frac{xy}{x-y}=c$$

36. $$\left(3x^2y^4+2xy\right)dx=\left(x^2-2x^3y^3\right)dy$$

$$\left(3x^2y^2+2\frac{x}{y}\right)dx=\left(\frac{x^2}{y^2}-2x^3y\right)dy$$

$$y^2.3x^2dx+2x^3.ydy+y.\frac{2x\,dx}{y^2}-\frac{x^2}{y^2}dy=0$$

$$d\left(x^3.y^2\right)+d\left(\frac{x^2}{y}\right)=0$$

$$\therefore x^3y^2+\frac{x^2}{y}=c$$

37. $d\left(\frac{\phi(x)}{y}\right)=dx$

$$\frac{\phi(x)}{y}=x+c$$

$$\phi(x)=y(x+c)$$

38. $\frac{dx}{dy}=x+y+1$

$$\frac{dx}{dy}-x=y+1$$

I.F = $e^{\int -1\,dy}=e^{-y}$

Sol is $x.e^{-y}=\int e^{-y}(y+1)dy$

$$x\,e^{-y}=-e^{-y}+\int y\,e^{-y}dy$$

$$x\,e^{-y}=-e^{-y}-y.e^{-y}+\int e^{-y}dy$$

$$x\,e^{-y}=-2\,e^{-y}-y\,e^{-y}+c$$

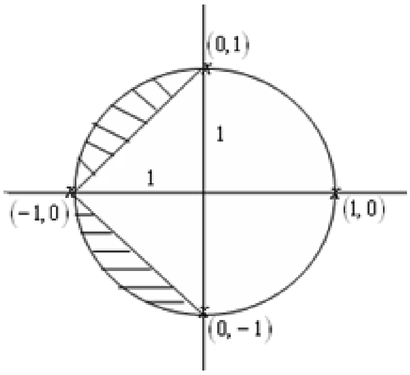
$$x=-2-y+ce^{y}$$

$$x=c.e^{y}-y-2$$

39. If $y>0\Rightarrow y=x+1$

If $y<0\Rightarrow -y=x+1$

$$y=-x-1$$



Required area = $2\left[\frac{\pi(1)^2}{4}-\frac{1}{2}\right]$

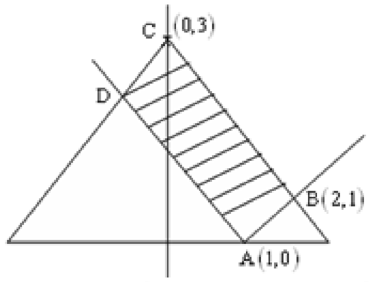
$=\frac{\pi}{2}-1$

40. Solve $|x-1|=3-|x|$

$x-1=3-x$

$2x=4$

$x=2$



$B=(2,1)$

$\therefore AB=\sqrt{1+1}=\sqrt{2}$

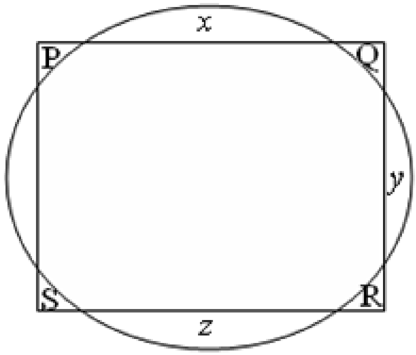
$BC=\sqrt{4+4}=\sqrt{8}$

Required area is Area of rectangle ABCD = $\sqrt{2}.\sqrt{8}.\sqrt{16}=4$

41. $r\rightarrow$ radius of circle $a\rightarrow$ sides of square

$x+y+z+w=k$ say

$p+q+r+s=k$



Now $\pi r^2 - k = ax^2 - k$

$\pi r^2 = a^2$

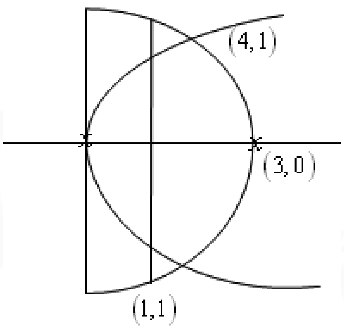
$\frac{r^2}{a^2} = \frac{1}{\pi}$

$\frac{r}{a} = \frac{1}{\sqrt{\pi}}$

42. $x = 3 - 2x \Rightarrow 3x = 3 \Rightarrow x = 1$

$\therefore y^2 = 1 \Rightarrow y = \pm 1$

Required area is $\int_{-1}^{+1} \left(3 - 2y^2 - y^2\right) dy$



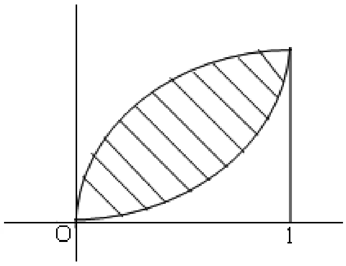
$2\int_0^1 \left(3 - 3y^2\right) dy$

$2\left[3y - y^3\right]_0^1 = \int -2 = (4)$

43. $y - \sin^{-1} x = \pm\sqrt{x - x^2}$ $\quad$ $x - x^2 \geq 0$

$y = \sin^{-1} x + \sqrt{x - x^2}$ $\quad$ $x(1 - x) \geq 0$

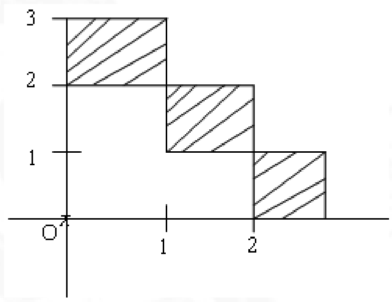
$y=\sin^{-1}x-\sqrt{x-x^2}$ $\qquad x(x-1)\le 0 \qquad 0\le x\le 1$



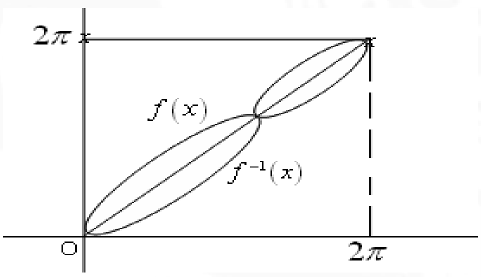
Required area $\int_0^1 2\sqrt{x-x^2}\,dx$

$=\frac{\pi}{4}$

44. Required $4\times$ Area in first quadrant $=4\times 3=12$ sq.units



45. Required area is $\int_D^{2\pi}(x+\sin x)dx=2\pi^2$ sq.units



Area bounded by $f(x)$ and $f^{-1}(x)$ is same on other respective Domains.

46. Let $y=f(x), f(0)=1$

$f'(x)=f(x)+\int_0^1 f(x)dx$ …………(1)

Differentiating. $f''(x)=f'(x)\Rightarrow\dfrac{f''(x)}{f'(x)}=1$

$\Rightarrow \log_e f'(x)=x+\log c$

$f'(x)=c.e^x$

$f(x)=c.e^x+d$

$f(0)=c+d=1$

$\therefore f(x)=c.e^x+1-c$

Substitute in (1) we get $c=\dfrac{2}{3-e}$ $\therefore f(x)=\dfrac{2\,e^x-e+1}{3-e}$

47. Differentiate the given equation twice.

$$\int_0^x y(t)\,dt+x\,y(x)=\int_0^x t\,y(t)\,dt+(x+1)xy(x)$$

$$y(x)+x.\frac{dy}{dx}+y(x)=x\,y(x)+(2x+1)y(x)+\left(x^2+x\right)y'(x)$$

$$2.y(x)=3x\,y(x)+y(x)+x^2y'(x)$$

$$\frac{y'(x)}{y(x)}=\frac{1}{x^2}-\frac{3}{x}$$

$$\log_e y(x)=\frac{-1}{x}-3\log_e c+\log_e c$$

$$\log_e(x)=-\frac{1}{x}+\log_e\left(c/x^3\right)$$

$$y(x)=e^{-1/x}.\frac{c}{x^3}$$

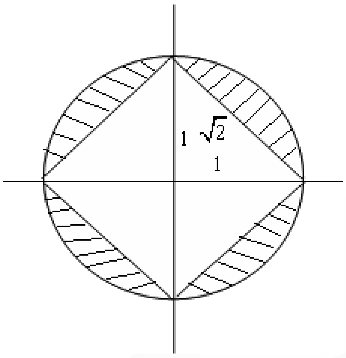
48. If $e^{\int\frac{2}{x}}=e^{\int\frac{2}{x}}=e^{2\log_e x}=x^2$

$$y.x^2=\int x^2.x\;\;dx$$

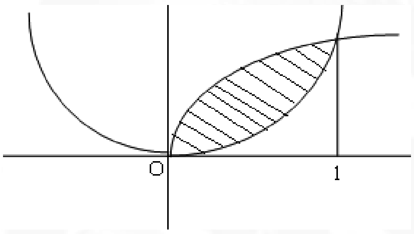
$$y.x^2=\frac{x^4}{4}+c$$

49. Area of circle – Area of square

$\pi - 2$

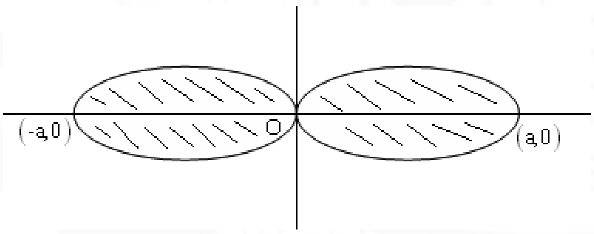


50. Required area = $\int_0^1 \left(\sqrt{x} - x^2\right) dx$



$$\left(\frac{x^{3/2}}{3/2} - \frac{x^3}{3}\right)_0^1 = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$$

51. Required area = $4\int_0^a x\sqrt{a^2 - x^2}\, dx$



Put $x = a\sin\theta$

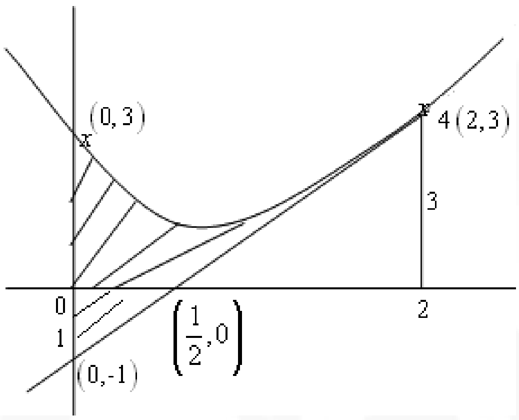
$$= 4\int_0^{\pi/2} a\sin\theta . a\cos\theta . a\cos\theta d\theta \qquad \cos\theta = t$$

$$= 4a^3 \int_0^{\pi/2} \sin\theta . \cos^2\theta d\theta \qquad \sin\theta d\theta = dt$$

$$4a^3\int_1^0 t^2(-dt) = 4a^3\int_0^1 t^2 dt = \frac{4a^3}{3}$$

52. $\frac{dy}{dx} = 2x-2$

$\left(\frac{dy}{dx}\right)_{m(2,3)} = 2$



Equation of target $y-3 = 2(x-2)$

$y = 2x-1$

Required area $= \left(\int_0^2 (x^2-2x+3)dx\right) - \frac{1}{2}\left(\frac{3}{2}\right)3 + \frac{1}{2}.\frac{1}{2}(1)$

$= \left(\frac{x^3}{3} - x^2 + 3x\right)_0^2 - \frac{9}{4} + \frac{1}{4}$

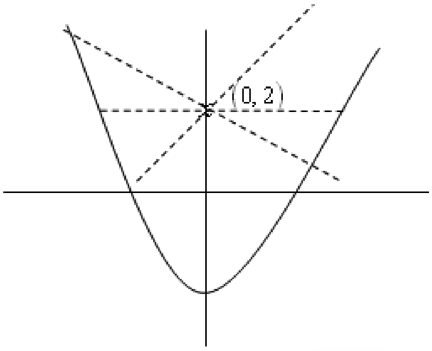
$= \frac{8}{3} - 4 + 6 - 2$

$= 8/3$ sq.units

53. The lines $y = ax+2$ are concurrent at $(0,2)$

Thus bounded area is minimum.

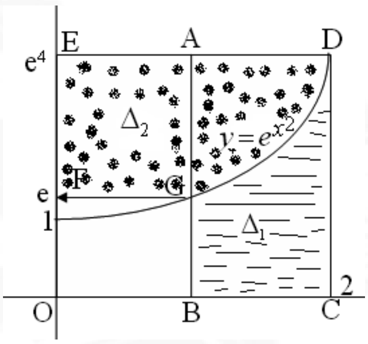
Iff $a = 0$, If a is either positive or negative area tend to increasing.



54. $\Delta_1 = \int_1^2 e^{x^2} dx = a \ldots\ldots\ldots(1)$

$\therefore \Delta_2 = (\text{Area of AGFE}) + (\text{Area of ABCD}) - \Delta_1$

$= (e^4 - e) + (2-1)e^4 - a$

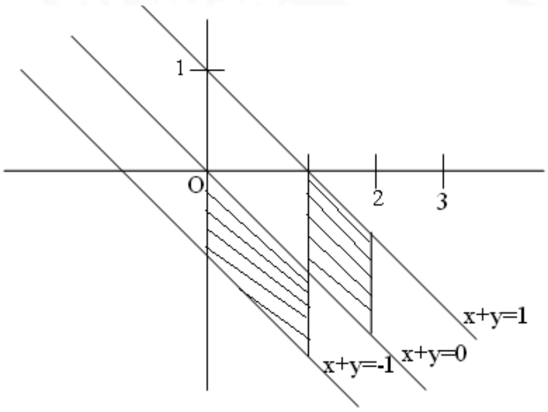


$= 2e^4 - e - a$

55. Shaded region is the region in which P – lies required area

$$= 4\left(\frac{1}{2}.1.\sqrt{2}\sin \pi/4\right)$$

$= 2$ sq.units



56. $xdy = ydx - 3\sqrt{y^2 - x^2}\,dx$.

$$3\sqrt{y^2 - x^2}\,dx = ydx - xdy$$

$$\frac{3x}{x^2}dx = \frac{ydx - xdy}{x^2\sqrt{\left(\frac{y}{x}\right)^2 - 1}}$$

$$\int \frac{3}{x}dx = -\int \frac{d(y/x)}{\sqrt{(y/x)^2 - 1}}$$

$$3\log_e x = -\cosh^{-1}(y/x) + \log c$$

$$\log_e x^3 = \log \frac{c}{\left(\frac{y + \sqrt{y^2 - x^2}}{x}\right)}$$

$$x^2\left(y + \sqrt{y^2 - x^2}\right) = c$$

57. Differentiating 2 times

58. $\frac{1}{y^3}\frac{dy}{dx} + \frac{1}{y^2}\left(\frac{1}{x}\right) = 1$

Put $\frac{1}{y^2} = z$

$$\frac{-2}{y^3}\frac{dy}{dx} = \frac{dz}{dx}$$

$$\frac{1}{y^3}\frac{dy}{dx} = \frac{-1}{2}\frac{dt}{dx}$$

$$\frac{-1}{2}\frac{dz}{dx} + \left(\frac{1}{x}\right)z = 1$$

$$\frac{dz}{dx} + \left(-\frac{2}{x}\right)z = (-2)$$

If $= e^{\int -\frac{2}{x}dx} = e^{-2\log_e x} = \frac{1}{x^2}$

$$z.\frac{1}{x^2}=\int\frac{1}{x^2}.(-2)dx$$

$$\frac{1}{x^2y^2}=\frac{2}{x}+c$$

$$\therefore \frac{1}{x}-\frac{1}{2x^2y^2}=c$$

59. $$2xydy=\left(x^2+1\right)dx+y^2dx$$

$$2xydy-y^2dx=\left(x^2+1\right)dx$$

$$\frac{2xy\,dy-y^2dx}{x^2}=\left(1+\frac{1}{x^2}\right)dx$$

$$d\left(\frac{y^2}{x}\right)=\left(1+\frac{1}{x^2}\right)dx$$

$$\frac{y^2}{x}=x-\frac{1}{x}+c$$

$$y^2=x^2-1+cx$$

$$y^2+1=x^2+cx$$

60. $$\left.\begin{array}{l} f''(x)=g''(x)\\ \Rightarrow f'(x)=g'(x)+c_1\\ f(x)=g(x)+c_1x+c_2\end{array}\right\}\text{from given condition } \mathrm{C}_1=-2;\,C_2=-2$$

$$f(4)-g(4)=4C_1+C_2=-10$$