MATHEMATICS: Max.Marks : 60

SECTION I
Single Correct Answer Type

This section contains **10 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

41. $\int\left(\sqrt{\cot x}\; e^{\sqrt{\sin x}}\; \sqrt{\cos x}\right)\left(1+e^{\sqrt{\sin x}}\right)dx =$

A) $\frac{\left(1+e^{\sqrt{\sin x}}\right)^2}{2}+c$ B) $2e^{\sqrt{\sin x}}+c$

C) $\frac{e^{\left(1+\sqrt{\sin x}\right)}}{2}+c$ D) $\left(1+e^{\sqrt{\sin x}}\right)^2+c$

42. $\int\frac{\sec x\left(2+\sec x\right)}{\left(1+2\sec x\right)^2}dx$

A) $\frac{\cos x}{\left(1+2\cos x\right)^2}+c$ B) $\frac{\sin x}{\left(2+\cos x\right)}+c$ C) $\frac{-\sin x}{\left(2+\cos x\right)^2}+c$ D) $\frac{\cos x}{2+\cos x}+c$

43. The value of $\int_0^{\infty}\frac{\ell nx}{x^2+2x+4}\,dx$ equals to

A) $\frac{\pi}{3\sqrt{3}}\ell n2$ B) $\frac{\pi}{\sqrt{3}}\ell n2$ C) $\frac{\pi}{2\sqrt{3}}$ D) $\frac{\pi}{2\sqrt{2}}\ell n2$

44. Consider a real valued continuous function f such that

$f(x)=\sin x+\int_{-\pi/2}^{\pi/2}\left(\sin x+tf(t)\right)dt$ then the difference of maximum, minimum value of f in $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$ is

A) $\pi+1$ B) $2(\pi+1)$ C) π D) cann't be determined

45. The value of $\int_0^{\infty}\frac{\tan^{-1}2016x-\tan^{-1}504x}{x}dx$

A) $\frac{\pi}{2}\ln 2$ B) $\pi\ln 2$ C) $\frac{\pi}{8}\ln 2$ D)0

46. The value of $\int_{-1}^{1}\frac{2x^{1007}+x^{3023}+4x^{2016}\left(\tan^{-1}x\right)^{2015}}{1+x^{2016}}dx$

A) 0 B) $\frac{1}{504}+\frac{\pi}{4}$ C) $\frac{1}{504}+\frac{\pi}{2016}$ D) $\frac{\pi}{504}+\frac{1}{4}$

47. $\lim_{n\to\infty}\left({}^{2n}C_n\right)^{\frac{1}{n}}=$

A) 1 B) 4 C) e^2 D) e^4

48. $p = 2016^{\left(\frac{\log_{10}\left(\frac{2015}{x}-1\right)}{-\log_{10}2016}\right)}$ then $\int \ln 10 \log_{10}^{(1/p)} dx =$

A) $(x-2015)\ln(2015-x)-x\ln x+c$ B) $(x-2015)\ln(2015-x)+x-x\ln x+c$

C) $(2015-x)\ln(2015-x)-x\ln x+c$ D) $(2015-x)\ln(2015-x)+x\ln x+c$

49. $\int \frac{(1+x)\left[(1-x+x^2)(1+x+x^2)+x^2\right]}{(1+2x+3x^2+4x^3+3x^4+2x^5+x^6)}dx = f(x)+c$, c is integration constant, f (0) = 0, then

$f(5) =$

A) 5 B) ln5 C) $\frac{1}{5}\ell n5$ D) ln6

50. Let f (x) = tan x + 2 tan 2x + 4 tan 4x + 8 cot 8x then primitive of f (x) w.r to x is

A) 8 ln |sin 8x| + c B) ln |sin8x| +c

C) ln |sin x | +c D) ln |sec x| +c

SECTION II

Multiple Correct Answer(s) Type

This section contains **5 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE are correct.**

51. If $\int \frac{\cos x - \sin x + 1 - x}{e^x + \sin x + x} dx = \log_e (f(x)) + g(x) + c$ where c is integration constant and f(x) is positive $f^1(0) = 2, g(0) = 0$ then

A) f (x) + g (x) is an even function

B) f (x) - g (x) is an increasing function

C) $|f(0) + g(2)| = 1$

D) If a function h(x) is such that h (x) + e^x = f(x) +g (x) then maximum value of h (x) is 1

52. Consider the function $f(x) = \begin{cases} \int_0^x (4 + |t-2|)\, dt & x > 3 \\ ax^2 + bx & x \le 3 \end{cases}$ and if f(x) is differentiable at x = 3 then

A) 27 ab = 1 B) f (1) = 85/18 C) f (4) = 20 D) $\int_4^6 (f^1(x) + 2)\, dx = 18$

53. Let f (x), g(x) be any two continuous functions, a is non zero real number b is positive real number but not equal to 1 then the value of $\int_{-a}^{a} \frac{f(x)}{b^{g(x)}+1} dx$ is

A) independent of f , if f even , g odd

B) independent of g, if f even , g odd

C) independent of f, g if both f, g even

D) independent of f,g if both f, g odd

54. Let f : $[0, \infty) \to R$ be a continuous strictly increasing function, such that $f^3(x) = \int_0^x t\ f^2(t)\,dt,\ \forall x \geq 0$, then which of the following is / are true

A) f (x) is an onto function

B) $\int_0^1 f(x)\,dx = \frac{1}{18}$

C) Number of solutions of 6f(x) = e^x is 2

D) The graph of y = 6 f(x) intersects the graph of the curve $y = 3x^2 + 2x + 2$ at one point

55. Let $I_1 = \int \frac{\sin^2 x + \sin x}{1 + \sin x + \cos x} dx$, $I_2 = \int \frac{\cos^2 x + \cos x}{1 + \sin x + \cos x} dx$ and if c is any arbitrary constant, then which of the following is/are correct

A) $2I_1 + \sin x + \cos x - x = c$ B) $I_1 - I_2 + \sin x + \cos x = c$

C) $I_1 + I_2 - x = c$ D) $2I_2 + \sin x - x - \cos x = c$

SECTION III

Integer Answer Type

This section contains **5 questions.** The answer to each question is single digit integer, ranging from 0 to 9 (*both inclusive*).

56. $\underset{n\to\infty}{Lt} \frac{3}{n}\left(1 + \sqrt{\frac{n}{n+3}} + \sqrt{\frac{n}{n+6}} + \sqrt{\frac{n}{n+9}} + \ldots\ldots\ldots + \sqrt{\frac{n}{4n-3}}\right) =$

57. If $\int \frac{1}{5\cos^2 x - 4\cos x \sin x - 2\sin^2 x} dx = \frac{1}{2\sqrt{14}} \ln \left| \frac{k + \tan x + 1}{k - \tan x - 1} \right| + c$ then integer part of k^2 is

58. If $\left(\int \cot^2 x \operatorname{cosec}^4 x \, dx\right) = a\cot^3 x + b\cot^5 x + c$ then $\frac{1}{|a|} + \frac{1}{|b|}$ is equal to

59. If $\int_{\sin x}^{1} t^2 f(t) dt = 1 - \sin x$ where $x \in \left(0, \frac{\pi}{2}\right)$ then $f\left(\frac{1}{\sqrt{2}}\right)$ is equal to

60. If $\int_2^8 \frac{dx}{x\sqrt{(x-2)(8-x)}} dx = \frac{\pi}{k}$ then the absolute value of k is