# Sri Chaitanya IIT Academy, India

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A right Choice for the Real Aspirant

ICON CENTRAL OFFICE, MADHAPUR-HYD

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| Sec: Sr. IPLCO | JEE ADVANCED | DATE : 13-12-15 |
| TIME : 02:00 PM TO 05: 00 PM | 2012_P2 MODEL | MAX MARKS : 198 |

## KEY & SOLUTIONS

## PHYSICS

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | B | 2 | C | 3 | A | 4 | B | 5 | D | 6 | D |
| 7 | B | 8 | A | 9 | B | 10 | D | 11 | B | 12 | A |
| 13 | A | 14 | C | 15 | BD | 16 | AC | 17 | ABC | 18 | CD |
| 19 | BC | 20 | ABC | | | | | | | | |

## CHEMISTRY

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 21 | A | 22 | C | 23 | A | 24 | C | 25 | B | 26 | B |
| 27 | A | 28 | C | 29 | A | 30 | C | 31 | A | 32 | B |
| 33 | C | 34 | A | 35 | BC | 36 | ABC | 37 | ABC | 38 | ABC |
| 39 | BCD | 40 | CD | | | | | | | | |

## MATHEMATICS

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 41 | A | 42 | C | 43 | A | 44 | A | 45 | C | 46 | B |
| 47 | B | 48 | B | 49 | D | 50 | B | 51 | B | 52 | D |
| 53 | B | 54 | D | 55 | BCD | 56 | AB | 57 | AD | 58 | BCD |
| 59 | BC | 60 | ABCD | | | | | | | | |

# MATHS

43. (a)

$$\frac{\pi}{2}-\cos^{-1}\cos\left(2-\underbrace{\frac{2}{x^2+5|x|+3}}_{0<\downarrow<2}\right)=\cot\cot^{-1}\left(\frac{2}{9|x|}-2\right)+\frac{\pi}{2}$$

$$\frac{\pi}{2}-2+\frac{2}{x^2+5|x|+3}=\frac{2}{9|x|}-2+\frac{\pi}{2}$$

$$\Rightarrow |x|^2-4|x|+3=0$$

$$|x|=1,3\Rightarrow x=\pm1,\pm3$$

44. $\angle B\neq\frac{\pi}{2}$ as $4\sin A\cos B=1$ & $\angle A\neq\frac{\pi}{2}$ as $\tan A$ is real $\Rightarrow \angle C=\frac{\pi}{2}$

There fore angles are in the ratio 1:2:3

R=1.

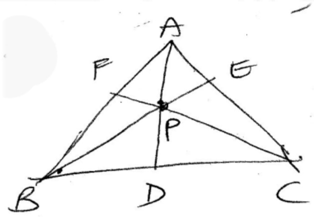
45. Using menelau's theorem in $\Delta ADC$

$$\frac{AP}{PD}\times\frac{-DB}{CB}\times\frac{CE}{EA}=-1 \qquad -(1)$$

Using menelau's theorem in $\Delta ABD$

$$\frac{AF}{FB}\times-\frac{BC}{DC}\times\frac{DP}{AP}=-1 \qquad -(2)$$

From (1), (2) $\frac{AF}{FB}+\frac{AE}{EC}=+\frac{DC}{BC}\times\frac{AP}{DP}+\frac{AP}{PD}\times\frac{BD}{BC}$




$$=+\frac{AP}{PD}\left(\frac{DC}{BC}+\frac{BD}{BC}\right)=+\frac{AP}{PD}\times1$$

$$\therefore\frac{AF}{FB}+\frac{AE}{EC}-\frac{AP}{PD}=0$$

46. $\cos1-\cos^{-1}1=0,\ \cot1-\cot^{-1}1=1$

$\sin1-\sin^{-1}1=1,\ \sec1-\sec^{-1}1=0$

$\tan 1 - \tan^{-1} 1 = 0$, $\cos ec 1 - \cos ec^{-1} 1 = 1$

**49&50.** (1) $\frac{1}{(a+x)}\frac{1}{(a+y)} = \frac{1}{2a^2+(x+y)a+1} < 1$

$$Tan^{-1}\frac{1}{(a+x)} + Tan^{-1}\frac{1}{(a+y)} = Tan^{-1}\frac{1}{a}$$

using idea of above problem

$$5Tan^{-1}\frac{1}{8} + 2Tan^{-1}\frac{1}{18} + 3Tan^{-1}\frac{1}{57}$$

$$= \quad 3\left(Tan^{-1}\frac{1}{1+7} + Tan^{-1}\frac{1}{7+50}\right) + 2\left(Tan^{-1}\frac{1}{8} + Tan^{-1}\frac{1}{18}\right)$$

$$= \quad 3Tan^{-1}\frac{1}{7} + 2\left(Tan^{-1}\frac{1}{8} + Tan^{-1}\frac{1}{18}\right)$$

$$= \quad 2\left(Tan^{-1}\frac{1}{5+2} + Tan^{-1}\frac{1}{5+13}\right) + Tan^{-1}\frac{1}{7} + 2Tan^{-1}\frac{1}{8}$$

$$= \quad 2\left(Tan^{-1}\frac{1}{3+2} + Tan^{-1}\frac{1}{3+5}\right) + Tan^{-1}\frac{1}{7} \quad = \quad 2Tan^{-1}\frac{1}{3} + Tan\frac{1}{7}$$

$$= 2Tan^{-1}\frac{1}{3} + Tan^{-1}\frac{1}{3} + Tan^{-1}\frac{1}{7}$$

$$= Tan^{-1}\frac{1}{3} + Tan^{-1}\frac{1}{1+2} + Tan^{-1}\frac{1}{2+5}$$

$$= Tan^{-1}\frac{1}{3} + Tan^{-1}\frac{1}{2}$$

$$= Tan^{-1}\frac{1}{1+2} + Tan^{-1}\frac{1}{1+1}$$

$$= Tan^{-1}(1) = \frac{\pi}{4}$$

51) Ans: (B)

Sol: $\bar{A}C = \theta = AB$

$CD = \sin\theta \; and \; OD = \cos\theta$

Therefore $AD = 1 - \cos\theta$

Therefore area of trapezoid

$$ABCD = \frac{1}{2}(AB + CD) \times AD$$

$$= \frac{(\theta + \sin\theta)(1-\cos\theta)}{2} = (\theta + \sin\theta)\sin^2\frac{\theta}{2}$$

52 Ans: (D)

Sol: $\Delta ABQ$ and $DCQ$ are similar, so

$$\frac{AB}{CD}=\frac{AQ}{DQ}=\frac{AQ}{AQ-AD}\Rightarrow\frac{\theta}{\sin\theta}=\frac{AQ}{AQ-(1-\cos\theta)}$$

or, $AQ=\dfrac{\theta(1-\cos\theta)}{\theta-\sin\theta}$

$$\therefore \lim_{\theta\to 0^+} AQ=\lim_{\theta\to 0^+}\frac{\theta(1-\cos\theta)}{\theta-\sin\theta}=\lim_{\theta\to 0^+}\frac{1-\cos\theta+\sin\theta}{1-\cos\theta}$$

$$=\lim_{\theta\to 0^+}\frac{2\sin\theta+\theta\cos\theta}{\sin\theta}=3$$

53. Let $\cot\frac{A}{2}=x$

$\cot\frac{B}{2}=y$

$\cot\frac{C}{2}=z$

$x=\cot\frac{A}{2}=\frac{s-a}{r}$

$y=\cot\frac{B}{2}=\frac{s-b}{r}$

$z=\cot\frac{C}{2}=\frac{s-c}{r}$

$x+y+z=\frac{3s-(a+b+c)}{r}=\frac{s}{r}$

$$\therefore\quad x^2+(2y)^2+(3z)^2=\left[\frac{6}{7}\cdot(x+y+z)\right]^2$$

or, $13x^2+160y^2+405z^2-72(xy+yz+zx)=0$

$\Rightarrow (3x-12y)^2+(4y-9z)^2+(18z-2x)^2=0$

$\Rightarrow 3x=12y,\ 4y=9z,\ 18z=2x$

$\Rightarrow x=4y,\ y=\frac{9}{4}z,\ x=9z$

$x:y:z=9z:\frac{9}{4}z:z=9:\frac{9}{4}:1=36:9:4$

$\frac{x}{36}=\frac{y}{9}=\frac{z}{4}\Rightarrow\frac{s-a}{36}=\frac{s-b}{9}=\frac{s-c}{4}.\Rightarrow\frac{a}{13}=\frac{b}{40}=\frac{c}{45}.$

54. $\dfrac{\cot\frac{A}{2}}{\cot\frac{B}{2}}+\dfrac{\cot\frac{B}{2}}{\cot\frac{C}{2}}+\dfrac{\cot\frac{C}{2}}{\cot\frac{A}{2}}$

$=\dfrac{x}{y}+\dfrac{y}{z}+\dfrac{z}{x}$

$=4+\dfrac{9}{4}+\dfrac{1}{9}$

$=\dfrac{144+81+4}{36}=\dfrac{229}{36}$

57: $\Delta=\dfrac{1}{2}ah_1 \Rightarrow a=\dfrac{2\Delta}{h_1}, b=\dfrac{2\Delta}{h_2}, c=\dfrac{2\Delta}{h_3}$

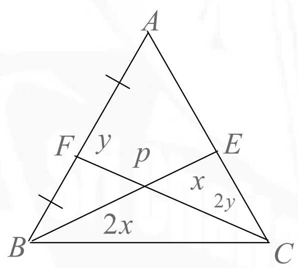
$\Delta=\dfrac{1}{2}ap_1+\dfrac{1}{2}bp_2+\dfrac{1}{2}cp_3$

$=\Delta\left(\dfrac{p_1}{h_1}+\dfrac{p_2}{h_2}+\dfrac{p_3}{h_3}\right)$

$\dfrac{p_1}{h_1}+\dfrac{p_2}{h_2}+\dfrac{p_3}{h_3}=1$

Since r = 1, some circles touch BC and some circles meets no side

58: $\tan B\ \dfrac{\frac{y}{2x}+\frac{y}{x}}{1-\frac{y^2}{2x^2}}=\dfrac{3xy}{2x^2-y^2}$




$\tan\angle PBF=\dfrac{y}{2x}$

$\tan\angle PBC=\dfrac{y}{x}$

$\cot B=\dfrac{2x^2-y^2}{3xy},\ \cot C=\dfrac{2y^2-x^2}{3xy}$

$\cot B=\cot C=\dfrac{x^2+y^2}{3xy}\geq\dfrac{2}{3}$

59. If $h_1, h_2, h_3$ are the altitudes drawn to the sides $a.b$ & $c$, then

$\frac{1}{4} = \frac{1}{h_1} = \frac{a}{2\Delta}$ and $\frac{1}{12} = \frac{1}{h_2} = \frac{b}{2\Delta}$.

So, $a > b$ and $\frac{1}{p} = \frac{c}{2\Delta} > \frac{a-b}{2\Delta} = \frac{1}{6}$. Also, $\frac{1}{p} < \frac{a+b}{2\Delta} = \frac{1}{3}$. $\therefore\ 3 < p < 6$

60. Let $ABC$ be the triangle in which $AB = AC$. Let $I, P$ respectively be the incentre and the ortho–centre of the triangle.

$AI = r \operatorname{cosec} \frac{A}{2}$, $AP = 2R\cos A$

$r \operatorname{cosec} \frac{A}{2} = 2R\cos A + r$