



Sri Chaitanya IIT Academy, India

A.P, TELANGANA, KARNATAKA, TAMILNADU, MAHARASHTRA, DELHI, RANCHI

A right Choice for the Real Aspirant
ICON CENTRAL OFFICE, MADHAPUR-HYD

Sec: Sr. IPLCO

Date: 01-08-15

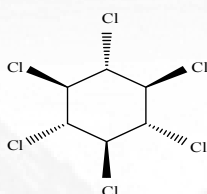
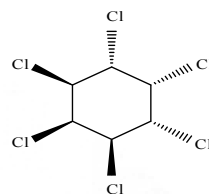
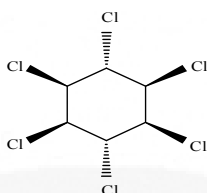
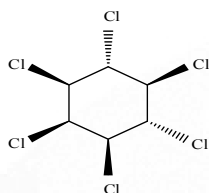
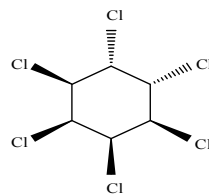
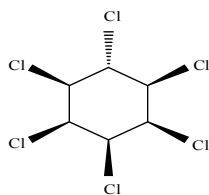
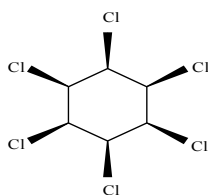
Time: 9:00 AM to 12:00 Noon

RPTM-1

Max.Marks: 360

KEY SHEET

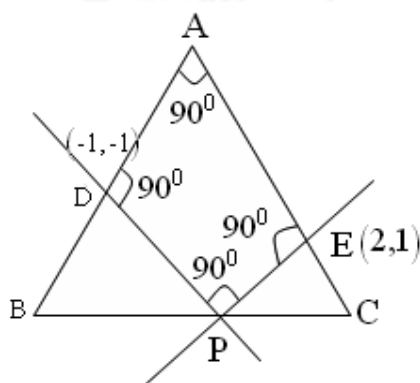
PHYSICS		CHEMISTRY		MATHS	
Q.NO	ANSWER	Q.NO	ANSWER	Q.NO	ANSWER
1	1	31	3	61	3
2	2	32	4	62	2
3	3	33	1	63	4
4	1	34	2	64	2
5	2	35	3	65	4
6	1	36	4	66	2
7	4	37	3	67	3
8	1	38	3	68	4
9	2	39	3	69	3
10	3	40	4	70	3
11	4	41	3	71	2
12	2	42	2	72	3
13	4	43	3	73	3
14	2	44	3	74	1
15	3	45	2	75	2
16	1	46	4	76	2
17	2	47	1	77	3
18	2	48	1	78	3
19	2	49	2	79	3
20	3	50	4	80	2
21	2	51	3	81	2
22	3	52	1	82	4
23	4	53	2	83	4
24	1	54	4	84	1
25	3	55	1	85	3
26	3	56	3	86	3
27	4	57	4	87	3
28	4	58	4	88	3
29	1	59	3	89	3
30	3	60	4	90	2



MATHS- HINTS

- 61) Clearly $x + y + 2 = 0$,
 $x - y - 1 = 0$ are perpendicular to each other

$$\therefore \angle BAC = 90^\circ$$



$\therefore A$ is the ortho centre of $\Delta^{le} ABC$

$$\text{Mid point of } BC = P = \text{circum centre} = \left(\frac{-1}{2}, \frac{-3}{2} \right)$$

$$\therefore PA^2 = DE^2 = \sqrt{9+4} = \sqrt{13}$$

- 62) Equation of line passing through P (3, 1) is $y - 1 = m(x - 3) \Rightarrow mx - y - 3m + 1 = 0$

Its distance from origin is $\left| \frac{3m-1}{\sqrt{m^2+1}} \right| = f(m)$ say

$f(m)$ is maximum Iff $m = -3$. Then the line is $y - 1 = -3(x - 3)$

$$3x + y = 10$$

$$\therefore \text{Required area} = \frac{1}{2} \times \frac{10}{3} \times 10 = \frac{50}{3} \text{ sq. units}$$

- 63) Suppose AB – subtends θ at C

$$\text{Then } \tan \theta = \left| \frac{\frac{h}{9} - \frac{h}{4}}{1 + \frac{h^2}{36}} \right| = \left| \frac{5h}{36 + h^2} \right| = \left| \frac{5}{h + \frac{36}{h}} \right|$$

$$AM \geq GM \Rightarrow h + \frac{36}{h} \geq 2\sqrt{h \cdot \frac{36}{h}} = 12 \quad \therefore \frac{1}{h + \frac{36}{h}} \leq \frac{1}{12}$$

$$\therefore \tan \theta \text{ is maximum. Iff } h + \frac{36}{h} = 12$$

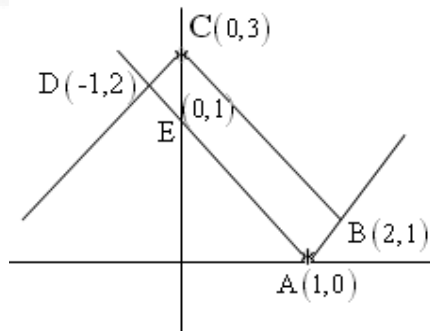
$$\Rightarrow h^2 - 12h + 36 = 0$$

$$(h - 6)^2 = 0 \Rightarrow h = 6$$

- 64) The lines are $y = |x - 1|$, $y = -|x| + 3$

They intersect at B(2, 1) and D(-1, 2)

$$\text{Area of react angle ABCD} = \sqrt{8} \times \sqrt{2} = 4$$



$$\text{Area of } \triangle CDE = \frac{1}{2} \times (2) \times 1 = 1$$

$$\therefore R_1 = 1, R_2 = 4 - 1 = 3 \Rightarrow \frac{R_1}{R_2} = \frac{1}{3}$$

65) Let the equation of the line passing through (0, 0) is $y = mx$. Which meets $2x + y = 2$ at A

$$\therefore A = \left(\frac{2}{2+m}, \frac{2m}{2+m} \right) \text{ and } y = mx \text{ meets } x - 2y + 2 = 0 \text{ in B. Where}$$

$$B = \left(\frac{2}{2m-1}, \frac{2m}{2m-1} \right)$$

Let (h, k) both mid point of AB

$$\therefore k = mh \Rightarrow m = k/h$$

$$\text{and } \frac{\frac{2}{2+m} + \frac{2}{2m-1}}{2} = h$$

$$\frac{1}{2+m} + \frac{1}{2m-1} = h$$

$$\therefore \frac{1}{2 + \frac{k}{h}} + \frac{1}{2\left(\frac{k}{h}\right) - 1} = h$$

$$\therefore \text{Locus of } (h, k) \text{ is } \frac{1}{2 + \frac{y}{x}} + \frac{1}{\frac{2y}{x} - 1} = x$$

$$\therefore \frac{x}{2x+y} + \frac{x}{2y-x} = x$$

$$2y - x + 2x + y = (2y - x)(2x + y)$$

$$3y \rightarrow x = 4xy + 2y^2 - 2x^2 - xy$$

$$\Rightarrow 2x^2 - 3xy - 2y^2 + x + 3y = 0$$

66) Clearly $OC = 10$

$$CP = 10$$

Shortest path from O to P

Which does not go inside the circle is

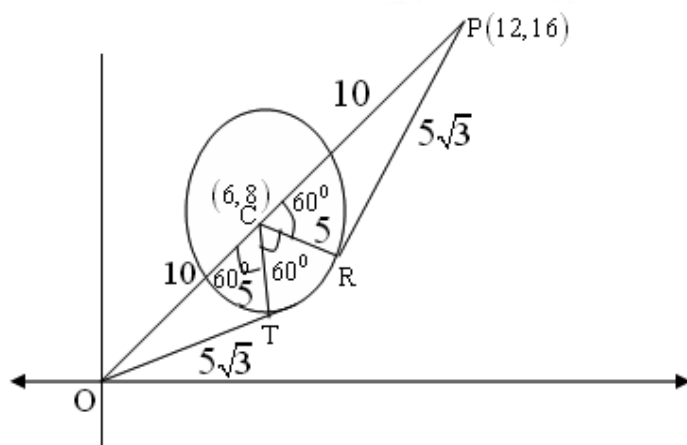
$$\overline{OT} + \widehat{TR} + \overline{RP}$$

When \overline{OT} = length of tangent from O to the circle = $5\sqrt{3}$

\overline{RP} = length of tangent from O to the circle = $5\sqrt{3}$

\widehat{TR} = arc length TR

$$= 5\frac{\pi}{3}$$



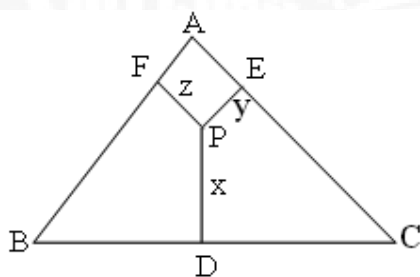
Required answer is $5\sqrt{3} + 5\sqrt{3} + \frac{5\pi}{3} = \frac{5\pi}{3} + 10\sqrt{3}$

67) Area of $\Delta^{le}ABC$ $\Delta = \frac{1}{2}BC(x) + \frac{1}{2}(AC)y + \frac{1}{2}(AB)z \dots\dots\dots(1)$

Let $\lambda = \frac{BC}{PD} + \frac{CA}{PE} + \frac{AB}{PF}$

$$\therefore \lambda = \frac{a}{x} + \frac{b}{y} + \frac{c}{z}$$

Now $2\Delta(\lambda) = (ax + by + cz)\left(\frac{a}{x} + \frac{b}{y} + \frac{c}{z}\right)$



Let $BC = a$ and $PD = x$

$AC = b$ $PE = y$

$AB = c$ $PF = z$

$$2\Delta\lambda \equiv (ax + by + cz) \left(\frac{a}{x} + \frac{b}{y} + \frac{c}{z} \right)$$

$$a^2 + b^2 + c^2 + ab \left(\frac{x}{y} + \frac{y}{x} \right) + bc \left(\frac{y}{z} + \frac{z}{y} \right) + ac \left(\frac{x}{z} + \frac{z}{x} \right)$$

$$\geq a^2 + b^2 + c^2 + 2ab + 2bc + 2ac = (a + b + c)^2$$

$$\left(\because \frac{x}{y} + \frac{y}{x} \geq 2, \frac{y}{z} + \frac{z}{y} \geq 2, \frac{x}{z} + \frac{z}{x} \geq 2 \right)$$

$$\therefore \lambda \geq \frac{(a + b + c)^2}{2\Delta}$$

$$\lambda \text{ is minimum iff } \frac{x}{y} + \frac{y}{x} = 2 \Rightarrow x = y \text{ and } \frac{y}{z} + \frac{z}{y} = 2 \Rightarrow y = z$$

$$\therefore \lambda \text{ is minimum iff } x = y = z$$

$$\therefore P \text{ is incentre of } \Delta^{le} ABC$$

68) Let A = (0, 0), B = (4, 0), C(0, 3).

Circumcentre of circle S is $(2, 3/2)$ and circumradius = $\frac{5}{2}$

$$\text{Equation of S is } (x - 2)^2 + (y - 3/2)^2 = \frac{25}{4} \dots\dots\dots(1)$$

If circle S_1 is having radius r_1 and touching AB and AC

$$\Rightarrow \text{Its centre is } (r_1, r_1).$$

$$S_1 \text{ and } S \text{ touch internally } \Rightarrow \sqrt{(r_1 - 2)^2 + (r_1 - 3/2)^2} = |r_1 - 5/2|$$

$$\Rightarrow r_1^2 - 2r_1 = 0 \Rightarrow r_1 = 2$$

If S_2 is having radius r_2 and touching AB and AC \Rightarrow centre of $S_2 = (r_2, r_2)$

$$S_2 \text{ touches S externally } \Rightarrow (r_2 - 2)^2 + (r_2 - 3/2)^2 = (r_2 + 5/2)^2$$

$$\Rightarrow r_2^2 - 12r_2 = 0 \Rightarrow r_2 = 12 \Rightarrow r_1 r_2 = 24$$

69) Let r be the radius of the circle and A = (0, 0) AB is along x - axis. AD is along y - axis.

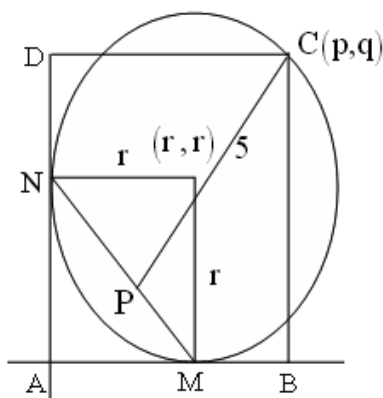
$$\text{Equation of circle is } (x - r)^2 + (y - r)^2 = r^2 \dots\dots\dots(1)$$

Equation of MN is $x + y = r$

Now \perp^{lar} distance from $C(p, q)$ to the above line is 5

$$\Rightarrow \left| \frac{p+q-r}{\sqrt{2}} \right| = 5 \Rightarrow (p+q-r)^2 = 50 \dots\dots\dots(2)$$

$$(p, q) \text{ lies on circle (1)} \Rightarrow p^2 + q^2 - 2r(p+q) + r^2 = 0$$



$$\Rightarrow (p+q-r)^2 - 2pq = 0$$

$$50 - 2pq = 0 \Rightarrow pq = 25$$

\therefore Area of rectangle ABCD is 25.

70) $AB = \sqrt{1+3} = 2$ $BC = \sqrt{1+3} = 2$, $AC = 2$

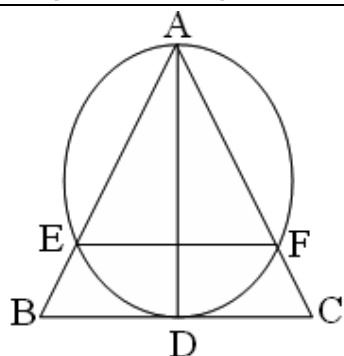
$\therefore \Delta^{le} ABC$ is equilateral triangle.

$$\text{In } \Delta^{le} AEF, \frac{EF}{\sin A} = AD$$

$$EF = AD \sin 60^\circ$$

$$= \sqrt{3} \cdot \frac{\sqrt{3}}{2} = 3/2$$

$$\therefore EF = 3/2$$



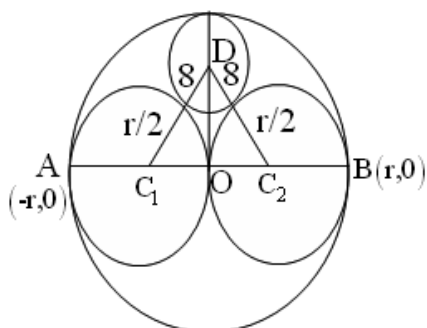
$$\left(\because \frac{AD}{AB} = \sin 60^\circ, AD = \cancel{2} \frac{\sqrt{3}}{\cancel{2}} = \sqrt{3} \right)$$

- 71) Let $A = (-r, 0), B = (r, 0)$ where r is radius of circle having AB as diameter.

$$OD^2 = \left(\frac{r}{2} + 8 \right)^2 - \left(\frac{r}{2} \right)^2$$

But $OD = |r - 8|$

$$\therefore (r - 8)^2 = \left(\frac{r}{2} + 8 \right)^2 - \frac{r^2}{4}$$



$$r^2 - 16r + 64 = \frac{r^2}{4} + 8r + 64 - \frac{r^2}{4}$$

$$r^2 - 24r = 0 \Rightarrow r = 24$$

$$\therefore AB = 48$$

- 72) Let x both sides of $\Delta^{le} DEF$

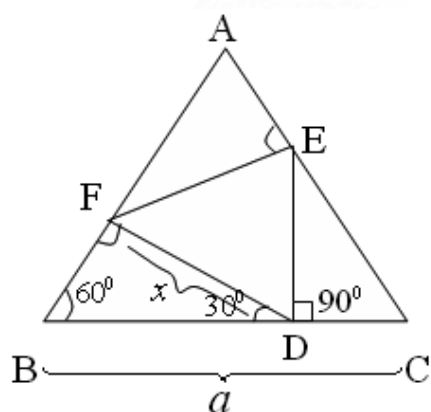
a be the side of $\Delta^{le} ABC$.

$$\frac{x}{BD} = \cos 30^\circ \Rightarrow BD = \frac{2x}{\sqrt{3}}$$

$$\frac{DC}{x} = \cot 60^\circ \Rightarrow DC = \frac{x}{\sqrt{3}}$$

$$\therefore BD + DC = \frac{3x}{\sqrt{3}} = x\sqrt{3}$$

$$a = x\sqrt{3}$$



$$\therefore \frac{\text{Area of } \triangle DEF}{\text{Area of } \triangle ABC} = \frac{\frac{\sqrt{3}}{4} x^2}{\frac{\sqrt{3}}{4} a^2} = \frac{x^2}{3x^2} = \frac{1}{3}$$

73) Let the required equation of the chord is $y = mx \equiv \overrightarrow{OP}$

Circle cuts y-axis at $(0,6) \equiv Q$

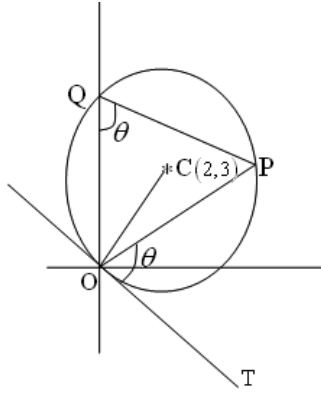
Let $\angle OQP = \theta$ where $\tan \theta = 7/4$

Let OT be the tangent of the circle at origin.

$\therefore \angle POT = \theta$ By alternate segment theorem.

Slope of $OC = 3/2 \Rightarrow \text{Slope of } OT = -2/3$

$$\therefore \tan \theta = \frac{m + 2/3}{1 - \frac{2m}{3}} = 7/4 \Rightarrow m = \frac{1}{2}$$



\therefore Equation of \overline{OP} is $y = \frac{1}{2}x \Rightarrow x - 2y = 0$

74) Let θ be the inclination of line PAB.

Any point on this line is $(1 + r \cos \theta, -2 + r \sin \theta)$

Substitution circle $x^2 + y^2 - x - y = 0$

$$\therefore r^2 + r(\cos \theta - 5 \sin \theta) + 6 = 0$$

\therefore roots of this equation are PA, PB

$$PA + PB = \cos \theta - 5 \sin \theta \Rightarrow \text{maximum value of } PA + PB \text{ is } \sqrt{26}$$

$$PA \times PB = 6$$

$$\frac{PA + PB}{2} \geq \sqrt{PA \cdot PB} \Rightarrow PA + PB \geq 2\sqrt{6}$$

$$PA + PB \geq \sqrt{24}$$

$$\therefore \sqrt{24} \leq PA + PB \leq \sqrt{26}$$

$$\text{Range of } PA + PB \text{ is } [\sqrt{24}, \sqrt{26}]$$

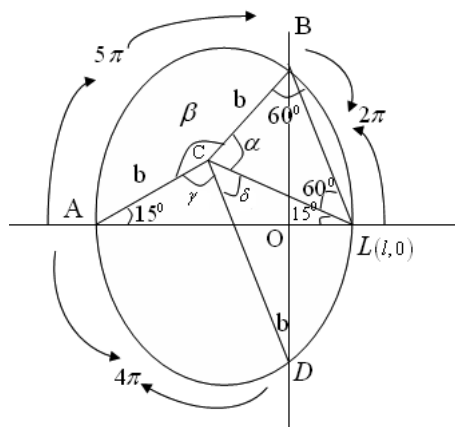
75) $6\alpha = 2\pi \Rightarrow \alpha = \frac{\pi}{3} = 60^\circ$

$$\text{Ily } \beta = 150^\circ, \gamma = 120^\circ$$

$$\Rightarrow \delta = 30^\circ$$

$$\text{In } \Delta^{\text{le}} ACL \quad \angle ACL = 150^\circ$$

$$\angle BLO = 75^\circ$$



$$\cos 75^{\circ} = \frac{l}{6} \Rightarrow l = 3 \cancel{\not{6}} \times \frac{\sqrt{3}-1}{2\sqrt{2}} = \frac{3(\sqrt{3}-1)}{\sqrt{2}}$$

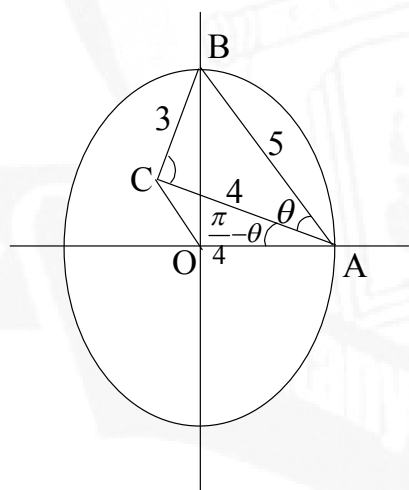
$$\therefore \text{coordination of centre } C \equiv (l + 6\cos 165^\circ, 6\sin 165^\circ) = \left(-3\sqrt{2}, \frac{3\sqrt{3}-1}{\sqrt{2}}\right)$$

76) Let $OC = r$, $C = (x, y)$

$$\text{In } \Delta^{le} ABC = \cos \theta = 4/5; \sin \theta = 3/5$$

$\ln \Delta^{le} OAC$

$$r^2 = 16 + \frac{25}{2} - 2 \times 4 \times \frac{5}{2} \cos(45 - \theta)$$



$$r^2 = \frac{1}{2}$$

Locus of C is $x^2 + y^2 = \frac{1}{2}$

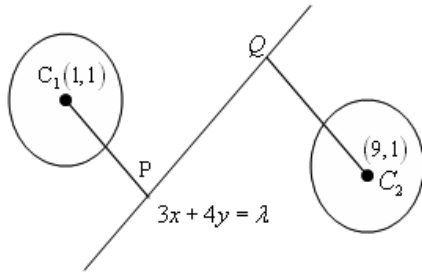
77) $S = x^2 + y^2 - 2x - 2y + 1 = 0$

Centre $C_1 = (1, 1), r_1 = 1$

$S' = x^2 + y^2 - 18x - 2y + 78 = 0$

$C_2 = (9, 1), r_2 = 2$

$C_1P > 1 \quad C_2Q > 2$



And C_1, C_2 lies on opposite side of given line.

$\Rightarrow (7 - \lambda)(31 - \lambda) < 0 \Rightarrow 7 < \lambda < 31 \dots\dots\dots(1)$

$C_1P > 1 \Rightarrow \lambda < 2, \lambda > 12 \dots\dots\dots(2)$

$C_2Q > 2 \Rightarrow \lambda < 21, \lambda > 41 \dots\dots\dots(3)$

$\therefore 12 < \lambda < 21.$

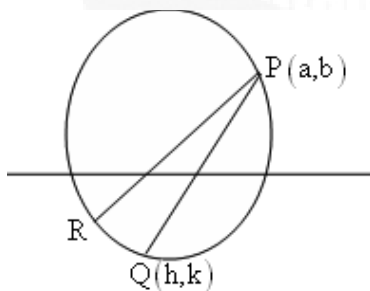
78) Let other end of chord be $Q(h, k)$

Mid point of $PQ = \left(\frac{h+a}{2}, \frac{k+b}{2} \right)$ which lies on x-axis $\Rightarrow \frac{k+b}{2} \Rightarrow k = -b$

$\therefore Q(h, -b)$ lies on the circle

$h^2 - ah + 2b^2 = 0$

This has two distinct real roots $\Rightarrow \text{Disc} > 0$



$\therefore a^2 - 8b^2 > 0$

$$\Rightarrow a^2 > 8b^2$$

79) required line is common chord of $(x - \alpha)^2 + (y - \beta)^2 = d^2$

$$\text{and } x^2 + y^2 = a^2$$

$$\text{i.e. } \alpha x + \beta y = a^2 - \frac{d^2}{2}$$

80) Let $P = (x_1, y_1)$

Let the inclination of the line \overline{PAB} is θ any point on this line is $(x_1 + r \cos \theta, y_1 + r \sin \theta)$

Substitute this point in given curve.

$$a(x_1 + r \cos \theta)^2 + 2h(x_1 + r \cos \theta)(y_1 + r \sin \theta) + b(y_1 + r \sin \theta)^2 = 1$$

$$r^2(a \cos^2 \theta + b \sin^2 \theta + 2h \sin \theta \cos \theta) + 2r(ax_1 \cos \theta + by_1 \sin \theta + hx_1 \sin \theta + hy_1 \cos \theta)$$

$$+ ax_1^2 + 2hx_1y_1 + by_1^2 - 1 = 0$$

It has two roots PA and PB.

$$PA \times PB = \frac{ax_1^2 + 2hx_1y_1 + by_1^2 - 1}{a \cos^2 \theta + b \sin^2 \theta + 2h \sin \theta \cos \theta}$$

It is independent of θ iff $a = b, h = 0$

Under this condition $ax^2 + 2hxy + by^2 = 1$ represents

$$ax^2 + ay^2 = 1 \Rightarrow x^2 + y^2 = \frac{1}{a}$$

It is a circle of radius $\frac{1}{\sqrt{a}}$.

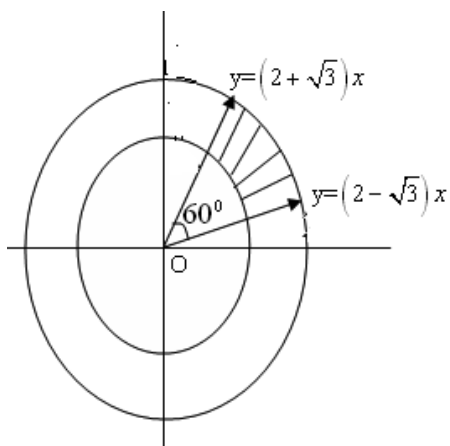
81) (a, b) lies between two circle $x^2 + y^2 = 4$ and $x^2 + y^2 = 9$

$$b^2 - 4ab + a^2 \leq 0 \Rightarrow (b - 2a)^2 - 3a^2 \leq 0$$

$$\Rightarrow (b - 2a - \sqrt{3}a)(b - 2a + \sqrt{3}a) \leq 0$$

$$(b - (2 + \sqrt{3})a)(b - (2 - \sqrt{3})a) \leq 0$$

$$\therefore (a, b) \text{ satisfying } (y - (2 + \sqrt{3})x)(y - (2 - \sqrt{3})x) \leq 0$$



$$(2 - \sqrt{3})x \leq y \leq (2 + \sqrt{3})x$$

$$\text{Required area } \frac{9\pi}{3} - \frac{4\pi}{3} = 3\pi - \frac{4\pi}{3} = \frac{5\pi}{3}$$

82) Equation of \overline{AB} is

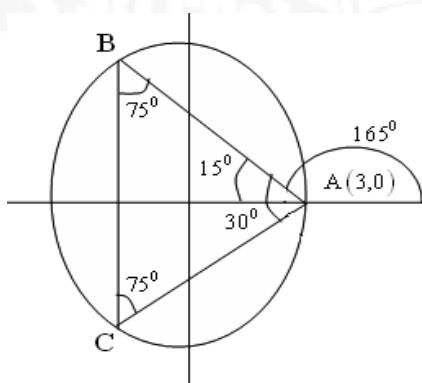
$$y - 0 = \tan 165^\circ (x - 3)$$

$$y = -\tan 15^\circ (x - 3)$$

$$y = (\sqrt{3} - 2)x - 3(\sqrt{3} - 2)$$

$$x = \frac{y + 3(\sqrt{3} - 2)}{\sqrt{3} - 2}$$

$$x = \frac{y}{\sqrt{3} - 2} + 3$$



$$\text{Substitution circle } x^2 + y^2 = 9$$

$$\therefore \text{Solve with } y = x \Rightarrow B = (1, 1)$$

$$\text{Solve with } y = -2x \Rightarrow C = \left(-\frac{1}{2}, 1\right)$$

$$\therefore \text{Area of } \Delta^{le} ABC = \frac{1}{2} \begin{vmatrix} 0 & 2 \\ 3 & 2 \\ \frac{1}{2} & 2 \end{vmatrix} = \left| \frac{1}{2}(-3) \right| = 3/2$$

84) Given two circles are point circle $\Rightarrow P = (\sqrt{2}, \sqrt{3})$

$$Q = (-\sqrt{3}, \sqrt{2})$$

Required circle is having P, Q as ends of diameter

$$(x - \sqrt{2})(x + \sqrt{3}) + (y - \sqrt{3})(y - \sqrt{2}) = 0$$

85) Let the equation of a chord be $y = mx + c$

It cuts the curve $3x^2 - y^2 - 2x + 4y = 0$ at A and B making the curve homogeneous to get the pair of lines OA and OB.

$$3x^2 - y^2 - 2x \left(\frac{y - mx}{c} \right) + 4y \left(\frac{y - mx}{c} \right) = 0$$

$$3(x^2 - cy^2 - 2xy + 2mx^2 + 4y^2 - 4mxy) = 0$$

$$(3c + 2m) + (-c + 4) = 0$$

$$2c + 2m + 4 = 0$$

$$c + m + 2 = 0$$

$$c = -m - 2$$

$$\therefore y = mx - m - 2$$

$$y + 2 - m(x - 1) = 0$$

This equation represents family of lines passing through intersection of $y + 2 = 0, x - 1 = 0$

i.e., (1, -2)

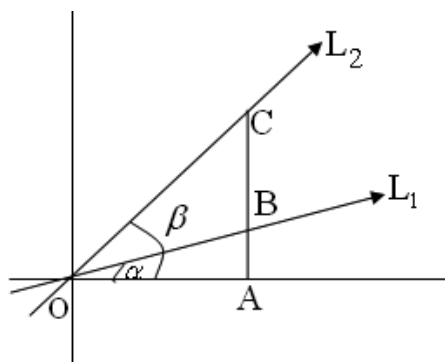
86) $\tan \alpha = \frac{AB}{OA}$

$$\tan \beta = \frac{AC}{OA} = \frac{2AB}{OA}$$

$$\tan \beta = 2 \tan \alpha$$

\therefore Slope of L_1 is m

Slope of L_2 is $2m$.



$$m + 2m = \frac{-2h}{b} \Rightarrow m = \frac{-2h}{3b}$$

$$m \times 2m = a/b \Rightarrow 2m^2 = a/b$$

$$2 \frac{4h^2}{ab} = \frac{a}{b}$$

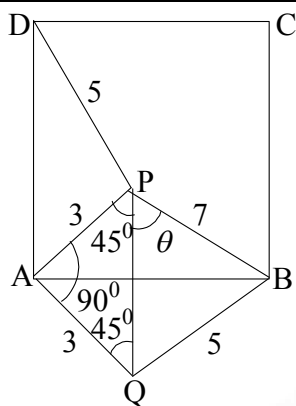
$$\frac{h^2}{ab} = \frac{9}{8}$$

87) rotate $\triangle^{le}APD$ about A through an angle 90° . So that P – goes to Q.

$\therefore \triangle^{le}APQ$ is Isosceles right angled triangle $PQ = \sqrt{9+9} = 3\sqrt{2}$.

$BQ = 5$.

$$\begin{aligned} \therefore \text{Area of } \triangle^{le}BPQ &= \sqrt{\frac{12+3\sqrt{2}}{2} \cdot \frac{3\sqrt{2}-2}{2} \cdot \frac{3\sqrt{2}+2}{2} \cdot \frac{12-3\sqrt{2}}{2}} = \frac{21}{2} \\ &= \frac{1}{2} PQ \cdot PB \cdot \sin \theta \end{aligned}$$



$$\therefore \frac{1}{2} \times 3\sqrt{2} \cdot 7 \cdot \sin \theta = \frac{21}{2} \Rightarrow \sin \theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45^\circ$$

$$\angle APB = 90^\circ \Rightarrow AB = \sqrt{9 + 49} = \sqrt{58}$$

$$\therefore \text{Area of } ABCD = 58.$$

- 88) $P(x, y)$ moves such that sum of its distances from given lines is 2, then locus of P is a rectangle of sides

$$\frac{2}{\sin \theta / 2}, \frac{2}{\cos \theta / 2} \text{ and area is } \frac{2d^2}{\sin \theta} = \frac{8}{\sin \theta} (\because d = 2)$$

Where θ is angle between given lines

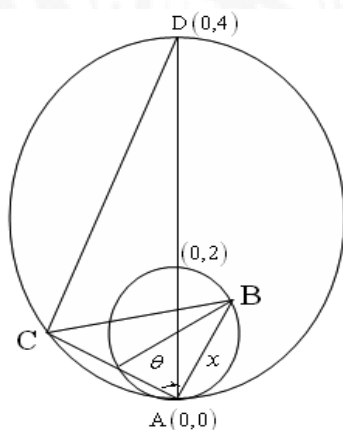
$$\text{Clearly } \sin \theta = \frac{1}{2}$$

$$\therefore \text{required area} = \frac{8}{\left(\frac{1}{2}\right)} = 16 \text{ sq. units}$$

- 89) Let $AB = x = AC = BC$

AC cuts small circle at E

$$x = AC = 4 \cos \theta = 2(2 \sin \theta) = 2(AE)$$



$$AE = \frac{x}{2}$$

$$BE^2 = x^2 + \frac{x^2}{4} - \frac{x^2}{2} = \frac{3x^2}{4} \Rightarrow BE = \frac{\sqrt{3}}{2}x = 2 \sin 60^\circ \Rightarrow x = 2$$

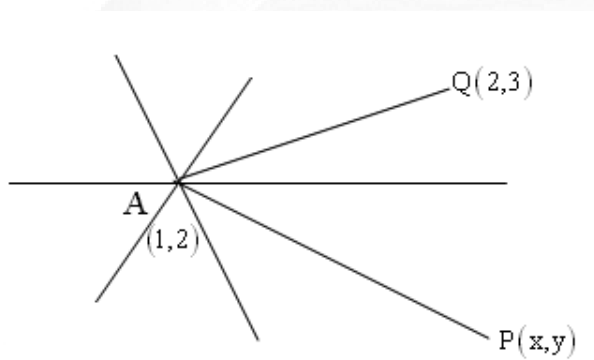
90) Given family passing through intersection of

$$x - 2y + 3 = 0 \dots\dots\dots(1)$$

$$2x - 3y + 4 = 0 \dots\dots\dots(2)$$

$$2x - 4y + 6 = 0$$

$$y - 2 = 0 \Rightarrow y = 2 \Rightarrow x = 1$$



Concurrent point is $(1, 2) = A$

Let $Q = (2, 3)$

Let $P(x, y)$ is its Image

$$AQ = AP$$

$$= AP^2 = AQ^2$$

$$(x-1)^2 + (y-2)^2 = 1+1$$

$$x^2 - 2x + 1 + y^2 - 4y + 4 = 2$$

$$x^2 + y^2 - 2x - 4y + 3 = 0$$