MATHEMATICS

1. Let $L_1 : \vec{r} = \vec{i} + 5\vec{j} + 5\vec{k} + t_1\left(4\vec{i} - 4\vec{j} + 5\vec{k}\right)$ and $L_2 : \left(2\vec{i} + 4\vec{j} + 5\vec{k}\right) + t_2\left(8\vec{i} - 3\vec{j} + \vec{k}\right)$ be two lines then

   1) L_1 is parallel to L_2

   2) L_1 is perpendicular to L_2

   3) Angle between L_1 and L_2 is 45^0

   4) Angle between L_1 and L_2 is $\cos^{-1}\left(\frac{49}{\sqrt{4218}}\right)$

2. The perpendicular distance from a point P with position vector $5\vec{i} + \vec{j} + 3\vec{k}$ to the line $\vec{r} = \left(3\vec{i} + 7\vec{j} + \vec{k}\right) + t\left(\vec{j} + \vec{k}\right)$ is

   1) 3 2) 6 3) 9 4) 12

3. Let $\vec{v} = 2\vec{i} + \vec{j} + \left(-\vec{k}\right)$ and $\vec{w} = \vec{i} + 3\vec{k}$ and $\vec{u}$ is a unit vector, then maximum value of scalar triple product $\left[\vec{u}\ \vec{v}\ \vec{w}\right]$ is

   1) -1 2) $\sqrt{10} + \sqrt{16}$ 3) $\sqrt{59}$ 4) $\sqrt{60}$

4. Let there be two points A and B on the curve $y = x^2$ in the plane OXY satisfying $\overrightarrow{OA}.\vec{i} = 1$ and $\overrightarrow{OB}.\vec{i} = -2$ then $\left|2\overrightarrow{OA} - 3\overrightarrow{OB}\right| =$ ____

1) $\sqrt{14}$ 2) $2\sqrt{51}$ 3) $2\sqrt{41}$ 4) 5

5. The value of a so that the volume of the parallelopipped formed by $\vec{i} + a\vec{j} + \vec{k}, \vec{j} + a\vec{k}$ and $a\vec{i} + \vec{k}$ becomes minimum $(a > 0)$

1) -3 2) 3 3) $\frac{1}{\sqrt{3}}$ 4) $\sqrt{3}$

6. If $\vec{a},\vec{b},\vec{c}$ are unit vectors then maximum value of $\left|\vec{a} - \vec{b}\right|^2 + \left|\vec{b} - \vec{c}\right|^2 + \left|\vec{c} - \vec{a}\right|^2 =$

1) 4 2) 9 3) 8 4) 6

7. If $\vec{a} = \vec{i} - \vec{j} - \vec{k}$, $\vec{a}.\vec{b} = 1$ and $\vec{a} \times \vec{b} = -\vec{j} + \vec{k}$ then $\vec{b} =$ ____

1) $\vec{i} + \vec{j} - \vec{k}$ 2) $-2\vec{j} + \vec{k}$ 3) $\vec{i}$ 4) $-2\vec{i} + \vec{k}$

8. If $\bar{a},\bar{b}$ are two vectors such that $\left|\vec{a}\right| = 1, \left|\vec{b}\right| = 2$ and $\left|\vec{a} - 2\vec{b}\right| = 4$ then $\left|\vec{a} + 3\vec{b}\right| =$

1) 8 2) $\sqrt{\frac{51}{2}}$ 3) $\frac{\sqrt{19}}{2}$ 4) $\sqrt{\frac{77}{2}}$

9. Let ABCD be a parallelogram such that $\overrightarrow{AB}=\vec{q}, \overrightarrow{AD}=\vec{p}$ and $\angle BAD$ is acute angle. If $\vec{r}$ is the vector that coincides with the altitude directed from B to the side $\overline{AD}$ then

1) $\vec{r}=-\vec{q}+\left(\frac{\vec{p}.\vec{q}}{\vec{p}.\vec{p}}\right)\vec{p}$ 2) $\vec{r}=\vec{q}-\frac{(\vec{p}.\vec{q})}{\vec{p}.\vec{p}}\vec{p}$

3) $\vec{r}=-3\vec{q}+\frac{3(\vec{p}.\vec{q})}{\vec{p}.\vec{p}}\vec{p}$ 4) $\vec{r}=3\vec{q}-\frac{3(\vec{p}.\vec{q})}{\vec{p}.\vec{p}}\vec{p}$

10. If four points $A(1,0,3), B(-1,3,4), C(1,2,1), D(k,2,5)$ are coplanar then $k=$ ___

1) 1 2) 2 3) 0 4) -1

11. The equation of the plane passing through the point (1, -1, 2) and parallel to the plane $3x+4y-5z=0$ is

1) $3x+4y-5z+11=0$ 2) $3x+4y-5z=11$

3) $6x+8y-10z=1$ 4) $3x+4y-5z=2$

12. If M denotes the mid-point of the line joining $A(4i+5j-10k)$ and $B(-i+2j+k)$, then equation of the plane through M and perpendicular to AB is

1) $r.(-5i-3j+11k)+\frac{135}{2}=0$ 2) $r.\left(\frac{3}{2}i+\frac{7}{2}j-\frac{9}{2}k\right)+\frac{135}{2}=0$

3) $r.(4i+5j-10k)+4=0$ 4) $r.(-i+2j+k)+4=0$

13. Equation of the plane through (3, 4, -1) which is parallel to the plane $r.(2i-3j+5k)+7=0$ is

1) $r.(2i-3j+5k)+11=0$ 2) $r.(3i+4j-k)+11=0$

3) $r.(3i+4j-k)+7=0$ 4) $r.(2i-3j+5k)-7=0$

14. A line passes through the points (6, -7, -1) and (2, -3, 1). If the angle α which the line makes with the positive direction of $x-$axis is acute, the direction cosines of the line are

1) $2/3,-2/3,-1/3$ 2) $2/3,2/3,-1/3$

3) $2/3,-2/3,1/3$ 4) $2/3,2/3,1/3$

15. The lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{-k}$ and $\frac{x-1}{k}=\frac{y-4}{2}=\frac{z-5}{1}$ are coplanar if

1) k = 1 or -1 2) k = 0 or -3 3) k = 3 or -3 4) k = 0 or -1

16. Two lines $x=ay+b, z=cy+d$ and $x=a'y+b', z=c'y+d'$ will be perpendicular, if and only if

1) $aa'+bb'+cc'=0$ 2) $(a+a')(b+b')(c+c')=0$

3) $aa'+cc'+1=0$ 4) $aa'+bb'+cc'+1=0$

17. A line makes the same angle θ with each of the x and z axis. If the angle β, which it makes with $y-$axis, is such that $\sin^2\beta = 3\sin^2\theta$, then $\cos^2\theta$ equals

1) 3/5 2) 1/5 3) 2/3 4) 2/5

18. The angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is

1) 45^0 2) 30^0 3) 0^0 4) 90^0

19. The distance between the line $r = 2i - 2j + 3k + \lambda(i - j + 4k)$ and the plane $r.(i + 5j + k) = 5$ is

1) 3/10 2) 10/3 3) 10/9 4) $10/3\sqrt{3}$

20. Let $\bar{a} = i + j + k, \bar{b} = i - j + 2k$ and $\bar{c} = xi + (x-2)j - k$. If the vector $\bar{c}$ lies in the plane of $\bar{a}$ and $\bar{b}$, then x equals

1) 0 2) 1 3) -4 4) -2

21. The distance of the point (1, -5, 9) from the plane $x - y + z = 5$ measured parallel to straight line $x = y = z$ is

1) $10\sqrt{3}$ 2) $5\sqrt{3}$ 3) $3\sqrt{10}$ 4) $3\sqrt{5}$

22. The equation of the plane which contains two parallel lines $\frac{x+1}{3}=\frac{y-2}{2}=\frac{z}{1}$ and $\frac{x-3}{3}=\frac{y+4}{2}=\frac{z-1}{1}$ is

1) $3x+2y+z=14$ 2) $8x+y-26z+6=0$

3) $4x-6y+z=53$ 4) None of these

23. The point P is the intersection of the straight line joining the points $Q(2,3,5)$ and $R(1,-1,4)$ with the plane $5x-4y-z=1$. If S is the foot of the perpendicular drawn from the point T(2, 1, 4) to QR, then length of the line segment PS is

1) $1/\sqrt{2}$ 2) $\sqrt{2}$ 3) 2 4) $2\sqrt{2}$

24. The reflection of the point A(1, 0, 0) in the line $\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$ is

1) $(3,-4,-2)$ 2) $(5,-8,-4)$ 3) $(1,-1,-10)$ 4) $(2,-3,8)$

25. The lines whose vector equations are $r=a+tb,\ r=c+t'd$ are coplanar if

1) $(a-b).c\times d=0$ 2) $(a-c).b\times d=0$

3) $(b-c).a\times d=0$ 4) $(b-d).a\times c=0$

26. The length of the shortest distance between the lines $r=3i+5j+7k+\lambda(i-2j+k)$ and $r=-i-j-k+\mu(7i-6j+k)$ is

1) 83 2) $\sqrt{6}$ 3) $\sqrt{3}$ 4) $2\sqrt{29}$

27. A line segment has length 63 and direction ratios are 3, -2, 6. If the line makes an obtuse angle with x-axis, the components of the line vector are

1) 27, -18, 54 2) -27, 18, -54 3) -27, 18, 54 4) 27, -18, -54

28. The foot of the perpendicular from the origin to the join of A(-9, 4, 5) and B(11, 0,-1) divides AB in the ratio

1) 2 : 3 2) 3 : 2 3) 1 : 1 4) None of these

29. The shortest distance between the lines $\frac{x-3}{2}=\frac{y+15}{-7}=\frac{z-9}{5}$ and $\frac{x+1}{2}=\frac{y-1}{1}=\frac{z-9}{-3}$ is

1) $4\sqrt{5}$ 2) $4\sqrt{17}$ 3) $4\sqrt{3}$ 4) $8\sqrt{2}$

30. Equation of the line through the point (2, -1,1) and the intersection of the lines $2x-y-4=0=y+2z, x+3z-4=0=2x+5z-8$ is

1) $x+y+z=2, x+2y=0$ 2) $x-y-z=2, x+2z=4$

3) $x+2y+z=1, x+2z=4$ 4) $x+2y+z=1, x+2y=0$