MATHS

31. Z_1 and Z_2 are two complex numbers represented by the points A and B in the Argand plane Z_1 and Z_2 are the roots of $Z^2+pZ+q=0$. If $\angle AOB=\alpha, \alpha \neq 0$ and O is origin and $OA=OB$, then $\frac{p^2}{q}$ is equal to

1) $4\cos^2\left(\frac{\alpha}{2}\right)$ 2) $4\sin^2\left(\frac{\alpha}{2}\right)$ 3) $2\cos\alpha$ 4) $2\sin\alpha$

32. If w is complex cube root of 1, then

$$\cos\left(\left((1-w)(1-w^2)+(2-w)(2-w^2)+........+(10-w)(10-w^2)\right)\frac{\pi}{900}\right)=$$

1) -1 2) 0 3) 1 4) $\frac{\sqrt{3}}{2}$

33. If Z is a complex number such that $-\frac{\pi}{2} \leq \operatorname{Arg} Z \leq \frac{\pi}{2}$ *then* which of the following in equality is true.

1) $|z-\bar{z}| \leq |z||\operatorname{Arg}z-\operatorname{Arg}\bar{z}|$ 2) $|z-\bar{z}| \leq |\operatorname{Arg}z-\operatorname{Arg}\bar{z}|$

3) $|z-\bar{z}| > |z||\operatorname{Arg}z-\operatorname{Arg}\bar{z}|$ 4) $|z-\bar{z}| = 1$

34. If $\omega \neq 1$ is a cube root of 1 and $|z-1|^2 + 2|z-\omega|^2 = 3|z-\omega^2|^2$ then z lies on

1) a straight line
2) a parabola
3) An ellipse
4) a rectangular hyperbola

35. The least value of $n \in N$ such that $(1+i)^n \pi = (1-i)^n 2\, Sin^{-1}\left(\frac{1+x^2}{2x}\right)$ $(x>0)$

1) 2
2) 4
3) 8
4) 16

36. If 1, w, $w^2, ... w^{n-1}$ *are* n^{th} *roots of* 1 *then value of* $\frac{1}{2-w} + \frac{1}{2-w^2} + ... + \frac{1}{2-w^{n-1}}$ equals to

1) $\frac{1}{2^n - 1}$
2) $\frac{n(2^n - 1)}{2^n + 1}$
3) $\frac{(n-2)2^{n-1}+1}{2^n - 1}$
4) $\frac{n2^{n-1}}{2^n - 1}$

37. The area of the figure formed by the roots of $z^5 = (z-1)^5$ in Argand plane is

1) $32\cos\frac{2\pi}{5}$
2) $1-\cos^2\frac{2\pi}{5} - \sin^2\frac{2\pi}{5}$
3) $\cot\frac{\pi}{5}$
4) $2\cot\frac{\pi}{5}$

38. If α is non real fifth root of 1 then $\log_2\left|1+\alpha+\alpha^2+\alpha^3-\frac{1}{\alpha}\right|$

1) 0
2) $\frac{1}{2}$
3) 1
4) 2

39. If 1, $\alpha_1, \alpha_2, ...\alpha_{99}$ *are roots of* $Z^{100}=1$ then $\sum_{1\le i<j\le 99} \alpha_i\alpha_j$ equal to

1) 0
2) 1
3) 99
4) 100

40. If $\cos x+2\cos y+3\cos z=\sin x+2\sin y+3\sin z=0$ then the value of $\sin 3x+8\sin 3y+27\sin 3z$ is

1) sin(x+y+z)
2) 3sin(x+y+z)
3) 18 sin (x+y+z)
4) sin(x+2y+z)

41. If $z_1, z_2 \in \mathbb{C}, {z_1}^2+{z_2}^2 \in IR, z_1({z_1}^2-3{z_2}^2)=2$ *and* $z_2(3z_1^2-z_2^2)=11$ then $z_1^2+z_2^2$ is

1) 10
2) 12
3) 5
4) 8

42. Locus of z if Arg $(z-(1+i)) = \begin{cases} \frac{3\pi}{4} \text{ if } |z| \le |z-2| \\ -\frac{\pi}{4} \text{ if } |z| > |z-4| \end{cases}$ is

1) Straight lines passing through (2,0)

2) straight lines passing through (2,0),(1,1)

3) a line segment

4) a set of two rays

43. The number of complex numbers z satisfying

$|z-3-i| = |z-9-i| \, and \, |z-3+3i| = 3 \, are$

1) 1 2) 2 3) 4 4) 0

44. z_1 *and* z_2 *lie* on a circle with centre at origin. The point of intersection of the tangents at z_1 *and* z_2 is given by

1) $\frac{\overline{z_1} + \overline{z_2}}{2}$ 2) $\frac{2z_1z_2(\overline{z_1} - \overline{z_2})}{(z_2\overline{z_1} - z_1\overline{z_2})}$ 3) $\frac{1}{2}\left(\frac{1}{z_1} + \frac{1}{z_2}\right)$ 4) $\frac{z_1 + z_2}{2\overline{z_1 z_2}}$

45. Let S_1 *and* S_2 are concentric circles with radius 1 and 8/3 respectively, having centre at (3,0), on the Argand plane. If z satisfies the in equality $\log_{\frac{1}{3}}\left(\frac{|z-3|^2+2}{11|z-3|-2}\right)>1$ then

1) z lies outside S_1 but inside S_2
2) z lies inside of both S_1 *and* S_2
3) z lies outside both of S_1 *and* S_2
4) z *lies on* S_1

46. If (a,b) lies on $x^2+y^2=100$ and z_1, z_2 lies on curve $z^2+z^{-2}=12$ then

$$\frac{|az_1-bz_2|^2+|bz_1+az_2|^2}{10\left(|z_1|^2+|z_2|^2\right)}=$$

1) 1
2) 10
3) 100
4) $\sqrt{10}$

47. The locus of z such that Arg$\left(\frac{z-z_1}{z-z_2}\right)=\frac{\pi}{4}$ where $z_1=10+6i, z_2=4+6i$ is part of a circle whose radius is

1) $\sqrt{2}$
2) $2\sqrt{2}$
3) $3\sqrt{2}$
4) 1

48. If $|z|=1$ *and* $w=\dfrac{z-1}{z+1},(z\neq -1)$ then Re(w) is

1) 0 2) $\dfrac{1}{|z+1|^2}$ 3) $\dfrac{z}{|z+1|}\dfrac{1}{|z+1|^2}$ 4) $\dfrac{\sqrt{2}}{|z+1|^2}$

49. $P=\{z:\operatorname{Im} z\geq 1\}$ $Q=\{z:|z-2-i|=3\}$ $R=\{z:\operatorname{Re}(1-i)z=\sqrt{2}\}$ If $Z\in P\cap Q\cap R$ then

$|z+1-i|^2+|z-5-i|^2=$

1) 23 2) 36 3) 72 4) 34

50. Let a, b be complex numbers and $b\neq 0$. If α *and* β are roots of the equation

$x^2+ax+b^2=0$ such that $|\alpha|=|\beta|=r$, then the value of $\dfrac{a^2}{b^2}$ is

1) $2+\dfrac{2}{r^2}\operatorname{Re}\left(\alpha\overline{\beta}\right)$ 2) 2

3) $2+\dfrac{2}{r^2}I_m\left(\alpha\overline{\beta}\right)$ 4) $2+\dfrac{2}{r^2}\operatorname{Re}\left(\alpha\beta\right)$

51. Let a,b,c be distinct complex numbers with $|a|=|b|=|c|$. If a root of the equation $aZ^2+bZ+c=0$ has modulus equal to 1, then

1) $b^2=ac$
2) $b^2=2ac$
3) $b^2=4ac$
4) $a^2=4bc$

52. If r_1 and r_2 are the distances of points on the curve $10\left(Z\overline{Z}\right)-3i\left(Z^2-\left(\overline{Z}\right)^2\right)-16=0$ which are at maximum and minimum distance from the origin then the value of $r_1+r_2=$

1) 1
2) 2
3) 3
4) 4

53. If $A\left(z_1\right),B\left(\overline{z}_1\right)$ are the vertices of a regular polygon whose centre at origin and if $\frac{\operatorname{Im}\left(\overline{z}_1\right)}{\operatorname{Re}\left(z_1\right)}=1-\sqrt{2}$ then the number of sides of the polygon is

1) 6
2) 8
3) 12
4) 10

54. If z_1 is a complex numbers satisfying $|z+1| = z+2+2i$ then the value of $|z_1| + \mathrm{Arg}(z_1)$ equals

1) $2+\frac{\pi}{4}$
2) $\frac{\sqrt{17}}{2} - \tan^{-1}(4)$
3) $\frac{\sqrt{3}}{2} - \tan^{-1}(4)$
4) $\frac{1}{2} - \tan^{-1} 2$

55. If the complex numbers z_1, z_2, z_3 represent the vertices of an equilateral triangle such that $|z_1 - 1| = |z_2 - 1| = |z_3 - 1|$ then the value of $\frac{9 - z_1^2 - z_2^2 - z_3^2}{z_1z_2 + z_2z_3 + z_3z_1}$ equals

1) 0
2) 1
3) 2
4) 3

56. Let z_1, z_2, z_3 and z_4 be the roots of the equation $z^4 + z^3 + 2 = 0$ then the values of $\underset{r=1}{\overset{4}{\pi}} (2z_r + 1)$ equals

1) 28
2) 29
3) 30
4) 31

57. If z is a complex number such that $|z-2i| \le \sqrt{2}$ then the maximum value of $\left[|3+i(z-i)|\right]$ equals (where [ ] is GIF)

1) 1 2) 2 3) 3 4) 4

58. If a,b,c are complex numbers such that $|a|=2, |b|=3, |c|=4$ then maximum value of $|a-b|^2+|b-c|^2+|c-a|^2$ equals

1) 58 2) 29 3) 42 4) 87

59. Let $z=x+iy$ where $x,y \in I$. Area of the octagon whose vertices are the roots of the equation $(z\bar{z})|z^2-\bar{z}^2|=1200$ equals

1) 14 2) 27 3) 17 4) 62

60. Let a complex number z, with minimum argument is lying on the cure $|z+4i|=2$ then $\left|\arg(z)+\sum_{r=0}^{17}\left(\frac{z}{2\sqrt{3}}\right)^r\right|=$

1) $5-\frac{\pi}{3}$ 2) $3-\frac{\pi}{3}$ 3) $\frac{\pi}{3}$ 4) $\frac{2\pi}{3}$