Sri Chaitanya IIT Academy, India

A.P, TELANGANA, KARNATAKA, TAMILNADU, MAHARASHTRA, DELHI, RANCHI

A right Choice for the Real Aspirant

ICON CENTRAL OFFICE, MADHAPUR-HYD

Sec: Sr. IPLCO	JEE ADVANCED	DATE : 03-01-16
TIME : 09:00 AM TO 12: 00 Noon	2013_P1 MODEL	MAX MARKS : 180

KEY & SOLUTIONS

PHYSICS

1	**C**	2	**A**	3	**A**	4	**C**	5	**B**	6	**A**
7	**B**	8	**C**	9	**B**	10	**A**	11	**C**	12	**ACD**
13	**BD**	14	**AC**	15	**AB**	16	**5**	17	**7**	18	**3**
19	**2**	20	**2**								

CHEMISTRY

21	**B**	22	**D**	23	**B**	24	**C**	25	**A**	26	**A**
27	**B**	28	**D**	29	**D**	30	**A**	31	**ACD**	32	**ACD**
33	**B**	34	**ABC**	35	**AB**	36	**2**	37	**2**	38	**1**
39	**5**	40	**2**								

MATHEMATICS

41	**D**	42	**C**	43	**B**	44	**C**	45	**A**	46	**D**
47	**B**	48	**D**	49	**C**	50	**A**	51	**CD**	52	**ABCD**
53	**BD**	54	**ABD**	55	**ABC**	56	**4**	57	**2**	58	**3**
59	**4**	60	**6**								

MATHS

41. $a_p = t+s-p; b_{t+s} = -(t+s)$ is true for any natural number p but only the given t and s.

42. $s_k + a_k = \sum_{r=1}^{n} a_r$ is fixed

Given $a_1, a_2....a_n$ are in H.P

$\Rightarrow \frac{a_1+s_1}{a_1}, \frac{a_2+s_2}{a_2}, ..., \frac{a_n+s_n}{a_n}$ are in A.P

43. $\sum_{m=1}^{20} {}^{20}C_m \left(\sum_{k=1}^{m} \left(\sum_{p=k}^{m} \frac{m!}{p!(m-p)!} \cdot \frac{p!}{k!(p-k)!} \right) \right)$

$= \sum_{m=1}^{20} {}^{20}C_m \left(\sum_{k=1}^{m} \left(\sum_{p=k}^{m} {}^{m-k}C_{p-k} \right) \frac{m!}{k!(m-k)!} \right)$

$= \sum_{m=1}^{20} {}^{20}C_m \left(\sum_{k=1}^{m} 2^{m-k} \cdot {}^{m}C_k \right)$

$= \sum_{m=1}^{20} {}^{20}C_m \left((1+2)^m - 2^m \right) = \sum_{m=1}^{20} \left({}^{20}C_m 3^m - {}^{20}C_m 2^m \right) = 4^{20} - 3^{20}$ ends with 75.

44. let x be the sum of the sequence without number 35 and n be the total number of distinct values in the sequence. So, $\frac{35+x}{n} = 53$. We know that $\frac{x}{n-1} = 54 \Rightarrow x = 54n - 54$

Hence $35 + (54n-54) = 53 \Rightarrow n = 19$. The sum of the numbers is $53 \times 19 = 1007$.

For highest possible number in the sequence, the other numbers in the sequence have to be lowest possible values. So 1+2+3+...+17+35+ H(Highest possible number) = 1007

This gives153 + 35 + H = 1007.

45. $3+4+5+5+7+8+9+9 = 50$

46. $R_n f_n = \left(5\sqrt{5}+11\right)^{2n+1} \left(5\sqrt{5}-11\right)^{2n+1} = 4^{2n+1} \Rightarrow R_3 f_3 = 2^{14}$

47. use $55^5 = 23m+8; 17^4 = 23k+8$

48. $I(m,n) = \lim_{x\to\infty} \int_{-x}^{x} \frac{dt}{(t^2+m^2)(t^2+n^2)} = \frac{\pi}{mn(m+n)}$

49. $a_n b_n = \left((x)^{\frac{1}{2^n}} - (y)^{\frac{1}{2^n}} \right) \left((x)^{\frac{1}{2^n}} + (y)^{\frac{1}{2^n}} \right) = (x)^{\frac{1}{2^{n-1}}} - (y)^{\frac{1}{2^{n-1}}} = b_{n-1}$

So, $a_n b_n a_{n-1} b_{n-1} a_{n-2} b_{n-2} .... a_2 b_2 = b_{n-1} b_{n-2} b_{n-3} .... b_1$

$\Rightarrow a_n b_n a_{n-1} a_{n-2} .... a_2 = b_1 \Rightarrow a_n a_{n-1} a_{n-2} .... a_2 a_1 = \frac{a_1 b_1}{b_n} = \frac{x-y}{b_n}$

50. $a(a^2+10ab+5b^2)^2-b(5a^2+10ab+b^2)^2=(a-b)^5$.

51. we have $1072<10(2a+19d)<1162$ and $a+5d=32$ gives d=5, a=7

52. $a_1+a_4+a_7+......+a_{16}=147 \Rightarrow a_1+a_{16}=49$

Again $a_1+a_4+a_7+a_{10}+...+a_{16}=a_1+a_1+3d+a_1+6d+...+a_1+15d$

$=6a_1+45d=147 \Rightarrow 2a_1+15d=49$

$a_1+a_6+a_{11}+a_{16}=a_1+a_1+5d+a_1+10d+a_1+15d$

$=4a_1+30d=2(2a_1+15d)=2(49)=98$

Now using $AM \geq GM$

$$\frac{a_1+a_2+...+a_{16}}{16} \geq (a_1a_2a_3...a_{16})^{\frac{1}{16}} \Rightarrow \frac{8(a_1+a_{16})}{16} \geq (a_1a_2a_3...a_{16})^{\frac{1}{16}} \Rightarrow \left(\frac{49}{2}\right)^{16} \geq a_1a_2a_3...a_{16}$$

53.

Let $g_i=ar^{i-1}$ we have a $a(1+r^3)=-49,\ ar(1+r)=14$

$a(1+r)(1-r+r^2)=-49$. $\quad \therefore \frac{1-r+r^2}{r}=-\frac{7}{2} \Rightarrow 2r^2+5r+2=0$

$\therefore r=-2,-\frac{1}{2}$ If $r=-2$, then $ar(1+r)=14 \Rightarrow a=7$.

$\therefore$ The GP is $7,-14,\ 28,-56$.

54.

Given $\left(3\sqrt{3}+5\right)^{2n+1}=\alpha+\beta$ …. (1)

Let $\beta'=\left(3\sqrt{3}-5\right)^{2n+1}$ …. (2)

$\alpha+\beta-\beta'=\left(3\sqrt{3}+5\right)^{2n+1}-\left(3\sqrt{3}-5\right)^{2n+1}$

$=2\left[{}^{2n+1}C_1\left(3\sqrt{3}\right)^{2n}5+{}^{2n+1}C_3\left(3\sqrt{3}\right)^{2n-2}(5)^3+...+{}^{2n+1}C_{2n+1}5^{2n+1}\right]$

$\alpha+\beta-\beta'=10k$, But $-1<\beta-\beta'<1$

$\therefore \beta-\beta'$ is an integer

$\therefore \beta-\beta'=0$ $\quad \therefore \alpha$ divisible by 10.

$$\Rightarrow (\alpha+\beta)^2 = \left[\left(3\sqrt{3}+5\right)^2\right]^{2n+1} = \left(52+30\sqrt{3}\right)^{2n+1} = 2^{2n+1}\left(26+15\sqrt{3}\right)^{2n+1}$$

$\therefore (\alpha+\beta)^2$ divisible by 2^{2n+1}

55.

T_p of $AP = \dfrac{1}{q(p+q)} = A+(p-1)D$ … (i)

T_q of $AP = \dfrac{1}{p(p+q)} = A+(q-1)D$ … (ii)

$\dfrac{1}{T_{p+q}} = A+(p+q-1)D$ and $\dfrac{1}{T_{pq}} = A+(pq-1)D$.

solving Eqs. (i) and (ii), we get $A = D = \dfrac{1}{pq(p+q)}$

$\therefore \dfrac{1}{T_{p+q}} = A+(p+q-1)D = (p+q)D = \dfrac{1}{pq}$

and $\dfrac{1}{T_{pq}} = A+f(p+q-1)D = pqD = \dfrac{1}{p+q}$

$\Rightarrow$ $T_{p+q} = pq$ and $T_{pq} = p+q$

Also, $\because pq > p+q$ i.e, $T_{p+q} > T_{pq}$

56. $\dfrac{(a_{10})^2}{(11\times 13\times 2\times 17\times 19)^2} = 4$

57. $A = \dfrac{25+n^2}{2}, G = 5n, H = \dfrac{50n^2}{25+n^2}$ should be natural numbers, n=15

58. $\dfrac{C_1}{C_0} + 2^2.\dfrac{C_2}{C_1} + 3^2.\dfrac{C_3}{C_2} + ...17^2.\dfrac{C_{17}}{C_{16}} = \sum_{r=1}^{17} r(18-r)$ =969

59. Let $A = a+b, B = c+d$

$\therefore A, B > 0,\ A+B = 4$

$\therefore (a+b)(c+d) \le 4.$

Hence $0 \le K \le 4.$

60. $1, \dfrac{{}^nC_1}{2}, \dfrac{{}^nC_2}{2^2}$ are in A.P $\Rightarrow n = 8$. The power, on x, in any term is $\dfrac{16-3r}{4}$.

For integer power of x, $r \in \{0,4,8\}$. So, there are 6 terms with non-integer powers