Sri Chaitanya IIT Academy, India

A.P, TELANGANA, KARNATAKA, TAMILNADU, MAHARASHTRA, DELHI, RANCHI

A right Choice for the Real Aspirant

ICON CENTRAL OFFICE, MADHAPUR-HYD

Sec: Sr. IPLCO **Date: 05-12-15**

Time: 9:00 AM to 12:00 Noon **RPTM-13** **Max.Marks: 360**

KEY SHEET

PHYSICS		CHEMISTRY		MATHS	
Q.NO	ANSWER	Q.NO	ANSWER	Q.NO	ANSWER
1	2	31	4	61	3
2	3	32	2	62	2
3	4	33	2	63	3
4	3	34	2	64	4
5	1	35	3	65	3
6	2	36	2	66	2
7	2	37	3	67	4
8	3	38	4	68	4
9	2	39	2	69	3
10	4	40	4	70	2
11	1	41	4	71	2
12	4	42	1	72	1
13	1	43	1	73	4
14	3	44	4	74	2
15	1	45	4	75	2
16	2	46	2	76	2
17	4	47	4	77	1
18	3	48	3	78	1
19	1	49	4	79	2
20	3	50	3	80	3
21	2	51	4	81	3
22	2	52	4	82	3
23	4	53	2	83	3
24	3	54	3	84	1
25	3	55	1	85	1
26	1	56	3	86	3
27	4	57	1	87	2
28	2	58	2	88	4
29	4	59	2	89	3
30	1	60	1	90	2

MATHS

61. let $a=\tan^2\alpha, b=\tan^2\beta$

$$Q=\frac{(a+1)^2}{b}+\frac{(b+1)^2}{a}=\left(\frac{a^2}{b}+\frac{b^2}{a}+\frac{1}{a}+\frac{1}{b}\right)+2\left(\frac{a}{b}+\frac{b}{a}\right) \rightarrow \text{Requirement}$$

Using $[AM \ge GM]$

$$\Rightarrow \frac{a^2}{b}+\frac{b^2}{a}+\frac{1}{a}+\frac{1}{b}\ge 4 \qquad ...(1)$$

$$\Rightarrow 2\left(\frac{a}{b}+\frac{b}{a}\right)\ge 4 \qquad ....(2)$$

62. $$\beta=\frac{\sin\theta}{\cos 3\theta}+\frac{\sin 3\theta}{\cos 9\theta}+\frac{\sin 9\theta}{\cos 27\theta}$$

$$\beta=\frac{\sin\theta\cos\theta}{\cos 3\theta\cos\theta}+\frac{\sin 3\theta\cos 3\theta}{\cos 9\theta\cos 3\theta}+\frac{\sin 9\theta\cos 9\theta}{\cos 27\theta\cos 9\theta}$$

$$\therefore 2\beta=\frac{\sin 2\theta}{\cos 3\theta\cos\theta}+\frac{\sin 6\theta}{\cos 9\theta\cos 3\theta}+\frac{\sin 18\theta}{\cos 27\theta\cos 9\theta}$$

$$=\frac{\sin(3\theta-\theta)}{\cos 3\theta\cos\theta}+\frac{\sin(9\theta-3\theta)}{\cos 9\theta\cos 3\theta}+\frac{\sin(27\theta-9\theta)}{\cos 27\theta\cos 9\theta}$$

$\therefore \alpha = 2B$

63. The maximum value exist if

$2a^2-1-\cos^2 x=0$ or $\cos^2 x=2a^2-1$ or $\sin^2 x=2-2a^2$ or $2a^2+\sin^2 x=2$

Now

$$\left|\sqrt{2a^2+\sin^2 x}-\sqrt{2a^2-1-\cos^2 x}\right|=\left|\sqrt{2}-0\right|\le A$$

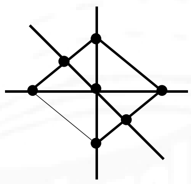
$\therefore A=\sqrt{2}$

64. $$8x^3+4x^2-4x-7=8\left(x-\cos\frac{2\pi}{7}\right)\left(x-\cos\frac{4\pi}{7}\right)\left(x-\cos\frac{6\pi}{7}\right)$$

Differentiate w.r.t x on both sides and put x = 1

65. (1, 0) (-1, 0) (0, 1) (0, -1)

$\left(\frac{-1}{2},\frac{1}{2}\right)\left(\frac{1}{2},\frac{-1}{2}\right)$



66. Let

$a=\sin x, b=\cos x$ and $c=\sin x+\cos x$

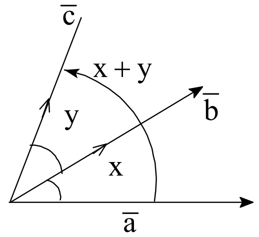
$c\in\left[-\sqrt{2},\sqrt{2}\right]$

$E=\left|a+b+\frac{a}{b}+\frac{b}{a}+\frac{1}{a}+\frac{1}{b}\right|$ note that $c^2=2ab+1$

$$\therefore E=\left|c-1+\frac{2}{c-1}+1\right|$$

$E_{min}=\left|-2\sqrt{2}+1\right|=2\sqrt{2}-1$ when $c=1-2\sqrt{2}$

67.



Let $\overline{a},\overline{b},\overline{c}$ be unit vectors in a plane such that

$\overline{a}.\overline{b}=\cos x;\overline{b}.\overline{c}=\cos y;\overline{a}.\overline{c}=\cos(x+y)$

$\left|\overline{a}+\overline{b}+\overline{c}\right|^2\ge 0$

$\Rightarrow \cos x+\cos y+\cos(x+y)\ge -\frac{3}{2}$

68. $\left(\sqrt{p}+\sqrt{q}+\sqrt{r}\right)^2=p+q+r+2\sqrt{pq}+2\sqrt{qr}+2\sqrt{rp}$

$\le p+q+r+p+q+q+r+r+p$ (AM – GM Inequality)

$\sqrt{p}+\sqrt{q}+\sqrt{r}\le\sqrt{3}\sqrt{p+q+r}\le 4\sqrt{3}$

69. $\tan^4 x+\cot^4 x=3\sin^2 y-1$

$\text{LHS}\ge 2;\text{RHS}\le 2$

Each must be 2

$\tan^2 x=1\Rightarrow x=\pm\frac{\pi}{4};\sin^2 y=1\Rightarrow y=\pm\frac{\pi}{2}$

Points are $\left(\frac{\pi}{4};\frac{\pi}{2}\right);\left(\frac{\pi}{4};\frac{-\pi}{2}\right);\left(\frac{-\pi}{4};\frac{\pi}{2}\right);\left(\frac{-\pi}{4};\frac{-\pi}{2}\right)$

70. The given equation is $\left(\sqrt{3}\sin x+\cos x\right)^{\sqrt{\sqrt{3}\sin 2x-\cos 2x+2}}=4$

$\Rightarrow\left[2\sin\left(x+\frac{\pi}{6}\right)\right]^{2\sin\left(x+\frac{\pi}{6}\right)}=4$

$\Rightarrow\left[2\sin\left(x+\frac{\pi}{6}\right)\right]=\pm 2\Rightarrow x+\frac{\pi}{6}=2n\pi\pm\frac{\pi}{2};x=2n\pi\pm\frac{\pi}{2}-\frac{\pi}{6}$

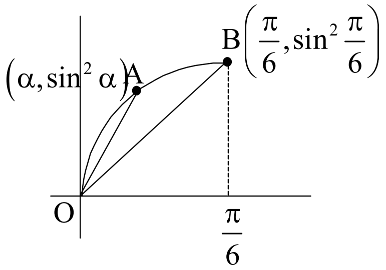
71. we have $1+2\cos 2a_k=1+2\left(1-2\sin^2 a_k\right)=3-4\sin^2 a_k=\frac{\sin 3a_k}{\sin a_k}$

Put $a_k=\frac{3^k\pi}{3^n+1}\Rightarrow a_{k+1}=3a_k$

Given product $=\frac{\sin 3a_n}{\sin a_1}=\frac{\sin\left(\frac{3^{n+1}\pi}{3^n+1}\right)}{\sin\left(\frac{3\pi}{3^n+1}\right)}=1$

72. Conceptual

73.



Slope of OA > slope of OB

74. $\frac{\cos(A+B+C)}{\cos A\cos B\cos C}=1-\sum\tan A\tan B$

75. $x\sin a+y\sin 2a+z\sin 3a=\sin 4a$

$\Rightarrow x+2y\cos a-4z\sin^2 A+3z=4\cos A\cos 2A$

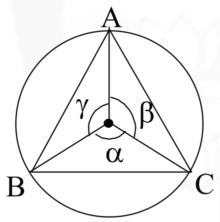
Put $t=\cos A$

Then equation reduces to $8t^3-4zt^2-(2y+x)t+z-x=0$

$\Rightarrow$ roots are $\cos a,\cos b,\cos c$

76. Difference of the roots of both the equations are same.

77.



Clearly, $\angle A=\frac{\alpha}{2},\angle B=\frac{\beta}{2},\angle C=\frac{\gamma}{2}$

$\therefore \alpha+\beta+\gamma=2\pi$

$A.M=\frac{1}{3}\left[\cos\left(\alpha+\frac{\pi}{2}\right);\cos\left(\beta+\frac{\pi}{2}\right);\cos\left(\gamma+\frac{\pi}{2}\right)\right]$

$=-\frac{4}{3}\sin A\sin B\sin C$

$\therefore A=B=C=\frac{\pi}{3}$

Least $A.M.=-\frac{4}{3}\left(\frac{\sqrt{3}}{2}\right)^3=-\frac{\sqrt{3}}{2}$

78. $16(\sin^5 x+\cos^5 x)-11(\sin x+\cos x)=0$

$\Rightarrow(\sin x+\cos x)(4\sin x\cos x-1)(4\sin x\cos x+5)=0$

We have $\sin x+\cos x=0, 4\sin x\cos -1=0$

There are 6 solutions in $[0,2\pi]$

79. $\text{Tan}\left(\frac{\pi}{4}+\frac{y}{2}\right)=\text{Tan}^3\left(\frac{\pi}{4}+\frac{x}{2}\right)$

$$\frac{1+\tan\frac{y}{2}}{1-\tan\frac{y}{2}}=\left(\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}\right)^3$$

Square both sides, we get $\frac{1+\sin y}{1-\sin y}=\left(\frac{1+\sin x}{1-\sin x}\right)^3$

Using componendo and dividend

$$\frac{2\sin y}{2}=\frac{\left(3+\sin^2 x\right)}{1+3\sin^2 x}\sin x$$

80. $1-\cos x=\frac{\sqrt{3}}{2}|x|+a$

Both sides of the equations are even functions

i.e., $y=1-\cos x$ and $y=\frac{\sqrt{3}}{2}|x|+a$ should not meet any where for $x\in R^+$ at the point

$P=\left(\frac{2\pi}{3},\frac{3}{2}\right)$

They touch each other

$$\Rightarrow\frac{3}{2}=\frac{\sqrt{3}}{2}.\frac{2\pi}{3}+a\Rightarrow a=\frac{3}{2}-\frac{\pi}{\sqrt{3}}$$

$$\therefore a\in\left(\frac{3}{2}-\frac{\pi}{\sqrt{3}},\infty\right)$$

81. Draw graph

82. $0<\alpha<\beta<\gamma<\frac{\pi}{2}\Rightarrow\sin\alpha<\sin\beta<\sin\gamma$

$$f(x)=(x-\sin\beta)(x-\sin\gamma)+(x-\sin\gamma)(x-\sin\alpha)+(x-\sin\alpha)(x-\sin\beta)$$

$$f(\sin\alpha)>0,f(\sin\beta)<0;f(\sin\gamma)>0$$

By intermediate value property. It has real and unequal roots.

83. $\left(\sin^2 x-1\right)^2+\left(\cos^2 y-1\right)^2+\left(\sin x-\cos y\right)^2=0$

$$\Rightarrow\sin^2 x=1;\cos^2 y=1;\sin x=\cos y$$

84. $y=\frac{1}{3}\left[\sin x+\left[\sin x+(\sin x)\right]\right]$

$$\Rightarrow y=[\sin x]\quad\text{and}\quad[y]+[y]=2\cos x\Rightarrow[y]=\cos x$$

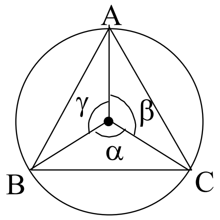
Use three case

Case (i) : $-1\le\sin x\le 0$

Case (ii) : $0\le\sin x<1$

Case (ii) : sinx = 1

85.



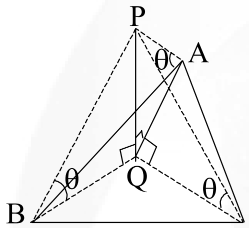
Arc (BC) = 3 $\Rightarrow \frac{\ell}{r} = \alpha \Rightarrow 3 = r\alpha$

Similarly $4 = r\beta; 5 = r\gamma$

$\Rightarrow 3+4+5 = r(\alpha+\beta+\gamma) \Rightarrow r = \frac{6}{\pi}$

Area $(\Delta ABC) = \Delta OAB + \Delta OBC + \Delta OCA$

86.



Let PQ be the pole that stands in the triangular park ABC Then,

$\angle PAQ = \angle PBQ = \angle PCQ = \theta$ (say).

In rt. ΔAPQ, we have $\frac{PQ}{AQ} = \tan\theta \Rightarrow AQ = PQ\cot\theta .....(1)$

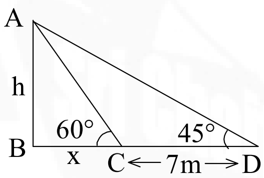
In rt. ΔBPQ, we have $\frac{PQ}{BQ} = \tan\theta \Rightarrow BQ = PQ\cot\theta .....(2)$

In rt. ΔCPQ, we have $\frac{PQ}{CQ} = \tan\theta \Rightarrow CQ = PQ\cot\theta .....(3)$

From (1), (2) & (3), we get $AQ = BQ = CQ = R(\text{say})$

$\therefore Q$ is the circumcentre of ΔABC

87.



Let the height of the pole be h metres. i.e. AB = h metres. Let BC = x metres.

We have $\angle ACB = 60°, \angle ADB = 45° \text{ and } CD = 7m.$

In rt. $\Delta ABC,$ we have $\frac{AB}{BC} = \tan 60° \Rightarrow \frac{h}{x} = \sqrt{3} \Rightarrow x = \frac{h}{\sqrt{3}} \quad ....(1)$

In rt. ΔABD, we have $x = h - 7 \quad ...(2)$

From (1) & (2) we have $h = \frac{7\sqrt{3}}{\sqrt{3}-1} = \frac{7\sqrt{3}}{2}(\sqrt{3}+1)m$

88.

Let the height of the tower be h metres *i.e.* $PQ = h$ metres.

In equilateral ΔPAM we have:

$PN \perp AM$.

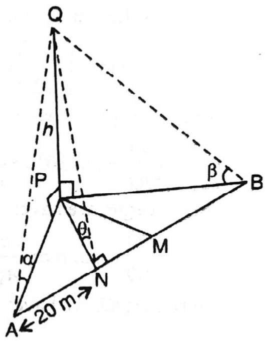
$\therefore\ AN = NM = 20$ m. *i.e.* $AM = 40$ m and

so $PA = PM = AM = 40$ m.

Now, in rt. ΔPAN we have:

$PN = \sqrt{PA^2 - AN^2}$

$= \sqrt{40^2 - 20^2} = \sqrt{1200} = 20\sqrt{3}$ m



Let the angle of elevation of the top of the tower at N be θ. Then, $\theta = \tan^{-1} 2$ (given)

$\therefore\quad \angle PNQ = \theta = \tan^{-1} 2$.

In rt. ΔPNQ we have:

$$\frac{PQ}{PN} = \tan\theta \Rightarrow \frac{h}{20\sqrt{3}} = \tan(\tan^{-1} 2) \Rightarrow \frac{h}{20\sqrt{3}} = 2$$

$\Rightarrow\quad h = 40\sqrt{3}$ m.

i.e. The height of the tower $= h = 40\sqrt{3}$ m.

89. we have $\sum_{m=1}^{6} \operatorname{cosec}\left(\theta + \frac{(m-1)\pi}{4}\right)\operatorname{cosec}\left(\theta + \frac{m\pi}{4}\right) = 4\sqrt{2}$

$$\sum_{m=1}^{6} \frac{\sin\left(\left(\theta + \frac{m\pi}{4}\right) - \left(\theta + \frac{(m-1)\pi}{4}\right)\right)}{\sin\left(\theta + \frac{(m-1)\pi}{4}\right)\sin\left(\theta + \frac{m\pi}{4}\right)} = 4$$

$$\sum_{m=1}^{6}\left[\cot\left(\theta + \frac{(m-1)\pi}{4}\right) - \cot\left(\theta + \frac{m\pi}{4}\right)\right] = 4$$

By telescopic, we get $\theta = \frac{\pi}{12}$ or $\frac{5\pi}{12}$

90.

On squaring both sides, we get

$1 + \sin\pi(1 - x) = |\log|x||^3 + 1$

$\Rightarrow\ \sin\pi x = (|\ln|x||)^3$

There are 6 solutions 3 right side of y – axis and 3 left side of y – axis.