16-08-15_Sr. IPLCO_JEE-ADV_(2012_P2)_RPTA-3_Key &Sol's



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A right Choice for the Real Aspirant ICON CENTRAL OFFICE, MADHAPUR-HYD

 Sec: Sr.IPLCO
 JEE-ADVANCE
 Date: 16-08-15

 Time: 3 Hours
 2012-P2-Model
 Max Marks: 198

KEY & SOLUTIONS

PHYSICS

1	A	2	D	3	D	4	С	5	A	6	D
7	D	8	С	9	A	10	С	11	A	12	A
13	В	14	С	15	AC	16	ACD	17	ВС	18	AB
19	CD	20	CD								

CHEMISTRY

											
21	В	22	A	23	С	24	A	25	A	26	В
27	D	28	В	29	В	30	С	31	A	32	С
33	В	34	С	35	CD	36	ACD	37	ABCD	38	ABC
39	ВС	40	AC	6 9					W	D_{ij}	

MATHS

41	D	42	D	43	C	44	В	45	A	46	A
47	C	48	D	49	D	50	C	51	A	52	D
53	A	54	C	55	BCD	56	AC	57	ABC	58	BD
59	ВС	60	ABD								

MATHS

41.

Sol:- If ellipse and hyperbola intersects at (x,y) then

$$x^2 = \frac{\ell_2^2}{e_e^2}, e_e = \text{Eccentricity of ellipse}$$

$$(SP)(S'P) = \ell_1^2 - e_e^2 x^2 = \ell_1^2 - \ell_2^2$$

42. Transverse axis is the equation of the angle bisector containing point (2,3), which is given by

$$\frac{3x-4y+5}{5} = \frac{12x+5y-40}{13}$$

or
$$21x + 77y = 265$$

- 43. conceptual
- 44. Point lies on $25x^2 + 9y^2 = 450$ and is outside $3x^2 + 5y^2 = 32$
- 45.

Sol:- $\frac{x^2}{169} + \frac{y^2}{25} = 1$ Equation of normal at the point $(13\cos\theta, 5\sin\theta)$ is

(1)
$$\frac{13x}{\cos\theta} - \frac{5y}{\sin\theta} = 144$$
. It passes through (0,6), So

$$(15 + 72\sin\theta) = 0$$

$$\Rightarrow \sin \theta = -\frac{5}{24}$$

$$\Rightarrow \theta = 2\pi - \sin^{-1}\left(\frac{5}{24}\right),$$

and
$$\pi + \sin^{-1} \frac{5}{24}$$

Also the y-axis is one of the normals.

- 46.
- 47.
- 48.

Paragraph Question:-

Paragraph - I

49-50

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Sol:- (ON)(OQ) =
$$2c^2 \Rightarrow c = \sqrt{2}$$

Equation of hyperbola is (x-1)(y-2) = 2. Its foci (-1,0) (3,4) and equation of directrix is x + y - 5 = 0, x + y - 7 = 0 is the equation of latus rectum of ellipse and tangents at end points of latus rectum always intersects on corresponding directrix.

Paragraph - II

51. Solving the curves given (eliminating x^2), we have

$$\frac{r^2 - y^2}{16} + \frac{y^2}{9} = 1$$

$$\Rightarrow y^2 = \frac{144 - 9r^2}{7}$$

Solving the curves given (eliminating y²), we have

$$\frac{x^2}{16} + \frac{r^2 - x^2}{9} = 1$$

$$\Rightarrow x^2 = \frac{16r^2 - 144}{7}$$

If ABCD is a square, then

$$x^2 = y^2$$

or
$$\frac{144-9r^2}{7} = \frac{16r^2-144}{7}$$

or
$$25r^2 = 288$$

or
$$r = \frac{12}{5}\sqrt{2}$$

52. Tangents of slope m to circle and ellipse is $y = mx \pm \sqrt{r^2m^2 + r^2}$ and $y = mx \pm \sqrt{16m^2 + 9}$, respectively. For common tangent, $r^2m^2 + r^2 = 16m^2 + 9$

Also if A'B'C'D' is a square, then

$$m = \pm 1$$

$$\Rightarrow r^2 + r^2 = 25$$

$$\Rightarrow r = 5 / \sqrt{2}$$

Paragraph - III

- 53. Equation of the ellipse is $2x^2 + 6y^2 = 9$
- 54.
- 55. Conceptual

Let the coordinates of point A are (ct, c/t) 56.

So, the slope of normal at A will be t^2

And normal will be parallel to BC.

So, t will be $\pm\sqrt{2} \Rightarrow c = \pm 2$

57. a,b,c are in A.P \Rightarrow ax + by + c = 0 are concurrent at (1,-2)

∴ centre of auxiliary circle = $(-\alpha, -\beta)$ = (1, -2)

Radius of aux. circle = 2; Length of major axis = 4 = 2A

$$\therefore \frac{1}{SP} + \frac{1}{SQ} = \frac{2A}{B^2} \Rightarrow B = \sqrt{3}$$
, hence $e = \frac{1}{2}$

Other asymptote is the image of y = 2x in the line x = y .i.e, x = 2y58.

 \Rightarrow Hyperbola is (x-2y)(2x-y) = K

: It passes through $(3, 4) \Rightarrow K = -10$

∴angle between asymptotes = 2 sec⁻¹ e

$$\Rightarrow \tan^{-1}\left(\frac{3}{4}\right) = 2\sec^{-1}e \Rightarrow e = \frac{\sqrt{10}}{3}$$

59. conceptual

60.

The equation of chord joining P and Q is $x + yt_1t_2 = c(t_1 + t_2)$ (i)

And the equation of chord joining Q and R is $x + yt_2t_3 = c(t_2 + t_3)$ (ii)

Let Eq. (i) be parallel to $y = m_1x$ and Eq. (ii) be parallel to $y = m_2x$.

$$m_1 = -\frac{1}{t_1 t_2} \qquad \text{and} \qquad m_2 = -\frac{1}{t_2 t_3}$$

$$\frac{m_1}{m_2} = \frac{t_3}{t_1} i e, t_3 = \left(\frac{m_1}{m_2}\right) t_1 \qquad \dots (iii)$$

Again the equation to the third side RP is

on to the third side RP is
$$x + yt_3t_1 = c(t_3 + t_1)$$

$$x + y\left(\frac{m_1}{m_2}\right)t_1^2 = c\left(\frac{m_1}{m_2}t_1 + t_1\right)$$
or

$$ym_1t_1^2 - ct_1(m_1 + m_2) + xm_2 = 0$$
(iv)

t₁ being a parameter. Since, t₁ is real the envelope of Eq.(iv) is given by the discriminate of Eq.(iv) = 0 ie, $c^2(m_1 + m_2)^2 - 4ym_1.xm_2 = 0$ or $4m_1m_2xy = c^2(m_1 + m_2)^2$

 m_1 and m_2 are roots of $x^2 - 6x + 1 = 0$...

$$m_1 + m_2 = 6, m_1 m_2 = 1$$

Then from Eq. (v), $4xy = c^2(6)^2$ Then $xy = 9c^2$:

$$\lambda = 9$$