MATHS

01. If the function $f(x)=\frac{\log_{\cos x}^{\cos 2x}}{\log_{\sin x}^{\sin 2x}}$ is continuous from the right of '0' then $f(0)=$

1) $\frac{1}{16}$ 2) $\frac{1}{8}$ 3) 4 4) $\frac{1}{4}$

02. $f(x)=\frac{ax^2+bx+c}{x-2}, x\neq 2$ and $f(2)=8$. $g(x)=\frac{ax^2+bx+c}{x-1}$ and $g(1)=-8$. If f, g are continuous functions on R then the value of $\frac{b}{c}$ is

1) 2 2) $\frac{-3}{2}$ 3) $-\frac{4}{3}$ 4) -3

03. $f(x)=\begin{cases} x^2 & ,x<0 \\ -x^2 & ,x\geq 0 \end{cases}$ and $g(x)=|x-1|$. The number of points at which the composite function gof is not differentiable is

1) 3 2) 4 3) 0 4) 1

04. If $f(x) =$ min $\{1+x, \sqrt{1-x}\}$ then the number of points at which f is not differentiable is

1)0 2)2 3)1 4)4

05. f , g are two continuous functions such that $f(1)=4, f(2)=3;\ g(1)=1$ and $g(2)=2$. Then there is always a, $"c" \in (1,2)$ such that $3f(c)-4g(c)=\lambda$ if $\lambda=$

1)4 2)10 3)$\frac{1}{2}$ 4)9

06. The function $f(x) = x^2 \sin\frac{1}{x} + \left|x^2 - 3x + 2\right| + 1, x \neq 0; f(0) = 3$ is

1)discontinuous at $x=0$

2)Non –differentiable at $x=0$

3)continuous but non-differentiable at $x=0$

4)Differentiable at $x=0$

07. If $f:R \to R$ is continuous and $f(x)-f(y)=[x](x-y)$ whenever $[x]=[y]$ and $f(1)=2$ Then $f(5)=$

([p] Denotes greatest integer not exceeding p)

1)4 2)6 3)10 4)12

08. f is a real function such that $f(x+y)=e^x f(y)+e^y f(x)$,$\forall x, y \in R$ if $f'(0)=4$ then $f(3)=$

1)$12e^3$ 2)$3e^2$ 3)$12e^2$ 4)$3e^3$

09. A function f continuous on R satisfies

(i) $f(1)=2$ (ii) $f'(x)=2$ if $x<0$

(iii) $f'(x)=1$ *if* $0<x<1$ (iv) $f'(x)=-2$ *if* $x>1$

Then $f(5)=$

1)-4 2)4 3)-6 4)6

10. If $f(x) = 2x + \log x$ and $g = f^{-1}$ then $g'(2) =$

1) e 2) $\frac{5}{2}$ 3) $\frac{1}{3}$ 4) 3

11. If $f(x) = \cos^{-1}\left(\frac{x - x^{-1}}{x + x^{-1}}\right)$ then $f'(-2) =$

1) $\frac{2}{5}$ 2) $\frac{-2}{5}$ 3) $\frac{1}{5}$ 4) $\frac{-1}{5}$

12. A real function f is defined by $f(x+2y) = f(x) + mf(y)$, $\forall x, y \in R$ where ‘ m’ is a non zero constant. If $f'(0) = 3$ and $f(2) = 6$ then m =

1) 1 2) 2 3) 3 4) 4

13. If $f(x) = \begin{cases} a + (x-b)^2, & |x-b| \le k \\ c + |x-b|, & |x-b| > k \end{cases}$ is differentiable everywhere and a, b ,c are constants, then a-c =

1) 1 2) $\frac{1}{2}$ 3) $\frac{1}{4}$ 4) $\frac{1}{3}$

14. $f:R\to R$ is such that $f\left(x+y^3\right)=f(x)+\left(f(y)\right)^3, \forall x,y\in R$. The number of such functions 'f' which are differentiable at "0" is

1)1 2)2 3)3 4)0

15. If $f(x)=1-\frac{x^2}{2}+\frac{x^3}{6}$ for $x\le 0$ and $f(x)=(x+1)e^{-x}$ for x > 0 then

1)f is not differentiable 2) f' is not differentiable

3) f'' is differentiable 4) f'' is not differentiable

16. If $f:(-1,1)\to(-1,1)$ is defined such that

$f(x)+f(y)=f\left(\frac{x\sqrt{1-y^4}+y\sqrt{1-x^4}}{1+x^2y^2}\right), \forall x,y\in(-1,1)$ and $f'(0)=1$ then $f'\left(\frac{1}{\sqrt{3}}\right)=$

1) $\frac{\sqrt{2}}{3}$ 2) $\frac{\sqrt{3}}{2}$ 3) $\frac{1}{2\sqrt{2}}$ 4) $\frac{3}{2\sqrt{2}}$

17. If $f(x)=\frac{1-\cos(x-\sin x)}{x^6}, x\ne 0$ is continuous at 0, then $f(0)=$

1) $\frac{1}{6}$ 2) $\frac{1}{72}$ 3) $\frac{1}{36}$ 4) $\frac{1}{24}$

18. If $f(x)=\underset{n\to\infty}{Lt}\dfrac{\ln(x+2)-x^{2n}\sin(x^2-1)}{x^{2n}+1}, x>-2, n\in N$ then the number of points of discontinuity of f(x) is

1)0 2)2 3)1 4)Infinite

19. The number of distinct integral values of k for which the function $f(x)=\sqrt{\dfrac{\log_e(x^2+kx+k+1)}{x^2+k}}$ is continuous on R is

1)5 2)4 3)3 4)Infinite

20. If $f'(x)=\dfrac{1}{\sqrt{1+x^3}}, x>0$ and g is the inverse of f then $g''(x)=$

1)$\dfrac{3}{2}g^2(x)$ 2)$\dfrac{3x^2}{2\sqrt{1+x^3}}$ 3)$\dfrac{3g^2(x)}{2\sqrt{1+g^3(x)}}$ 4)$\dfrac{2\sqrt{1+x^3}}{3x^2}$

21. Consider the following statements

i)If f is a continuous function defined on $[0,1]$ such that $f(0)=f(1)$ then $\exists c\in\left[0,\frac{1}{2}\right]$ such that $f(c)=f\left(c+\frac{1}{2}\right)$.

ii)The equation $x^2=x\sin x+\cos x$ has at least two distinct real roots.

iii)If f is a continuous function defined on $[a,b]$ and $f(a),f(b)$ have same sign then the equation $f(x)=0$ may have a real root between a and b.

iv)If f is defined on an open interval containing 'a' and $Rf'(a)$ and $Lf'(a)$ both exist finitely but are not equal then f is continuous at a.

$\left(Rf'(a)=f'_{+}(a);Lf'(a)=f'_{-}(a)\right)$.

The number of true statements among the above four is

1)1 2)2 3)3 4)4

22. $f:(-\infty,0]\rightarrow[1,\infty)$ is defined by $f(x)=2^{x(x-1)}$. Then $\underset{x\rightarrow 2^{20}}{Lt} f^{-1}(x)$

1) is 5 2)is -4 3)is -5 4)does not exist

23. How many of the following statements are true

i)If f is a real function such that $f(x+y)=f(x)+f(y),\forall x,y\in R$ then either f is continuous everywhere or f is continuous nowhere.

ii)If f is a real function such that $f(xy)=f(x)+f(y),\forall x,y\in R^{+}$ then either f is continuous everywhere on R^{+} or f is continuous nowhere on R^{+}.

iii)If f is a real function such that $f(x+y)=f(x)f(y),\forall x,y\in R$ then either f is continuous everywhere or f is continuous nowhere.

iv)If f is a real function such that $f(x+y)+\cos(x-y)=2f(x)f(y),\forall x,y\in R$ then f is continuous everywhere.

1)4 2)0 3)1 4)3

24. Let a real function f be defined by $f(x)=\begin{cases} x^b \sin\frac{1}{x} & ,x>0 \\ 0 & ,x\le 0 \end{cases}$ where $b\in R$ is a constant.

Then the set of all real values of ' b' for which the function f' is differentiable on R but the function f'' is discontinuous is

1) $(4,+\infty)$ 2) $(3,+\infty)$ 3) $(3,4)$ 4) $(3,4]$

25. $f:R\to R$ is a function such that $f(x+y)=f(x)+f(y)+2xy-3$,$\forall x,y\in R$ and $f'(0)=-1$. Then $f(5)=$

1)23 2)22 3)24 4)21

26. $f:R\to R$ is a function such that $(f(x))^2=1,\forall x\in R$. What is the number of such functions which are continuous at all real values of x except at x = 0 ?

1)6 2)4 3)2 4)Infinite

27. f(x) is a cubic polynomial function whose derivative vanishes at x = 1 and x = 3. If $f(1)=6$ and $f(3)=2$ then $f(x)=0$ has

1)a unique real root which is rational 2)three distinct real roots

3)one irrational root and two imaginary roots 4) two distinct real roots

28. Let $f(x)=\begin{cases} \dfrac{\sin x - x}{x} & , x<0 \\ \int_0^x [t]\,dt & , x \geq 0 \end{cases}$ where $[t]$ represents the greatest integer not exceeding t. Then f is discontinuous at

1)all positive integral values of x 2)0 only

3)all non negative integral values of x 4)no real value of x

29. $f:[a,b]\to(0,+\infty)$ is a continuous function (a<b) then which of the following numbers may not belong to the range of f?

1) $\left(f(a)\right)^{\frac{1}{2016}}\left(f(b)\right)^{\frac{2015}{2016}}$ 2) $\frac{3f(a)+4f(b)}{7}$

3) $\frac{2f(a)f(b)}{f(a)+f(b)}$ 4) $\log_e\left(f(a)+f(b)\right)$

30. If a curve $y=f(x)$ defined by $y^2+\sin y+x^2=4$ passes through the point (-2,0) then $\left(\frac{d^2y}{dx^2}\right)_{x=-2}$ is

1) 15 2) -34 3) -16 4) 18