Sri Chaitanya IIT Academy, India

A.P, TELANGANA, KARNATAKA, TAMILNADU, MAHARASHTRA, DELHI, RANCHI

A right Choice for the Real Aspirant

ICON CENTRAL OFFICE, MADHAPUR-HYD

Sec: Sr. IPLCO **Date: 14-08-15**

Time: 9:00 AM to 12:00 Noon **RPTM-3** **Max.Marks: 360**

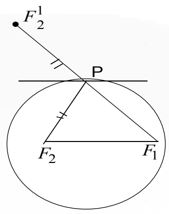
KEY SHEET

PHYSICS			CHEMISTRY			MATHS	
Q.NO	ANSWER		Q.NO	ANSWER		Q.NO	ANSWER
1	1		31	2		61	3
2	1		32	3		62	1
3	4		33	1		63	2
4	3		34	1		64	1
5	1		35	2		65	2
6	2		36	4		66	3
7	1		37	3		67	2
8	3		38	1		68	4
9	2		39	2		69	4
10	2		40	4		70	2
11	4		41	1		71	2
12	1		42	3		72	4
13	1		43	2		73	3
14	3		44	2		74	3
15	3		45	3		75	2
16	1		46	4		76	3
17	3		47	1		77	3
18	4		48	2		78	1
19	3		49	4		79	3
20	4		50	3		80	3
21	4		51	4		81	2
22	4		52	2		82	2
23	1		53	3		83	2
24	1		54	4		84	2
25	3		55	4		85	1
26	4		56	3		86	1
27	2		57	2		87	3
28	2		58	1		88	3
29	3		59	4		89	1
30	3		60	3		90	2

MATHS

61. They form a square of with side $=\frac{32}{\sqrt{7}}$

62. Let F_2' be reflection of F_2 w.r to a tangent at P. Then F_2' lies on circle with F_1 as centre and $F_1F_2' = F_1P + PF_2 = 2a$, as radius. $\therefore$ locus is a circle with radius = 2a.



63. The asymptotes of hyperbola are $x+2y=0,\ x-2y=0$. The tangent at any point $A(2\sec\theta,\tan\theta)$ is given by

$x\frac{\sec\theta}{2} - y\frac{\tan\theta}{1} = 1$. Then the points Q, R are

$(2(\sec\theta+\tan\theta),\sec\theta+\tan\theta)$, $(2(\sec\theta-\tan\theta),\tan\theta-\sec\theta)$

$\therefore CQ.CR = 5$

64. Note that maximum value of $\frac{6}{\sqrt{9\cos^2\theta+3\sin^2\theta}} = 3$

$\therefore$ least length of intercept of tangent $x\frac{\cos\theta}{2} + y\frac{\sin\theta}{3} = 1$

Made by circle $x^2+y^2=25$ is $2\sqrt{25-9}=8$

65. Since origin (0,0) is centre of the conic, it bisects all chords passing through it.

$\therefore m=0, n=0$

66. The normal at a point 't' on $xy=c^2$ meet the curve again at $\frac{-1}{t^3}$

67. $\because \left|SP - S^1P\right| = 2a$ we have $2a=3, 2ae = SS^1 = 5$

$\therefore e = \frac{5}{3}$ Now e_1= eccentricity of conjugate hyperbola is calculated by $\frac{1}{e^2}+\frac{1}{e_1^2}$ =1 $\Rightarrow e_1^2 = \frac{5}{4}$ $\therefore 16\ e_1 = 20$

68. Smallest chord is minor axis. Centre is (0,0)

put $x = r\cos\theta, y = r\sin\theta$ in the equation of curve

which gives $r^2 = \frac{2}{2+\sin 2\theta}$ $\therefore \min imum\ of\ 2r = \sqrt{\frac{8}{3}}$

69. $3(x+1)^2 + 2(y-1)^2 = 6$. ie $\frac{(x+1)^2}{2}+\frac{(y-1)^2}{3} = 1$

The triangle with maximum area on major anis

$=\frac{1}{2}(2\sqrt{3})\sqrt{2} = \sqrt{6}$

70. Slope of normal at $P = \frac{a}{b}$, Slope of normal at $Q = \frac{-a}{b}$

Angle between them is $\frac{\pi}{4}$ implies $1 = \frac{4a^2b^2}{\left(a^2-b^2\right)^2}$

$\Rightarrow (a^2-b^2)^2 = 4a^2b^2$

$\Rightarrow e^4 + 4e^2 - 4 = 0$ $\therefore e^2 = 2\sqrt{2}-2$

71. Equation of chord with mid point $\left(\frac{1}{3},\frac{1}{6}\right)$ is $2x+2y=1$

For the point $(\lambda, \lambda+1)$ in smaller region,

$\lambda^2 + 2(\lambda+1)^2 - 2 < 0$ and $2\lambda + 2(\lambda+1) - 1 > 0$ $\Rightarrow \lambda \in \left(\frac{-1}{4}, 0\right)$

72. $a^2m^2 + b^2 = \frac{a^2}{m^2}, m \neq 0 \Rightarrow \frac{b^2}{a^2} = \frac{1}{m^2} - m^2 > 0 \Rightarrow m \in (-1,1)$

and $\frac{1}{m^2} - m^2 \neq 1$ $\Rightarrow \therefore m\ cannot\ be \left\{0, \pm\sqrt{\frac{\sqrt{5}-1}{2}}\right\}$

73. The length of RS = 2b for the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1.$

If T is foot of $\perp$ from origin on RS

Whose equation $y-0=m(x-ae), where\ m=\sqrt{\frac{r^2-b^2}{a^2-r^2}}$,

Then $RT^2=r^2-\frac{a^2m^2e^2}{1+m^2}=b^2\ and\ RS=2RT$

74. Use $\frac{1}{SP}+\frac{1}{SQ}=\frac{2}{l}$ where $l=$ semilatusrectum

75. Equation of any tangent to hyperbola $\frac{x}{a}\sec\theta-\frac{y}{b}\tan\theta=1$ The equation of pair of lines joining the points of intersection of this line with director circle $x^2+y^2=a^2-b^2$ to origin is

$x^2+y^2-(a^2-b^2)(\frac{x}{a}\sec\theta-\frac{y}{b}\tan\theta)^2=0$. For this pair of lines

Product of slopes $=\frac{\text{c}oeff\ x^2}{\text{c}oeff\ y^2}=\frac{1-\left(\frac{a^2-b^2}{a^2}\right)\sec^2\theta}{1-\left(\frac{a^2-b^2}{b^2}\right)\tan^2\theta}=\frac{b^2}{a^2}$

76. If any tangent of an ellipse, meet the tangents at the ends of major axis, meet at T,T^1 then circle on TT^1 as diameter passes through foci.

$\therefore$ The distance between the foci is the intercept made by given circle on the line $y=x$, which is 8 units.

77. $\frac{x^2}{4}+y^2=1$ let P (h, h-5) be any point on the given line. Then chord of contact of P to the ellipse is

$\frac{xh}{4}+y(h-5)-1=0$ ie $h(x-4y)-20y-4=0$

Which always pass through $\left(\frac{4}{5},\frac{-1}{5}\right)$ for all h

78. The product of Perpendiculars form foci to any tangent of the ellipse is b^2

For ellipse given $a^2 = 16,\ e^2 = \frac{16-b^2}{16}$

$\therefore$ foci are $\left(\pm\sqrt{16-b^2}, 0\right)$

foci of hyperbola given as $(\pm 3, 0)$

$\therefore\ 16 - b^2 = 9 \Rightarrow b^2 = 7$

79. Since P, Q,R,S are conormal points then sum of eccentric angles, of these points is odd multiple of π. Since P,Q,R,T are concyclic then sum of eccentric angles of those points is even multiple of π $\therefore$ If θ_4, θ_5 denote eccentric angles of S,T then $|\theta_4 - \theta_5| = (2K+1)\pi$ for some integer K. $\Rightarrow (0,0)$ is mid point of ST.

80. The equation of normal at (x_1, y_1) on hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

is $\frac{a^2 x}{x_1} + \frac{b^2 y}{y_1} = a^2 + b^2$ i.e $a^2 y_1 x + b^2 x_1 y - \left(a^2 + b^2\right) x_1 y_1 = 0$

$\therefore$ the lines, $a^2 y_i x + b^2 x_i x - \left(a^2 + b^2\right) x_i y_i = 0,\ i = 1,2,3$

will be concurrent if $\begin{vmatrix} x_1 & y_1 & x_1 y_1 \\ x_2 & y_2 & x_2 y_2 \\ x_3 & y_3 & x_3 y_3 \end{vmatrix} = 0$

81. $SP = e\ PM \quad \Rightarrow x^2 + y^2 = 2\left(\frac{x+y+1}{\sqrt{2}}\right)^2$

$\Rightarrow$ equation of hyperbola is $2xy + 2x + 2y + 1 = 0$

.i e $xy + x + y + 1 = \frac{1}{2}$, $\therefore$ Equation of asymptotes are $xy + x + y + 1 = 0$

82. Let $p(x_1 y_1)$ be mid point of chord which make an angle $45°$ with x-axis.

Its equation is $2xx_1 - 3yy_1 = 2x_1^2 - 3y_1^2$

Since it has slope 1, $2x_1 = 3y_1$

$\therefore$ locus of P is line with slope $2/3$

83. Here $e^4 + e^2 = 1 \Rightarrow e^2 = \dfrac{\sqrt{5}-1}{2} = 2\sin 18° \Rightarrow \lambda = 2$, $\therefore \lambda^2 - \lambda - 2 = 0$

84. $y = mx + \sqrt{a^2m^2 + b^2}$ must pass through $(-2,0)$ when $m = 2$

$\Rightarrow 4a^2 + b^2 = 16$, now $AM \geq GM$, gives $|ab| \leq 4$

85. using $\tan\dfrac{B}{2} = \dfrac{\Delta}{s(s-b)}, \tan\dfrac{C}{2} = \dfrac{\Delta}{s(s-c)}$ for a ΔABC

$\cot\dfrac{B}{2}\cot\dfrac{c}{2} = 4$ gives $AB + AC = 10$. $\therefore$ locus of A is an ellipse with B,C as foci , major $axis = 10$

86. Any point on the ellipse $(5\cos\theta, 3\sin\theta)$ will be at $\sqrt{9+16\cos^2\theta}$ distance from axis,

$\therefore\ 9 + 16\cos^2\theta = 13 \Rightarrow \cos^2\theta = \dfrac{1}{4}\tan^2\theta = 3$

Now tangent at $(5\cos\theta, 3\sin\theta)$ form an area

$\dfrac{1}{2}(5\sec\theta)(3\cos ec\theta)$ with coordinate axes.

$\therefore$ required area $= 4 \times \dfrac{1}{2} \times 5\ \ \sec\theta\ 3\cos ec\theta = \dfrac{60}{\sin 2\theta} = 40\sqrt{3}$

87. Tangent to $y^2 = 4ax, y = mx + \dfrac{a}{m}$ it is a tangent to

$4xy = -a^2 \Rightarrow for\ \ 4x\left(mx + \dfrac{a}{m}\right) = -a^2$

$\Delta = 0 \Rightarrow m = 1$

88. $m_1m_2 = \dfrac{y^2 + b^2}{x_1^2 - a^2}$

$\dfrac{y^2+4}{x^2-9} = 4 \Rightarrow y^2 + 4 = 4x^2 - 36 \Rightarrow 4x^2 - y^2 = 40$

$\therefore$ locus is hyperbola $\dfrac{x^2}{10} - \dfrac{y^2}{40} = 1$ whose Eccentricity $\sqrt{\dfrac{10+40}{10}} = \sqrt{5}$

89. Tangent to the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{\lambda^2a^2} = 1$ at $p(a\cos\theta, \lambda\ a\sin\theta)$

$\dfrac{x\cos\theta}{a} + \dfrac{y\sin\theta}{\lambda a} = 1$ must pass through (g,f)

$\Rightarrow \lambda(a - g\cos\theta) - f\sin\theta = 0 \Rightarrow \sin\theta = \dfrac{(a - g\cos\theta)\lambda}{f}$

If $a = g$ then $\sin\theta$ $= (1-\cos\theta)\dfrac{g\lambda}{f} \Rightarrow \cot\dfrac{\theta}{2} = \dfrac{g\lambda}{f}$

90. orthocenter=in center =circumcenter

The orthocenter of the triangle lies again on the same curve

$\therefore$ circumcentre = $(\alpha, 6)$ lies on $xy = 36$ $\therefore\ \alpha = 6$