



Sri Chaitanya IIT Academy, India

A.P, TELANGANA, KARNATAKA, TAMILNADU, MAHARASHTRA, DELHI, RANCHI

A right Choice for the Real Aspirant

ICON CENTRAL OFFICE, MADHAPUR-HYD

Sec: Sr. IPLCO

Time: 9:00 AM to 12:00 Noon

RPTM-4

Date: 22-08-15

Max.Marks: 360

KEY SHEET

PHYSICS		MATHS		CHEMISTRY	
Q.NO	ANSWER	Q.NO	ANSWER	Q.NO	ANSWER
1	3	31	1	61	1
2	2	32	2	62	2
3	3	33	1	63	2
4	2	34	1	64	1
5	4	35	2	65	1
6	1	36	3	66	2
7	1	37	2	67	1
8	3	38	3	68	4
9	2	39	2	69	3
10	1	40	3	70	3
11	2	41	3	71	4
12	4	42	4	72	2
13	2	43	1	73	2
14	4	44	2	74	1
15	4	45	1	75	4
16	1	46	2	76	2
17	3	47	3	77	1
18	1	48	1	78	4
19	4	49	2	79	1
20	2	50	1	80	1
21	3	51	1	81	3
22	1	52	3	82	3
23	3	53	2	83	1
24	3	54	2	84	1
25	4	55	3	85	1
26	2	56	4	86	4
27	1	57	2	87	2
28	4	58	4	88	2
29	1	59	4	89	3
30	1	60	4	90	3

MATHS

31.

$$\begin{aligned}
 OA=OB, \angle AOB &= \alpha \Rightarrow Z_2 = Z_1 \operatorname{cis} \alpha \\
 \Rightarrow Z_2 - Z_1 \cos \alpha &= Z_1 i \sin \alpha \\
 \Rightarrow Z_2^2 + Z_1^2 \cos^2 \alpha - 2Z_1 Z_2 \cos \alpha &= -Z_1^2 \sin^2 \alpha \\
 \Rightarrow Z_1^2 + Z_2^2 &= 2Z_1 Z_2 \cos \alpha \\
 (Z_1 + Z_2)^2 &= 2Z_1 Z_2 (1 + \cos \alpha) \\
 \Rightarrow \frac{p^2}{q} &= 2(1 + \cos \alpha) = 4 \cos^2 \frac{\alpha}{2}
 \end{aligned}$$

$$32. \quad \sum_{r=1}^{10} (r-w)(r-w^2) = 450 \text{ since } w+w^2 = -1, w^3 = 1$$

$$33. \quad z = re^{i\theta}, \bar{z} = re^{-i\theta}$$

$$\left| \frac{z}{r} - \frac{\bar{z}}{r} \right| = |2i \sin \theta| \leq |\arg z - \arg \bar{z}|$$

$$\Rightarrow |z - \bar{z}| \leq |z| |\arg z - \arg \bar{z}|$$

$$34. \quad \text{put } z = x + iy$$

$$35. \quad \left(\frac{1+i}{1-i} \right)^n = \frac{2}{\pi} \sin^{-1} \left(\frac{x^2+1}{2x} \right), x > 0$$

$$\Rightarrow \left(\frac{1+i}{1-i} \right)^n = \frac{2}{\pi} \cdot \frac{\pi}{2}$$

\Rightarrow least value of n is 4

$$36. \quad \frac{z^n - 1}{z - 1} = (z - w)(z - w^2) \dots (z - w^{n-1})$$

Take 'log' both sides and diff w.r. to z both sides and put $z = 2$

$$37. \quad z^5 = (z-1)^5 \Rightarrow |z| = |z-1| = 1, \operatorname{Re}(z) = \frac{-1}{2}$$

\therefore The roots lie on line $\operatorname{Re} |z| = \frac{-1}{2}$

$$38. \quad \log_2 \left| (\alpha + \alpha^2 + \alpha^3 + 1) - \alpha^4 \right|$$

$$\log_2 |-2\alpha^4| = \log_2^2 = 1$$

$$39. \quad z^{100} - 1 = (z-1)(z-\alpha_1)(z-\alpha_2)\dots(z-\alpha_{99}) = 0$$

$$\text{Sum of roots} = 1 + \alpha_1 + \alpha_2 + \dots + \alpha_{99} = 0$$

$$\text{Sum of roots taken two at a time} = 0$$

$$\Rightarrow \sum_{1 \leq i < j \leq 99} \alpha_i \alpha_j + \alpha_1 + \alpha_2 + \dots + \alpha_{99} = 0$$

$$\Rightarrow \sum \alpha_i \alpha_j = 1$$

$$40. \quad \alpha = cis x, \beta = cis y, \gamma = cis z \quad \text{Given}$$

$$\alpha + 2\beta + 3\gamma = 0 \Rightarrow \alpha^3 + 8\beta^3 + 27\gamma^3 = 18\alpha\beta\gamma$$

$$\Rightarrow \sin 3x + 8\sin 3y + 27\sin 3z = 18\sin(x+y+z)$$

$$z_1(z_1^2 - 3z_2^2) = 2 \quad - (1)$$

41.

$$z_2(3z_1^2 - z_2^2) = 11 \quad - (2)$$

$$\text{Multiply (2) by } i \text{ add to (1)} \Rightarrow (z_1 + iz_2)^3 = 2 + 11i$$

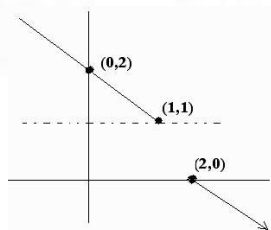
$$\text{Multiply (2) by } i \text{ subtract from (1)} \Rightarrow (z_1 - iz_2)^3 = 2 - 11i$$

$$\Rightarrow (z_1 + iz_2)^3 (z_1 - iz_2)^3 = 125$$

$$\Rightarrow (z_1^2 + z_2^2)^3 = 125 \Rightarrow z_1^2 + z_2^2 = 5$$

$$42. \quad \text{Arg}(z - (1+i)) = \begin{cases} \frac{3\pi}{4} & x \leq 1 \\ -\frac{\pi}{4} & x > 2 \end{cases}$$

\therefore The locus of z is Set of two rays



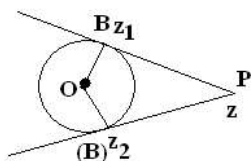
$$43. \quad x = 6 \text{ and } (x-3)^2 + (y+3)^2 = 9 \Rightarrow y = -3$$

44. $OA \perp PA$

$$\frac{z_1}{z_1} + \frac{z - z_1}{z - z_1} = 0$$

Similarly $OB \perp PB \Rightarrow \frac{z_2}{z_2} + \frac{z - z_2}{z - z_2} = 0$

On eliminating \bar{z} we get $z = \frac{2z_1 z_2}{z_1 + z_2}$



45. $\ln \left(\frac{|z-3|^2 + 2}{11|z-3|-2} \right) > 1$

$$\Rightarrow \frac{|z-3|^2 + 2}{11|z-3|-2} < \frac{1}{3}$$

$$\Rightarrow 3t^2 + 6 < 11t - 2$$

$$\Rightarrow 3t^2 - 11t + 8 < 0$$

$$(3t - 8)(t - 1) < 0$$

$$1 < t < 8/3$$

$$1 < |z-3| < 8/3$$

46. For any $z_1, z_2 \in \mathbb{C}$, $|az_1 - bz_2|^2 + |bz_1 + az_2|^2 = (a^2 + b^2)(|z_1|^2 + |z_2|^2)$

47. The locus of z is part of $|z - 7 - 9i| = 3\sqrt{2}$

48. $w = \frac{z-1}{z+1}$

$$\Rightarrow z = \frac{1+w}{1-w}$$

$$|z| = 1$$

$$\Rightarrow |1+w| = |1-w|$$

$\Rightarrow w$ lies on imaginary axis

$$\therefore \operatorname{Re}(w) = 0$$

49. A(-1, 1) B(5, 1) are diameter ends of a circle and P(z) is on the circle

$$|z+1-i|^2 + |z-5-i|^2 = AB^2 = 36$$

50.

$$\alpha + \beta = -a$$

$$\alpha\beta = b^2$$

$$\frac{a^2}{b^2} = \frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2$$

$$= 2 + \frac{2}{r^2} \operatorname{Re}(\alpha\bar{\beta})$$

51.

$$b^2 = ac$$

52.

$$\text{The equation is } 10(x^2 + y^2) - 3i(2x)(2iy) - 6 = 0$$

$$\Rightarrow 5(x^2 + y^2) + 6xy - 8 = 0 \rightarrow (1)$$

Let $(r \sin \theta, r \cos \theta)$ be the point on (1)

$$\text{then } 5r^2 + 6r^2 \sin \theta \cos \theta = 0 \Rightarrow r^2 = \frac{8}{5 + 3 \sin 2\theta}$$

$$\therefore r_1 + r_2 = 2 + 1 = 3$$

$$53. \quad z_1 = r.e^{i\theta} \Rightarrow \tan \theta = \sqrt{2} - 1 \Rightarrow \theta = \frac{\pi}{8} \Rightarrow 2\theta = \frac{\pi}{4} \Rightarrow n = 8$$

$$54. \quad z_1 = \frac{1}{2} - 2i \Rightarrow G.E. = \frac{\sqrt{17}}{2} - \tan^{-1}(4)$$

$$55. \quad G = S \Rightarrow \frac{z_1 + z_2 + z_3}{3} = 1 \Rightarrow z_1 + z_2 + z_3 = 3 \Rightarrow \sum z_1^2 + 2 \sum z_1 z_2 = 9 \Rightarrow \frac{9 - \sum z_1^2}{\sum z_1 z_2} = 2$$

56.

$$z^4 + z^3 + 2 = (z - z_1)(z - z_2)(z - z_3)(z - z_4)$$

$$\text{put } z = \frac{-1}{2}$$

$$57. \quad \text{Maximum value of } |3 + i(3 - i)| \text{ is } 2\sqrt{2}$$

$$G.E. = [2\sqrt{2}] = 2$$

$$58. \quad G.E. \leq 3(4 + 9 + 16) \leq 87$$

$$59. \quad \text{Put } z = x + iy \text{ and find the vertices, Area} = 62 \text{ sq.u}$$

$$60. \quad \text{Clearly } z = -\sqrt{3} - 3i \Rightarrow \frac{z}{2\sqrt{3}} = -\left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)$$