Sri Chaitanya IIT Academy, India

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A right Choice for the Real Aspirant

ICON CENTRAL OFFICE, MADHAPUR-HYD

Sec: Sr. IPLCO	JEE ADVANCED	DATE : 06-12-15
TIME : 3:00	2013_P1 MODEL	MAX MARKS : 180

KEY & SOLUTIONS

PHYSICS

1	**A**	2	**B**	3	**B**	4	**D**	5	**D**	6	**C**
7	C	8	**B**	9	**D**	10	**C**	11	**A,B,C,D**	12	**A,C,D**
13	**A,C,D**	14	**A,C,D**	15	**A,C,D**	16	**3**	17	**4**	18	**2**
19	**3**	20	**1**								

CHEMISTRY

21	**D**	22	**C**	23	**A**	24	**A**	25	**B**	26	**A**
27	**D**	28	**C**	29	**B**	30	**B**	31	**AB**	32	**ABC**
33	**D**	34	**BD**	35	**C**	36	**4**	37	**3**	38	**1**
39	**5**	40	**2**								

MATHEMATICS

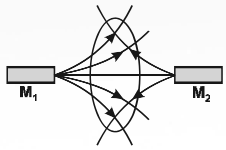
41	**C**	42	**C**	43	**A**	44	**A**	45	**A**	46	**B**
47	**A**	48	**B**	49	**B**	50	**D**	51	**AD**	52	**ABC**
53	**C**	54	**AB**	55	**AB**	56	**3**	57	**4**	58	**3**
59	**1**	60	**8**								

PHYSICS

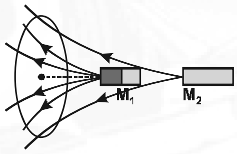
1. $e = \frac{BdA}{dt}$

$= \frac{Bd}{dt}\left(\pi r^2\right) = B2\pi r\frac{dr}{dt}$

2. From the given instant to the instant when M_1 passes through centre of ring, magnetic field is towards right and is increasing in magnitude, so from Lenz's law, the induced current has to be in anti-clockwise direction.



Form the instant of crossing M_1 through centre of ring to collision of M_1 with M_2, the magnetic field is towards left and is decreasing in magnitude. So, from Lenz's law the induced current has to be in anti-clockwise direction.



3. At t = 2s, B = 4T and dB/dt = 2T/s

$\phi = BA$

$A = 20\times30\ cm^2$

anddA/dt = d/dt [20×(40–5t)] = –100cm^2/s

$\frac{d\phi}{dt} = B\frac{dA}{dt} + A\frac{dB}{dt}$

= 0.08 V

$|e| = \left|-\frac{d\phi}{dt}\right| = 0.08V$

4. The two loops are connected in such a way that the currents induced in the loops are always equal in magnitude but opp1osite in direction. That is if current in left loop is clockwise, it is anticlockwise in right loop and viceversa. Thus the emfs induced in the two loops will oppose each other.

The emf induced in first loop

$\varepsilon_1 = \frac{d}{dt}\left(a_2 B\right) = a^2\frac{dB}{dt}$

$= a^2\frac{d}{dt}\left(B\sin\omega t\right)$

The emf induced in second loop

$\varepsilon_2 = \frac{d}{dt}\left(b^2 B\right) = b^2\frac{dB}{dt}$

$= b^2\frac{d}{dt}\left(B_0\sin\omega t\right) = b^2 B_0\omega\cos\omega t$

Net emf induced,

$\varepsilon = \varepsilon_1 - \varepsilon_2 = \left(a^2 - b^2\right) B_0 \omega \cos \omega t$

Total resistance of current = 4 (a + b) r

$\therefore$ Instantaneous current at time t

$i = \frac{\varepsilon}{R} = \frac{\left(a^2 - b^2\right) B_0 \omega \cos \omega t}{4(a+b)r}$

Maximum value of current induced $(\cos \omega t = 1)$

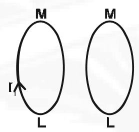
$i_0 = \frac{\left(a^2 - b^2\right) B_0 \omega}{4(a+h)r} = \frac{(a-b) B_0 \omega}{4r}$

Here a = 0.20 m, b = 0.10 m, $B_0 = 10^3$ T,

$r = 50 \times 10^{-3}\ \Omega/m$, $\omega = 100\ rad/s$

$\therefore i_0 = \frac{[(0.20)-(0.10)] \times 10^{-3} \times 100}{4 \times 50 \times 10^{-3}} = 0.5A$

5. Let the coefficients of self – inductance and mutual inductance be L and M, respectively. Flux linked with coil 1 is due to its own current and due to current in coil 2.



Due to current I_1, then flux generated in coil 1 is LI_1 out of which 40% is linked with other, so the remaining flux linked with 1 is 0.6 LI_1, in addition to this, the flux linked with coil 1 due to coil 2 would be same as 40% of LI_1.

So, $0.6LI_1 + 0.4\ LI_1 = \phi_1$

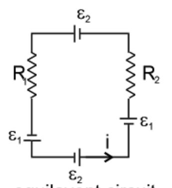
$\Rightarrow LI_1 = 10^{-6} \Rightarrow L = 1\mu H.$

$MI_1 = 0.4\ LI_1 = 0.4 \times 10^{-6}$

$\Rightarrow M = 0.4\mu\ H$

$e = \frac{2MI_1}{0.3} = \frac{2 \times 0.4 \times 10^{-6}}{0.3} = \frac{8}{3}\mu V$

6.



equilavent circuit

$i = \frac{2\varepsilon_1 + 2\varepsilon_2}{R_1 + R_2} = \frac{\varepsilon}{R_1 + R_2}$

Where $\varepsilon = \frac{d\phi}{dt}$ is the net emf in the circuit.

$\therefore V_1 - V_2 = (\varepsilon - iR_1) - (\varepsilon - iR_2) = \frac{\varepsilon(R_2 - R_1)}{R_1 + R_2}$

9. When capacitor is shorted,

$\tan 60^\circ = \omega L/R$

$\Rightarrow \omega L = \sqrt{3} \times 100$

When inductor is shorted, $\tan 60^\circ = \frac{1}{\omega CR}$

$\Rightarrow 1/\omega C = \sqrt{3} \times 100$

For L-C-R circuit, $Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = 100\,\Omega$

$I = V/Z = (200/100)A = 2A$

$P = I^2R = 4 \times 100 = 400W$

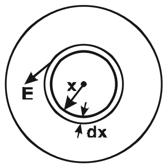
10.

$$\frac{\omega L_1}{R_1} = \frac{4}{3} \Rightarrow \phi_1 = \tan^{-1}\left(\frac{4}{3}\right) \Rightarrow \phi_1 = 53^\circ$$

$$\Rightarrow \phi_2 = \tan^{-1}\frac{\omega L_2}{R_2} = \tan^{-1}\left(\frac{3}{4}\right) \Rightarrow \phi_2 = 37^\circ$$

$$\phi_2 - \phi_1 = \frac{16\pi}{180}$$

12. Here, due to induced electric field, charge element of disc experiences a force which causes a torque and hence the disc rotates.



$dB/dt = 2\beta$

Induced electric field at the location of ring is

E = $\frac{2\alpha \times x}{2} = \beta x$

dq = α ×2πx dx

dτ = dq × E × x dx

= 2πσBx^3 dx

τ = $\int_0^R 2\pi\sigma\beta x^3 dx = \frac{MR^2}{2} \times \alpha$

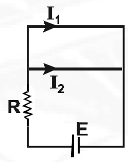
$\Rightarrow \alpha = \frac{\pi\sigma\beta\, R^2}{M}\ for\ t > t_0$

13. For given circuit, at the steady, state

$I = I_1 + I_2$ and $L_1I_1 = L_2I_2$

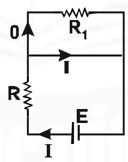
So, $I_1 = \frac{E}{R} \times \frac{L_2}{L_1 + L_2}$

and $I_1 = \frac{E}{R} \times \frac{L_1}{L_1 + L_2}$



When R_1 is connected, then at the steady state, circuit an be redrawn as current through L_1 is 0

Current through L_2 is E/R



Similarly, you can proceed for the case when R_2 is connected.

14. $v = L\frac{di}{dt}$ and $\frac{di}{dt}$ is same for both coils.

$\frac{v_1}{v_2} = \frac{L_1}{L_2} = \frac{8}{2} = 4$ or $\frac{v_2}{v_1} = \frac{1}{4} = 4$

(D) is true

Instantaneous power = vi = $L\frac{di}{dt}(i)$ is same for both coils.

$\therefore L_1 i_1 = L_2 i_2$

$\therefore \frac{i_1}{i_2} = \frac{L_2}{L_1} = \frac{2}{8} = \frac{1}{4}$

(A) is true

Energy store $W = \frac{1}{2}Li^2$

$\therefore \frac{W_1}{W_2} = \frac{L_1}{L_2} \times \left(\frac{i_1}{i_2}\right)^2 = \frac{8}{2}\left(\frac{1}{4}\right)^2 = \frac{1}{4}$

(C) is true

15. When S_1 is closed and (S_2, S_3) are open then, circuit is simply L-R circuit in which current is growing with time. Current in circuit at t = ∞, I = E/R, so magnetic energy stored in the inductor at t = ∞ is $LI^2/2 = LE^2/2R^2$. When S_2 is closed and S_1 is opened the inductor gets shorted but there is no potential difference across the inductor and hence current through inductor for any time t< 0 is zero. When S_2, S_3 both are closed and S_1 is opened, then battery gets disconnected from circuit and is apart of open circuit, so no current flows through battery in this situation.

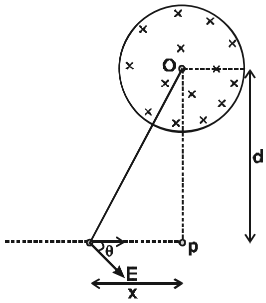
16.

At a distance x from P on a line perpendicular to OP, the induced electric field would be.

$E \times 2\pi\sqrt{x^2+d^2} = \pi R^2 \times \frac{dB}{dt}$

$\Rightarrow E = \frac{KR^2}{2} \times \frac{1}{\sqrt{x^2+d^2}}$

The force experienced by the charged particle due to induced electric field is, $\vec{F} = q\vec{E}$



The component of this electric force along the line perpendicular to OP is,

$$\vec{F}_x = qE\cos\theta = q\times\frac{KR^2}{2}\times\frac{d}{x^2+d^2}$$

Using work – energy theorem,

$\Delta K = W_{\text{external agent}} + W_{\vec{F}}$

$0 = W_{\text{external agent}} + \int -F_x \, dx$

$\Rightarrow W_{\text{external agent}} = \int_0^{\infty} F_x \, dx$

$$= \frac{q\times KR^2\times d}{2}\int_0^{\infty}\frac{dx}{x^2+d^2}$$

$$= q\times\frac{\pi R^2}{4}\times K$$

So, $\alpha = 3$.

17. When the frame has turned through at angle θ,

$\Phi = BA\cos\theta$

where $A = \int_0^y 2x\,dy = \frac{2}{\sqrt{k}}\int_0^y \sqrt{y}\,dy = \frac{4}{3\sqrt{k}}y^{3/2}$

Since $y = \frac{1}{2}at^2$

$$\therefore \Phi = \frac{B}{3}\sqrt{\frac{2}{K}}a^{3/2}t^3\cos\theta$$

By Faraday's law, $E_{ind} = \frac{d\Phi}{dt}$

or $E_{ind} = \frac{Ba^{3/2}}{3}\sqrt{\frac{2}{K}}\left[t^3\sin\theta\left(\frac{d\theta}{dt}\right) - 3t^2\cos\theta\right]$

When the frame turns through $\pi/4$,

$$t = \frac{\theta}{\omega} = \frac{\pi}{4\omega}$$

$$R = \lambda 2x = \lambda.2\sqrt{\frac{y}{k}} = \frac{2\lambda}{\sqrt{k}}\sqrt{\frac{a}{2}}t = \sqrt{\frac{2a}{k}}\frac{\lambda\pi}{4\omega}$$

$$I = \frac{E_{ind}}{R} = \frac{\pi^2(\pi-12)Ba^{3/2}\sqrt{k}\;\;4\omega}{192\omega^2\sqrt{k}\sqrt{2a}\;\;\pi\lambda} = \frac{Ba\pi(\pi-12)}{48\sqrt{2}\;\omega\lambda}$$

18.

First send current through larger loop and calculate flux through smaller one to calculate M.

We get $\left\{\frac{\mu_o i}{2b}\right\}\pi a^2 = Mi \Rightarrow M = \frac{\mu_o \pi a^2}{2b}$.

Now for the current going through smaller loop.

Emf in larger loop = $M\frac{di}{dt} = \frac{\mu_o \pi a^2}{2b}(4t) = \frac{2\mu_o \pi a^2 t}{b}$

$\varepsilon = \frac{q}{c} + iR$

$\Rightarrow \frac{d\varepsilon}{dt} = \frac{i}{c} + \frac{di}{dt}R$

$\frac{2\mu_o \pi a^2}{b} = \frac{i}{c} + \frac{di}{dt}R$

$\Rightarrow$ long time current = i when $\frac{di}{dt} = 0$

$= \frac{2\mu_o \pi a^2 c}{b}$

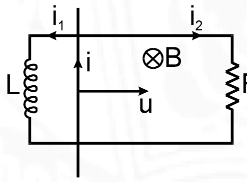
19.

$I_{upper\,branch} = \frac{60}{4} = 12A$

$I_{lower\,branch} = \frac{60}{12} = 5A$

r.m.s current trough the source $= \sqrt{12^2 + 5^2} = 13A$

20. Let i_1 and i_2 be the current through L and R at any time t



$\therefore i = i_1 + i_2 \Rightarrow \frac{B\ell v}{R} = i_2$ and $Bv\ell = L\frac{di_1}{dt}$

Force on conducting rod = $m\frac{dv}{dt}$ = $-i\ell B = -\left(i_1 + \frac{B\ell v}{R}\right)B\ell$

$\Rightarrow \quad m\,dv = -\ell B\, i_1 dt - \frac{B^2\ell^2}{R} v\,dt$

$\Rightarrow \quad m\int dv = -\ell B \int i_1\, dt - \frac{B^2\ell^2}{R}\int v\,dt$

$\Rightarrow \quad m(v_f - u) = -\ell B\, Q - \frac{B^2\ell^2}{R} x$

(v_f = velocity, when it has moved a distance 'x')

$\Rightarrow Q = \frac{-\frac{B^2\ell^2}{R}\times x - m(v_1 - u)}{b\ell} = 1C$