MATHS **Max.Marks:80**

SECTION-1

(SINGLE CORRECT CHOICE TYPE)

Section-I (Single Correct Answer Type, Total Marks: 24) contains 8 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONLY ONE is correct. For each question you will be awarded 3 marks if you darken ONLY the bubble corresponding to the correct answer and zero marks if no bubble is darkened. In all other cases, minus one (-1) mark will be awarded.

41. If the roots of the equation $z^3-(2\cos\alpha+1)z^2+(2\cos\alpha+1)z-1=0$ are the affixes of the vertices of a triangle in the argand diagram $(0<\alpha<\pi)$ perimeter of the triangle when triangle has maximum area is

A) $4\sqrt{3}$ B) $3\sqrt{3}$ C) $2\sqrt{3}$ D) 6

42. The minimum distance between the points z_1 and z_2 where z_1 and z_2 are the points on $\left|\frac{z-1}{z-2}\right|=2$ and $\left|\frac{z-2}{z-1}\right|=2$ respectively, is ________

A) 1 B) 0 C) $2/3$ D) $1/3$

43. Regular hexagon ABCDEF is inscribed in a circle of radius 1. A point P is taken on the circle of radius 2, having the same centre. Then the value of

$PA^2+PB^2+PC^2+PD^2+PE^2+PF^2=$ _____________

A) 18 B) 12 C) 24 D) 30

44. If z and w are two complex numbers simultaneously satisfying the equation

$z^3 + w^5 = 0$ and $z^2(\bar{w})^4 = 1$ then.

A) z & w both are purely real

B) z & w both are purely imaginary

C) z is purely real & w is purely imaginary

D) z is purely imaginary and w is purely real.

45. If $A(z_1) \& B(z_2)$ be two complex numbers satisfying the equation $z + (\sqrt{3} + i)t - i = 0$ & $\sqrt{3}z + (\sqrt{3} + i)\lambda - \sqrt{3}i = 0$ respectively, $\forall \lambda, t \in R$.

Let for some $z_1 \& z_2$, Arg $(z_1) = \pi/4$ & Arg $(z_2) = -\frac{3\pi}{4}$ then area of the triangle.

ABC, is (affix of the vertex C is 0+i) is _____

A) $\frac{\sqrt{3}+1}{2}$ B) $\frac{\sqrt{3}+1}{4}$

C) $\frac{\sqrt{3}+2}{4}$ D) $\frac{\sqrt{3}+2}{2}$

46. Tangent drawn to the circle $|z| = 2$ at $A(z_1)$ and normal drawn to the circle $|z| = 2$ at $B(z_2)$, intersect at $P(z)$ then the length of the line segment PA, is _________

A) $\frac{z_1 i \operatorname{Im}(\overline{z}_1 z_2)}{\operatorname{Re}(\overline{z}_1 z_2)}$ B) $\frac{z_2 i \operatorname{Im}(\overline{z}_1 z_2)}{\operatorname{Re}(\overline{z}_1 z_2)}$ C) $\frac{i \operatorname{Im}(\overline{z}_1 z_2)}{\operatorname{Re}(\overline{z}_1 z_2)}$ D) $\frac{2z_2 i \operatorname{Im}(\overline{z}_1 z_2)}{\operatorname{Re}(\overline{z}_1 z_2)}$

47. Let n be a positive integer, $z^{n+1} - z^n - 1 = 0$ has a root z_1, satisfying $|z_1| = 1$ then $(n+2)$ is divisible by

A) 7 B) 5 C) 4 D) 6

48. Let Arg $(z_k) = \left(\frac{2k+1}{n}\right)\pi$ where $k = 1, 2, 3.., n$.

If Arg $(z_1 z_2 z_3 ... z_n) = \pi$ then n must be of the form

A) 4m, $m \in I$ B) $2m - 1, m \in I$ C) $2m, m \in I$ D) $3m - 1, m \in I$

SECTION-2

(MORE THAN ONE TYPE)

Section - II (Multiple Correct Answers Type, Total Marks: 16) contains 4 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONE or MORE may be correct. For each question you will be awarded 4 marks if you darken ALL the bubble(s) corresponding to the correct answer(s) ONLY and zero marks otherwise. There are no negative marks in this section.

49. If $2\cos\theta = x + \frac{1}{x}$ then

A) $\frac{x^6+1}{x^5+x} = \frac{\cos 3\theta}{\cos 2\theta}$ B) $\frac{x^8+1}{x^7+x} = \frac{\cos 4\theta}{\cos 3\theta}$

C) $\frac{x^4+1}{x^3+x} = \frac{\cos 2\theta}{\cos\theta}$ D) $\frac{x^{10}+1}{x^9+x} = \frac{\cos 5\theta}{\cos 4\theta}$

50. Let the vertices of a triangle ABC, are represented by $A(z_1), B(z_2), C(z_3)$ in the anti clockwise sense. $A'(z_4), B'(z_5)$, $C'(z_6)$ are the exterior points of triangle ABC such that triangles $A'BC$, $B'AC$, $C'AB$ are equilateral triangles. If the centroids of triangles $A'BC$, $B'AC$, $C'AB$, are represented by z_7, z_8, z_9 respectively. Then

A) $z_7 = \frac{z_2(3+i\sqrt{3}) + z_3(3-i\sqrt{3})}{6}$

B) $z_7 + z_8 + z_9 = z_1 + z_2 + z_3$

C) triangle, formed by joining the points whose affixes are z_7, z_8, z_9 is an equilateral triangle

D) $z_4 = \frac{z_2(1+i\sqrt{3}) + z_3(1-i\sqrt{3})}{4}$

51. If $\sin\alpha+\sin\beta+\sin\gamma=0=\cos\alpha+\cos\beta+\cos\gamma$ Then. Which of the following statement(s) is/are true?

A) $\sin 2\alpha+\sin 2\beta+\sin 2\gamma=0$ B) $\sin 3\alpha+\sin 3\beta+\sin 3\gamma=3\sin(\alpha+\beta+\gamma)$

C) $\sin 4\alpha+\sin 4\beta+\sin 4\gamma=2\sum\sin 2(\alpha+\beta)$ D) $\sin(\alpha+\beta)+\sin(\beta+\gamma)+\sin(\gamma+\alpha)=0$

52. For the points z_1, z_2, z_3, z_4 in complex plane. $|z_1|<1$, $|z_2|=1$ & $|z_3|\le 1$ &

$z_3=\dfrac{z_2(z_1-z_4)}{\bar{z}_1 z_4-1}$ Then possible values of $|z_4|$_________

A) 2 B) $2/5$ C) $1/3$ D) $5/2$

SECTION-3

[INTEGER TYPE]

Section-III (Integer Answer Type, Total Marks: 24) contains 6 questions. The answer to each of the questions is a single-digit integer, ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the ORS. For each question you will be awarded 4 marks if you darken ONLY the bubble corresponding to the correct answer and zero marks otherwise. There are no negative marks in this section.

53. Let z_1, z_2, z_3 be complex numbers such that $|z_1|=|z_2|=|z_3|=3$ & $(z_1\ne z_3)$

Then find the value of $\dfrac{|z_1-z_2|.|z_1-z_3|}{\min\{|az_2+(1-a)z_3-z_1|\}}(a\in R)=$ _________

54. Let z represents a variable point in complex plane such that $z-z_1$ is real, where z_1 is a fixed point in the same plane. Let "m" be the number of "z" values such that $|z|=\lambda$, where $\lambda>|\text{Im}(z_1)|$ & let "n" be the number of "z" values such that $|z|=\lambda$, where $\lambda=|\text{Im}(z_1)|$ then value of $m+n=$__________

55. The least possible degree of a polynomial equation, with real coefficients having $2\omega^2, 3+4\omega, 3+4\omega^2, 5-\omega-\omega^2$ as roots is _______

(ω, ω^2 are non-real cube roots of 1).

56. A curve is defined as $2|z+\bar{z}| = 32+(z-\bar{z})^2$. Two spiders, one male and other female were moving together along the curve. The female spider suddenly realizes that the male spider is a rogue spider and immediately tries to get away as far as possible from it. Hence it moved onto the another point on the curve. The maximum distance between two final points when both spiders try out all possibilities, is k. Then the value of $\left[\frac{\sqrt{k}}{3}\right] =$ __________[.] Greatest integer function

57. If α, β, γ are the roots of $x^3 - 3x^2 + 3x + 7 = 0$ and w is a non-real cube root of 1, and the value of $\frac{\alpha-1}{\beta-1} + \frac{\beta-1}{\gamma-1} + \frac{\gamma-1}{\alpha-1} = \left(w^q\right)p \ \ (p, q \in z)$ then number of ordered pairs (p,q) such that $p+q=15$ is _______

58. A function f is defined by $f(z) = (4+i)z^2 + az + \gamma, \forall z$

(Where $\alpha, \gamma \in C$). If f(1) and $f(i)_1$ both are real then the smallest possible value of $|\alpha| + |\gamma| = k$, find the value of $\left[\frac{k^2}{2}\right]$, ([.] greatest integer function)

SECTION-4

[Matrix Matching Type]

Section-IV (Matrix-Match Type, Total Marks: 16) contains 2 questions. Each question has four statements (A, B, C and D) given in Column I and five statements (p, q, r, s and t) in Column II. Any given statement in Column I can have correct matching with ONE or MORE statement(s) given in Column II. For example, if for a given question, statement B matches with the statements given in q and r, then for the particular question, against statement B, darken the bubbles corresponding to q and r in the ORS. For each question you will be awarded 2 marks for each row in which you have darkened ALL the bubble(s) corresponding to the correct answer(s) ONLY and zero marks otherwise. Thus, each question in this section carries a maximum of 8 marks. There are no negative marks in this section.

59. Matching

Column –I	Column –II
A) If $z_1, z_2 \ldots .\ z_n$ lie on the circle $\|z\| = 2$ then the value of $\|z_1 + z_2 + ... + z_n\| - 4\left\|\frac{1}{z_1} + \frac{1}{z_2} + ... + \frac{1}{z_n}\right\| =$ ______	P) 1
B) Suppose z is a complex number such that $z^n = (1+z)^n = 1, (n \in N)$ then the least value of n = ______	Q) 2
C) If ω is a non-real cube root of 1, then the magnitude of $\sqrt{\omega} + \sqrt{\omega^2} =$ _______	R) 0
D) Solutions of $z^4 + 4iz^3 - 6z^2 - 4iz - i = 0$ are the vertices of a convex polygon. Whose area is k then $\left[\frac{k^4}{5}\right] =$ _______ ([.] greatest integer function.)	S) 6

60. Z is a complex number satisfying $|z-2-3i|=\alpha \quad (\alpha \in I)$ and z_m and z_M be the corresponding complex numbers for which $|z+1+i|$ is minimum and maximum respectively.

Column –I	Column –II
A) If $\arg\left(\dfrac{Z_m+1+i}{Z_M-2-3i}\right)=0$ then "α" can be	P) 4
B) If $\arg\left(\dfrac{Z_m+1+i}{Z_M-2-3i}\right)=\pi$ then "α" can be	Q) 6
C) Possible value of α, for which maximum value of principle arg(z), can be obtained, is _____	R) 7
D) When $\alpha=2$, then $\|M-m\|=$ ___	S) 8