MATHEMATICS: Max.Marks : 60

SECTION I
Single Correct Answer Type

This section contains **10 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

41. Let f: $R \to R$ be defined by $f(x) = 3^{-|x|} - 3^{x} + \operatorname{sgn}\left(e^{-x} + 3\right)$ where R is set of real numbers and $\operatorname{sgn} x$ denotes signum function of x, then which one is correct.

A) f is injective but not surjective

B) f is surjective but not injective

C) f is injective as well as surjective

D) f is neither injetive nor surjective

42. Let a function f defined by $f : R \to R$ and $f(x) = \begin{cases} m - x, x \le 1 \\ 2mx + 1, x > 1 \end{cases}$. If the function is onto then the range of m is

A) $[-2, \infty)$ B) $[-2, 0)$ C) $[-2, 2]$ D) $(-\infty, -2]$

43. Which of the following function is not identical with the other three functions ([.] G.I.F, {.} FPF) (sgn is signum function)

A) $f(x) = \frac{2}{\pi}\left(\sin^{-1}\{x\} + \cos^{-1}\{x\}\right)$ B) $g(x) = \sec^2\left[\{x\}\right] - \tan^2\left[\{x\}\right]$

C) $h(x) = \sin^2(\ln x) + \cos^2(\ln x)$ D) $\phi(x) = \operatorname{sgn}\left(|x| + 1\right)$

44. The equation $\sin^{-1} x = |x-a|$ will have at least one solution if 'a' lies in the interval

A) $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ B) $[-1,1]$ C) $\left[1,1+\frac{\pi}{2}\right]$ D) $\left[1-\frac{\pi}{2},1+\frac{\pi}{2}\right]$

45. Consider a function f defined on the set of non-negative integers such that $f(0)=1,\ f(1)=0$ and $f(n)+f(n-1)=n\ f(n-1)+(n-1)f(n-2)$ for $n \ge 2$ then $f(6)$ equals

A) 140 B) 44 C) 45 D) 265

46. If $f: R-\{-1\} \to R$ and f (which is not a constant function and $f(x) \ne x$) is a differentiable function satisfies $f\left(x+f(y)+x f(y)\right)=y+f(x)+yf(x),\ \forall x,y \in R-\{-1\}$ then the value of 2010 $\{1+f(2009)\}$ is

A) 1 B) 2 C) 0 D) $\frac{1}{2}$

47. Period of the function $f(x)=\cos\left(2\pi\{2x\}\right)+\sin\left(2\pi\{2x\}\right)$ (where $\{.\}$ denotes fractional part of x)

A) 1 B) $\frac{\pi}{2}$ C) ½ D) π

48. Let $f(x)$ be a function given by $f:[0,2] \to \left[\frac{1}{7},\frac{2}{7}\right] \cup [1,4)$ and satisfies $3x - f(x) + 1 = 0$ for $0 \le x < 1$ and $x - 7f(x) = 0$ for $1 \le x \le 2$ then the sum of solutions of the equation $f(x) = f^{-1}(x)$

A) 1 B) 0 C) 5/6 D) 2

49. Let $f(x)$ be a polynomial of degree 8 satisfying $f(r) = \frac{1}{r}, r = 1,2,3,......9$ and,

$$g(x) = \begin{cases} \left(\frac{x}{1}-1\right)\left(\frac{x}{2}-1\right)\left(\frac{x}{3}-1\right)...\left(\frac{x}{9}-1\right) & ,x \ne 0 \\ 1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{9} & ,x = 0 \end{cases}$$

then the value of $\frac{g(-1)}{f(10)}$ equals

A) 50 B) 45 C) 55 D) 5

50. The domain of $f(x)$ is $(0,1)$ therefore the domain of $y = f\left(e^x\right) + f\left(\ln|x|\right)$ is

A) $\left(\frac{1}{e},1\right)$ B) $(-e,-1)$ C) $\left(-1,-\frac{1}{e}\right)$ D) $(-e,-1) \cup (1,e)$

SECTION II
Multiple Correct Answer(s) Type

This section contains **5 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE are correct.**

51. If α is the number of solutions of the equation $|x| = \ln(x-[x])$ (where [.] denotes greatest integer function) and if the value of $\underset{x\to\alpha}{lt} \frac{xe^{ax} - b\sin x}{x^3}$ is L which is finite then

A) $a+b=1$

B) $a-b=1$

C) L is a root of $6x^2-7x+1=0$

D) None of a,b,L is zero

52. Let $f: R \to R,\ f(x) = \frac{x^2+bx+1}{x^2+2x+b}$, and if the function $f(x)$ and $\frac{1}{f(x)}$ have the same bounded set as their range then the value of b cannot be

A) $\frac{1}{2}$

B) 2

C) $2\sqrt{3}-2$

D) $\sqrt{3}+1$

53. Let $k \in N$ and $a \in R^+,\ a \neq 1$ then $\underset{n\to\infty}{lt}\, n^k\left(a^{\frac{1}{n}}-1\right)\left(\sqrt{\frac{n-1}{n}}-\sqrt{\frac{n+1}{n+2}}\right)$ is

A) 0 if $k \in \{1,2\}$

B) $-\ln a$ if $k=3$

C) $\ln a$ if $k=2$

D) $\ln a$ if $k=3$

54. Let $f: R \to R$ defined by $f(x) = \begin{cases} \{x\} & , x \in Q \\ x & , x \in R - Q \end{cases}$ ({.} is fractional part function) then $\underset{x \to a}{lt} f(x)$ exists if

A) $a = 0$ B) $a = 1$ C) $a = \frac{2}{3}$ D) $a = \frac{3}{\pi}$

55. Let $f(x) = \begin{cases} x+3 & if & x \in [-4,-2) \\ 1 & if & x \in [-2,2) \\ 3-x & if & x \in [2,4] \end{cases}$ and $g(x) = \begin{cases} x+6, & x < 0 \\ 2x+6, & x \geq 0 \end{cases}$ then which is true

A) $gof(x)$ is an even function

B) Range of $fog(x) is [-1,1]$

C) $\underset{x \to -2}{lt} fog(x) = -1$

D) The equation $gof(x) = k$ will have at least one solution if $k \in [5,8]$

SECTION III

Integer Answer Type

This section contains **5 questions.** The answer to each question is single digit integer, ranging from 0 to 9 (*both inclusive*).

56. Let $f(x)=a\sin x+b.\sqrt[3]{x}+6$ where a and b are real numbers. If $f\left(\log_{30}\left(\log_{15}^{30}\right)\right)=7$ then the value of $f\left(\log_{30}\left(\log_{30}^{15}\right)\right)$ is

57. It $f(x)=\ln\left(\dfrac{x^2+e}{x^2+1}\right)$ and if the range of $g(x)=\sqrt{\sin(f(x))}+\sqrt{\cos(f(x))}$ is $(a,b]$ then the value of a^4+b^4 equals.

58. Let $f(x)$ be a real valued function defined on the interval $[-2,2]$ as follows $f(x)=\begin{cases}1, & -2\le x\le -1\\ x+2, & -1<x<1\\ 1-x, & 1\le x\le 2\end{cases}$ then the number of solutions of the equation $\{f(x)\}=\dfrac{1}{2}$ (where $\{.\}$ is fractional part function)

59. Let $f(x)=\left\{x+\left[\log_2(1+x)\right]\right\}+\left\{x+\left[\left(\log_2\left(1+x^2\right)\right)\right]\right\}+\ldots+\left\{x+\left[\log_2\left(1+x^{10}\right)\right]\right\}$ then the number of solutions of the equation $f(x)=x$ is ([.] is G.I.F, {.}FPF)

60. Let $A=\{1,2,3,4\}$ number of functions from A to A that are satisfying $fof(x)=x\ \forall x\in A$ is k then $k-5$ equals