

MATHEMATICS:**Max. Marks : 60****SECTION – I****(MULTIPLE CORRECT CHOICE TYPE)**

This section contains **8 multiple choice questions**. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONE OR MORE is/ are correct**

41. If the smallest positive solution of the equation $\sqrt{\sin(1-x)} = \sqrt{\cos x}$ is $\frac{a}{2} + \frac{b\pi}{c}$ then

$$(a, b, c \in \mathbb{N})$$

- A) $a + b = 8$ B) $bc = 28$ C) $\left[\frac{b}{c}\right] = 1$ D) $a < c$

42. Let A, B, C be the angles in $\left(0, \frac{\pi}{2}\right)$ and if $\tan A, \tan B, \tan C$ are integers such that

$\frac{\sqrt{2} \tan A + \tan B}{\sqrt{2} \tan B + \tan C}$ is a rational number then which of the following is always, an integer

- A) $\frac{2 \tan^2 A + \tan^2 B}{2 \tan^2 B + \tan^2 C}$ B) $\frac{\tan^2 A + \tan^2 B + \tan^2 C}{\tan A + \tan B - \tan C}$
C) $\frac{\tan^2 A + 2 \tan^2 B}{\tan^2 B + 2 \tan^2 C}$ D) $\frac{\tan^2 A + \tan^2 B + \tan^2 C}{\tan A + \tan C - \tan B}$

43. $\tan(\log_2 6) - \tan(\log_2 3) - \tan 1$ equals

A) $\frac{\tan^2 1 \tan(\log_2 3) + \tan 1 \tan^2(\log_2 3)}{1 - \tan 1 \tan(\log_2 3)}$ B) $\tan(1 + \log_2 3) \cdot \tan(\log_2 3) \cdot \tan 1$

C) $(\tan(\log_2 6) + \tan(\log_2 3)) \tan 1$ D) to a negative real number

44. If $\frac{\tan 3A}{\tan A} = k$ ($k \neq 1$) then which is / are true

A) $\frac{\cos A}{\cos 3A} = \frac{k-1}{2}$ B) $\frac{\sin 3A}{\sin A} = \frac{2k}{k-1}$ C) $k < \frac{1}{3}$ D) $k > 3$

45. If the equation $|\sin 2x| + |\cos 2x| = |\sin y|$, $x \in R$, $y \in [-2\pi, 2\pi]$ is satisfied for 'p' number of 'y' values then the equation(s) having number of solutions $\geq p$ is/are

A) $|\cos x| = 2[x]$, ([] is G.I.F), $x \in [-2\pi, 2\pi]$

B) $\sin \pi x = |\ln|x||$

C) $|\cos x| = \sin x$, $0 \leq x \leq 4\pi$

D) $\frac{\sqrt{3}-1}{\sin x} + \frac{\sqrt{3}+1}{\cos x} = 4\sqrt{2}$, $0 < x < \frac{\pi}{2}$

46. Which is / are correct

A) $\frac{\sin 3\alpha}{\cos 2\alpha} > 0$ for $\alpha \in \left(\frac{3\pi}{8}, \frac{23\pi}{48}\right)$ B) $\frac{\sin 3\alpha}{\cos 2\alpha} < 0$ for $\alpha \in \left(\frac{13\pi}{48}, \frac{14\pi}{48}\right)$

C) $\frac{\sin 2\alpha}{\cos \alpha} < 0$ for $\alpha \in \left(-\frac{\pi}{2}, 0\right)$ D) $\frac{\sin 2\alpha}{\cos \alpha} > 0$ for $\alpha \in \left(\frac{13\pi}{48}, \frac{14\pi}{48}\right)$

47. Let $f(\theta) = \left(\cos \theta - \frac{\cos \pi}{8}\right) \left(\cos \theta - \cos \frac{3\pi}{8}\right) \left(\cos \theta - \cos \frac{5\pi}{8}\right) \left(\cos \theta - \frac{\cos 7\pi}{8}\right)$ then

A) Number of solutions of $f(\theta) = 0$ in $[0, \pi]$ is 8

B) maximum value of $f(\theta)$ ($\theta \in R$) is $1/8$

C) maximum value of $f(\theta)$ ($\theta \in R$) is $1/4$

D) $8f(0) = 1$

48. If the number of solutions of the equation

$(\sin x - 1)^3 + (\cos x - 1)^3 + (\sin x)^3 = (2\sin x + \cos x - 2)^3$ in $[0, 2\pi]$ is n_1 and if the value of $[x + y] - 2005$ is n_2 where $x + \sin y = 2014$

and $x + 2014\cos y = 2013$, $0 \leq y \leq \frac{\pi}{2}$ then

A) $n_1 < n_2$

B) $n_2 = 2n_1 - 1$

C) $n_1^2 = n_2$

D) $\left[\frac{n_1 + n_2}{n_2 - n_1} \right] = 3$ ([] is GIF)

SECTION - II**(COMPREHENSION TYPE)**

This section contains **4 groups of questions**. Each group has 2 multiple choice questions based on a paragraph. Each question has 4 choices A), B), C) and D) for its answer, out of which **ONLY ONE** is correct.

Paragraph for Questions 49 and 50

If $\alpha = \frac{\pi}{7}$, $\beta = 3\alpha$, $\gamma = 5\alpha$ and if $\cos \alpha, \cos \beta, \cos \gamma$ are roots of $8x^3 - 4x^2 - 4x + 1 = 0$ then

49. The value of $\sin \frac{\alpha}{2} \sin \frac{\beta}{2} \sin \frac{\gamma}{2} =$

- A) $\frac{1}{4}$ B) $\frac{\sqrt{7}}{4}$ C) $\frac{\sqrt{7}}{8}$ D) $\frac{1}{8}$

50. The value of $\sec \alpha \sec \beta + \sec \beta \sec \gamma + \sec \gamma \sec \alpha =$

- A) -4 B) 4 C) 8 D) 1

Paragraph for Questions 51 and 52

Consider the cubic equation

$x^3 - (\sin \theta + \cos \theta(1 + \sin \theta))x^2 + \sin \theta \cos \theta(1 + \sin \theta + \cos \theta)x - \sin^2 \theta \cos^2 \theta = 0$ whose roots are λ, μ, δ then

51. The maximum value of $\lambda^2 + \mu^2 + \delta^2$ equals

- A) $\frac{5}{4}$ B) $\frac{5}{2}$ C) 1 D) $\frac{6}{5}$

52. If $\frac{\pi}{4} < \theta < \frac{\pi}{2}$ and if $\lambda < \mu < \delta$ then the maximum value of $\lambda + \delta$ equals

- A) $\frac{\sqrt{3}}{2}$ B) $\frac{3\sqrt{3}}{4}$ C) $\sqrt{3}$ D) $\frac{\sqrt{3}}{4}$

Paragraph for Questions 53 and 54

Let $A = \{(x, y) / x^2 + y^2 \leq 16\}$, $B = \{(x, y) / x^2 + y^2 \geq 12\}$,

$C = \{(x, y) / \sin(x + y) \geq 0\}$, $D = \{(x, y) / \sin(x + y) \leq 0\}$ then answer the following

53. The area of the region formed by the points (x, y) satisfying $A \cap B \cap C$ equals

- A) 2π B) $\frac{3\pi}{2}$ C) $\frac{\pi}{2}$ D) $\frac{3\pi}{2} - \sin^{-1}\left(\frac{2}{3}\right)$

54. The area of the region bounded by the points (x, y) satisfying $A \cap B \cap D$ equals

- A) π B) 2π C) $\frac{3\pi}{2}$ D) $\frac{3\pi}{2} + \sin^{-1}\left(\frac{2}{3}\right)$

Paragraph for Questions 55 and 56

Let the value of $\tan\left(\frac{19\pi}{24}\right) = a + \sqrt{a} - \sqrt{b} - \sqrt{ab}$ where $b > a > 0 (a, b \in \mathbb{N})$ then

55. The value of $\cos \frac{2\pi}{15} \cos \frac{4\pi}{15} \cos \frac{8\pi}{15} \cos \frac{14\pi}{15}$ equals

- A) $\frac{b}{a^2}$ B) $\frac{1}{a^2}$ C) $\frac{1}{b^2}$ D) $\frac{a+b}{b^3}$

56. The value of $\prod_{r=0}^3 \left(1 + \cos\left(2r+1\right)\frac{\pi}{8}\right)$ equals

- A) $\frac{1}{b^3}$ B) $\frac{1}{2a+b}$ C) $\frac{1}{2b+a}$ D) $\frac{b}{a+b}$

SECTION – III

(MATRIX MATCH TYPE)

This section contains **4 multiple choice questions**. Each question has matching lists. The codes for the lists have choices (A), (B), (C), and (D) out of which **ONLY ONE** is correct.

57. If x, y, z are such that $\left(\cos x^2 + \frac{1}{\cos^2 x}\right)(1 + \tan^2 2y)(3 + \sin 3z) = 4$ match the following of

List-I with List-II

	List – I		List – II
(A)	x can be	(P)	Integral multiple of $\frac{\pi}{2}$
(B)	y can be	(Q)	Integral multiple of π
(C)	z can be	(R)	Integral multiple of $\frac{\pi}{3}$
(D)	$x + y$ can be	(S)	Integral multiple of $\frac{\pi}{6}$

A) A-PQRS, B-PQRS, C-PS, D-PQRS

B) A-PQRS, B-PQRS, C-S, D-PQRS

C) A-PQRS, B-PQRS, C-PQS, D-PS

D) A-PQRS, B-PQRS, C-PQRS, D-PQRS

58. Match the following

	List – I		List – II
(A)	The equation $3\sin\theta + 4\cos\theta = 5(x^2 + ax + 1)$ ($\theta, x \in R$) has real solution then 'a' can take the value(s)	(P)	-7
(B)	If the number of solutions of $ \ln x + \sin\pi x = 0$ is n then $n - 5$ equals	(Q)	-4
(C)	If the sum of solutions $\cos^5 x + \sin^3 x = 1$ in $x \in [0, 2\pi]$ is $\frac{a\pi}{b}$ ($a, b \in N$ and relative prime) then $b - a - 1$ equals	(R)	-8
(D)	Let $\frac{\sin^4 x}{a} + \frac{\cos^4 x}{b} = \frac{1}{a+b}$ ($0 \leq x \leq \frac{\pi}{2}$) then the value of $\frac{\sin^4 x}{a^2} - \frac{\cos^4 x}{b^2}$ equals	(S)	0

A) A-PQRS, B-Q, C-Q, D-S

B) A-PQRS, B-Q, C-R, D-Q

C) A-PQRS, B-QR, C-QR, D-RS

D) A-PQRS, B-QRS, C-RS, D-Q

59.

	List – I		List – II
(A)	If the range of 'a' which is a parameter, for which the equation $a \sin \frac{x}{2} = \sin x + \sin \frac{3x}{2}$ ($\cos x \neq 1$) possesses at least one solution is $[l, m]$ then $\left \frac{m}{l} \right $ equals	(P)	3
(B)	The number of distinct real solutions of the equation $\tan\left(\frac{2\pi x}{x^2 + x + 1}\right) = -\sqrt{3}$ is equal to	(Q)	8
(C)	Number of distinct values of 'a' $\in N$ and $1 \leq a \leq 100$ satisfying $\int_0^x (t^2 - 8t + 39) dx = x \cdot \sin \frac{\pi a}{x}$	(R)	5
(D)	If $\sum_{n=1}^{\infty} \frac{\tan\left(\frac{\theta}{2^n}\right)}{2^{n-1} \cdot \cos\left(\frac{\theta}{2^{n-1}}\right)} = \frac{a}{\sin 2\theta} - \frac{b}{2\theta}$ then a+b equals	(S)	4

A) A-S,B-R,C-Q,D-S

B) A-S,B-Q,C-R,D-P

C) A-S,B-R,C-P,D-Q

D) A-S,B-R,C-Q,D-P

60.

	List – I		List – II
(A)	If $\sin \frac{\pi}{14}$ is a root of cubic equation $8x^3 - 4x^2 - 4x + \alpha = 0$ then the value of $\left[\frac{\alpha}{2}\right]$ equals where $[]$ is GIF	(P)	1
(B)	Number of positive integral divisors of the reciprocal of the number $\prod_{r=1}^7 \sin\left(\frac{(2r-1)\pi}{14}\right)$ equals	(Q)	3
(C)	The number of distinct real solutions of $\cos\left(\frac{\sqrt{2}+1}{2}x\right) \cdot \cos\left(\frac{\sqrt{2}-1}{2}x\right) = 1$ is P then P is less than	(R)	7
(D)	The number of distinct solutions of $\int_0^x \cos(t+x^2)dt = \sin x, 2 \leq x \leq 3$ is less than	(S)	0

A) A-S,B-R,C-QR,D-QR

B) A-S,B-R,C-QR,D-PR

C) A-S,B-R,C-PQ,D-PS

D) A-S,B-P,C-R,D-QR