PART-III_MATHEMATICS **Max Marks : 60**

Section-1
(One or More options Correct Type)

This section contains 10 multiple choice equations. Each question has four choices (**A**) (**B**)(**C**) and (**D**) out of which **ONE or MORE** are correct.

41. Let $\bar{a},\bar{b},\bar{c}$ be three non-zero vectors satisfying $(x+1)\bar{a}-\bar{b}=2\bar{c}$ and $\bar{a}-x\bar{b}=3\bar{c}$. If the vectors $\bar{a}$ and $\bar{c}$ have opposite direction then x cannot be

A) 1 B) 2 C) 3 D) 4

42. The vector $\overrightarrow{OA}=\hat{i}-2\hat{j}+2\hat{k}$ is rotated about O through 90^0, while doing so it passes over positive $x-axis$ and $\overrightarrow{OB}$ is its new position. Now $\overrightarrow{OB}$ is rotated about 'O' through 90^0, while doing so it passes over positive $y-axis$ and $\overrightarrow{OC}$ is its new position then

A) $\overrightarrow{OB}=\frac{4i+j-k}{\sqrt{2}}$ B) $\overrightarrow{OB}=\frac{i+4j-k}{\sqrt{2}}$

C) $\overrightarrow{OC}=\frac{-4i+17j+k}{\sqrt{34}}$ D) $\overrightarrow{OC}=\frac{4i+j+k}{\sqrt{2}}$

43. Given $\bar{a},\bar{b},\bar{c}$ 3 vectors and $|\bar{a}|=|\bar{b}|=1$ and $|\bar{a}+\bar{b}|=\sqrt{3}$. If $\bar{c}$ is a vector such that $\bar{c}-\bar{a}-2\bar{b}=3(\bar{a}\times\bar{b})$ then

A) $\bar{a}.\bar{c}=2$ B) $\bar{a}.\bar{b}+\bar{b}.\bar{c}+\bar{c}.\bar{a}=5$ C) $|\bar{c}|=\sqrt{34}$

D) $\bar{b}$ and $\bar{c}$ are perpendicular to each other.

44. Let $OABC$ be a regular tetrahedran of each edge unity. If

$\overrightarrow{OC} = p\left(\overline{a}+\overline{b}\right)+q\left(\overline{a}\times\overline{b}\right)$ then which is/are true

A) Its volume is $\frac{1}{6\sqrt{2}}$

B) $p=\frac{1}{3}$

C) p is irrational

D) $|q|=\frac{2\sqrt{2}}{3}$

45. Let 0 be origin and position vectors of A,B,C respectively are $\bar{i}+2\bar{j}+2\bar{k}$,

$2\bar{i}+3\bar{j}+6\bar{k}$ and $3\bar{i}+4\bar{j}+12\bar{k}$. Let D be point such that $\overrightarrow{OD}$ is equally inclined

to $\overrightarrow{OA},\overrightarrow{OB},\overrightarrow{OC}$ then which is/are correct.

A) $\alpha+\beta+\gamma=\frac{1}{\sqrt{3}}$ if $\overrightarrow{OD}=\alpha\bar{i}+\beta\bar{j}+\gamma\bar{k},\left|\overrightarrow{OD}\right|=1$

B) $\alpha+\beta+\gamma=\frac{-1}{\sqrt{3}}$ if $\overrightarrow{OD}=\alpha\bar{i}+\beta\bar{j}+\gamma\bar{k},\left|\overrightarrow{OD}\right|=1$

C) Minimum possible volume of tetrahedron ABCD is $\frac{1}{3}$ if $\left|\overrightarrow{OD}\right|=\sqrt{3}$

D) Maximum possible volume of tetrahedron ABCD is 1 if $\left|\overrightarrow{OD}\right|=\sqrt{3}$

46. If S is the set of all values of α such that the two lines $x=2, \frac{y}{1-\alpha}=\frac{z}{\alpha}$ and $x=1, \frac{y}{2-\alpha}=\frac{z}{3(1-\alpha)}$ are coplanar then which is/are false

A) $-\frac{1}{2} \in S$

B) The lines are perpendicular for some $\alpha \in S$

C) The lines are parallel for two values of $\alpha \in S$

D) The two lines are intersecting lines for some $\alpha \in S$

47. If $\bar{a}$ is not parallel to $\hat{n}$ and if the projection of the line $\bar{r}=\bar{a}+t\bar{b}$ on the plane $\bar{r}.\hat{n}=5$ is $\bar{r}=x\bar{a}+y\hat{n}+z\left(\hat{n}\times\left(\bar{b}\times\hat{n}\right)\right)$ and if $x+y=2$ then $|\bar{a}|$ can not be

A) 5 B) 4 C) 3 D) 1

48. The line x+2y+z=2015, 6x+8y+3z=2016 is parallel to the plane $\alpha x+\mu y+z=0$

A) for $\mu=0, \alpha=-2$

B) for $\alpha=1, \mu=2$

C) for $\mu=\alpha=1$

D) for $\alpha=\mu=-1$

49. If y=2x, z=5 and y=-2x, z=-5 are two lines then

A) The acute angle between the lines is greater than $\frac{\pi}{4}$

B) Both lines are parallel to z-axis

C) The lines are intersecting lines

D) Locus of point equidistant from the two lines is $2xy+25z=0$

50. Let O be origin, $y=x^2+x+1$ is a parabola in x.y plane of $\mathbb{R}^3$ space. P and Q lie on the parabola and $\overrightarrow{OP}=2\hat{i}+p\hat{j},\overrightarrow{OQ}=q\bar{i}+13\bar{j}, p>O, q<O$, and if R is on arc PQ of the parabola such that Δ PQR has maximum area then

A) Distance of origin from R is $\sqrt{2}$

B) R lies on the line $\frac{x}{2}=\frac{y-2}{2}=\frac{z-\frac{1}{2}}{1}$

C) R lies on one of the angle bisector planes of co-ordinate planes

D) The distance of R from the plane $x+4=0$ is 5

Section-2
(Integer Value Correct Type)

This section contains 10 questions. The answer to each question is a **single digit integer, ranging** from 0 to 9 (both inclusive).

51. Let $\bar{a},\bar{b},\bar{c}$ be 3 unit vectors and if $f\left(\bar{a},\bar{b},\bar{c}\right)=\left|2\bar{a}-3\bar{b}\right|^2+\left|2\bar{b}-3\bar{c}\right|^2+\left|2\bar{c}-3\bar{a}\right|^2$. If the maximum value of $f\left(\bar{a},\bar{b},\bar{c}\right)$ is M then the value of $\dfrac{M-f\left(\hat{i},\hat{j},\hat{k}\right)}{6}$ equals

52. The shortest distance from $(1,2,3)$ to $2x^2+8y^2+2z^2-4xy-4yz-2zx=0$ is d then the value of $[3d]$ is $([\ ]\, is\, GIF)$

53. Let $\bar{a}$ be a unit vector and $\bar{c}$ is a vector such that $\sqrt{2}\le\left|\bar{c}\right|\le\sqrt{6}$ and if $\bar{a}+\bar{a}\times\bar{b}=\bar{c}$ and if the range of values of volume of tetrahedran is $[\alpha,\beta]$ then the value of $3(\beta-\alpha)$ equals

54. The distance of the point $(3,8,2)$ from the line $\dfrac{x-1}{2}=\dfrac{y-3}{4}=\dfrac{z-2}{3}$ measured parallel to the plane $3x+2y-2z+15=0$ is

55. Let $\bar{r}$ be a position vector of a variable point in Cartesian OXY plane such that $\bar{r}.\left(10\bar{j}-8\bar{i}-\bar{r}\right)=40$ and $P=\min\left\{\left|\bar{r}+2\bar{i}-3\bar{j}\right|^2\right\}$ then the value of $\beta-\alpha$ if $P=\beta-2\sqrt{\alpha},\ (\alpha,\beta\in N)$ equals

56. The number of pieces into which the cube $C=\{(x,y,z)/o\le x,y,z\le 1\}$ is cut by the 3 planes $x=y$ and $y=z$ and $z=x$, simultaneously is

57. The nearest of the lines $\frac{x-2}{7}=\frac{y-2}{-6}=\frac{z+1}{1}$ and $\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}$ to the plane 2x+3y+4z-1=0 meets xy plane at $P(\alpha,\beta,\lambda)$ then the value of $\alpha+2\beta+\lambda$ equals

58. Let $\bar{a},\bar{b},\bar{c}$ be three vectors having magnitudes 1,2,3 respectively such that $\left|\left[\bar{a}\bar{b}\bar{c}\right]\right|=6$. If $\bar{d}$ is a unit rector coplanar with $\bar{b}$ and $\bar{c}$ such that $\bar{b}.\bar{d}=1$ then the value of $\left|(\bar{a}\times\bar{c}).\bar{d}\right|^2+\left|(\bar{a}\times\bar{c})\times\bar{d}\right|^2$ equals

59. A square ABCD of diagonal 2a is folded along the diagonal AC so that planes DAC, BAC are at right angles, the shortest distance between DC and AB is $\frac{ka}{\sqrt{3}}$ so that k equals

60. The distance of the plane $x+2y-z=2$ from the point $(2,-1,3)$ as measured in the direction with d.r.s 2,2,1 is