Sri Chaitanya IIT Academy, India

A.P, TELANGANA, KARNATAKA, TAMILNADU, MAHARASHTRA, DELHI, RANCHI

A right Choice for the Real Aspirant

ICON CENTRAL OFFICE, MADHAPUR-HYD

Sec: Sr. IPLCO | JEE ADVANCED | DATE : 09-08-15
TIME : 3:00 | 2013_P1 MODEL | MAX MARKS : 180

KEY & SOLUTIONS

PHYSICS

1	**A**	2	**C**	3	**C**	4	**A**	5	**B**	6	**B**
7	**B**	8	**B**	9	**C**	10	**B**	11	**AD**	12	**BCD**
13	**AC**	14	**ABD**	15	**ABCD`**	16	**2**	17	**5**	18	**3**
19	**6**	20	**6**								

CHEMISTRY

21	**D**	22	**C**	23	**C**	24	**B**	25	**D**	26	**C**
27	**B**	28	**A**	29	**D**	30	**D**	31	**ABCD**	32	**A**
33	**ABCD**	34	**CD**	35	**BCD**	36	**1**	37	**1**	38	**6**
39	**9**	40	**6**								

MATHEMATICS

41	**B**	42	**A**	43	**D**	44	**B**	45	**D**	46	**B**
47	**A**	48	**A**	49	**C**	50	**B**	51	**ABC**	52	**ABCD**
53	**ABD**	54	**BC**	55	**B**	56	**3**	57	**1**	58	**5**
59	**4**	60	**2**								

MATHS

41. Let O is the intersection of AC and BD. R is one extremity of the chord containing B and D. So, the angles AOR and ROC are right angles and AR=RC. Further AC and RC are radii hence the triangle ARC is equilateral.

42. Center of such circle in the case of parabolas $y^2 = 4ax, x^2 = 4ay$ is $\left(\frac{3a}{2}, \frac{3a}{2}\right)$

43. The parabolas are $y^2 = 4\sin^2\alpha(x+\sin^2\alpha)$ and $y^2 = 4\cos^2\alpha(x+\cos^2\alpha)$, hence the locus is $x+\cos^2\alpha+\sin^2\alpha = 0 \Rightarrow x+1=0$

44. The x-axis touches at A(1, 0) and x=y touches at B(1, 1). Hence the equation to the curve through these points is given by $y(y-x)+k(x-1)^2 = 0$. For this to represent a parabola, 4k=1. The equation is $x^2-4xy+4y^2-2x+1=0$. Vertex $\left(\frac{13}{25}, \frac{4}{25}\right)$, focus $\left(\frac{3}{5}, \frac{1}{5}\right)$

45. Taking the new axes as $X = \frac{4x+3y}{5}, Y = \frac{3x-4y}{5}$, we see that the parabola can be $Y = \pm\frac{X^2-25}{10}$ with the required condition $|X| \le 200$

46. Eliminating t, we get $(3x-4y+2)^2 = 16x+12y-27$. Vertex is $\left(\frac{21}{25}, \frac{113}{100}\right)$. Hence k=29 and the area is 3π

47. If P(t) is the point and Q(T) is another point in the question, we have $T = t+\frac{2}{t}$ and $\tan\theta = \left|\frac{T-t}{1+Tt}\right| \Rightarrow \frac{2}{t(t^2+3)} = \pm\tan\theta$. As $t(t^2+3) = \pm 2\cot\theta$ can have only one real root, there will be only one such point P.

48. Assume the point $P\left(-4\cos\frac{\pi}{4}-\sin\frac{\pi}{12}, 2\sin\frac{\pi}{4}+\cos\frac{\pi}{12}\right)$ as origin and line joining it to the centre as x-axis, the equation to the circle becomes $x^2+y^2-2x=0$, center is $A_1(1,0)$ and the second circle has the equation $x^2+y^2-2\sqrt{2}y=0$ center $A_2(0,\sqrt{2})$. Similarly $A_3(-2,0)$, $A_4(0,-2\sqrt{2})$ etc.

49. If AB=14, AD=8, radius of circle is 5, the points C, E and V are given by $\left(17\frac{1}{3},8\right)$, (9, 8) and (8, 1) respectively. Required area = area of triangle CEV – area of minor segment CEV of the circle.

50. Let the parabola is $y^2 = 4x$. The equations $(x-t^2)^2+(y-2t)^2+2\lambda(x-yt+t^2)=0$ and $(x-1)^2+y^2+2\mu(mx-y-m)=0$ should represent the same circle touching $mx-y-m=0$ at (1, 0) and the parabola at $(t^2,2t)$. Eliminating λ and μ, we get $mt^3-3t^2-3mt+1=0$ will give three values of t for any given m.

51. Let A(t), B(s), C(p), D(q) are the points on the parabola $y^2=4x$. P is (h, k)
So we have p, q, s, t roots of $r^4+(4+2g)r^2+4fr+c=0$. So, p+q+t+s=0,
$ts(p+q)+pq(t+s)=-4f$ As AB is diameter, we have $t+s=-f$ hence $ts-pq=-4$
If the line $2x-(t+s)y+2ts=0$ passes through P(h, k) then
$2h-(t+s)k+2ts=0 \Rightarrow 2h+(p+q)k+2(pq-4)=0$ hence CD passes through $Q(h-4,-k)$

52. Take the circle as $x^2+y^2=25$. Chords can be on either of side or on the same side of the center, take them $y=\pm 3,\ y=\pm 4$. Two of the tangents are perpendicular and other two are NOT.

53. For the ends of normal chords to be lattice points, the combinations are $(1,\pm 2),(9,\pm 6)$ and $(4,\pm 4),(9,\pm 6)$

54. We have a,b,c are roots of $x^3-7x-6=0$ and p,q,r are roots of $x^3-7x+6=0$ and
$\Delta = 2(a-b)(b-c)(c-a)(p-q)(q-r)(r-p)=-800$ because $b<p<q<a<c<r$

55. Let the circles are $x^2+y^2+2ax-k^2=0,\ x^2+y^2+2bx-k^2=0$ intersecting at $A(0,k),B(0,-k)$
If $P(\alpha,\beta),Q(\gamma,\delta)$ and their mid point $R(x_1,y_1)$ and slope of AP and BQ is m, then we have
$\alpha,0$ are roots of $x^2+(mx+k)^2+2ax-k^2=0$ hence $\alpha=-\frac{2(mk+a)}{1+m^2}$
Similarly, $\gamma,0$ are roots of $x^2+(mx-k)^2+2bx-k^2=0$ hence $\gamma=-\frac{2(-mk+b)}{1+m^2}$
This gives $\alpha+\gamma=2x_1=-\frac{2(a+b)}{1+m^2}$
$\beta-k=\alpha m,\delta+k=\gamma m \Rightarrow y_1=mx_1$. Eliminating m, we get $x^2+y^2+(a+b)x=0$

56. The equation to the parabola can be given by $(x\sin\alpha - y\cos\alpha + \cos\alpha)^2 = k(x\cos\alpha + y\sin\alpha - \sin\alpha)$. If it is to be touched by x-axis, then $(x\sin\alpha + \cos\alpha)^2 = k(x\cos\alpha - \sin\alpha)$ should have equal roots.

So, $k = \dfrac{4\sin\alpha}{\cos^2\alpha}$ and the magnitude of equal root is $\left|\dfrac{2-\cos^2\alpha}{\sin\alpha\cos\alpha}\right|$, square of whose minimum value is 8.

57. If $a > b$ are the radii of the externally touching circles and direct common tangents include 60^0, we have $b = \dfrac{a}{3} = \dfrac{2}{3}$. Starting with next largest circle, the total area of the circles is $\dfrac{4\pi}{9}\left(1+\dfrac{1}{9}+\dfrac{1}{81}+.....\right) = \dfrac{\pi}{2}$. Area of the quadrilateral formed with the tangents and the radii of largest circle is $4\sqrt{3}$. So, the required area is $A = 4\sqrt{3} - \left(\dfrac{1}{3}\times 4\pi + \dfrac{\pi}{2}\right) = 4\sqrt{3} - \dfrac{11}{6}\pi$

58. The line $y = 8 + m(x+3)$ should not have intersection with the parabola $y = 2x^2 + 3x + 22$
Hence we should have $m^2 + 18m - 103 < 0$

59. We have $t_1 + t_2 + t_3 = 0$ and $\dfrac{2}{t_2}\dfrac{2}{t_3} = -1 \Rightarrow t_2 t_3 = -4$. Area $|(t_1 - t_2)(t_2 - t_3)(t_3 - t_1)| = 70$ gives $3,1,-4;\ -3,-1,4$ for t_1, t_2, t_3. Fourth vertex is either $(8,-12)$ or $(8,12)$

60. Shortest normal chord makes and angle $\mathrm{Tan}^{-1}\sqrt{2}$ with positive x-axis.