PHYSICS

21. 1cm of the scale at $30°C$

$=\left\{1+\left(18\times10^{-6}\times20\right)\right\}cm$

$\therefore$ 60cm of the scale at $30°C$

$=60\left\{1+36\times10^{-5}\right\}=60.02cm$

22. $-\frac{dT}{dt}=\frac{K_2A}{CL}(T-T_a)$ through radiation & $-\frac{dT}{dt}=\frac{K_2A}{CL}(T-T_a)$ through conduction net rate of fall of temperature is

$\left(-\frac{dT}{dt}\right)_{net}=\left(R_1+\frac{K_2A}{CL}\right)(T-T_a)$

$-\int\frac{dT}{\left(T-T_a\right)}=\int K_1+\frac{K_2A}{CL}\int dt$

$-l\,\mathrm{n}\left(T-T_a\right)|_{400}^{T}=\left(K_1+\frac{K_2A}{CL}\right)t$

$l\,\mathrm{n}\left(\frac{T-T_a}{400-T_a}\right)=-t\left(K_1+\frac{K_2A}{CL}\right)$

$\frac{T-300}{100}=e^{-t\left[K_1+\frac{K_2A}{CL}\right]}$

$T\left(t\right)=300+100e^{-t\left[K_1+\frac{K_2A}{CL}\right]}$

23. At the interface between ice and water $\boldsymbol{T_x}=0°\boldsymbol{C}$. Then $R_1T_2+R_2T_1=0$, or $k_1T_1/L_1+k_2T_2/L_2=0$. Not only that, $L_1+L_2=L$, so

$k_1T_1L_2+(L-L_2)k_2T_2=0$,

So $$L_2=\frac{(1.42m)(1.67W/m.K)(-5.20°C)}{(1.67W/m.K)(-5.20°C)-(0.502W/m.K)(3.98°C)}=1.15m.$$

24. a. $6T_1=3T_2=2T_4=T_3=1800K$

$T_1=300\,K;\ T_2=600\,K$

$T_4=900\,K;\ T_3=1800\,K$

$4\to1$ and $2\to3$ are isochoric processes in which work done = 0

$W_{12}=P\left(V_2-V_1\right)=nR\left(T_2-T_1\right)=2\times R\left(600-300\right)=600R$

$W_{34}=P\left(V_4-V_3\right)=nR\left(T_4-T_3\right)=2\times R\left(900-1800\right)=-1800R$

$W_{Total}=600R-1800R=-1200R=-10000\,J$

25. c.

$\frac{\Delta Q}{\Delta t}=\frac{KA\Delta T}{\Delta x}$ $\qquad\Delta Q=KA\left(\frac{\Delta T}{\Delta x}\right)\Delta t$

Assuming the thickness of the spheres to be small, we have for smaller sphere: (rate of heat flow) (time) = (volume of ice melted) (ρL)

$i.e.,\ K_1\left(4\pi r^2\right)\frac{\Delta\theta}{d}.16=\frac{4}{3}\pi r^3\rho L\qquad(i)$

For larger sphere:

$$K_2\left[4\pi(2r)^2\right]\frac{\Delta\theta}{d/4}.25=\frac{4\pi}{3}(2r)^3\rho L \qquad (ii)$$

Dividing Eq. (ii) by Eq. (i),

$K_2/K_1=8/25$.

26. b. Evidently the initial temperature of the water contained in the vessel (Mg) is $80°C$, and the temperature of the water passed into it is $60°C$, as the final temperature of the mixture tends to attain a value of $60°C$.

$$M\times1(80-70)=m\times10\times1(70-60)$$

or, $M/m=10$

since the heat exchanged after a long time is 800 cal.

$$(Mg)(1cal/g°C)(80-60°C)=800\,cal$$

or, $M=40g$

$\Rightarrow m=4g$

27. b. Heat given to the metal

$$dQ=P\,dt=C_P(t)dT \qquad (i)$$

At constant pressure in time interval at

Given $T=T_0\left[1+a(t-t_0)\right]^{1/4}$

$$\frac{dT}{dt}=\frac{T_0}{4}\left[1+a(t-t_0)\right]^{-3/4}\times a \qquad (ii)$$

From Eqs. (i) and (ii)

$$C_P(T)=\frac{P}{\left(\frac{dT}{dt}\right)}=\frac{4P\left[1+a(t-t_0)\right]^{3/4}}{T_0a}=\frac{4PT^3}{aT_0^4}$$

28. b. Let *m* be the mass of ice.

Rate of heat given by the burner is constant.

In the first 50 min

$$\frac{dQ}{dt}=\frac{mL}{t_3}=\frac{m\ kg\times(80\times4.2\times10^3)J/kg}{(50\min)} \qquad (i)$$

From 50 min to 60 min

$$\frac{dQ}{dt}=\frac{(m+5)S_{H_2O}\Delta\theta}{t_2}$$

$$=\frac{(m+5)kg(4.2\times10^3)J/kg\times2°C}{10\min} \qquad (ii)$$

From Eqs. (i) and (ii)

$$\frac{80m}{50}=\frac{2(m+5)}{10}$$

$$7m=5\Rightarrow m=\frac{5}{7}kg=0.7\,kg$$

29. Freezing water to ice is a process with Q being negative volume of system increases $\Rightarrow W_{system}$ is positive using $Q=\Delta U+W\Rightarrow\Delta U$ is negative.

30. $\frac{x-0}{100}=\frac{-x-32}{180}\Rightarrow x=-\frac{80}{7}°C$ or $\frac{80}{7}°F$

31. (A) Area under the curve is equal to number of molecules of the gas sample

Area $=\frac{av_0}{2}=N$

(B) $V_{avg}=\frac{\int VdN}{\int dN}=\frac{\int_0^{v_0}\frac{a}{V_0}VdV}{\int_0^{v_0}\frac{a}{v_0}VdV}=\frac{V^{2/3}}{\frac{V^2}{2}}\Big|_0^{V_0}=\frac{2}{3}V_0$

(C) $V_{rms}^2=\frac{\int V^2dN}{\int dN}=\frac{\int_0^{v_0}V^2\frac{a}{V_0}VdV}{\int_0^{v_0}\frac{a}{V_0}VdV}=\frac{V^{4/4}}{V^2/2}\Big|_0^{V_0}=\frac{V_0^2}{2}$

(D) Area under curve from $\frac{V_0}{2}$ to V_0 is $\frac{3}{4}$ of total area

32. Conceptual

33. Rate of heat conduction through rod = rate of the heat lost from right end of the rod.

$$\frac{KA(T_1-T_2)}{L}=eA\sigma\left(T_2^4-T_S^4\right)\qquad (i)$$

Given that $T_2=T_S+\Delta T$

$$T_2^4=(T_S+\Delta T)^4=T_S^4\left(1+\frac{\Delta T}{T_S}\right)^4$$

Using binomial expansion, we have

$$T_2^4=T_S^4\left(1+4\frac{\Delta T}{T_S}\right)\quad (\text{as } \Delta T<<T_S)$$

$$T_2^4-T_S^4=4(\Delta T)(T_S^3)$$

Substituting in Eq. (i), we have

$$\frac{K(T_1-T_S-\Delta T)}{L}=4e\sigma T_S^3\Delta T$$

$$\frac{K(T_1-T_S)}{L}=\left(4e\sigma T_S^3+\frac{K}{L}\right)\Delta T$$

$$\Delta T=\frac{K(T_1-T_S)}{(4e\sigma LT_S^3+K)}$$

Comparing with the given relation, proportionality constant

$$=\frac{K}{4e\sigma LT_S^3+K}$$

34. Let *l* be side of cube at initial temperature and *d* the depth of cube submerged. Then according law of floatation

Weight of solid = weight of liquid displaced

$\therefore$ $Mg=l^2d\rho_lg$

But $l' = l(1+\alpha_s \Delta T), \rho_l = \dfrac{\rho_l}{1+\gamma_l \Delta T}$

Substituting these values in Eq. (iii), we get

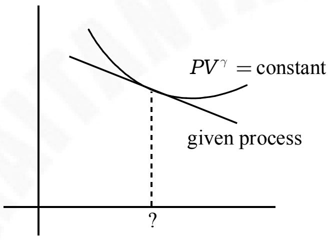
$l^2 d\rho_l g = l^2 (1+\alpha_s\Delta T)^2 \dfrac{\rho_l}{1+\gamma_l \Delta T} g \quad \Rightarrow 1+\gamma_l\Delta T = (1+\alpha_s \Delta T)^2$

As $\alpha_s \Delta T << 1$, using binomial theorem

$1+\gamma_l \Delta T = 1+2\alpha_s \Delta T \ \Rightarrow \gamma_l = 2\alpha_s$

35. The given process will be tangent to an adiabat as shown. We have to find that point

of tangency as it obeys $dQ = 0$. At this point dQ changes sign.



Equation of process is $\dfrac{P}{P_0} + \dfrac{V}{V_0} = 1 \Rightarrow P = P_0\left(1 - \dfrac{V}{V_0}\right)$

Slope of process $= -\dfrac{P_0}{V_0}$

Let us assume required volume is V_1

Slope of adiabat at this volume is $-\dfrac{\gamma P_1}{V_1} = -\dfrac{\gamma P_0\left(1-\dfrac{V_1}{V_0}\right)}{V_1}$

But this should be equal to $-\dfrac{P_0}{V_0}$

$$\Rightarrow -\frac{P_0}{V_0} = -\frac{\gamma P_0\left[1-\dfrac{V_1}{V_0}\right]}{V_1}$$

$$\Rightarrow \frac{V_1}{V_0} = \gamma\left[1-\frac{V_1}{V_0}\right] \quad \Rightarrow V_1 = \frac{V_0\ \gamma}{1+\gamma} = \frac{5V_0}{8}$$

36. The heat current is given as

$i = -KA\dfrac{dT}{dx}$

Putting $K = \alpha\sqrt{T}$, where $\alpha =$ constant, We have

$i = -\alpha\sqrt{T} \qquad A\dfrac{dT}{dx}$

or $\quad idx = -\alpha A\sqrt{T}dT$

Integrating both sides

$i\int_0^x dx = -\alpha A\int_{T_1}^{T}\sqrt{T}dT$

or $ix = -\frac{2}{3}\alpha A\left(T^{3/2} - T_1^{3/2}\right)$

or $ix = \frac{2}{3}\alpha A\left(T_1^{3/2} - T^{3/2}\right)$

putting $x = l$ and $T = T_2$, we have

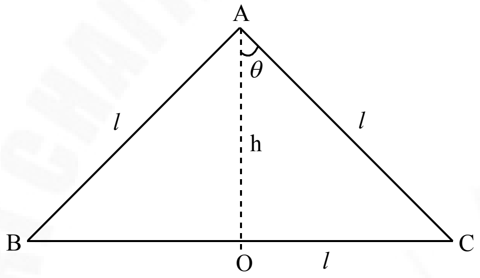
$il = \frac{2}{3}\alpha A\left(T_1^{3/2} - T_2^{3/2}\right)$

On dividing eqn (i) by eqn. (ii),

$\frac{x}{l} = \frac{T_1^{3/2} - T^{3/2}}{T_1^{3/2} - T_2^{3/2}}$

or $T = T_1\left[1 + \frac{x}{l}\left[\left(T_2 / T_1\right)^{3/2} - 1\right]\right]$

37.



$h^2 = l^2 - l^2/4 = \frac{3}{4}l^2$

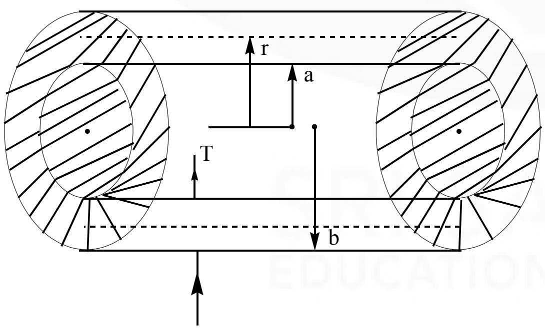
$2h = \frac{dh}{dT} = \frac{3}{4}2l\frac{dl}{dT}$

$y_{cm} = \frac{1}{3}h$

$\frac{dy_{cm}}{dT} = \frac{1}{3}\frac{dh}{dT} = \frac{1}{3}\cdot\frac{3}{4}\ \frac{l}{h}\ \frac{dl}{dT}$

$= \frac{1}{4}\ 2\tan 30°\ l_0\alpha = \frac{1}{2\sqrt{3}}2\times 4\sqrt{3}10^{-6}$

38.



At any instant let "P" be the power entered into the cylinder, $P = K2\pi rl\left(\frac{-dT}{-dr}\right)$;

where "r" is the radius of any cylindrical layer between 'a' and 'b'.

$P = K2\pi l\Delta T\ l\text{n}\left(b/a\right)$.

ΔT is the instantaneous temperature different of body with surroundings.

Then $P = \pi a^2 Sl \dfrac{dT}{dt} = K2\pi l \Delta T \; ln\left(b/a\right)$

or $$t = \frac{\pi a^2 Sl}{K2Tl\,ln\left(b/a\right)} ln\frac{(T_0 - T_2)}{(T_0 - T_1)}$$

$$t = \frac{Sa^2}{2K} ln\left(\frac{b}{a}\right) ln\frac{(T_0 - T_1)}{(T_0 - T_2)}$$

39. $P_1V = N_1kT_1$

$P_2V = N_2kT_2$ and $N_2 = 0.8N_1 + 2(0.2N_1) = 1.2N_1$

$$\frac{P_2}{P_1} = (1.2)\left(\frac{T_2}{T_1}\right) = (1.2)(1.01) \approx 1.21$$

$U_1 = \dfrac{f_1}{2} N_1kT_1$ and $U_2 = \dfrac{f_1}{2} 0.8N_1kT_2 + \dfrac{f_2}{2}(2)(0.2N_1)kT_2$

Where $f_1 = 5$ and $f_2 = 3$ for diatomic and monoatomic gas respectively.

$$\Rightarrow \frac{U_2}{U_1} = 1.04\frac{T_2}{T_1} = 1.05$$

$$C_V = \frac{n_1C_{V_1} + n_2C_{V_2}}{n_1 + n_2} = \frac{0.8\left(\frac{5R}{2}\right) + 0.4\left(\frac{3R}{2}\right)}{1.2} = \frac{13R}{6}$$

% increase in $C_V = \left(\dfrac{13/6}{5/2} - 1\right) \times 100 = \dfrac{-2}{15} \times 100$

$$C_P = C_V + R = \frac{19R}{6}$$

% increase in $C_P = \left(\dfrac{19/6}{7/2} - 1\right) \times 100 = \dfrac{-2}{21} \times 100$

Difference $= \left(\dfrac{2}{15} - \dfrac{2}{21}\right) \times 100 = \dfrac{200 \times 6}{15 \times 21} = \dfrac{80}{21} \approx 4$