MATHS

61. If $f(x)=\int_0^1 |x-t|\,dt, x\in[0,1]$, then $\int_0^1 f(x)\,dx=$

1) 1 2) $\frac{1}{2}$ 3) $\frac{1}{3}$ 4) $\frac{1}{4}$

62. If f(x) is monotonic and differentiable function, then $\int_{f(a)}^{f(b)} 2x\left(b-f^{-1}(x)\right)dx=$

1) $\int_a^b f^2(x)\,dx$ 2) $\int_a^b \left(f^2(x)-f^2(a)\right)dx$

3) $\int_a^b \left(f^2(x)-f^2(b)\right)dx$ 4) $\int_a^b \left(f^2(x)+f^2(b)\right)dx$

63. Let $f(x)$ be differentiable function such that $f(x)=x^2+\int_0^x e^{-t} f(x-t)\,dt$ then $f(1)=$

1) 1 2) 2 3) $\frac{1}{3}$ 4) $\frac{4}{3}$

64. $\int_0^{\frac{\pi}{4}} \frac{\sin x+\cos x}{3+\sin 2x}dx=$

1) $\ln 3$ 2) $\frac{1}{2}\ln 3$ 3) $\frac{1}{3}\ln 3$ 4) $\frac{1}{4}\ln 3$

65. $\int_1^e \frac{(1+\ln x)^2 \, dx}{1+\ln x^{x+1}+\left(\ln x^{\sqrt{x}}\right)^2} =$

1) e 2) $\ln(e-1)$ 3) $\ln(e+1)$ 4) $\frac{\pi}{2}$

66. $\int_0^{\pi} \frac{dx}{(5+4\cos x)^2} =$

1) $\frac{3\pi}{7}$ 2) $\frac{5\pi}{17}$ 3) $\frac{5\pi}{27}$ 4) $\frac{5\pi}{26}$

67. $\int_{-1}^{1} \frac{(x+2)\,dx}{\left(2x^2+4x+3\right)^2} =$

1) $\frac{1}{2\sqrt{2}}+\frac{1}{3}\tan^{-1}\left(2\sqrt{2}\right)$ 2) $\frac{1}{2\sqrt{2}}-\frac{1}{3}\tan^{-1}\left(2\sqrt{2}\right)$

3) $\frac{1}{2\sqrt{2}}\tan^{-1}\left(2\sqrt{2}\right)+\frac{1}{3}$ 4) $\frac{1}{3}-\frac{1}{2\sqrt{2}}\tan^{-1}\left(2\sqrt{2}\right)$

68. $\int_0^{\frac{\pi}{2}} \sec\left(x-\frac{\pi}{6}\right)\sec\left(x-\frac{\pi}{3}\right)dx =$

1) $\ln\sqrt{3}$ 2) $\ln 3$ 3) $2\ln 3$ 4) $3\ln 3$

69. $\int_0^{\pi} \frac{xdx}{a^2\cos^2 x + b^2\sin^2 x} =$ (a > 0 and b > 0)

1) $\frac{\pi^2}{a^2+b^2}$ 2) $\frac{\pi^2}{2\left(a^2+b^2\right)}$ 3) $\frac{\pi^2}{ab}$ 4) $\frac{\pi^2}{2ab}$

70. $\int_0^{\frac{\pi}{2}} \frac{1+2\cos x}{(2+\cos x)^2} dx =$

1) 2 2) $\frac{1}{2}$ 3) $\frac{\pi}{4}$ 4) $\frac{1}{4}$

71. $\int_0^{\frac{\pi}{4}} \frac{dx}{\cos^4 x - \cos^2 x\sin^2 x + \sin^4 x} =$

1) 2π 2) π 3) $\frac{\pi}{2}$ 4) $\frac{\pi}{4}$

72. A cubic polynomial function $f(x)$ vanishes at $x=-2$ and has local extrema at $x=-1$ and $x=\frac{1}{3}$. If $\int_{-1}^{1} f(x)dx = \frac{14}{3}$, then $f(1) =$

1)1 2)2 3)3 4)4

73. If $f(x)=\sin x+\int_0^{\frac{\pi}{2}}\sin x\cos t\, f(t)\,dt$, then $\int_0^{\frac{\pi}{2}} f(x)\,dx=$

1) 1 2) 2 3) $\frac{\pi}{2}$ 4) $\frac{1}{2}$

74. let $f(x)$ be a function such that $f'(x)=f(x), f(0)=1$ and $g(x)$ be a function such that $f(x)+g(x)=x^2$. Then $\int_0^1 f(x)g(x)\,dx=$

1) $e-\frac{e^2}{2}-\frac{5}{2}$ 2) $e+\frac{e^2}{2}-\frac{3}{2}$ 3) $e-\frac{e^2}{2}-\frac{3}{2}$ 4) $e+\frac{e^2}{2}+\frac{5}{2}$

75. If $I_n=\int_0^1 x^n \tan^{-1} x\,dx$, then $5I_4+3I_2=$

1) $\frac{\pi}{2}$ 2) $\frac{\pi}{2}-1$ 3) $\frac{\pi}{2}-\frac{1}{2}$ 4) $\frac{\pi}{2}-\frac{1}{4}$

76. $\int_0^{\frac{\pi}{4}} e^{\sec x}\frac{\sin\left(x+\frac{\pi}{4}\right)}{\cos x(1-\sin x)}dx=$

1) $\frac{1}{\sqrt{2}}\left(e^{\sqrt{2}}-e\right)$ 2) $\left(1+\frac{1}{\sqrt{2}}\right)e^{\sqrt{2}}-\frac{e}{\sqrt{2}}$

3) $\left(1-\frac{1}{\sqrt{2}}\right)e^{\sqrt{2}}+\frac{e}{\sqrt{2}}$ 4) $\frac{1}{\sqrt{2}}\left(e^{\sqrt{2}}+1\right)$

77. Let $f(x)=\int_0^x \frac{dr}{\sqrt{1+r^3}}$ and $g(x)$ be the inverse function of $f(x)$. Then

1) $2g''=g^2$ 2) $2g''=3g^2$ 3) $3g''=2g^2$ 4) $g''=g^2$

78. Let $S_n=\sum_{k=1}^{n}\frac{n}{n^2+kn+k^2}$ and $T_n=\sum_{k=0}^{n-1}\frac{n}{n^2+kn+k^2}$. Then

1) $S_n<\frac{\pi}{3\sqrt{3}}$ 2) $S_n>\frac{\pi}{3\sqrt{3}}$ 3) $T_n<\frac{\pi}{3\sqrt{3}}$ 4) $S_n=\frac{\pi}{3\sqrt{3}}$

79. $f(x)=\int_{-1}^{x} t\left(e^t-1\right)(t-1)(t-2)^3(t-3)^5\,dt$ has a local maximum at x =

1) 0 2) 1 3) 2 4) 3

80. $\lim_{x\to+\infty}(1+x)\int_0^x e^{t^2-x^2}\,dt=$

1) 1 2) $\frac{1}{2}$ 3) $\frac{1}{3}$ 4) 2

81. $\lim_{n\to\infty}\frac{\left(\sqrt{1}+\sqrt{2}+\sqrt{3}+....+\sqrt{n}\right)\left(\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+...+\frac{1}{\sqrt{n}}\right)}{1+2+3+...+n}=$

1) 2 2) 3 3) $\frac{7}{3}$ 4) $\frac{8}{3}$

82. The minimum value of $f(x)=\int_0^4 e^{|x-t|}dt,\ 0\le x\le 4$, is

1) 1 2) e 3) e^2-1 4) $2(e^2-1)$

83. $\int_0^2 [\tan^{-1}x]dx+\int_0^2[\cot^{-1}x]dx=$ $([p]\text{denotes greatest integer not exceeding p})$

1) $1-\cot 2$ 2) $1+\cot 2$ 3) $2(1+\cot 2)$ 4) $2(1-\cot 2)$

84. $\int_0^1 \sin^{-1}\left(2x\sqrt{1-x^2}\right)dx=$

1) $\pi-2$ 2) $\pi+2$ 3) $2(\sqrt{2}-1)$ 4) $2\sqrt{2}$

85. If $f(x)=ax^2+bx+c,\ \int_0^4 f(x)dx=1,\ \int_0^4 xf(x)dx=2,\ \int_0^4 x^2f(x)dx=3,$ then in the interval $[0,4]$ the equation $f(x)=0$ has

1) No root 2) Exactly one root

3) atmost one root 4) atleast one root

86. Let $f(x)$ be a differentiable function. If $f(0)=0$, $f(1)=1$, then the minimum value of $\int_0^1 \left(f'(x)\right)^2 dx$ is

1) 0 2) $\frac{1}{2}$ 3) 1 4) 2

87. $\int_0^1 \tan^{-1}\left(1-x+x^2\right)dx =$

1) $\frac{\pi}{2}$ 2) $\frac{\pi}{4}$ 3) $\ln 2$ 4) $\frac{\pi}{2}-\ln 2$

88. $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x}dx =$

1) $\frac{\pi}{2}$ 2) π^2 3) 0 4) $\frac{\pi^2}{4}$

89. Let $f(x)$ be an even function, $I_1 = \int_0^{\frac{\pi}{2}} f(\cos 2x)\cos x dx$, $I_2 = \int_0^{\frac{\pi}{4}} f(\sin 2x)\cos x dx$. Then $\frac{I_1}{I_2} =$

1) 1 2) $\frac{1}{2}$ 3) $\frac{1}{\sqrt{2}}$ 4) $\sqrt{2}$

90. If $\int \frac{\cos 9x+\cos 6x}{2\cos 5x-1}dx = A\sin 4x + B\sin x + C$, (where A,B are constants and C is the constant of integration) then A+B =

1) $\frac{1}{2}$ 2) $\frac{3}{4}$ 3) $\frac{5}{4}$ 4) $\frac{7}{4}$