

MATHS

41. $f(x) = 1 - x - x^3 \Rightarrow f'(x) = -1 - 3x^2 < 0 \Rightarrow f$ is decreasing

$$G.E \Rightarrow f(f(x)) > f(1-5x) \Rightarrow f(x) < 1-5x$$

$$\Rightarrow 1 - x - x^3 < 1 - 5x \Rightarrow x^3 - 4x > 0 \Rightarrow x(x+2)(x-2) > 0 \Rightarrow x \in (-2, 0) \cup (2, \infty)$$

43. Clearly $f(x) = \{x\}$

A, C, D applicable

44. $4 = -a - c + d$ and $1 = 3a + c + d \Rightarrow 4a + 2c = -3$

Also $4 = a + u + v$ and $1 = -3a - u + v \Rightarrow 4a + 2u = 3$

45.
$$\left. \begin{array}{l} \text{coeff of } x^3 = 0 \Rightarrow [k] = 1, 4 \\ \text{coeff of } x = 0 \Rightarrow \{x\} = \frac{1}{2}, \frac{1}{3} \end{array} \right\} \Rightarrow 1 + \frac{1}{2}, 1 + \frac{1}{3}, 4 + \frac{1}{2}, 4 + \frac{1}{3} \Rightarrow k = \frac{3}{2}, \frac{4}{3}, \frac{9}{2}, \frac{13}{3}$$

46. Conceptual

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48. $\Delta = 0 \Rightarrow 16 - 4(3)(k+1) = 0 \Rightarrow k = \frac{1}{3}$ only

49 & 50.

n(A)	n(B)	$f: A \rightarrow B$	$f: B \rightarrow A$
6	2	2^6	6^2
5	3	3^5	5^3
4	4	4^4	4^4
3	5	5^3	3^5
2	6	6^2	2^6
Max = $4^4 = 256$;		Min = $6^2 = 36$	

53 & 54

$$\begin{aligned} \text{Consider } N &= (1+x)^{1/2} - (1+ax)(1+bx)^{-1} = \left(1 + \frac{1}{2}x - \frac{1}{8}x^2 \dots\right) - (1+ax)(1-bx+b^2x^2\dots) \\ &= \left(\frac{1}{2} + b - a\right)x + \left(-\frac{1}{8} - b^2 + ab\right)x^2 \dots \end{aligned}$$

$$\text{Clearly } a - b = \frac{1}{2} \text{ \& } ab - b^2 = \frac{1}{8}$$

$$\therefore b = \frac{1}{4} \text{ \& } a = \frac{3}{4} \quad \therefore \sin \alpha = \frac{3}{4} \text{ \& } \sin \beta = \frac{1}{4}$$

No. of solutions = 2 in both cases.