MATHEMATICS: Max. Marks : 60

SECTION – I
(MULTIPLE CORRECT CHOICE TYPE)

This section contains **8 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONE OR MORE is/ are correct**

41. The value of $\int_{-\infty}^{a} \frac{\sin^{-1} e^{x} + \sec^{-1} e^{-x}}{\left(\tan^{-1} e^{a} + \tan^{-1} e^{x}\right)\left(e^{x} + e^{-x}\right)} dx,\ a \in R$, is

A) Independent of a B) Depends on a

C) $\frac{\pi}{2}\ell n 2$ D) $\frac{\pi}{2}\ell n\left(2\tan^{-1} e^{a}\right)$

42. If $I_n = \int_{0}^{1}\left(1+x+x^2+.....+x^{n-1}\right)\left(1+3x+5x^2+....+(2n-3)x^{n-2}+(2n-1)x^{n-1}\right)dx, \forall\ n \in N$

then

A) $I_3 + I_4 = I_5$ B) I_n is a perfect square

C) $I_{n+1} - I_n = 2n+1, \forall n \in N$ D) $I_{n+1} - I_n = 2n-1, \forall n \in N$

43. The value of $L = \underset{n\to\infty}{Lt} \frac{\left(\sum_{r=1}^{2n}\sqrt{r}\right)\left(\sum_{r=1}^{3n}\frac{1}{\sqrt{r}}\right)}{\left(\sum_{r=1}^{5n} r\right)}$ is less than

A) $\sqrt{3}$ B) 2 C) $\frac{32}{75}$ D) $\sqrt{2}$

44. Let $f(x)=\int_0^x e^t \sin(x-t)\,dt$ then

A) $f''\left(\frac{\pi}{2}\right)-f'\left(\frac{\pi}{2}\right)=-1$ B) $f''(0)-f'(0)=1$

C) $(f''(x)-f(x))_{max}=\sqrt{2}$ D) $f''\left(\frac{\pi}{2}\right)-f'\left(\frac{\pi}{2}\right)=1$

45. If $f(x)=\int x^{\sin x}(\sin x+x\cos x\ \ell nx+1)dx$ and $f\left(\frac{\pi}{2}\right)=\frac{\pi^2}{4}$ then

A) $f\left(\frac{\pi}{2}\right)<f\left(\frac{\pi}{4}\right)$ B) $f\left(\frac{\pi}{6}\right)=\left(\frac{\pi}{6}\right)^{3/2}$ C) $f\left(\frac{2}{3}\right)=f\left(\frac{3}{2}\right)$ D) $f(\pi)<f(2\pi)$

46. Let f (x) be a differentiable function so that $f^1(x)$ is continuous function and $\int\frac{(f(x)-f^1(x))e^x}{(e^x+f(x))^2}dx$ = g (x) + c, where c is integration constant, then which of the following is / are true?

A) $g(x)=\frac{1}{1+e^x}$ *if* $f(x)=e^{2x}$

B) $g(\pi)=g(2\pi)$ *if* $f(x)=\sin x$

C) $g'(x)=0$ has atleast one solution in $\left(\frac{\pi}{2},\frac{3\pi}{2}\right)$ if f(x) = cosx

D) g (x) is a bounded function when f(x) = e^x

47. Let the value of $\int_0^{\infty} \frac{\ell nx}{x^2+2x+4}\,dx$ be I. Then

A) $\sqrt{3}I < \pi \ell n2$ B) $\frac{3\sqrt{3}I}{\pi} < \ell n3$ C) $I = \frac{\pi}{\sqrt{3}} \ln 8$ D) $\sqrt{3}I = \frac{\pi \ell n2}{3\sqrt{3}}$

48. If $f(x) = \int \frac{2+\sqrt{x}}{\left(x+1+\sqrt{x}\right)^2}\,dx$ and f (0) = 0 and $g(x) = \left(x+1+\sqrt{x}\right) f(x)$ then

A) Integer part of g(4) is 8

B) $f(9) = \frac{18}{13}$

C) The graph of g(x) intersect the line y = 2x +3 at two points.

D) $\int_0^4 g(x)\,dx = 16$

SECTION - II
(COMPREHENSION TYPE)

This section contains **4 groups of questions.** Each group has 2 multiple choice questions based on a paragraph. Each question has 4 choices A), B), C) and D) for its answer, out of which **ONLY ONE** is **correct.**

Paragraph for Questions 49 and 50

let $f: R \to R$ be a differentiable function such that $f(x) = x^2 + \int_0^x e^{-t}\left(f(x-t)\right)dt$. Then

49. $\int_0^1 f(x)\,dx =$

A) $\frac{1}{4}$ B) $\frac{-1}{12}$ C) $\frac{5}{12}$ D) $\frac{12}{7}$

50. f(x) is

A) one-one but not onto B) onto but not one – one

C) both one- one and onto D) neither one – one nor onto

Paragraph for Questions 51 and 52

If $\int\frac{x^3-6x^2+11x-6}{\sqrt{x^2+4x+3}}dx=\left(lx^2+mx+n\right)\sqrt{x^2+4x+3}+\lambda\int\frac{dx}{\sqrt{x^2+4x+3}}$ then

51. Value of $n+\lambda=$

A) $3l+6m$ B) $6m-3l$ C) $3l-6m$ D) $6l+3m$

52. $\int_0^7 3\left(lx^2+mx\right)-\left(\lambda+17\right)dx=$

A) 0 B) $\frac{49}{3}$ C) $\frac{343}{3}$ D) $\frac{-343}{3}$

Paragraph for Questions 53 and 54

Let $I_1=\int_1^2\tan^{-1}x^2dx$ *and* $\int_1^2\left(\cot^{-1}x^2+\tan^{-1}\frac{1-x^2}{1+x^2}\right)dx=I_2$ and if $aI_1+bI_2=\frac{3\pi}{4}$ and also

$I_1=2\tan^{-1}a_1-\frac{\pi}{4}-\frac{1}{\sqrt{2}}\left(\tan^{-1}\frac{3}{2\sqrt{2}}+\ln\left(\frac{a_2+a_3\sqrt{2}}{a_4}\right)^{\frac{1}{2}}\right)$ (where $a_1\in I$, a_2,a_3,a_4 are co primes, a,b are real numbers) then

53. $a_1 + a_2 - a_3 - a_4 =$

A) 0 B) 7 C) 5 D) 11

54. a+b =

A)7 B)3 C)-2 D)1

Paragraph for Questions 55 and 56

Let g(x) = $\int \sin(a+1)x.(\sin x)^{a-1} dx = f(x) + c$, g (0) = 0 where c is integration constant, and a is integer>1

55. $f\left(\frac{\pi}{2}\right)$ when a=50 is

A) 0 B) -1 C) $\frac{1}{50}$ D) $-\frac{1}{50}$

56. Which of the following differs from other options when a=100?

A) $f\left(\frac{\pi}{5}\right)$ B) $f\left(\frac{\pi}{10}\right)$ C) $f\left(\frac{\pi}{4}\right)$ D) $f\left(\frac{\pi}{15}\right)$

SECTION – III

(MATRIX MATCH TYPE)

This section contains **4 multiple choice questions.** Each question has matching lists. The codes for the lists have choices (A), (B), (C), and (D) out of which **ONLY ONE** is correct.

57. Match List –I with List –II

List –I	**List –II**
P) If $f(x)=\lim_{n\to\infty}\sum_{k=1}^{n}\frac{n}{n^2+k^2x^2}$, x>0 , then value of f(1) is	1) $\frac{\pi}{2}$
Q) The value of $\int_0^{\frac{1}{\sqrt{2}}}\frac{x^2}{\sqrt{1-x^2}\left(1+\sqrt{1-x^2}\right)}dx$ is	2) $-\frac{\pi}{2}$
R) Let $\int_0^{f(x)} t^2dt = x\cos\pi x$ then the value of	3) $\frac{\pi}{4}$
$\frac{\pi^2}{8}\int_{f^2(9)f'(9)}^{1}\left\lvert\sin\pi x\right\rvert dx$ is	
S) Value of $\int_{-\frac{1}{2}}^{\frac{1}{2}}\left(\sin^{-1}\left(3x-4x^3\right)-\cos^{-1}\left(4x^3-3x\right)\right)dx$	4) $\frac{\pi}{4}-\frac{1}{\sqrt{2}}$

A) P-4 Q-1 R-3 S-2
B) P-1 Q-3 R-2 S-4
C) P-2 Q-4 R-1 S-3
D) P-3 Q-4 R-1 S-2

58. **List –I** **List –II**

P) $\underset{n\to\infty}{Lt}\sum_{i=1}^{n}\frac{3}{n}\sin\left(2\pi+\frac{3\pi i}{n}\right)$ is equal to 1) $\frac{2}{e}$

Q) $\int_0^{\infty}[x]e^{-x}dx$, [.] integer part, is equal 2) $\frac{1}{\pi}$

R) If $\underset{n\to\infty}{Lt}\sum_{i=1}^{n}\left(\frac{\ln 2i-\ln n}{n}\right)=\ln k$ then k 3) $\frac{2}{\pi}$

S) $\underset{n\to\infty}{Lt}\sum_{i=1}^{n}\frac{i}{n^2}\sin\left(\frac{i^2\pi}{n^2}\right)$ is equal to 4) $\frac{1}{e-1}$

A) P-3 Q-4 R-1 S-2

B) P-3 Q-4 R-2 S-1

C) P-2 Q-1 R-4 S-3

D) P-2 Q-4 R-4 S-2

59. Given $\int_0^{\infty} e^{-x^2}\,dx = \frac{\sqrt{\pi}}{2}$ then match list-I with List-II

List-I	**List-II**
P) $\int_0^{\infty} e^{-3x^2}\,dx$ is equal to	1) $\frac{\sqrt{\pi}}{6}$
Q) $\int_{-\infty}^{\infty} e^{-3x^2}\,dx$ is equal to	2) $\frac{1}{2}\sqrt{\frac{\pi}{3}}$
R) $\int_0^{\infty} \frac{1}{x^2} e^{-\frac{1}{x^2}}\,dx$ is equal to	3) $\frac{\sqrt{3}\pi}{3}$
S) $\int_0^{\infty} e^{-9x^2}\,dx$ is equal to	4) $\frac{\sqrt{\pi}}{2}$

A) P – 4 Q – 1 R – 2 S – 3

B) P – 3 Q – 4 R – 1 S – 2

C) P – 1 Q – 3 R – 2 S – 4

D) P – 2 Q – 3 R – 4 S – 1

60. Match list – I with List – II

List – I	**List – II**
P) The value of $\int_0^1 \frac{1}{\sqrt{x}+\sqrt[3]{x}}dx = a+b\ell n2$ Where a, b are integers then $\lvert a+b\rvert$	1) 4
Q) If $\int_0^{\ell n10} \frac{e^x\sqrt{e^x-1}}{e^x+8}dx = a-b\pi$ then $\frac{a}{b}=$ (a,b relatively prime)	2) 0
R) $\frac{\int_0^{\pi/2}(\cos x)^{\sqrt{2}+1}dx}{\int_0^{\pi/2}(\cos x)^{\sqrt{2}-1}dx} = a-\sqrt{b}$ *then* $\lvert a-b\rvert$	3) 1
S) If $\int_0^2 \frac{\ell n(1+2x)}{1+x^2}dx = \left(\tan^{-1}a\right)\ell n\sqrt{b}$ *then* $\lvert b-a\rvert$	4) 3

A) P – 3 Q – 1 R – 4 S – 2

B) P – 1 Q – 2 R – 3 S – 4

C) P – 3 Q – 1 R – 2 S – 4

D) P – 1 Q – 3 R – 3 S – 2