MATHS **Max.Marks:80**

SECTION-1
(SINGLE CORRECT CHOICE TYPE)

Section-I (Single Correct Answer Type, Total Marks: 24) contains 8 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONLY ONE is correct. For each question you will be awarded 3 marks if you darken ONLY the bubble corresponding to the correct answer and zero marks if no bubble is darkened. In all other cases, minus one (-1) mark will be awarded.

41. Let $S_1 = \{(x,y) : \log_{4-x}(16x - y^2) \text{ is defined}\}$

$$S_2 = \left\{(x,\, y) : \frac{xy(x^2 - 3x + 2)}{x^2 - 7x + 12} > 0\right\}$$

Then area bounded by $S_1 \cap S_2$ is__________

A) $\frac{8}{3}$ B) $\frac{64}{3}$ C) $\frac{32}{3}$ D) none

42. A square ABCD is inscribed in a circle of radius 4. A point P moves inside the circle such that $d(P,\, AB) \le \min\{d(P,\, BC), d(P,\, DA)\}$ where $d(P,\, AB)$ is the distance of a point P from line AB. The area of region covered by the moving point P, is (in square units).

A) 4π B) 8π C) $8\pi - 16$ D) $4\pi - 4$

43. If $y_1(x)$ (not identically zero) is a solution of the differential equation $\frac{dy}{dx} + f(x)y = 0$ then a solution of the differential equation $\frac{dy}{dx} + f(x)y = r(x)$ is __

A) $\frac{1}{y(x)}\int y_1(x)dx$ B) $y_1(x)\int \frac{r(x)}{y_1(x)}dx$

C) $\int r(x)y_1(x)dx$ D) none of these

44. Two lines drawn from the point $P(4, 0)$ divide the area, bounded by the curve $y = \sqrt{2}\sin\left(\frac{\pi x}{4}\right)$ and x-axis between the lines $x = 2$ and $x = 4$ into three equal parts. Sum of the slopes of the drawn lines is equal to_________

A) $\frac{-2\sqrt{2}}{\pi}$ B) $\frac{-\sqrt{2}}{\pi}$ C) $\frac{-2}{\pi}$ D) $\frac{-4\sqrt{2}}{\pi}$

45. The primitive of the differential equation $\left(2xy^4e^y + 2xy^3 + y\right)dx + \left(x^2y^4e^y - x^2y^2 - 3x\right)dy = 0$ is_____

A) $x^2e^y + \frac{x^2}{y} - \frac{x}{y^3} = k$ B) $x^2e^y - \frac{x^2}{y} + \frac{x}{y^3} = k$

C) $x^2e^y + \frac{x^2}{y} + \frac{x}{y^3} = k$ D) $x^2e^y - \frac{x^2}{y} - \frac{x}{y^3} = k$

46. The area of the region $\left\{(x, y): x^2 + y^2 \le 5,\ \left||x| - |y|\right| \ge 1\right\}$ is_________ (in square units)

A) $4\left\{\pi - \tan^{-1}\left(\frac{24}{7}\right)\right\} - 4$ B) $10\pi - 4 - 20\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$

C) $10\pi + 4 - 20\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ D) $2\left\{\pi - \tan^{-1}\left(\frac{24}{7}\right)\right\} - 1$

47. Let $y(x)$ be a solution of the differential equation $(1+e^x)y'+ye^x=1$. If $y(0)=2$, then which of the following statements is (are) true?

A) $y(4)=0$

B) $y(-2)=0$

C) $y(x)$ has a critical point in the interval $(-1,0)$

D) $y(x)$ has no critical point in the interval $(-1,0)$

48. Tangent drawn at any point P on a curve, meets x-axis at Q such that x coordinate of the circumcentre of triangle POQ is half of the y coordinate of the circumcentre of the triangle POQ (where O is origin). Then the differential equation to such a curve, is________

A) $\frac{dy}{dx}=\frac{x+2y}{2x-y}$ B) $\frac{dy}{dx}=\frac{2y-x}{2x+y}$ C) $\frac{dy}{dx}=\frac{2x+y}{2x-y}$ D) none

SECTION-2

(MORE THAN ONE TYPE)

Section - II (Multiple Correct Answers Type, Total Marks: 16) contains 4 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONE or MORE may be correct. For each question you will be awarded 4 marks if you darken ALL the bubble(s) corresponding to the correct answer(s) ONLY and zero marks otherwise. There are no negative marks in this section.

49. If $P(x,y)$ be any point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and S be the secrtorial area bounded by the curve the x-axis and the line joining the origin to P, then

A) $x=a\cos\left(\frac{2s}{ab}\right)$ B) $y=b\sin\left(\frac{s}{ab}\right)$ C) $S=\frac{ab}{2}\cos^{-1}\left(\frac{x}{a}\right)$ D) $S=ab\sin^{-1}\left(\frac{y}{b}\right)$

50. Which of the following is/are true?

A) If $y = f(x)$ is a strictly monotonic function in (a,b) with $f'(x) \neq 0$, then the area bounded by $y = f(x)$, the lines $x = a, x = b$ & $y = f(c)$ where $c \in (a,b)$ is minimum when $c = \frac{a+b}{2}$

B) If $y = f(x)$ is a strictly monotonic function in (a,b) with $f'(x) \neq 0$, then the area bounded by the lines $x = a, x = b$ & $y = f(c)$ where $c \in (a,b)$ is minimum when $c = \frac{2a+b}{3}$ or $c = \frac{a+2b}{3}$

C) If the area bounded by $f(x) = \frac{x^3}{3} - x^2 + a$ & the lines $x = 0, x = 2$ & the x-axis is minimum when $a = 2/3$

D) If the area bounded by $f(x) = \frac{x^3}{3} - x^2 + a$ & the lines $x = 0, x = 2$ & the x-axis is minimum when $a = \frac{28}{81}$

51. Consider the differential equation $(x-2)(x-3)\frac{dy}{dx} + 2y = (x-1)(x-2)$

A) All solutions do not tend to a finite limit as $x \to 2$

B) All solutions tend to a finite limit as $x \to 2$

C) All solutions tend to a finite limit as $x \to 3$

D) No solution tends to a finite limit as $x \to 3$

52. Consider the curves $y^2 = |x|$ & $x^2 = |y|$ then

A) equation of the common tangent drawn to $y^2 = x$ & $x^2 = y$ is $x + y + \frac{1}{4} = 0$

B) equation of the common tangent drawn to $y^2 = -x$, $x^2 = -y$ is $x + y - \frac{1}{4} = 0$

C) area of the quadrilateral, formed by the common tangents (other than coordinate axes) to the curves $y^2 = |x|$ & $x^2 = |y|$ taken in pairs is $\frac{1}{8}$ square units

D) area bounded by the lines $|x| + |y| = \frac{1}{4}$ is $\frac{1}{8}$

SECTION-3
[INTEGER TYPE]

Section-III (Integer Answer Type, Total Marks: 24) contains 6 questions. The answer to each of the questions is a single-digit integer, ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the ORS. For each question you will be awarded 4 marks if you darken ONLY the bubble corresponding to the correct answer and zero marks otherwise. There are no negative marks in this section.

53. Let $y = f(x)$ be a curve passing through (e, e^e) which satisfy the differential equation $(2ny + xy\log_e x)dx - (x\log_e(x))dy = 0$ $(x > 0, y > 0)$ *and* $g(x) = \underset{n\to\infty}{Lt} \frac{1}{50} f(x)$ then

$\int_{1/e}^{e} g(x)\,dx = -------$

54. If the solution of the differential equation

$\left(y+x\sqrt{xy}\,(x+y)\right)dx+\left(y\sqrt{xy}\,(x+y)-x\right)dy=0$ & $y(1)=1$ is $\frac{x^2+y^2}{n}+m\left(\tan^{-1}\sqrt{\frac{x}{y}}\right)=1+\frac{\pi}{2}$ then the value of $m+n=$ ………..

55. If $y=f(x)$ is the solution of the differential equation $x\left(y^3-x\right)dy=y\left(x+y^3\right)dx$ and $f(1)=(-2)^{1/3}$ and $f^{-1}(-2)=k$. Then $|k|=$__________

56. If the solution of equation

$\frac{dy}{dx}=y+\int_0^2 ydx$ is $y(x)$, given that y(0)=1. Then the value of

$[|y(2)|]=$________ ([.] *denotes greatest* int *eger function*)

57. A continuous function $f:R\to R$, satisfying the equation $f(x)=\left(1+x^2\right)\left(1+\int_0^x \frac{f^2(t)}{1+t^2}dt\right)$

If the area of triangle formed by tangent drawn to the curve $y=f(x)$ at $x=1$ with the coordinate axes is Δ then the value of $\left[\frac{\Delta}{3}\right]=$ ([.] greatest integer function)

58. Area of the region, enclosed between the curves $x=y^2-1$ and $x=|y|\sqrt{1-y^2}$ is___

SECTION-4

[Matrix Matching Type]

Section-IV (Matrix-Match Type, Total Marks: 16) contains 2 questions. Each question has four statements (A, B, C and D) given in Column I and five statements (p, q, r, s and t) in Column II. Any given statement in Column I can have correct matching with ONE or MORE statement(s) given in Column II. For example, if for a given question, statement B matches with the statements given in q and r, then for the particular question, against statement B, darken the bubbles corresponding to q and r in the ORS. For each question you will be awarded 2 marks for each row in which you have darkened ALL the bubble(s) corresponding to the correct answer(s) ONLY and zero marks otherwise. Thus, each question in this section carries a maximum of 8 marks. There are no negative marks in this section.

59.

Column-I	Column-II
A) Area bounded between curve $f(x) = \cos^{-1}(\cos x)$ $0 \le x \le 2\pi$ with the tangent drawn to the curve $f(x) = \lvert\cos x\rvert$ at $x = \pi$ is	P) $\frac{\pi^2}{2}$
B) $y = f(x)$ be a function such that $f(x) = \min\left\{\sqrt{x(2-x)}, 2-x\right\}$, then area bounded by $y = f(x)$ and x-axis is	Q) $(\pi - 1)^2$
C) Area bounded by the curve $\lvert y\rvert = \sin^{-1}(\sin x)$, $2\pi < x < 3\pi$ is	R) $\frac{\pi^2}{4}$
D) Area bounded between curves $y = \tan^{-1}(\tan x)$, $\pi < x < 2\pi$ and the lines $y = \pi - x$, $y = -x + 2\pi$, is	S) $\frac{\pi}{4} + \frac{1}{2}$

60. The differential equation, (column-II) for the corresponding family of curves given in column -I is (where a and b are arbitrary constants)

A) $y = ae^{3x} + be^{5x}$	P) $\frac{d^2y}{dx^2} + 16y = 0$
B) $xy = ae^{x} + be^{-x} + x^2$	Q) $\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 15y = 0$
C) $y = ax^2 + bx$	R) $x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - xy + x^2 - 2 = 0$
D) $y = a\sin(4x+b)$	S) $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$