Sri Chaitanya IIT Academy, India

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A right Choice for the Real Aspirant

ICON CENTRAL OFFICE, MADHAPUR-HYD

Sec: Sr. IPLCO	JEE ADVANCED	DATE : 08-11-15
TIME : 02:00 AM TO 05: 00 PM	2013_P2 MODEL	MAX MARKS : 180

KEY & SOLUTIONS

PHYSICS

1	**B**	2	**AB**	3	**BCD**	4	**ABC**	5	**ABC**
6	**ABD**	7	**ACD**	8	**BCD**	9	**B**	10	**A**
11	**B**	12	**B**	13	**C**	14	**B**	15	**A**
16	**D**	17	**D**	18	**B**	19	**C**	20	**A**

CHEMISTRY

21	**ABC**	22	**BCD**	23	**ABCD**	24	**BCD**	25	**ABCD**
26	**ABC**	27	**ABCD**	28	**ABCD**	29	**B**	30	**C**
31	**C**	32	**C**	33	**C**	34	**B**	35	**A**
36	**C**	37	**C**	38	**A**	39	**D**	40	**B**

MATHEMATICS

41	**AC**	42	**ABC**	43	**ABD**	44	**BC**	45	**ABD**
46	**ABCD**	47	**ABCD**	48	**ABD**	49	**C**	50	**B**
51	**B**	52	**C**	53	**A**	54	**B**	55	**A**
56	**D**	57	**D**	58	**A**	59	**D**	60	**C**

MATHS

41. $I=\int_{-\infty}^{a}\frac{\left(\sin^{-1}e^{x}+\cos^{-1}e^{x}\right)}{\left(\tan^{-1}e^{a}+\tan^{-1}e^{x}\right)}\frac{e^{x}}{e^{2x}+1}\,dx=\frac{\pi}{2}\int_{-\infty}^{a}\frac{1}{\left(\tan^{-1}e^{a}+\tan^{-1}e^{x}\right)}\frac{e^{x}}{e^{2x}+1}dx$

Put $\tan^{-1}e^{x}=t,$ ten $I=\frac{\pi}{2}\log\left(t+\tan^{-1}e^{a}\right)\Big|_{0}^{\tan^{-1}e^{a}}$

$=\frac{\pi}{2}\ell n2$

42. $I_n=\int_0^1\left(1+x+x^2+......x^{n-1}\right)\left(1+3x+5x^2+....+(2n-3)x^{n-2}+(2n-1)x^{n-1}\right)dx$

Put $x=t^2$ then $I_n=2\int_0^1\left(t+t^3+t^5+.......+t^{2n-1}\right)\left(1+3t^2+5t^4+...........+(2n-1)t^{2n-2}\right)dt$

Now put $t+t^3+t^5+.........+t^{2n-1}=u$

Then $I_n=2\int_0^n u\,du=n^2$

$\therefore I_n=n^2\ \ \forall n\in N$

43. $L=\frac{\left(\int_0^2\sqrt{x}\ dx\right)\left(\int_0^3\frac{1}{\sqrt{x}}\ dx\right)}{\int_0^5 x\ dx}=\frac{16\sqrt{2}}{25\sqrt{3}}$

44. $f(x)=e^{x}\int_0^x e^{-t}\sin t\ dt$

$f'(x)=e^{x}\ e^{-x}\ \sin x+f(x)$

$f'(x)-f(x)=\sin x$

$f''(x)-f'(x)=\cos x$

$f''(x)-f(x)=\sin x+\cos x$

45. Given integrand is derivative of $x.x^{\sin x}$

1) $\left(\frac{\pi}{4}\right)^{1+\frac{1}{\sqrt{2}}}>\left(\frac{\pi}{4}\right)^{2}$ 2) $f\left(\frac{\pi}{6}\right)=\left(\frac{\pi}{6}\right)^{1/2+1}$ 4) $f(\pi)=\pi,\ f(2\pi)=2\pi$

46. $I=\int\frac{\left(f(x)-f'(x)e^{x}\right)}{\left(e^{x}+f(x)\right)^{2}}dx=\int\frac{e^{-x}\left(f(x)-f'^{(x)}\right)}{\left(1+e^{-x}f(x)\right)^{2}}dx$

Put $\frac{1}{1+e^{-x}f(x)}=t$

Then $I=\frac{1}{1+e^{-x}f(x)}+c$

$\therefore g(x)=\frac{1}{1+e^{-x}f(x)}$

1) $g(x)=\frac{1}{1+e^{x}}$ if $f(x)=e^{2x}$

2) $g(\pi)=g(2\pi)$ if $f(x)=\sin x$ as $g(x)=\frac{1}{1+e^{-x}\sin x}$

3) $g(x)=\frac{1}{1+e^{-x}\cos x}$

$\because g\left(\frac{\pi}{2}\right)=g\left(\frac{3\pi}{2}\right)$ by Rolles theorem $g'(c)=0$ $for\ some\ c\ \ such\ that\ \frac{\pi}{2}<c<\frac{3\pi}{2}$

4) g (x) s bounded when $f(x)=e^{x}$ $\sin ce$ $g(x)=\frac{1}{2}$

47. $I=\int_0^{\infty}\frac{\ell nx}{x^2+2x+4}dx=\frac{\pi\ell n2}{3\sqrt{3}}$

48. $\int\frac{\left(\frac{2}{x^2}+\frac{1}{x\sqrt{x}}\right)}{\left(1+\frac{1}{\sqrt{x}}+\frac{1}{x}\right)^2}dx$, put $1+\frac{1}{\sqrt{x}}+\frac{1}{x}=t$

Then $f(x)=\frac{2x}{x+\sqrt{x}+1}$

g (x) = 2x

49 - 50

$f(x)=x^2+\int_0^x e^{-t}f(x-t)\,dt$

$=x^2+e^{-x}\int e^{t}f(t)\,dt$

$\Rightarrow f^1(x)=2x+x^2$

$\Rightarrow f(x)=\frac{x^3}{3}+x^2$

f (x) is onto but not one-one

$$\int_0^1 f(x)\,dx = \frac{1}{12}+\frac{1}{3}=\frac{5}{12}$$

51 – 52

Differentiations both sides

$$\frac{x^3-6x^2+11x-6}{\sqrt{x^2+4x+3}} = \left(\ell x^2+mx+n\right)\frac{x+2}{\sqrt{x^2+4x+3}}+\left(2\ell x+m\right)\sqrt{x^2+4x+3}+\frac{\lambda}{\sqrt{x^2+4x+3}}$$

$$\Rightarrow x^3-6x^2+11x-6=\left(\ell x^2+mx+x\right)\left(x+2\right)+\left(2\ell x+m\right)\left(x^2+4x+3\right)+\lambda$$

Comparing like terms, gives $\ell=\frac{1}{3},\ m=\frac{-14}{3}\ n=37\ \ \lambda=-66$

51. $3\ell-6m+\lambda+n=1+28-66+37=0$

52. $3\ell x^2+3mx-\lambda-17=x^2-14x+49=\left(x-7\right)^2$

53-54

$$I_2=\int_1^2\left(\frac{\pi}{2}-tan^{-1}x^2+\tan^{-1}1-tan^{-1}x^2\right)dx=\frac{3\pi}{4}-2I_1\Rightarrow a=2,\ b=1$$

Again, $I_1=\int_1^2 tan^{-1}x^2dx=x\tan^{-1}x^2\Big|_1^2-\int_1^2\frac{2x^2}{x^4+1}dx$

$$2\tan^{-1}4-\frac{\pi}{4}-\int_1^2\frac{x^2+1+\left(x^2-1\right)}{x^4+1}\,dx$$

Now use $\int\frac{x^2+1}{x^4+1}dx=\int\frac{1+\frac{1}{x^2}}{\left(x-\frac{1}{x}\right)^2+2}\,dx$

$$\int\frac{x^2-1}{x^4+1}dx=\int\frac{1-\frac{1}{x^2}}{\left(x+\frac{1}{x}\right)^2-2}\,dx$$

55-56

$$\left(\sin ax\ .\ \cos x+\cos a\,x.\ \sin x\right)\left(\sin x\right)^{a-1}$$

$$=\ \sin ax.\left(\sin x\right)^{a-1}\cos x+\cos ax.\left(\sin x\right)^a=\frac{1}{a}\frac{d}{dx}\left(\sin ax.\left(\sin x\right)^a\right)$$

$$\therefore f\left(x\right)=\frac{\sin ax\left(\sin x\right)^a}{a}$$

When a = 50, f (x) = $\frac{\sin 50x.(\sin x)^{50}}{50}$

When a = 100, f(x) = $\frac{\sin 100x.(\sin x)^{100}}{100}$

57. (P) $f(x)=\frac{\tan^{-1}x}{x}$

(R) $f^2(9)f'(9)=-1$

(S) use $\sin^{-1}(-x)=-\sin^{-1}x$ *and* $\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}$

58. P) $3\int_0^1\sin(2\pi+3\pi x)dx=3\int_0^1\sin 3\pi x dx=\frac{2}{\pi}$

Q) $I=\int_0^\infty [x]\,e^{-x}dx=\int_0^1 0+\int_1^2 e^{-x}dx+\int_2^3 2e^{-x}dx+.......$

$$= e^{-1}+e^{-2}+e^{-3}+........=\frac{1}{e-1}$$

R) $\underset{n\to\infty}{Lt}\frac{1}{n}\sum_{i=1}^{n}\ell n\left(\frac{2i}{n}\right)=\int_0^1(\ell n2+\ell nx)\,dx=\ell n\frac{2}{e}\Rightarrow k=\frac{2}{e}$

S) $\int_0^1 x\sin\pi x^2dx=\frac{1}{\pi}$

60. P) put $x=t^6 then\, I=\int_0^1\frac{6t^5}{t^3+t^2}=5-6\log 2$

Q) $I=\int_0^{1n10}\frac{e^x\sqrt{e^x-1}}{e^x+8}dx\ \ put\ e^x-1=t^2$

Then $I=6-\frac{3\pi}{2},\ \ \frac{a}{b}=4$

R) $I_1=\int_0^{\pi/2}(\cos x)^{\sqrt{2}+1}dx,\ \ I_2=\int_0^{\pi/2}(\cos x)^{\sqrt{2}-1}dx,$

Now $I_1=\int_0^{\pi/2}(\cos x)^{\sqrt{2}}\cos x\,dx$ use integration by parts

$I_1=\sqrt{2}\ I_2-\sqrt{2}\ I_1\Rightarrow\frac{I_1}{I_2}=2-\sqrt{2}$

S) $I=\int_0^a\frac{\ell n(1+ax)}{1+x^2}dx\ =(\tan^{-1}a)\left(\ell n\sqrt{1+a^2}\right)$