PART-III_MATHEMATICS Max Marks : 60

Section-1
(Only one Option correct Type)

This section contains 10 Multiple Choice questions. Each Question has Four choices (A), (B), (C) and (D). Out of Which **Only One is correct**

41. Let $\bar{a},\bar{b},\bar{c}$ be three vectors given by $\bar{a}=i+j+2k,\bar{b}=2i+j-k,\bar{c}=5i+3j$ and if $\bar{r}$ is a unit vector then the maximum possible value of $\left\|\left[\bar{r}\,\bar{a}\,\bar{b}\right]\right\|+\left\|\left[\bar{r}\,\bar{b}\,\bar{c}\right]\right\|$ equals

A) $2\sqrt{35}$ B) $3\sqrt{7}$ C) $5\sqrt{7}$ D) $10\sqrt{7}$

42. If $\bar{a}+\bar{b}+\bar{c}=\bar{p},\bar{a}\times\bar{b}=\bar{q},\bar{b}\times\bar{c}=\bar{r},\bar{a}.\bar{p}=1,\bar{b}.\bar{p}=1$ and $|\bar{p}|^2=3$ then which is false

A) $\bar{a}=\frac{1}{3}(\bar{p}+2\bar{p}\times\bar{q}+\bar{p}\times\bar{r})$ B) $\bar{b}=\frac{1}{3}(\bar{p}+\bar{p}\times\bar{q}+\bar{p}\times\bar{r})$

C) $\bar{p}\times\bar{q}=\bar{a}-\bar{b}$ D) $\left[\bar{a}\,\bar{b}\,\bar{c}\right]=\left[\bar{p}\,\bar{q}\,\bar{r}\right]$

43. Let $\bar{a},\bar{b},\bar{c}$ be unit vectors and are non-coplanar such that $\left[\bar{a}\times(\bar{b}\times\bar{c})\ 2\bar{b}\times(\bar{c}\times\bar{a})\ 3\bar{c}\times(\bar{a}\times\bar{b})\right]=(\lambda^3-\lambda)\left[\bar{a}\,\bar{b}\,\bar{c}\right],\lambda$ is real, then number of possible values of λ

A) 0 B) 1 C) 2 D) 3

44. If A, B,C,D are four points in space and satisfying $\left|\overrightarrow{AB}\right| = 3, \left|\overrightarrow{BC}\right| = 7, \left|\overrightarrow{CD}\right| = 11$ and $\left|\overrightarrow{DA}\right| = 9$ then the value of $\overrightarrow{AC}.\overrightarrow{BD}$

A) 0 B) 21 C) 15 D) 27

45. If the vectors $-\bar{i} + c\bar{j} + b\bar{k}, c\bar{i} - \bar{j} + a\bar{k}, b\bar{i} + a\bar{j} - \bar{k}$ are coplanar vectors and $|a| \le 1$ then the maximum value of $\left|a\bar{i} + b\bar{j} + c\bar{k}\right|^2$ is

A) 3 B) 12 C) 18 D) 8

46. The resultant of the two vectors $\bar{a}$ and $\bar{b}$ is $\bar{c}$. If $\bar{c}$ trisects the angle between $\bar{a}$ and $\bar{b}$ and if $|\bar{a}| = 6, \left|\bar{b}\right| = 4$ then $|\bar{c}|$ equals (no two vectors are parallel)

A) 3 B) 4 C) 5 D) 6

47. If the vector $\bar{r}$ satisfies $\bar{r} \times \bar{a} + \left(\bar{r}.\bar{b}\right)\bar{c} = \bar{d}$ be given by $\bar{r} = \lambda\bar{a} + \bar{a} \times \dfrac{\bar{a} \times \left(\bar{d} \times \bar{c}\right)}{\left(\bar{a}.\bar{c}\right)|\bar{a}|^2}$ then $\lambda =$ ($\bar{a}, \bar{b}, \bar{c}, \bar{d}$ are non – zero vectors and $\bar{a}$ is not perpendicular to $\bar{c}$)

A) $\dfrac{\bar{a}.\bar{c}}{|\bar{a}|^2}$ B) $\dfrac{\bar{a}.\bar{b}}{|\bar{a}|^2}$ C) $\dfrac{\bar{c}.\bar{d}}{|\bar{a}|^2}$ D) $\dfrac{\bar{r}.\bar{a}}{|\bar{a}|^2}$

48. The plane which contains the line $3x+y=1, z=4$ and parallel to the line $x+y+z+1=0, y+2z=1$ cuts the x, y, z axes respectively at $(\alpha,0,0),(0,\beta,0),(0,0,\gamma)$ then $\alpha^2+\beta^2+\gamma^2$ equals

A) 10 B) 14 C) 19 D) 21

49. Given a tetrahedran ABCD with AB=12, CD=6,. If the shortest distance between the skew lines AB and CD is 8 and the angle between them is $\frac{\pi}{6}$ then the volume of tetrahedran is

A) 12 B) 36 C) 48 D) 72

50. If $\overline{x}$ and $\overline{y}$ be two unit vectors inclined at an angle 60^0 so that $\overline{x}+\overline{y}=\overline{a}$ and $\overline{x}\times\overline{y}=\overline{b}$ such that $\overline{x}=\alpha\overline{a}+\beta\left(\overline{a}\times\overline{b}\right)$ then the value of $\frac{1}{\beta}+2\alpha$ is

A) 1 B) 2 C) 4 D) 5

Section-2
(Paragraph Type)

This section contains 3 paragraphs each describing theory, experiment, data etc. Six questions relate to three paragraphs with two questions on each paragraph. Each question pertaining to a particular paragraph should have **only one correct answer** among the four choices A, B, C and D.

Paragraph for Questions 51 & 52

A plane P contains the line $L_1 : \frac{y}{b} + \frac{z}{c} = 1, x = 0$ and is parallel to the line $L_2 : \frac{x}{a} - \frac{z}{c} = 1, y = 0$ then

51. Equation of the plane P is

A) $\frac{x}{a} - \frac{y}{b} + \frac{z}{c} + 1 = 0$ B) $\frac{x}{a} - \frac{y}{b} - \frac{z}{c} + 1 = 0$

C) $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} + 1 = 0$ D) $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} - 1 = 0$

52. If the shortest distance between the lines L_1 and L_2 is ¼ then the shortest distance from origin to the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ is

A) ¼ B) ½ C) 1 D) 1/8

Paragraph For Questions 53 & 54

If the three lines $L_1 : x = y = z, L_2 : x = \frac{y}{2} = \frac{z}{3}, L_3 : \frac{x-1}{a} = \frac{y-1}{b} = \frac{z-1}{c}$ from a triangle of area $\sqrt{6}$ square units

53. If (α, β, γ) is the point of intersection of L_2 and L_3 then $|\alpha + \beta + \gamma|$ equals

A) 4 B) 6 C) 0 D) 12

54. The possible acute angle between L_2 and L_3 is

A) $\frac{\pi}{3}$ B) $\frac{\pi}{6}$ C) $\cos^{-1}\left(\frac{22}{7\sqrt{10}}\right)$ D) $\frac{\pi}{2}$

Paragraph For Questions 55 & 56

The line of greatest slope on an inclined plane P_1 is the line in the plane P_1 which is perpendicular to the line of intersection of the plane P_1 and a horizontal plane P_2

55. Assuming the plane $2x - 3y + 4z = 0$ to be horizontal, the direction cosines of the line of greatest slope in the plane $x - 2y + 3z = 0$ are

A) $\frac{4}{\sqrt{21}}, \frac{-1}{\sqrt{21}}, \frac{-2}{\sqrt{21}}$ B) $\frac{4}{\sqrt{21}}, \frac{1}{\sqrt{21}}, \frac{-2}{\sqrt{21}}$ C) $\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}$ D) $\frac{4}{\sqrt{21}}, \frac{2}{\sqrt{21}}, \frac{-1}{\sqrt{21}}$

56. The coordinates of a point on the plane $2x+y-5z=0$ which is $2\sqrt{11}$ units away from the line of intersection of $2x+y-5z=0$ and $4x-3y+7z=0$ are

A) $(6,2,-2)$ B) $(3,1,-1)$ C) $(6,-2,2)$ D) $(1,3,-1)$

Section-3

(Matching List Type)

This section contains four questions, each having two matching lists (List-1 & List-II). The options for the **correct match** are provided as (A), (B),(C) and (D) out of which **ONLY ONE** is correct.

57. Match the following

	COLUMN – I	COLUMN – II
(A)	A line perpendicular to $x+2y+2z=0$ and passes through $(0,1,0)$. The perpendicular distance of this line from origin is k then $[k]$ equals ([] is GIF)	(p) 3
(B)	The value of λ for which the plane $x-y+z+1=0$, $\lambda x+3y+2z-3=0, 3x+\lambda y+z-2=0$ from a triangular prism is	(q) 2
(C)	If the circumcentre of the triangle whose vertices are $(3,2,-5),(-3,8,-5)$ and $(-3,2,1)$ is $(-1,\lambda,-3)$ then λ equals	(r) 4
(D)	If the four planes $my+nz=0, nz+lx=0, lx+my=0$ and $lx+my+nz=p$ from a tetrahedran whose volume is $\frac{\lambda p^3}{3lmn}$ then λ equals	(s) 0

A) A–s, B–r, C–r, D–q B) A–s, B–p, C–r, D–q

C) A–p, B–q, C–r, D–s D) A–p, B–r, C–s, D–q

58. Let $L_1 : x + y + z - 4 = 0 = 2x - y + z - 3,\ L_2 : 3x - y + z - 4 = 0 = \lambda x - z + 3$

	COLUMN – I	COLUMN – II
(A)	L_1, L_2 are coplanar then λ equals	(p) 2
(B)	If $((\alpha, \beta, \gamma)$ lies on both L_1 and L_2 then $\alpha - \beta + 3\gamma$ equals	(q) 1
(C)	If L_1 is parallel to the plane $x + y + pz - 7 = 0$ then p equals	(r) 6
(D)	If L_1, L_2 are coplanar and if acute angle between L_1, L_2 is $\cos^{-1}\left(\frac{\sqrt{7}}{k\sqrt{3}}\right)$ then k equals	(s) -1

A) A–s, B–r, C–q, D–p
B) A–s, B–p, C–q, D–r
C) A–s, B–q, C–p, D–r
D) A–s, B–p, C–r, D–q

59. Match the following

	COLUMN – I		COLUMN – II
(A)	If $\vec{a}+\vec{b}=\hat{j}$ and $2\vec{a}-\vec{b}=3\hat{i}+\frac{\hat{j}}{2}$, then cosine of the angle between $\vec{a}$ and $\vec{b}$ is	(p)	1
(B)	If $\lvert\vec{a}\rvert=\lvert\vec{b}\rvert=\lvert\vec{c}\rvert$, angle between each pair of vectors is $\frac{\pi}{3}$ and $\lvert\vec{a}+\vec{b}+\vec{c}\rvert=\sqrt{6}$, then $\lvert\vec{a}\rvert=$	(q)	$5\sqrt{3}$
(C)	Area of the parallelogram whose diagonals represent the vectors $3\hat{i}+\hat{j}-2\hat{k}$ and $\hat{i}-3\hat{j}+4\hat{k}$ is	(r)	7
(D)	If $\vec{a}$ is perpendicular to $\vec{b}+\vec{c}$, $\vec{b}$ is perpendicular to $\vec{c}+\vec{a}$, $\vec{c}$ is perpendicular to $\vec{a}+\vec{b}$, $\lvert\vec{a}\rvert=2$, $\lvert\vec{b}\rvert=3$ and $\lvert\vec{c}\rvert=6$ then $\lvert\vec{a}+\vec{b}+\vec{c}\rvert$ is	(s)	$\frac{-3}{5}$

A) A–s, B–p, C–q, D–r
B) A–s, B–q, C–p, D–r
C) A–s, B–r, C–p, D–q
D) A–r, B–q, C–p, D–s

60.

	COLUMN – I		COLUMN – II
(A)	The line $\vec{r}=\left(\hat{i}+\hat{j}\right)+t\left(\hat{i}-\hat{k}\right)$ where $'t'$ is scalar passes through the point	(p)	$-\hat{i}-\hat{j}+2\hat{k}$
(B)	The line $\vec{r}=\left(\hat{i}+\hat{j}\right)+t\left(\hat{i}-\hat{k}\right)$ where $'t'$ is scalar and the plane $\vec{r}.\left(2\hat{i}+\hat{j}+\hat{k}\right)=2$ intersect at the point	(q)	$-\hat{i}+\hat{j}+2\hat{k}$
(C)	The point on the line $\vec{r}=\left(\hat{i}+\hat{j}\right)+t\left(\hat{i}-\hat{k}\right)$ where $'t'$ is scalar, which is at a distance of 3 units from the point having position vector $\hat{i}$ is/are	(r)	$-2\hat{i}+\hat{j}+3\hat{k}$
(D)	The volume of the parallelepiped having adjacent sides $\hat{i}+\hat{k}, 2\hat{i}+\hat{j}+\hat{k}$ and $\vec{c}$ is 4 cubic units then $\vec{c}$ may be	(s)	$\hat{j}+\hat{k}$

A) A–qrs, B–s, C–q, D–q

B) A–qrs, B–q, C–s, D–q

C) A–qrs, B–p, C–s, D–r

D) A–qrs, B–prs, C–qs, D–q