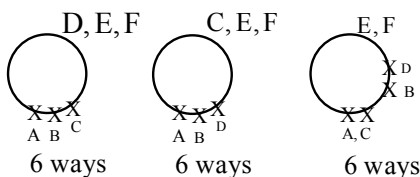


MATHS

41. $1^2 + 2^2 + 3^2 + \dots + 9^2 = 285$



43. $2\ 333333 = 7$

$$2233333 =$$

$$2223333 =$$

$$222233 = 1$$

44. $x_1 + x_2 + \dots + x_n = n^2, \quad x_i \geq i$

$$x_1 - 1 + x_2 - 2 + \dots + x_n - n = n^2 (1 + 2 + 3 + \dots + n)$$

$$t_1 + t_2 + t_3 + \dots + t_n = n^2 - \left(n \frac{(n+1)}{2} \right)$$

$$n^2 - n \left(n \frac{(n+1)}{2} \right) + n - 1 \quad C_{n-1}$$

45. $N = 9 \cdot 10^5 \cdot 5 = 2^5 \cdot 3^2 \cdot 5^6$

46. Sum of all divisors of $(2^n - 1)(2^{n-1})$

$$= P^1 2^{n-1}$$

$$= (P^0 + P^1)(2^0 + 2^1 + \dots + 2^{n-1}) = [1 + 2^n - 1](1 + 2 + \dots + 2^{n-1})$$

$$= 2^n \frac{(2^n - 1)}{2 - 1} = 2^n (2^n - 1)$$

47. $f(n) = n! \left\{ \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \dots + \frac{(-1)^n}{n!} \right\}$ $f(n-1) = (n-1)! \left\{ \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \dots + \frac{(-1)^{n-1}}{(n+1)!} \right\}$

$$f(n-2) = (n-2)! \left\{ \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \dots + \frac{(-1)^{n-2}}{(n-2)!} \right\}$$

48. 18 items are

AC AC AC AC AC AD AD AD AD and
BC BC BC BC BD BD BD BD BD

A, B, C, D should contribute 4 each

$$\left({}^5C_4 \cdot {}^4C_0 \cdot {}^4C_0 \cdot {}^5C_4\right)^2 + \left({}^5C_4 \cdot {}^4C_0\right)^2 + \left({}^5C_3 \cdot {}^4C_1\right)^2 + \dots + \left({}^5C_0 \cdot {}^4C_4\right)^2$$

$$= 25 + 1600 + 3600 + 400 + 1 = 5626$$

49. $x_1 + x_2 + x_3 + x_4 + x_5 = 6$

coefficient in $6! \left(x^1 + \frac{x^2}{2!}\right)^5 =$ coefficient x^6 in $6!(e^x - 1)^5$

$$= 6! \frac{5}{2} = {}^6C_2 \cdot 5! = 1800$$

Also equal to no. of onto functions from a set of 6 elements to a set of 5 elements.

50. $x_1 x_2 x_3 P = mp = 120 = 2^3 \cdot 3^1 \cdot 5^1$

$$\therefore \lambda = {}^{3+4-1}C_3 \cdot {}^{1+4-1}C_3 \cdot {}^{1+4-1}C_3$$

$$= {}^6C_3 \cdot {}^4C_3 \cdot {}^4C_3$$

$$= 320$$

52.

A) $a_{11} + a_{22} + a_{33} = 0$ remaining '6' elements can be filled in 7^6 ways

$$\left. \begin{array}{l} (-3, 0, 3), (-2, 0, 2), (-1, 0, 1) \\ (-3, 1, 2), (3, -1, -2) \end{array} \right\} \rightarrow 3! \cdot 5 = 30$$

$$(-2, 1, 1)(2, -1, -1) \rightarrow 3 \cdot 2 = 6$$

$$(0, 0, 0) \rightarrow \frac{1}{37}$$

B) Each of 9 elements can be filled in 7 ways

C) 3 elements can be filled 7^3 ways

D) 6 elements can be filled 7^6 ways

53. R_1, R_2, R_3

contain 1 person each

$${}^2C_1 \cdot {}^2C_1 \cdot {}^2C_1 \cdot 3! = P$$

$$\frac{8.6}{16} = \frac{48}{16} = 3$$

54. $Ans = \frac{n(n-1)(n-2)(n-3)}{8} = 3 \times nC_4$

55. B NN AAA can be arranged in $3!$

56. 1 letter is to be selected from TION

57. $1/2(\text{Total no of divisors})$

59. A) $3^6 - {}^3C_1 2^6 + {}^3C_2 1^6 = 540$

B) x_1, x_2, x_3 can be assigned in 2^3 ways and x_4, x_5, x_6 can be in 3^3 ways

$$\therefore \text{Total no of ways} = 2^3 \times 3 = 216$$

C) Number of invertible functions is zero, since it is not possible to have one-one functions.

D) All functions are many one functions = 729

60. A) ${}^2C_1 \cdot {}^5P_2 = 2 \cdot 20 = 40$

B) $\frac{6!}{3!3!} = \frac{720}{6 \cdot 6} = 20$

C) ${}^{22-1}C_{4-1} = {}^{21}C_3$

D) ${}^{23-7-2}C_{10-7} = {}^{14}C_3$