MATHS **Max.Marks:80**

SECTION-1

(SINGLE CORRECT CHOICE TYPE)

Section-I (Single Correct Answer Type, Total Marks: 24) contains 8 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONLY ONE is correct. For each question you will be awarded 3 marks if you darken ONLY the bubble corresponding to the correct answer and zero marks if no bubble is darkened. In all other cases, minus one (-1) mark will be awarded.

41. Let $\vec{a}=2i-4j+9k;\ \vec{b}=25i+16j+4k$ are given vectors. The vectors $\vec{c}=i+xj+x^2k;x\in\mathbb{R}$ and $\vec{d}=y^2i+yj+k;y\in\mathbb{R}$ satisfy $\vec{c}.\vec{d}=3;\left(\vec{a}\times\vec{c}\right).\left(\vec{b}\times\vec{d}\right)=-3$. Number of vector pairs $\left(\vec{c},\vec{d}\right)$ possible is

A) 2 B) 1 C) 0 D) infinite

42. The plane $3x+8y+15z+91=0$ is rotated through a right angle about its line of intersection with plane $5x+17y+29z-2=0$. The plane in its new position cuts negative y-axis, positive x, z axes so that the magnitudes of the intercepts are same. If k is the distance of the new plane from origin, then the value of [k], where [•] denotes greatest integer function, is

A) 48 B) 84 C) 106 D) 73

43. If the length of any edge of a regular tetrahedron is 1 unit and θ is the angle between any edge and a face not containing that edge, then $\cos\theta$ is equal to

A) $\frac{1}{\sqrt{3}}$ B) $\frac{1}{3\sqrt{3}}$ C) $\frac{\sqrt{10}}{3\sqrt{3}}$ D) $\frac{2}{3\sqrt{3}}$

44. In a regular hexagon ABCDEF, M is midpoint of DE, N is midpoint of AM, P is midpoint of BC. If $\overrightarrow{NP} = x\overrightarrow{AB} + y\overrightarrow{AF}$ then x and y satisfy

A) $2x + y = 2$ B) $4x - 4y = 5$

C) $4x + 4y = 3$ D) $2x - y = 1$

45. Perpendiculars are dropped from the points on the line $\frac{x+2}{2} = \frac{y+1}{-1} = \frac{z}{3}$ on to the plane $x + y + z = 3$. The feet of perpendiculars lie on a line sum of whose direction ratios is

A) 4 B) 8 C) 0 D) 9

46. The line passing through the point $(1,3,2)$ and perpendicular to the lines $\vec{r} = (1+2t)i + (2+t)j + (t-1)k$ and $\vec{r} = (2i + 6j + k) + s(i + 2j + 3k)$ is given by

A) $\vec{r} = (1+t)i + (3+5t)j + (2t+3)k$ B) $\vec{r} = (1+t)i + (3-5t)j + (2+3t)k$

C) $\vec{r} = (1+t)i + (3-5t)j + (2t+3)k$ D) $\vec{r} = (1+t)i + (3+5t)j + (2+3t)k$

47. If $\vec{r} = \frac{1}{2}\left(\vec{b}\times\vec{c}\right) + \frac{1}{3}\left(\vec{c}\times\vec{a}\right) + \frac{1}{6}\left(\vec{a}\times\vec{b}\right)$, $\left[\vec{a}\ \vec{b}\ \vec{c}\right] = 1$ then the value of $\vec{r}\cdot\left(\vec{a}+\vec{b}+\vec{c}\right) =$

A) 2 B) 4 C) 1 D) 6

48. The area of the figure formed by the points $(-1,\ -1,\ 1)$ and $(1,\ 1,\ 1)$ and their mirror images on the plane 3x + 2y + 6z + 1 =0 is,…..

A) $\frac{20\sqrt{33}}{29}$ B) $\frac{21\sqrt{33}}{29}$ C) $\frac{\sqrt{73}}{29}$ D) $\frac{4\sqrt{73}}{7}$

SECTION-2

(MORE THAN ONE TYPE)

Section - II (Multiple Correct Answers Type, Total Marks: 16) contains 4 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONE or MORE may be correct. For each question you will be awarded 4 marks if you darken ALL the bubble(s) corresponding to the correct answer(s) ONLY and zero marks otherwise. There are no negative marks in this section.

49. Let $f(x)=1+\frac{x^3}{3!}+\frac{x^6}{6!}+...;\ g(x)=\frac{x^2}{2!}+\frac{x^5}{5!}+\frac{x^8}{8!}+...;\ h(x)=\frac{x}{1!}+\frac{x^4}{4!}+\frac{x^7}{7!}+...$ are some real valued functions. Let $\vec{a}=f(x)i+g(x)j+h(x)k;\quad \vec{b}=f'(x)i+g'(x)j+h'(x)k;$ $\vec{c}=f''(x)i+g''(x)j+h''(x)k$ then which of the following statements is/are TRUE?

A) The vectors $\vec{b},\vec{c},\vec{a}$ form a right handed system

B) The vectors $\vec{a},\vec{c},\vec{b}$ form a right handed system

C) $\left[\vec{a}\ \vec{b}\ \vec{c}\right]=-1$

D) $\left[\vec{a}\ \vec{b}\ \vec{c}\right]=1$

50. The planes $ax+4y+z=0$, $2y+3z-1=0$, $3x-bz+2=0$

A) Will have no common point if $ab=15, b\neq 5$

B) Will meet on a line if $ab\ =\ 15,\ a\ =\ 3$

C) Will meet at a point if $ab\neq 15$

D) Will have no common point if $ab\ =\ 15, a\neq 3$

51. If θ is the angle between the medians drawn from the acute angles in a right angled isosceles triangle, then which of the following is/are possible?

A) $\cos\theta = \frac{3}{5}$ B) $\sin\theta = \frac{3}{5}$ C) $\cos\theta = -\frac{3}{5}$ D) $\cos\theta = -\frac{4}{5}$

52. AB, AC, AD are adjacent edges of a parallelopiped. The vector along the diagonal AP is given by $\vec{a}$ so that $|\vec{a}| = 3$. The vector areas of the faces containing A, B, C and A, B, D are respectively given by $\overrightarrow{AB} \times \overrightarrow{AC} = \vec{b}; \overrightarrow{AB} \times \overrightarrow{AD} = \vec{c}$. The projections of the edges AB and AC on the diagonal AP are both equal to 1. Then which of the following can be correct?

A) $\overrightarrow{AC} = \frac{\vec{a}}{3} - \frac{1}{9}\left(\vec{a} \times (2\vec{b} - \vec{c})\right)$ B) $\overrightarrow{AD} = \vec{a} - \frac{1}{3}\left(\vec{a} \times (\vec{b} + 2\vec{c})\right)$

C) $\overrightarrow{AB} = \vec{a} + \frac{1}{3}\left(\vec{a} \times (2\vec{b} + \vec{c})\right)$ D) $\overrightarrow{AD} = \frac{\vec{a}}{3} + \frac{1}{9}\left(\vec{a} \times (\vec{b} - 2\vec{c})\right)$

SECTION-3
[INTEGER TYPE]

Section-III (Integer Answer Type, Total Marks: 24) contains 6 questions. The answer to each of the questions is a single-digit integer, ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the ORS. For each question you will be awarded 4 marks if you darken ONLY the bubble corresponding to the correct answer and zero marks otherwise. There are no negative marks in this section.

53. In tetrahedron ABCD the face ABC is an equilateral triangle and the face DBC is perpendicular to it. Given that $\angle DAC = \frac{\pi}{3}$, $|AD| = 6$, and the angle between the lines $\overrightarrow{AD}$ & $\overrightarrow{BC}$ is equal to $\cos^{-1}\left(\frac{1}{4}\right)$. If α is angle between $\overrightarrow{AB}$ & $\overrightarrow{AD}$ then $8\cos\alpha =$

54. P is a point on the segment of the line joining (3, 3, 5) and (4, 6, 7) dividing it in the ratio m : 1and also projections of OP (O origin) on coordinate axes are $\frac{32}{9},\frac{42}{9},\frac{55}{9}$, then $|(2mi+j-k).(i+2mj+3k)|=$

55. Let the direction cosines of a straight line L passing through the origin and intersecting both the lines $\frac{x+1}{1}=\frac{y-1}{-2}=\frac{z+1}{2}$, $x+1=y+2=\frac{-z}{2}$ are p,q,r. Then the value of $\frac{p-q}{r+p}=$……

56. If $\hat{\alpha},\hat{\beta}$ are two non-collinear unit vectors and $\vec{r}$ is a vector such that $\vec{r}.\hat{\alpha}=0$ and $3(\vec{r}\times\hat{\beta})=5(\vec{r}\times\hat{\alpha})-\hat{\beta}$ then the value of $\frac{1}{|\vec{r}|}=$……

57. If the acute angle which the line of intersection of the planes $2x+y+z=0$ and $x+y+2z=0$ makes with positive y-axis is $\operatorname{Tan}^{-1}\alpha$ then the value of $9\alpha^2-1=$

58. If $P(m,n,p)$ is the point common to the line $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z+2}{2}$ and the plane $3x-4y+5z+1=0$, and the shortest distance between the lines $\vec{r}=3mj+pk+t(mi-j+k)$ and $\vec{r}=-2mi-nj+2nk+s(-3i+pj+4k)$ is $\sqrt{10T}$ then $\left|\frac{T}{9}\right|=$

SECTION-4

[Matrix Matching Type]

Section-IV (Matrix-Match Type, Total Marks: 16) contains 2 questions. Each question has four statements (A, B, C and D) given in Column I and five statements (p, q, r, s and t) in Column II. Any given statement in Column I can have correct matching with ONE or MORE statement(s) given in Column II. For example, if for a given question, statement B matches with the statements given in q and r, then for the particular question, against statement B, darken the bubbles corresponding to q and r in the ORS. For each question you will be awarded 2 marks for each row in which you have darkened ALL the bubble(s) corresponding to the correct answer(s) ONLY and zero marks otherwise. Thus, each question in this section carries a maximum of 8 marks. There are no negative marks in this section.

59. Let the position vectors of the points P, Q, R are given by

$\overrightarrow{OP} = ai + bj + ck$; $\overrightarrow{OQ} = bi + cj + ak$; and $\overrightarrow{OR} = ci + aj + bk$ such that

$\overrightarrow{OP}.\overrightarrow{OQ} = \overrightarrow{OQ}.\overrightarrow{OR} = \overrightarrow{OR}.\overrightarrow{OP} = 1$ and $\left|\overrightarrow{PQ}\right| = \left|\overrightarrow{QR}\right| = \left|\overrightarrow{RP}\right| = 1$

	Column I		**Column II**
A)	The distance from origin, at which the points P, Q, R lie is	P)	$\sqrt{\frac{7}{2}}$
B)	$\left\|\overrightarrow{OP}.i + \overrightarrow{OQ}.j + \overrightarrow{OR}.k\right\|$	Q)	$3\sqrt{\frac{3}{2}}$
C)	If $9\left\|\left(\overrightarrow{QP}.i\right)\left(\overrightarrow{RQ}.j\right)\left(\overrightarrow{PR}.k\right)\right\| \le M$ to be true, then M can be	R)	$\sqrt{\frac{3}{2}}$
D)	$\left\|\left[\overrightarrow{OP}\quad \overrightarrow{OQ}\quad \overrightarrow{OR}\right]\right\| =$	S)	$\frac{1}{2}\sqrt{\frac{7}{2}}$

60.

Column I		Column II
A)	If $\vec{x}$ and $\vec{y}$ are two unit vectors inclined at an angle 60^0 so that $\vec{x}+\vec{y}=\vec{a}$ and $\vec{x}\times\vec{y}=\vec{b}$ then $\vec{x}=p\vec{a}+q(\vec{a}\times\vec{b})$ then $\frac{1}{q}+4p=$	P) 0
B)	If $\vec{a},\vec{b},\vec{c}$ are three non coplanar non zero vectors and $\vec{r}$ is any vector in space , then $\left(2\vec{a}\times 3\vec{b}\right)\times\left(\vec{r}\times 4\vec{c}\right)+\left(3\vec{b}\times 2\vec{c}\right)\times\left(\vec{r}\times 4\vec{a}\right)+\left(2\vec{c}\times 4\vec{a}\right)\times\left(\vec{r}\times 3\vec{b}\right)=\frac{16\lambda(\lambda-2)}{5}[\vec{a}\,\vec{b}\,\vec{c}]\vec{r}$ implies $\lambda=$	Q) 2
C)	If $m=\begin{bmatrix}\frac{\vec{a}\times\vec{b}}{12} & \frac{\vec{b}\times\vec{c}}{13} & \frac{\vec{a}+\vec{b}+\vec{c}}{\sqrt{14}}\end{bmatrix}$ where $\lvert\vec{a}\rvert=3, \lvert\vec{b}\rvert=5, \lvert\vec{c}\rvert=6$, $\vec{a}\perp\vec{c}, \vec{b}\perp\vec{c}$ and $\vec{b}.\vec{a}=1$, then the value of $1+m$	R) 5
D)	If $\vec{a}\times(\vec{b}\times\vec{c})=pq\vec{b}-(r+1)\vec{c}$, $\vec{c}\times(\vec{a}\times\vec{b})=4\vec{a}-6\vec{b}$ and $\vec{b}\times(\vec{a}\times\vec{c})=r\vec{a}-(p+q)\vec{c}$ then the value of $3p-2q$	S) –3