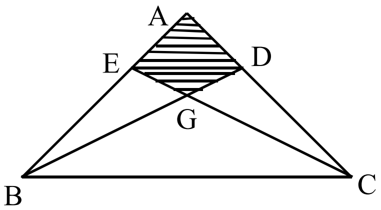
MATHS

49. required area $= 2\left(\Delta^{le} EAG\right)$



$= 2 \times \frac{1}{2} AE \times EG$

$= \frac{1}{2} AB \times \frac{1}{3} CE$

$= \frac{1}{6} \times 4 \times \sqrt{4^2 - 2^2} = \frac{4\sqrt{3}}{3}$

51. $\frac{1}{x^2}\frac{dx}{dy} - \frac{1}{x} y^3 = y^7$ if $= e^{\frac{y^4}{4}}$

Solution is $\frac{-e^{\frac{y^4}{4}}}{x} = y^4 e^{\frac{y^4}{4}} - 4e^{\frac{y^4}{4}} - e^{1/4}$ & $y = -1 \Rightarrow x = 1/4$ & $\frac{dy}{dx}$ at $\left(1/4, -1\right)$ is $= \frac{-16}{5}$

52. I.F $= e^{-x}$

$ye^{-x} = \int e^{-x}(\cos x - \sin x)\,dx$

$\Rightarrow ye^{-x} = e^{-x}\sin x + c$

Since y is bounded when $x \to \infty,\ c = 0$

$\therefore y = \sin x$

Required area is $= \int_0^{\pi/4} (\cos x - \sin x)dx = \sqrt{2} - 1$

53. $\int_{\pi/4}^{B} f(x)\,dx = \beta \sin\beta + \frac{\pi}{4}\cos\beta + \sqrt{2}\beta$ differentiate both sides, with respect to "β",

two times.

54. Differentiate the given statement $x^2 f(x) = \int_0^x (1-t)f(t)\,dt$ again differentiate,

$\frac{f'(x)}{f(x)} = \frac{1-3x}{x^2}$ and $f(1) = 1 \Rightarrow f(x) = \frac{1}{x^3} e^{\left(1 - 1/x\right)}$

55. $\tan\theta = -\frac{dx}{dy}$ the given equation becomes

$$y\left(x^2\left(-\frac{dx}{dy}\right)+\frac{dy}{dx}\right)=x\left(y^2+1\right)$$

$$\Rightarrow y\left(\frac{dy}{dx}\right)^2-x\left(y^2+1\right)\frac{dy}{dx}-yx^2=0$$

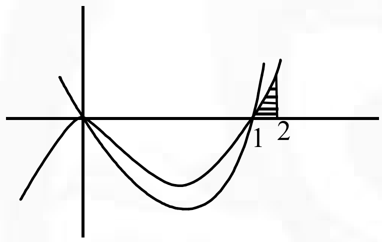
$$\Rightarrow \frac{dy}{dx}=xy \text{ (or) } \frac{dy}{dx}=\frac{x}{y}$$

$$\Rightarrow y=ke^{x^2/2} \text{ (or) } \log\left(y^2\right)=x^2-\log k$$

$$\log\left(ky^2\right)=x^2 \text{ (or) } \log y=\frac{x^2}{2}+c \text{ (or) } x^2-y^2=e$$

59. $f(x)=x^2-x$

$g(x)=x^3-x^2$



$A=-1/12$

$$\int_1^2\left(x^2-x\right)dx=5/6$$

60 TO 61. Solving differential equation $y=\frac{4x^3}{3\left(1+x^2\right)}$

$$\frac{dy}{dx}=\frac{4x^2\left(3+x^2\right)}{3\left(1+x^2\right)^2}>0\forall x>0\left(x\neq 0\right)$$

$$A=\frac{2}{3}-\frac{4}{3}\int_0^1\frac{x^3}{1+x^2}dx$$

$$=\frac{2}{3}\,ln2$$

63. $\frac{dy}{dx}=\frac{y}{x^2}$ & passes through (1, 3).

$$\Rightarrow y=3e^{1-1/x}$$

Since $f'(x)>0\forall x\in R-\{0\}$, $\lim\limits_{x\to\infty} f(x)=3e$

64. $\cos y\dfrac{dy}{dx}+x\sin y=x\sin^2 y$ solving, $\sin y=\dfrac{1}{1+ce^{x^2/2}}$

65. $C=\left(\dfrac{1}{2},\dfrac{1}{4}\right)$ area of triangle $=\dfrac{27}{8}$ area between the line & parabola is $=\dfrac{9}{2}$

66. $[|b|=2\Rightarrow P(x)=x^3-12x+16$

Roots of $P(x)=0$ are -4, 2

$=\int_{-4}^{2}P(x)\,dx=\int_{-4}^{2}\left(x^3-12x+16\right)dx=108]$

67. $A=\int_2^a\left(\dfrac{1}{x}-\dfrac{1}{2x-1}\right)dx=ln\left(\dfrac{4}{\sqrt{5}}\right)$

$\Rightarrow ln\left(\dfrac{a^2}{2a-1}\right)=ln\left(\dfrac{64}{15}\right)$

$\Rightarrow 15a^2-128a+64=0$

$\Rightarrow a=8, a=\dfrac{8}{15}$

68. Required area

$=\int_{-1/2}^{0}\left((1-x)-\left(\dfrac{-2x^2+5x+3}{3}\right)\right)dx+\int_0^1\left(\left(\dfrac{-2x^2+5x+3}{3}\right)-(1-x)\right)dx=\dfrac{17}{36}$

69. (line equation passing through the points $(0, 3)$ & $(5, -2)$is $x+y=3$, which is tangent to $y=\dfrac{c}{x+1}\Rightarrow c=4$

$\therefore$required area is $\int_1^2\dfrac{4}{x+1}dx=4\,l\text{n}\left(3/2\right)=4l\text{n}\left(1+1/2\right)$

$=\lambda\left\{\dfrac{1/2}{1}-\dfrac{\left(1/2\right)^2}{2}+\dfrac{\left(1/2\right)^3}{3}.....\right\}=2\lambda\,l\text{n}\left(3/2\right)$

$\Rightarrow \lambda=2$