Sri Chaitanya IIT Academy, India

A.P, TELANGANA, KARNATAKA, TAMILNADU, MAHARASHTRA, DELHI, RANCHI

A right Choice for the Real Aspirant

ICON CENTRAL OFFICE, MADHAPUR-HYD

Sec: Sr. IPLCO	JEE ADVANCED	DATE : 08-11-15
TIME : 3:00	2013_P1 MODEL	MAX MARKS : 180

KEY & SOLUTIONS

PHYSICS

1	C	2	A	3	B	4	B	5	B
6	C	7	B	8	D	9	D	10	C
11	ABC	12	ABC	13	ACD	14	ABC	15	ABCD
16	2	17	6	18	2	19	6	20	6

CHEMISTRY

21	C	22	D	23	D	24	C	25	C
26	C	27	C	28	D	29	B	30	A
31	ABCD	32	BC	33	ABC	34	ABCD	35	ABCD
36	8	37	4	38	5	39	6	40	3

MATHEMATICS

41	D	42	B	43	A	44	B	45	B
46	A	47	B	48	A	49	D	50	C
51	BCD	52	BCD	53	BD	54	B	55	ABC
56	2	57	3	58	8	59	2	60	4

MATHS

41. PUT $1+e^{\sqrt{\sin x}}=t$

$$\left(e^{\sqrt{\sin x}}.\frac{1}{2\sqrt{\sin x}}.\sqrt{\cos x}.\sqrt{cosx}\right)dx=dt$$

$$I=2\int t\,dt$$

$$=t^2+c$$

42. $I=\int\frac{2\cos x+1}{(2+\cos x)^2}dx=\frac{\sin x}{2+\cos x}+c$

$$\frac{d}{dx}\left(\frac{\sin x}{a\cos x+b}\right)=\frac{b\cos x+a}{(a\cos x+b)^2}$$

43. Put x = 2t and write *l*n 2t = *l*n2 + *l*n t and separate I as two different integrals

44. We have $f(x)=\sin x+\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(\sin x+t\,f(t))dt$

Then f(x) is of the form

$$=(\pi+1)\sin x+A$$

Where $A=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}t\,f(t)dt$

Now from A, $A=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}t((\pi+1)\sin t+A)dt$

$$\Rightarrow A=2(\pi+1)$$

$$\therefore f(x)=(\pi+1)\sin x+2(\pi+1)$$

$$f_{\max}=3\pi+3$$

$$f_{\max}=\pi+1$$

45. let $\int_0^{\infty}\frac{\tan^{-1}ax-\tan^{-1}x}{x}dx=I(a)$

Then $\frac{dI}{da}=\int_0^{\infty}\frac{1}{1+a^2x^2}dx=\frac{\pi}{2a}$

$$\Rightarrow I(a)=\frac{\pi}{2}\ln a$$

46. Given integrand is an odd function.

47. $L=\underset{n\to\infty}{\mathrm{lt}}\left(\frac{2n!}{n!n!}\right)^{\frac{1}{n}}$

$$L=\left(\frac{(n+1)(n+2).....(n+n)}{\lfloor n}\right)^{\frac{1}{n}}$$

Then $\log L=\frac{1}{n}\left(\ln\left(\frac{n+1}{2}\right)+.....+\ln\left(\frac{n+n}{n}\right)\right)$

$$=\frac{1}{n}\sum\log\left(1+\frac{n}{r}\right)$$

$$\ln L=\int_0^1\ln\left(1+\frac{1}{x}\right)dx=\int_0^1(\ln(x+1)-\ln x)dx$$

=ln4

$$\Rightarrow L=4$$

48. P = $2016^{\log_{2016}\left(\frac{x}{2015-x}\right)}$

$$\therefore\frac{1}{p}=\frac{2015-x}{x}$$

$$I=\int\log_{10}.\log_{10}\left(\frac{2015-x}{x}\right)dx$$

$$\int(\ln(2015-x)-\ln x)dx$$

(x-2015) ln (2015-x) – x ln x + c

49. $I(x)=\int\frac{(1+x)(1+x^2)^2}{(1+x)^2(1+x^2)^2}\,dx=\int\frac{1}{1+x}dx=\ln(1+x)+c$

50. f (x) = cot x

51. $\int\frac{(e^x+\cos x+1)-(e^x+\sin x+x)}{e^x+\sin x+x}dx=\log_e(e^x+\sin x+x)-x+c$

$$\therefore f(x)=e^x+\sin x+x;\ g(x)=-x$$

$$\therefore f(x)+g(x)=e^x+\sin x$$

$f(x)-g(x)=e^x+\sin x+2x$

h (x) = sin x

52. $\int_0^x 4+|t-2|\,dt=\int_0^2 (6-t)+\int_2^x (2+t)\,dt=\frac{x^2}{2}+2x+4$

$\therefore\ f(x)=\begin{cases}\frac{x^2}{2}+2x+4 & x>3\\ ax^2+bx & x\le 3\end{cases}$

And f is differentiable $\Rightarrow a=\frac{1}{18},\ b=\frac{14}{3}$

53. If g is odd then $I=\int_{-a}^{a} f(x)\,dx$, if f , g both odd then I = 0

54. $f^3(x)=\int_0^x tf^2(t)\,dt$

Differentiating both sides, gives $f^1(x)=\frac{x}{3}$ as $f\uparrow$

$\Rightarrow f(x)=\frac{x^2}{6}$

55. $I_1+I_2=\int 1dx=x+c$ -------------- (1)

$I_1-I_2=\int(\sin x-\cos x)\,dx=-\cos x-\sin x+c$ ---------- (2)

(1) + (2) gives $2I_1=x-\sin x-\cos x+c$

(1) - (2) gives $2I_2=x+\sin x+\cos x+c$

56. $\frac{3}{n}\left(\frac{\sqrt{1}}{1+0}+\sqrt{\frac{1}{1+\frac{3}{n}}}+\sqrt{\frac{1}{1+\frac{6}{n}}}+......+\sqrt{\frac{1}{1+\frac{3(n+1)}{n}}}\right)$

$\therefore$ the given limit $=\int_0^3 \frac{1}{\sqrt{1+x}}\,dx$

57. $I=\int\frac{\sec^2 x}{5-4\tan x-2\tan^2 x}\,dx$

Put $\tan x=t$

Then $I=\int\frac{1}{5-4t-2t^2}\,dt=\frac{1}{2\sqrt{14}}\ln\left|\frac{\sqrt{\frac{7}{2}}+t+1}{\sqrt{\frac{7}{2}}-t-1}\right|+c$

$\Rightarrow k^2 = \frac{7}{2}$

58. $\int \cot^2 x \cos ec^4 x dx$

$= \int \cot^2 x \left(1+\cot^2 x\right) \cos ec^2 dx$

$= \int \left(\cot^2 x + \cot^4 x\right) \cos ec^2 dx$

$= -\frac{\cot^3 x}{3} - \frac{\cot^5 x}{5} + c$

59. given $-\sin^2 x f\left(\sin x\right)\cos x = -\cos x$

$\Rightarrow f\left(\sin x\right) = \frac{1}{\sin^2 x} \qquad xt\left(0, \frac{\pi}{2}\right)$

$\Rightarrow f\left(t\right) = \frac{1}{t^2} \qquad where\ t \in \left(0,1\right)$

$\Rightarrow f\left(\frac{1}{\sqrt{2}}\right) = 2$

60. $x = 2\cos^2\theta + 8\sin^2\theta$

$I = 2\int_0^{\frac{\pi}{2}} \frac{\sec^2\theta}{2+8\tan^2\theta} d\theta = \frac{\pi}{4}$