MATHEMATICS **Max Marks: 80**

SECTION – I

(Straight Objective Type)

This section contains 7 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

47. Let $\vec{p} = y^3 i + x^2 j + xk;\ \vec{q} = z^3 i + y^2 j + yk;\ \vec{r} = x^3 i + z^2 j + zk$ are such that $\vec{p}.\vec{a} = \vec{q}.\vec{a} = \vec{r}.\vec{a} = 64$ and $(xi + yj + zk).(i + j + k) = 12$ where $\vec{a} = i - 12j + 48k$. Then the value of $\left|(xi + yj - zk).\{(zi - xj + yk) \times (-yi + zj + xk)\}\right| =$

A) 316 B) 384 C) 256 D) 294

48. The acute angle between the line joining the points $(-1,2,3),(-3,5,-3)$ and a line parallel to $\frac{x-2}{-1} = \frac{y-4}{2} = \frac{z+5}{-2}$ is

A) $\cos^{-1}\left(\frac{20}{21}\right)$ B) $\cos^{-1}\left(\frac{1}{14}\right)$ C) $\cos^{-1}\left(\frac{3}{7}\right)$ D) $\cos^{-1}\left(\frac{2}{21}\right)$

49. If the line L_1, has direction ratios 1, 2, 3 and passing through the point $(2, 9, 13)$, is coplanar with another line L_2 whose direction ratios are $-1,2,-3$ and passing through $(-3,7,p)$, then the value of p is

A) – 1 B) 1 C) –2 D) 2

50. If $\vec{x},\vec{y}$ are two non-collinear vectors and ABC is a triangle with side lengths a, b, c satisfying $(15a-12b)\vec{x}+(20c-15a)\vec{y}+(12b-20c)(\vec{x}\times\vec{y})=\vec{0}$ then ΔABC is

A) a right-angled triangle B) an obtuse-angled triangle

C) a acute-angled triangle D) an isosceles triangle

51. Let $\hat{c}$ is a unit vector making complementary angles with $\vec{a}\ \&\ \vec{b}$ and $|\vec{a}|=a,|\vec{b}|=b$. Equation of the plane through the line of intersection of the planes $\vec{r}.\vec{a}=p\ ;\ \vec{r}.\vec{b}=q$ and passing through the point $C(\hat{c})$ is given by $\vec{r}.\{\hat{c}\times(\vec{b}\times\vec{a})+q\vec{a}-p\vec{b}\}=\alpha p+\beta q$ for some scalars α,β. Then the point (β,α) lies on

A) $x^2+y^2=a^2+b^2$ B) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ C) $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$ D) $\frac{x^2}{b^2}-\frac{y^2}{a^2}=1$

52. ABCD is a parallelogram L is a point on BC so that $\frac{BL}{LC}=\frac{1}{2}$, M is a point on DC so that $\frac{DM}{MC}=2$, AL intersects BD at P and AM intersects BD in Q. Then $\frac{|\overrightarrow{PQ}|}{|\overrightarrow{BD}|}=$

A) $\frac{2}{5}$ B) $\frac{1}{3}$ C) $\frac{7}{20}$ D) $\frac{3}{4}$

53. The vectors $i+j-3k, 2i+j-k$ are position vectors of two points. Equation to a plane is given by $\vec{r}.(3i-2j-6k)-5=0$. Then the points

A) lie on same side of the plane and equidistant from the plane

B) lie on either side of the plane and equidistant from the plane

C) lie on same side of the plane and at unequal distances from the plane

D) lie on either side of the plane and at unequal distances from the plane

SECTION – II

Multiple Correct Answer Type

This section contains 4 multiple correct answer(s) type questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONE OR MORE** is/are correct.

54. Which of the following points is/are NOT on the plane passing through the $(2, 3, -1)$ and containing the lines whose direction cosines are proportional to $1, -2, 2$ and $2, 3, -1$?

A) $(-5, -7, 2)$ B) $(3, -6, 6)$ C) $(6, 5, 3)$ D) $(26, -3, 17)$

55. Let O is the circumcentre of the triangle ABC. D, E, F are midpoints of sides BC, CA, AB respectively. P, Q, R are centroids of triangles ADC, BEA, CFB. Identify the true statements from the following.

A) If $OR \perp AD$, then $AB = BC$ B) If $OP \perp AE$, then $BA \perp AC$

C) If $OQ \perp BE$, then $AB = AC$ D) If $OP \perp AD$, then $CB = AC$

56. Which of the following planes contains the projection of the line $\frac{x+1}{-1}=\frac{y}{3}=\frac{z-1}{3}$ on the plane $x-2y+2z=6$?

A) $2x+y+2=0$ B) $12x+5y-z+11=0$

C) $3y-z+1=0$ D) $6x+17y-13z+47=0$

57. Let $\vec{a}\,\&\,\vec{c}$ be unit vectors and $|\vec{b}|=\sqrt{\frac{11}{2}}$. The angle between $\vec{a}\,\&\,\vec{c}$ is $\cos^{-1}\left(\frac{5}{16}\right)$. If $\vec{b}-2\vec{c}=\lambda\vec{a}$ then $|4\lambda|$ can be

A) 3 B) 8 C) 2 D) 4

SECTION – III

[Linked Comprehension Type]

This section contains 2 paragraphs. Based upon one of paragraphs 2 multiple choice questions and based on the other paragraph 3 multiple choice questions have to be answered. Each of these questions has four choices (A), (B),(C) and (D) out of which **ONLY ONE** is correct.

Paragraph for Questions Nos. 58 to 60

The vertices of a ΔABC are $A(3,1,3)$, $B(0,2,2)$ and $C(2,-1,5)$. Points D and E divide AB, CA in the ratio $2:1$ respectively. P is a point in the space such that the perpendicular from it to the plane of ΔABC meets at F, where F is the point of intersection of $\overrightarrow{CD}$ and $\overrightarrow{BE}$. Also the distance of P from the plane of ΔABC is $2\sqrt{2}$

58. Which of the following can be $\overrightarrow{FP}$?

A) $7i+7j+7k$ B) $\frac{7}{2}(j+k)$ C) $2(j+k)$ D) $(j+k)$

59. For the above P from Question 58, the volume of the tetrahedron PABC is

A) $\frac{4}{7}$ cubic units B) $\frac{40}{7}$ cubic units C) $\frac{7}{3}$ cubic units D) $\frac{14}{3}$ cubic units

60. Vector equation to the line $\overleftrightarrow{AP}$ is $\vec{r} =$

A) $3i+j+k+t(13i-16j-12k)$

B) $3i+j+3k+t(13i-16j-12k)$

C) $3i+j+3k+t(13i+16j+12k)$

D) $3i+j+3k+t(13i-12j-16k)$

Paragraph for Questions Nos. 61 to 62

Let $\vec{a}=2xi+yj+k;\vec{b}=3i-2j+(x-5y)k$ and $\vec{c}=x(i-j)+yk;\vec{d}=xi+y(k-j)$ are four vectors such that $\vec{a}.\vec{b}=3(\vec{c}.\vec{d})$. Given that x,y are integers, answer the following questions.

61. Which of the following is a false statement?

A) $x-y$ is divisible by 3 but not 6

B) x^3-y^3 is divisible by 7 but not 5

C) $x-y$ is divisible by 9 but not 12

D) $x^3-(y+1)^3$ is divisible by 2 but not 4

62. Sum of all possible values of $(\vec{a}+\vec{b}).k$ is

A) 54 B) 56 C) 8 D) 28

SECTION – IV
(INTEGER ANSWER TYPE)

This section contains 7 questions Answer to each of the questions is a single digit integer ranging from '0' to '9'. The bubble corresponding to the correct answer is to be darkened in the ORS.

63. Let $\vec{p}$ is a vector satisfying the equation $\vec{p}+\vec{p}\times\vec{b}=\vec{a}$ where $|\vec{b}|=|\vec{a}|=1$ and $\vec{b}\perp\vec{a}$. Then the value of $\frac{4}{|\vec{p}|^2}$ is

64. The plane $ax+by+cz+1=0$ makes an angle of 60^0 with the line $x=y=z$. It makes 45^0 with the line $x=y-z=0$ and its distance from the point $(2,1,1)$ is 3 units. Finally, if it makes an acute angle $\frac{\pi}{k}, k\in N$ with the plane $x=0$, then the value of $2k=$

65. Let $\vec{a},\vec{b},\vec{c}$ are three mutually perpendicular vectors of same magnitude. If $\vec{r}$ satisfies the equation $\vec{a}\times\{(\vec{r}-\vec{b})\times\vec{a}\}+\vec{b}\times\{(\vec{r}-\vec{c})\times\vec{b}\}+\vec{c}\times\{(\vec{r}-\vec{a})\times\vec{c}\}=\vec{0}$ then $\vec{r}=m(\vec{a}+\vec{b}+\vec{c})$ will be true for $\frac{1}{|m|}=$

66. If $(2t-2,\ 3-4t\ ,t-4)$ and $(-2-s,\ 3+2s,\ 3s-4)$ $(t,s\in R)$ are points on two different lines, the acute angle between which is same as the acute angle between the vectors $i+2j-2k$ and $2i-j+pk$. Then the value of $p^2=$

67. If $\vec{a},\vec{b},\vec{c},\vec{d}$ are position vectors of the vertices A, B, C, D of a cyclic quadrilateral ABCD, then the value of $\frac{\left|\vec{a}\times\vec{b}+\vec{b}\times\vec{d}+\vec{d}\times\vec{a}\right|}{\left(\vec{b}-\vec{a}\right).\left(\vec{d}-\vec{a}\right)}+\frac{\left|\vec{d}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{d}\right|}{\left(\vec{b}-\vec{c}\right).\left(\vec{d}-\vec{c}\right)}=$

68. If the perpendicular distance from one corner of a unit cube to the diagonal not passing through it is μ, then the value of $3\mu^2-1$ is

69. A plane consisting of all points that are equidistant from the points $A(-4,2,1)$ and $B(2,-4,3)$ is intersected by the line whose equation is $\frac{x-1}{1}=\frac{y+2}{3}=\frac{z-7}{-1}$ at $C(p+1,q+1,r+2)$, then the value of $p+q+r=$