

MATHEMATICS:**Max. Marks : 60****SECTION – I****(MULTIPLE CORRECT CHOICE TYPE)**

This section contains **8 multiple choice questions**. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONE OR MORE is/ are correct**

41. Let $f(x) = 1 - x - x^3$ then real value of x satisfying the inequality

$$1 - f(x) - f^3(x) > f(1 - 5x) \text{ is}$$

- A) 3 B) 4 C) $5/2$ D) 6

42. The solution set of the equation $||x - 2012| + \log_2^a| = 3$ has exactly four elements then range of a is

- A) $\left(0, \frac{1}{8}\right)$ B) $\left(\frac{1}{8}, \infty\right)$ C) $\left(-\infty, -\frac{1}{8}\right)$ D) none

43. Let $f(x) = \min(\{x+1\}, \{x-1\}, \{x+2\}), \forall x \in \mathbb{R}$ where $\{.\}$ denotes the fractional part function then which is necessarily true

A) $\lim_{x \rightarrow \frac{\pi}{2}} f(x) = \frac{\pi}{2} - 1$

B) $\lim_{x \rightarrow 2} f(x) = 1$

C) $f(2x)$ is periodic function having period $\frac{1}{2}$

D) $f(x) = f(-x)$ has infinite number of solutions

44. If $a(>0), c, d, u, v$ are non-zero constants and if the graphs of $f(x) = |ax + c| + d$, and

$g(x) = -|ax + u| + v$ intersect at exactly two points (1,4) and (3,1) then

A) $4a + 2c = -3$ B) $4a + 2u = 3$ C) $\frac{u+c}{a} = -4$ D) $2a + c = 1$

45. Let $f(x) = ([k]^2 - 5[k] + 4)x^3 - (6\{k\}^2 - 5\{k\} + 1)x - \tan x \cdot \text{sgn } x$ be an even function then k can be (where $x \in R, [.]$ is G.I.F and $\{.\}$ denotes fractional part of function) (sgn is signum)

A) $3/2$ B) $7/2$ C) $4/3$ D) $13/3$

46. Which is correct

A) $\lim_{n \rightarrow \infty} \left(\frac{2n^2 - 3}{2n^2 - n + 1} \right)^{\frac{n^2 - 1}{n}} = \sqrt{e}$

B) $\lim_{x \rightarrow \infty} x \left(\frac{1}{e} - \left(\frac{x}{x+1} \right)^x \right) = -\frac{1}{2e}$

C) $\lim_{x \rightarrow 1} \sin(\sin^{-1} x) = 1$

D) $\lim_{x \rightarrow 0^+} (1^{1/x} + 2^{1/x} + \pi^{1/x})^x = \pi$

47. Possible integral values of x which can be in the domain of definition of the function $f(x) = \ln(ax^3 + (a+b)x^2 + (b+c)x + c)$ if $b^2 - 4ac < 0$ and $a > 0$ is__

- A) -1 B) 0 C) 1 D) 2

48. Let $f: R \rightarrow [1, \infty)$ be defined as $f(x) = \log_{10}(\sqrt{3x^2 - 4x + k + 1} + 10)$. If $f(x)$ is a surjection then k can be

- A) 1/3 B) 2/3 C) 4/3 D) 1

SECTION - II
(COMPREHENSION TYPE)

This section contains **4 groups of questions**. Each group has 2 multiple choice questions based on a paragraph. Each question has 4 choices A), B), C) and D) for its answer, out of which **ONLY ONE** is correct.

Paragraph for Question 49 and 50

Let A and B are two sets such that $A \cup B = \{1, 2, 3, 4, 5, 6\}$, $A \cap B = \{4, 5\}$ then

49. The maximum number of possible functions from set A to set B is N then sum of digits of N is

- A) 9 B) 8 C) 10 D) 13

50. The minimum number of possible functions from set B to set A is M then product of digits of M is

- A) 18 B) 10 C) 24 D) 60

Paragraph for Question 51 and 52

Let $f(x) = \frac{1}{x}$, $g(x) = \frac{1}{4x^2 - 1}$, $h(x) = \frac{5x}{x+2}$ be three functions and $\phi(x) = h(g(f(x)))$

51. If the domain of $\phi(x)$ is $R - \{a_1, a_2, a_3, \dots, a_n\}$ then the value of $\left(\sum_{i=1}^n a_i\right) + n$ equals

- A) 4 B) 5 C) 6 D) 7

52. If the range of $\phi(x)$ is $R - A$ where R is set of real numbers then the number of integers in set A is

- A) 5 B) 6 C) 7 D) 8

Paragraph for Question 53 and 54

Consider the limit given by $\lim_{x \rightarrow 0} \frac{1}{x^3} \left(\frac{1}{\sqrt{1+x}} - \frac{1+x \sin \alpha}{1+x \sin \beta} \right)$ exists and has finite value

where $\alpha, \beta \in [-\pi, 2\pi]$ then

53. The number of value of α is

- A) 2 B) 3 C) 4 D) 0

54. The number of values of β is

- A) 1 B) 4 C) 0 D) 2

Paragraph for Question 55 and 56

Let $f: R \rightarrow [1, \infty)$ be a quadratic function which is surjective, such that

$f(2+x) = f(2-x)$ and $f(1) = 2$. If $g: (-\infty, \ln 2] \rightarrow [1, 5]$ is given by $g(\ln x) = f(x)$ then

55. $g^{-1}(x)$ is given by

- A) $\ln(2 - \sqrt{x-1})$ B) $\ln(2 + \sqrt{x-1})$ C) $\ln(2 - \sqrt{1-x})$ D) $\ln(2 + \sqrt{1-x})$

56. The sum of the values of x satisfying the equation $f(x) = 5$ is

- A) 2 B) 4 C) 6 D) -2

SECTION – III
(MATRIX MATCH TYPE)

This section contains **4 multiple choice questions**. Each question has matching lists. The codes for the lists have choices (A), (B), (C), and (D) out of which **ONLY ONE** is correct.

57. Let $\lim_{x \rightarrow \infty} (2^x + e^x + a^x)^{1/x} = L$ then match the range of values of 'a' with 'L' value (e is exponent a is constant)

List – I**List – II**

(P) $a \in \left(\frac{\pi}{2}, e\right)$

(1)

e

(Q) $a \in (e, \infty)$

(2)

a

(R) $a \in (0, e)$

(3)

a/2

(S) $a \in \{2e\}$

(4)

2e

A) P-1,Q-2,R-1,S-4

B) P-2,Q-1,R-2,S-4

C) P-2,Q-1,R-1,S-4

D) P-1,Q-2,R-4,S-1

58. Match the following

List – I**List – II**

(P) $\lim_{n \rightarrow \infty} \left(\frac{1 + \sqrt[n]{16}}{2} \right)^n =$

(1) 1

(Q) If $\lim_{x \rightarrow 0} \frac{\ln \left(\cot \left(\frac{\pi}{4} - \beta x \right) \right)}{\tan \alpha x} = 1$ then $\frac{\alpha}{\beta} =$

(2) 2

(R) $\lim_{x \rightarrow 0} \frac{\pi \sin \left(\pi \cos^2 \left(\tan (\sin x) \right) \right)}{\sin^2 (\pi x)} =$

(3) 3

(S) If $\lim_{x \rightarrow \infty} \left[(x^5 + 10x^4 + 3)^c - x \right]$ is finite non-zero, for certain value of c then value of the limit is

(4) 4

A) P-4,Q-1,R-2,S-2

B) P-2,Q-4,R-1,S-1

C) P-4,Q-2,R-1,S-1

D) P-4,Q-2,R-1,S-2

59.

List – I

List – II

- (P) The number of values of a such that $\lim_{x \rightarrow a} \left[\sin^{-1} \left(\frac{2x}{1+x^2} \right) \right]$ (where $[.]$ is G.I.F) doesn't exist is (1) 4
- (Q) If $f(x) = \left\{ x + \left[\log_2(2016+x) \right] \right\} + \left\{ x + \left[\log_2(2016+x^2) \right] \right\} + \dots + \left\{ x + \left[\log_2(2016+x^{10}) \right] \right\}$ then $f(e)$ equals (where $[.]$ is G.I.F $\{.\}$ F.P.F) (2) 8
- (R) The number of solutions of $f(f(f(x))) = \frac{x}{4}$ where $f(x) = 4x(1-x), 0 \leq x \leq 1$ is (3) 7
- (S) The number of elements in the range of $f(x) = \text{sgn}([|\sin x| + |\cos x|])$ is, k then $4k$ (where $[.]$ is G.I.F, sgn is signum) (4) 5
- A) P-4, Q-1, R-2, S-3 B) P-4, Q-2, R-3, S-1
- C) P-4, Q-3, R-2, S-1 D) P-1, Q-3, R-2, S-4

60.

List – I

List – II

- The number of integers in the range of the function,
- (P) $f(x) = \cos^3 x - 6 \cos^2 x + 11 \cos x - 6, (x \in R)$ (1) 13
- (Q) $\lim_{x \rightarrow 0} \frac{1 - \cos x \cos 2x \cos 3x \cos 4x}{x^2}$ equals (2) 24
- (R) $A = \{1, 3, 5, 7\}, B = \{2, 4, 6, 8\}$ then the number of functions from A to B such that $i + f(i) < 10$ (3) 25
- Let $f(x)$ be a function such that
- (S) $f(x+2) + 6f(x) = 5f(x+1) \forall x \in R$ where (4) 15
- $f(0) = 2, f(1) = 5$ then $f(2)$ equals
- A) P-3, Q-4, R-2, S-1 B) P-3, Q-4, R-1, S-2
- C) P-3, Q-1, R-4, S-2 D) P-3, Q-4, R-4, S-1