MATHEMATICS **Max Marks: 80**

SECTION – I

(Straight Objective Type)

This section contains 7 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

47. Let z be a complex number, satisfies $|\arg(z)| < \pi/2$ and $3\bar{z} + 2|z + 2i| + 9i = 0$ then real part of ln(z) = _____

A) $\sqrt{20}$ B) $\ell n\left(\sqrt{20}\right)$ C) $\ell n\left(\sqrt{29}\right)$ D) $\ell n\left(\sqrt{40}\right)$

48. Condition on complex constant α and β, such that the equation $z^2 + \alpha z + \beta = 0$ have one of the roots on unit circle $|z| = 1$

A) $|\alpha - \bar{\alpha}\beta| = \left|1 - |\beta|^2\right|$ B) $|\alpha + \bar{\alpha}\beta| = \left|1 - |\beta|^2\right|$ C) $|\alpha - \bar{\alpha}\beta| = \left|1 + |\beta|^2\right|$ D) none

49. If $z^n - 1 = 0$ & $z^m - 1 = 0$ have only one common root $(m, n \in N)$, Then

A) m,n are co-primes B) m & n are primes

C) one of m and n is even and the other is odd

D) atleast one of m & n is prime

50. If z_1 and z_2 are two complex numbers satisfying $\frac{z_1}{2z_2}+\frac{2z_2}{z_1}=i$ and if origin, z_1, z_2 form two non-similar triangles and if α,β are the least angles in the two triangles then the value of $\cot\alpha+\cot\beta=k$, where k = _____

A) $\sqrt{5}$ B) $2\sqrt{5}$ C) 1 D) 2

51. If exactly one of the roots of the equation $z^2+az+b=0$. $(a,b\in C)$ is purely imaginary, then.

A) $(\bar{b}-b)^2=-(a\bar{b}+\bar{a}b)(a+\bar{a})$ B) $(\bar{b}-b)^2=-(a\bar{b}-\bar{a}b)(a-\bar{a})$

C) $(\bar{b}-b)^2=-(a\bar{b}-\bar{a}b)(a+\bar{a})$ D) $(\bar{b}-b)^2=-(a\bar{b}-\bar{a}b)(a-\bar{a})$

52. Find the modulus of the complex number z, satisfying $\text{Arg}(z+1)=\frac{\pi}{6}$ and $\text{Arg}(z-1)=\frac{2\pi}{3}$ is

A) 2 B) $\frac{1}{2}$ C) 1 D) $\sqrt{2}$

53. If $z_1 = (8+i)\sin\theta + (7+4i)\cos\theta$ & $z_2 = (1+8i)\sin\theta + (4+7i)\cos\theta$ & $z_1 z_2 = a + ib(a,b \in R)$

Then the maximum value of $a+b$ is M and the minimum value of $a+b$ is m.

Where

A) $M+m=125$ B) $M-m=125$ C) $30 = M-m$ D) $130 = M+m$

SECTION – II

Multiple Correct Answer Type

This section contains 4 multiple correct answer(s) type questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONE OR MORE** is/are correct.

54. The roots of the equation $z^4 + az^3 + (12+9i)z^2 + b\ z = 0$ are the vertices of a square then

A) "a" can be $6+2i$ B) "a" can be -6-2i

C) "b" can be $9+13i$ D) "b" can be -9-13i

55. If α,β are roots of the equation

$z + 1/z = 2(\cos\theta + i\sin\theta),\ (0<\theta<\pi)$ then which of the following are true.

A) $|\alpha - i| = \sqrt{2}$ B) $|\beta - i| = \sqrt{2}$

C) $|\alpha - i| = 2$ D) $|\beta - i| = 2$

56. If for any two complex numbers z_1, z_2 $(|z_2| \neq 1)$,

$\sqrt{z_1} - i\sqrt{z_2} = |z_2|\sqrt{z_1} + i|z_1|\sqrt{z_2}$ then which of the following options are correct.

A) $z_1\bar{z}_2 + 1 = 0$ B) $z_1 + z_2 = 0$ C) $\text{Arg}\left(\frac{z_1}{z_2}\right) = \pi$ D) $\text{Arg}\left(\frac{z_1}{z_2}\right) = \pi/2$

57. If the equation $az^2 + z + 1 = 0$ has purely imaginary root where $a = \cos\theta + i\sin\theta, (i = \sqrt{-1})$ then the interval in which the function $f(x) = x^3 - 3x^2 + 3(1+\cos\theta)x + 5$ is increasing, is

A) $(-\infty, -2)$ B) $(2,5)$ C) R-(1,2) D) $(5, \infty)$

SECTION – III

[Linked Comprehension Type]

This section contains 2 paragraphs. Based upon one of paragraphs 2 multiple choice questions and based on the other paragraph 3 multiple choice questions have to be answered. Each of these questions has four choices (A), (B),(C) and (D) out of which **ONLY ONE** is correct.

PASSAGE-1

Let z be a complex number and k be a non zero real number. Consider the sets.

$A = \{z : |\text{Im}(z)| = k - |\text{Re}(z) - k|\}$

$B = \{z : |z - k| > |z - 2k|\}$

C= circle, inscribed in the geometrical figure formed by A.

58. Radius of the circle "C" is

A) $K/2$ B) $K/\sqrt{2}$ C) K D) $\frac{3K}{2}$

59. Number of points of contact of C, with A, that belong to B is _____

A) 0 B) 2 C) 3 D) 4

PASSAGE-2

Let $A(z_1 = e^{i\theta})$ be a point on the unit circle $|z| = 1$ Points E,A,C,B (clock wise) are taken on the same circle such that C is the image of A, with respective to real axis, $|\underline{AOE} = \alpha$ and A, O, B are collinear points (O-origin) and $a = e^{i\alpha}$.

Two parallelograms OCDB, OAHE are formed now OE is extended such that EF=EH and G is the midpoint of FH

60. The affix of the point D is ___

A) $z_1 + \bar{z}_1$ B) $z_1 - \frac{1}{z_1}$ C) $\frac{1}{z_1} - z_1$ D) none

61. If OH is perpendicular to OC, then $\theta =$ _____

A) $\frac{\pi-\alpha}{2}$ B) $\frac{2\pi-\alpha}{4}$ C) $\frac{\pi+\alpha}{4}$ D) $\frac{\pi-\alpha}{4}$

62. The affix of the point G, is ____

A) $\frac{z+3az}{2}$ B) $\frac{z+2az}{2}$ C) $\frac{2az+3az}{5}$ D) $\frac{z+2az}{3}$

SECTION – IV

(INTEGER ANSWER TYPE)

This section contains 7 questions Answer to each of the questions is a single digit integer ranging from '0' to '9'. The bubble corresponding to the correct answer is to be darkened in the ORS.

63. If Z be a complex number such that $z \in C-R,$ and $\frac{z^2+z+1}{z^2-z+1}$=real, then maximum value of $|z-3-4i| =$_____

64. For the equation $z^6-z^3-2450=0$ where z is a complex number, the number of roots having positive real part is α, negative real part is β, positive imaginary part is γ, negative imaginary part is δ the $(\alpha+\beta)-(\gamma+\delta) =$ _____

65. If $a=\cos\left(\frac{\pi}{2n}\right)+i\sin\left(\frac{\pi}{2n}\right)$ and $z=\lim_{n\to\infty}\frac{\pi}{2n}\left(1+a+a^2+a^3+.....+a^{n-1}\right)$ then the value of $|z|^2=$_____

66. Let a be a complex number with $|a|=1$ and arg(a)= θ. If the roots of the quadratic equation $az^2+z+1=0$ are purely imaginary then the value of $\cos^2\theta+\cos\theta+6=0$ is __

67. The area of region bounded by the curves

(i) $|z-z_1|=|z-z_3|$

(ii) $|\mathrm{Re}(z)-\mathrm{Re}(z_1)|=|\mathrm{Re}(z)-\mathrm{Re}(z_3)|$

(iii) $|z-z_2|-|z-z_1|=|z_1-z_2|$

Where $z_1=1+i, z_2=2+i, z_3=-3+3i$ is $\frac{p}{q}$ (p,q are coprime) then the value of p+q = _____.

68. The number of complex number(s) z, satisfying the equation $\bar{z}+z^6=i(\bar{z}-z^6)$ is_____

69. Let $A(z_1)\,\&\,B(z_2)$ be two fixed points then locus of z, such that $|z-z_1|-3|z-z_2|=0$, is a circle and its centre divides the line segment AB in the ratio $\lambda:1$ externally, then $\lambda=$_____