Sri Chaitanya IIT Academy, India

A.P, TELANGANA, KARNATAKA, TAMILNADU, MAHARASHTRA, DELHI, RANCHI

A right Choice for the Real Aspirant

ICON CENTRAL OFFICE, MADHAPUR-HYD

Sec: Sr. IPLCO **Date: 17-10-15**

Time: 9:00 AM to 12:00 Noon **RPTM-9** **Max.Marks: 360**

KEY SHEET

MATHS		CHEMISTRY		PHYSICS	
Q.NO	ANSWER	Q.NO	ANSWER	Q.NO	ANSWER
1	4	31	2	61	3
2	2	32	2	62	1
3	3	33	2	63	4
4	3	34	3	64	1
5	3	35	2	65	3
6	2	36	2	66	4
7	3	37	3	67	1
8	4	38	3	68	2
9	1	39	1	69	1
10	4	40	3	70	3
11	1	41	4	71	3
12	1	42	3	72	3
13	1	43	4	73	3
14	1	44	2	74	4
15	2	45	1	75	2
16	3	46	3	76	1
17	1	47	4	77	1
18	4	48	1	78	2
19	4	49	2	79	2
20	4	50	1	80	1
21	1	51	3	81	4
22	2	52	4	82	3
23	1	53	2	83	1
24	2	54	3	84	3
25	2	55	3	85	4
26	4	56	1	86	3
27	2	57	2	87	1
28	3	58	4	88	2
29	3	59	2	89	4
30	2	60	2	90	1

MATHS

1. $$\cos\theta = \frac{32+12+5}{\sqrt{16+16+25}\sqrt{64+9+1}} - \frac{49}{\sqrt{57\times 74}} = \frac{49}{\sqrt{4218}}$$

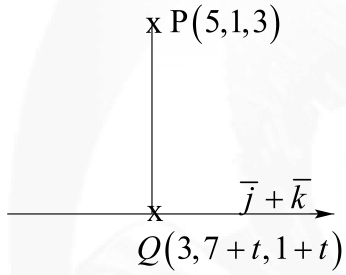
$$\theta = \cos^{-1}\left(\frac{49}{4218}\right)$$

2. Clearly $\overline{PQ}.\left(\vec{j}+\vec{k}\right)=0$

$$\left(-2\vec{i}+(6+t)\vec{j}+(-2+t)\vec{k}\right).\left(\vec{j}+\vec{k}\right)=0$$

$$\Rightarrow 6+t-2+t=0$$

$$2t+4=0 \Rightarrow t=-2$$



$$\therefore Q=(3,5,-1)$$

$$\therefore QP=\sqrt{4+16+16}=\sqrt{36}=6$$

3. $$\left[\vec{u}\ \ \vec{v}\ \ \vec{w}\right]=\vec{u}.\vec{v}\times\vec{w}$$

$$=|\vec{u}||\vec{v}\times\vec{w}|\cos\theta$$

(θ= angle between $\vec{u},\vec{v}\times\vec{w}$)

$$\le|\vec{u}|\,|\vec{v}\times\vec{w}|$$

$$=|\vec{v}\times\vec{w}|$$

$$=\sqrt{59}.$$

4. Let $\overline{OA}=x_1\vec{i}+y_1\vec{j}$

$$\overline{OB}=x_2\vec{j}+y_2\vec{j}$$

$$\overline{OA}.\vec{i}=1\Rightarrow x_1=1\Rightarrow y_1=1^2=1$$

$$\overline{OB}.\vec{i}=-2\Rightarrow x_2=-2\Rightarrow y_2=4.$$

$\therefore \overrightarrow{OA} = \vec{i} + \vec{j}, \overrightarrow{OB} = -2\vec{i} + 4\vec{j}$

$\left|2\overrightarrow{OA} - 3\overrightarrow{OB}\right| = \left|8\vec{i} - 10\vec{j}\right| = \sqrt{164} = 2\sqrt{41}$

5. Volume = $f(a) = \begin{vmatrix} 1 & a & 1 \\ 0 & 1 & a \\ a & 0 & 1 \end{vmatrix}$

$f(a) = 1 - a(-a^2) + (-a)$

$f(a) = a^3 - a + 1$

$f'(a) = 3a^2 - 1 = 0 \Rightarrow a = \pm\frac{1}{\sqrt{3}}$

$f''(a) = 6a > 0$ If $a = \frac{1}{\sqrt{3}}$

$\therefore$ $f(a)$ is minimum If $a = \frac{1}{\sqrt{3}}$

6. $\left|\vec{a} + \vec{b} + \vec{c}\right|^2 \geq 0$

$\Rightarrow |\vec{a}|^2 + \left|\vec{b}\right|^2 + |\vec{c}|^2 + 2\left(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}\right) \geq 0$

$\Rightarrow \vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} \geq \frac{-3}{2}$

Now $\left|\vec{a} - \vec{b}\right|^2 + \left|\vec{b} - \vec{c}\right|^2 + |\vec{c} - \vec{a}|^2$

$= 2\left(|\vec{a}|^2 + \left|\vec{b}\right|^2 + |\vec{c}| - \vec{a}.\vec{b} - \vec{b}.\vec{c}.\vec{c}.\vec{a}\right)$

$\leq 2(3 - (-3/2)) = 6 + 3 = 9$

7. Let $\vec{b} = \alpha\vec{i} + \beta\vec{j} + \gamma\vec{k}$

$\vec{a}.\vec{b} = 1 \Rightarrow \alpha - \beta - \gamma = 1$ …………..(1)

$\vec{a} \times \vec{b} = -\vec{j} + \vec{k}$

$$\Rightarrow\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 1 & -1 & -1\\ \alpha & \beta & \gamma\end{vmatrix}=-\vec{j}+\vec{k}$$

$$(\beta-\gamma)\vec{i}-(\gamma+\alpha)\vec{j}+(\beta+\gamma)\vec{k}=\vec{j}+\vec{k}$$

$$\left.\begin{aligned}\beta-\gamma&=0\\ \Rightarrow\gamma+\alpha&=1\\ \beta+\gamma&=1\end{aligned}\right\}\Rightarrow\alpha=1,=0,\ \gamma=0$$

$$\therefore\ \vec{b}=\vec{i}$$

8. $$16=\left|\vec{a}-2\vec{b}\right|^2=\left|\vec{a}\right|^2+4\left|\vec{b}\right|^2-4\vec{a}.\vec{b}$$

$$16=1+16-4\left(\vec{a}.\vec{b}\right)\Rightarrow4\left(\vec{a}.\vec{b}\right)=1$$

$$\vec{a}.\vec{b}=\frac{1}{4}$$

$$\left|\vec{a}+3\vec{b}\right|^2=\left|\vec{a}\right|^2+9\left|\vec{b}\right|^2+6\,\vec{a}.\vec{b}$$

$$=1+36+\cancel{6}^{3}.\frac{1}{\cancel{4}^{2}}=\frac{74+3}{2}=\frac{77}{2}$$

9. $$\overrightarrow{AM}=\alpha\vec{p}$$

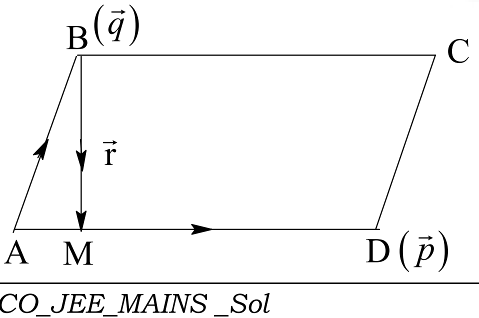
$$\overrightarrow{AM}=\overrightarrow{AB}+\overrightarrow{BM}=\vec{q}+\vec{r}$$

$$\alpha\vec{p}=\vec{q}+\vec{r}$$

$$\alpha\vec{p}.\vec{p}=(\vec{q}+\vec{r}).\vec{p}$$

$$\alpha\vec{p}.\vec{p}=\vec{q}.\vec{p}$$

$$\alpha=\frac{\vec{q}.\vec{p}}{\vec{p}.\vec{p}}\Rightarrow\vec{r}=-\vec{q}+\alpha\vec{p}$$



$$\vec{r} = -\vec{q} + \frac{(\vec{q}.\vec{p})}{\vec{p}.\vec{p}}\vec{p}$$

10. $\left[\overrightarrow{AB} \quad \overrightarrow{AC} \quad \overrightarrow{AD}\right] = 0$

$$\begin{vmatrix} -2 & 3 & 1 \\ 0 & 2 & -2 \\ k-1 & 2 & 2 \end{vmatrix} = 0 \Rightarrow k = -1$$

11. Equation of any plane parallel to the plane $3x+4y-5z=0$ is $3x+4y-5z=K$.

If it passes through (1, -1, 2), then $3-4-5(2)=K \Rightarrow K=-11$

So the required equation is $3x+4y-5z+11=0$

12. Middle point M of AB is $M\left(\frac{1}{2}(4i+5j-10k-i+2j+k)\right)$

$$= M\left(\frac{3}{2}i+\frac{7}{2}j-\frac{9}{2}k\right)$$

Also $AB = -i+2j+k-(4i+5j-10k)$

$$= -5i-3j+11k$$

So the plane passing through M and perpendicular to the direction AB is

$$\left[r-\left(\frac{3}{2}i+\frac{7}{2}j-\frac{9}{2}k\right)\right](-5i-3j+11k)=0$$

$$r.(-5i-3j+11k)+135/2=0$$

13. Equation of any plane parallel to the give plane is

$$r.(2i-3j+5k)+\lambda=0$$

If $r = xi+yj+zk$, we get $2x-3y+5z+\lambda=0$

This plane passes through the point (3, 4, -1)

if $2\times3-3\times4+5(-1)+\lambda=0$ or if $\lambda=11$ and hence the equation of the required plane is $r.(2i-3j+5k)+11=0$

14. The direction cosines of the given line are proportional to

$2-6, -3+7, 1+1$, i.e. $-2, 2, 1$

The direction cosines are therefore $=\frac{\mp}{3},\frac{\pm 2}{3},\frac{\pm 1}{3}$.

Since the angle α which the line makes with positive direction of $x-$axis is acute, $\cos\alpha > 0 \Rightarrow \cos\alpha = 2/3$

Thus, required direction cosines are 2/3, -2/3, -1/3.

15. The lines are coplanar if $\begin{vmatrix} 2-1 & 3-4 & 4-5 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$

$$\Rightarrow \begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ 1 & 2 & 1-k \\ k & k+2 & k+1 \end{vmatrix} = 0$$

$$\Rightarrow 2(k+1)-(1-k)(k+2)=0 \Rightarrow k=0 \text{ or } -3$$

16. Lines can be written as $\frac{x-b}{a}=\frac{y}{1}=\frac{z-d}{c}$ and $\frac{x-b'}{a'}=\frac{y}{1}=\frac{z-d'}{c'}$ which will be perpendicular if and only if $aa'+1+cc'=0$

17. We have $\cos^2\theta+\cos^2\theta+\cos^2\beta=1$

$$\Rightarrow 2\cos^2\theta=\sin^2\beta=3\sin^2\theta=3\left(1-\cos^2\theta\right)$$

$$\Rightarrow 5\cos^2\theta=3 \Rightarrow \cos^2\theta=3/5$$

18. Lines can be written as $\frac{x}{3}=\frac{y}{2}=\frac{z}{-6}$ and $\frac{x}{2}=\frac{y}{-12}=\frac{z}{-3}$

Since $3\times 2+2\times(-12)+(-6)(-3)=0$

The lines are perpendicular.

19. Direction ratios of the line are 1, -1, 4 and of the normal to the plane are 1, 5,1

Since $1\times 1+(-1)(5)+4\times 1=0$

Normal to the plane is perpendicular to the line and thus the line is parallel to the plane and the distance between them is the distance of any point on the line from the plane and is equal to $\left|\frac{2(1)+(-2)(5)+3\times 1-5}{\sqrt{1+25+1}}\right|=\frac{10}{\sqrt{27}}=\frac{10}{3\sqrt{3}}$

20. Since the three vectors are coplanar $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ x & x-2 & -1 \end{vmatrix} = 0$

$$\Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ 1 & -2 & 1 \\ x & -2 & -1-x \end{vmatrix} = 0$$

$$\Rightarrow -2(-1-x)+2=0$$

$$\Rightarrow x=-2$$

21. Equation of line through (1, -5, 9) parallel to the line $x=y=z$ is

$$\frac{x-1}{1}=\frac{y+5}{1}=\frac{z-9}{1}=r(\text{say})$$

Any point P on this line is $(r+1, r-5, r+9)$

which lies on the given plane if $r+1-(r-5)+r+9=5 \Rightarrow r=-10$

coordinate of P are $(-9,-15,-1)$

Distances of P from (1, -5, 9)

$$=\sqrt{(-9-1)^2+(+5-15)^2-(-9-1)^2}=10\sqrt{3}$$

22. Let the equation of the plane containing the first line be

$$a(x+1)+b(y-2)+cz=0$$

where $3a+2b+c=0$ other line will also lie on the plane if the point (3, -4, 1) lies on the plane.

$$a(3+1)+b(-4-2)+c=0$$

$$\Rightarrow \quad 4a-6b+c=0$$

Solving, we get $\frac{a}{8}=\frac{b}{1}=\frac{c}{-26}$ and the required equation is $8x+y-26z+6=0$.

23. Equation of QR is $\frac{x-2}{1}=\frac{y-3}{4}=\frac{z-5}{1}$

Let the coordinates of P be $(r+2, 4r+3, r+5)$ As P lies on the plane $5x-4y-z=1$

$5(r+2)-4(4r+3)-(r+5)=1$

$\Rightarrow r=-2/3 \Rightarrow P\left(\frac{4}{3},\frac{1}{3},\frac{13}{3}\right)$

Let the coordinates of S be $(t+2, 4t+3, t+5)$

As TS is perpendicular to QR

$1(t+2-2)+4(4t+3-1)+1(t+5-4)=0$

$\Rightarrow t=-\frac{1}{2} \Rightarrow S\left(\frac{3}{2},1,\frac{9}{2}\right)$

Hence $PS=\sqrt{\left(\frac{3}{2}-\frac{4}{3}\right)^2+\left(1-\frac{1}{3}\right)^2+\left(\frac{9}{2}-\frac{13}{3}\right)^2}$

$=\sqrt{\frac{9}{18}}=\frac{1}{\sqrt{2}}$

24. Any point P on the given line is $(2r+1,-3r-1,8r-10)$

So the direction ratios of AP are $2r,-3r-1,8r-10$.

Now AP is perpendicular to the given line if $2(2r)-3(-3r-1)+8(8r-10)=0$

$\Rightarrow 77r-77=0 \Rightarrow r=1$

and thus the coordinates of P, the foot of the perpendicular from A on the line are (3, -4, -2).

Let $B(a,b,c)$ be the reflection of A in the given line. Then P is the mid-point of AB

$$\frac{a+1}{2}=3, \frac{b}{2}=-4, \frac{c}{2}=-2$$

$\Rightarrow a=5, b=-8, c=-4$

Thus the coordinates of required point are (5, -8, -4).

25. The given lines are coplanar, if the normal to the plane containing these lines is perpendicular to both of them. Since the given lines are parallel to the vectors b and d, the normal to the plane is parallel to $b\times d$, which is perpendicular to the line joining the points on the plane with position vectors a and c

$\Rightarrow (a-c).b\times d = 0$, which is the required condition for the given lines to the coplanar.

26. Taking $a_1 = 3i+5j+7k, b_1 = i-2j+k$

$a_2 = -i-j-k,\ b_2 = 7i-6j+k$

We get $|b_1\times b_2| = |4i+6j+8k|$

$= \sqrt{4^2+6^2+8^2} = \sqrt{116}$ and $|(a_2-a_1).b_1\times b_2| = 1-4\times4-6-8\times81 = 116$

Hence the length of shortest distance is $\dfrac{116}{\sqrt{116}} = 2\sqrt{29}$

27. Let the components of the line segment vector be a,b,c, then

$$a^2+b^2+c^2 = (63)^2 \text{ ....(1)}$$

also $\dfrac{a}{3} = \dfrac{b}{-2} = \dfrac{c}{6} = \lambda(\text{say})$ then $a = 3\lambda, b = -2\lambda$ and $c = 6\lambda$

and from (1) we have $9\lambda^2+4\lambda^2+36\lambda^2 = (63)^2 \Rightarrow 49\lambda^2 = (63)^2$

$\Rightarrow \lambda = \pm\dfrac{63}{7} = \pm 9.$

Since $a = 3\lambda < 0$ as the line makes an obtuse angle with x-axis, $\lambda = -9$ and the required components are -27, 18, -54.

28. Let D be the foot of the perpendicular from the origin to the join of A and B and divide AB in the ratio $k:1$, then the coordinates of D are $\left(\dfrac{11k-9}{k+1}, \dfrac{0k+4}{k+1}, \dfrac{-k+5}{k+1}\right)$

So that the direction cosines of OD are proportional to $11k-9, 4, 5-k$

and direction cosines of AB are proportional to $11+9, 0-4, -1-5$

Since OD is perpendicular to AB.

$10(11k-9)-2(4)-3(5-k) = 0$

$\Rightarrow 110k-90-8-15+3k = 0$

$\Rightarrow 113k = 113 \Rightarrow k = 1$

29. Let l,m,n be the direction cosines of the line of shortest distance, then as it is perpendicular to the given lines $2l-7m+5n = 0; 2l+m-3n = 0$

$\Rightarrow \frac{l}{16} = \frac{m}{16} = \frac{n}{16}$

$\Rightarrow \frac{l}{1} = \frac{m}{1} = \frac{n}{1} = \frac{\sqrt{l^2 = m^2 + n^2}}{\sqrt{1+1+1}}$

$\Rightarrow l = m = n = 1/\sqrt{3}$.

Now the shortest distance between the given lines is the projection of the join of the points (3, -15, 9) and (-1, 1, 9) on the line of shortest distance. Hence the required distance = $\frac{1}{\sqrt{3}}\left|(3+1)+(-15-1)+9-9\right| = 4\sqrt{3}$.

30. Equation of a line through the intersection of the given lines is

$2x - y - 4 + \lambda(y + 2z) = 0$

$= x + 3z - 4 + \mu(2x + 5z - 8)$

for all values of λ and μ.

If it passes through the point (2, -1, 1) $\Rightarrow \lambda = -1$

and $2 + 3 - 4 + \mu(4 + 5 - 8) = 0 \Rightarrow \mu = -1$

$\Rightarrow \lambda = 1, \mu = -1$ and the required equation is therefore ; $x - y - z = 2, x + 2z = 4$