Sri Chaitanya IIT Academy, India

A.P, TELANGANA, KARNATAKA, TAMILNADU, MAHARASHTRA, DELHI, RANCHI

A right Choice for the Real Aspirant

ICON CENTRAL OFFICE, MADHAPUR-HYD

Sec: Sr. IPLCO	JEE ADVANCED	DATE : 03-01-16
TIME : 02:00 PM TO 05: 00 PM	2013_P2 MODEL	MAX MARKS : 180

KEY & SOLUTIONS

PHYSICS

1	**BC**	2	**ABC**	3	**ABC**	4	**ABD**	5	**D**	6	**ABCD**
7	**AB**	8	**BD**	9	**B**	10	**D**	11	**B**	12	**C**
13	**A**	14	**A**	15	**A**	16	**A**	17	**A**	18	**B**
19	**B**	20	**B**								

CHEMISTRY

21	**BD**	22	**BD**	23	**ABCD**	24	**BD**	25	**AB**	26	**AB**
27	**ABCD**	28	**BC**	29	**B**	30	**C**	31	**A**	32	**A**
33	**C**	34	**A**	35	**A**	36	**C**	37	**A**	38	**D**
39	**D**	40	**C**								

MATHEMATICS

41	**AD**	42	**C**	43	**ACD**	44	**C**	45	**AC**	46	**ABCD**
47	**ABCD**	48	**ACD**	49	**B**	50	**A**	51	**C**	52	**C**
53	**C**	54	**B**	55	**A**	56	**B**	57	**C**	58	**D**
59	**D**	60	**B**								

MATHS

41. $t_r = \sum_{k=1}^{r} t_k - \sum_{k=1}^{r-1} t_k = \frac{(r+1)(r+2)(r+3)}{12} - \frac{r(r+1)(r+2)}{12} = \frac{(r+1)(r+2)}{4}$

$\therefore \sum_{r=1}^{n} \frac{1}{t_r} = 4\sum_{r=1}^{n}\left(\frac{1}{r+1} - \frac{1}{r+2}\right) = \frac{2n}{n+2}$

42. First differences are in G.P

$\therefore S = (2^1 - 1) + (2^2 - 1) + (2^3 - 1) + ... + (2^{100} - 1)$

$= \frac{2(2^{100} - 1)}{2-1} - 100 = 2^{101} - 102$

44. $b_n = \sum_{r=0}^{n} \frac{r}{^nC_r} = \sum_{r=0}^{n} \frac{n-(n-r)}{^nC_r} = na_n - b_n \Rightarrow c_p = \frac{a_p}{b_p} = \frac{2}{p}$. Hence (p, q)=(3, 6), (6, 3), (4, 4)

45. The 1st elements of $s_1, s_2, s_3, s_4, ...,$ are $\frac{1^2-1}{1}, \frac{2^2-1}{2}, \frac{3^2-1}{3}, ...$

s_i must be a set starting at $\frac{t^2-1}{i}$

$s_{20} = \left\{\frac{399}{20}, \frac{399+20}{20}, \frac{399+40}{20}, ...20\,term\right\}$

$\Rightarrow$ third term is $\frac{439}{20}$

$s_{20} = \frac{20}{2}\left[\frac{2 \times 399}{20} + 19 \times 1\right] = 589.$

46. $m = 3, n = 2, A = 60$

47. Let d be the common difference of the AP

$a_7 = a_{16} - a_9 = 7d$. $\therefore$ the AP is d, 2d, 3d, ……, 16d

a_4, a_6, a_9 is a GP with Common ratio $\frac{3}{2}$, hence A is false

a_1, a_2, a_4 and a_2, a_4, a_8 are two G.P's, both with Common ratio = 2, hence B is false

a_1, a_4, a_{16} is the only GP with Common ratio = 4, hence C is false.

48. $C_{n-3}, C_{n-2}, C_{n-1}$ are in AP hence C_1, C_2, C_3 are in AP

So, $2C_2 = C_1 + C_3 \Rightarrow n(n-1) = n + \frac{n(n-1)(n-2)}{6} \Rightarrow n = 2 \text{ or } 7$

49. x_1, x_2, x_3, x_4 are in G.P $\therefore \frac{x_2}{x_1} = \frac{x_4}{x_3} \Rightarrow \frac{b^2}{ac} = \frac{q^2}{pr} = 1$

50. Given $x_1, x_2, \frac{1}{x_3}, \frac{1}{x_4}$ are in A.P

$$\Rightarrow x_2 - x_1 = \frac{1}{x_4} - \frac{1}{x_3} \Rightarrow \frac{x_2 - x_1}{x_3 - x_4} = \frac{1}{x_3 x_4}$$

$$\Rightarrow \frac{\frac{b^2}{a^2} - \frac{4c}{a}}{\frac{a^2}{p^2} - \frac{4r}{p}} = \frac{p^2}{r^2} \quad \Rightarrow \frac{b^2 - 4ac}{q^2 - 4pr} = \frac{p^2}{r^2} \times \frac{a^2}{p^2} = \frac{a^2}{r^2}$$

51. $S_n = \sum_{k=1}^{n} \frac{4k}{4k^4 + 1} = \sum_{k=1}^{n} \frac{k}{k^4 + \frac{1}{4}}$

$$= \frac{1}{2} \sum_{k=1}^{n} \frac{2k}{\left(k^2 - k + \frac{1}{2}\right)\left(k^2 + k + \frac{1}{2}\right)} = \frac{1}{2} \sum_{k=1}^{n} \frac{1}{\left(k^2 - k + \frac{1}{2}\right)} - \frac{1}{\left(k^2 + k + \frac{1}{2}\right)}$$

53. $a\left(\frac{1}{r} + 1 + r\right) = \alpha S$.......................(1)

$a^2\left(\frac{1}{r^2} + 1 + r^2\right) = S^2$.......................(2)

$\Rightarrow a\left(\frac{1}{r} - 1 + r\right) = \frac{S}{\alpha}$.......................(3)

$\Rightarrow 2a = S\left(\alpha - \frac{1}{\alpha}\right) = S\left(\frac{\alpha^2 - 1}{\alpha}\right)$

So, $\frac{\left(\alpha^2 - 1\right)^2}{4\alpha^2}\left(\frac{1}{r^2} + 1 + r^2\right) = 1$

$\Rightarrow \left(r - \frac{1}{r}\right)^2 + 3 = \frac{4\alpha^2}{\left(\alpha^2 - 1\right)^2}$..................(4)

$\Leftrightarrow 3\alpha^4 - 10\alpha^2 + 3 < 0$

$\Leftrightarrow \frac{1}{3} < \alpha^2 < 3$

But $\alpha^2 = 1 \Rightarrow a = 0$. Not possible.

54. Putting $\alpha^2 = 2$

$\left(r - \frac{1}{r}\right)^2 = 5$

$\Rightarrow r^2 - \sqrt{5}r - 1 = 0$

$\Rightarrow r = \frac{\sqrt{5} + 3}{2} \qquad [\because r > 1]$

55, 56

Let p be the common ratio of the G.P and d be the common difference of AP

$a_5p - a_5 = 4d; a_5p^2 - a_5p = 7d \Rightarrow p = \frac{7}{4}$

$\Rightarrow d = \frac{3}{16}(a_1 + 4d) \Rightarrow 4d = 3a_1 \;\; \Rightarrow d = \frac{3a_1}{4}$

$\therefore a_1$ is a multiple of 4, then d is an integer and therefore all terms of AP are integers.

The AP is $\frac{4}{3}d, \frac{7}{3}d, \frac{10}{3}d \ldots\ldots\ldots$

$T_r = \frac{(3r+1)d}{3} \;\; \Rightarrow 4T_r = T_{4r+1}$

$\therefore T_5, T_9, T_{16}$ are in GP $\Rightarrow T_{21}, T_{37}, T_{65}$ are in GP.

$\Rightarrow T_{85}, T_{149}, T_{261}$ are in GP and so on.

57. P) If A_p is p^{th} term of the AP, then $(p-q) = \frac{A_p - A_q}{d} = A_p\frac{(1-k)}{d}$ where k is common ratio of GP

Then $(q-r) = A_pk\frac{(1-k)}{d}; (r-s) = A_pk^2\frac{(1-k)}{d} \Rightarrow p-q, q-r, r-s$ are in G.P

Q) ln x, ln y, ln z are in G.P

$\Rightarrow \ln(\ln x), \ln(\ln y), \ln(\ln z)$ are in A.P$\Rightarrow 2x + \ln(\ln x), 3x + \ln(\ln y), 4x + \ln(\ln z)$ are in A.P.

R) $n!, 3\times n!, (n+1)!$ are in G.P implies n=8.

S) the first term is x, and $(2n-1)^{th}$ term is y for the progressions

$t_n = \frac{t_1 + t_{2n-1}}{2} = \frac{x+y}{2}$ for the AP, $t_n = \sqrt{t_1t_{2n-1}} = \sqrt{xy}$ for GP, $\frac{1}{t_n} = \frac{\frac{1}{t_1} + \frac{1}{t_{2n-1}}}{2} = \frac{x+y}{2xy}$ for HP

Hence the terms are in GP

58. P) We have $T_r = \frac{4r+1}{5^r r(r-1)}$, where $r \geq 2$,

$= \frac{5r-(r-1)}{5^r r(r-1)} = \frac{1}{5^{r-1}(r-1)} - \frac{1}{5^r r}$,

$\therefore \sum_{r=2}^{\infty} T_r = \left[\left(\frac{1}{5^1.1} - \frac{1}{5^2.2}\right) + \left(\frac{1}{5^2.2} - \frac{1}{5^3.3}\right) + \left(\frac{1}{5^3.3} - \frac{1}{5^4.4}\right) + \ldots = \frac{1}{5}\right.$

Q) We have, $T_r = \frac{8r}{4r^4+1} = \frac{8r}{(2r^2+2r+1)(2r^2-2r+1)} = 2\left[\frac{1}{(2r^2-2r+1)} - \frac{1}{(2r^2+2r+1)}\right]$,

$\therefore \sum_{r=1}^{\infty} T_r = 2\left[\left(\frac{1}{1} - \frac{1}{5}\right) + \left(\frac{1}{5} - \frac{1}{13}\right) + \left(\frac{1}{13} - \frac{1}{25}\right) + \ldots\right] = 2$.

R) The general term is

$$t_n = \frac{n}{1.3.5...(2n+1)} = \frac{1}{2}\left[\frac{(2n+1)-1}{1.3.5...(2n-1)(2n+1)}\right] = \frac{1}{2}\left[\frac{1}{1.3.5...(2n-1)} - \frac{1}{1.3.5...(2n+1)}\right]$$

$= \frac{1}{2}[T_{n-1} - T_n]$ where

$$T_n = \frac{1}{1.3.5...(2n+1)} \therefore S_n = \sum_{n=1}^{n} t_n = \frac{1}{2}[T_0 - T_1 + T_1 - T_2 + ... - T_n]$$

$= \frac{1}{2}(T_0 - T_n) = \frac{1}{2} - \left[1 - \frac{1}{1.3.5...(2n+1)}\right]$. Hence the sum is $S = \lim_{n\to\infty} S_n = \frac{1}{2}$

59. P) Note that $P = a(b-c)(a+b+c),\ Q = b(c-a)(a+b+c),\ R = c(a-b)(a+b+c)$

So, $Px^2 + Qx + R = 0$ has a root equal to 1 and it has equal roots, hence P=R hence $b = \frac{2ac}{a+c}$.

Q) a, b, c are in HP $\Rightarrow \frac{s}{a}-1, \frac{s}{b}-1, \frac{s}{c}-1$ are in AP $\Rightarrow \frac{s-a}{a}, \frac{s-b}{b}, \frac{s-c}{c}$ are in AP $\Rightarrow \frac{a}{b+c-a}, \frac{b}{c+a-b}, \frac{c}{a+b-c}$ are in HP.

R) By componendo and dividendo, we obtain that

$$\frac{a}{bk} = \frac{b}{ck} = \frac{c}{dk} \Rightarrow \frac{a}{b} = \frac{b}{c} = \frac{c}{d}$$

60. P) $\frac{a+b}{ab} + \frac{a+b}{ab} + ... + \frac{a+b}{ab} = 10\left(\frac{a+b}{ab}\right) = \frac{40}{5} = 8$

Q) $T_r = \frac{1}{r^2}(1+2+3+......+r)^2 = \frac{r^2+2r+1}{4}$ $\qquad T_7 = 16$

R) Given $\dfrac{\frac{n}{2}[2a_1 + (n-1)d_1]}{\frac{n}{2}[2a_2 + (n-1)d_2]} = \frac{5n+3}{3n+3}$, Replace ‘n’ with 2n-1

$$\frac{[2a_1 + (2n-1-1)d_1]}{[2a_2 + (2n-1-1)d_2]} = \frac{5(2n-1)+3}{3(2n-1)+3}$$

$$\frac{a_1 + (n-1)d_1}{a_2 + (n-1)d_2} = \frac{10n-2}{6n+1}$$

For n =10,

$$\frac{a_1 + 9d_1}{a_2 + 9d_2} = \frac{68}{61}$$

S) $\dfrac{\left(\frac{a}{b}+\frac{b}{a}\right) + \left(\frac{b}{c}+\frac{c}{b}\right) + \left(\frac{c}{a}+\frac{a}{c}\right)}{6} \geq 1$