Sri Chaitanya IIT Academy, India

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A right Choice for the Real Aspirant

ICON CENTRAL OFFICE, MADHAPUR-HYD

Sec: Sr.IPLCO	JEE–ADVANCE	Date: 20-09-15
Time: 3 Hours	2011-P1-Model	Max Marks: 240

PAPER-I

KEY & SOLUTIONS

CHEMISTRY

1	D	2	C	3	B	4	C	5	A	6	B
7	A	8	ABCD	9	ABC	10	BD	11	ACD	12	A
13	C	14	D	15	A	16	D	17	5	18	9
19	7	20	8	21	8	22	8	23	8		

PHYSICS

24	C	25	B	26	A	27	D	28	C	29	C
30	B	31	BD	32	AD	33	A	34	BC	35	B
36	C	37	C	38	C	39	D	40	1	41	1
42	5	43	5	44	2	45	1	46	2		

MATHS

47	B	48	C	49	B	50	B	51	2	52	C
53	A	54	ABC	55	AB	56	BD	57	A	58	C
59	B	60	A	61	B	62	A	63	6	64	9
65	2	66	9	67	3	68	6	69	8		

MATHS

47. $\sin\left(x_{n+1}-x_n\right)+2^{-(n+1)}\sin x_n \sin x_{n+1}=0$

$\Rightarrow \dfrac{\sin\left(x_{n+1}-x_n\right)}{\sin x_n \sin x_{n+1}}=-2^{-(n+1)} \Rightarrow \cot x_{n+1}=\cot x_n+2^{-(n+1)} \Rightarrow \cot x_n=1-\dfrac{1}{2^n}$

48. $f(x)=x^3-2x^2-3x+3$, $g(x)=[h(x)]^2;\ h(x)=x^2-x-2$

49. The equation $f(x)=2x^3+5x^2-6x-2=0$

We see that $f(-4)f(-3)<0; f(-1)f(0)<0; f(1)f(2)<0$

Sum of fractional parts = sum of the roots – (sum of the integral parts) = $-\dfrac{5}{2}-(-4-1+1)=1.5$

50. Slopes of the tangents at P and Q are given by $-\dfrac{4}{3}\cot\theta$ and $-\cot\theta$ respectively.

Hence $\operatorname{Tan}A=\left|\dfrac{\cot\theta}{3+4\cot^2\theta}\right|$

51. we have $f(x)=e^x;\ g(x)=1+k\tan^{-1}x$

Both are increasing functions for given k, and f(0)=g(0). Hence there is only solution.

52. For the curve $y=\ln|f(x)|$ is decreasing, we should have $f'(x)>0, f(x)<0$ or $f'(x)<0, f(x)>0$

53. $g(x)=f(\sin x)+f(\cos x)$

$\Rightarrow g''(x)=f''(\sin x)\cos^2 x+f''(\cos x)\sin^2 x-f'(\sin x)\sin x-f'(\cos x)\cos x$

54. Conceptual

55. We have $f'(x)=\dfrac{4}{x}+2x-a$

Solving $f'(x)=0$, we get $x_1=\dfrac{a-\sqrt{a^2-32}}{4};\ x_2=\dfrac{a+\sqrt{a^2-32}}{4}$

For $x_1\in(0,1]$, we should have $a\ge 6$

Now, $g(a)=f(x_1)-f(x_2)=4(\ln x_1-\ln x_2)+(x_1^2-x_2^2)-a(x_1-x_2)$

This gives $g(a)=8\ln(a-\sqrt{a^2-32})-20\ln 2+\dfrac{a\sqrt{a^2-32}}{4}$

Now we can check that $g'(a)$ is positive for $a > 6$

Hence g(a) is increasing in $[6,\infty)$. So, $\min g(a) = g(6) = 3 - 4\ln 2$

56. $f(x) = (4a-5)(x+\log 5) + 2(a-8)\cot\frac{x}{2}\sin^2\frac{x}{2}$

$\Rightarrow f(x) = (4a-5)(x+\log 5) + (a-8)\sin x$

$\therefore f'(x) = (4a-5) + (a-8)\cos x$

If f(x) does not have critical points, then f'(x) =0 does not have any solution in R. Now, solving $f'(x) = 0 \Rightarrow \cos x = \frac{4a-5}{8-a}$

$\Rightarrow \left|\frac{4a-5}{8-a}\right| \le 1 \quad [\because |\cos x| \le 1]$

$\Rightarrow -1 \le \frac{4a-5}{8-a} \le 1$

$\Rightarrow -1 \le a \le \frac{13}{5}$

57. Suppose $(3p, p^3)$ is on $27y = x^3$. Tangent at P is $y = p^2x - 2p^3$. If this line touches $y = (x+a)^2$, then the equation $p^2x - 2p^3 = (x+a)^2$ should have equal roots. Hence $(2a - p^2)^2 = 4(a^2 + 2p^3)$.

This gives $p^2(p^2 - 8p - 4a) = 0$ for some real p other than p=0. Hence $64 + 16a \ge 0 \Rightarrow a \ge -4$.

58, 59 & 60

$f(x) = xe^{-x}$

We have $pe^{-p} = qe^{-q} \Rightarrow pe^m = q, q - p = m$

Hence $p = \frac{m}{e^m - 1}; q = \frac{me^m}{e^m - 1}$

$f'(x) = (1-x)e^{-x} = 0 \Rightarrow \alpha = 1$. So, $QR = q - 1, PR = 1 - p$. Hence $QR - PR = q + p - 2$

We have $p + q - 2 = \frac{m}{e^m - 1} + \frac{me^m}{e^m - 1} - 2 = \frac{(m-2)e^m + m + 2}{e^m - 1} > 0$

$f''(x) = (x-2)e^{-x} = 0 \Rightarrow \beta = 2$

61&62

We have $f(x)=\begin{vmatrix} xe^{bx}, & x\le 0 \\ x+bx^2-x^3, & x>0 \end{vmatrix}$

So, $f'(x)=\begin{vmatrix} (1+bx)e^{bx}, & x\le 0 \\ 1+2bx-3x^2, & x>0 \end{vmatrix}$ and $f''(x)=\begin{vmatrix} (2+bx)e^{bx}, & x\le 0 \\ 2b-6x, & x>0 \end{vmatrix}$

For $f'(x)$ to be increasing, we should have $b+2x>0, x\le 0;\ 2b-6x>0, x>0$

So, the interval is $\left(-\frac{2}{b},\frac{b}{3}\right)$. We have $g(b)=\frac{b}{3}+\frac{2}{b}$. Hence $g'(b)=\frac{1}{3}-\frac{2}{b^2}$ and

$g(b)=\left(\sqrt{\frac{b}{3}}-\sqrt{\frac{2}{b}}\right)^2+\frac{2\sqrt{2}}{\sqrt{3}}$

63. $f(x)=x^4-\frac{244}{3}x^3+2ax^2$ will not have any local maxima if

$f'(x)=4x^3-244x^2+4ax=0$ has only one real root. So, $4x^2-244x+4a=0$ should have complex roots. Number of values for a is 69.

64. Min 0, Max $\frac{1}{8}$

65. $\lim\limits_{x\to\pm\infty}\left(\frac{y}{x}\right)=\lim\limits_{x\to\pm\infty}\left[\frac{\sqrt{1+x^2}}{x}\sin\left(\frac{1}{x}\right)\right]=0$ and $\lim\limits_{x\to\infty}y=1, \lim\limits_{x\to-\infty}y=-1$

66. $f\left(\frac{3}{2}\right)=0\Rightarrow \lim\limits_{x\to\frac{3}{2}}\left|x^2-3x\right|+a\le 0$ Hence, $|4k|=9$

67. $y=a\sqrt{x}+bx$ has slope $\frac{1}{2}$ at $(9,0)$. Hence $a=-3; b=3$. So, $\lambda=\frac{3}{5}$

68. The tangent at $P(2,1)$ to curve $x=\sec^2 t, y=\cot t$ is given by

$x+2y=4$. It meets the curve again at $Q\left(5,-\frac{1}{2}\right)$. Hence $PQ=\frac{3\sqrt{5}}{2}$

69. $f(x)=\frac{195x(x^2+4)}{9x^4+97x^2+144}=\frac{13(5x)(3x^2+12)}{(3x^2+12)^2+(5x)^2}$ can be written as $f(x)=\frac{13}{\frac{3x^2+12}{5x}+\frac{5x}{3x^2+12}}$ for

$x\ne 0$ Then $\left|\frac{3x^2+12}{5x}\right|\ge\frac{12}{5}$. Hence $|f(x)|\le\frac{60}{13}$