PART-III_MATHEMATICS **Max Marks : 60**

Section-1
(One or More options Correct Type)

This section contains 10 multiple choice equations. Each question has four choices (**A**) (**B**)(**C**) and (**D**) out of which **ONE or MORE** are correct.

41. Let $A=\{1^2,3^2,5^2,.........\}$.If 9 elements selected from set A (without repetition) to make a 3×3 matrix then det (A) will be divisible by

A) 9 B) 36 C) 8 D) 64

42. Which of the following statements are *FALSE*?

A) If A and B are square matrices of the same order such that ABAB = 0, it follows that BABA = 0.

B) Let A and B be different n × n matrices with real numbers. If $A^3 = B^3$ and $A^2B = B^2A$, then $A^2 + B^2$ is invertible.

C) If A is a square, non-singular and symmetric matrix, then $\left(\left(A^{-1}\right)^{-1}\right)^{-1}$ is skew symmetric.

D) The matrix of the product of two invertible square matrices of the same order is also invertible.

43. The system of equation is $x - y\cos\theta + z\cos 2\theta = 0$, $x\cos 2\theta - y + z\cos\theta = 0$ $x\cos 2\theta - y\cos\theta + z = 0$ has non-trivial solution for θ equals to

A) $\frac{8\pi}{3}$ B) $\frac{\pi}{6}$ C) $\frac{2\pi}{3}$ D) $\frac{\pi}{12}$

44. If both the roots of the equation $x^2 - 2ax + a^2 + a - 3 = 0$ in the variable x are less than 3 then 'a' can be

A) 2 B) 5/2 C) $\sqrt{3}$ D) -7

45. Complete set of real values of a for the equation $9^x + a.3^x + 1 = 0$ has

A) two real solutions, is $(-\infty,-2)$

B) no real solution, is $(-2,\infty)$

C) exactly one real solution, is {-2}

D) at least one real solution, is $(-\infty,-2]$

46. If $a_1 < a_2 < a_3 < a_4 < a_5 < a_6$, then the equation $(x-a_1)(x-a_3)(x-a_5)+3(x-a_2)(x-a_4)(x-a_6)=0$ has

A) three real roots B) a root in $(-\infty,a_1)$

C) a root in (a_1,a_2) D) a root in (a_5,a_6)

47. Let $a_1, a_2, a_3........$be real numbers which are in arithmetic progression with common difference $d \neq 0$. Then

A) $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_5 & a_6 & a_7 \end{bmatrix}$ is singular B) $B = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_2 & a_3 & a_4 \\ a_2 & a_4 & a_6 \end{bmatrix}$ is non-singular

C) The system of equations $a_1x + a_2y + a_3z = 0$, $a_3x + a_1y + a_2z = 0$, $a_4x + a_5y + a_6z = 0$ has unique solution

D) The system of equations $a_1x + a_2y + a_3z = 0$, $a_4x + a_5y + a_6z = 0$, $a_7x + a_8y + a_9z = 0$ has infinitely many solutions

48. If by eliminating x between the equations $x^2 + ax + b = 0$ and $xy + l(x + y) + m = 0$, a quadratic equation in y is formed whose roots are the same as those original quadratic in x, then which of the following may be correct?

A) $a = 2l$ B) $b = m$ C) $b + m = al$ D) $a + b = l$

49. Let $|a| < |b|$ and $a, b \in R$ are the roots of the equation $x^2 - |\alpha| x - |\beta| = 0.$ If $|\alpha| < b - 1,$ then the equation $\log_{|a|}\left(\frac{x}{b}\right)^2 - 1 = 0$ has at least one

A) root lying between $(-\infty, a)$ B) roots lying between (b, ∞)

C) negative root D) positive root

50. Given $|ax^2+bx+c| \le |Ax^2+Bx+C|$, $\forall x \in R$, a,b,c A,B,C $\in$ R and d=$b^2-4ac>0$ and D=$B^2-4AC>0$. Then which of the following statements are true

A) $|a| \le |A|$

B) $|d| \le |D|$

C) $|a| \ge |A|$

D) if D,d are not necessarily positive then roots of $ax^2+bx+c=0$ and $Ax^2+Bx+C=0$ may not be equal

Section-2
(Integer Value Correct Type)

This section contains 10 questions. The answer to each question is a **single digit integer, ranging** from 0 to 9 (both inclusive).

51. Let $P(x)=x^2+bx+c$, where b and c are integer. If $P(x)$ is a factor of both x^4+6x^2+25 and $3x^4+4x^2+28x+5$, find the value of P(1).

52. The set of real parameter 'a' for which the equation $x^4-2ax^2+x+a^2-a=0$ has all real solutions, is given by $\left[\frac{m}{n},\infty\right)$ where m and n are relatively prime positive integers, then the value of $(m+n)$ is

53. Sum of non-real roots of $(x^2+x-2)(x^2+x-3)=12$ is k, then $|k|=$

54. If α,β,γ are the roots of the equation $x^3+px^2+qx+r=0$, and they are in G.P. such that β satisfy the equation $px+k_1q=0$ and α,γ satisfy the equation $pqx^2-k_2(q-p^2)qx+p^2r=0$ then the value of k_1+k_2 is

55. If the equation $x^4+px^3+qx^2+rx+5=0$ has four positive real roots, then the minimum value of pr/10 is

56. A be set of 3 × 3 matrices formed by entries 0, –1, and 1 only. Also each of 1, -1, 0 occurs exactly three times in each matrix. The number of symmetric matrices with trace (A) = 0 is k, then $\frac{k}{6}=$..........

57. Let A_n, ($n \in N$) be a matrix of order $(2n-1)\times(2n-1)$, such that $a_{ij}=0,\ \forall i \neq j$ and $a_{ij}=n^2+i+1-2n,\ \forall\ i=j$ where a_{ij} denotes the element of i^{th} row and j^{th} column of A_n. Let $T_n=(-1)^n\times$ (sum of all the elements of A_n). Find the value of $\left[\frac{\sum_{n=1}^{102}T_n}{520200}\right]$, where [.] represents the greatest integer function.

58. If $A=\begin{bmatrix}1 & -1 & 1\\ 0 & 2 & -3\\ 2 & 1 & 0\end{bmatrix}$ and B = (adj A) and C = 5A, then find the value of $\frac{|\text{adj } B|}{|C|}$.

59. If $-2 \le [x] < 2,\ -1 \le [y] < 1, -3 \le [z] < 3$, where $[x]$ denotes greatest integer function and the minimum value of $\begin{vmatrix}[x]+1 & [y] & [z]\\ [x] & [y]+1 & [z]\\ [x] & [y] & [z]+1\end{vmatrix}$ is equals to K then $|K|$ equals to

60. Let $x_1, x_2, x_3, x_4, x_5 > 0$ and A = $[a_{ij}]_{5 \times 5}$ matrix such that $a_{ij} = \begin{cases}|x_i - x_j| & i \ne j\\ x_i, & i = j\end{cases}$

if $(x_1^2 - x_3x_5)(x_2^2 - x_3x_5) \le 0, (x_2^2 - x_4x_1)(x_3^2 - x_4x_1) \le 0$, $(x_3^2 - x_5x_2)(x_4^2 - x_5x_2) \le 0$

$(x_4^2 - x_1x_3)(x_5^2 - x_1x_3) \le 0, (x_5^2 - x_2x_4)(x_1^2 - x_2x_4) \le 0$, then the sum of the digits of the number det A is ________________ (given that $x_3 = 3$)