MATHEMATICS

Max Marks: 80

SECTION – I

(Straight Objective Type)

This section contains 7 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

47. The coordinates of a point $P(x,y)$ are functions of time 't' the locus of point $P(x,y)$ is a curve given by $\frac{dx}{dt}+\frac{dy}{dt}=t$ and $\frac{dx}{dt}-2\frac{dy}{dt}=t^2$ at any instant of time ''t'' the locus of point $P(x,y)$ is a curve given by (assume $x(0)=0, y(0)=0$)

A) $(x+y)^3=(x-2y)^3$

B) $x=\frac{t^2}{6}-\frac{t^3}{9};\ y=\frac{t^2}{3}+\frac{t^3}{9}$

C) $8(x+y)^3=9(x-2y)^2$

D) $9(x+y)^3=8(x-2y)^2$

48. The solution of the differential Equation $\frac{\sqrt{x}dx+\sqrt{y}dy}{\sqrt{x}dx-\sqrt{y}dy}=\sqrt{\frac{y^3}{x^3}}$ is given by

A) $\frac{3}{2}\log\left(\frac{y}{x}\right)+\log\left(\frac{x^{3/2}+y^{3/2}}{x^{3/2}}\right)+\left(\tan^{-1}\left(\frac{y}{x}\right)\right)^{3/2}+c=0$

B) $\frac{2}{3}\log\left(\frac{y}{x}\right)+\log\left(\frac{x^{3/2}+y^{3/2}}{x^{3/2}}\right)+\tan^{-1}\left(\frac{y}{x}\right)+c=0$

C) $\frac{2}{3}\left(\frac{y}{x}\right)+\log\left(\frac{x+y}{x}\right)+\tan^{-1}\left(\frac{y^{-3/2}}{x^{3/2}}\right)+c=0$

D) $\frac{1}{2}\log\left(x^3+y^3\right)+\tan^{-1}\left(\frac{y}{x}\right)^{3/2}=c/2$

49. Let ABC be a triangle with vertices $A\left(6, 2\sqrt{3}+2\right), B(4, 2)$ and $C(8, 2)$. Let R be the region consisting of all these points P inside triangle ABC which satisfy $d(P, BC) \geq \max.\{d(P, AB), d(P, AC)\}$ where $d(P, L)$ denotes the distance of the points P from the line L. Then the area of the region R is (in square units)

A) $\frac{2\sqrt{3}}{3}$ B) $\frac{4\sqrt{3}}{3}$ C) $2\sqrt{3}$ D) $4\sqrt{3}$

50. The solutions of the differential equation $\frac{dy}{dx} = \frac{1}{xy\left(x^2 \sin y^2 + 1\right)}$ is______

(c – being an arbitary constant)

A) $x^2\left(\cos y^2 - \sin y^2 - 2ce^{-y^2}\right) = 2$ B) $x^2\left(\cos y^2 - \sin y^2 - 2e^{-y^2}\right) = 4c$

C) $y^2\left(\cos x^2 - \sin x^2 - 2ce^{-y^2}\right) = 2$ D) $y^2\left(\cos x^2 - \sin x^2 - ce^{-y^2}\right) = 4c$

51. Suppose a solution of the differential equation $\left(xy^3 + x^2y^7\right)\frac{dy}{dx} = 1$ satisfies the initial condition $y\left(\frac{1}{4}\right) = 1$ then the value of $\frac{dy}{dx}$ when $y = -1$ is_______

A) $-3/20$ B) $\frac{-20}{3}$ C) $\frac{-5}{16}$ D) $\frac{-16}{5}$

52. A function $y = f(x)$ satisfies the differential equation $\frac{dy}{dx} - y = \cos x - \sin x$ with initial condition that y is bounded when $x \to \infty$, then the area enclosed by $y = f(x)$, $y = \cos x$ and the y-axis, in first quadrant is________

A) $\sqrt{2}-1$ B) $\sqrt{2}$ C) 1 D) $\frac{1}{\sqrt{2}}$

53. Let $f(x)$ be a non-negative continuous function such that the area bounded by the curve $y = f(x)$, x-axis and the ordinates $x = \frac{\pi}{4}$, $x = \beta > \pi/4$ is $\beta \sin\beta + \frac{\pi}{4}\cos\beta + \sqrt{2}\beta$. Then $f'\left(\pi/2\right) =$_____

A) $\frac{\pi}{2} - \sqrt{2} - 1$ B) $\frac{\pi}{2} + \sqrt{2} - 1$ C) $\frac{-\pi}{2}$ D) $1 - \frac{\pi}{4} + \sqrt{2}$

SECTION – II

Multiple Correct Answer Type

This section contains 4 multiple correct answer(s) type questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONE OR MORE** is/are correct.

54. Let $f:(0,\infty) \to (0,\infty)$ be a differentiable function satisfying

$x\int_0^x (1-t)f(t)\,dt = \int_0^x tf(t)\,dt \;\forall x \in R \;\&\; f(1) = 1$ then

A) $f(2) = \frac{1}{8}e^{1/2}$ B) $f(2) = \frac{1}{8}e^{-1/2}$ C) $f(3) = \frac{1}{27}e^{2/3}$ D) $f(3) = \frac{1}{27}e^{-2/3}$

55. The normal at a general point (a, b) on curve makes an angle θ with positive direction of x-axis, which satisfies $b\left(a^2\tan\theta-\cot\theta\right)=a\left(b^2-1\right)$ then equation of the curve, can be

A) $y=e^{x^2/2}+c$ B) $\log_e\left(ky^2\right)=x^2$ C) $y=ke^{x^2/2}$ D) $x^2+y^2=k$

56. Let $y=f(x)$ be a polynomial such that

$\int_{x+\sqrt{5}}^{x+\sqrt{2}} f(x)dx=ax+b \quad \forall x\in R \quad (a\neq 0,\, b\in R)$ and $f(0)=3,\ f(1)=5$ then

Which of the following is/are true?

A) $y=f(x), f:R\to R$ is one-one and on to

B) $y=f(x), f:R\to R$ is neither one-one nor on to

C) Area, bounded by the curve $y=f(x)$, between the lines $x=0,\ x=2$ x-axis is 10 square units

D) Area, bounded by the curve $y=f(x)$, between the lines $x=0,\ x=2$, x-axis is 8 square units

57. Given a function g which has a derivative $g^1(x)$ for every real x and which satisfies the following conditions $g^1(0)=2$ & $g(x+y)=e^x g(y)+e^y g(x)$ for all x and y, then which of the following is/are correct

A) $g(2x)=2e^x g(x)$ and $g(3x)=3e^{2x}g(x)$

B) $g(nx)=ne^{(n-1)x}g(x)$, n is a positive integer

C) $g(0)=0$ & $\lim_{h\to 0}\frac{g(h)}{h}=2$

D) There is a constant c such that $g^1(x)=g(x)+ce^x$, for all x

SECTION – III

[Linked Comprehension Type]

This section contains 2 paragraphs. Based upon one of paragraphs 2 multiple choice questions and based on the other paragraph 3 multiple choice questions have to be answered. Each of these questions has four choices (A), (B),(C) and (D) out of which **ONLY ONE** is correct.

Paragraph for Questions Nos. 58 to 60

$f(x)$ is a polynomial function such that $f(x^2+1)=f(x^2)+2x^2$ and $f(0)=0$

$g(x)=xf(x)$ then

58. If $\int_0^1 g(x)dx = A$ then

A) $A=2\int_0^1 f(x)dx \neq 0$ B) $|A|=1$

C) $A+|A|=0$ D) $\frac{1}{2}<|A|<1$

59. Area bounded by $\min\{f(x),\, g(x)\}$, $x=2$ & x-axis in first quadrant, is

A) 12 B) $5/6$ C) $11/2$ D) $4/3$

Paragraph for Questions Nos. 61 to 62

A curve $y=f(x)$ satisfies the differential equation $(1+x^2)\frac{dy}{dx}+2yx=4x^2$ and passes through the origin

60. The function $y=f(x)$

A) Is strictly increasing $\forall x \in R$

B) Is such that it has a local minima but not local maxima

C) Is such that it has a local maxima but not local minima

D) Has no inflection point

61. The area enclosed by $y = f^{-1}(x)$, the x-axis & the ordinate at $x = \frac{2}{3}$ is

A) $2\,ln\ 2$ B) $\frac{4}{3}ln\ 2$ C) $\frac{2}{3}ln\ 2$ D) $\frac{1}{3}ln\ 2$

62. For the function $y = f(x)$ which one of the following does not hold good?

A) $f(x)$ is a rational function

B) $f(x)$ has the same domain and same range

C) $y = f(x)$ is a injective function

D) $g(x)$ is the inverse of $f(x)$, (if it exists) then $g'\left(\frac{2}{3}\right) = \frac{4}{3}$

SECTION – IV
(INTEGER ANSWER TYPE)

This section contains 7 questions Answer to each of the questions is a single digit integer ranging from '0' to '9'. The bubble corresponding to the correct answer is to be darkened in the ORS.

63. Slope of the tangent at any point (x, y) on the curve $y = f(x)$ is $\frac{y}{x^2}$ and the curve passes through the point $(1, 3)$ then $\lim_{x\to\infty} f(x) = k.e$, where k = ______

64. The differential equation of the family of curves $c(y+c)^2 = x^3$ (where c is – an arbitary constant) is $mx\left(\frac{dy}{dx}\right)^3 + ny\left(\frac{dy}{dx}\right)^2 - 27x = 0$ then $|m+n| =$ ______

65. Let A and B be the points of intersection of the parabola $y=x^2$ and the line $y=x+2$ and let C be the point on the parabola where the tangent line is parallel to the graph of $y=x+2$ then $\frac{\text{(area of the parabolic segment cut from the parabola by the line)}}{\text{area of the triangle ABC}}=\frac{m}{n}$ (where m,n are integers $HCF(m,n)=1$)then $m+n=$

66. Let $P(x)=x^3-3b^2x+16$ if $P(x)=0$ has all integral roots (no complex root) and local minimum value of $P(x)$ is 0. And the area bounded by the curve $y=P(x)$, & x-axis, is k then $\left[\frac{\sqrt{k}}{2}\right]$__________(in square units), ([.] *denotes greatest integer function*)

67. For what value of a $(a>2)$ is the area of the region bounded by $y=\frac{1}{x}, y=\frac{1}{2x-1}, x=2$ & $x=a$ equal to $ln\left(\frac{4}{\sqrt{5}}\right)$?

68. Let $F(x)=\int_x^{x^2+\frac{\pi}{6}} 2\cos^2 t\,dt$ for all $x\in\mathbb{R}$ and $f:\left[0,\frac{1}{2}\right]\to[0,\infty)$ be a continuous function. For $a\in\left[0,\frac{1}{2}\right]$, if $F'(a)+2$ is the area of the region bounded by $x=0,\ y=0,\ y=f(x)$ and $x=a$, then $f(0)$ is

69. If c is a constant such that, the line joining the points $(0,3)$, $(5,-2)$ is a tangent to the curve $y=\frac{c}{x+1}$ then the area of the region bounded by the curve and x-axis and $x=1,\ x=2$ is $\lambda\left\{1-\frac{1}{2.2}+\frac{1}{2^2.3}-\frac{1}{2^3.4}+.....\infty\right\}$ where λ equals to__________