Sri Chaitanya IIT Academy, India

A.P, TELANGANA, KARNATAKA, TAMILNADU, MAHARASHTRA, DELHI, RANCHI

A right Choice for the Real Aspirant

ICON CENTRAL OFFICE, MADHAPUR-HYD

Sec: Sr. IPLCO **Date: 19-09-15**

Time: 9:00 AM to 12:00 Noon **RPTM-7** **Max.Marks: 360**

KEY SHEET

PHYSICS		MATHS		CHEMISTRY	
Q.NO	ANSWER	Q.NO	ANSWER	Q.NO	ANSWER
1	3	31	1	61	3
2	4	32	2	62	3
3	2	33	2	63	2
4	4	34	4	64	1
5	1	35	4	65	3
6	3	36	4	66	4
7	3	37	2	67	2
8	4	38	2	68	4
9	2	39	3	69	2
10	2	40	1	70	4
11	4	41	3	71	3
12	1	42	2	72	2
13	3	43	2	73	4
14	4	44	1	74	3
15	2	45	1	75	2
16	4	46	2	76	3
17	1	47	3	77	1
18	1	48	4	78	4
19	4	49	3	79	4
20	3	50	1	80	3
21	2	51	1	81	1
22	1	52	1	82	2
23	2	53	2	83	2
24	2	54	4	84	1
25	4	55	4	85	2
26	2	56	3	86	2
27	4	57	1	87	2
28	3	58	4	88	4
29	1	59	1	89	2
30	4	60	1	90	1

MATHS

31. $\left(\frac{dy}{dx}\right)_{(0,0)} = \tan 45^0 = 1$

$\left(3ax^2 + 2bx + c\right)_{(0,0)} = 1 \Rightarrow c = 1$

$(1,0)$ lies on the curve $\Rightarrow a + b + 1 = 0 \Rightarrow a + b = -1 \ldots\ldots\ldots\ldots(1)$

$\left(\frac{dy}{dx}\right)_{(1,0)} = 0 \Rightarrow \left(3ax^2 + 2bx + c\right)_{(1,0)} = 0$

$3a + 2b + c = 0$

$3a + 2b + 1 = 0 \ldots\ldots\ldots\ldots\ldots(2)$

Solve (1) & (2) $\quad 2a + 2b + 2 = 0$

$a - 1 = 0 \quad \Rightarrow a = 1$

$\therefore b = -2$

$(a,b,c) = (1,-2,1)$

32. $ay^2 = x^3 \Rightarrow 2ay.\frac{dy}{dx} = 3x^2$

$\frac{dy}{dx} = \frac{3x^2}{2ay}$

Slope of normal at a point $p(x_1\ y_1)$ is $\frac{-1}{\left(\frac{dy}{dx}\right)_{(x_1\ y_1)}} = \frac{-2ay_1}{3x_1^{\ 2}}$

But normal makes equal intercepts has the slope = -1

$\therefore \frac{-2ay_1}{3x_1^{\ 2}} = -1 \Rightarrow y_1 = \frac{3x_1^{\ 2}}{2a} \ldots\ldots\ldots.(1)$

But $(x_1 y_1)$ lies on $ay^2 = x^3$

$\Rightarrow ay_1^{\ 2} = x_1^{\ 3}$

$\therefore a\frac{9.x_1^{\ 4}}{4a^2} = x_1^{\ 3}$

$\Rightarrow x_1 = \frac{4a}{9}$

33. $xy^n = a^{n+1}$

$\log_e x + n\log_e y - (n+1)\log_e a$

$\frac{1}{x} + \frac{n}{y}\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{y}{nx}$

Length of subnormal = $y \times \frac{dy}{dx} = \frac{-y^2}{nx} = \text{constant}$

$\Rightarrow \dfrac{-y^2.y^n}{n.a^{n+1}}$ is constant $\Rightarrow n+2=0 \Rightarrow n=-2$

34. $x=y^2, xy=k \Rightarrow y^3=k \Rightarrow y=k^{1/3}$

$$\Rightarrow x=k^{2/3}$$

Point of intersection of two curve is $P\left(k^{2/3},k^{1/3}\right)$

Slope of tangent of $x=y^2$ is $\dfrac{dy}{dx}=\dfrac{1}{2y}$

$$\left(\frac{dy}{dx}\right)_P=\frac{1}{2k^{1/3}}=m_1$$

Slope of tangent of $xy=k$ is $\dfrac{dy}{dx}=\dfrac{-k}{x^2}$

$$\left(\frac{dy}{dx}\right)_P=\frac{-k}{k^{4/3}}=\frac{-1}{k^{1/3}}=m_2$$

Since two curves at orthogonally $m_1m_2=-1$

$$\therefore \frac{1}{2k^{1/3}}.\frac{-1}{k^{1/3}}=-1$$

$\therefore 2k^{2/3}=1 \Rightarrow 8k^2=1$

35. Any point on $\dfrac{x^2}{a^2}+\dfrac{y^2}{4}=1$ is $P\left(a\cos\alpha, 2\sin\theta\right)$

Slope of tangent at P $\rightarrow \dfrac{-2}{a}.\dfrac{\cos\theta}{\sin\theta}=m_1$

P – lies on $y^3=16x \Rightarrow \cos\theta=\dfrac{\sin^3\theta}{2a}$

Slope of tangent at P on $y^3=16x \rightarrow \dfrac{4}{3\sin^2\theta}=m_1$

Now $m_1m_2=-1 \Rightarrow \dfrac{8\cos\theta}{3a\sin^3\theta}=+1$

$\Rightarrow \dfrac{8\sin^3\theta}{3a.2a.\sin^3\theta}=1 \Rightarrow a^2=\dfrac{4}{3}$

$3a^2=4$

36. $\dfrac{dy}{dx}=3x^2-2ax+1>0 \ \forall x\in R$

$\Rightarrow$ Its Disc <0

$\Rightarrow 4a^2-4(3)<0 \Rightarrow a^2-3<0$

$$\Rightarrow \sqrt{3}<a<\sqrt{3}$$

37. Tangent at $P(a,b)$ is $xa^2+yb^2=c^3$

It meets the curve again at $Q(a_1,b_1)$

$\therefore$ Slope of $PQ = \dfrac{b_1 - b}{a_1 - a} = \dfrac{-a^2}{b^2}$

$b_1 b^2 + a_1 a^2 = c^3$ …………(1)

(a,b) lies on the curve $\Rightarrow a^3 + b^3 = c^3$

(a_1, b_1) lies on the curve $\Rightarrow a_1^{\,3} + b_1^{\,3} = c^3$

$\therefore a^3 + b^3 = a_1^{\,3} + b_1^{\,3}$

$a^3 - a_1^{\,3} = b_1^{\,3} - b^3 \Rightarrow (a - a_1)\left(a^2 + a_1^{\,2} + aa_1\right) = -(b - b_1)\left(b^2 + b_1^{\,2} + bb_1\right)$

$\therefore \dfrac{b - b_1}{a - a_1} = -\dfrac{a^2 + a_1^{\,2} + aa_1}{b^2 + b_1^{\,2} + bb_1} = +\dfrac{a^2}{b^2}$

$\dfrac{a^2 + a_1^{\,2} + aa_1}{a^2} = +\dfrac{b^2 + b_1^{\,2} + bb_1}{b^2}$

$1 + \dfrac{a_1^{\,2}}{a^2} + \dfrac{a_1}{a} = +1 + \dfrac{b_1^{\,2}}{b^2} + \dfrac{b_1}{b}$

$\left(\dfrac{a_1^{\,2}}{a^2} - \dfrac{b_1^{\,2}}{b^2}\right) = \left(\dfrac{b_1}{b} - \dfrac{a_1}{a}\right)$

$\therefore \left(\dfrac{a_1}{a} - \dfrac{b_1}{b}\right)\left(\dfrac{a_1}{a} + \dfrac{b}{b}\right) = -\left(\dfrac{a_1}{a} - \dfrac{b_1}{b}\right)$

$\therefore \dfrac{a_1}{a} + \dfrac{b_1}{b} = -1$

38. $\dfrac{dy}{dx} = 2\,e^{2-2x} + (2x-1)e^{2-2x}(-2) = 0$

$e^{2-2x}\left(2 + (2x-1)(-2)\right) = 0$

$1 - 2x + 1 = 0 \Rightarrow x = 1$

Clearly $\left(\dfrac{d^2y}{dx^2}\right)_{(x=1)}$ is < 0

$\therefore$ y is maximum at $x = 1 \Rightarrow y_{(x=1)} = 1$

$\therefore$ Point of maximum = (1, 1)

Tangent at (1, 1) is $y - 1 = 0(x-1) \Rightarrow y = 1$

39. $2y^3 = ax^2 + x^3$

$6y^2 . \dfrac{dy}{dx} = 2ax + 3x^2$

$\dfrac{dy}{dx} = \dfrac{2ax + 3x^2}{6y^2}$

$\left(\dfrac{dy}{dx}\right)_{(a,a)} = \dfrac{2a^2 + 3a^2}{6a^2} = 5/6$

Tangent at (a,a) is $y-a=\frac{5}{6}(x-1)$

$$6y-6a=5x-5a$$
$$5x-6y=-a$$
$$\frac{x}{(-a/5)}+\frac{y}{(a/6)}=1$$

$\therefore \alpha=-a/5,\ \beta=a/6$

$$\alpha^2+\beta^2=61\Rightarrow\frac{a^2}{25}+\frac{a^2}{36}=61$$
$$\Rightarrow\frac{a^2}{25.36}\times 61=61$$
$$a^2=25\times 36$$
$$|a|=5\times 6=30$$

40. $\left(\frac{dy}{dx}\right)_{(1,1)}=\frac{-1}{2}$

Tangent at (1, 1) $\rightarrow y-1=\frac{-1}{2}(x-1)$

$$y=\frac{-x+3}{2}$$

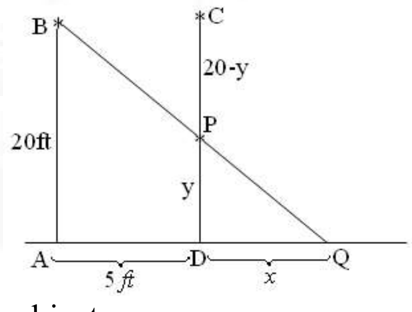
Solve with curve $4x^3-17x^2+22x-9=0$

$$(x-1)(4x^2-13x+9)=0$$
$$(x-1)(x-1)(4x-9)=0$$
$$\Rightarrow x=9/4\Rightarrow y=3/8$$
$$\therefore(9/4,3/8)$$

41. The rate at which object falls freely due to gravity = $\sqrt{2\times 32\times 16}=32$ ft/sec

Now y = distance to be travelled by the object

$\therefore\frac{dy}{dt}=-32$ ft/sec



Let Q both shadow of the object.

$\therefore\frac{20}{y}=\frac{5+x}{x}\Rightarrow\frac{20}{y}=1+\frac{5}{x}$ where $y=4\Rightarrow 4=\frac{5}{x}$

$$\Rightarrow x=5/4$$

$$\frac{-20}{y^2}.\frac{dy}{dt}=\frac{-5}{x^2}.\frac{dx}{dt}\Rightarrow\frac{dx}{dt}=4.\frac{x^2}{y^2}.\frac{dy}{dt}$$

$$=4\times\frac{25}{16}\times\frac{1}{16}\times(-32)$$

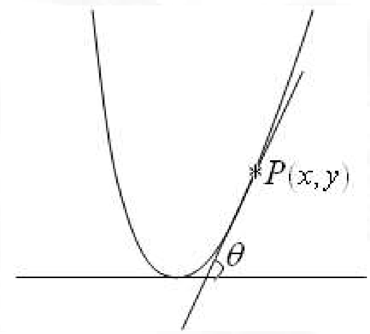
$$=\frac{-25}{2}=-12.5$$

42. Velocity of the point P is $\sqrt{\left(\frac{dy}{dt}\right)^2+\left(\frac{dx}{dt}\right)^2}=4$

$$\left(\frac{dy}{dt}\right)^2+\left(\frac{dx}{dt}\right)^2=16$$

$$\left(\frac{d}{dt}\left(2x^2\right)\right)^2+\left(\frac{dx}{dt}\right)^2=16\Rightarrow\left(4x.\frac{dx}{dt}\right)^2+\left(\frac{dx}{dt}\right)^2=1$$

$$\left(\frac{dx}{dt}\right)^2\left(16x^2+1\right)=1$$



$$\frac{dx}{dt}=\frac{1}{\sqrt{16x^2+1}}$$

$$\left(\frac{dx}{dt}\right)_{(1,2)}=\frac{1}{\sqrt{17}}$$

Slope of tangent at $P(x,y)=\tan\theta=\left(\frac{dy}{dx}\right)=4x$

$$\theta=\tan^{-1}(4x)$$

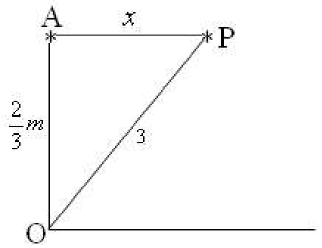
$$\frac{d\theta}{dt}=\frac{1}{1+16x^2}\times4.\frac{dx}{dt}$$

$$\left(\frac{d\theta}{dt}\right)_P=\frac{4}{17\sqrt{17}}\text{rad/sec}$$

43. $\frac{dx}{dt}=15mph$

$1hr\rightarrow15m$

$\frac{2}{60}hr \to ?$



$(x)_{\text{when, 2-mint}} = \frac{2}{60} \times \frac{15}{1} = \frac{1}{2}hr$

Now $S^2 = x^2 + \frac{4}{9}$

$2S.\frac{ds}{dt} = 2x.\frac{dx}{dt}$

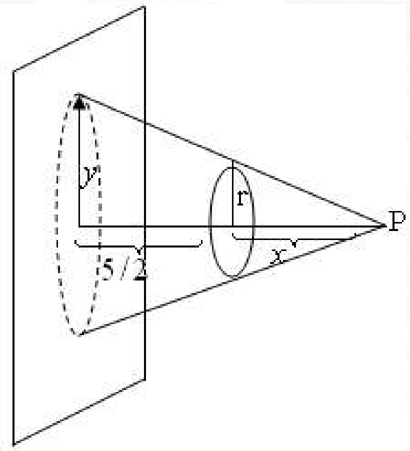
$\frac{ds}{dt} = \frac{x}{S}.\frac{dx}{dt}$

$\frac{ds}{dt} = \frac{x}{\sqrt{x^2 + \frac{4}{9}}}.\frac{dx}{dt}$

$= \frac{\frac{1}{2}}{\sqrt{\frac{1}{4} + \frac{4}{9}}}.15$

$\frac{6}{2} \times 3 = 9\ mph$

44 Let $r -$ both radius of circular disc.



$\pi r^2 = 10$

$r = \sqrt{\frac{10}{\pi}} ft$

$\frac{dx}{dt} = 5\ ft/\sec$

$y \to$ radius of shadow of the dis $\Rightarrow \frac{dy}{dt} = ?$

$$\frac{y}{r}=\frac{\frac{5}{2}+x}{x}$$

$$\frac{y}{r}=\frac{5}{2x}+1\ \ldots\ldots(1)$$

When $x=20-\frac{5}{2}=\frac{35}{2}$

$$y=r\left(\frac{5}{35}+1\right)$$

$$=r.\frac{40}{35}=r.\frac{8}{7}$$

diff. (t)

$$-\frac{1}{r}.\frac{dy}{dt}=\frac{-5}{2x^2}.\frac{dx}{dt}$$

$$\frac{dy}{dt}=\frac{5\,r}{2x^2}.5$$

$$=\frac{25}{2}\times\sqrt{\frac{10}{\pi}}\times\frac{4}{(35)^2}$$

Now S=Area of shadow

$$S=\pi y^2$$

$$\frac{ds}{dt}=2\pi y.\frac{dy}{dt}$$

$$=2\pi.r.\frac{8}{7}.\frac{25.\sqrt{10}}{\sqrt{\pi}}.\frac{2}{(35)^2}$$

$$=2\pi.\frac{10}{\pi}\times\frac{8}{7}\times\frac{25\times 2}{35\times 35}=\frac{320}{343}$$

45. $b=2R\sin B\Rightarrow\delta b=2R\cos B\delta B$

$c=2R\sin C\Rightarrow\delta c=2R\sin C\delta C$

$$\delta b\sec B+\delta c\sec=2R(\delta B+\delta C)$$

$$=0$$

$$(\therefore A+B+C=\pi\quad \delta B+\delta C=0)$$

46. $f''(x)<0\Rightarrow f'(x)$ is decreasing $\forall x\in(0,2)$

Now $g'(x)=2f'\left(\frac{x}{2}\right).\frac{1}{2}-f'(2-x)<0$

$$f'\left(\frac{x}{2}\right)-f'(2-x)<0$$

$$f'\left(\frac{x}{2}\right)<f'(2-x)$$

$$\Rightarrow\frac{x}{2}>2-x$$

$$x+\frac{x}{2}>2$$

$\frac{3x}{2} > 2$

$\Rightarrow x > \frac{4}{3}$

$\therefore x \in \left(\frac{4}{3}, 2\right)$

47 $f'(x) = 3x^2 + 2(a+2)x + 3a > 0 \ \forall x \in R$.

Disc $< 0 \Rightarrow 4(a+2)^2 - 4(9a) < 0$

$a^2 + 4a + 4 - 9a < 0 \Rightarrow a^2 - 5a + 4 < 0$

$(a-1)(a-4) < 0$

48. $f'(x) = 1 - \sin x \geq 0$

$\therefore f(x)$ is monotonically increasing

$\forall x$ (except $x = (4n+1)\frac{\pi}{2} |n \in Z|$

Where $f'(x) = 0$

$f(0) = 1 - a < 0 \ (\because a > 1)$

$\therefore y = f(x)$ should cross the x-axis at one – point.

$\therefore$ $f(x) = 0$ has one root in $(0, \infty)$

49. $f(x) = x^{\frac{1}{x}}$

$\log_e f(x) = \frac{\log_e x}{x} \Rightarrow \frac{f'(x)}{f(x)} = \frac{x.\frac{1}{x} - \log_e x}{x^2} > 0$

$f'(x) = \frac{(1 - \log_e x)}{x^2} f(x) > 0$

$1 - \log_e x > 0 \Rightarrow \log_e x < 1 \Rightarrow x < e$

$\therefore f(x)$ is increasing for $x \in (0, e)$

$f(x)$ is decreasing for $x > e \Rightarrow (e, \infty)$

$e < \pi \Rightarrow f(e) > f(\pi)$

$e^{-\frac{1}{e}} > \pi^{\frac{1}{\pi}} \Rightarrow e^{\pi} > \pi^{e}$

$a > b$

50. $f'(x) = 3\left((x-a)^2 + (x-b)^2 + (x-c)^2\right) > 0$

$\therefore f(x)$ is increasing $\forall \, x \in R$

As $f(x) \to -\infty$ as $x \to -\infty$

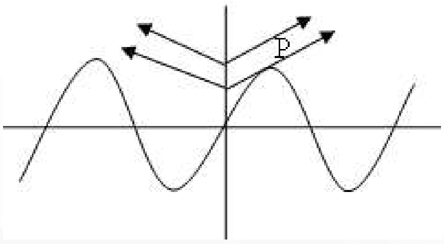
$f(x) \to +\infty$ as $x \to +\infty$

$\therefore$ It crosses $x - axis$ at one point.

51. $\sin x = \frac{1}{2}|x| + \frac{a}{2}$

$y = \sin x,\ y = \frac{1}{2}x + a/2\left(x > 0\right)$

$\frac{dy}{dx} = \sin x\ :\ Slope = \frac{1}{2}$



$\cos x = \frac{1}{2} \Rightarrow x = \frac{\pi}{3}, \Rightarrow y = \frac{\sqrt{3}}{2} \Rightarrow$ given equation has a solution $\sqrt{3} = \frac{\pi}{3} + a$

$$a = \left(3\sqrt{3} - \pi\right)/3$$

No solution . If $a > \frac{3\sqrt{3} - \sqrt{\pi}}{3}$

Range of $\left(\frac{3\sqrt{3} - \pi}{3}, \infty\right)$

52. C.S.Inequality.

$\left|ax + by + cz\right| \le \sqrt{a^2 + b^2 + c^2}\sqrt{x^2 + y^2 + z^2}$

$\left|ax + by + cz\right| \le 1$

53. $f\left(x\right) = ax^3 + bx^2 + cx + d$

$f\left(-2\right) = 0 \Rightarrow -8a + 4b - 2c + d = 0$ ………(1)

$f'\left(x\right) = 3ax^2 + 2bx + c$

$f'\left(-1\right) = 0 \Rightarrow +3a - 2b + c = 0$ …………..(2)

$f'\left(\frac{1}{3}\right) = 0 \Rightarrow \frac{a}{3} + \frac{2b}{3} + c = 0$

$a + 2b + 3c = 0$……..(3)

$\int_{-1}^{+!} ax^3 + bx^2 + cx + d = \frac{14}{3} \Rightarrow \frac{2b}{3} + 2d = \frac{14}{3} \to (4)\ \Rightarrow a = 1, b = 1, c = -1, d = 2$

54. $x = 2\cos\theta,\ y = 2\sin\theta$. Satisfily $x^2 + y^2 = 4$

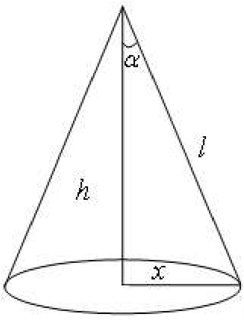
$\therefore 12x + 5y + 6 = 24\cos\theta + 10\sin\theta + 6$

Minimum $6 - \sqrt{576 - 100}$

$6 - 26 = -20$

55. $\frac{h}{l} = \sin\alpha \Rightarrow h = l\cos\alpha$

$$\frac{r}{l} = \sin\alpha \Rightarrow r = l\sin\alpha$$



$$v = \frac{1}{3}\pi r^2 h$$

$$= \frac{\pi}{3}.l^2 \sin^2\alpha . l\cos\alpha$$

$$v' = \frac{\pi l^3}{3}\left\{2\sin\alpha\cos^2\alpha - \sin^3\alpha\right\} = 0 \Rightarrow \tan^2\alpha = 2$$

$$\tan\alpha = \sqrt{2}$$

$$\alpha = \tan^{-1}\sqrt{2}$$

56. $x + y = k$

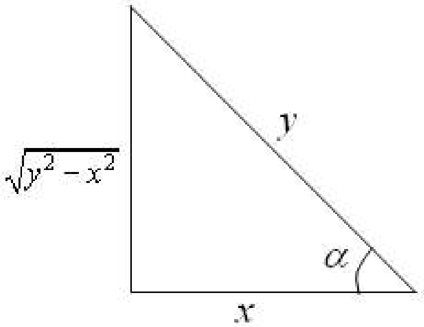
Area $= \frac{1}{2}x\sqrt{y^2 - x^2} = \Delta$

$$\Delta = \frac{1}{2}x\sqrt{(k-x)^2 - x^2}$$

$$\Delta = \frac{x}{2}\sqrt{k^2 - 2kx}$$

$$\Delta' = \frac{\sqrt{k^2 - 2kx}}{2} + \frac{x(-2k)}{4\sqrt{k^2 - 2kx}} = 0$$

$$k^2 - 2kx - kx = 0 \Rightarrow k^2 = 3kx \Rightarrow x = k/3$$



$$\Rightarrow y = k - k/3 = 2k/3.$$

Now $\cos\alpha = \frac{x}{y} = \frac{\frac{k}{3}}{\frac{2k}{3}} = \frac{1}{2} \Rightarrow \alpha = 60^0$

57. $f'(x)=\frac{a}{x}+2bx+1$

$f'(1)=a+2b+1=0$ …………..(1)

$f'(3)=\frac{a}{3}+6b+1=0$ …………(2)

Solve (1) and (2)

$3a+6b+3=0$

$\frac{a}{3}+6b+1=0$

$3a-\frac{a}{3}+2=0$

$\frac{8a}{3}=-2$

$a=-3/4 \Rightarrow b=\frac{-1}{8}$

58. f – should be continuous on [0, 1]

$\Rightarrow \underset{x\to 0+}{lt}\ x\,\alpha \log_e x=0$

$\underset{x\to 0+}{lt}\ \frac{\log_e x}{x^{-\alpha}}=\underset{x\to 0+}{lt}\ \frac{\frac{1}{x}}{-\alpha x^{-\alpha-1}}=0$

$\underset{x\to 0+}{lt}\ x^{\alpha}=0$

Then α can be any positive real number

$\therefore\ \alpha=\frac{1}{2}$

59. $f(x)=2ax^3+3bx^2+6cx$

$f(0)=0, f(1)=2a+3b+6c=0$

$\therefore f$ is continuous of (0, 1]

F is differentiable on (0, 1)

$f(0)=f(1)$

$\therefore$ by rolles theorem $\exists\lambda\in(0,1)$ such that $f'(\lambda)=0\Rightarrow\left(ax^2+bx+c\right)_{x=\lambda}=0$

60. f is continuous I [1, 3]

f is differentiable in (1, 3) by L.M.V.C

There exist c such that $f'(c)=\frac{f(3)-f(1)}{3-1}$

$\frac{1}{c}=\frac{\log 3}{2}\Rightarrow c=\frac{2}{\log_e 3}$

$c=2\log_3 e$