Sri Chaitanya IIT Academy, India

A.P, TELANGANA, KARNATAKA, TAMILNADU, MAHARASHTRA, DELHI, RANCHI

A right Choice for the Real Aspirant

ICON CENTRAL OFFICE, MADHAPUR-HYD

Sec: Sr.IPLCO	JEE–ADVANCE	Date: 27-09-15
Time: 3 Hours	2014-P2-Model	Max Marks: 180

PAPER-II
KEY & SOLUTIONS

PHYSICS

1	**B**	2	**B**	3	**C**	4	**C**	5	**B**	6	**A**
7	**C**	8	**C**	9	**B**	10	**D**	11	**B**	12	**A**
13	**C**	14	**C**	15	**A**	16	**B**	17	**A**	18	**A**
19	**C**	20	**A**								

CHEMISTRY

21	**D**	22	**D**	23	**D**	24	**C**	25	**D**	26	**C**
27	**C**	28	**B**	29	**C**	30	**D**	31	**C**	32	**A**
33	**A**	34	**B**	35	**A**	36	**B**	37	**A**	38	**C**
39	**D**	40	**B**								

MATHS

41	**D**	42	**B**	43	**C**	44	**B**	45	**B**	46	**C**
47	**C**	48	**D**	49	**C**	50	**D**	51	**B**	52	**C**
53	**B**	54	**D**	55	**B**	56	**B**	57	**D**	58	**B**
59	**D**	60	**C**								

MATHS

41. Let $ab+bc+ca=x$

$\Rightarrow 2b^2+2(c-2)b-4c+c^2+x=0$

Since $b\in R,$

$\therefore c^2-4c+2x-4\le 0$

Since $c\in R$

$\therefore x\le 4$

42. Let (α,β) be point on the curve such that the tangent drawn at (α,β) passes through (0, 7)

$y^1=4x-4\Rightarrow y^1_{(\alpha,\beta)}=4\alpha-4$

Tangent at (α,β) is $y-\beta=(4\alpha-4)(x-\alpha)$pass through (0,-7)

$\Rightarrow -7-\beta=(4\alpha-4)(0-\alpha)$

But $\beta=2\alpha^2-4\alpha-5\,\therefore$ It follows that $\alpha^2=1$

$\Rightarrow \alpha=\pm 1$

So, $x_1=1,\ x_2=-1$

So, $3x_1-2x_2=5$.

43. According to the given condition, we have

$$\left|am^2+bm+c\right|=am^2+bm+c$$

i.e. $am^2+bm+c>0$

$\Rightarrow$ if $a<0$, the m lies in (α,β)

and if $a>0$, then m does not lies in (α,β)

Hence, option (c) is correct, since

$$\frac{|a|}{a}=1\Rightarrow a>0$$

And in that case m does not lie in (α,β).

44. Putting $3^x=y,$ we have

$$(2a-4)y^2-(2a-3)y+1=0$$

This equation must have real solution

$\Rightarrow \quad (2a-3)^2 - 4(2a-4) \geq 0$

$\Rightarrow \quad 4a^2 - 20a + 25 \geq 0$

$\Rightarrow \quad (2a-5)^2 \geq 0.$ This is true.

$y = 1$ satisfies the equation

Since 3^x is positive and $3^x \geq 3^0$, $y \geq 1$

Product of the roots $= 1 \times y > 1$

$\Rightarrow \quad \dfrac{1}{2a-4} > 1$

$\Rightarrow \quad 2a - 4 < 1 \Rightarrow a < \dfrac{5}{2}$

Sum of the roots $= \dfrac{2a-3}{2a-4} > 1$

$\Rightarrow \quad \dfrac{(2a-3)-(2a-4)}{2a-4} > 0$

$\Rightarrow \quad \dfrac{1}{2a-4} > 0 \Rightarrow a > 2$

$\Rightarrow \quad 2 < a < \dfrac{5}{2}$

45. $(x+3)^2 + y^2 = 13$

$x + 3 = \pm 2, y = \pm 3 \; or \; x + 3 = \pm 3, y = \pm 2$

46. Hint: X = AB + BA $\Rightarrow X^T = X$

and Y = AB – BA $\Rightarrow Y^T = -Y$

Now, $(XY)^T = Y^T \times X^T = -YX.$

47. Multiply by y,z and x in rows 1,2 and 3 respectively and then take common y, z and x from column 1,2 and 3 respectively, then

$$\begin{vmatrix} y^3+1 & y^3 & y^3 \\ z^3 & z^3+1 & z^3 \\ x^3 & x^3 & x^3+1 \end{vmatrix} = 11$$

$$\Rightarrow \begin{vmatrix} 1 & 0 & y^3 \\ -1 & 1 & z^3 \\ 0 & -1 & x^3+1 \end{vmatrix} = 11 \qquad \left(C_1 \to C_1 - C_2 \text{ and } C_2 \to C_2 - C_3\right)$$

$$\Rightarrow 1\left(x^3+1+z^3\right)+y^3(1)=11 \Rightarrow x^3+y^3+z^3=10$$

So solution are (1,1,2) , (1,2,1) or (2,1,1)

48. $A = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}$

$AA^T = I$ (i)

Now, $C = ABA^T$

$\Rightarrow$ $A^T C = BA^T$ (ii)

Now $A^T C^n A = A^T C.C^{n-1}A = BA^T C^{n-1} A$ (from (ii))

$= BA^T C.C^{n-2}A = B^2 A^T C^{n-2} A = \ldots\ldots\ldots\ldots$

$= B^{n-1}A^T CA = B^{n-1}BA^T A = B^n = \begin{bmatrix} 1 & 0 \\ -n & 1 \end{bmatrix}$

49. $$\Delta = \begin{vmatrix} X & SX & tX \\ X_1 & SX_1 + S_1X & tX_1 + t_1X \\ X_2 & SX_2 + 2S_1X_1 + S_2X & tX_2 + 2t_1X_1 + t_2X \end{vmatrix}$$

$$\begin{pmatrix} C_2 \leftarrow C_2 - SC_1 \\ C_3 \leftarrow C_3 - C_1 \end{pmatrix}$$

$$= \Delta = \begin{vmatrix} X & 0 & 0 \\ X_1 & S_1X & t_1X \\ X_2 & 2S_1X_1 + S_2X & 2t_1X_1 + t_2X \end{vmatrix}$$

$$= S^2 \begin{vmatrix} S_1 & t_1 \\ 2S_1X_1 + S_2X & 2t_1X_1 + t_2X \end{vmatrix}$$

$$= \; = X^3 \le \begin{vmatrix} S_1 & t_1 \\ S_2 & t_2 \end{vmatrix} \left(R_2 \leftarrow R_2 - 2X_1R_1\right)$$

$\therefore n = 3.$

50. Since AB = B and BA = A

$\therefore$ A and B both are idempotent

$(A-B)^2 = A^2 - AB - BA + B^2 = A - B - A + B = 0$

$\therefore$ A – B is nilpotent

51 & 52 . Let $x^4 + ax^3 + bx^2 + cx + d$

$= (x-x_1)(x-x_2)(x-x_3)(x-x_4)$

Let $(x-x_1)(x-x_2) = x^2 + px + q$

and $(x-x_3)(x-x_4) = x^2 + px + r$

$\therefore$ $q = x_1x_2$ and $r = x_3x_4$

$\therefore$ $x^4 + ax^3 + bx^2 + cx + d$

$= x^4 + 2px^3 + (p^2+q+r)x^2 + p(q+r)x + qr$

$\therefore$ $a = 2p, b = p^2 + q + r, c = p(q+r), d = qr$

Clearly, $a^3 - 4ab + 8c = 0$

If $a = 2 \quad \Rightarrow b - c = 1$

Investigating the nature of the cubic equation of $'a'$.

Let $f(a) = a^3 - 4ab + 8c$

$f'(a) = 3a^2 - 4b$

If $b < 0 \Rightarrow f'(a) > 0$

The equation $a^3 - 4ab + 8c = 0$ hence only one real root.

53. $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 2 \\ 2 & 2 & 3 \end{bmatrix}\begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 26 \\ 20 \\ 29 \end{bmatrix} \Rightarrow 29^2 \neq 20^2 + 26^2$

Similarly, $Q\begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \\ 13 \end{bmatrix}$, and $R\begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 15 \\ 8 \\ 17 \end{bmatrix}$

54. detP = 1, detQ = 1, detR = 1

det(PQ) = 1, det(QR) = 1, det(RP) = 1

det(PQR) = 1

57. A) $\lambda > 2$

$64-4(\lambda-2)(\lambda+4)<0$

$\Rightarrow \quad (\lambda+6)(\lambda-4)>0$

$\lambda<-6 \text{ or } \lambda>4$

$\therefore$ The least positive integral value of λ is 5

(B) Roots are of opposite signs

$\Rightarrow \quad a^2-14a+48<0$

$(a-6)(a-8)<0$, so a can be 7

The equation is $x^2+100x-1=0$

$\therefore$ discriminant = D = $100^2+4>0$

$\therefore$ Roots are real

C)

Let $f(x)=ax^2+2bx+4c-16$

Clearly $f(-2)=4a-4b+4c-16$

$=4(a-b+c-4)>0$

$=f(x)>0, \forall x\in R$

$\Rightarrow f(0)>0 \Rightarrow 4c-16>0$

$\Rightarrow c>4$

(D) $\because |x^2-x-6|=x+2$

$\Rightarrow \quad |(x-3)(x+2)|=x+2$

$\Rightarrow \quad |x-3||x+2|=x+2$

$$\Rightarrow \begin{cases} (x-3)(x+2)=x+2, & x<-2 \\ -(x-3)(x+2)=x+2, & -2\le x<3 \\ (x-3)(x+2)=x+2, & x>3 \end{cases}$$

$$\Rightarrow \begin{cases} x=4, & x<-2 \\ x=-2,2 & -2\le x<3 \\ x=4, & x<3 \end{cases}$$

Hence, $x=-2,2,4$

N =3

58. $f(x) = 2x^3 - 3(k+2)x^2 + 12kx - 7$

$f'(x) = 6[x^2 - (k+2)x + 2a] = 6(x-k)(x-2)$

(A) For f(x) to have only one real root $k = 2$ or $f(k)\, f(2) > 0 \Rightarrow k = 0, 1, 2, 3, 4, 5$

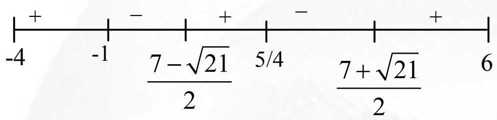
(B) For f(x) to have two equal roots, $k \neq 2$ and $f(k)\, f(2) = 0 \Rightarrow k = -1$.

(C) for f(x) to be invertible $f'(x) \geq 0\ \forall\ x \in R \Rightarrow k = 2$

(D) for f(x) to have three real and distinct roots, $k \neq 2$ and $f(k)\, f(2) < 0$

$(2k^3 - 3(k+2)k^2 + 12k^2 - 7)(16 - 12(k+2) + 24k - 7) < 0$

$\Rightarrow (k^3 - 6k^2 + 7)(4k - 5) > 0 \Rightarrow (k+1)(k^2 - 7k + 7)(4k - 5) > 0.$



$\Rightarrow k = -4, -3, -2, 6$

59. (A) $(I+A)^8 = {}^8C_0 I + {}^8C_1 IA + {}^8C_2 IA^2 + \dots\dots\dots + {}^8C_8 IA^8$

$= {}^8C_0 I + {}^8C_1 A + {}^8C_2 A + \dots\dots + {}^8C_8 A^8$

$= I + A\left({}^8C_1 + {}^8C_2 + \dots\dots + {}^8C_8\right)$

$= I + A(2^8 - 1) \Rightarrow \lambda = 2^8 - 1$

(B) $\left|\operatorname{adj}\left(A^{-1}\right)\right| = |A^{-1}|^2 = \frac{1}{|A|^2}$

$\left|\left(\operatorname{adj}\left(A^{-1}\right)\right)^{-1}\right| = \frac{1}{|\operatorname{adj}A^{-1}|} = |A|^2 = 2^2 = 4$

(C) $|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

$$\Rightarrow \quad |B| = \begin{vmatrix} a_{11} & \lambda^{-1}a_{12} & \lambda^{-2}a_{13} \\ \lambda a_{21} & a_{22} & \lambda^{-1}a_{23} \\ \lambda^2 a_{31} & \lambda a_{32} & a_{33} \end{vmatrix} = \frac{1}{\lambda^3}\begin{vmatrix} \lambda^2 a_{11} & \lambda a_{12} & a_{13} \\ \lambda^2 a_{21} & \lambda a_{22} & a_{23} \\ \lambda^2 a_{31} & \lambda a_{32} & a_{33} \end{vmatrix} = |A|$$

Hence, $|A| = |B| \Rightarrow \lambda = 1$.

(D) A diagonal matrix is commutative with every square matrix, if it is a scalar matrix.

So every diagonal element is 4.

$\therefore$ $|A| = 64$.

60. (A) Expand along C_1 to obtain

$$p(\theta) = \left(-\sqrt{2}\right)(-1) + (-1)(-2\sin\theta\cos\theta) + (-1)\left(\sin^2\theta - \cos^2\theta\right)$$

$$= \sqrt{2} + \sin 2\theta + \cos 2\theta = \sqrt{2} + \sqrt{2}\sin\left(2\theta + \frac{\pi}{4}\right)$$

$\therefore$ range of $p(\theta)$ is $\left[0, 2\sqrt{2}\right]$.

(B) Applying $R_2 \to R_2 + 4R_1, R_3 \to R_3 + 7R_1$, we get

$$q(\theta) = \begin{vmatrix} \sin 2\theta & -1 & 1 \\ \cos 2\theta + 4\sin 2\theta & 0 & 1 \\ 2 + 7\sin 2\theta & 0 & 2 \end{vmatrix} = 2\cos 2\theta + 8\sin 2\theta - 2 - 7\sin 2\theta$$

$= 2\cos 2\theta + \sin 2\theta - 2$

As $2\cos 2\theta + \sin 2\theta$ lies between $-\sqrt{5}$ to $\sqrt{5}$, we get range of $q(\theta)$ is $\left[-\sqrt{5} - 2, \sqrt{5} - 2\right]$.

(C) Using $C_1 \to C_1 + C_3$, we get

$$r(\theta) = 2\cos\theta \begin{vmatrix} 1 & \sin\theta & \cos\theta \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{vmatrix} = 2\cos\theta$$

$\therefore$ range of $r(\theta)$ is $[-2, 2]$

(D) Taking $\sec^2\theta$ common from R_1, we get

$$s(\theta) = \sec^2\theta \begin{vmatrix} 1 & \cos^2\theta & \cos^2\theta \\ \cos^2\theta & \cos^2\theta & \operatorname{cosec}^2\theta \\ 1 & \cos^2\theta & \cot^2\theta \end{vmatrix}$$

$R_3 \to R_3 - R_1$, we get

$$s(\theta) = \sec^2\theta \begin{vmatrix} 1 & \cos^2\theta & \cos^2\theta \\ \cos^2\theta & \cos^2\theta & \operatorname{cosec}^2\theta \\ 0 & 0 & \cot^2\theta - \cos^2\theta \end{vmatrix}$$

$$= \sec^2\theta\left(\cot^2\theta - \cos^2\theta\right)\left(\cos^2\theta - \cos^4\theta\right)$$

$$= \left(\cot^2\theta - \cos^2\theta\right)\sin^2\theta = \cos^2\theta - \cos^2\theta\sin^2\theta = \cos^4\theta$$

$\therefore$ range of $s(\theta)$ is $[0, 1]$