Sri Chaitanya IIT Academy, India

A.P, TELANGANA, KARNATAKA, TAMILNADU, MAHARASHTRA, DELHI, RANCHI

A right Choice for the Real Aspirant

ICON CENTRAL OFFICE, MADHAPUR-HYD

Sec: Sr. IPLCO JEE ADVANCED DATE : 13-12-15
TIME : 3:00 2012_P1 MODEL MAX MARKS : 210

KEY & SOLUTIONS

PHYSICS

1	C	2	B	3	A	4	D	5	A	6	C
7	D	8	B	9	B	10	D	11	BCD	12	AC
13	ABD	14	ACD	15	ABD	16	6	17	1	18	1
19	5	20	4								

CHEMISTRY

21	C	22	D	23	A	24	B	25	C	26	A
27	D	28	D	29	C	30	A	31	BC	32	ABC
33	ABD	34	AB	35	ABC	36	3	37	3	38	2
39	2	40	4								

MATHEMATICS

41	D	42	C	43	C	44	C	45	B	46	D
47	C	48	B	49	D	50	D	51	ACD	52	ABCD
53	ABCD	54	BC	55	ABD	56	1	57	1	58	2
59	1	60	4								

MATHS

41. $\sin^{-1}x\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$; $\cos^{-1}y\in[0,\pi]$

$\sec^{-1}z\in\left[0,\frac{\pi}{2}\right)\cup\left(\frac{\pi}{2},\pi\right]$

$\Rightarrow \sin^{-1}x+\cos^{-1}y+\sec^{-1}z\le\frac{\pi}{2}+\pi+\pi=\frac{5\pi}{2}$

Also $t^2-\sqrt{2\pi}t+3\pi$

$=t^2-2\sqrt{\frac{\pi}{2}}t+\frac{\pi}{2}-\frac{\pi}{2}+3\pi=\left(t-\sqrt{\frac{\pi}{2}}\right)+\frac{5\pi}{2}\ge\frac{5\pi}{2}$

The given inequality holds

$\Leftrightarrow x=1,\ y=-1,\ z=-1$

LHS = RHS $=\frac{5\pi}{2}$

$\Rightarrow x=1, y=-1, z=-1$ and

$t=\sqrt{\frac{\pi}{2}}\Rightarrow \cos^{-1}\left(\cos 5t^2\right)=\cos^{-1}\left(\cos\left(\frac{5\pi}{2}\right)\right)=\frac{\pi}{2}$

$\cos^{-1}\left(\min\{x,y,z\}\right)=\cos^{-1}(-1)=\pi$

45. In ΔABP $\frac{\sin\left(B+\frac{A}{2}\right)}{AP}=\frac{SinC}{AB}=\frac{\sin C}{2\sin C}=\frac{1}{2}$

$\Rightarrow AP=2\sin\left(B+\frac{A}{2}\right), BQ=2\sin\left(C+\frac{B}{2}\right)\ and\ CR=2\sin\left(A+\frac{C}{2}\right)$

$\sum AP\cos\frac{A}{2}=2\sin\left(B+\frac{A}{2}\right)\cos\frac{A}{2}+2\sin\left(C+\frac{B}{2}\right)\cos\frac{B}{2}+2\sin\left(A+\frac{C}{2}\right)\cos\frac{C}{2}$

$=2\left(\sin A+\sin B+\sin C\right)$

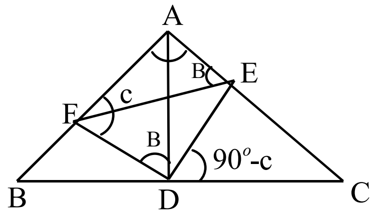
but $\sin A+\sin B+\sin C\le\frac{3\sqrt{3}}{2}$

$\therefore\le 3\sqrt{3}$

47. $\angle FDE=180^o-A$

$\angle FED=90^o-B$

$\angle EFD=90^o-C$



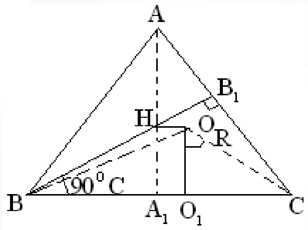
By sine rule in ΔDEF

$$\frac{\sin A}{EF}=\frac{\cos B}{FD}=\frac{\cos C}{DE}$$

$$\Delta = ADC+\Delta ADB$$

$$\Rightarrow \frac{\Delta}{EF}=\frac{1}{2}\left(b\frac{DE}{EF}+c\frac{DF}{EF}\right)=\frac{1}{2}\left(\frac{b\cos c}{\sin A}+\frac{c\cos B}{\sin A}\right)=R$$

49. d



Diag:

$OC=R$, $OB_1=R\cos A, HA_1=OO_1$

$BA_1=C\cos B$

$HA_1=BA_1\cot C$

From these relation we get

$\tan B\tan C=3$

In Δle, ABC, $\tan A+\tan B+\tan C=\ \tan A\tan B\tan C$

$\Rightarrow \tan B+\tan C=2\tan A$

In Δle ABC scalence acute $(\tan B-\tan C)^2>0$ for $\underline{|B}\neq\underline{|C}$

A.M > G.M. $(\tan B+\tan C)^2>4\tan B\tan C$

$$\Rightarrow 4\tan^2 A>12\Rightarrow \tan A>\sqrt{3}\Rightarrow A\in\left(\frac{\pi}{3},\frac{\pi}{2}\right)$$

50. $\sum \frac{a}{r_1}=\sum \frac{\cos\frac{A}{2}}{\cos\frac{B}{2}\cos\frac{C}{2}}=\sum\left(\tan\frac{A}{2}+\tan\frac{B}{2}\right)$

$-2\sum \tan\frac{A}{2}-4\sum\frac{r_1}{2s}-4\left(\frac{r_1+r_2+r_3}{a+b+c}\right)$

53. $\because \angle A=\alpha$ a constant. If 'I' is in-centre of ΔABC

$BIC-90^0+\frac{\alpha}{2}$ which is also fixed chord.

Hence 'I' lies on a fixed circle of which BC is a fixed chord

$\because \angle A=\alpha$ If 'H' is orthocentre $\angle BHC=180^0-\alpha$ which is fixed.

Hence, 'H' lies on a circle of which BC is fixed chord.

$\therefore$ $\angle A=\alpha, \angle HGK=\alpha$ where H,K are points of trisection of base BC which are fixed.

$\therefore$ The fixed line segment H,K subtends a constant angle α at a variable point G.

Hence, locus of centroid is also lies on circle.

55. (A, B, D)

$-\frac{\pi}{2}\le \sin^{-1}x\le\frac{\pi}{2}$

$\Rightarrow \left[\sin^{-1}x\right]\in\{-2,-1,0,1\}$

$1+\left[\sin^{-1}x\right]\in\{-1,0,1,2\}$

$0\le\cos^{-1}x\le\pi$

$\left[\cos^{-1}x\right]\in\{0,1,2,3\}$

$1+\left[\sin^{-1}x\right]>\left[\cos^{-1}x\right]$ only if $1+\left[\sin^{-1}x\right]=1$ and $\left[\cos^{-1}x\right]=0$

or $1+\left[\sin^{-1}x\right]=2$ and $\left[\cos^{-1}x\right]=0$ or 1.

Case I

$1+\left[\sin^{-1}x\right]=1$ and $\left[\cos^{-1}x\right]=0$

$\Rightarrow \left[\sin^{-1}x\right]=0 \quad 0\le\cos^{-1}x<1$

$0\le\sin^{-1}x<1$ and $\cos 1<x\le 1$

$0\le x<\sin 1$

$\Rightarrow \cos 1<x<\sin 1$ … (i)

Case II

$1+\left[\sin^{-1}x\right]=2$ and $\left[\cos^{-1}x\right]=0$ or 1

$\left[\sin^{-1} x\right] = 1 \qquad 0 \le \cos^{-1} x < 2$

$1 \le \sin^{-1} x \le \frac{\pi}{2} \qquad \cos 2 < x \le 1$

$\sin 1 \le x \le 1$

$\Rightarrow \quad \sin 1 \le x \le 1 \quad \ldots$ (ii)

from (i) & (ii)

$\Rightarrow \quad \cos 1 < x \le 1.$

56. Let us denote the nth terms of the series by t_n then we have

$$t_n = \cot^{-1} 2n^2 = \tan^{-1} \frac{1}{2n^2}$$

$$= \tan^{-1}\left(\frac{2}{4n^2}\right) = \tan^{-1}\left(\frac{2}{4n^2 - 1 + 1}\right)$$

$$= \tan^{-1}\left[\frac{(2n+1)-(2n-1)}{(2n-1)(2n+1)+1}\right]$$

$$= \tan^{-1}(2n+1) - \tan^{-1}(2n-1) \quad \text{or}$$

$$t_n = \tan^{-1}(2n+1) - \tan^{-1}(2n-1)$$

Putting $n = 1, 2, 3, \ldots etc.$ in the above equation we have each terms as

$$t_1 = \tan^{-1} 3 - \tan^{-1} 1$$

$$t_2 = \tan^{-1} 5 - \tan^{-1} 3$$

$$t_3 = \tan^{-1} 7 - \tan^{-1} 5$$

....

....

$$t_n = \tan^{-1}(2n+1) - \tan^{-1}(2n-1)$$

....

adding, $S_n = \tan^{-1}(2n+1) - \tan^{-1} 1$

as $n \to \infty$, $2n+1 \to \infty$ and $\tan^{-1} 1 = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

$K = 1$.

58. We have

$\therefore \quad r = OI$

$\Rightarrow \quad R\sqrt{1-8\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}} = 4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

$\Rightarrow \quad \sqrt{1-x} = 4x$

Where $x = \sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

On solving, we get $x = \frac{\sqrt{2}-1}{4}$

Now use $\cos A + \cos B + \cos C = 1 + 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

59. $\sin^{-1} x \in \left(0, \frac{\pi}{2}\right)$

$\cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x \Rightarrow \cos^{-1} x \in \left(0, \frac{\pi}{2}\right)$

$\Rightarrow \sin\left(\cos^{-1} x\right) = \cos\left(\sin^{-1} x\right) = \frac{1}{\sqrt{1-x^2}}$

Thus $\cos^{-1}\left(\sin\left(\cos^{-1} x\right)\right) + \sin^{-1}\left(\cos\left(\sin^{-1} x\right)\right) = \frac{\pi}{2}$

$\Rightarrow \tan\left(\frac{\pi}{4}\right) = 1.$