



APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

LOGIC CIRCUIT DESIGN (ECT-203)

MODULE 2 **NOTES**

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3	Sum of Product and Product of Sum	SUM OF PRODUCT PRODUCT OF SUM	https://youtu.be/MEUyTF9Fv2I
4	SOP and POS Numerical Problems	SUM OF PRODUCT PRODUCT OF SUM	https://youtu.be/_4RFZ0s3Qks
5	NAND NOR Implementation	NAND NOR IMPLEMENTATION	https://youtu.be/TNj8W5XXH-Q
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MODULE 2 (TOPIC-1)

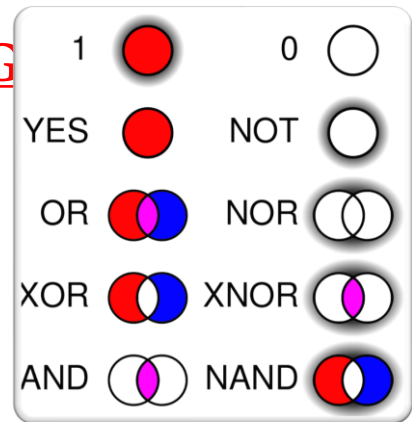
THE LOGIC GATES

https://youtu.be/MY8pLZ_MOI

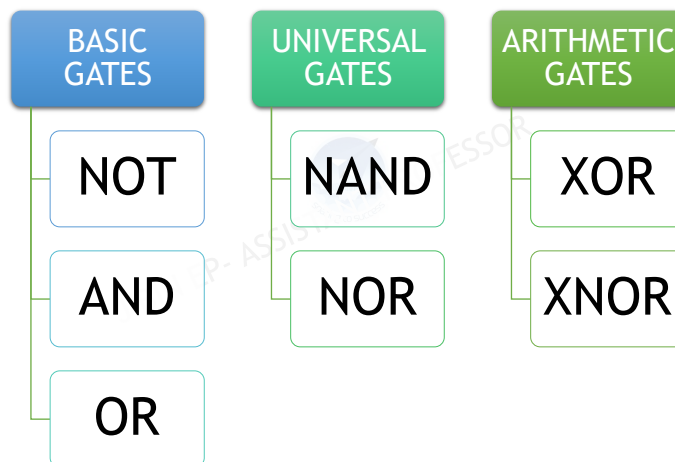


DEFENITION OF LOGIC G

- It is a physical device which preforms logic operation on one or more logical input and produces a single output

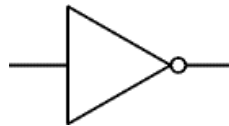


Classification of Logic Gates



BASIC GATES

1. NOT GATE (INVERTER)



A NOT gate can only have one input and the output is the inverse of the input..

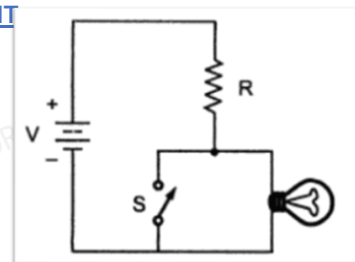
1. TRUTH TABLE

INPUT (A)	OUTPUT (Y)
0	1
1	0

2. BOOLEAN EXPRESSION

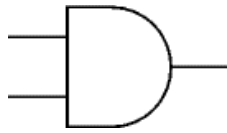
$$Y = \bar{A}$$

4. SWITCHING CIRCUIT



INPUT	OUTPUT
Switch Open	Lamp ON
Switch Close	Lamp OFF

2. AND GATE



An AND gate can have two or more inputs, its output is true if all inputs are true. The output Q is true if input A AND input B are both true

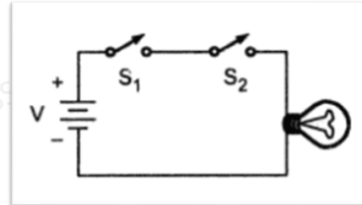
1. TRUTH TABLE

INPUT (A)	INPUT (B)	OUTPUT(Y)
0	0	0
0	1	0
1	0	0
1	1	1

2. BOOLEAN EXPRESSION

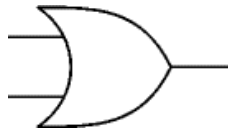
$$Y=AB$$

3. SWITCHING CIRCUIT



INPUT S1	INPUT S2	OUTPUT
Open	Open	Lamp OFF
Open	Close	Lamp OFF
Close	Open	Lamp OFF
Close	Close	Lamp ON

3. OR GATE



An OR gate can have two or more inputs, its output is true if at least one input is true.

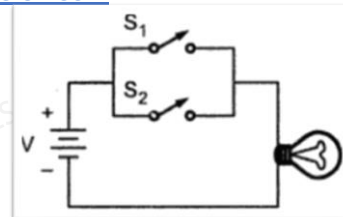
1. TRUTH TABLE

INPUT (A)	INPUT (B)	OUTPUT(Y)
0	0	0
0	1	1
1	0	1
1	1	1

2. BOOLEAN EXPRESSION

$$Y=A+B$$

3. SWITCHING CIRCUIT



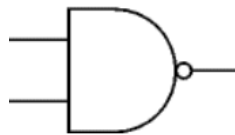
INPUT S1	INPUT S2	OUTPUT
Open	Open	Lamp OFF
Open	Close	Lamp ON
Close	Open	Lamp ON
Close	Close	Lamp ON

UNIVERSAL GATES

NAND and NOR gates are widely known to be universal logic gates, meaning that any other logic gate can be made from NAND or NOR gates

3. NAND GATE

(NOT + AND)



This is an AND gate with the output inverted, A NAND gate can have two or more inputs, its output is true if NOT all inputs are true.

1. TRUTH TABLE

INPUT (A)	INPUT (B)	OUTPUT(Y)
0	0	1
0	1	1
1	0	1
1	1	0

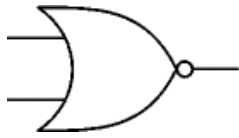
2. BOOLEAN EXPRESSION

$$Y = \overline{AB}$$



3. NOR GATE

(NOT + OR)



This is an OR gate with the output inverted, A NOR gate can have two or more inputs, its output is true if no inputs are true

1. TRUTH TABLE

INPUT (A)	INPUT (B)	OUTPUT(Y)
0	0	1
0	1	0
1	0	0
1	1	0

2. BOOLEAN EXPRESSION

$$Y = \overline{A + B}$$



ARITHMETIC GATES

ARITHMETIC GATES

3.XOR GATE (EXCLUSIVE OR GATE)



This is an arithmetic gate, which produces low output when the inputs are same

1. TRUTH TABLE

INPUT (A)	INPUT (B)	OUTPUT(Y)
0	0	0
0	1	1
1	0	1
1	1	0

2. BOOLEAN EXPRESSION

$$Y = A \oplus B$$



3.XNOR GATE (EXCLUSIVE NOR GATE)

This is an arithmetic gate, which produces high output when the inputs are same

1. TRUTH TABLE

INPUT (A)	INPUT (B)	OUTPUT(Y)
0	0	1
0	1	0
1	0	0
1	1	1

2. BOOLEAN EXPRESSION

$$Y = A \odot B$$



MODULE 2 (TOPIC-2)

BOOLEAN ALGEBRA

<https://youtu.be/djWQBrgtrxs>  YouTube

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Inversion Law

This law uses the NOT operation. The inversion law states that double inversion of variable results in the original variable itself.

- $A + \bar{\bar{A}} = 1$

AND Law

These laws use the AND operation. Therefore they are called AND laws

- $A \cdot 0 = 0$
- $A \cdot 1 = A$
- $A \cdot A = A$
- $A \cdot \bar{A} = 0$

OR Law

These laws use the OR operation. Therefore they are called OR laws.

- $A + 0 = A$
- $A + 1 = 1$
- $A + A = A$
- $A + \bar{A} = 1$

Commutative Law

Any binary operation which satisfies the following expression is referred to as a commutative operation. Commutative law states that changing the sequence of the variables does not have any effect on the output of a logic circuit.

- $A \cdot B = B \cdot A$
- $A + B = B + A$

Associative Law

It states that the order in which the logic operations are performed is irrelevant as their effect is the same.

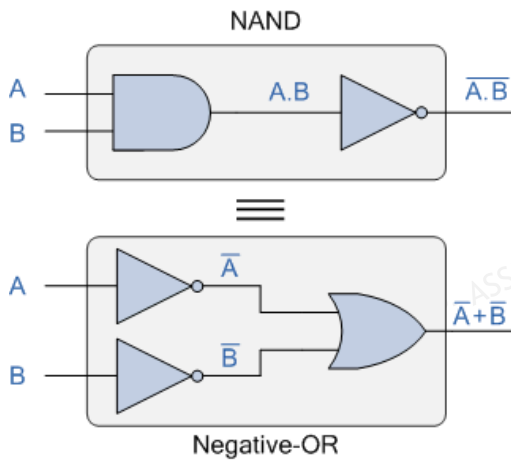
- $(A \cdot B) \cdot C = A \cdot (B \cdot C)$
- $(A + B) + C = A + (B + C)$

Distributive Law

Distributive law states the following conditions:

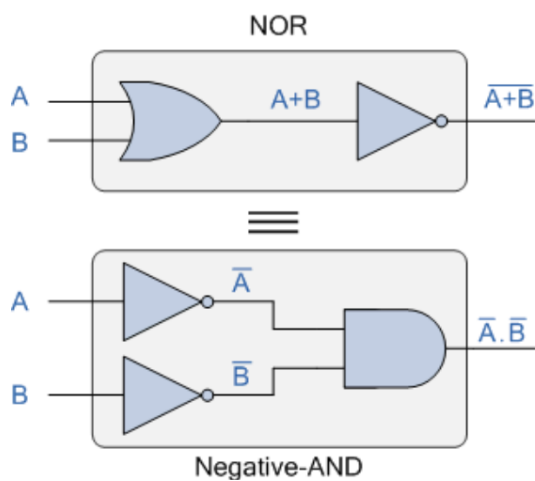
- $A \cdot (B + C) = (A \cdot B) + (A \cdot C)$
- $A + (B \cdot C) = (A + B) \cdot (A + C)$

DeMorgan's First Law Implementation using Logic Gates



$$\overline{A \cdot B} = \overline{A} + \overline{B}$$

DeMorgan's Second Theorem



$$\overline{A + B} = \overline{A} \cdot \overline{B}$$

MODULE 2 (TOPIC-3,4)

SUM OF PRODUCT PRODUCT OF SUM

<https://youtu.be/FOGczanvVBk>
https://youtu.be/_4RFZ0s3Qks



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Sum Of Product (SOP)

- Each SOP Consist of two or more product terms(AND) that **ORed** together

$$1. \quad f(A, B, C) = \overline{A} B C + A \overline{B} \overline{C}$$

product terms

$$2. \quad f(P, Q, R, S) = \overline{P} Q + Q R + R S$$

Product terms

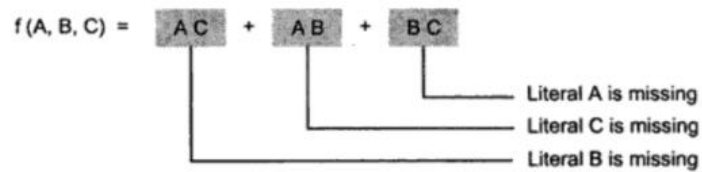
Canonical SOP

➡ **Example 2.3 :** Convert the given expression in standard SOP form.

$$f(A, B, C) = AC + AB + BC$$

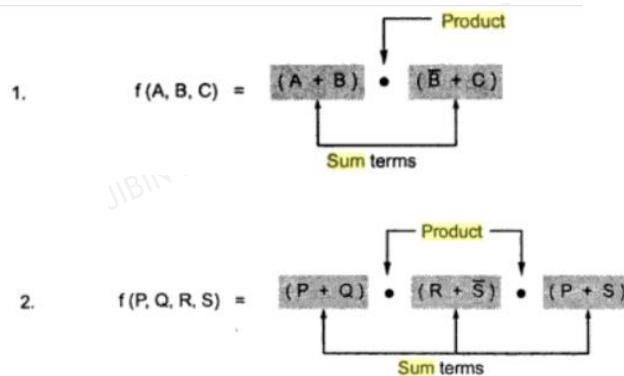
Solution :

Step 1 : Find the missing literal/s in each product term



Product of Sum (POS)

- Each POS Consist of Two or more Sum Terms(OR) that **AND** ed Together



Canonical POS

►►► **Example 2.5 :** Convert the given expression in standard POS form.

$$f(A, B, C) = (A + B)(B + C)(A + C)$$

Solution :

Step 1 : Find the missing literal/s in each **sum** term

$$f(A, B, C) = (A + B) \quad (B + C) \quad (A + C)$$

Literal B is missing
Literal A is missing
Literal C is missing

Mintem and Maxterm

- Each individual term in Standard SOP is called **Mintem**
- Each individual term in Standard POS is called **Maxterm**

Variables			Minterms	Maxterms
A	B	C	m_i	M_i
0	0	0	$\bar{A} \bar{B} \bar{C} = m_0$	$A + B + C = M_0$
0	0	1	$\bar{A} \bar{B} C = m_1$	$A + B + \bar{C} = M_1$
0	1	0	$\bar{A} B \bar{C} = m_2$	$A + \bar{B} + C = M_2$
0	1	1	$\bar{A} B C = m_3$	$A + \bar{B} + \bar{C} = M_3$
1	0	0	$A \bar{B} \bar{C} = m_4$	$\bar{A} + B + C = M_4$
1	0	1	$A \bar{B} C = m_5$	$\bar{A} + B + \bar{C} = M_5$
1	1	0	$A B \bar{C} = m_6$	$\bar{A} + \bar{B} + C = M_6$
1	1	1	$A B C = m_7$	$\bar{A} + \bar{B} + \bar{C} = M_7$

MODULE 2 (TOPIC-5)

NAND NOR IMPLEMENTATION

<https://youtu.be/TNj8WSXXH-Q>



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CONTENTS

NAND

- NOT
- AND
- OR
- NOR

NOR

- NOT
- AND
- OR
- NAND

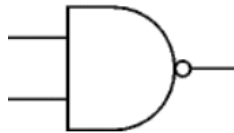
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NAND GATE

3. NAND GATE

(NOT + AND)



This is an AND gate with the output inverted, A NAND gate can have two or more inputs, its output is true if NOT all inputs are true.

1. TRUTH TABLE

INPUT (A)	INPUT (B)	OUTPUT(Y)
0	0	1
0	1	1
1	0	1
1	1	0

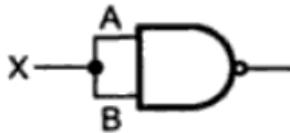
2. BOOLEAN EXPRESSION

$$Y = \overline{AB}$$



NOT FUNCTION

An inverter can be made from NAND gate by connecting all the input together and creating, in effect a single common input



PROOF

$$Y = \overline{AB}$$

$$Y = \overline{XX}$$

DEMORGANS LAW

$$Y = \bar{X} + \bar{X}$$

REFERENCE : $A+A=A$

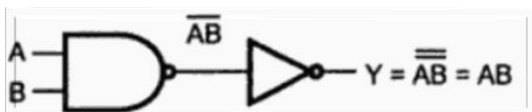
$$Y = \bar{X}$$

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AND FUNCTION

An AND Function is Generated by simply inverting the output of NAND Gate



A	B	AB
0	0	0
0	1	0
1	0	0
1	1	1



A	B	AB	AB-bar-bar
0	0	1	0
0	1	1	0
1	0	1	0
1	1	0	1

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OR FUNCTION

An OR Function is Generated by simply inverting both the inputs of NAND Gate

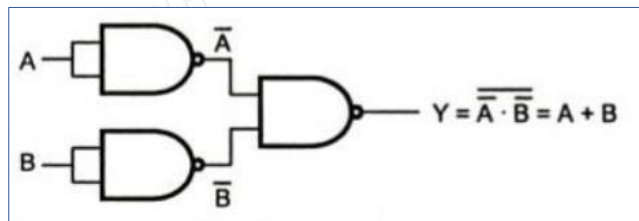
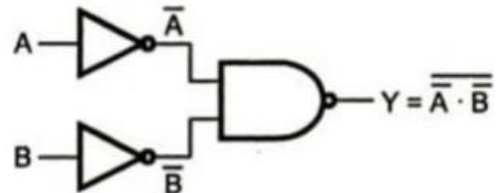
$$Y = A + B$$

$$= \overline{\overline{A}} + \overline{\overline{B}}$$

$$= \overline{\overline{A} \cdot \overline{B}}$$

$$[\overline{\overline{A}} = A]$$

DeMorgan's Theorem 1



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NOR FUNCTION

An NOR Function is Generated by simply inverting both the inputs of NAND Gate and invert the output

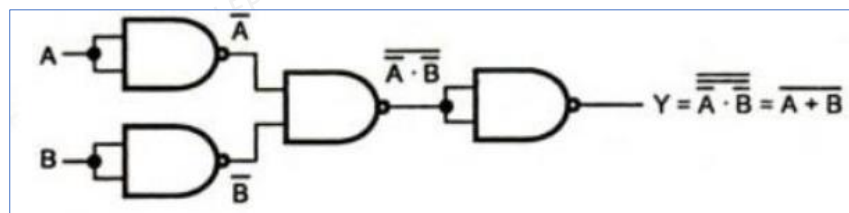
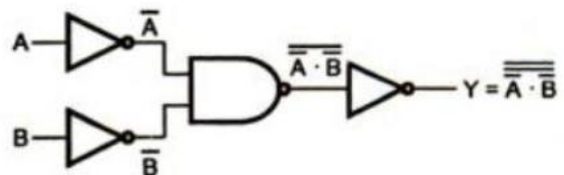
$$Y = \overline{A + B}$$

$$= \overline{\overline{A} \cdot \overline{B}}$$

$$= \overline{\overline{\overline{A} \cdot \overline{B}}}$$

DeMorgan's Theorem 2

$$[\overline{\overline{A}} = A]$$



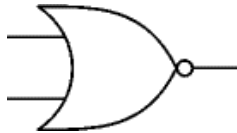
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NOR GATE

3. NOR GATE

(NOT + OR)



This is an OR gate with the output inverted, A NOR gate can have two or more inputs, its output is true if no inputs are true

1. TRUTH TABLE

INPUT (A)	INPUT (B)	OUTPUT(Y)
0	0	1
0	1	0
1	0	0
1	1	0

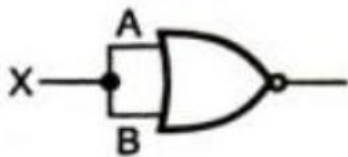
2. BOOLEAN EXPRESSION

$$Y = \overline{A + B}$$



NOT FUNCTION

An inverter can be made from NOR gate by connecting all the input together and creating, in effect a single common input



PROOF

$$Y = \overline{A + B}$$

$$Y = \overline{X + X}$$

REFERENCE : $A + A = A$

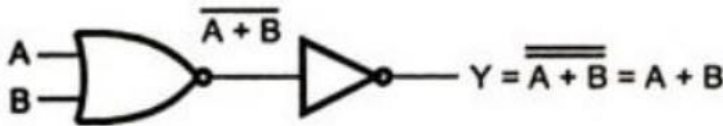
$$Y = \bar{X}$$

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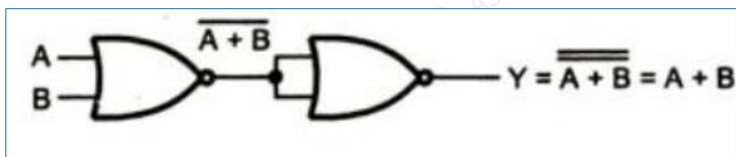
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OR FUNCTION

An OR Function is Generated by simply inverting the output of NOR Gate



A	B	$A \cdot B$
0	0	0
0	1	0
1	0	0
1	1	1



A	B	$\bar{A} + \bar{B}$	$\overline{\bar{A} + \bar{B}}$
0	0	1	0
0	1	1	0
1	0	1	0
1	1	0	1

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AND FUNCTION

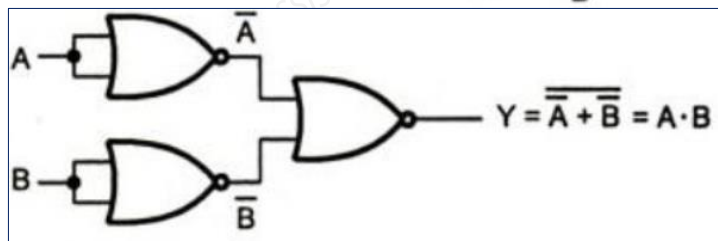
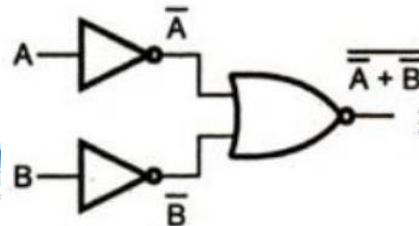
An AND Function is Generated by simply inverting both the inputs of NOR Gate

$$Y = A \cdot B$$

$$= \overline{\overline{A}} \cdot \overline{\overline{B}}$$

$$[\overline{\overline{A}} = A]$$

$$= \overline{\overline{A} + \overline{B}} \quad \text{DeMorgan's Theorem 2}$$



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NAND FUNCTION

An NAND Function is Generated by simply inverting both the inputs of NOR Gate and invert the output

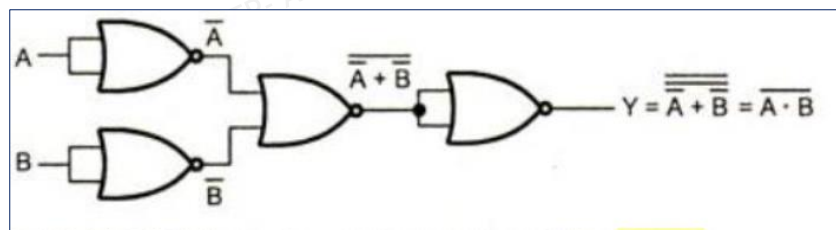
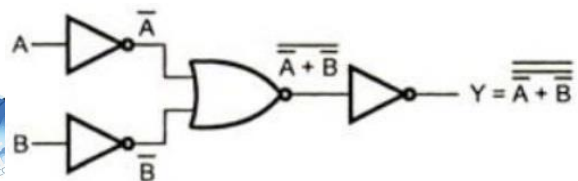
$$Y = \overline{A \cdot B}$$

$$= \overline{\overline{A} + \overline{B}}$$

DeMorgan's Theorem 1

$$= \overline{\overline{\overline{A} + \overline{B}}}$$

$$[\overline{\overline{A}} = A]$$

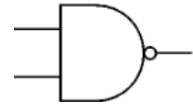


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CONCLUSION

3.NAND GATE



NOT

- Connecting all the input together

AND

- Inverting the output of NAND Gate

OR

- Inverting the Inputs of NAND Gate

NOR

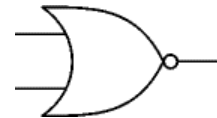
- Inverting the Inputs of NAND Gate and invert the output

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CONCLUSION

3.NOR GATE



NOT

- Connecting all the input together

OR

- Inverting the output of NOR Gate

AND

- Inverting the Inputs of NOR Gate

NAND

- Inverting the Inputs of NOR Gate and invert the output

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MODULE 2 (TOPIC-6,7,8)

KARNAUGH MAP

<https://youtu.be/ezp0B2F6L0k>

<https://youtu.be/UoxRJkMJ-pl> <https://youtu.be/ZMARz-Xr0F4>



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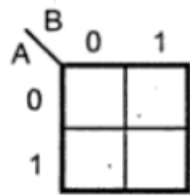
Karnaugh Map

- We have simplified the Boolean functions using Boolean postulates and theorems. It is a time consuming process and we have to re-write the simplified expressions after each step.
- To overcome this difficulty, **Karnaugh introduced a method for simplification of Boolean functions in an easy way.** This method is known as Karnaugh map method or K-map method. It is a graphical method, which consists of 2^n cells for 'n' variables. The adjacent cells are differed only in single bit position.

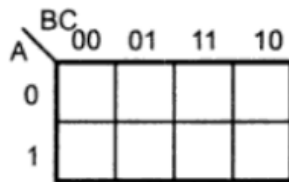
TYES OF KMAP

2 Variable	<ul style="list-style-type: none"> • SOP • POS
3 Variable	<ul style="list-style-type: none"> • SOP • POS
4 Variable	<ul style="list-style-type: none"> • SOP • POS
5 Variable	<ul style="list-style-type: none"> • SOP • POS

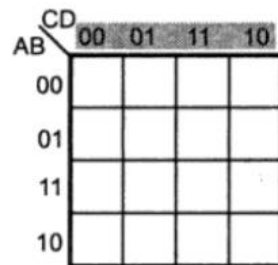
TYPES OF KMAP



2-Variable map
(4 cells)



3-Variable map
(8 cells)



4-Variable map
(16 cells)

Values of CD
in gray code
sequence

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	$\bar{A}\bar{B}\bar{C}\bar{D}$ 0	$\bar{A}\bar{B}\bar{C}D$ 1	$\bar{A}\bar{B}C\bar{D}$ 3	$\bar{A}\bar{B}CD$ 2
$\bar{A}B$	$\bar{A}B\bar{C}\bar{D}$ 4	$\bar{A}B\bar{C}D$ 5	$\bar{A}BC\bar{D}$ 7	$\bar{A}BCD$ 6
AB	$AB\bar{C}\bar{D}$ 12	$AB\bar{C}D$ 13	$ABC\bar{D}$ 15	$ABCD$ 14
$A\bar{B}$	$A\bar{B}\bar{C}\bar{D}$ 8	$A\bar{B}\bar{C}D$ 9	$A\bar{B}C\bar{D}$ 11	$A\bar{B}CD$ 10

4-Variable map
(d)

TYPES OF KMAP

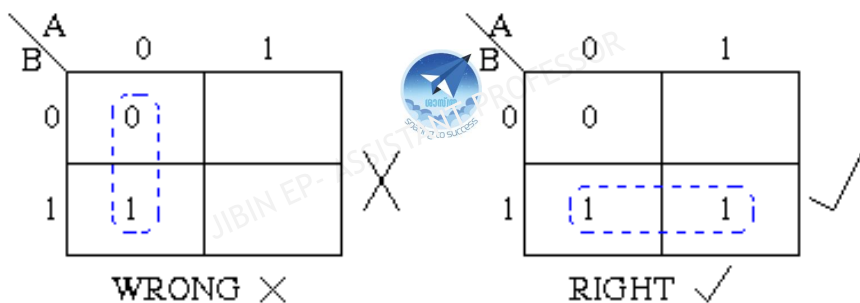
		CDE							
		000	001	011	010	110	111	101	100
A B	00								
	01								
	11								
	10								

5-variable Karnaugh map (Gray code)

RULES OF KMAP

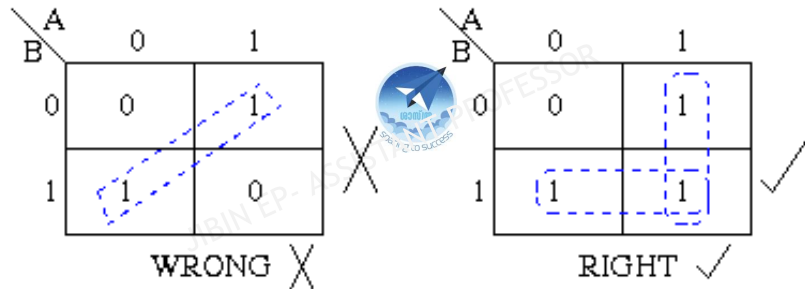
RULE 1

Groups may not include any cell containing a **zero**



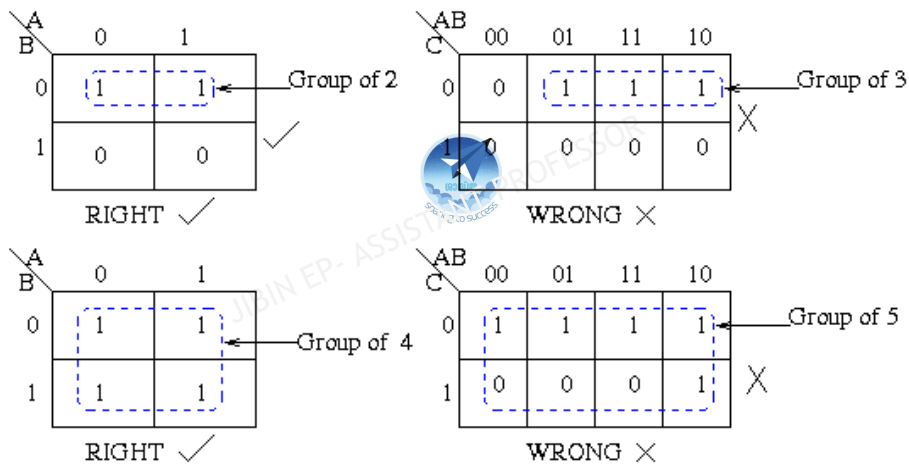
RULE 2

Groups may be horizontal or vertical, but not diagonal



RULE 3

Groups must contain **1, 2, 4, 8, 16** or in general 2^n cells.



RULE 4

Each group should be as large as possible.

$\backslash AB$	00	01	11	10
C				
0	1	1	1	1
1	0	0	1	1

RIGHT ✓

$\backslash AB$	00	01	11	10
C				
0	1	1	1	1
1	0	0	1	1

WRONG ✗

(Note that no Boolean laws broken, but not sufficiently minimal)

RULE 5

Each cell containing a one must be in at least one group.

$\backslash AB$	00	01	11	10
C				
0	0	0	1	1
1	0	0	0	1



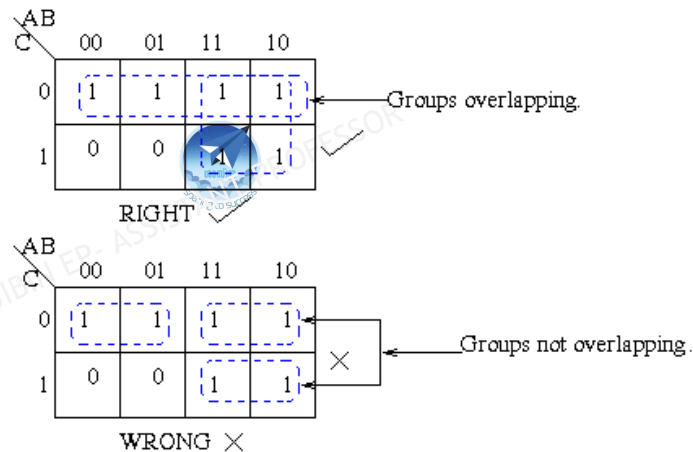
Group I

Group II

1 present in at least one group.

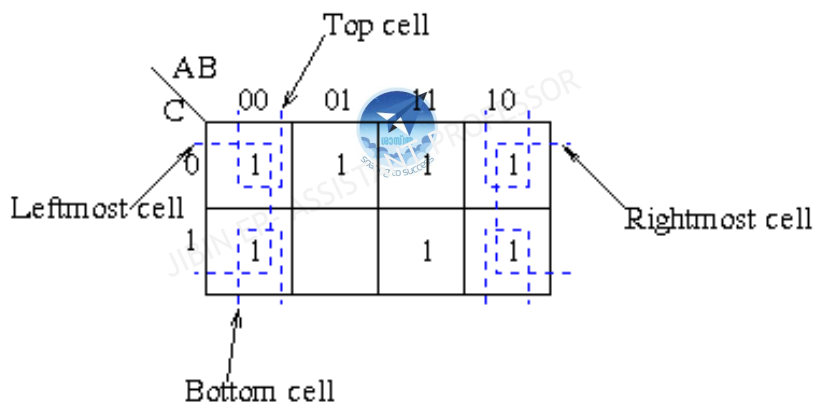
RULE 6

Groups may overlap



RULE 7

Groups may wrap around the table. The leftmost cell in a row may be grouped with the rightmost cell and the top cell in a column may be grouped with the bottom cell.



RULE 8

There should be as few groups as possible, as long as this does not contradict any of the previous rules.

AB \ C	00	01	11	10
0	1	1	1	1
1	0	0	1	1

RIGHT ✓

AB \ C	00	01	11	10
0	1	1	1	1
1	0	0	1	1

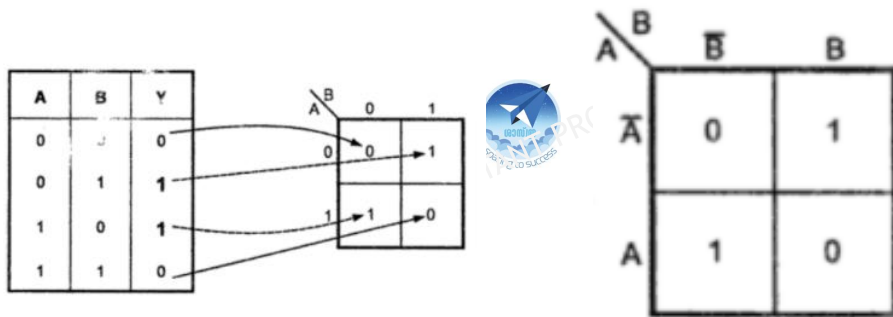
WRONG ✗

RULES OF KMAP

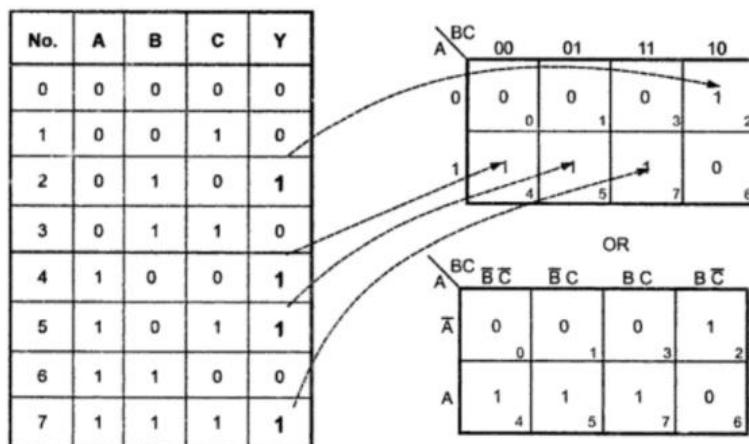
The Karnaugh map uses the following rules for the simplification of expressions by grouping together adjacent cells containing ones

- No zeros allowed.
- No diagonals
- Only power of 2 number of cells in each group.
- Groups should be as large as possible.
- Every one must be in at least one group.
- Overlapping allowed.
- Wrap around allowed.
- Only power of 2 number of cells in each group.

Representation of Truth Table on KMAP



Representation of Truth Table on KMAP



Representation of Truth Table on KMAP

No.	A	B	C	D	Y
0	0	0	0	0	1
1	0	0	0	1	0
2	0	0	1	0	0
3	0	0	1	1	1
4	0	1	0	0	1
5	0	1	0	1	1
6	0	1	1	0	0
7	0	1	1	1	1
8	1	0	0	0	0
9	1	0	0	1	0
10	1	0	1	0	1
11	1	0	1	1	0
12	1	1	0	0	1
13	1	1	0	1	0
14	1	1	1	0	1
15	1	1	1	1	1

CD	00	01	11	10
AB				
00	1 0	0 1	1 3	0 2
01	1 4	1 5	1 7	0 6
11	1 12	0 13	1 15	1 14
10	0 8	0 9	0 11	1 10

CD	00	01	11	10
AB				
00	1 0	0 1	1 3	0 2
01	1 4	1 5	1 7	0 6
11	1 12	0 13	1 15	1 14
10	0 8	0 9	0 11	1 10

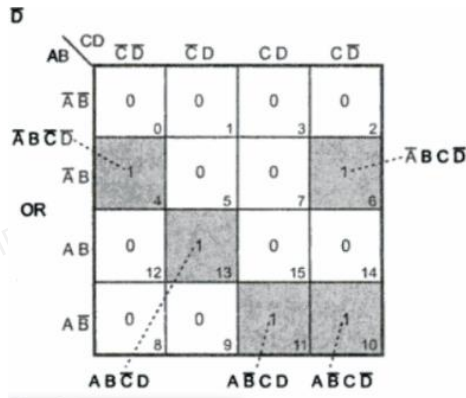
Representation of Boolean Expression on KMAP

Plot Boolean expression $Y = AB\bar{C} + ABC + \bar{A}\bar{B}C$ on the Karnaugh map.

BC	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
A				
\bar{A}	0 0	1 1	0 3	0 2
A	0 4	0 5	1 7	1 6

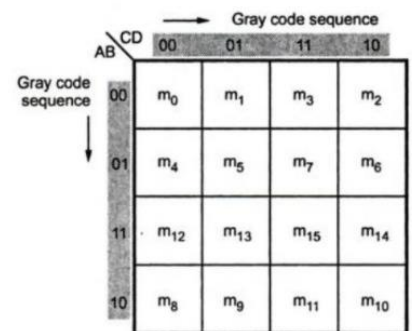
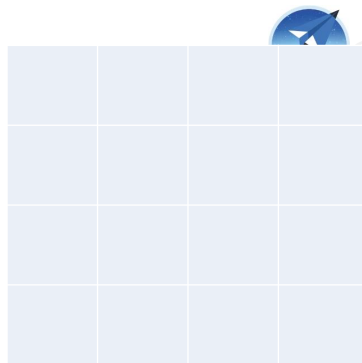
Representation of Boolean Expression on KMAP

$Y = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}C\bar{D} + \bar{A}B\bar{C}\bar{D} + A\bar{B}C\bar{D} + AB\bar{C}\bar{D}$ on the Karnaugh map.



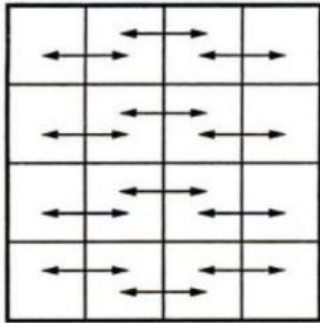
Representation of Boolean Expression on KMAP

$$\sum m(1,5,7,9,15)$$

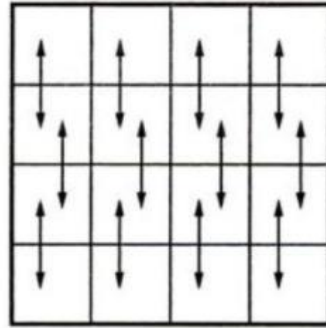


(d) 4 Variable map

Grouping of Cells



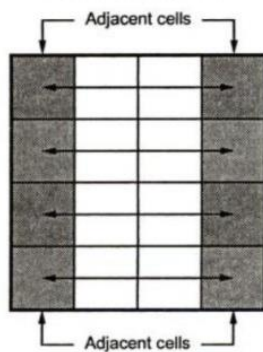
(a) Neighbouring cells in the row are adjacent



(b) Neighbouring cells in the column are adjacent

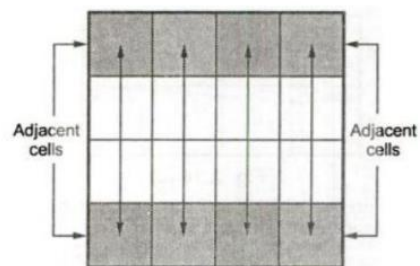
Grouping of Cells

(a) Neighbouring cells in the row are adjacent



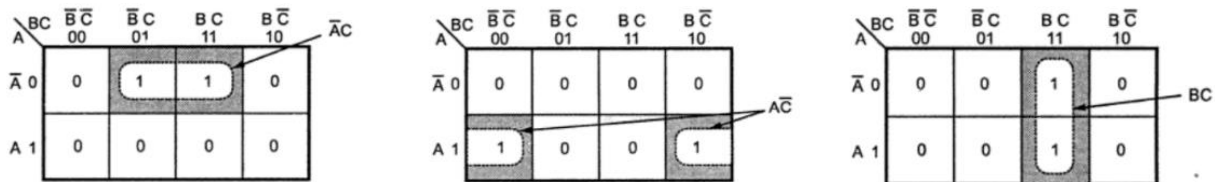
(c) Leftmost and corresponding rightmost cells are adjacent

(b) Neighbouring cells in the column are adjacent

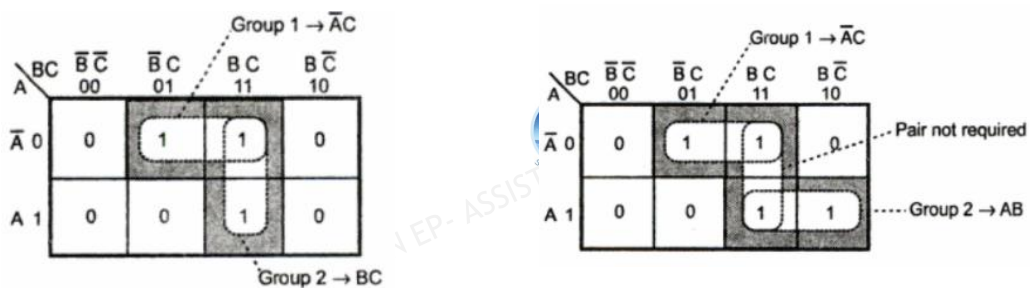


(d) Top and corresponding bottom cells are adjacent

Grouping of two adjacent ones



Grouping of two adjacent ones



Grouping of Four adjacent ones

BC	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
A	00	01	11	10
\bar{A} 0	0	0	0	0
A 1	1	1	1	1

Grouping of four adjacent ones in the row where A=1, resulting in the term A .

AB	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$ 00	0	0	1	0
$\bar{A}B$ 01	0	0	1	0
AB 11	0	0	1	0
$A\bar{B}$ 10	0	0	1	0

Grouping of four adjacent ones in the column where CD=1, resulting in the term CD .

Grouping of Four adjacent ones

AB	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$ 00	0	0	0	0
$\bar{A}B$ 01	0	1	1	0
AB 11	0	1	1	0
$A\bar{B}$ 10	0	0	0	0

Grouping of four adjacent ones in the middle 2x2 square, resulting in the term $Y = BD$.

(c) $Y = BD$

AB	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$ 00	0	0	0	0
$\bar{A}B$ 01	0	0	0	0
AB 11	1	0	0	1
$A\bar{B}$ 10	1	0	0	1

Grouping of four adjacent ones in the corners, resulting in the term $Y = A\bar{D}$.

(d) $Y = A\bar{D}$

Grouping of Four adjacent ones

		CD	$\bar{C}\bar{D}$	$\bar{C}D$	$C\bar{D}$	CD	$C\bar{D}$
AB	$\bar{A}\bar{B}$	00	1	0	0	1	1
	$\bar{A}B$	01	0	0	0	0	0
	AB	11	0	0	0	0	0
	$A\bar{B}$	10	1	0	0	1	1

Fig. 2.15 (e) $Y = \bar{B}\bar{D}$

Grouping of Four adjacent ones

		CD	$\bar{C}\bar{D}$	$\bar{C}D$	$C\bar{D}$	CD	$C\bar{D}$
AB	$\bar{A}\bar{B}$	00	0	0	0	0	0
	$\bar{A}B$	01	0	0	0	0	0
	AB	11	1	1	1	1	1
	$A\bar{B}$	10	0	1	1	1	1

Group 1 → AB
Group 2 → AD
Group 3 → AC

(f) $Y = AB + AD + AC$

1	1	1	1
	1	1	1

Grouping of Eight adjacent ones

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$ 00	0	0	0	0
$\bar{A}B$ 01	1	1	1	1
AB 11	1	1	1	1
$A\bar{B}$ 10	0	0	0	0

(a) $Y = B$

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$ 00	0	1	1	0
$\bar{A}B$ 01	0	1	1	0
AB 11	0	1	1	0
$A\bar{B}$ 10	0	1	1	0

(b) $Y = D$

Grouping of Eight adjacent ones

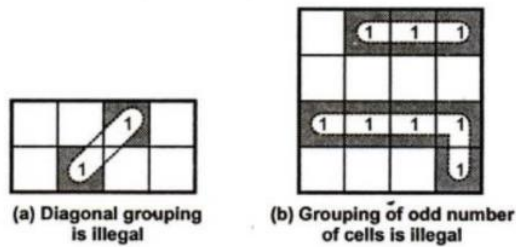
AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$ 00	1	1	1	1
$\bar{A}B$ 01	0	0	0	0
AB 11	0	0	0	0
$A\bar{B}$ 10	1	1	1	1

(c) $Y = \bar{B}$

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$ 00	1	0	0	1
$\bar{A}B$ 01	1	0	0	1
AB 11	1	0	0	1
$A\bar{B}$ 10	1	0	0	1

(d) $Y = \bar{D}$

Illegal Grouping



DON'T CARE TERMS (INCOMPLETELY SPECIFIED FUNCTIONS)

- In Some logic circuits, certain input conditions never occur. Therefore the corresponding output never appears
- In Such cases output level is not defined, it can be either HIGH or LOW
- These output levels are indicated by 'X' or 'd' in the truth tables and are called **don't care conditions**

DON'T CARE TERMS (INCOMPLETELY SPECIFIED FUNCTIONS)

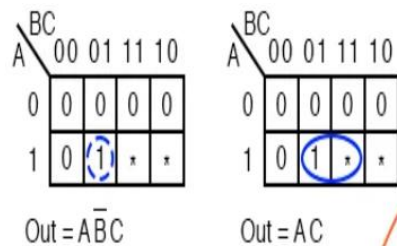
A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	X
1	1	1	X

Here the output levels are defined for input conditions from 000 to 101

For remaining two conditions of input, output is not defined. Hence these are called **Don't care conditions**

DON'T CARE TERMS

- When forming groups of cells, treat the don't care cell as either a 1 or a 0, or ignore the don't cares.
- This is helpful if it allows us to form a larger group than would otherwise be possible without the don't cares. There is no requirement to group all or any of the don't cares.
- Only use them in a group if it simplifies the logic.



Reduce the following Boolean Expression using KMAP

$$f(A, B, C) = \sum m(0,1,3,7)+d(2,5)$$

A \ BC				
	$\overline{B}\overline{C}$ 00	$\overline{B}C$ 01	$B\overline{C}$ 11	BC 10
\overline{A} 0	1 ₀	1 ₁	1 ₃	X ₂
A 1	0 ₄	X ₅	1 ₇	0 ₆



A \ BC				
	$\overline{B}\overline{C}$ 00	$\overline{B}C$ 01	$B\overline{C}$ 11	BC 10
\overline{A} 0	1	1	1	1
A 1	0	1	1	0

Groupings: \overline{A} (row 0), C (column 1 and 2)

Reduce the following Boolean Expression using KMAP

$$f(A, B, C, D) = \sum m(0,7,8,9,10,12)+d(2,5,13)$$

WX \ YZ				
	$\overline{Y}\overline{Z}$ 00	$\overline{Y}Z$ 01	YZ 11	$Y\overline{Z}$ 10
$\overline{W}\overline{X}$ 00	1	0	0	X
$\overline{W}X$ 01	0	X	1	0
WX 11	1	X	0	0
$W\overline{X}$ 10	1	1	0	1



WX \ YZ				
	$\overline{Y}\overline{Z}$ 00	$\overline{Y}Z$ 01	YZ 11	$Y\overline{Z}$ 10
$\overline{W}\overline{X}$ 00	1	0	0	1
$\overline{W}X$ 01	0	1	1	0
WX 11	1	1	0	0
$W\overline{X}$ 10	1	1	0	1

Groupings: $\overline{X}\overline{Z}$ (row 0, col 0 and 4), $\overline{W}XZ$ (row 1, col 2 and 3), $W\overline{Y}$ (row 2 and 3, col 0 and 1)

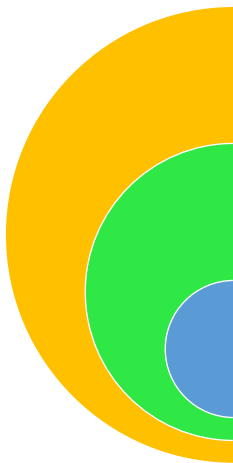
$$F(W, X, Y, Z) = \overline{X}\overline{Z} + \overline{W}XZ + W\overline{Y}$$

	00	01	11	10
00	0	0	1	1
01	0	0	1	1
11	0	0	0	1
10	0	1	1	1

KARNAUGH MAP (PART-2)

<https://youtu.be/ZMARz-Xr0F4>

CONTENTS



NUMERICAL PROBLEMS :
SOP KMAP (2,3,4 VARIABLES)

DON'T CARE CONDITIONS

NUMERICAL PROBLEMS
: DON'T CARE CONDITIONS

PREVIOUS VIDEO

Introduction to KMAP

Rules of KMAP

Representation of KMAP

Problems to workout

1

Using K-map simplify the Boolean function F as Sum of Products using the don't care conditions d.

$$F(w,x,y,z)=w'(x'y+ x'y' +xyz) + x'z'(y+w)$$

$$d(w,x,y,z)=w'x(y'z + yz) + wyz$$

2

Reduce the following expressions using K-map and implement the real minimal expression in universal logic.

$$1) F(A,B,C,D)=\sum m(0,1,2,3,5,7,8,9,10,12,13)$$

Problems to workout

3

- b) Simplify the given Boolean function $F(w, x, y, z) = \sum(2, 3, 12, 13, 14, 15)$
 i) Sum of Products and ii) Product of Sums (use K Map)



4

- Simplify the Boolean function $F(w, x, y, z) = \sum m(0, 5, 7, 8, 9, 10, 11, 14, 15)$

5

Example 3.10 : Reduce the following function using Karnaugh map technique and implement using basic gates

$$f(A, B, C, D) = \overline{A}\overline{B}D + ABC\overline{D} + \overline{A}BD + ABC\overline{D}$$

Problems to workout

6

➡ **Example 3.9 :** Simplify the logic function specified by the truth Table 3.1 using the Karnaugh map method. Y is the output variable, and A, B and C are the input variables.

A	B	C	Y
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Problems to workout

7

➡ **Example 3.8 :** Reduce the following function to its minimum sum of products form :

$$Y = \bar{A}\bar{B}\bar{C}D + \bar{A}B\bar{C}D + \bar{A}BCD + \bar{A}BC\bar{D} + AB\bar{C}\bar{D} + AB\bar{C}D + ABCD + A\bar{B}CD$$

8

➡ **Example 3.7 :** Reduce the following four variable function to its minimum sum of products form :

$$Y = \bar{A}BC\bar{D} + ABC\bar{D} + \bar{A}B\bar{C}D + \bar{A}B\bar{C}\bar{D} + \bar{A}B\bar{C}D + \bar{A}B\bar{C}\bar{D} + \bar{A}B\bar{C}D + \bar{A}B\bar{C}\bar{D}$$



	00	01	11	10
00	0	0	1	1
01	0	0	1	1
11	0	0	0	1
10	0	1	1	1



KARNAUGH MAP (PART-3)

<https://youtu.be/ezp0B2F6L0k>

SIMPLIFICATION OF POS EXPRESSION (MAXTERM KMAP)X

STPES TO SOLVE

1

- Plot the K-Map and place 0's in those cells corresponds to the 0s in the TT or Maxterms in the Product of Sum Expression

2

- Check the K-map for adjacent 0's and encircle those 0's

3

- Form the simplified POS Expression by taking POS Terms of all the groups

►►► **Example 2.21 :** Minimize the expression

$$Y = (A + B + \bar{C})(A + \bar{B} + \bar{C})(\bar{A} + \bar{B} + \bar{C})(\bar{A} + B + C)(A + B + C)$$

Solution : $(A + B + \bar{C}) = M_1$, $(A + \bar{B} + \bar{C}) = M_3$, $(\bar{A} + \bar{B} + \bar{C}) = M_7$,
 $(\bar{A} + B + C) = M_4$, $(A + B + C) = M_0$

C \ AB					
		00	01	11	10
0	0				
	1				

MODULE 2 (TOPIC-9)

VERILOG PROGRAMMING

JIBIN E

<https://youtu.be/TQNI363ZrA>  YouTube

Verilog Programming

- Verilog is a HARDWARE DESCRIPTION LANGUAGE (HDL).
- It is a language used for describing a digital system like a network switch or a microprocessor or a memory or a flip-flop.
- It means, by using a HDL we can describe any digital hardware at any level.
- Designs, which are described in HDL are independent of technology, very easy for designing and debugging, and are normally more useful than schematics, particularly for large circuits

Verilog Operators



1.Arithmetic Operations

Operator Symbol	Operations Performed
*	Multiply
/	Divide
+	Add
-	Subtract
%	Modulus
**	Power (Exponent)

2.Logic Operations

Operator Symbol	Operations Performed
!	Logical Negation
&&	Logical AND
	Logical OR

3.Relational Operator

Operator Symbol	Operations Performed
>	Greater than
<	Less than
>=	Greater than or equal
<=	Less than or equal

4.EquityOperator

Operator Symbol	Operations Performed
==	Equality
!=	Inequality

5.Bitwise Operations

Operator Symbol	Operations Performed
~	Bitwise Negation
&	Bitwise AND
	Bitwise OR
^	Bitwise XOR
~^ / ^~	Bitwise XNOR

6.Shift Operations

Operator Symbol	Operations Performed
>>	Right Shift
<<	Left Shift
>>>	Arithmetic Right Shift
<<<	Arithmetic Left Shift

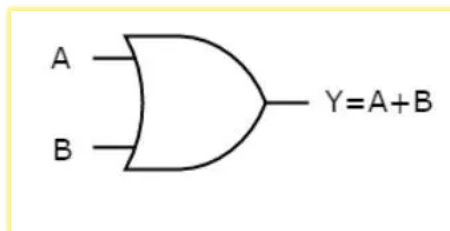
Gate Flow Modeling

- Designing a complex circuit using Basic Logic Gates is the goal of Gate Level Modeling
- We specify the gates of the circuit in our code. Verilog supports describing circuits using basic logic gates as predefined primitives.
- These primitives are instantiated like modules except that they are predefined in Verilog and do not need a module definition.

Data flow modeling

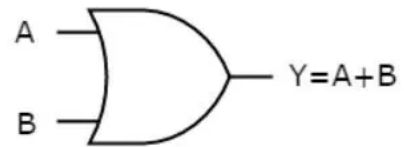
- Compared to gate-level modeling, dataflow modeling is a higher level of abstraction.
- Data Flow describes the circuits by their function rather than by their gate structure. That becomes handy because gate-level modeling becomes very complicated for large circuits because let's face it, a digital circuit with a bunch of gates can seem quite daunting.
- Hence, dataflow modeling is a very important way of implementing the design.
- We require the boolean logic equation and **Continuous Assignment Statement** to build the designs. The continuous assignments are made using the keyword assign.

OR GATE



Verilog code for OR gate using gate-level modeling


The output of the OR gate is high if at least one of the inputs is high else the output is low. Here's the logical representation of the OR gate.



```
module OR_2_gate_level(output Y, input A,
B);
  or(Y, A, B);
End module
```

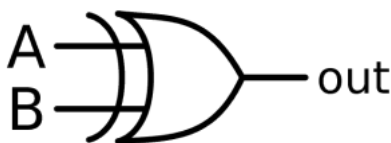
Verilog code for OR gate using Data-Flow modeling

The boolean equation of an OR gate is $Y = A + B$.



```
module OR_2_data_flow (output Y, input A,  
B);  
  Assign Y = A | B;  
End module
```

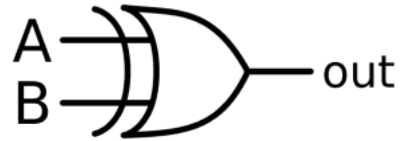
XOR GATE



Verilog code for XOR gate using Gate Level modeling

Logic Circuit of the XOR gate

Here's the logical representation of the XOR gate. It has a graphic symbol similar to that of the OR gate, except for the additional curved line on the input side.



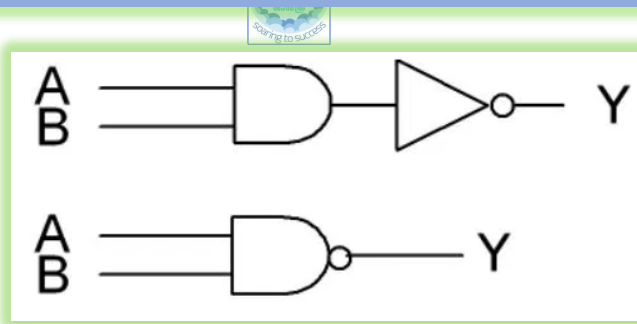
```
module XOR_2_gate_level(output Y, input A,
B);
xor (Y, A, B);
end module
```

Verilog code for OR gate using Data Flow modeling

The boolean equation of an XOR gate is
 $Y = (A \oplus B)$.

```
module XOR_2_data_flow (output Y, input A,
B); assign Y = A ^ B;
end module
```


NAND GATE



Verilog code for OR gate using Data Flow modeling

The Boolean equation for a NAND gate is $Y = (A.B)'$ or $\sim(A \& B)$.

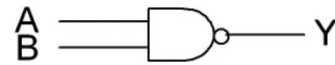
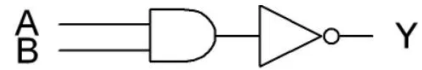


```
module NAND_2_data_flow (output Y, input A,
    B); assign Y = ~(A & B);
end module
```

Verilog code for NAND gate using Gate Level modeling

Logic Circuit of the NAND gate

The NAND gate is the complement of AND function. Its graphic symbol consists of an AND gate's graphic symbol followed by a small circle. Here's the logical representation of the NAND gate.



```
module NAND_2_gate_level(output Y, input
A,B); wire Yd;
and(Yd, A, B);
not(Y, Yd);
end module
```

Verilog code for NAND gate using Gate Level modeling

- **wire** in Verilog represents an electrical connection

```
wire Yd;
and(Yd, A, B);
not(Y, Yd);
endmodule;
```

- Here **and** is the operation performed on A, B, to get its output in Yd.
- Then Yd is passed through an inverter, and the output is obtained in Y.
- The compiler understands the and and the not operation the same way we do.
- endmodule terminates the module.

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<https://www.youtube.com/ShastraTechnicalInstitute>

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