PG Topic 1. Sets

Why? Counting; Enumeration; Algorithms; Induction and Recursion

DEFINITION: A collection of "things" (mathematical objects). $A = \{x, y, z\}$

Sets are normally denoted by using curly brackets: $\{0, 1\}$. $\{\text{True}, \text{False}\}$.

Label sets, often use capital letters:

We say "two sets are equal", "=", if they contain the same elements:

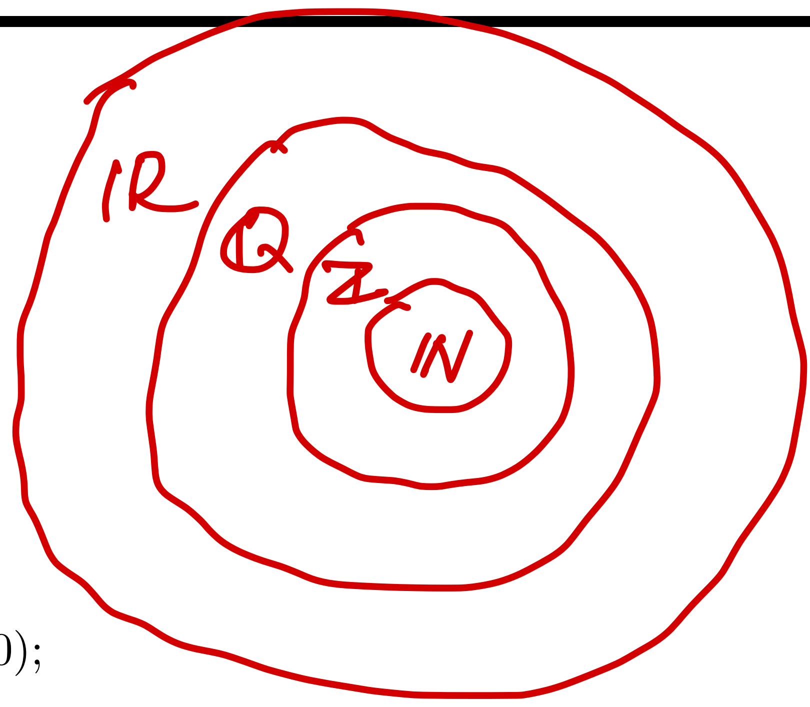
$B = \{x, y, z\}$. $A = B$. $C = \{x, z, y\}$.

In set theory, orders are NOT important:

$A = C$. $\{x, y, z\} = \{z, y, x\} = \{x, z, y\}$.

In set theory, a set cannot contain duplicates:

$\{x, x, y, z\}$ $\otimes$ $\{x, y, z\}$ $\checkmark$.



Important sets

- Natural numbers (counting numbers);

$\mathbb{N} = \{1, 2, 3, \dots\}$. $+$ $\times$.

- Integers (whole numbers, both positive and negative and 0);

$\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, 3, \dots\}$ $+$ $\times$ $-$.

- Fractions/Rational numbers (any number that can be written as a ratio of two integers;

$\mathbb{Q} = \{ \frac{a}{b} \mid a \text{ and } b \text{ are integers. } b \neq 0 \}$. $+$ $\times$ $-$ $\div$

- Real numbers (includes all rational and all numbers that cannot be written as a fraction, e.g., $\pi, \sqrt{2}, log(2)$.

e $\sqrt{5}$ $\log(3)\dots$

$a = 2$ $b = 1$ $\frac{a}{b} = 2$ ✓

$\mathbb{R} := \{\text{rationals, irationals}\}$.

$a = 1$ $b = 2$ $\frac{a}{b} = \frac{1}{2} = 0.5$

$a = 1$ $b = 3$ $\frac{a}{b} = \frac{1}{3} = 0.333\dots$

Notation $A = \{x, y, z\}$

- $\in$ means "belongs to";

$x \in A$. $x \in \{x, y, z\}$.

- $\notin$ means "does not belong to";

$d \notin A$. $d \notin \{x, y, z\}$.

- | or : mean "such that" or "given that";

$E = \{x \mid x \text{ is even}\} = \{x : x \text{ is even}\} = \{\cdots, -4, -2, 0, 2, 4, 6, \cdots\}$

$= \{x \mid x = 2k, k \in \mathbb{Z}\}$.

- $\subseteq$ means "is a subset of", or "is contained in or equal to";

$B = \{x, y\}$. $B \subseteq A$. $\mathbb{Q} \subseteq \mathbb{R}$.

- $\subset$ means "is a proper subset of". $A \subseteq A$. Recall: $\mathbb{Z} = \{\cdots, -3, -2, -1, 0, 1, 2, \cdots\}$

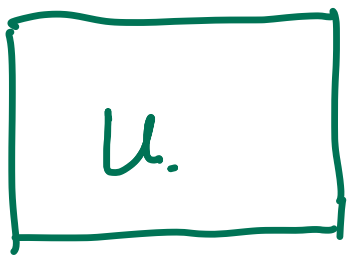
$B \subset A$. $\mathbb{Q} \subset \mathbb{R}$.

$A \not\subset A$

$C = \{x, y, z\}$. $A = C$. $A \subseteq C$. $A \subset C$ ✗

$A = \{x, y, z\}.$

Notation



- $\mathcal{U}$ means "universal set" (set of reference);
- $\emptyset$ or $\{\}$ mean empty set (set with no elements);
- $|A|$ means the size of set A (number of elements in A).

$|A| = 3. \quad |\emptyset| = 0.$

Can sets contain other sets as elements? Yes

$B = \{1, 2, 3\}. \quad C = \{4, 5\}.$

$D = \{B, C\} = \{\{1, 2, 3\}, \{4, 5\}\}.$

$|B| = 3. \quad |C| = 2. \quad |D| = 2.$

Set Operations

Let A be a set:

- A^c - complement set of A, i.e,. everything that is not in A but in $\mathcal{U}$.

EXAMPLE: $\mathcal{U} = \mathcal{N},\ A = \{2, 4, 6, ...\}$

- $A \backslash B$ - set difference (set minus, "A takeaway B"), i.e,. everything that is in B are removed from A.

EXAMPLE: $\mathcal{U} = \mathcal{N},\ A = \{1, 2, 3, 4, 5, 6\}, B = \{2, 3, 5, 7, 11\}$.

Set Operations

- $A \cup B$ - union of A and B, i.e,. everything that appears in either A or B or both.

EXAMPLE: $\mathcal{U} = \mathcal{N}$, $A = \{1, 2, 3, 4, 5, 6\}, B = \{2, 3, 5, 7, 11\}$.

- $A \cap B$ - intersection of A and B, i.e,. everything that appears in both A and B.

EXAMPLE: $\mathcal{U} = \mathcal{N}$, $A = \{1, 2, 3, 4, 5, 6\}, B = \{2, 3, 5, 7, 11\}$.

- $A \Delta B$ - symmetric difference between A and B, i.e,. everything that appears in either A or B but NOT in both.

EXAMPLE: $\mathcal{U} = \mathcal{N}$, $A = \{1, 2, 3, 4, 5, 6\}, B = \{2, 3, 5, 7, 11\}$.

Laws for set operation

- $A \cup A = A$
- $A \cap A = A$
- $A \cup B = B \cup A$
- $A \cap B = B \cap A$
- $A \cup (B \cup C) = (A \cup B) \cup C$
- $A \cap (B \cap C) = (A \cap B) \cap C$