1. Find:

(a)

$$\int_0^1 3x^3 + 2x^2 + x + 1dx$$

[3 marks]

(b)

$$\int e^{3x} + \sin(3x)dx$$

[2 marks]

[Total: 5 marks]

2. Let $f(x) = x + 2$ and $g(x) = x^2 + 1$. Find:

(a) $f^2(-2)$; **[2 marks]**

(b) $g^{-1}(x)$; **[1 mark]**

(c) $g \circ f(x)$; **[2 marks]**

(d) the values of a such that $f(a) = g(a)$. **[3 marks]**

[Total: 8 marks]

3. Solve the following inequalities:

(a) $|2x - 1| > 5$ **[3 marks]**

(b) $4 > \frac{3x-1}{2-x}$ **[3 marks]**

(c) $2x^2 - x - 3 < 0$ **[3 marks]**

[Total: 9 marks]

4. Find the derivatives:

(a)

$$\frac{d}{dx}\left(x^2 + 2x\right)\cos(2x)$$

[2 marks]

(b)

$$\frac{d}{dx}\left(3x^2 - 2x + 1\right)^{-1/2}$$

[3 marks]

[Total: 5 marks]

5. Using integration by parts, find

$$\int \left(\frac{3}{2}x^2 + 2x - 1\right) \ln(x) dx$$

[Total: 6 marks]

6. Let $M = \begin{pmatrix} -1 & 3 \\ -2 & 4 \end{pmatrix}$ and $N = \begin{pmatrix} 1 & 0 \\ -1 & 2 \end{pmatrix}$.

(a) Find:

i. MN; **[3 marks]**

ii. M^{-1}; **[3 marks]**

iii. $N^T + M^T$. **[2 marks]**

(b) Solve the following simultaneous equations using matrix methods **only**:

$$\begin{aligned} -x + 3y &= -\tfrac{1}{2} \\ -2x + 4y &= -2 \end{aligned}$$

[3 marks]

[Total: 11 marks]

7. Consider the vectors $\mathbf{u} = \begin{pmatrix} 0 \\ 1 \\ -3 \\ 2 \end{pmatrix}, \mathbf{v} = \begin{pmatrix} -1 \\ 2 \\ 0 \\ 3 \end{pmatrix}$.

(a) Find:

i. the vector $\mathbf{u} + \mathbf{v}$; **[1 mark]**

ii. the unit vector, $\hat{\mathbf{v}}$; **[2 marks]**

iii. the angle between $\mathbf{u}, \mathbf{v}$; **[4 marks]**

iv. the projection of $\mathbf{u}$ onto $\mathbf{v}$. **[3 marks]**

(b) Let $\mathbf{w} = (3, 2, 1, 0)^T$. Determine which vector, $\mathbf{u}$ or $\mathbf{v}$, is closer to $\mathbf{w}$ by calculating the Euclidean (L_2) distance. Now calculate the Manhattan (L_1) distance, does your answer change? Would you ever expect it to? **[6 marks]**

(c) By considering the basis vectors, $\mathbf{e}_1 = (1, -1, 0, 0)^T, \mathbf{e}_2 = (0, -1, 1, 0)^T$, $\mathbf{e}_3 = (0, 0, 1, -1)^T$ and $\mathbf{e}_4 = (0, 0, 0, -1)^T$. Find values of $\alpha, \beta, \gamma, \delta$ such that $\mathbf{w} = \alpha\mathbf{e}_1 + \beta\mathbf{e}_2 + \gamma\mathbf{e}_3 + \delta\mathbf{e}_4$ **[3 marks]**

[Total: 19 marks]

8. **(a)** Find and classify all the turning points for the following function,

$$f(x) = \frac{4}{3}x^3 - 9x.$$

[6 marks]

(b) Show that the function $g(x) = \ln(x)$ (where $\ln(x)$ is the natural log) has no turning points. Clearly explain why. **[2 marks]**

[Total: 8 marks]

9. Let $A = \{-3, -2, -1, 0, 1, 2, 3\}, B = \{-1, 1\}, C = \{0, 0.5, 1, 1.5, 2, 2.5, 3\}$. Find:

(a) $|A|$; **[1 mark]**

(b) $A \cap B$; **[2 marks]**

(c) $A \backslash B$; **[2 marks]**

(d) the Jaccard distance between A and C. **[4 marks]**

[Total: 9 marks]

10. In how many ways can the letters in the word COMBINATION be arranged? **[Total: 3 marks]**

11. There are 16 textbooks on a shelf

(a) In how many ways can 10 books be chosen from the shelf? **[3 marks]**

(b) Assuming that 10 of the books are maths textbooks and 6 are biology textbooks, in how many ways can 10 books be chosen so that 6 are maths textbooks and 4 are biology textbooks? **[4 marks]**

[Total: 7 marks]

12. Let A, B, C be three non-empty sets.

(a) By sketching Venn diagrams, shade the regions which correspond to:

i. $A \cap B \cap C$; **[2 marks]**

ii. $(B \triangle A) \cup C$. **[3 marks]**

(b) Let A contain 50 elements, B contain 40 elements and C contain 30 elements.

i. Give the maximum and minimum values for the number of elements in the union of all three sets $A \cup B \cup C$. **[2 marks]**

ii. If it is known that there are 10 elements in the intersection of each pair of sets, and the intersection of all three sets contains 5 elements. Find how many elements are in the union of all three sets $A \cup B \cup C$. **[3 marks]**

[Total: 10 marks]