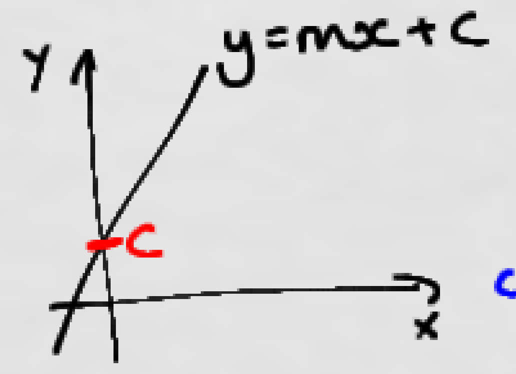
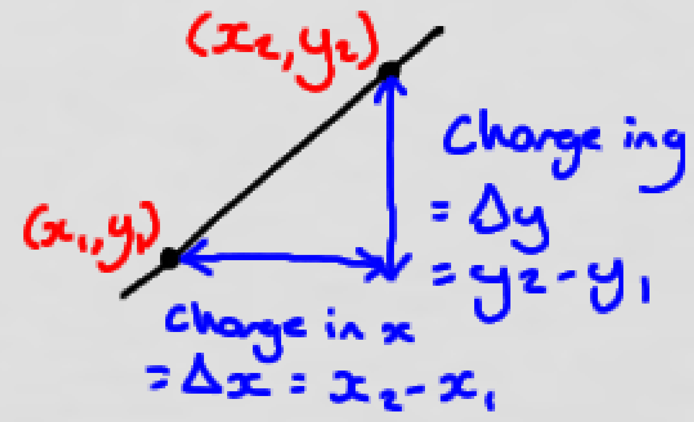
Straight line graphs (why? line of best fit, linear regression, proportionality)



x = variable (thing we change)

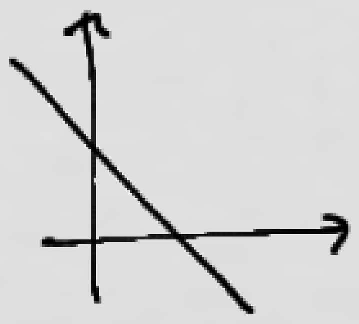
constants $\left\{ \begin{array}{l} m = \text{gradient | slope (for every one unit across, 1 go "m" units up | down)} \\ c = y\text{-intercept} \end{array} \right.$

-Calculating gradient: $m = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$ (Delta)

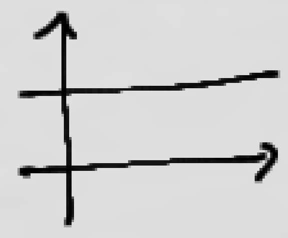


if $m > 0$ (+ve)

, $m < 0$ (-ve)



, $m = 0$

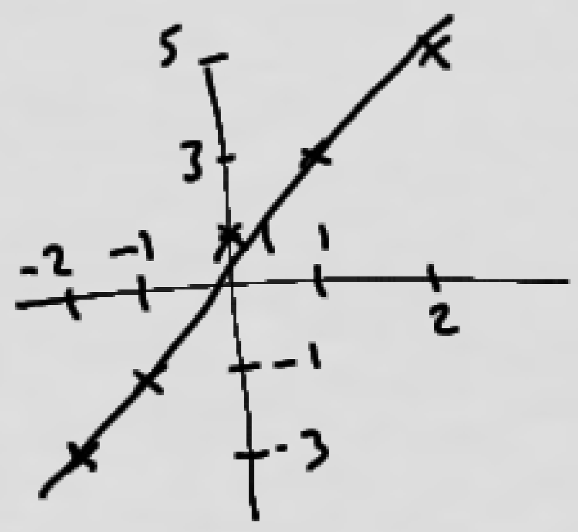


How to sketch:

① Working out coordinates

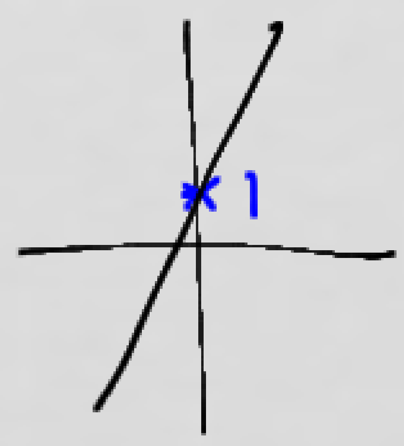
$y = 2x + 1$:

x	-2	-1	0	1	2
y	-3	-1	1	3	5



② looking at constants: (m & c)

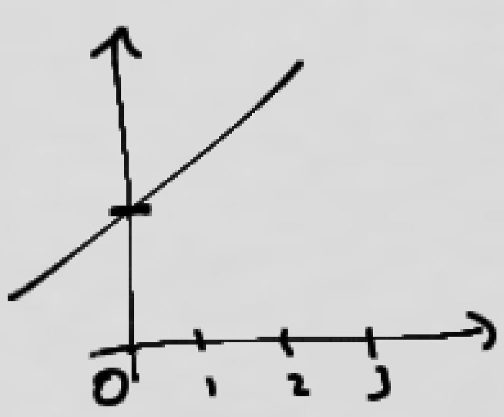
$y = \underbrace{2x}_{m} + \underbrace{1}_{c}$



Q: How many points do we need in order to find the equation of a straight line?

A: 2

Eg: Find the eqn. of the straight line passing through $\underset{x_1, y_1}{(0, 2)}$ & $\underset{x_2, y_2}{(2, 4)}$



A: · Find m: $\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 2}{2 - 0} = \frac{2}{2} = \underline{\underline{1}}$

· Find c: We can let $x = 0$ (as this is the x coordinate for my y-intercept)

$\rightarrow y = mx + c \overset{m=1}{\Rightarrow} y = x + c \rightarrow$ we plug in any coordinate that is on this line

eg. $(0, 2)$: $2 = 0 + c \Rightarrow \underline{\underline{2 = c}}$ $\boxed{y = x + 2}$

eg. $(2, 4)$: $4 = 2 + c$

$4 - 2 = c$

$\underline{\underline{2 = c}}$

In general, if we have two points (x_1, y_1) & (x_2, y_2). We can find the equation of the straight line by using the formula

$$\boxed{\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}}$$

Eg. Find the line connecting $\underset{(x_1,y_1)}{(-1,-1)}$ & $\underset{(x_2,y_2)}{(3,1)}$

$$\frac{y-(-1)}{x-(-1)} = \frac{1-(-1)}{3-(-1)}$$

$$\frac{y+1}{x+1} = \frac{1+1}{3+1}$$

$$\frac{y+1}{x+1} = \frac{1}{2} \Rightarrow y+1 = \frac{1}{2}(x+1) \Rightarrow \underline{\underline{y = \frac{1}{2}x - \frac{1}{2}}}$$

- Consider the line $y=2x+1$. How would we describe this to a non-mathematician?

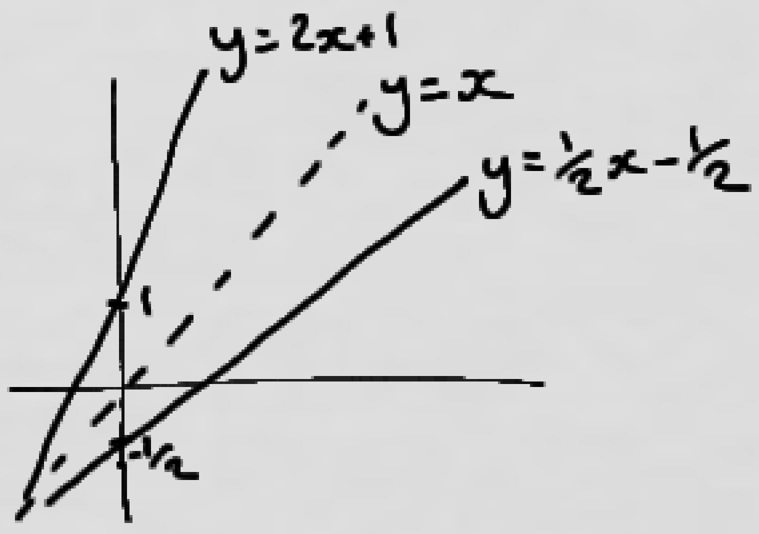
We take a number multiply by 2 then add 1.

How do we do the opposite (backwards)?

We do this by subtracting 1 then dividing by 2

We can write this mathematically as

$$y = \frac{(x-1)}{2} = \frac{1}{2}(x-1) = \frac{1}{2}x - \frac{1}{2} \leftarrow \text{new line} \quad , \quad y = 2x+1$$



We say that these lines are inverse functions of each other.

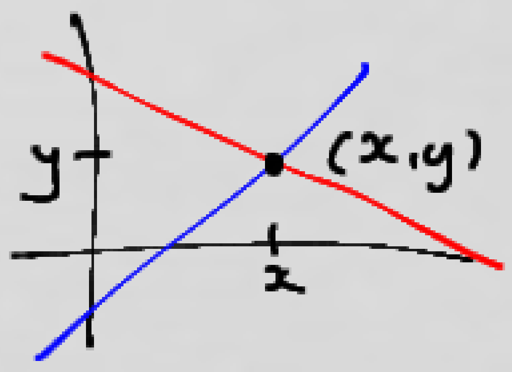
Eg, $x = \underline{\underline{2}}$: $y = 2x+1 = 2(2)+1 = \underline{\underline{5}}$

$x = \underline{\underline{5}}$: $y = \frac{1}{2}x - \frac{1}{2} = \frac{1}{2}(5) - \frac{1}{2} = \underline{\underline{2}}$

Graphically, the inverse of a straight line can be found by reflecting in the line $y=x$. Or mathematically by switching x & y in the equation and rearranging.

- Simultaneous Equations

Suppose I have 2 linear equations (straight line) and I want to find where they meet. I can do this by solving them simultaneously. That is finding values for the coordinate (x,y) that lies on both lines



How do we find this?

Eg., $y=2x+3$ & $y=x+4$ Where do these lines cross?

This is equivalent to me solving the following pair of simultaneous eqns.

① $y=2x+3$

② $y=x+4$

Method ①: get 'rid of' one variable

Take eqn. ② away from ①

$$y-y = 2x+3-(x+4)$$
$$0 = 2x+3-x-4$$
$$0 = x-1 \Rightarrow \underline{\underline{x=1}}$$

To find y we sub $x=1$ into ① or ②

Sub in to ①: $y=2(1)+3=\underline{\underline{5}}$

(check by subbing in to ②)

②: $y=(1)+4=\underline{\underline{5}}$ ✓

A., $(1,5)$

Method ②: substitution

Everywhere I see 'y' in Eqn ①, I'm going to replace with $x+4$ & rearrange

$$\underbrace{x+4}_{y} = 2x+3$$
$$4-3 = 2x-x$$
$$\underline{\underline{1 = x}} \Rightarrow \underline{\underline{y=5}}$$

Eg., $2y=4x-3$: $2y=4x-3$ ①

$y+3x=1$ $\Rightarrow y=1-3x$ ②

Using substitution: Eqn ② into Eqn ①

$$2\underbrace{(1-3x)}_{=y} = 4x-3$$
$$2-6x = 4x-3$$
$$2+3 = 4x+6x$$
$$5 = 10x$$
$$\Rightarrow \underline{\underline{x=\tfrac{1}{2}}} \Rightarrow \underline{\underline{y=-\tfrac{1}{2}}}$$

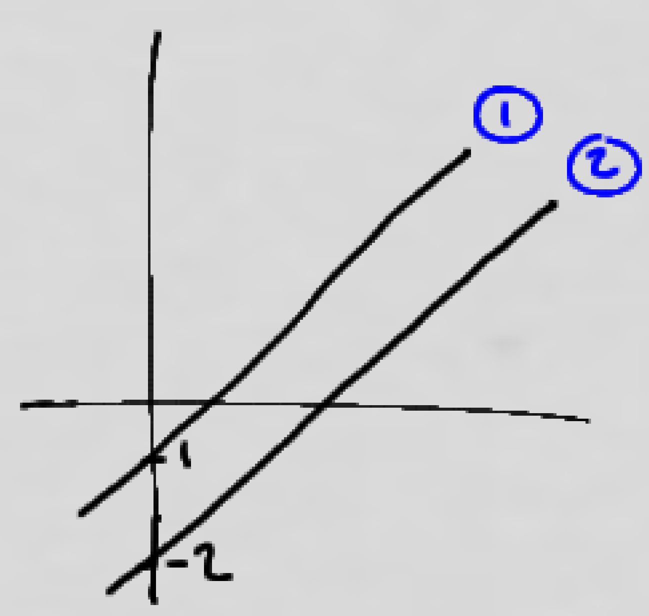
check!

Eg., $2y=2x-2 \Rightarrow 2y=2x-2$ ① $\longrightarrow 2y=2x-2 \overset{\div 2}{\Rightarrow} y=x-1$

$y=x-2$ $(\times 2) \Rightarrow 2y=2(x-2)$ ② $\longrightarrow y=x-2$

①-②: $2y-2y = 2x-2-2(x-2)$

$$0 = 2x-2-2x+4$$
$$\underline{\underline{0=2}}$$



With straight lines they either (i) meet at precisely one point

(ii) never meet (same gradient, different y-intercept) $\rightarrow$ parallel

(iii) meet at infinite points (same line)

$y=x+1$

$2y=2x+2$

$y-x=1$