Project Report

Faculty Of Engineering Ain Shams University

CSE332s: Design and Analysis of Algorithm

Team 12 Members

Name	ID	Task
Fathy Abdlhady Fathy	2001152	1 , 7 , 8
Mohamed Ayman Mohamed Soliman	2001048	1 , 7
Yousef Shawky Mohamed	2001500	7 , 8
Ahmed Youssef Mansour Mahfouz	2002238	6
Youssef Wael Hamdy Ibrahim Ashmawy	2001430	2 , 6
Omar Nader Ahmed Gamal El Din	2001714	3
Mark Ramy Fathy	2000923	4
Heba Maher Abdelrahman	2001400	4
Mohamed Mostafa Mahmoud Mohamed	2001299	6
Omar Saleh Mohamed	2001993	2 , 3
Abdallah Mohamed	2001803	5

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Task 1 : Tromino Tilings

1.1 Problem Description

Devise an algorithm for the following task: given a 2n × 2n (n > 1) board with one missing square, tile it with right trominoes of only three colors so that no pair of trominoes that share an edge have the same color. Recall that the right tromino is an L-shaped tile formed by three adjacent squares.
Use dynamic programing to solve this problem.

1.2 Detailed Assumptions

1. The board is a square with dimensions 2n x 2n, where n > 1 (minimum size is 4x4).
2. There is exactly one missing square on the board.
3. The right tromino is an L-shaped tile formed by three squares.
4. There are three distinct colors available for the trominoes (Red, Green, Blue, for example).
5. The missing square can be located anywhere on the board.

1.3 Detailed Solution

1.3.1 Description of Solution

1. we always put tromino of value 1 in the center of board
2. we divide board into four quarters and if divided board is bigger than 2x2 board do first step then second step again else we do third step
3. we do this step if board == 2x2 board if this is the first time we access this quarter then we see all other trominoes around it so that we place the right tromino the we record it because all quarters of same type in other divided boards will have the same tromino

1.3.2 Pseudo Code

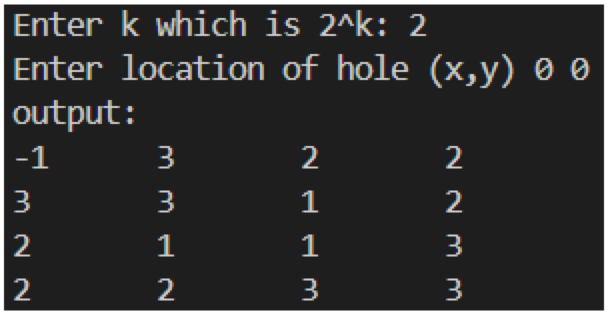
```
function tromino(grid_size, start_x, start_y, grid)
  // Base Case: single cell
  if grid_size == 1 then
    return

 // Find the location of the missing square
  missing_x = null
  missing_y = null
  for i = start_x to start_x + grid_size - 1 do
    for j = start_y to start_y + grid_size - 1 do
      if grid[i][j] != 0 and grid[i][j] != -1 then
        missing_x = i
        missing_y = j
        break 2; // Exit both loops after finding the first missing square
      endif
    endfor
  endfor

  // Handle different cases based on missing square location
  if grid_size == 2 then
    // Place tromino based on missing square quadrant
    if missing_x < start_x + 1 and missing_y < start_y + 1 then
      placeTromino(start_x + 1, start_y + 1, start_x + 1, start_y, start_x, start_y +
1, 1, grid)
    else if missing_x >= start_x + 1 and missing_y < start_y + 1 then
      placeTromino(start_x + 1, start_y + 1, start_x + 1, start_y, start_x, start_y -
1, 2, grid)
    else if missing_x < start_x + 1 and missing_y >= start_y + 1 then
      placeTromino(start_x + 1, start_y + 1, start_x, start_y + 1, start_x - 1,
start_y + 1, 3, grid)
    else
```

```
      placeTromino(start_x + 1, start_y + 1, start_x, start_y, start_x - 1, start_y -
1, 4, grid)

    endif

  else

    // Handle missing square in center quadrant

    if missing_x < start_x + grid_size / 2 and missing_y < start_y + grid_size / 2
then

      placeTromino(start_x + grid_size / 2, start_y + grid_size / 2 - 1, start_x +
grid_size / 2, start_y + grid_size / 2, start_x + grid_size / 2 - 1, start_y +
grid_size / 2, 0, grid)

    else if ... (similar checks for other quadrants with missing center square)

    endif


    // Recursively call tromino for sub-grids

    tromino(grid_size / 2, start_x, start_y + grid_size / 2, grid)

    tromino(grid_size / 2, start_x, start_y, grid)

    tromino(grid_size / 2, start_x + grid_size / 2, start_y, grid)

    tromino(grid_size / 2, start_x + grid_size / 2, start_y + grid_size / 2, grid)

  endif

end function


function placeTromino(x1, y1, x2, y2, x3, y3, quadrant, grid)

  // Update neighboring cells based on tromino placement and quadrant

  // ... (similar logic to original placeTrom function)


  // Set the quadrant value for all three tromino squares in the grid

  grid[x1][y1] = quadrant

  grid[x2][y2] = quadrant

  grid[x3][y3] = quadrant

end function
```

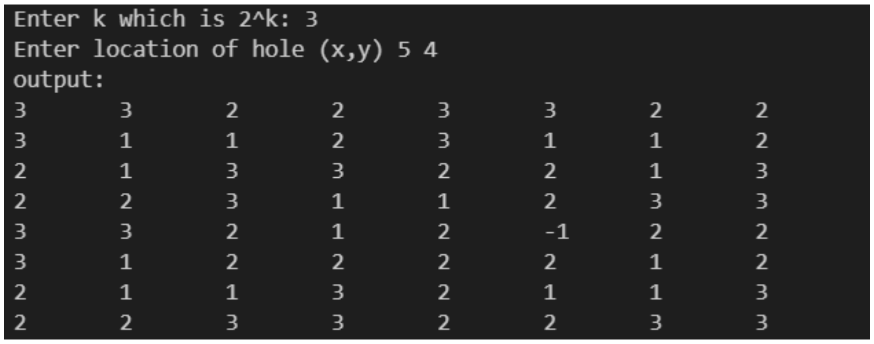
1.4 Complexity Analysis for the Algorithm

Base Case: the base case tromino(n = 1) takes constant time (O(1)).
Recursive Calls: The tromino function makes four recursive calls for sub-grids with size n/2. This seems similar to a logarithmic depth, but there's a crucial difference.
placeTrom function: While the number of iterations in placeTrom itself is constant, the key point is that placeTrom is called for every cell in the grid during the recursion. This happens because in each sub-grid, all the cells are checked for potential tromino placement based on the missing square's location.
Combining Time: The number of recursive calls grows with the grid size (n), leading to placeTrom being called for an increasing number of cells. This translates to the total work done growing proportionally to the square of the grid size (n^2).
T(n) = 4T(n/2) + n
Overall Time Complexity:
Using master theorem then **time complexity: O(n2)**

1.5 Sample Output of the Solution



```
Enter k which is 2^k: 2
Enter location of hole (x,y) 0 0
output:
-1      3       2       2
3       3       1       2
2       1       1       3
2       2       3       3
```



```
Enter k which is 2^k: 3
Enter location of hole (x,y) 5 4
output:
3       3       2       2       3       3       2       2
3       1       1       2       3       1       1       2
2       1       3       3       2       2       1       3
2       2       3       1       1       2       3       3
3       3       2       1       2       -1      2       2
3       1       2       2       2       2       1       2
2       1       1       3       2       1       1       3
2       2       3       3       2       2       3       3
```

1.6 Comparison with Another Algorithm

TABLE 1

	Dynamic Programming	Divide & Conquer
Time Complexity	$O(n^2)$	$O(n^2)$
Space Complexity	O(n)	O(n)
Readability	Complex and Difficult And not easy to understand	Conceptually simple and straightforward

1.7 Conclusion

By comparing the algorithms used to solve the problem we observe:
The dynamic programming approach is not any different compared to divide & conquer approach in time complexity and space complexity but when it comes to readability and implementation the divide & conquer is a lot easier compared to dynamic programming

Task 2 : Knight Tour Closed

2.1 Problem Description

Is it possible for a chess knight to visit all the cells of an 8 × 8 chessboard exactly once, ending at a cell one knight's move away from the starting cell? (Such a tour is called closed or re-entrant. Note that a cell is considered visited only when the knight lands on it, not just passes over it on its move.)
If it is possible design a greedy algorithm to find the minimum number of moves the chess knight needs.

2.2 Detailed Assumptions

1. Standard 8x8 chessboard is assumed.
2. Knight moves according to its L-shaped pattern.
3. Each square on the chessboard must be visited exactly once.
4. The tour must form a closed loop, returning to the starting square.
5. Backtracking is typically employed in the solution algorithm.
6. Some algorithms may introduce randomness for efficiency.
7. Heuristics are commonly used to guide the search process.
8. An optimal or any valid closed-circuit solution may be sought.

2.3 Detailed Solution

2.3.1 Description of Solution

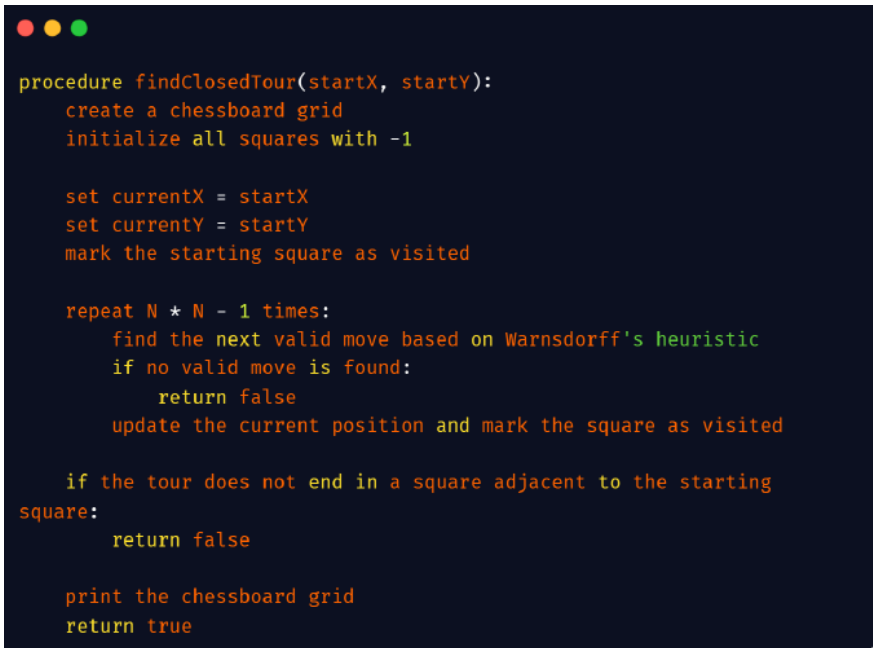
Warnsdorff's algorithm is a heuristic approach to solve the Knight's tour problem. It's based on the idea of always choosing the next move to be the one with the fewest onward moves (vertex with smallest degree). This heuristic helps to reduce the branching factor and improves the chances of finding a solution.

2.3.2 Pseudo Code

2.3.2.1 High-Level

1. Initialize an empty chessboard.
2. Start from a given square.
3. Mark the starting square as visited.
4. Find the next valid move using Warnsdorff's heuristic:
    a. Consider all possible moves from the current square.
    b. Choose the move with the fewest accessible squares.
5. Update the current position and mark it as visited.
6. Repeat steps 4-5 until all squares are visited or no further moves are possible.
7. Check if the tour is closed by verifying if the knight can move back to the starting square.
8. Print the solution if a closed tour is found.

2.3.2.1 In Details

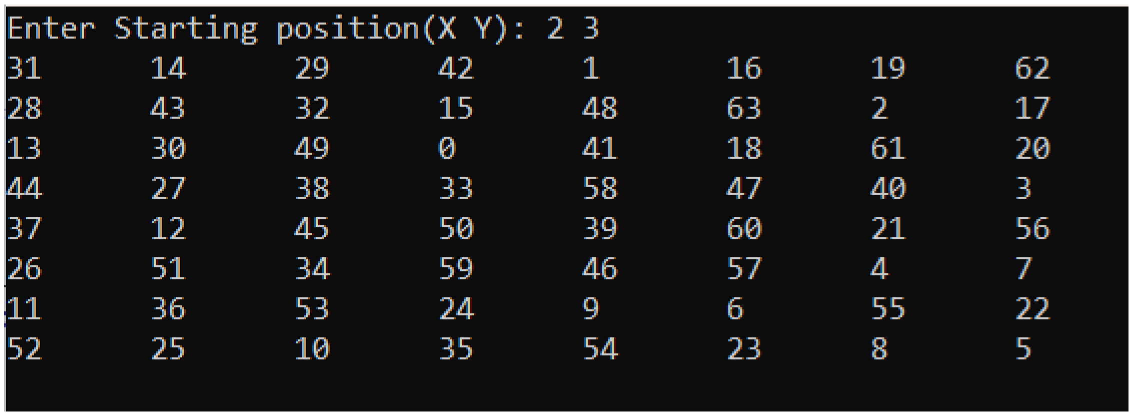


```
procedure findClosedTour(startX, startY):
    create a chessboard grid
    initialize all squares with -1

    set currentX = startX
    set currentY = startY
    mark the starting square as visited

    repeat N * N - 1 times:
        find the next valid move based on Warnsdorff's heuristic
        if no valid move is found:
            return false
        update the current position and mark the square as visited

    if the tour does not end in a square adjacent to the starting
square:
        return false

    print the chessboard grid
    return true
```

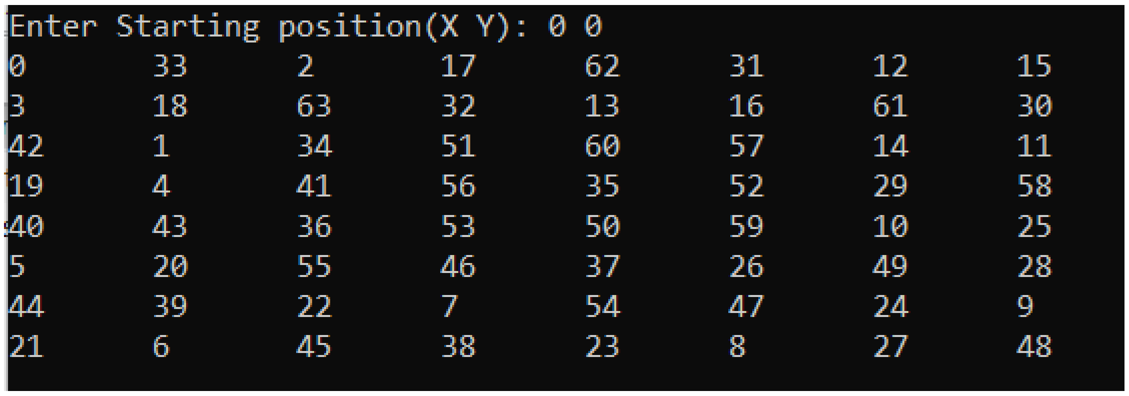
2.4 Complexity Analysis for the Algorithm

Time complexity: O(N^2)
Space Complexity: O(N^2))

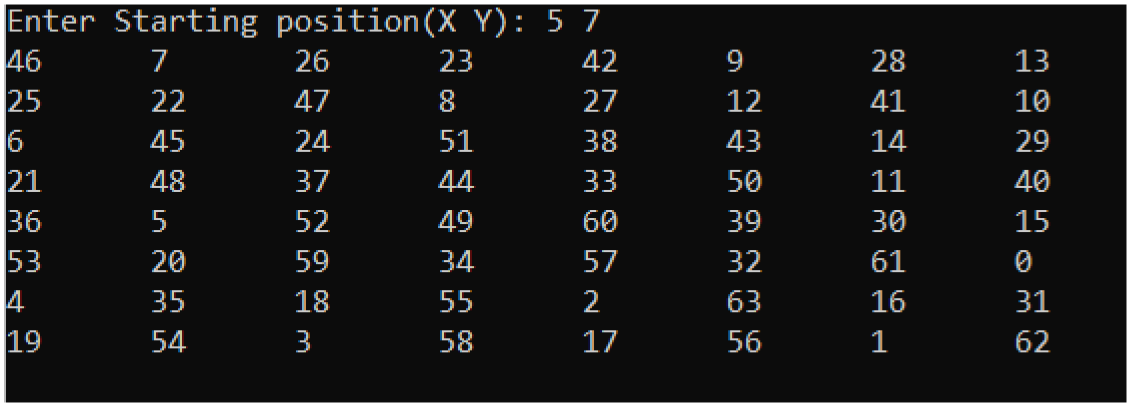
2.5 Sample Output of the Solution



```
Enter Starting position(X Y): 2 3
31      14      29      42      1       16      19      62
28      43      32      15      48      63      2       17
13      30      49      0       41      18      61      20
44      27      38      33      58      47      40      3
37      12      45      50      39      60      21      56
26      51      34      59      46      57      4       7
11      36      53      24      9       6       55      22
52      25      10      35      54      23      8       5
```



```
Enter Starting position(X Y): 0 0
0       33      2       17      62      31      12      15
3       18      63      32      13      16      61      30
42      1       34      51      60      57      14      11
19      4       41      56      35      52      29      58
40      43      36      53      50      59      10      25
5       20      55      46      37      26      49      28
44      39      22      7       54      47      24      9
21      6       45      38      23      8       27      48
```



```
Enter Starting position(X Y): 5 7
46      7       26      23      42      9       28      13
25      22      47      8       27      12      41      10
6       45      24      51      38      43      14      29
21      48      37      44      33      50      11      40
36      5       52      49      60      39      30      15
53      20      59      34      57      32      61      0
4       35      18      55      2       63      16      31
19      54      3       58      17      56      1       62
```

2.6 Comparison with Another Algorithm

TABLE 2

Criteria	Brute Force Approach	Warnsdorff's Heuristic
Advantages	Guarantees to find a solution if it exists.	More efficient for larger chessboards.
	Conceptually simple and straightforward	Often finds solutions faster.
		Lower space complexity.
Disadvantages	Exponential time complexity: O(8^(N^2)).	Not guaranteed to find solution always.
	Inefficient for large chessboards.	May fail for certain configurations.
		Possibility of not finding a solution.
Complexity	Time: O(8^(N^2)), Space: O(N^2).	Time: O(N^2), Space: O(N^2).

2.7 Conclusion

In conclusion, solving the Knight's Tour problem can be approached in two main ways: brute force and using Warnsdorff's heuristic.

The brute force method checks every possible move until it finds a closed tour or exhausts all options. It's easy to understand but can be very slow, especially for larger chessboards.

On the other hand, Warnsdorff's heuristic method uses a simple rule to guide the knight's movements, making it faster and more efficient. While it may not always find the best solution, it strikes a good balance between speed and accuracy.

Overall, for practical purposes, Warnsdorff's heuristic approach is often the better choice due to its speed and reliability, even though it might not always find the absolute best solution.

Task 3 : Security Switches

3.1 Problem Description

There is a row of n security switches protecting a military installation entrance. The switches can
be manipulated as follows:
(i) The rightmost switch may be turned on or off at will.
(ii) Any other switch may be turned on or off only if the switch to its immediate right is on and all the other switches to its right, if any, are off.
(iii) Only one switch may be toggled at a time.
Design a Dynamic Programing algorithm to turn off all the switches, which are initially all on, in the minimum number of moves. (Toggling one switch is considered one move.) Also find the minimum number of moves.

3.2 Comment

It is an optimization problem as we want the minimum number of moves using dynamic programming, we will go with a forward approach backward reasoning and as there are lots of possibilities it will be hard to draw the multistage graph so we will settle only with the recurrence relation.

3.3 Detailed Solution

3.3.1 Description of Solution

First analyze the problem and understand the recursion in it as it has overlapping problems, we can see that to turn off a switch we need its immediate right to be on and all the others to the right off

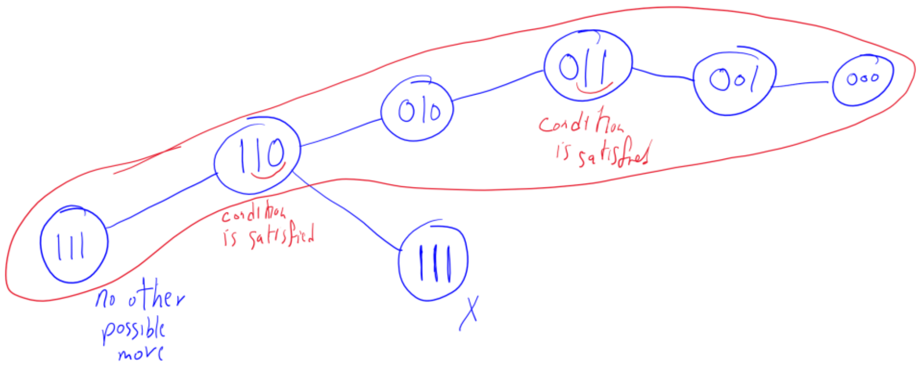


FIGURE 1

We can see the pattern to solve a switch (i) we need to solve for the switches i+2 and the rest on the right to be off and then toggle switch(i) getting 0100…. To get it to be 011111… we need M(n-2) moves again leading to 2 M(n-2) till now then we have to solve M(N-1) of the original problem +1 for the first switch that is a base case leading to the recurrence relation below.

Recurrence Relation:

$M(n) = M(n - 2) + 1 + M(n - 2) + M(n - 1)$

Or

$M(n) = M(n - 1) + 2M(n - 2) + 1$ for $n \geq 3$, $M(1) = 1$, $M(2) = 2$

- $M(1) = 1$ and $M(2) = 2$ are the base cases
- $M(n - 1)$: Number of moves needed to solve for (n - 1) switches.
- $M(n - 2)$: Number of moves needed to solve for (n - 2) switches, and we need to double it because we toggle the switches twice in that case.
- 1: Additional move needed to toggle the first switch

3.3.2 Pseudo Code

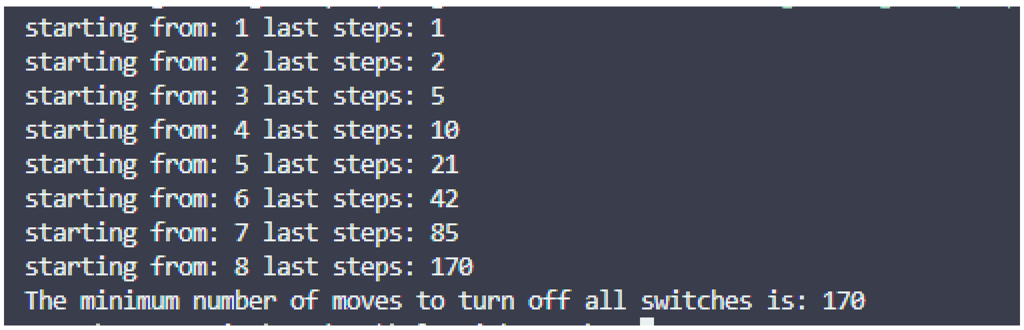
```
Function min_moves(n):
    If n is 1:
        Return 1
    If n is 2:
        Return 2

    Initialize an array dp of size n+1 with zeros
    Set dp[1] to 1
    Set dp[2] to 2

    Print "starting from: 1 last steps: " + dp[1]
    Print "starting from: 2 last steps: " + dp[2]

    For i from 3 to n:
        Set dp[i] to dp[i-1] + 2*dp[i-2] + 1
        Print "starting from: " + i + " last steps: " + dp[i]

    Return dp[n]
End Function
```

3.4 Complexity Analysis for the Algorithm

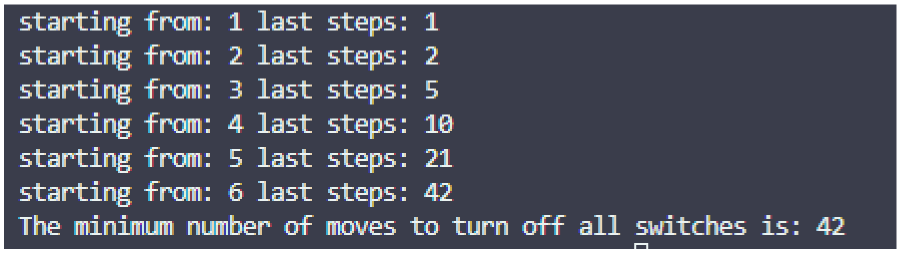
For this code the time complexity will be O(n) using the recurrence relation

The puzzle can be solved in the minimum of $\frac{2}{3} \times 2^n - \frac{1}{6}(-1^n) - \frac{1}{2}$ switch toggles.

3.5 Sample Output of the Solution



```
starting from: 1 last steps: 1
starting from: 2 last steps: 2
starting from: 3 last steps: 5
starting from: 4 last steps: 10
starting from: 5 last steps: 21
starting from: 6 last steps: 42
starting from: 7 last steps: 85
starting from: 8 last steps: 170
The minimum number of moves to turn off all switches is: 170
```



```
starting from: 1 last steps: 1
starting from: 2 last steps: 2
starting from: 3 last steps: 5
starting from: 4 last steps: 10
starting from: 5 last steps: 21
starting from: 6 last steps: 42
The minimum number of moves to turn off all switches is: 42
```

3.6 Another solution

Another Solution This is another solution that solves the puzzle using decease-and-conquer algorithm. The solution was taken from the book: “Algorithmic Puzzles by Anany Levitin”. We will number the switches left to right from 1 to n and denote the “on” and“off” states of a switch by a 1 and 0, respectively. To help ourselves with solving the general instance of the puzzle, let us start by solving its four smallest instances. Consider now the general instance of the puzzle represented by the bit string of n 1’s: 111. . . 1. Before we can turn off the first (leftmost) switch, the switches should be in the state 110. . . 0. Hence, to begin with, we need to turn off the last n−2 switches and do this in the minimum number of moves if we want to have an optimal algorithm. In other words, we should first solve the same problem as given for the last n−2 switches. This can be done recursively with the n = 1 and n = 2 instances solved directly. After that, we can toggle the first switch to get 010. . . 0. Now, before we can toggle the second switch we will need to pass through the state with all the switches following it “on,” which can be easily proved by mathematical induction. Getting all the switches from the third to the last one “on” can be achieved by reversing the optimal sequence of moves made previously to toggle the last n − 2 switches from the “on” position to the “off” position. This will give us 011. . . 1. Ignoring the first 0, we now have the n − 1 instance of the original puzzle, which can be solved recursively.

1
0

$n = 1$

1	1
0	1
0	0

$n = 2$

1	1	1
1	1	0
0	1	0
0	1	1
0	0	1
0	0	0

$n = 3$

1	1	1	1
1	1	0	1
1	1	0	0
0	1	0	0
0	1	0	1
0	1	1	1
0	1	1	0
0	0	1	0
0	0	1	1
0	0	0	1
0	0	0	0

$n = 4$

FIGURE 2

3.7 Conclusion

The approach described in the book "Algorithmic Puzzles by Anany Levitin" uses a decrease-and conquer strategy instead of dynamic programming.

TABLE 3

Criteria	Dynamic Programming	Decrease-and-Conquer
Base Cases	The base case is when the subproblem size is 1, and the solution for that subproblem is already known	The base cases are the smallest instances of the problem, which are solved directly
Solution Construction	The solution is constructed by recursively solving subproblems and combining the results	The solution is constructed by solving the smallest instances of the problem first, then using those solutions to solve larger instances

Task 4 : The Four- Peg Tower of Hanoi

4.1 Problem Description

There are eight disks of different sizes and four pegs. Initially, all the disks are on the first peg in order of size, the largest on the bottom and the smallest on the top. It's required to use divide and conquer algorithm to transfer all the disks to another peg by a sequence of moves.
The constraints are that only one disk can be moved at a time, and it is forbidden to place a larger disk on top of a smaller one.
Also it's required to use dynamic programming algorithm to transfer all the disks to another peg in 33 moves

4.2 Detailed Assumptions

1. The disks are labeled with consecutive integers starting from 1. Each disk is represented by an object of Disk class, which has a disk_num attribute.
2. The moves variable is used to keep track of the total number of moves performed during solving the Tower of Hanoi Problem.
3. The Tower of Hanoi problem is being solved optimally, it aims to minimize the number of moves required to transfer the disks from the starting peg to the end peg.
4. The line that assigns k to the optimal value of k that is resulted in the function of getOptimalDiskNum is commented in the code if divide and conquer method is used an the value of k will be assigned to n/2 where n is the number of disks
5. The line that assigns k to n/2 is commented in case of using dynamic programming and the value of k will be assigned to the value resulted in OptimalK array in the getOptimalDiskNum function

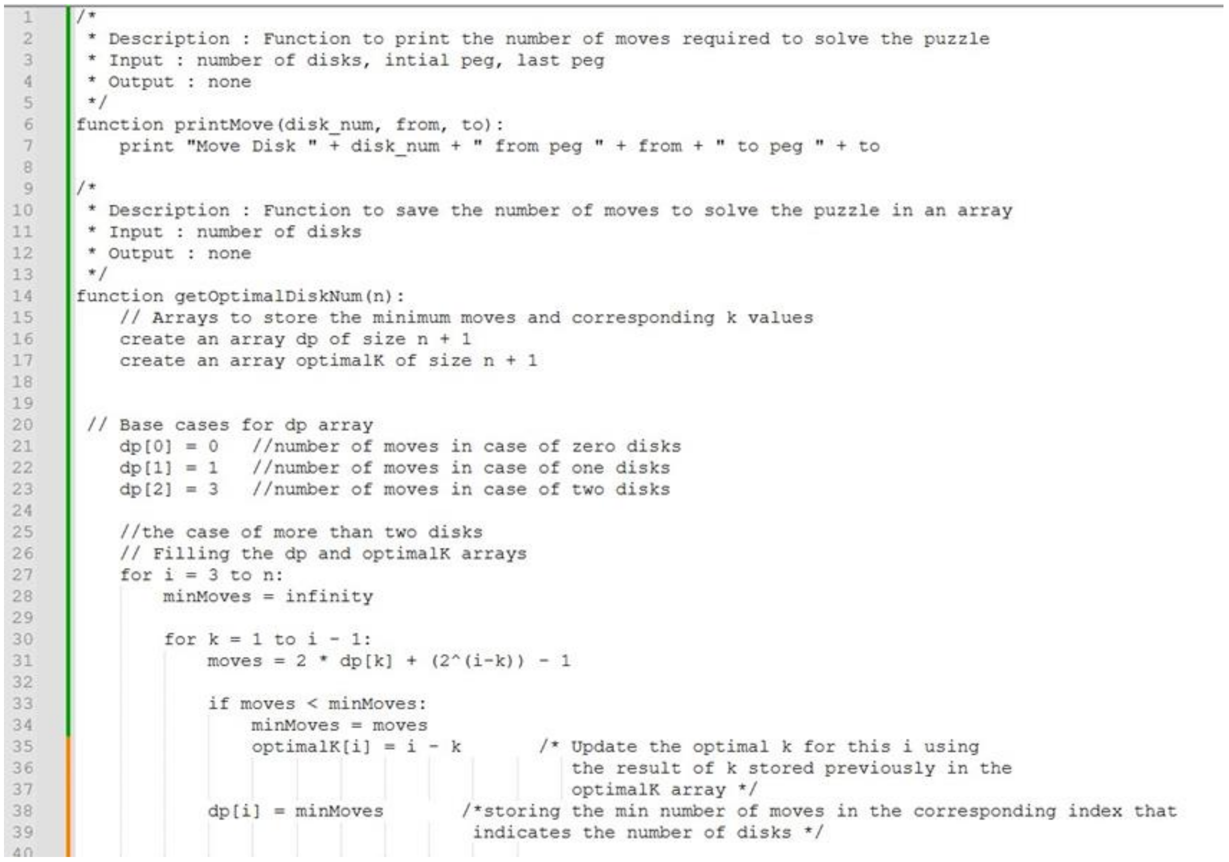
4.3 Detailed Solution

4.3.1 Description of Solution

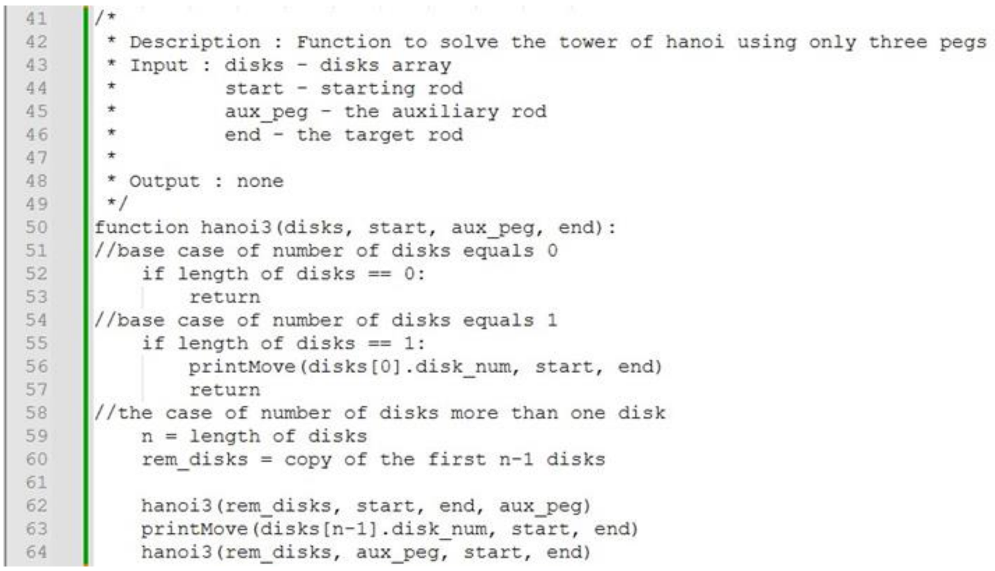
1. Initialize an array dp of size n+1 to store the number of moves required to transfer the disks from the starting peg to the end peg
2. Initialize an array optimalK of size n+1 to store to optimal value of k that will be used to divide the disks in dynamics programming technique while in divide and conquer the disks will be divided into halves
3. The code keep divide the disks into sub-problems until reaches the base case to transfer the disks to another peg

4.3.2 Pseudo Code

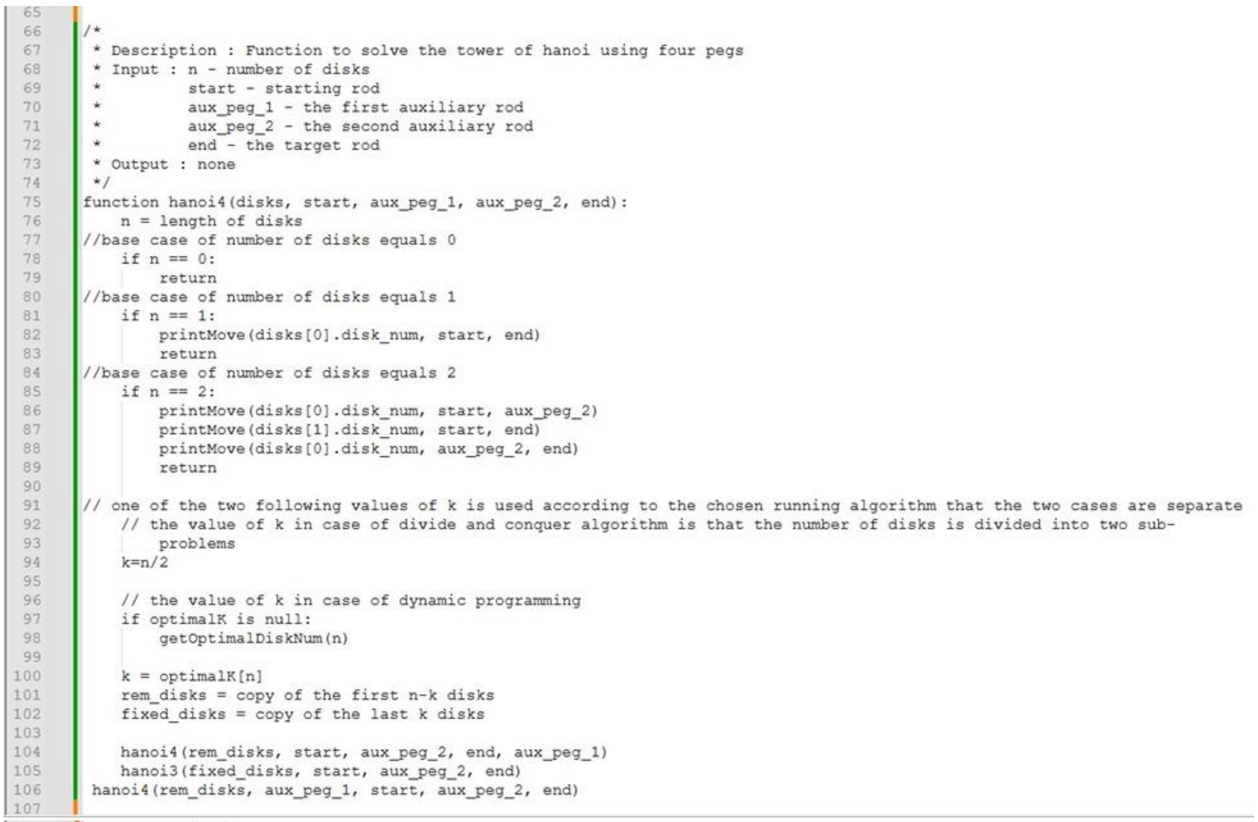
Both divide and conquer algorithm and dynamic programming algorithm are implemented in the following pseudocode



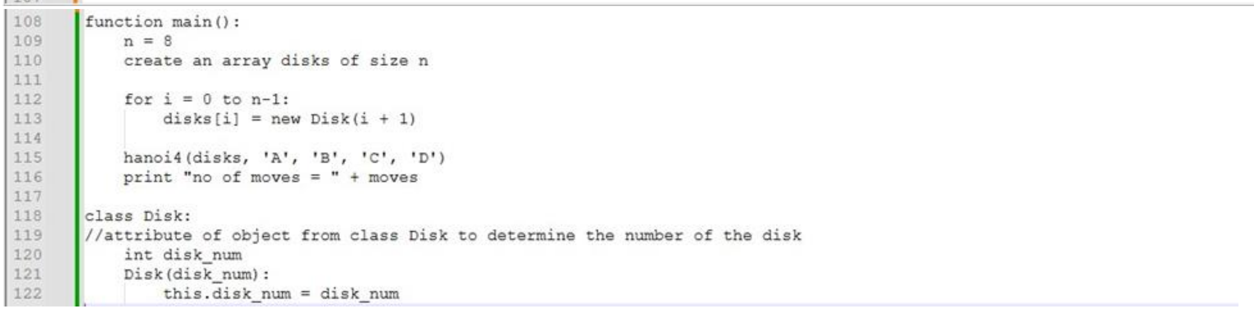
```
/*
 * Description : Function to print the number of moves required to solve the puzzle
 * Input : number of disks, intial peg, last peg
 * Output : none
 */
function printMove(disk_num, from, to):
    print "Move Disk " + disk_num + " from peg " + from + " to peg " + to

/*
 * Description : Function to save the number of moves to solve the puzzle in an array
 * Input : number of disks
 * Output : none
 */
function getOptimalDiskNum(n):
    // Arrays to store the minimum moves and corresponding k values
    create an array dp of size n + 1
    create an array optimalK of size n + 1


 // Base cases for dp array
    dp[0] = 0   //number of moves in case of zero disks
    dp[1] = 1   //number of moves in case of one disks
    dp[2] = 3   //number of moves in case of two disks

    //the case of more than two disks
    // Filling the dp and optimalK arrays
    for i = 3 to n:
        minMoves = infinity

        for k = 1 to i - 1:
            moves = 2 * dp[k] + (2^(i-k)) - 1

            if moves < minMoves:
                minMoves = moves
                optimalK[i] = i - k       /* Update the optimal k for this i using
                                             the result of k stored previously in the
                                             optimalK array */
            dp[i] = minMoves       /*storing the min number of moves in the corresponding index that
                                    indicates the number of disks */
```



```
/*
 * Description : Function to solve the tower of hanoi using only three pegs
 * Input : disks - disks array
 *         start - starting rod
 *         aux_peg - the auxiliary rod
 *         end - the target rod
 *
 * Output : none
 */
function hanoi3(disks, start, aux_peg, end):
//base case of number of disks equals 0
    if length of disks == 0:
        return
//base case of number of disks equals 1
    if length of disks == 1:
        printMove(disks[0].disk_num, start, end)
        return
//the case of number of disks more than one disk
    n = length of disks
    rem_disks = copy of the first n-1 disks

    hanoi3(rem_disks, start, end, aux_peg)
    printMove(disks[n-1].disk_num, start, end)
    hanoi3(rem_disks, aux_peg, start, end)
```



```
/*
 * Description : Function to solve the tower of hanoi using four pegs
 * Input : n - number of disks
 *         start - starting rod
 *         aux_peg_1 - the first auxiliary rod
 *         aux_peg_2 - the second auxiliary rod
 *         end - the target rod
 * Output : none
 */
function hanoi4(disks, start, aux_peg_1, aux_peg_2, end):
    n = length of disks
//base case of number of disks equals 0
    if n == 0:
        return
//base case of number of disks equals 1
    if n == 1:
        printMove(disks[0].disk_num, start, end)
        return
//base case of number of disks equals 2
    if n == 2:
        printMove(disks[0].disk_num, start, aux_peg_2)
        printMove(disks[1].disk_num, start, end)
        printMove(disks[0].disk_num, aux_peg_2, end)
        return

// one of the two following values of k is used according to the chosen running algorithm that the two cases are separate
    // the value of k in case of divide and conquer algorithm is that the number of disks is divided into two sub-
        problems
    k=n/2

    // the value of k in case of dynamic programming
    if optimalK is null:
        getOptimalDiskNum(n)

    k = optimalK[n]
    rem_disks = copy of the first n-k disks
    fixed_disks = copy of the last k disks

    hanoi4(rem_disks, start, aux_peg_2, end, aux_peg_1)
    hanoi3(fixed_disks, start, aux_peg_2, end)
 hanoi4(rem_disks, aux_peg_1, start, aux_peg_2, end)
```



```
function main():
    n = 8
    create an array disks of size n

    for i = 0 to n-1:
        disks[i] = new Disk(i + 1)

    hanoi4(disks, 'A', 'B', 'C', 'D')
    print "no of moves = " + moves

class Disk:
//attribute of object from class Disk to determine the number of the disk
    int disk_num
    Disk(disk_num):
        this.disk_num = disk_num
```

4.4 Complexity Analysis for the Algorithm

1. getOptimalDiskNum function:
    - Time Complexity: O(n^2)
    - Space Complexity: O(n)
2. hanoi3 function:
    - Time Complexity: O(2^n)
    - Space Complexity: O(n)
3. hanoi4 function:
    - Time Complexity: O(2^n)
    - Space Complexity: O(n)
4. main function:
    - Time Complexity: O(2^n)
    - Space Complexity: O(n)

The resulted Time Complexity: O(2^n)

The resulted Space Complexity: O(n)

4.5 Sample Output of the Solution

- **Case of five disks**

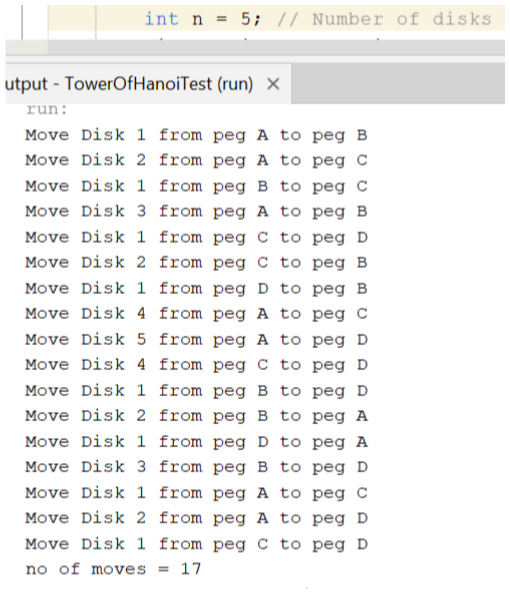
The five disks are divided into two sub-problems of size three and two disks based of the value of optimal k

The upper three disks are ordered using the four rods in the first call of function hanoi4

The lower two disks are ordered using the remaining three rods in the call of function hanoi3

The upper three disks move back again to be ordered above the bottom two disks using the four rods in the second call of function hanoi4

The number of moves is calculated based on dynamic programming that stores the result of the base case which are 0,1, and 2 disks in



```
int n = 5; // Number of disks
```

```
run:
Move Disk 1 from peg A to peg B
Move Disk 2 from peg A to peg C
Move Disk 1 from peg B to peg C
Move Disk 3 from peg A to peg B
Move Disk 1 from peg C to peg D
Move Disk 2 from peg C to peg B
Move Disk 1 from peg D to peg B
Move Disk 4 from peg A to peg C
Move Disk 5 from peg A to peg D
Move Disk 4 from peg C to peg D
Move Disk 1 from peg B to peg D
Move Disk 2 from peg B to peg A
Move Disk 1 from peg D to peg A
Move Disk 3 from peg B to peg D
Move Disk 1 from peg A to peg C
Move Disk 2 from peg A to peg D
Move Disk 1 from peg C to peg D
no of moves = 17
```

FIGURE 3

0, 1, and 3 moves then storing the number of moves in the dp array to be used in the next number of disks

- **Case of six disks**

The six disks are divided into two sub-problems of equal size of three disks each based of the value of optimal k

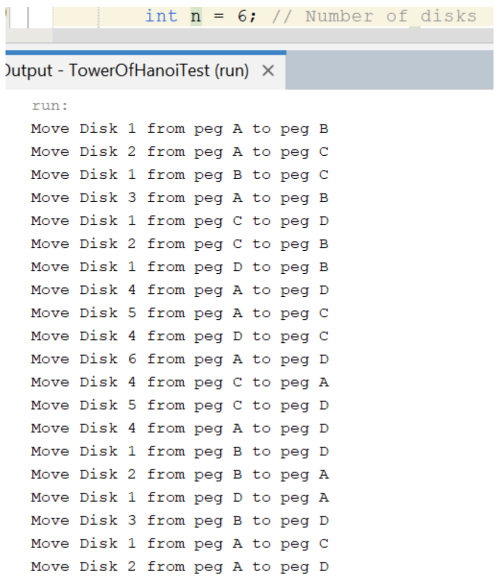
The upper three disks are ordered using the four rods in the first call of function hanoi4

The lower three disks are ordered using the remaining three rods in the call of function hanoi3

The upper three disks move back again to be ordered above the bottom three disks using the four rods in the second call of function hanoi4

The number of moves is calculated based on

dynamic programming that stores the result of the base cases which are 0,1, and 2 disks in 0, 1, and 3 moves then storing the number of moves in the dp array to be used in the next numberof disks



```
int n = 6; // Number of disks
```

```
run:
Move Disk 1 from peg A to peg B
Move Disk 2 from peg A to peg C
Move Disk 1 from peg B to peg C
Move Disk 3 from peg A to peg B
Move Disk 1 from peg C to peg D
Move Disk 2 from peg C to peg B
Move Disk 1 from peg D to peg B
Move Disk 4 from peg A to peg D
Move Disk 5 from peg A to peg C
Move Disk 4 from peg D to peg C
Move Disk 6 from peg A to peg D
Move Disk 4 from peg C to peg A
Move Disk 5 from peg C to peg D
Move Disk 4 from peg A to peg D
Move Disk 1 from peg B to peg D
Move Disk 2 from peg B to peg A
Move Disk 1 from peg D to peg A
Move Disk 3 from peg B to peg D
Move Disk 1 from peg A to peg C
Move Disk 2 from peg A to peg D
```

FIGURE 4

- **Case of eight disks**

The six disks are divided into two sub-problems of equal size of four disks each based of the value of optimal k

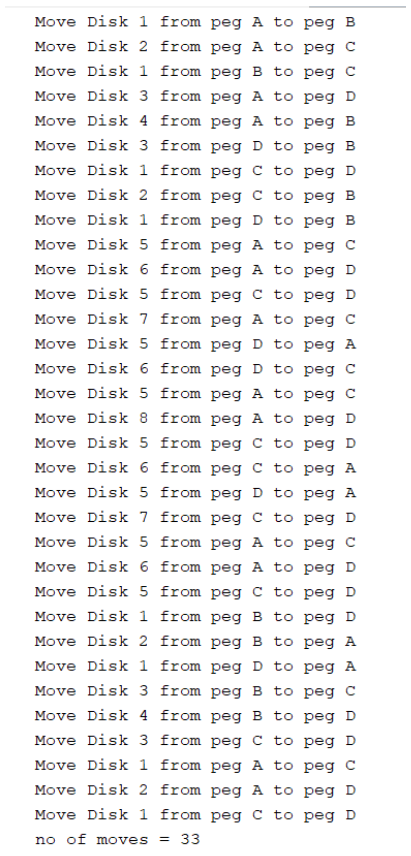
In the first call of hanoi4

The upper four disks are ordered by recursively calling hanoi4 and divide the four disks into two sub-problems of the same size of two disks each based on the value of optimal k

The upper two disks (disk 1 and 2) ordered on peg C using the four rods in the call of function hanoi4 while the lower two disks (disk 3 and 4) are ordered on peg B using the three rods in the call of function hanoi3 then the upper two disks are ordered above the lower two disks on peg B using the four rods in the call of function hanoi4

The lower four disks are ordered by calling hanoi3 and recursively reduce the problem by one into sub-problem until it reaches the base case which is two disks (disk 5 an disk 6) then order disk 7 then order disk 8 on peg D

In the second call of hanoi4



```
Move Disk 1 from peg A to peg B
Move Disk 2 from peg A to peg C
Move Disk 1 from peg B to peg C
Move Disk 3 from peg A to peg D
Move Disk 4 from peg A to peg B
Move Disk 3 from peg D to peg B
Move Disk 1 from peg C to peg D
Move Disk 2 from peg C to peg B
Move Disk 1 from peg D to peg B
Move Disk 5 from peg A to peg C
Move Disk 6 from peg A to peg D
Move Disk 5 from peg C to peg D
Move Disk 7 from peg A to peg C
Move Disk 5 from peg D to peg A
Move Disk 6 from peg D to peg C
Move Disk 5 from peg A to peg C
Move Disk 8 from peg A to peg D
Move Disk 5 from peg C to peg D
Move Disk 6 from peg C to peg A
Move Disk 5 from peg D to peg A
Move Disk 7 from peg C to peg D
Move Disk 5 from peg A to peg C
Move Disk 6 from peg A to peg D
Move Disk 5 from peg C to peg D
Move Disk 1 from peg B to peg D
Move Disk 2 from peg B to peg A
Move Disk 1 from peg D to peg A
Move Disk 3 from peg B to peg C
Move Disk 4 from peg B to peg D
Move Disk 3 from peg C to peg D
Move Disk 1 from peg A to peg C
Move Disk 2 from peg A to peg D
Move Disk 1 from peg C to peg D
no of moves = 33
```

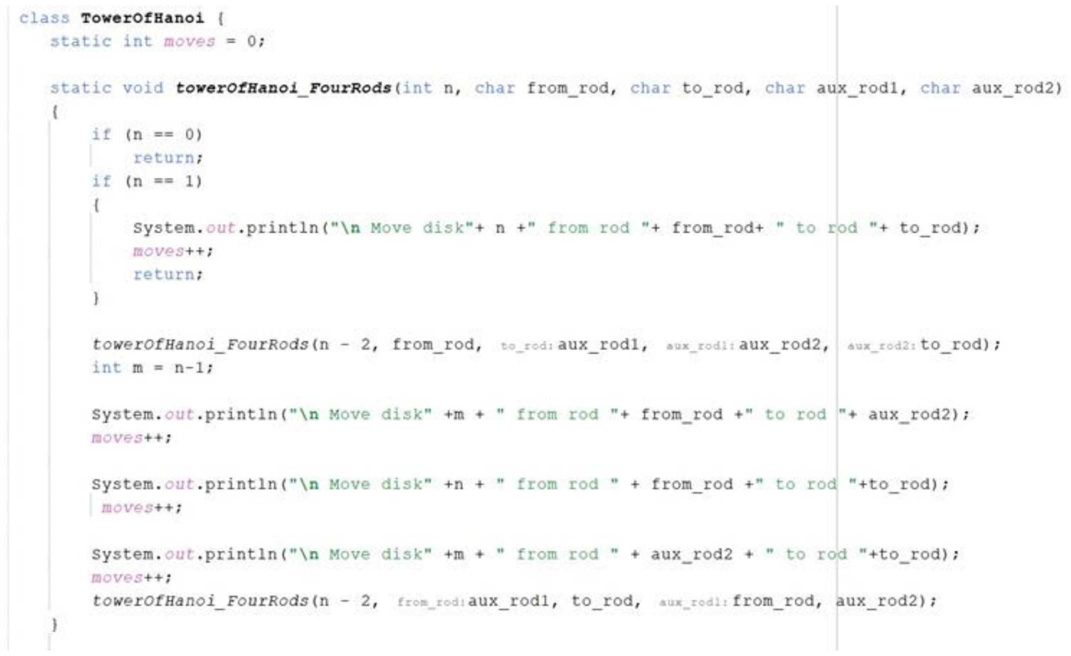
FIGURE 5

The upper four disks are ordered by recursively calling hanoi4 and again divide the four disks into two sub-problems of the same size of two disks each based on the value of optimal k

The upper two disks (disk 1 and 2) ordered on peg A using the four rods in the call of function hanoi4 while the lower two disks (disk 3 and 4) are ordered on peg D using the three rods in the call of function hanoi3 then the upper two disks are ordered above the lower two disks on peg D using the four rods in the call of function hanoi4.

4.6 Comparison with Another Algorithm

The solution using decrease and conquer



```
class TowerOfHanoi {
    static int moves = 0;

    static void towerOfHanoi_FourRods(int n, char from_rod, char to_rod, char aux_rod1, char aux_rod2)
    {
        if (n == 0)
            return;
        if (n == 1)
        {
            System.out.println("\n Move disk"+ n +" from rod "+ from_rod+ " to rod "+ to_rod);
            moves++;
            return;
        }

        towerOfHanoi_FourRods(n - 2, from_rod,  to_rod: aux_rod1,  aux_rod1: aux_rod2,  aux_rod2: to_rod);
        int m = n-1;

        System.out.println("\n Move disk" +m + " from rod "+ from_rod +" to rod "+ aux_rod2);
        moves++;

        System.out.println("\n Move disk" +n + " from rod " + from_rod +" to rod "+to_rod);
         moves++;

        System.out.println("\n Move disk" +m + " from rod " + aux_rod2 + " to rod "+to_rod);
        moves++;
        towerOfHanoi_FourRods(n - 2,  from_rod: aux_rod1, to_rod,  aux_rod1: from_rod, aux_rod2);
    }
```

TABLE 4

	Divide and Conquer	Decrease and Conquer
Advantages	Efficient for large problem size in case the number of disks increase	Simplicity as it reduces the problem by one until reaches the base case Lower Memory requirements as it often operate on the problem in place
Disadvantages	Overhead of combining sub-problems Increase memory usage	May not be efficient for large problems
Number of moves	Transfers 8 disks in 33 moves	Transfers 8 disks in 41 moves

4.7 Conclusion

In Conclusion, using Divide and Conquer with the Dynamic Programming approach to divide the problem to two sub-problems and calculate the minimum number of moves required for different number of disks and storing them in the memory leads to optimizing the solution that offers a modified approach to solving the Tower of Hanoi Problem using four pegs, allowing more efficient solution compared to the traditional approach.

4.8 References

The Four- Peg Tower of Hanoi Puzzle by Richard Johnsonbaugh.

Task 5 : Jumping Coins

5.1 Problem Description

“There are n coins placed in a row. The goal is to form n/2 pairs of them by a sequence of moves. On the first move a single coin has to jump over one coin adjacent to it, on the second move a single coin has to jump over two adjacent coins, on the third move a single coin has to jump over three adjacent coins, and so on, until after n/2 moves n/2 coin pairs are formed. (On each move, a coin can jump right or left but it has to land on a single coin. Jumping over a coin pair counts as jumping over two coins. Any empty space between adjacent coins is ignored.) Determine all the values of n which the problem has a solution and design an algorithm that solves it in the minimum number of moves for those n’s. Design greedy algorithm to find minimum number of moves.”

5.2 Detailed Assumptions

We assumed that in both solutions either the greedy or the brute force, the algorithm will start from the very first coin in the left most of the row and work its way to the right direction. As it’s arbitrary to start from any coin and work its way in any direction and there’s no constraints or cases that will go against this assumption, so there’s no problem to assume that we will start from that coin and in that direction.

5.3 Detailed Solution

5.3.1 Description of Solution

If we try by hand to solve this problem, we will find that some values of n work fine and other doesn't, some of the values that works are 4, 12 and 28. We need to observe the pattern or the mathematical formula, and after that use it to deduce the next values, then finally test those values to see if the pattern or the mathematical formula was right or wrong. After looking for some formulas and pattern, there's only one simple pattern or formula that satisfies the problem constraints, and it is as follows:
If we assume that the value of n is equal to the sum of two big piles of mathematically calculated number of fours, then you could put the 4, 12 and 28 as follows: 4 = (0) + (4), 12 = (4) + (4+4), 28 = (4+4+4) + (4+4+4+4) then you we can deduce a mathematical formula based on that, and it is as follows:
n = ((2^(i)-1) + (2^ (i))) * 4 = (2^(i+3)) - 4 where i is a non-negative integer.
Then for i=0 -> n = 4, for i=1 -> n=12, for i=2 -> n=28, for i=3 -> n=60, etc.…
If you try n=60, it will work. So, the deduced formula is right, hence that was one of the requirements, what are the values of n that satisfies the problem constraints? **(2^(i+3)) – 4 where i is a non-negative integer.**

5.3.2 Greedy Technique

5.3.2.1 Key Idea

Simply start from the left most of the row, set the number of moves to zero and start moving the first coin according to the constraints of the problem to form a pair in the left direction, then next ***choose the nearest coin to the empty space that it was left behind (greedy step)***, increment the number of moves then repeat the steps again until you form n/2 coin pairs in just ***n/2 moves which is logically the minimum amount of moves required to form n/2 coin pairs.***

5.3.2.2 Pseudo Code

5.3.2.2.1 High-Level

1. Set the current move to 0.
2. Start from the leftmost coin.
3. Iterate through the row until you reach the coin before last.
   3.1 For each single coin you find while iterating, start to the column to the left of it.
      3.1.1 Increase the current move by one.
      3.1.2 Set the count of jumped over coins for this specific coin to 0.
      3.1.3 If the current jumped over coins count equals the current move, then put the single coin on the current column to form a pair, then break out of this loop to go to the next nearest single coin to the previous one that we selected.
      3.1.4 Else if the current jumped over coins count doesn't equal the current move, then increase the jumped over coins by the number of coins in the current column and go to the next left column.

5.3.2.2.2 In Details

```
numberOfMoves <- 0

For i = 0 to n-1 {

            // skip empty spaces or coin pairs until you find the nearest coin
(greedy step)

            if(rowOfCoins[i] != 1)

                continue


            numberOfMoves <- numberOfMoves + 1

            jumpedOverCoins <- 0


            // making a coin move to form a pair of coins

            For j=i+1 to n-1 {

                if (jumpedOverCoins == numberOfMoves) {

                    rowOfCoins[i]  <- 0;

                    rowOfCoins[j] <- rowOfCoins[j] + 1

                    break

                }

                // counting the number of jumped over coins
                jumpedOverCoins <- jumpedOverCoins + rowOfCoins[j];
            }
}
```

5.3.3 Brute-Force Technique

5.3.3.1 Key Idea

Again simply start from the left most of the row, set the number of moves and start moving the first coin according to the constraints of the problem to form a pair in the right direction of the row, and if no place was found satisfying the constraints to form a coin pair then search in the left direction for a place, if also no place was found satisfying the constraints then we go back one step to the previous pair that was formed, unform it then try to find another different place for that previous singe coin to form a pair then go to the next single coin that we couldn't find a place for it to form a pair in the first place, then we try to find a place for it, eventually we will repeat the steps and make our way brute forcing it with those steps until we make n/2 coin pairs in just n/2 moves which is logically the minimum amount of moves required to form n/2 coin pairs.

5.3.3.2 Pseudo Code

5.3.3.2.1 High-Level

1. Start by calling the function to run at the left most position and operate in the right direction of the row.
2. Check if the position passed is valid, if not then return false.
3. Set the number of moves to 1 and make a flag that determines if we are done or not and set it to false.
4. Set the count of jumped over coins to 0.
5. Start iterating from the column to the to the left or right of the passed position as it was specified in the function call, and do the following in each iteration:
    - 5.1 If the current jumped over coins count equals the current move, then put the single coin at the specified position on the current column to form a coin pair, increase the current move number by one, set the flag to true, then finally break out of this loop to go to the next nearest single coin to the previous one that we selected.
    - 5.2 Else if the current jumped over coins are greater than the current move then breaks out of the loop.
    - 5.3 Else if the current jumped over coins count are less than the current move, then increase the jumped over coins by the number of coins in the current

column and go to the next left or right column as the direction was specified at the beginning of the loop in the function call.

6. Check if there's still any single coins left and if so, set the flag to false.
7. If the flag is true, then return true.
8. Determine the next position of the next single coin that will be moved to form a coin pair.
9. Call the function again with the new position calculated in the right direction (go to step 2) and set the returned value out of the function to the flag.
10. If the flag is true, then return true.
11. Call the function again with the new position calculated in the left direction (go to step 2) and set the returned value out of the function to the flag.
12. Return the flag value.

5.3.3.2.2 In Details

```
algorithmUsingBruteForce (position, currentMoveNumber, direction) {

        if ((position < 0) || (position > n))
            return false

        numberOfMoves  ← currentMoveNumber
        jumpedOverCoins  ← 0
        doneFlag ← false

        // check in the right direction the possibility to form a pair
        if (direction == "right")) {
            for i ← position+1 to n-1 {
                // move the coin to the column if the condition is satisfied
                // or break if exceeded
                If (jumpedOverCoins >= numberOfMoves) {
                    if (jumpedOverCoins == numberOfMoves) {
                        rowOfCoins[position]  ← 0
                        rowOfCoins[i]  ← rowOfCoins[i] + 1
                        currentMoveNumber  ← currentMoveNumber + 1
                        doneFlag  ← true
                    }
                    break
                }
                // or jump over the column and count the jumped over coins
                else if (jumpedOverCoins < numberOfMoves)
                    jumpedOverCoins  ← jumpedOverCoins + rowOfCoins[i]
            }
        }
        // check in the left direction the possibility to form a pair
        else if (direction == "left")) {
            for  i ← position-1 to 0 {
                // move the coin to the column if the condition is satisfied
                // or break if exceeded
                If (jumpedOverCoins >= numberOfMoves) {
                    if (jumpedOverCoins == numberOfMoves) {
                        rowOfCoins[position]  ← 0
                        rowOfCoins[i]  ← rowOfCoins[i] + 1
                        currentMoveNumber  ← currentMoveNumber + 1
                        doneFlag ← true
                    }
                    break
                }

                // or jump over the column and count the jumped over coins
                else if (jumpedOverCoins < numberOfMoves)
                    jumpedOverCoins  ← jumpedOverCoins + rowOfCoins[i]
            }
        }

        // check if there're still single coins
        For i in rowOfCoins {
            if (i == 1) {
                doneFlag ← false
                break
            }
        }
        // check if all are the coins are pairs
        // if so, then we are done
        if (doneFlag == true)
            return true

        // next single coin to be moved to form a pair
        int nextPosition ← position;
        for i ←position+1 to n-1{
            if (rowOfCoins[i] == 1){
                nextPosition ← i
                break
            }
        }

        doneFlag ← algorithmUsingBruteForce (nextPosition, currentMoveNumber,
"right")

        // still, there're single coins?
        If (doneFlag == true)
            return true

        doneFlag ← algorithmUsingBruteForce (nextPosition, currentMoveNumber,
"left")

        return doneFlag
    }
```

5.4 Complexity Analysis for the Algorithm

5.4.1 Greedy Technique

In general, you loop over n in the left direction skipping every empty space and every coin pair that was formed, until you find a single coin, then you start to iterate in the same left direction until you find the right spot to put that single coin so you form a coin pair, then repeat.

For the first coin, you will jump over one coin, for the second you will jump over two coins, etc.…

As an approximation we could say that the whole time is upper bounded by:
[1 + 2 + 3 + 4 + … + n/2] + n/2 where each number represents a coin that we iterated that number of times to find the right spot for it, except the last (n/2) it represents the skipped empty spaces and coin pairs that we looped through, but we didn't select any of them.

So, [1 + 2 + 3 + 4 + … + n/2] + n/2 = ((n/2) * (n/2 + 1))/2 + (n/2) = ((n^2) / 8) + (3n/4) which is O(n^2) in all cases.

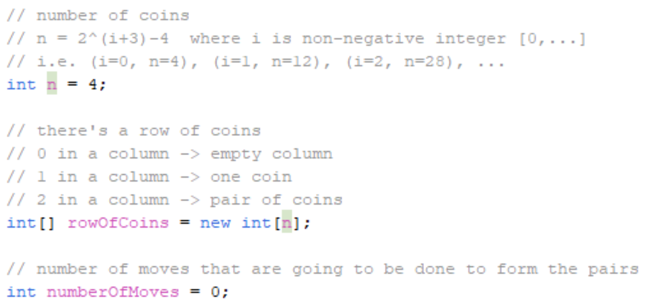
5.4.1 Brute Force Technique

It is expected that the brute force technique will have a very high running time, higher than the greedy technique by a lot as it goes through a lot of paths most of them are wrong until it reaches the right path that leads to the solution, but the greedy chooses the greediest path from the start which is likely to be the solution or an approximate to it.

But in our case, and due to the assumption that we start in both algorithms from the left most coin and work our way in the right direction, the brute force technique will reach the right path from the first try as if it was the greedy technique, and then it will stop.

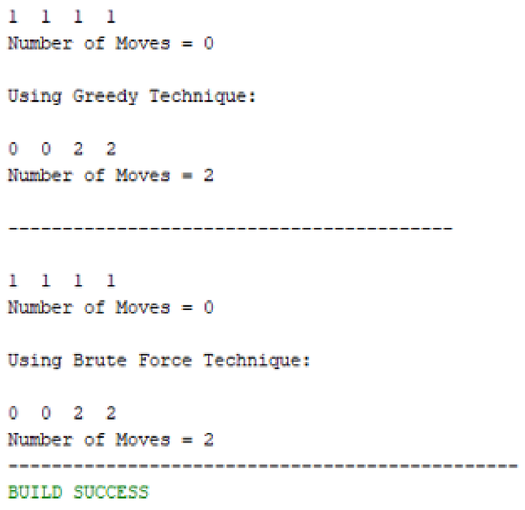
So, the brute force technique will be upper bounded by approximately the same running time as the greedy technique which is O(n^2) and that is in all cases.

5.5 Sample Output of the Solution



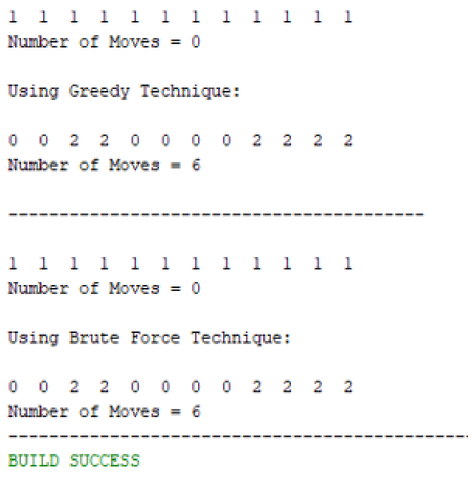
```
// number of coins
// n = 2^(i+3)-4  where i is non-negative integer [0,...]
// i.e. (i=0, n=4), (i=1, n=12), (i=2, n=28), ...
int n = 4;

// there's a row of coins
// 0 in a column -> empty column
// 1 in a column -> one coin
// 2 in a column -> pair of coins
int[] rowOfCoins = new int[n];

// number of moves that are going to be done to form the pairs
int numberOfMoves = 0;
```

Where n = 4



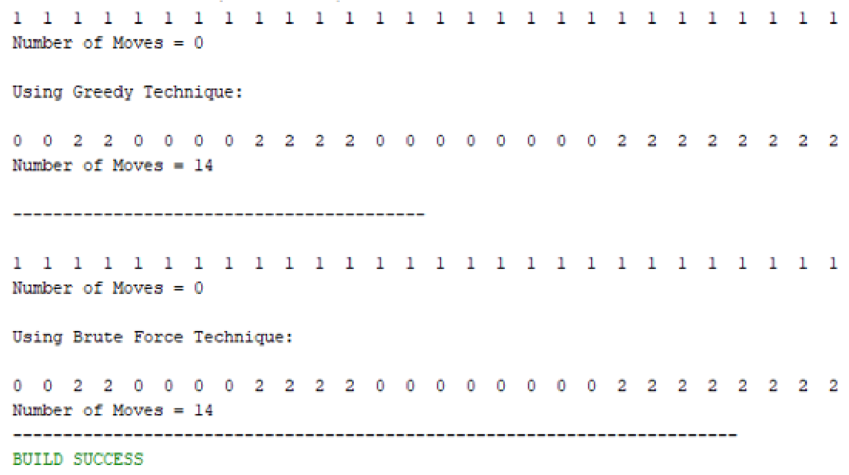
```
1  1  1  1
Number of Moves = 0

Using Greedy Technique:

0  0  2  2
Number of Moves = 2

-----------------------------------------

1  1  1  1
Number of Moves = 0

Using Brute Force Technique:

0  0  2  2
Number of Moves = 2
-----------------------------------------------
BUILD SUCCESS
```

Where n = 12



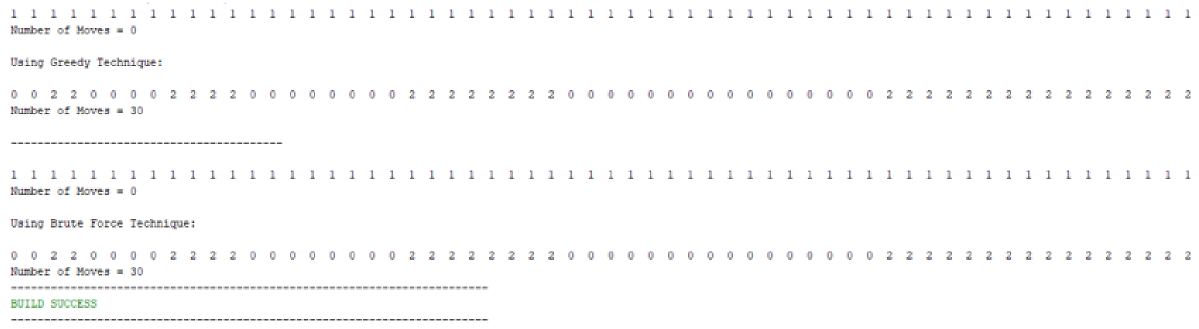
```
1  1  1  1  1  1  1  1  1  1  1  1
Number of Moves = 0

Using Greedy Technique:

0  0  2  2  0  0  0  0  2  2  2  2
Number of Moves = 6

-----------------------------------------

1  1  1  1  1  1  1  1  1  1  1  1
Number of Moves = 0

Using Brute Force Technique:

0  0  2  2  0  0  0  0  2  2  2  2
Number of Moves = 6
----------------------------------------------
BUILD SUCCESS
```

Where n = 28



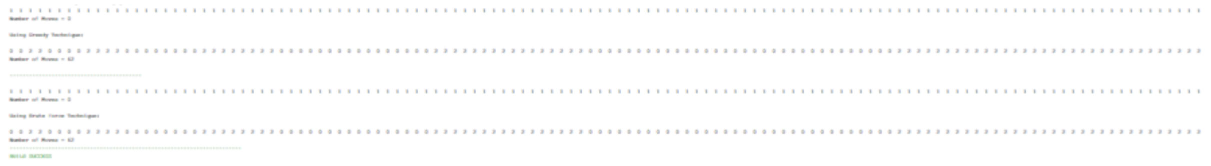
```
1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
Number of Moves = 0

Using Greedy Technique:

0  0  2  2  0  0  0  0  2  2  2  2  0  0  0  0  0  0  0  0  2  2  2  2  2  2  2  2
Number of Moves = 14

-----------------------------------------

1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
Number of Moves = 0

Using Brute Force Technique:

0  0  2  2  0  0  0  0  2  2  2  2  0  0  0  0  0  0  0  0  2  2  2  2  2  2  2  2
Number of Moves = 14
------------------------------------------------------------------------
BUILD SUCCESS
```

Where n = 60



```
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
Number of Moves = 0

Using Greedy Technique:

0 0 2 2 0 0 0 0 2 2 2 2 0 0 0 0 0 0 0 0 2 2 2 2 2 2 2 2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
Number of Moves = 30

-----------------------------------------

1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
Number of Moves = 0

Using Brute Force Technique:

0 0 2 2 0 0 0 0 2 2 2 2 0 0 0 0 0 0 0 0 2 2 2 2 2 2 2 2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
Number of Moves = 30
------------------------------------------------------------------------
BUILD SUCCESS
------------------------------------------------------------------------
```

Where n = 124



```
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
Number of Moves = 0

Using Greedy Technique:

0 0 2 2 0 0 0 0 2 2 2 2 0 0 0 0 0 0 0 0 2 2 2 2 2 2 2 2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
Number of Moves = 62

-----------------------------------------

1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
Number of Moves = 0

Using Brute Force Technique:

0 0 2 2 0 0 0 0 2 2 2 2 0 0 0 0 0 0 0 0 2 2 2 2 2 2 2 2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
Number of Moves = 62
------------------------------------------------------------------------
BUILD SUCCESS
```

5.6 Comparison Between the two techniques

As the two techniques has two different methods of finding the solution. But like discussed above, the two techniques will result in two different algorithms each will solve the problem and find the solution in approximately similar running time in all cases due to the problem nature and the assumption and implementations that were made.

5.7 Conclusion

The values of n that satisfies the problem constraints are equal to **(2^(i+3)) – 4, where i is a non-negative integer** and two algorithms were designed one using the greedy technique and the other one using the brute-force technique to solve the problem.

There might be in some cases, two different techniques that could produce two different algorithms each one of them has its own way of finding the solution to the problem but due to the problem nature, implementation, assumptions and so on, they could end up with the same time complexity, like in our case in the discussed problem in task 5.

In our case we saw the problem solved by two different techniques the first one is the main one that was required which is the greedy technique which produced a solution that was upper-bounded by ***O(n^2)*** in all cases, and the second technique was the brute-force technique which could have led to a much higher running time but as we said due to the assumptions, implementation and the problem nature that helped in our case, the algorithm based on the brute-force technique produced a solution that was upper-bounded by the same running time as the algorithm based on the greedy technique which is ***O(n^2).***

Task 6 : Six Knights

6.1 Problem Description

There are six knights on a 3 × 4 chessboard: the three white knights are at the bottom row, and the three black knights are at the top row.
Design an iterative improvement algorithm to exchange the knights to get the position shown on the right of the figure in the minimum number of knights moves, not allowing more than one knight on a square at any time.

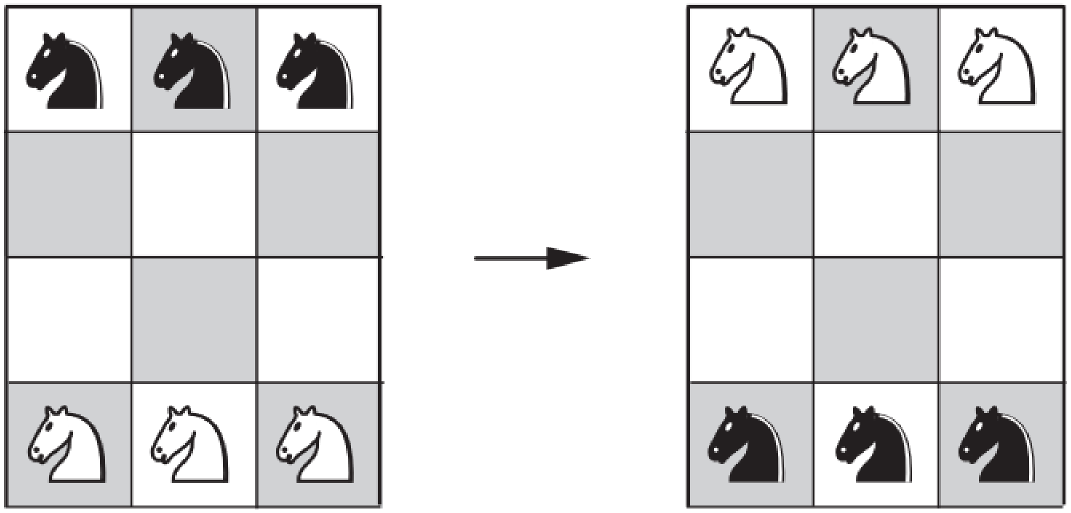


FIGURE 6

6.2 Detailed Assumptions

1. A chessboard of 4X3 is assumed.
2. Knights must follow an L-shaped pattern.
3. A knight can't go outside the board boundaries.
4. Each Knight placement in a board is a State.
5. Best first search is implemented using heuristics.
6. The heuristic is implemented using Manhattan distance.
7. Once in a path you can't visit the same state twice.

6.3 Detailed Solution

6.3.1 Description of Solution

1. An initial state is passed in the code.
2. The code starts by Generating moves for knights starting with the white knights.
3. Each state is stored inside a path and each path is stored inside priority queue that is sorted based on heuristic value.
4. The heuristic value of the path is the heuristic value of the last state of this path.
5. The top of the queue (the one with the lowest heuristic value) is hypothetically the closest board arrangement to our solution.
6. If our board arrangement is found its Heuristic will equal -1

6.3.2 Pseudo Code

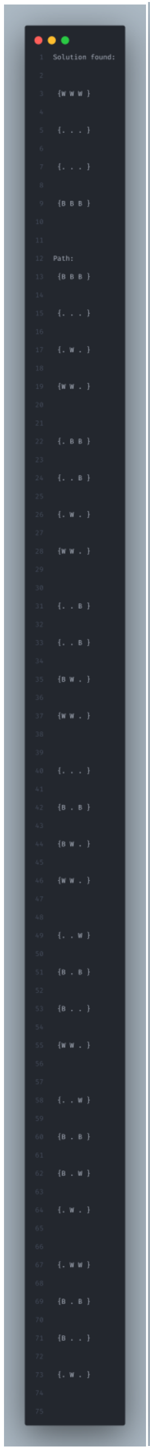
```
FUNCTION calculateHeuristic(board)
// Inputs: Board is the 2D vector representing the game board
// Output: Returns the heuristic value calculated based on the board state
    heuristic <= 0


    // Define goal positions for white and black knights
    whiteGoalPositions <= {{0, 0}, {0, 1}, {0, 2}}
    blackGoalPositions <= {{3, 0}, {3, 1}, {3, 2}}


    FOR i FROM 0 TO ROWS - 1 DO
        FOR j FROM 0 TO COLS - 1 DO
            IF board[i][j] == WHITE THEN
                minDist <= INT_MAX
                FOR EACH goalPos IN whiteGoalPositions DO
                    dist <= ABS(i - goalPos.first) + ABS(j - goalPos.second)
                    minDist <= MIN(minDist, dist)
                END FOR
                heuristic += minDist
            ELSE IF board[i][j] == BLACK THEN
                minDist <= INT_MAX
                FOR EACH goalPos IN blackGoalPositions DO

                    dist <= ABS(i - goalPos.first) + ABS(j - goalPos.second)

                    minDist <= MIN(minDist, dist)

                END FOR

                heuristic += minDist

            END IF

        END FOR

    END FOR


    IF isGoalState(board) THEN

        RETURN -1

    END IF


    RETURN heuristic / 3.0 + 3

END FUNCTION




FUNCTION isGoalState(board)

// Inputs: board is the 2D vector representing the game board

// Output: Returns boolean true if the board represents a goal state

    RETURN (board[0][0] == WHITE AND board[0][1] == WHITE AND board[0][2] == WHITE)
AND (board[3][0] == BLACK AND board[3][1] == BLACK AND board[3][2] == BLACK)

END FUNCTION


FUNCTION getKnightMoves(x, y, board)

// Inputs: x is the X coordinate of the knight, y is the Y coordinate of the knight,
board is the 2D vector representing the game board

// Output: Returns a list of valid knight moves from the given position

    moves <= []


    directions <= {{1, 2}, {-2, 1}, {-2, -1}, {-1, -2}, {1, -2}, {2, -1}, {2, 1}, {-
1, 2}}
```

```
    FOR EACH dir IN directions DO

        newX <= x + dir.first

        newY <= y + dir.second

        IF isEmptyAndWithinLimits(board, newX, newY) THEN

            moves.push({newX, newY})

        END IF

    END FOR


    RETURN moves

END FUNCTION


FUNCTION generateNextState(currentState, pq, currentPath)

// Inputs: currentState is the current state of the game, pq is the priority queue of
paths, currentPath is the current path

// Output: Generates and pushes next path to the priority queue

    FOR i FROM 0 TO ROWS - 1 DO

        FOR j FROM 0 TO COLS - 1 DO

            IF currentState.board[i][j] != 0 THEN

                moves <= getKnightMoves(i, j, currentState.board)

                FOR EACH move IN moves DO

                    nextBoard <= COPY currentState.board

                    SWAP nextBoard[i][j] AND nextBoard[move.first][move.second]

                    nextHeuristic <= calculateHeuristic(nextBoard)

                    nextState <= State(nextBoard, nextHeuristic)

                    IF NOT currentPath.hasDupeStates(nextState) THEN

                        currentPath.push(nextState)

                        pq.push(currentPath)

                    END IF

                END FOR

            END IF

        END FOR
    END FOR
END FUNCTION

// Performs Best-First Search to find the solution
FUNCTION bestFirstSearch(initialState)
// Inputs: initialState is the initial state of the game
// Output: Prints Path leading to solution if found
    pq <= Priority Queue of Paths
    initialHeuristic <= calculateHeuristic(initialState)
    beginningState <= State(initialState, initialHeuristic)
    initialPath <= Path(beginningState)
    pq.push(initialPath)

    WHILE NOT pq.empty() DO
        currentPath <= pq.top()
        currentState <= currentPath.path[currentPath.path.size() - 1]
        pq.pop()

        IF isGoalState(currentState.board) OR currentState.heuristic == -1 THEN
            PRINT "\n\nSolution found:"
            printBoard(currentState.board)
            PRINT "Path: "
            currentPath.printPath()
            PRINT "\nTotal moves: " + currentPath.length() - 1
            PRINT "\nTotal paths: " + pq.size()
            RETURN
        END IF

        IF currentPath.length() <= MAX_MOVES THEN
            generateNextState(currentState, pq, currentPath)
        END IF
    END WHILE


    PRINT "No solution found."
END FUNCTION


FUNCTION withinLimits(x, y)
// Inputs: x is the X coordinate of the knight, y is the Y coordinate of the knight
// Output: Returns boolean true if the coordinates are within the board limits
    RETURN (x >= 0 AND y >= 0) AND (x < ROWS AND y < COLS)
END FUNCTION


FUNCTION isEmptyAndWithinLimits(board, x, y)
// Inputs: board is the 2D vector representing the game board, x is the X coordinate
of the knight, y is the Y coordinate of the knight
// Output: Returns boolean true if the knight movement is within the board and the
target square is empty
    RETURN withinLimits(x, y) AND board[x][y] == 0
END FUNCTION
```

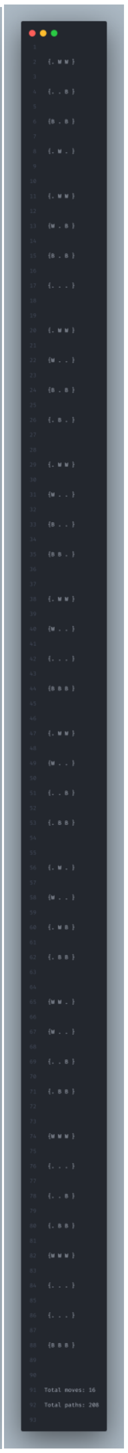
6.4 Complexity Analysis for the Algorithm

Time Complexity: O(N * Log(N)).
Space complexity: O(N).
Where N is the number of paths in priority queue.

6.5 Sample Output of the Solution



```
Solution found:

 {W W W }

 {. . . }

 {. . . }

 {B B B }


Path:
 {B B B }

 {. . . }

 {. W . }

 {W W . }


 {. B B }

 {. . B }

 {. W . }

 {W W . }


 {. . B }

 {. . B }

 {B W . }

 {W W . }


 {. . . }

 {B . B }

 {B W . }

 {W W . }


 {. . W }

 {B . B }

 {B . . }

 {W W . }


 {. . W }

 {B . B }

 {B . W }

 {. W . }


 {. W W }

 {B . B }

 {B . . }

 {. W . }
```



```

 {. W W }

 {. . B }

 {B . B }

 {. W . }


 {. W W }

 {W . B }

 {B . B }

 {. . . }


 {. W W }

 {W . . }

 {B . B }

 {. B . }


 {. W W }

 {W . . }

 {B . . }

 {B B . }


 {. W W }

 {W . . }

 {. . . }

 {B B B }


 {. W W }

 {W . . }

 {. . B }

 {. B B }


 {. W . }

 {W . . }

 {. W B }

 {. B B }


 {W W . }

 {W . . }

 {. . B }

 {. B B }


 {W W W }

 {. . . }

 {. . B }

 {. B B }

 {W W W }

 {. . . }

 {. . . }

 {B B B }


Total moves: 16
Total paths: 208
```

Each state Generated in turn generates more states each chain of states is a path so this code starts with initial state generates a new state and adds it to this state and it's a path so this path is put into Priority queue where it's sorted based on Heuristic values of each state each new iteration it takes top path of queue and then it generates new states for it and so on till we reach goal.

6.6 Comparison with Another Algorithm

TABLE 5

	Brute force Approach	**Best first search**
Advantages	Guarantees to find a solution if one exists	A faster approach then brute force and can work with any size of chessboard
Disadvantages	A huge time complexity	Not guaranteed to find a solution if heuristic is poorly chosen
Time Complexity	8^N where N is the number of knights.	O(N*Log(N)) where N is the number of paths in Priority queue

6.7 Conclusion

Solving the Six knights problem requires one of 2 approaches Brute force, Searching algorithms (best first, Hill climbing, A*).
The brute force method Generates all possible moves combinations till it reaches the goal, but can yield very high time complexity, and isn't feasible for bigger chessboards.
Best first algorithm is better than brute force if heuristic calculations are good and can reach solution a lot faster than brute force approach, this approach is an approximation so it might not get the best path each time and it might encounter convergence problems.

6.8 References

- Best first search
- Wiki
- Puzzle stack exchange
- Princeton university pages 3,4 and 5.
- Carnegie Mellon University.

Task 7 : Hitting a Moving Target

7.1 Problem Description

A computer game has a shooter and a moving target. The shooter can hit any of n > 1 hiding spot located along a straight line in which the target can hide. The shooter can never see the target; all he knows is that the target moves to an adjacent hiding sp ot between every two consecutive shots.
Design a Dynamic Programing algorithm that guarantees hitting the target

7.2 Detailed Assumptions

Target Movement

- The target moves between adjacent spots in a straight line.
- There's no limit on the number of times the target can move.

Shooter

- The shooter can take one shot at a time, aiming at any of the spots.
- The shot is always successful if aimed at the spot where the target currently is.

Other Assumptions

- The number of spots (n) is greater than or equal to 1 (there must be at least one spot).

7.3 Detailed Solution

7.3.1 Description of Solution

We stimulate the target movements and we store the bullets in each hiding spot Then move forward in our hitting spot until we hit the spot n-1 then we hit the way back until we hit the spot 2 with this we guarantee hitting the target

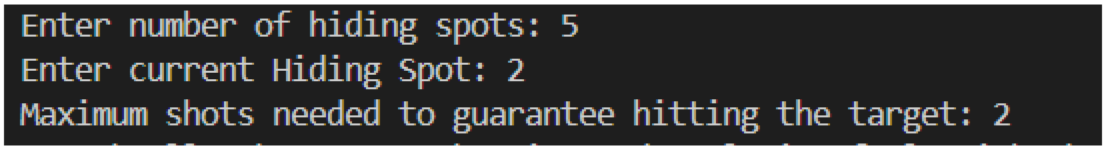
7.3.2 Pseudo Code

```
Function getspot(i, j, n):
    If i is less than 1:
        Return j
    Else if j is greater than n:
        Return i
    Else:
        Generate a random integer between 0 and 999 (inclusive)
        If the random integer is greater than or equal to 499:
            Return i
        Else:
            Return j
    End If
End Function


Function hittingAMovingTarget(n, currentHidingSpot):
    Initialize an array dp of size n+1 with zeros


    Initialize variables:
        k = 1
        count = n-1
        shotspot = 2


    Loop i from 0 to n+1:
        If count is 1:
            Increment dp[shotspot] by 1
            Set k to -1
            Reset count to n-1
        Else:
            Update dp[shotspot] as max(dp[shotspot-1]+1, dp[shotspot+1]+1)

        Print "shot = " + shotspot + ", hiding spot = " + currentHidingSpot

        If shotspot equals currentHidingSpot:
            Break out of the loop

        Decrement count
        Update currentHidingSpot using getspot(currentHidingSpot-1,
currentHidingSpot+1, n)

        If (shotspot is not 2 and shotspot is not n-1) or count is not 1:
            Increment shotspot by k

    Return dp[currentHidingSpot]
End Function
```

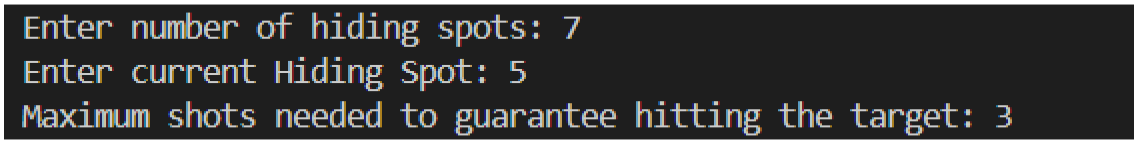
7.4 Complexity Analysis for the Algorithm

Adding the time complexities of initialization and filling the DP table, we get **O(n) + O(n) = O(2n),** which can be simplified to **O(n).**

7.5 Sample Output of the Solution



```
Enter number of hiding spots: 5
Enter current Hiding Spot: 2
Maximum shots needed to guarantee hitting the target: 2
```



```
Enter number of hiding spots: 7
Enter current Hiding Spot: 5
Maximum shots needed to guarantee hitting the target: 3
```

7.6 Comparison with Another Algorithm

TABLE 6

	Dynamic Programming	Greedy
Time Complexity	O(n)	O(n)
Space Complexity	O(n)	O(1)
Optimum Solution	Always minimum number of moves to hit target	A random number of moves not always minimum

7.7 Conclusion

We use two methods to solve the problem

The first method is dynamic programming, we can say that it is an efficient method in that it always out the minimum number of moves to hit targe but it is has a high time complexity and space complexity.

The second method is greedy algorithm it's advantage is it's time complexity and space complexity But it doesn't give a very efficient output

Task 8 : Fake Box

8.1 Problem Description

If you have 50 boxes that contains 50 pieces of metal all of the same known weight. one of these boxes contains fake metal pieces that weigh 1 kilogram less than the pieces in the rest of the boxes. You can use a digital scale only once to find this fake box.
Design a brute force algorithm to solve this problem.

8.2 Detailed Assumptions

1. If we assume that each real box weigh [a kilogram] , then fake box weigh [a-1 kilograms]
2. Then each real metal piece in real boxes weight $\frac{a}{50}$
3. Then each real metal piece in real boxes weight $\frac{a}{50}$

8.3 Detailed Solution

8.3.1 Description of Solution

1. we give each box a unique number from 1 to 50
2. we take a number of metal pieces from each box depending on box's number
Example: 1 piece of box 1 and 2 pieces of box 2 and so on until we take 50 pieces of box 50
3. we weigh the gathered pieces and store the result
4. by computing 50 probabilities of expected results and storing them
5. comparing the result the digital scale with computed results, we find the fake box

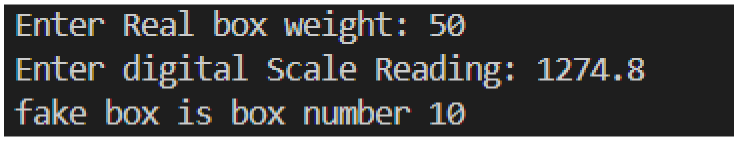
8.3.2 Pseudo Code

```
findFakeBox(array boxes,float digitalScaleReading ,double realMetalPieceWeigt, double
fakeMetalPieceWeigt)
       // boxes: array of number of pieces taken from each box
       float sum = 0;
       arrayOfProbablities[50]
       for i <- 1 to 50
              sum = 0;
              for j <- 0 to 49
                     if j+1 == i
                            sum += boxes[j] * fakeMetalPieceWeigt;
                     else
                            sum += boxes[j] * realMetalPieceWeigt;
              endfor
              arrayOfProbablities[i-1] = sum;
       endfor
       for i <- 0 to 49
              if digitalScaleReading == arrayOfProbablities[i]
                     return i+1;
endfor
```

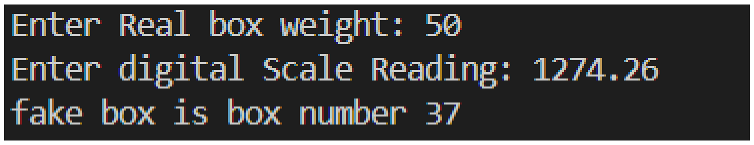
8.4 Complexity Analysis for the Algorithm

$T(n) = \Sigma 1\le i\le 50\ (\Sigma 0\le j\le 49\ 1) + \Sigma 0\le i\le 49\ = O(1)$

8.5 Sample Output of the Solution



```
Enter Real box weight: 50
Enter digital Scale Reading: 1274.8
fake box is box number 10
```



```
Enter Real box weight: 50
Enter digital Scale Reading: 1274.26
fake box is box number 37
```

8.6 Comparison with Another Algorithm

TABLE 7

	Brute Force	Divide & Conquer
Time Complexity	O(n^2)	O(n^2)
Space Complexity	O(1)	O(1)
Readability	Easy	Easy

8.7 Conclusion

In conclusion, we used two approaches to solve the problem

The brute force approach compare the scaled weight with each possibility of fake box number

On the other hand, the divide & conquer approach sort the possibilities and use binary search to find the fake box

But regardless of algorithms used all of them will lead to same efficiency of Brute force so brute force is better because of it is simplicity