SYDE 556/750

Simulating Neurobiological Systems
Lecture 5: Feed-Forward Transformation

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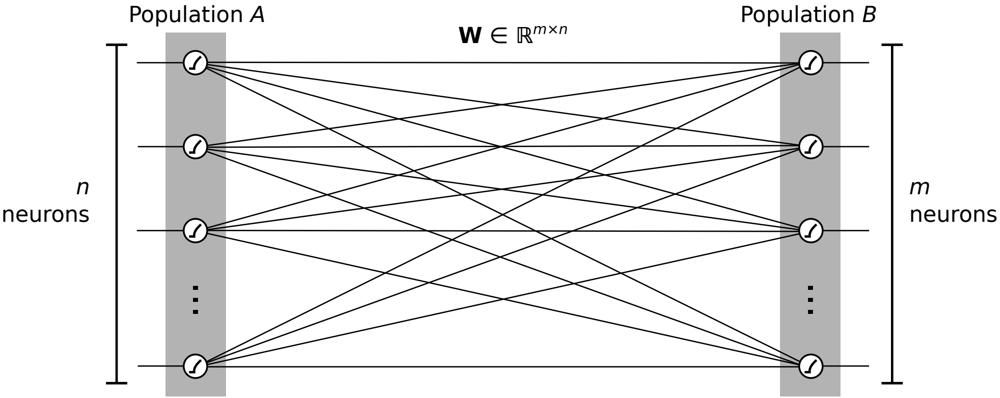
Introduction

- We've only talked about representation til now
  - What about computation?
- We start by focusing on the state of a network after learning and development
- A kind of hypothesis testing and generation

DALL-E AI Generated Art, 2022

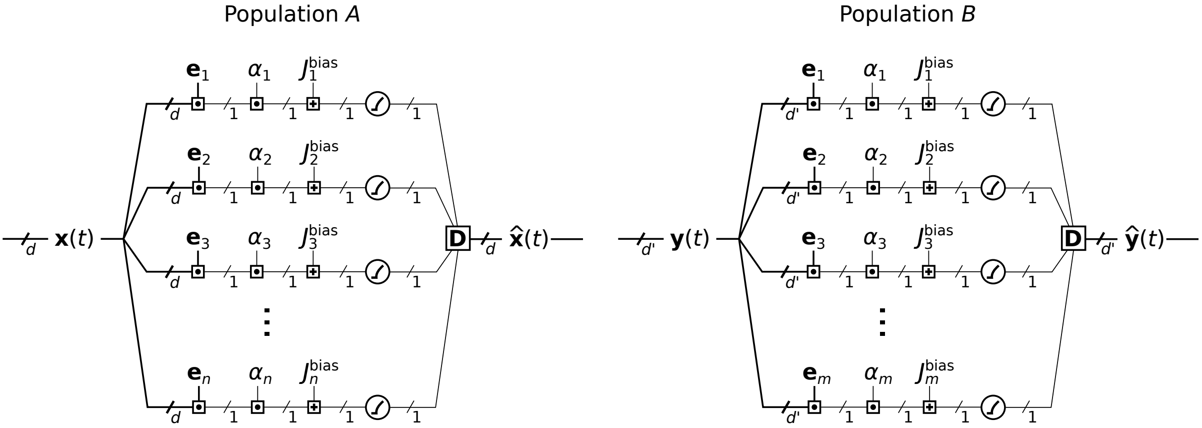
NEF Principle 2: Transformation



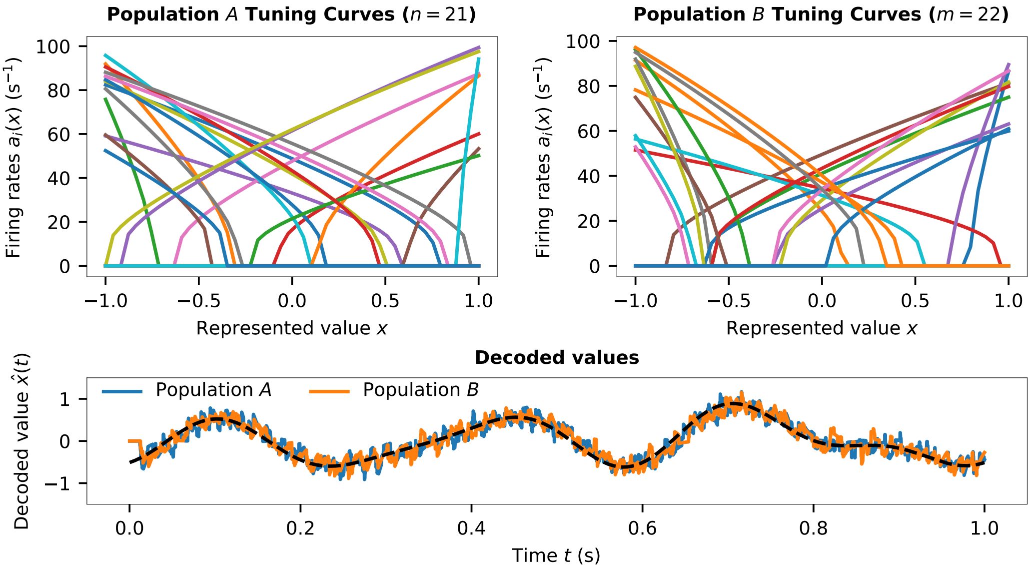
NEF Principle 2 – Transformation

Connections between populations describe *transformations* of neural representations. Transformations are functions of the variables represented by neural populations.

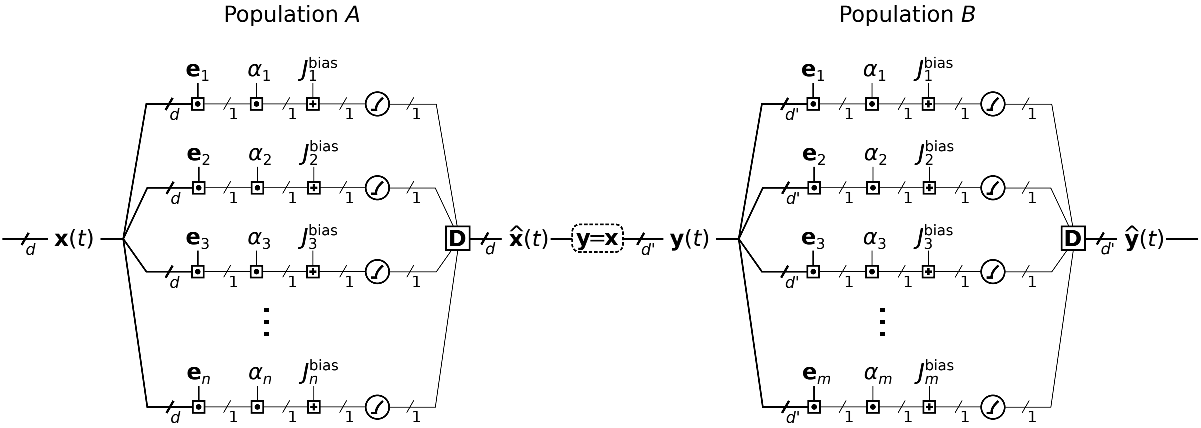
A Tale of Two Populations (I)



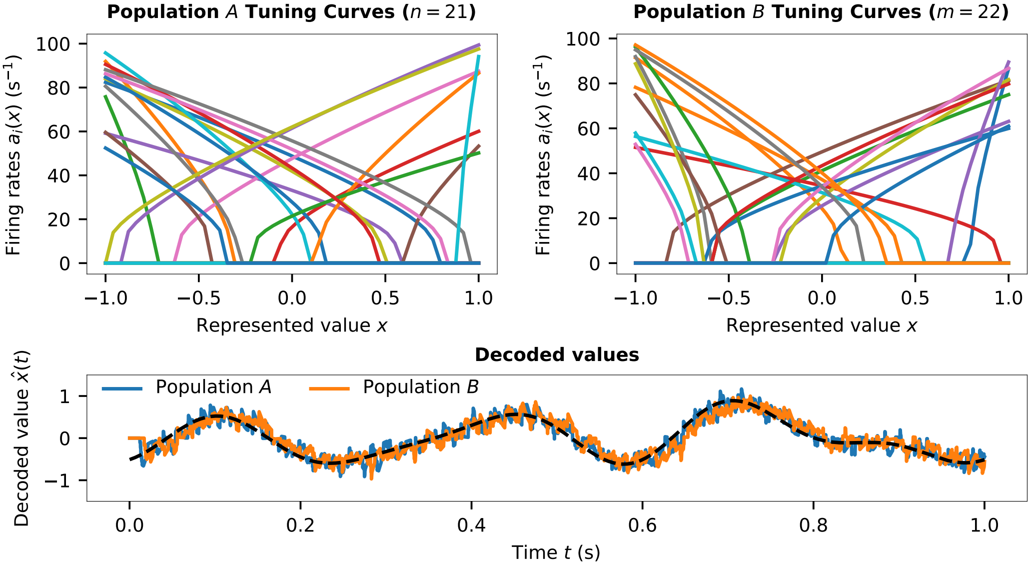
Communication Channel Experiment: Same input signal



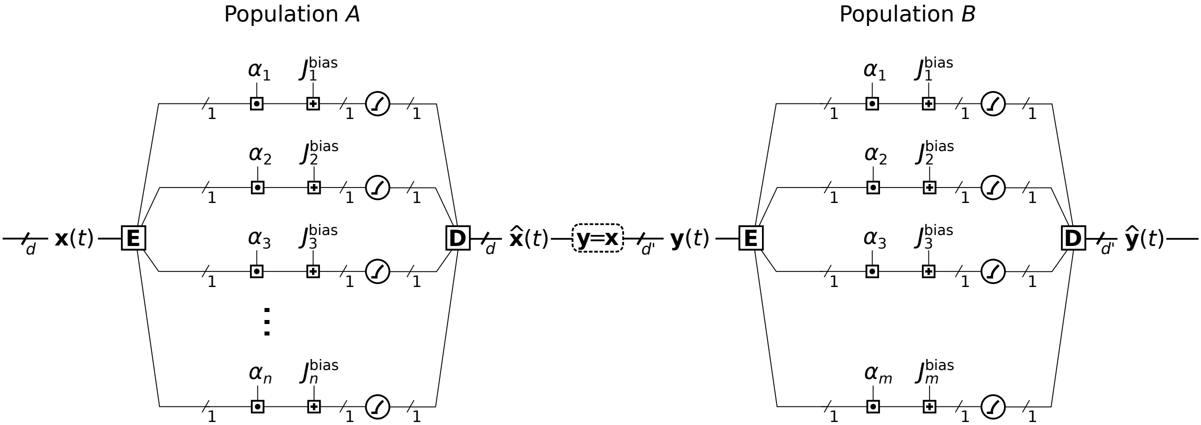
A Tale of Two Populations (II)



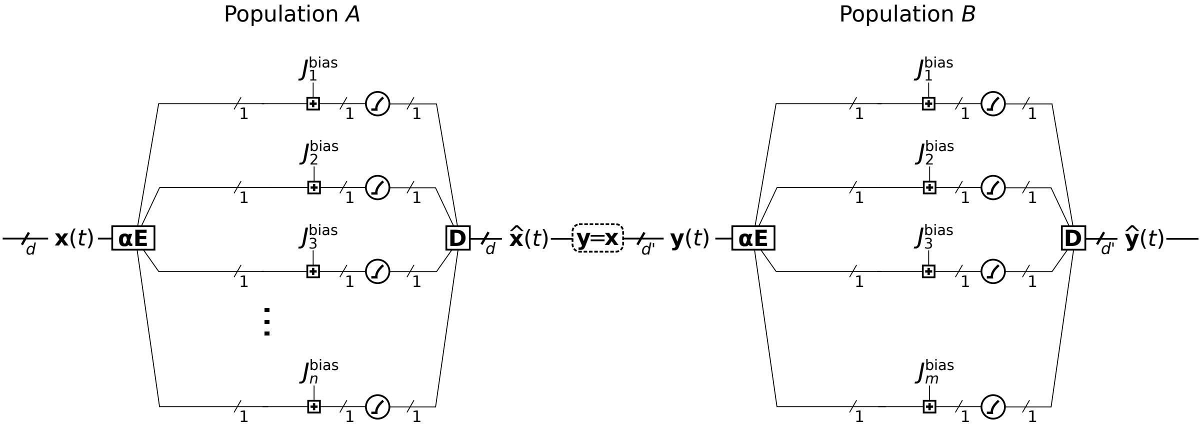
Communication Channel Experiment: Populations in series



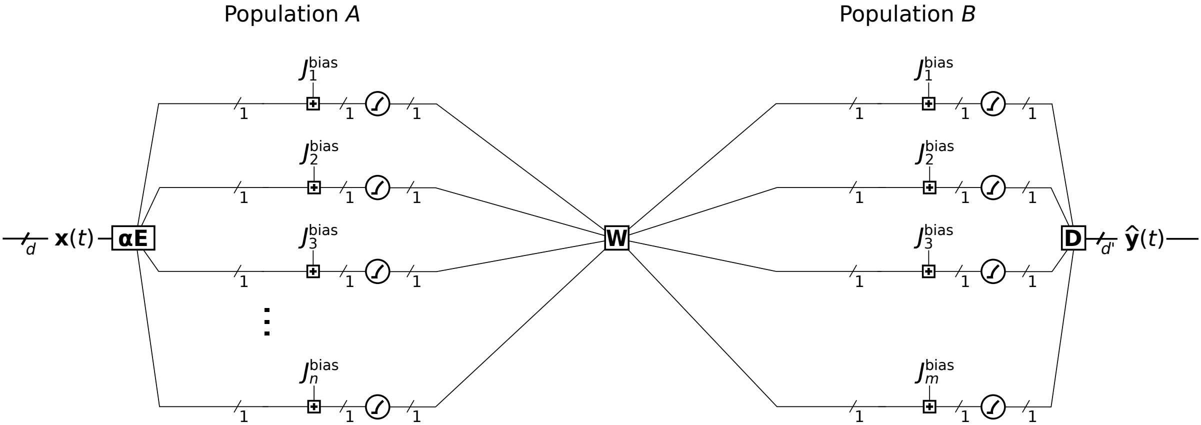
Computing Synaptic Weights: Step 1 – Encoding Matrix



Computing Synaptic Weights: Step 2 – Scaled Encoding Matrix



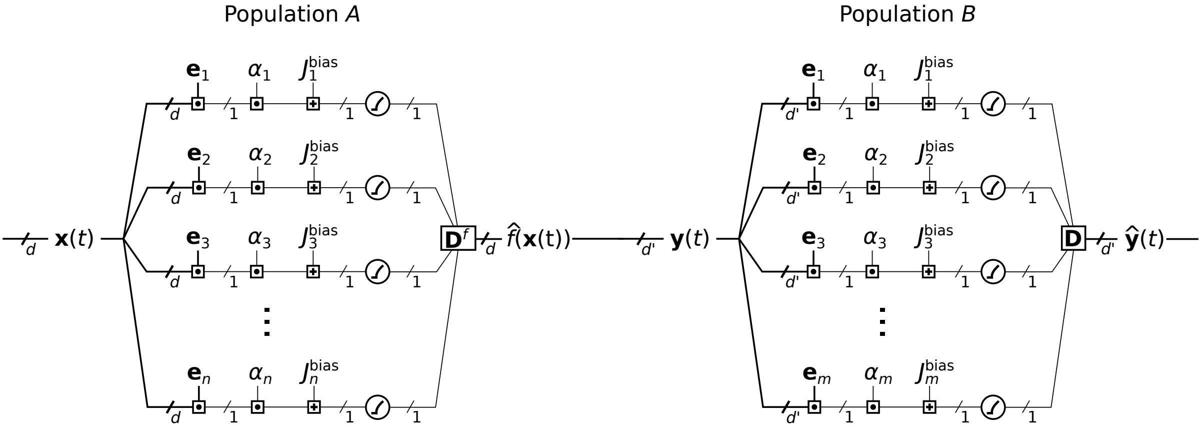
Computing Synaptic Weights: Step 3 – $\mathbf{W} = \mathbf{E}\mathbf{D}$



Computational Complexity

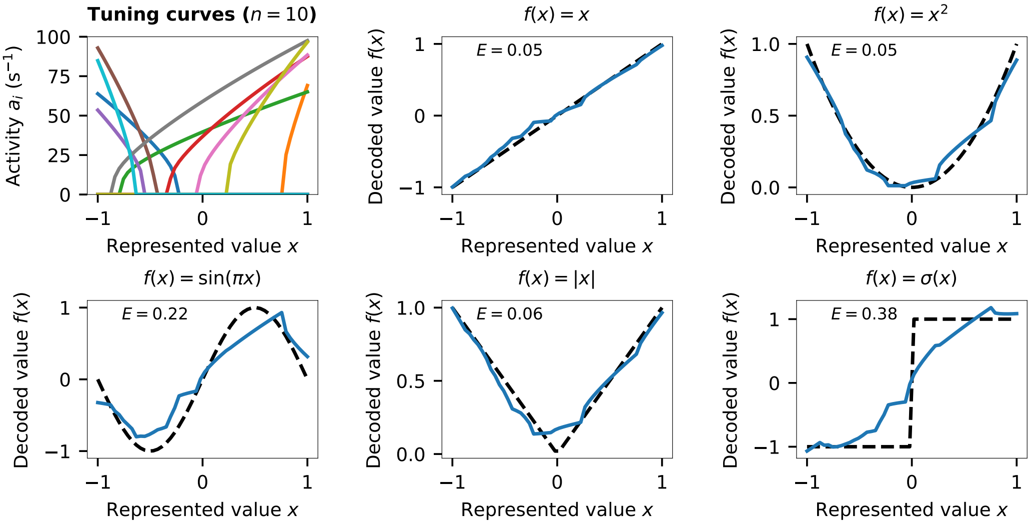
- Weights - multiplying $\mathbf{a} \in \mathbb{R}^n$ with $\mathbf{W} \in \mathbb{R}^{m \times n}$ is $\mathcal{O}(nm)$ i.e., $\approx \mathcal{O}(n^2)$
- Decoding - $\hat{\mathbf{x}} = \mathbf{D}\mathbf{a}$ is $\mathcal{O}(dn)$
- Encoding - $\mathbf{J} = \mathbf{E}\hat{\mathbf{x}} + \mathbf{J}_{\text{bias}}$ is $\mathcal{O}(dm)$
- Encoding/Decoding - $\mathcal{O}(d(n+m))$ or $\approx \mathcal{O}(dn)$ for $n = m$
- So if d is small we get a linear complexity $\mathcal{O}(n)$
- Therefore, sequential decoding and re-encoding saves a lot of time compared to using actual synaptic weights
- One reason why Nengo is so fast compared to other SNN simulators

Computing Functions

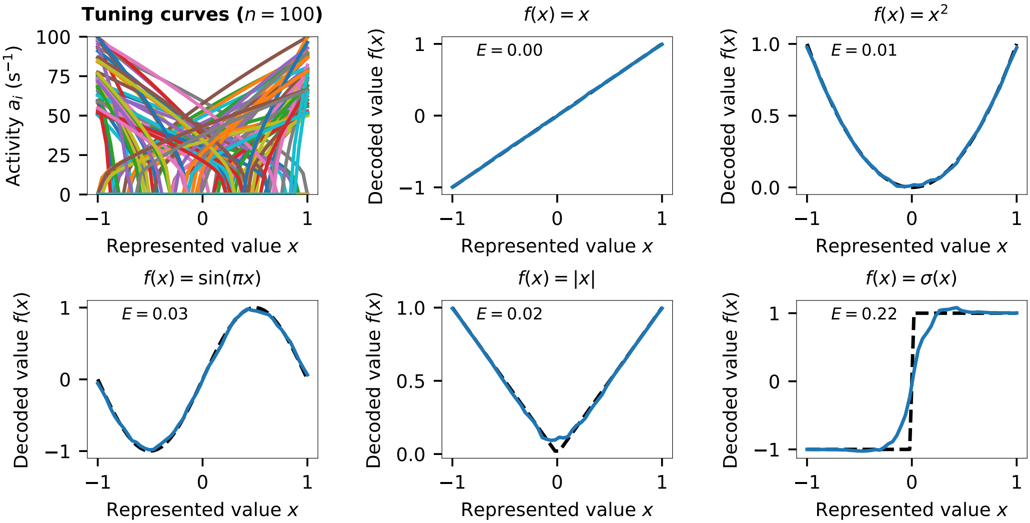


Function Decoder $\mathbf{D}^f = \left((\mathbf{A}\mathbf{A}^\top + N\sigma^2\mathbf{I})^{-1}\mathbf{A}\mathbf{Y}^\top\right)^\top$, where $(\mathbf{Y})_{ik} = (f(\mathbf{x}_k))_i$

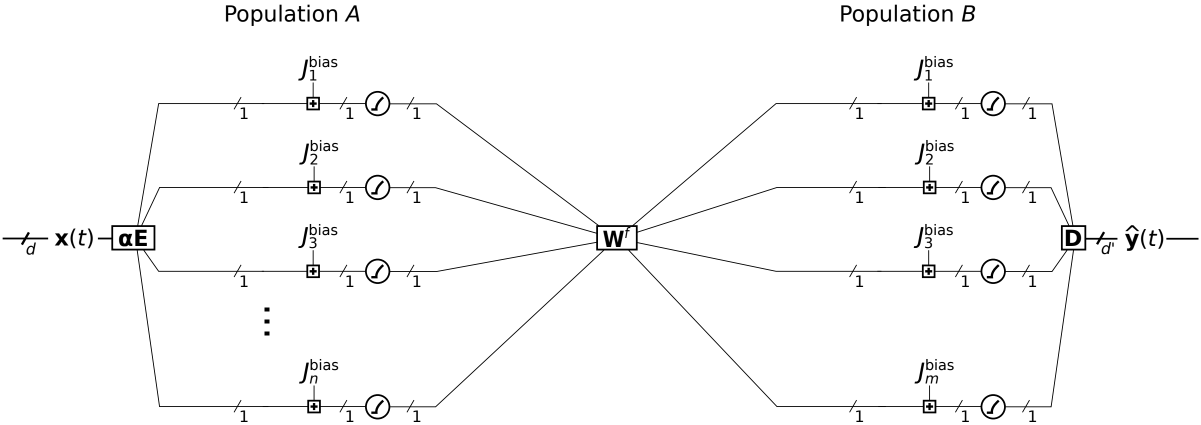
Decoding Functions – Using a Few Neurons



Decoding Functions – Using More Neurons



Computing Functions – Weight Matrix



$$\mathbf{W}^f = \mathbf{E}\mathbf{D}^f$$

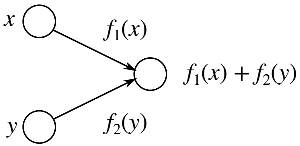
Computing Multivariate Functions

Homogenous population Heterogenous population

Linear connection Inh. connection Exc. connection

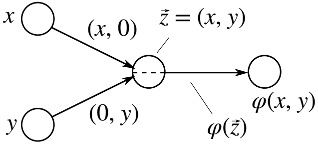
Linear Superposition

$$W^{f_1}\mathbf{a}_1(\mathbf{x}) + W^{f_2}\mathbf{a}_2(\mathbf{y})$$



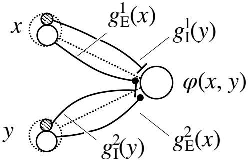
Nonlinear Functions

Multi-dimensional $\mathbf{z}$



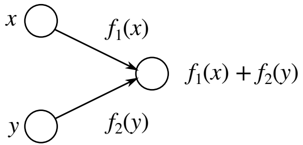
(Dendritic Computation)

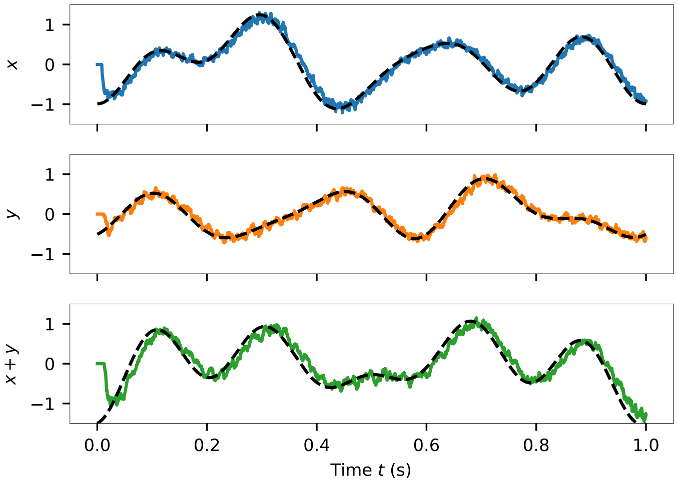
Exploit dendritic nonlinearity



Computing Multivariate Functions – Linear Superposition

Linear Superposition

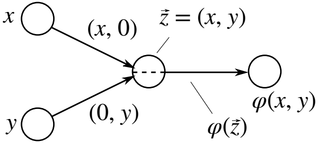




Computing Multivariate Functions – Multiplication

Nonlinear Functions

Multi-dimensional $\mathbf{z}$



Multiplication is useful...

- Gating of signals
- Attention effects
- Binding
- Statistical inference

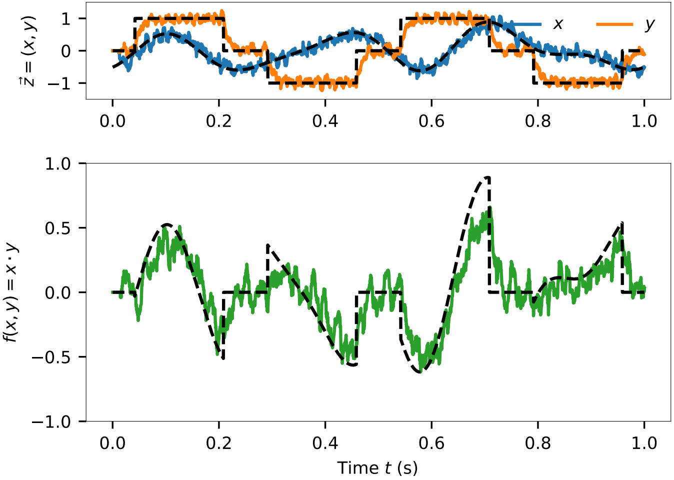


Image sources

Title slide
"Yellow Butterfly"
Author: Albert Bierstadt, circa 1890.
From Wikimedia.