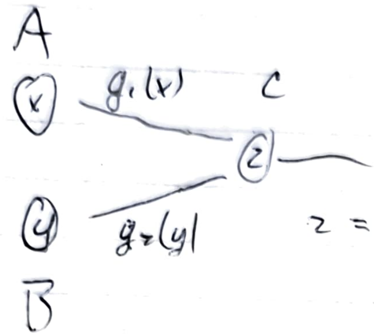
LECTURE 6



$z = g_1(x) + g_2(y)$

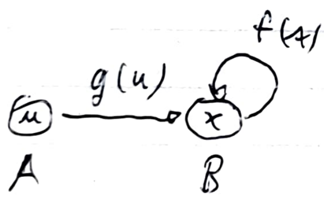
into z neurons:

$$J_i^C(t) = \langle w_i^A, (\alpha_{pre}^A * h(t)) \rangle + \langle w_i^B, (\alpha_{pre}^B * h(t)) \rangle$$

$$w_i^A = (E D^{g_1})_i$$

$$w_i^B = (E D^{g_2})_i$$

Recurrent Version



into x neurons

$$J_i^B = \langle w_i^A, (\alpha_{pre}^A * h)(t) \rangle + \langle w_i^B, (\alpha_{pre}^B * h)(t) \rangle$$

$$w_i^A = (E D^g)_i$$

$$w_i^B = (E D^f)_i$$

Feedback Examples

(1) $f(x) = x + 1$

$x = u + f(x) = u + x + 1$

$x - x = u + 1$ (?)

(2) $f(x) = -x$

$x = u - x$
$2x = u$
$x = \frac{u}{2}$

(3) $f(x) = x^2$

$x = u + x^2$
$x - x^2 = u$
$x(1 - x) = u$?

$x(t) = \left(g(u(t)) + f(x(t))\right) * h(t)$

$\mathcal{L}$

LAPLACE TRANSFORM

$$\mathcal{L}(f) = F(s) \int_0^\infty f(t) e^{-st}\, dt \qquad s = \sigma + i\omega$$

$$\mathcal{L}(f(t)) \Rightarrow F(s)$$

$$\mathcal{L}\left(\frac{df}{dt}\right) \quad \mathcal{L}(\dot{f}) = sF(s)$$

eqn $x(t) = h(t) * \left(g(u(t)) + f(x(t))\right)$

$$\mathcal{L}(h)? \qquad h(t) = \frac{1}{\tau} e^{-t/\tau}$$

$$\mathcal{L}(h) = \int_0^\infty h(t) e^{-st}\, dt = \int_0^\infty \frac{1}{\tau} e^{-t/\tau} e^{-st}\, dt$$

$$= \int_0^\infty \frac{1}{\tau} e^{-t(1/\tau + s)}\, dt$$

$$= \left[- \frac{1}{\tau(\frac{1}{\tau} + s)} \right]_0^\infty = \boxed{\frac{1}{1+s\tau} = H(s)}$$

$$\mathcal{L}(x(t)) = X(s) = H(s)\left(G(s) + F(s)\right).$$

$$X(s) = \frac{1}{1+s\tau}\left(G(s) + F(s)\right)$$

$$X(s) + s\tau X(s) = G(s) + F(s)$$

$$s\tau X(s) = G(s) + F(s) - X(s)$$

$$sX(s) = \frac{1}{\tau}\left(G(s) + F(s) - X(s)\right)$$

$$\frac{dx}{dt} = \dot{x}(t) = \frac{1}{\tau}\underbrace{\left(g(u(t)) + f(x(t)) - x(t)\right)}_{?}$$

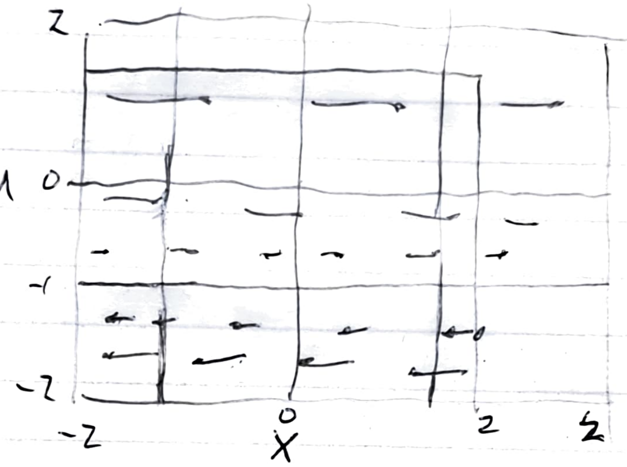
$$\dot{x}(t) = \phi(x, u)$$

RETURN TO EXAMPLES

① $f(x) = x + 1$

$$\dot{x}(t) = \frac{1}{\tau}\left(u(t) + \cancel{x(t)} + 1 - \cancel{x(t)}\right)$$

$$= \frac{1}{\tau}(u + 1)$$



Phase portrait

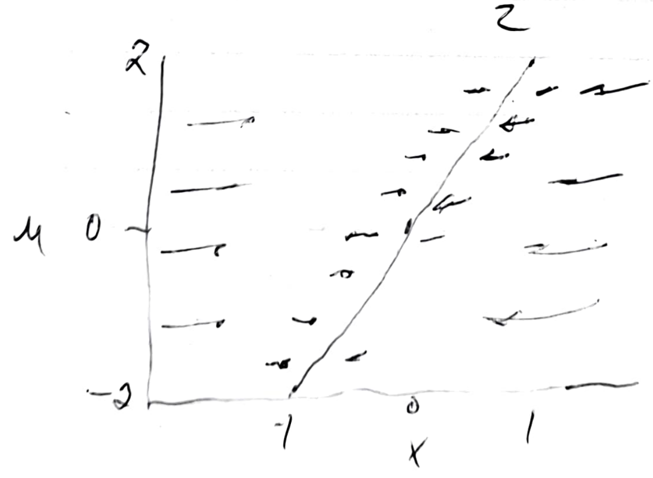
② $f(x) = -x$

$$\dot{x} = \frac{1}{\tau}\left(u + (-x) - x\right)$$

$$= \frac{1}{\tau}(u - 2x)$$

$u = 2x$

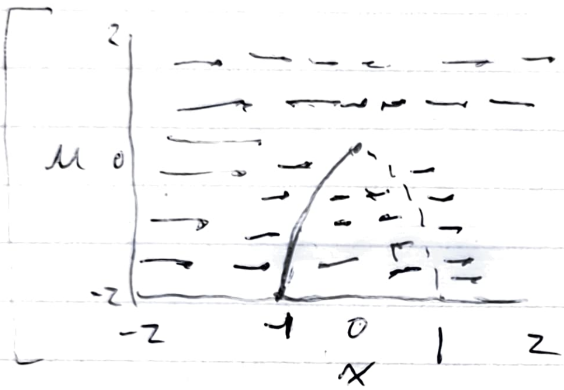
$x = \frac{u}{2}$



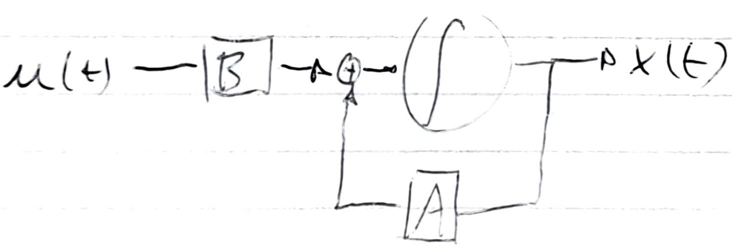
(3) $f(x) = x^2$

$$\dot{x} = \frac{1}{\tau}(u + \underbrace{x^2 - x})$$

$$\dot{x} = \frac{1}{\tau}\left(x - 1 + \frac{\sqrt{1-4u}}{2}\right)\left(x - 1 - \frac{\sqrt{1-4u}}{2}\right)$$



INVERT THIS ANALYSIS



LTI $\dot{x} = Ax + Bu$

NEURAL DYNAMICS $x = h * (A'x + B'u)$

$$\dot{x} = Ax + Bu$$

$$sX(s) = AX(s) + BU(s)$$

$$x = h * (A'x + B'u)$$

$$X(s) = H(s)\left(A'X(s) + B'U(s)\right)$$

$$H(s) = \frac{1}{1+s\tau}$$

$$X(s) = \frac{1}{1+s\tau}\left(A'X(s) + B'U(s)\right)$$

$$X(s) + s\tau X(s) = A'X(s) + B'U(s)$$

$$sX(s) = \frac{1}{\tau}\left(A'X(s) + B'U(s) - X(s)\right)$$

$$sX(s) = \underbrace{\frac{1}{\tau}(A'-I)}_{A}X(s) + \underbrace{\frac{1}{\tau}B'}_{B}U(s)$$

$$A = \frac{1}{\tau}(A'-I)$$

$$B = \frac{1}{\tau}B'$$

$$\left.\begin{aligned} A' &= \tau A + I \\ B' &= \tau B \end{aligned}\right]$$

MAPS DESIRED TO NEURAL DYNAMICS