SYDE 556/750

Simulating Neurobiological Systems
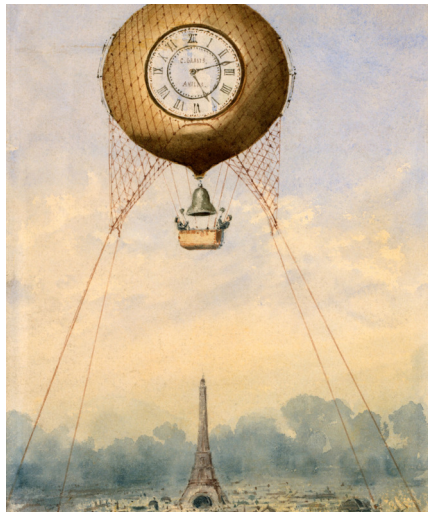**Lecture 4: Temporal Representations**

Chris Eliasmith

Sept 20 & 25, 2023

▶ Slide design: Andreas Stöckel
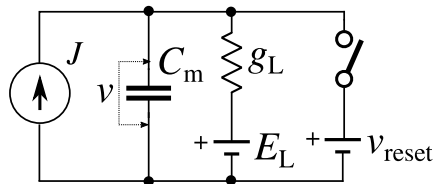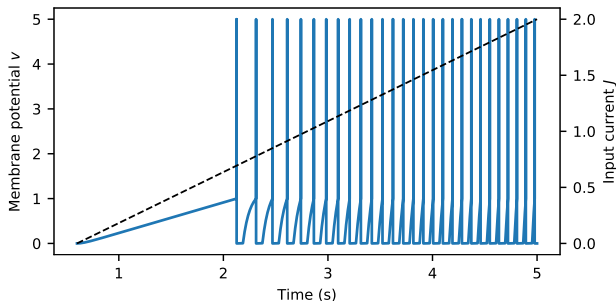▶ Content: Terry Stewart, Andreas Stöckel, Chris Eliasmith

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Reminder: The LIF Neuron

$$\frac{\mathrm{d}}{\mathrm{d}t}v(t) = -\frac{1}{\tau_{\mathrm{RC}}}\big(v(t) - J\big)\,, \qquad \text{if } v(t) < 1\,,$$

$$v(t) \leftarrow \delta(t - t_{\mathrm{th}})\,, \qquad\qquad \text{if } t = t_{\mathrm{th}}\,,$$

$$v(t) \leftarrow 0\,, \qquad\qquad\qquad \text{if } t > t_{\mathrm{th}} \text{ and } t \geq t_{\mathrm{th}} + \tau_{\mathrm{ref}}\,,$$
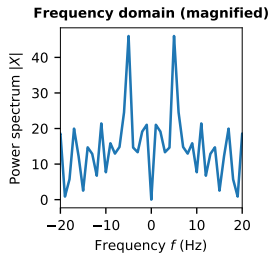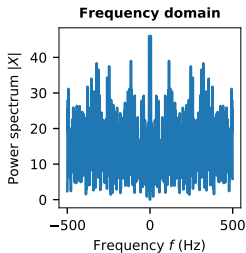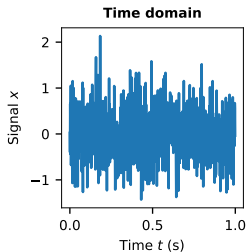
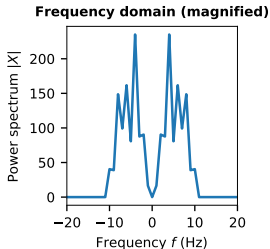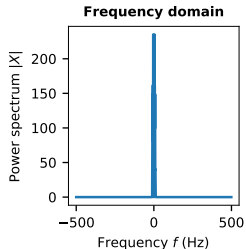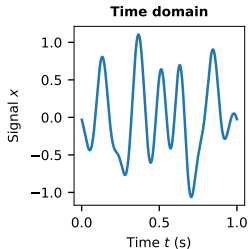# Temporal Decoding of Two Neurons - Weighted Spikes
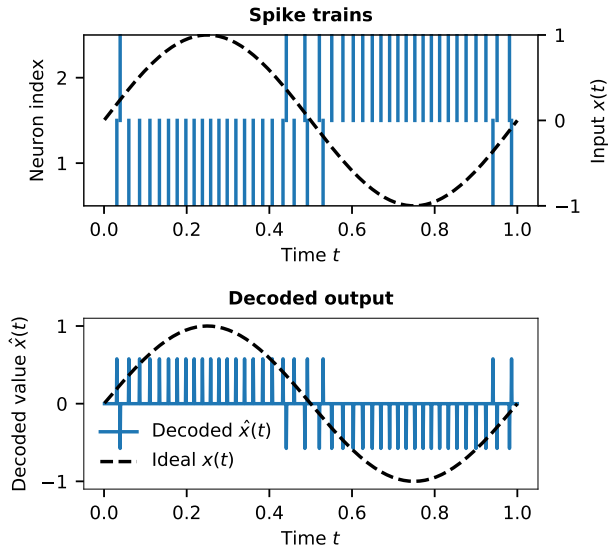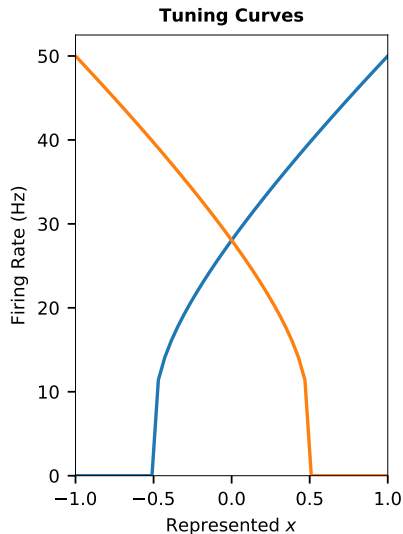
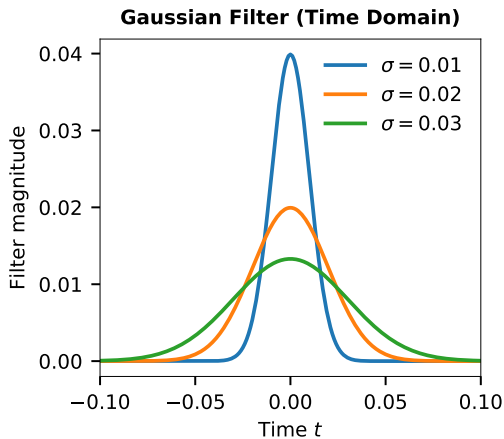# Temporal Decoding of One Hundred Neurons - Weighted Spikes
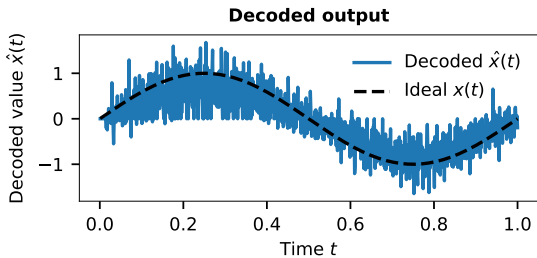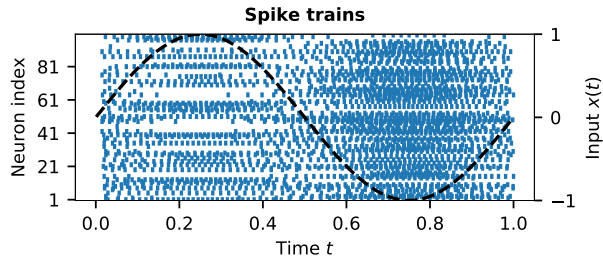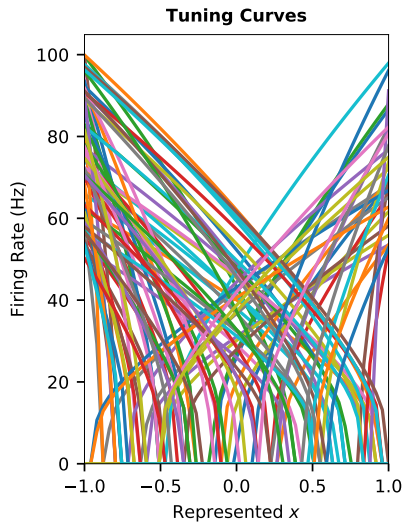
Random Signals

White Noise
(zero mean)

Bandlimited
White Noise
(zero mean,
10 Hz bandwidth)

Filtering by Convolution

Gaussian Filter (Time Domain)
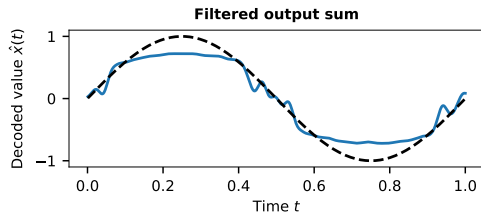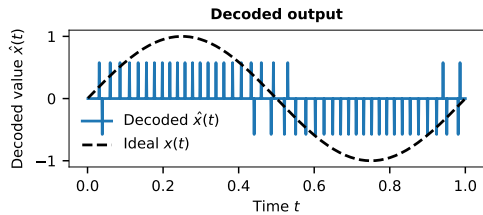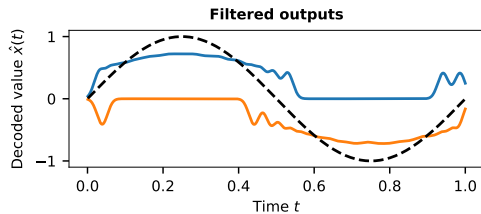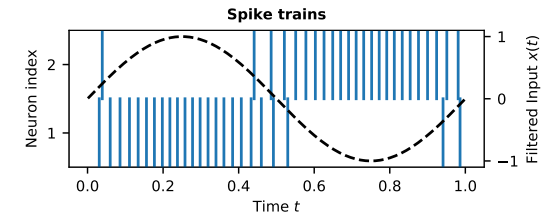
Gaussian Filter

$$h(t) = c \exp\left(\frac{-t^2}{\sigma^2}\right)$$

where c chosen s.t. $\int_{-\infty}^{\infty} h(t)\,\mathrm{d}t = 1$

Convolution

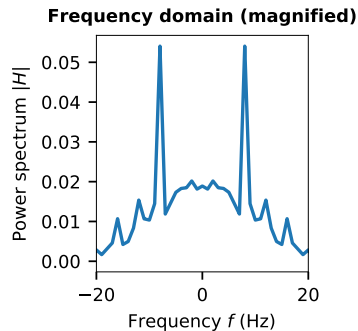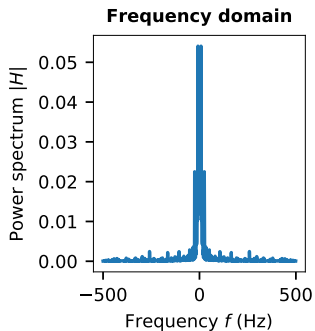$$\bigl(f * h\bigr)(t) = \int_{-\infty}^{\infty} f(t - t')h(t')\,\mathrm{d}t'$$

Filtering a Spike Train
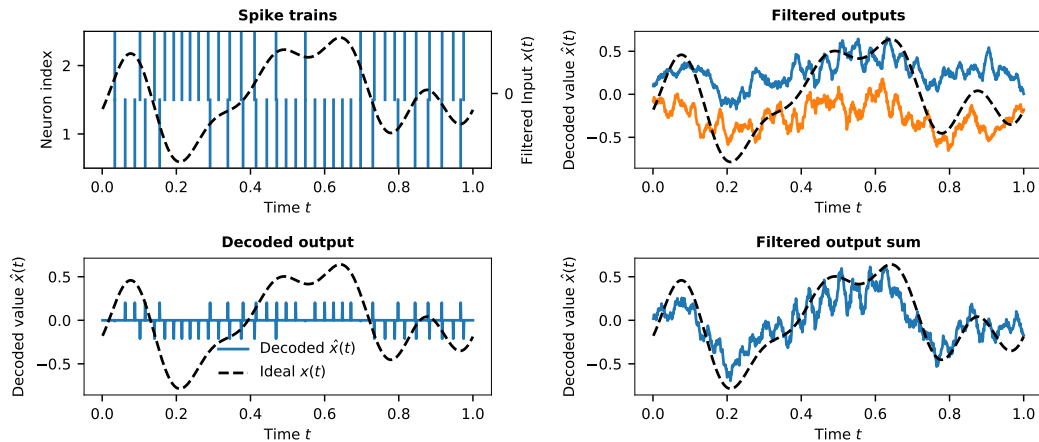
Filtering a Spike Train for a Random Signal
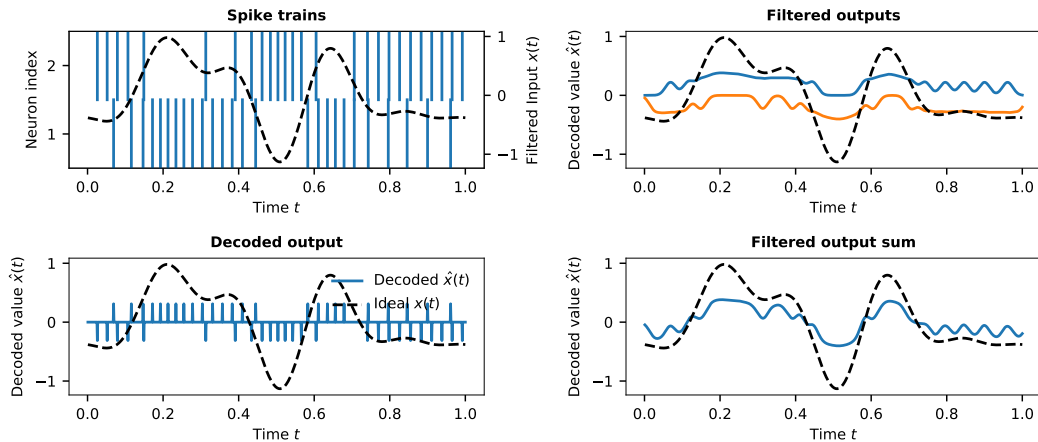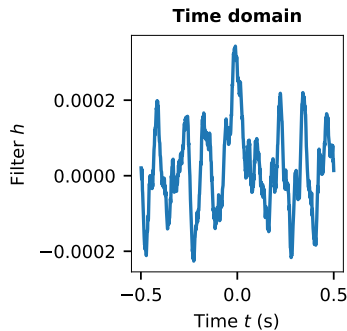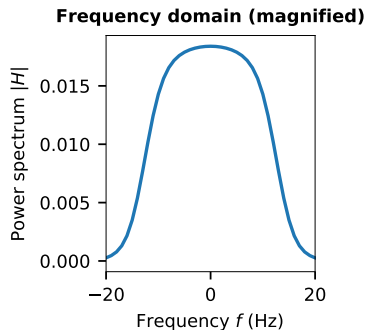
Optimal Filter

$$H(\omega) = \frac{X(\omega)\overline{R}(\omega)}{|R(\omega)|^2}$$

Filtering a Spike Train for a Random Signal (Optimal Filter)

Optimal Filter (Improved)
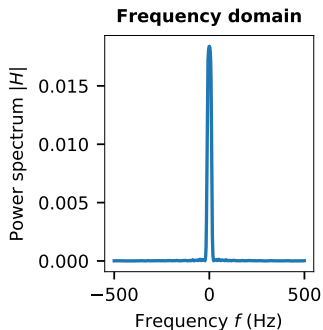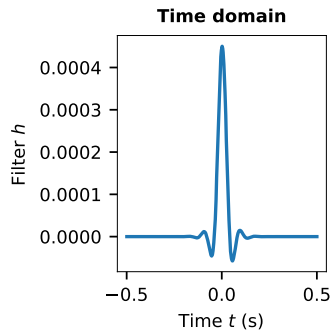
$$H(\omega) = \frac{X(\omega)\overline{R}(\omega) * W(\omega)}{|R(\omega)|^2 * W(\omega)}$$

Filtering a Spike Train for a Random Signal (Improved Optimal Filter)
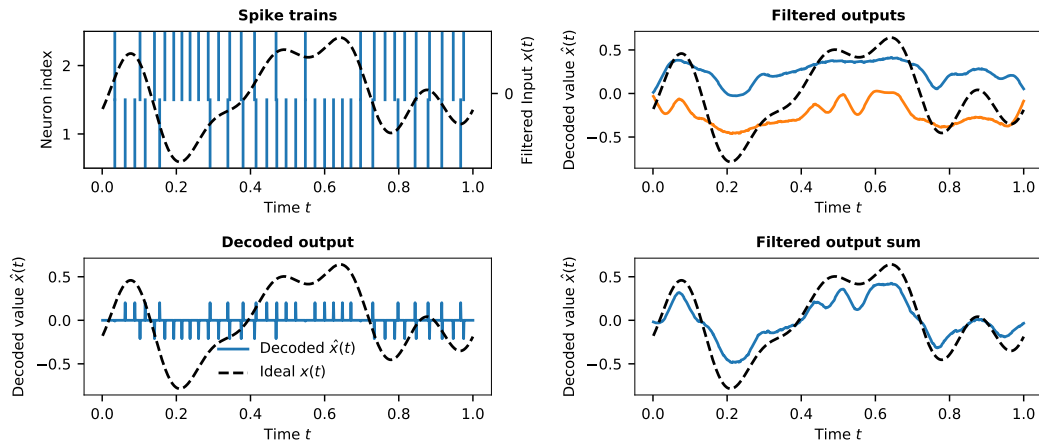
Pros and Cons of the Optimal Filter
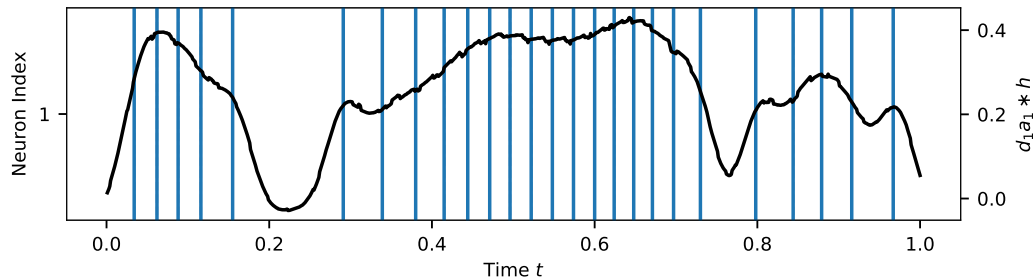
➕ **Precise**
Good for analysing data after the fact

➖ **Non-causal**
Does not describe a biological process

We need to find a mechanism that low-pass filters spikes over time!

Synapses as Filters

Post-synaptic currents (EPSCs, IPSCs) are low-pass filtered spike trains!

Exponential Low-Pass Filter (I)

Synaptic Filter (Time Domain)

$\tau = 5\text{ms}, n = 0$
$\tau = 10\text{ms}, n = 0$
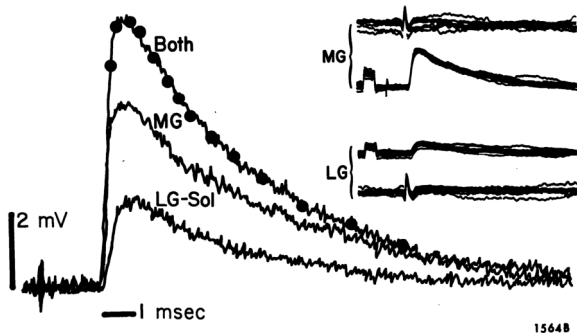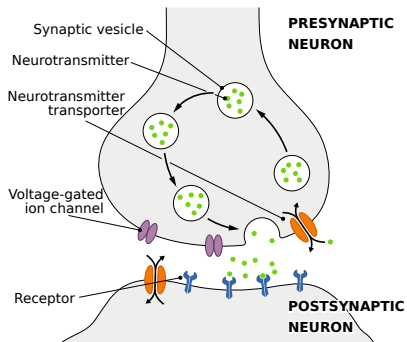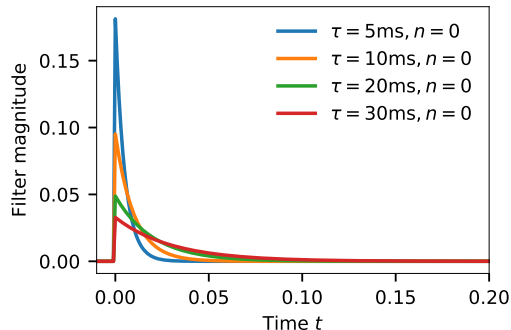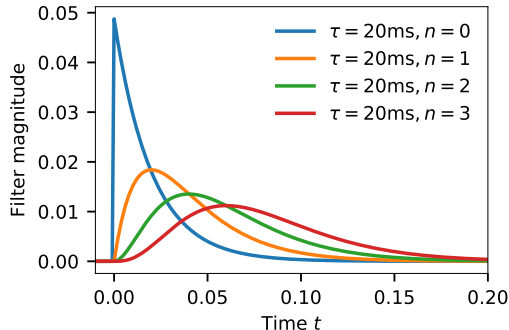$\tau = 20\text{ms}, n = 0$
$\tau = 30\text{ms}, n = 0$

Synaptic Filter (Time Domain)

$\tau = 20\text{ms}, n = 0$
$\tau = 20\text{ms}, n = 1$
$\tau = 20\text{ms}, n = 2$
$\tau = 20\text{ms}, n = 3$

$$h(t) = \begin{cases} c^{-1} t^n \exp^{-t/\tau} & \text{if } t \geq 0 \,, \\ 0 & \text{otherwise} \,, \end{cases} \qquad \text{where } c = \int_0^\infty t^n \exp^{-t/\tau} \, \mathrm{d}t \,.$$

Exponential Low-Pass Filter (II)
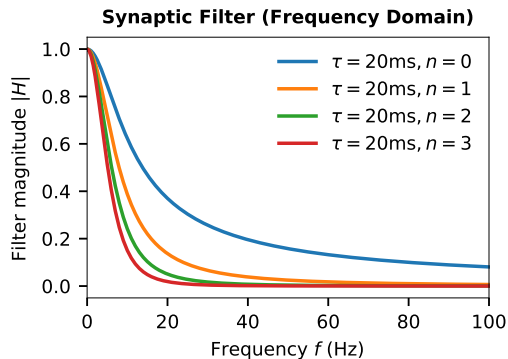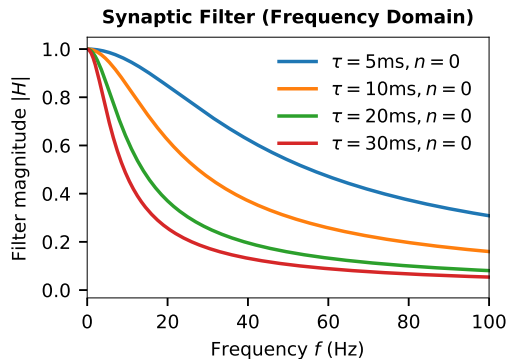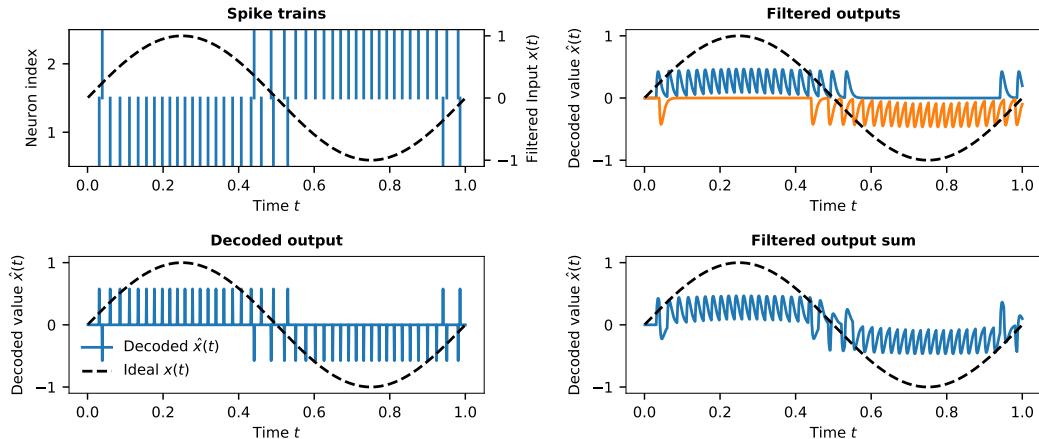

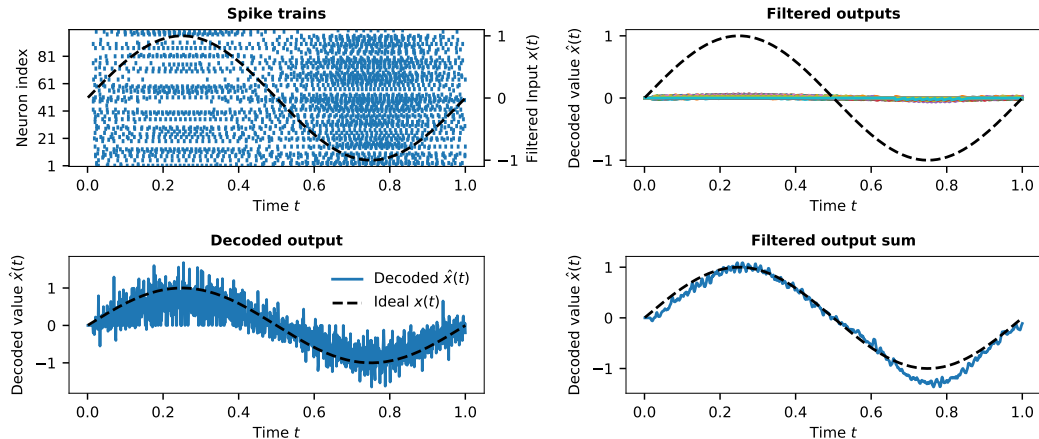$$h(t) = \begin{cases} c^{-1} t^n \exp^{-t/\tau} & \text{if } t \geq 0 \,, \\ 0 & \text{otherwise} \,, \end{cases} \qquad \text{where } c = \int_0^\infty t^n \exp^{-t/\tau} \, \mathrm{d}t \,.$$

Example: Synaptic Filter for Two Neurons

$\tau = 5\,\text{ms}, n = 1$

Example: Synaptic Filter for One Hundred Neurons

$\tau = 5\,\mathrm{ms}, n = 1$

Image sources

Title slide
"Captive balloon with clock face and bell, floating above the Eiffel Tower, Paris, France."
Author: Camille Grávis, between 1889 and 1900.
From Wikimedia.