

**SYDE 556/750**

**Simulating Neurobiological Systems**  
**Lecture 2: Neurons**

Chris Eliasmith

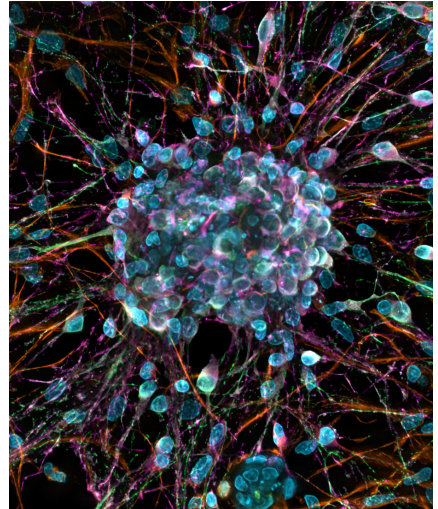
September 9, 2022

- ▶ Slide design: Andreas Stöckel
- ▶ Content: Terry Stewart, Andreas Stöckel, Chris Eliasmith

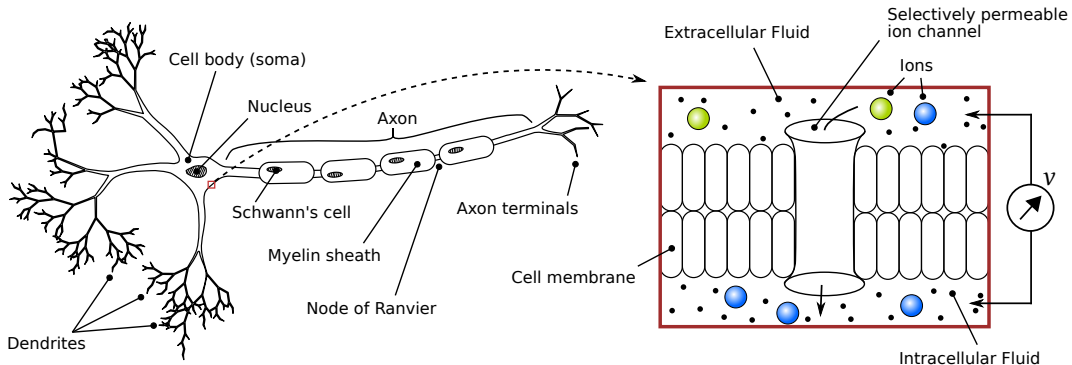


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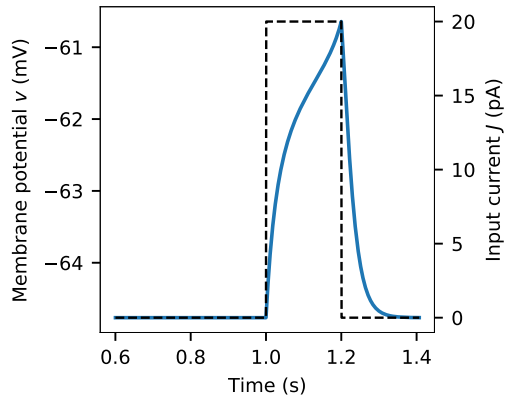
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# Textbook Neuron and Cell Membrane

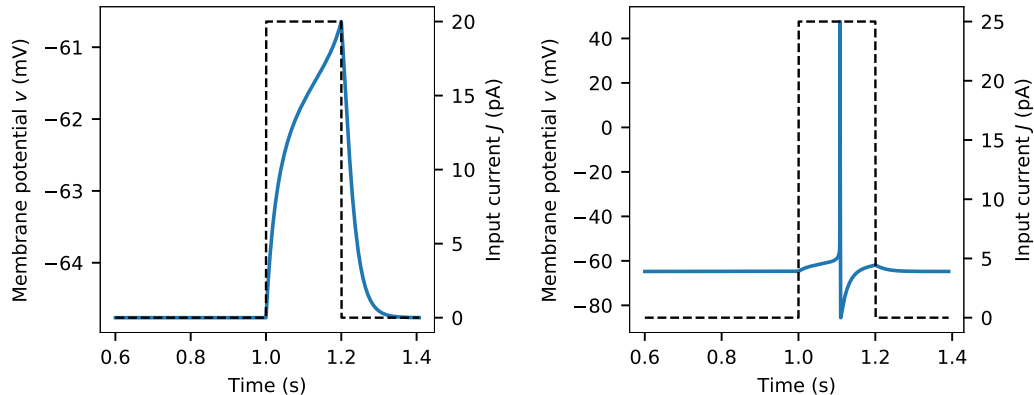


## Injecting a Current Into a Detailed Neuron Model



Computer simulation of an Hodgkin-Huxley type neuron with Traub kinematics (Roger D. Traub and Richard Miles, *Neuronal Networks of the Hippocampus*, Cambridge University Press, 1991)

## Injecting a Current Into a Detailed Neuron Model

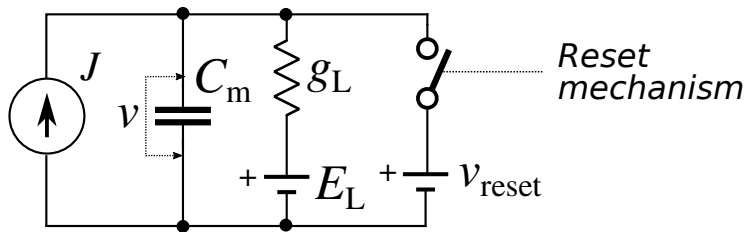


Computer simulation of an Hodgkin-Huxley type neuron with Traub kinematics (Roger D. Traub and Richard Miles, *Neuronal Networks of the Hippocampus*, Cambridge University Press, 1991)

## Basic High-Level Details (Lapicque, 1907)

1. The cell acts like a *capacitor*, i.e., the voltage increases while we're injecting a current.
2. The capacitor is *leaky*. As soon as we stop injecting a current, the voltage collapses back to the resting potential  $E_L$ .
3. As soon as the voltage surpasses a certain value, the *threshold potential*  $v_{th}$ , the cell will generate a spike.
4. Shortly after the spike has been produced, the voltage drops below the resting potential. During this period, the *refractory period* of length  $\tau_{ref}$ , we cannot get the neuron to spike again, even if we apply relatively large input currents  $J$ .

## The Leaky Integrate-and-Fire Equivalent Circuit

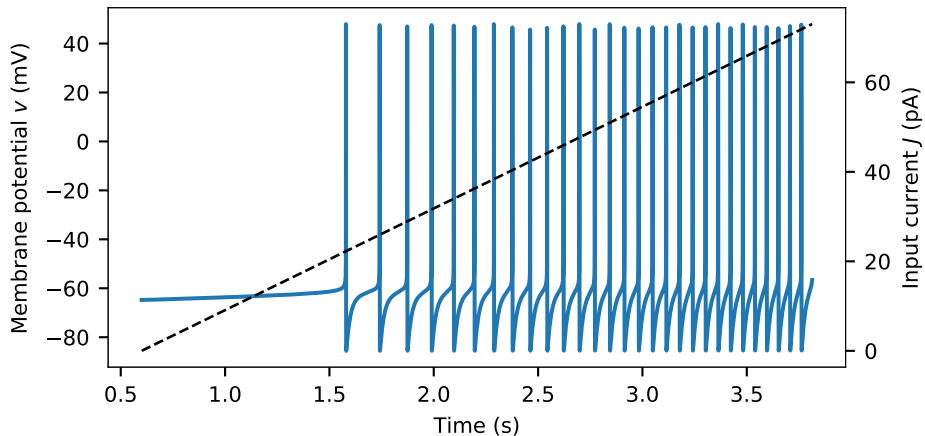


$$\frac{d}{dt}v(t) = \frac{1}{C_m} (g_L(E_L - v(t)) + J), \quad \text{if } v(t) < v_{\text{th}}.$$

if  $v(t) = v_{\text{th}}$  at  $t = t_{\text{th}}$ , output a spike ( $\delta(t - t_{\text{th}})$ ) and:

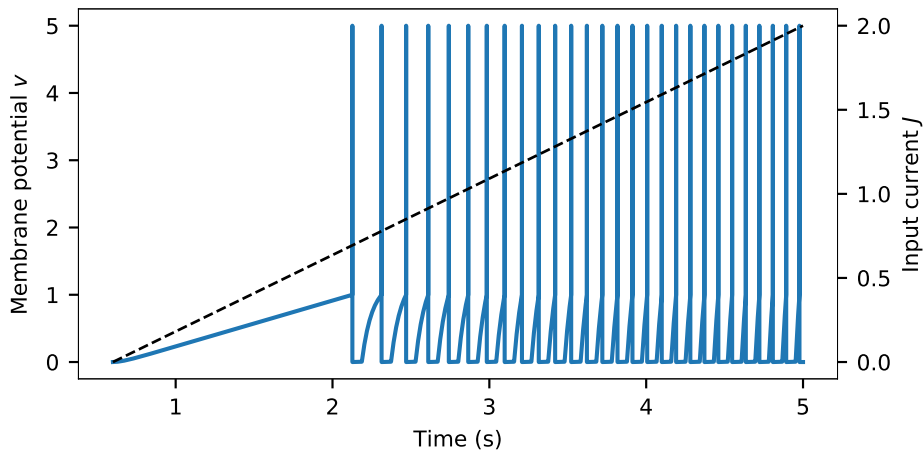
$$v(t) \leftarrow v_{\text{reset}}, \quad \text{if } t_{\text{th}} < t \leq t_{\text{th}} + \tau_{\text{ref}},$$

## Injecting a Current Ramp into a Detailed Neuron Model



Computer simulation of an Hodgkin-Huxley type neuron with Traub kinematics (Roger D. Traub and Richard Miles, *Neuronal Networks of the Hippocampus*, Cambridge University Press, 1991)

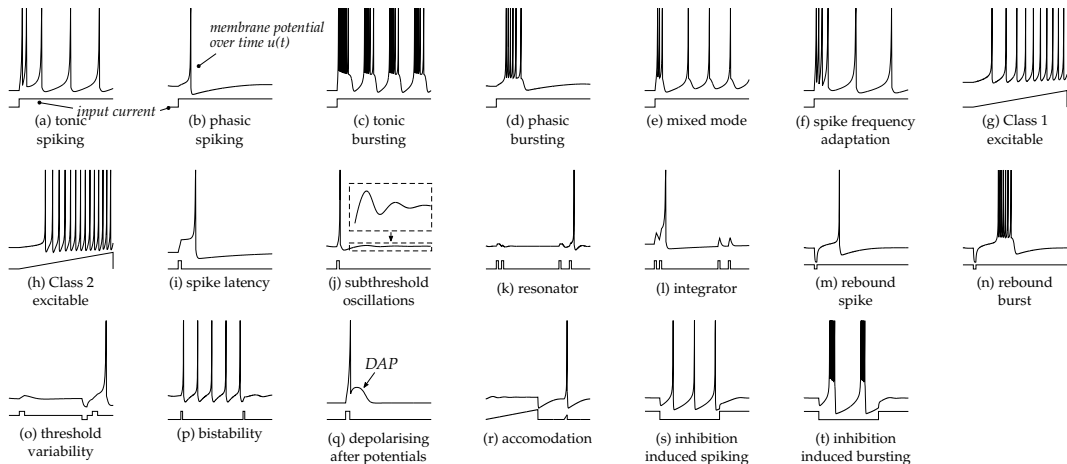
## Injecting a Current Ramp into a LIF Neuron Model



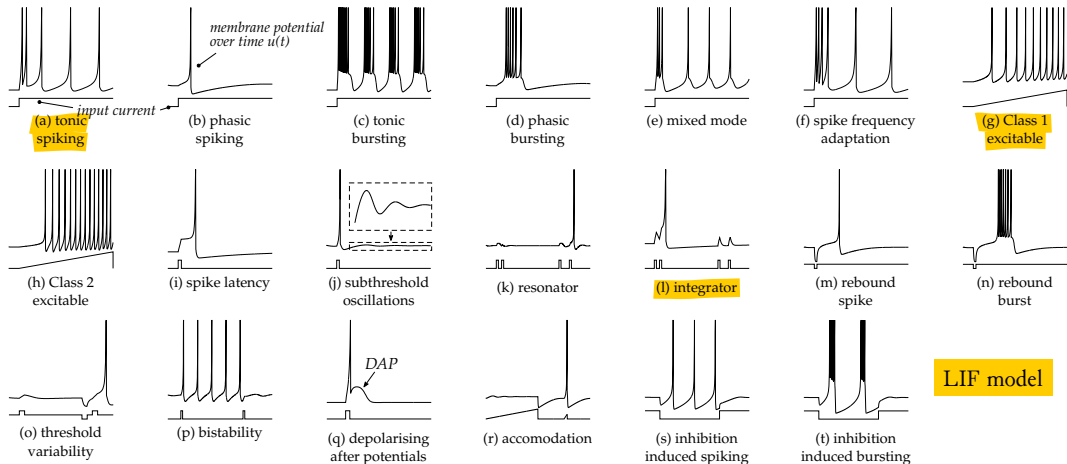
(note normalization to  $v_{th} = 1$ ,  $v_{reset} = E_L = 0$ )



# Limitations of the LIF Neuron Model



# Limitations of the LIF Neuron Model



## LIF Rate Approximation

- ▶ need to compute  $t_{\text{th}}$  (the time  $v(t_{\text{th}}) = v_{\text{th}}$ )
- ▶ assume:  $J$  is constant and  $v(0) = 0$ .
- ▶ also:  $g_L = 1/R$  and  $\tau_{\text{RC}} = RC_m$

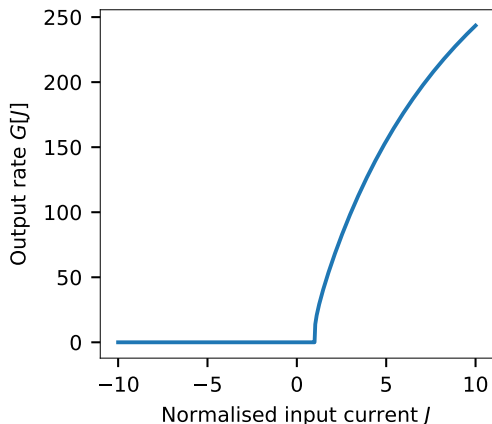
$$v(t) = - \int_0^t \frac{1}{\tau_{\text{RC}}} (v(t') - RJ) dt' = RJ \left( 1 - e^{-\frac{t}{\tau_{\text{RC}}}} \right) .$$

$$v_{\text{th}} = RJ \left( 1 - e^{-\frac{t_{\text{th}}}{\tau_{\text{RC}}}} \right) \Leftrightarrow 1 - \frac{v_{\text{th}}}{RJ} = e^{-\frac{t_{\text{th}}}{\tau_{\text{RC}}}} ,$$

$$t_{\text{th}} = -\tau_{\text{RC}} \log \left( 1 - \frac{v_{\text{th}}}{RJ} \right)$$

$$G[J] = \begin{cases} \frac{1}{\tau_{\text{ref}} - \tau_{\text{RC}} \log \left( 1 - \frac{v_{\text{th}}}{RJ} \right)} & \text{if } 1 - \frac{v_{\text{th}}}{RJ} > 0 , \\ 0 & \text{otherwise .} \end{cases}$$

# Artificial Rate Neurons: LIF

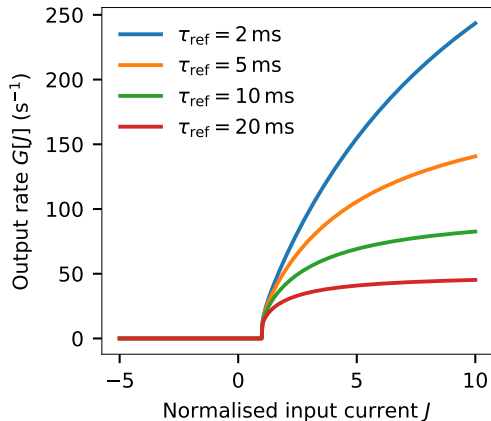
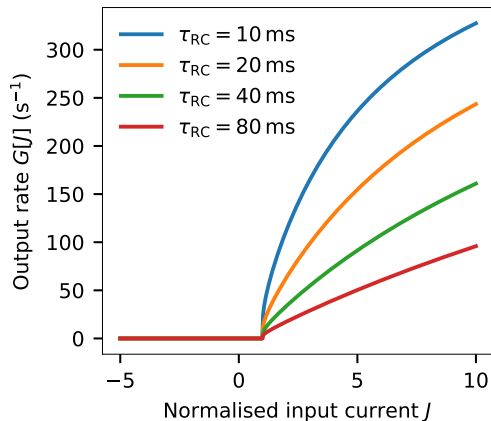


$$G[J] = \frac{1}{\tau_{\text{ref}} - \tau_{\text{RC}} \log \left(1 - \frac{1}{J}\right)}$$

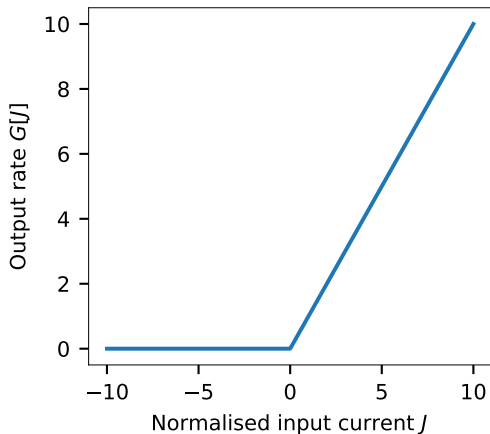
## Usefulness to neurobiological systems modellers:

- ⊕ Biologically motivated
- ⊕ Captures saturation effects
- Relatively slow to evaluate numerically (for machine-learning people)
- ⊖ Spike onset is smooth in noisy systems

## Exploring the LIF Rate Approximation



# Artificial Rate Neurons: ReLU

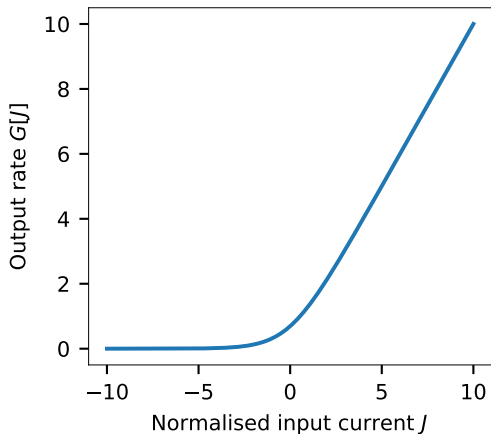


$$G[J] = \max\{0, J\}$$

## Usefulness to neurobiological systems modellers:

- ⊕ Fast to evaluate
- Rough approximation of the LIF response curve
- ⊖ Does not capture saturation effects
- ⊖ Spike onset is smooth in noisy systems

## Artificial Rate Neurons: Smooth ReLU (Softplus)

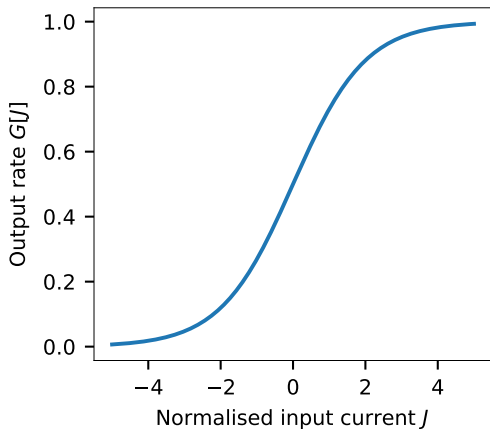


$$G[J] = \log(1 + \exp(J))$$

**Usefulness to neurobiological systems modellers:**

- ⊕ Models smooth spike onset
- Rough approximation of the LIF response curve
- ⊖ Does not capture saturation effects

## Artificial Rate Neurons: Logistic Function



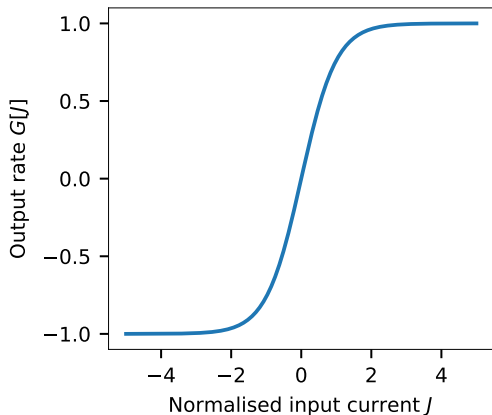
$$G[J] = \frac{1}{1 + e^{-J}}$$

**Usefulness to neurobiological systems modellers:**

- Models smooth spike onset and saturation (?)



# Artificial Rate Neurons: Hyperbolic Tangent



$$G[J] = \tanh(J) = \frac{e^J - e^{-J}}{e^J + e^{-J}}$$

**Usefulness to neurobiological systems modellers:**

- Models smooth spike onset and saturation (?)
- Negative rates

# Image sources

## **Title slide**

Image of rat primary cortical neurons in culture.

Author: ZEISS Microscopy, <http://www.zeiss.com/celldiscoverer>.

From Wikimedia.