SYDE 556/750

Simulating Neurobiological Systems
Lecture 9: Analysing Representations

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Nov 6, 2023

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- Content: Terry Stewart, Andreas Stöckel, Chris Eliasmith

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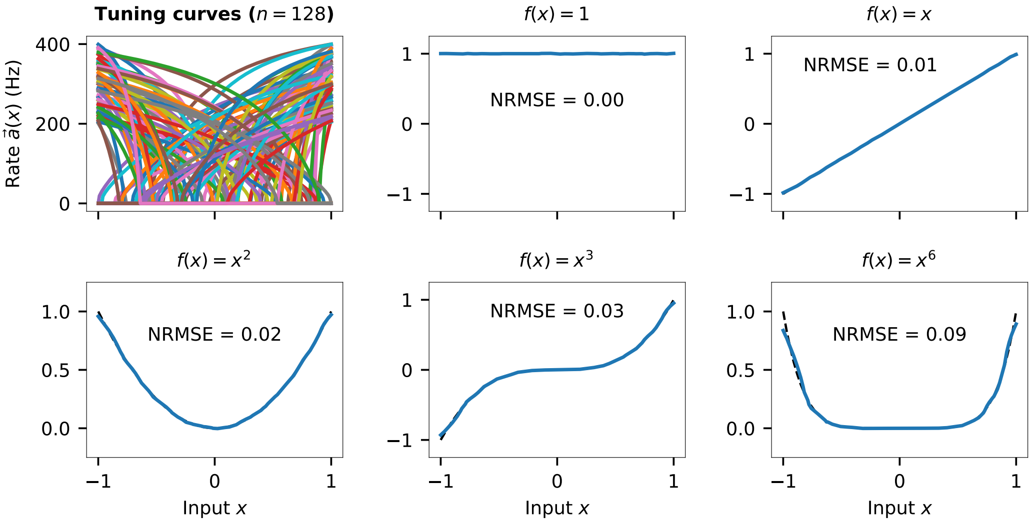
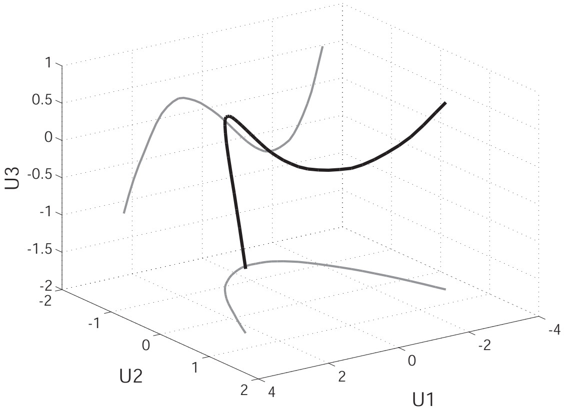
Decoding Polynomials

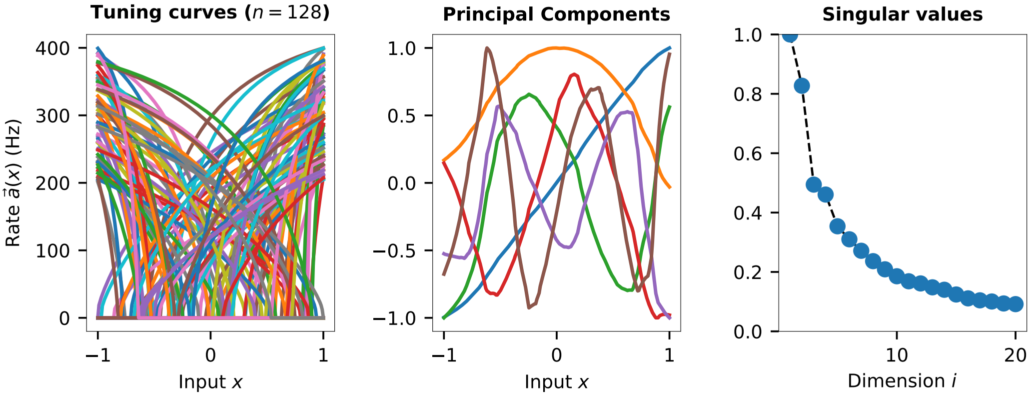


Projection of the Neuron Space to PCA

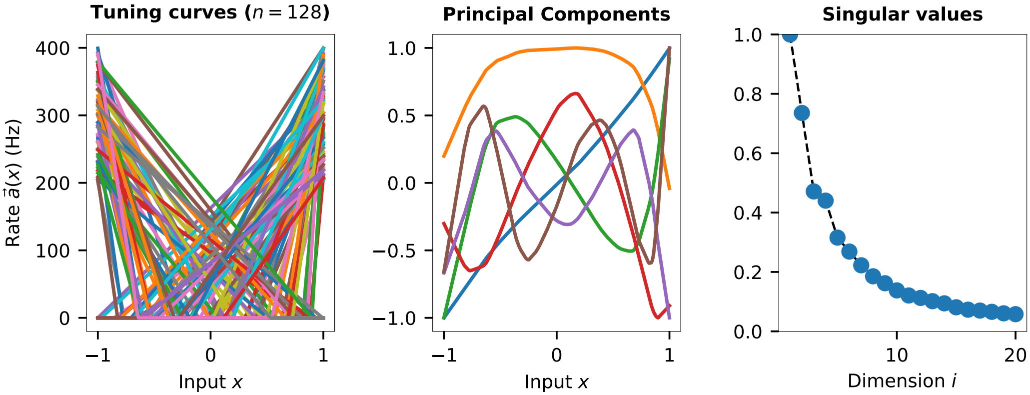


A subspace of neuron activity being projected onto the first few principle component planes. Notice that the axis scales are different, capturing the size of the singular value.

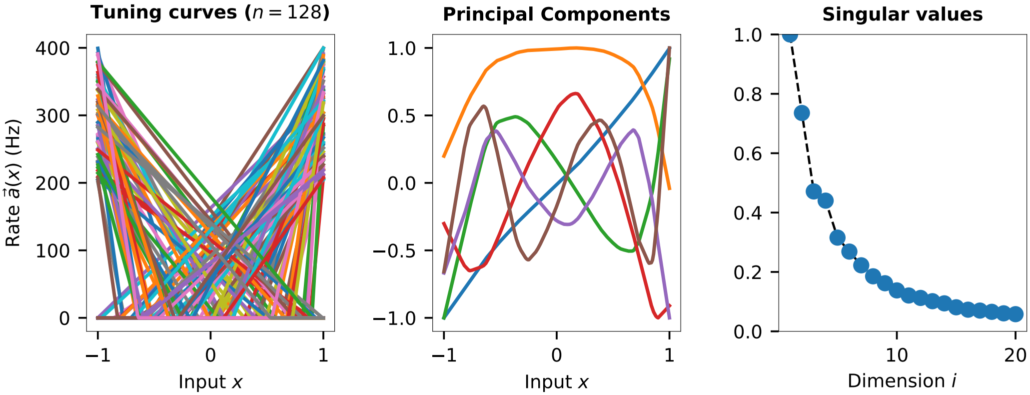
LIF Tuning Curve Principal Components



ReLU Tuning Curve Principal Components

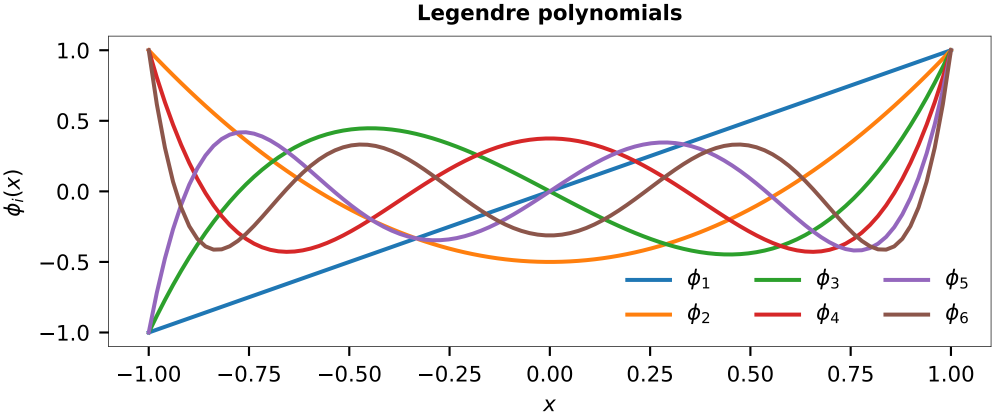


ReLU Tuning Curve Principal Components



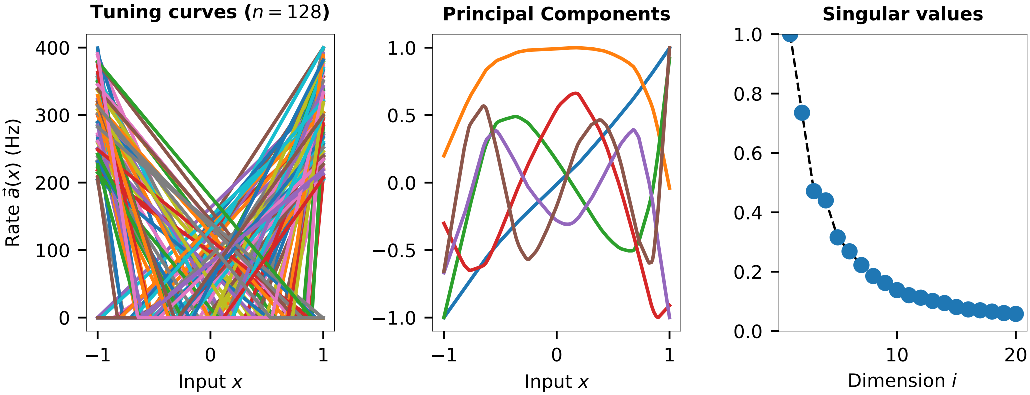
$\approx$ Legendre Basis

Reminder: Legendre Polynomials

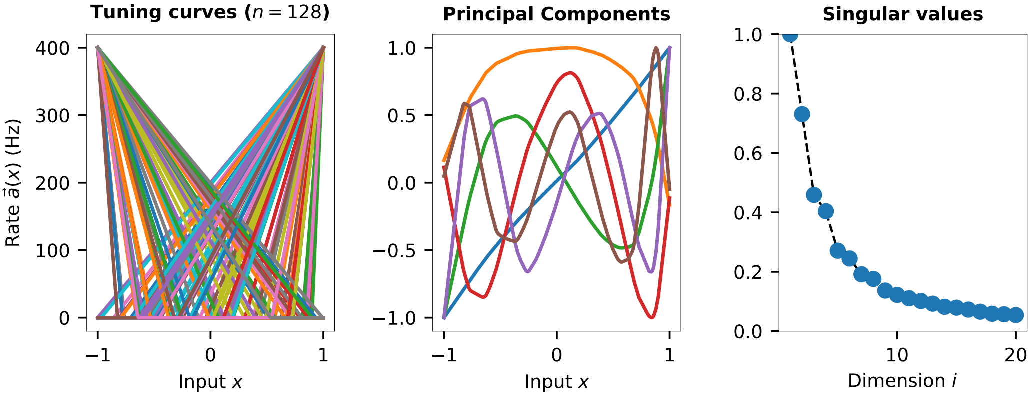


$$\varphi_i(x) = \frac{1}{2^i} \sum_{k=0}^{n} \binom{i}{k}^2 (x-1)^{i-k} (x+1)^k$$

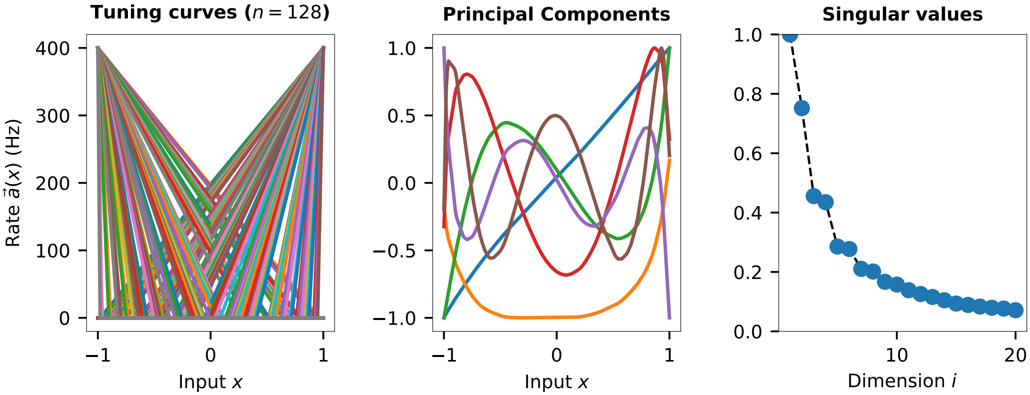
Modifying the Basis – Same Maximum Rate (I)



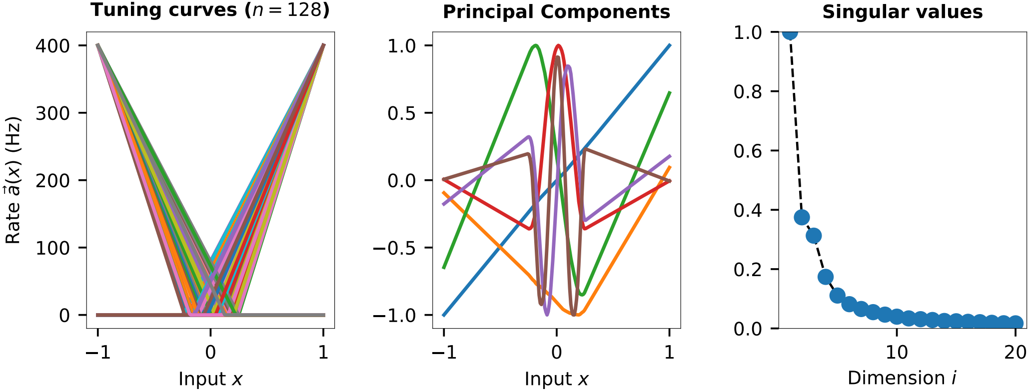
Modifying the Basis – Same Maximum Rate (I)



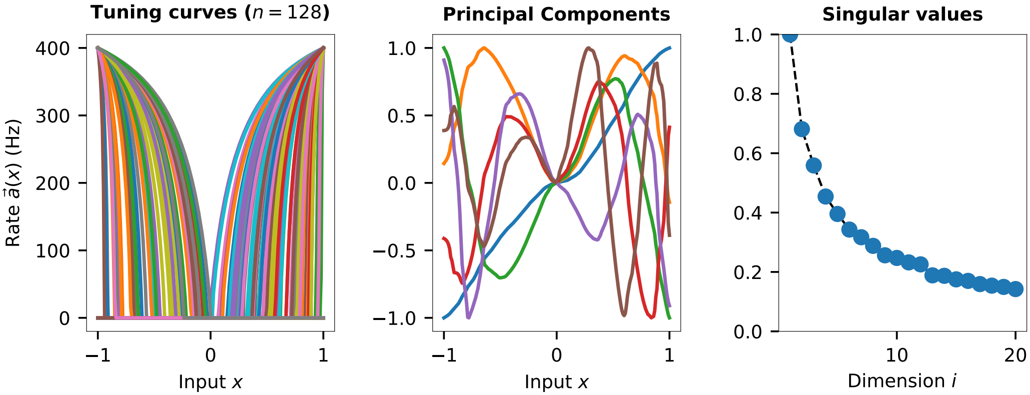
Modifying the Basis – Equidistant x-Intercepts (II)



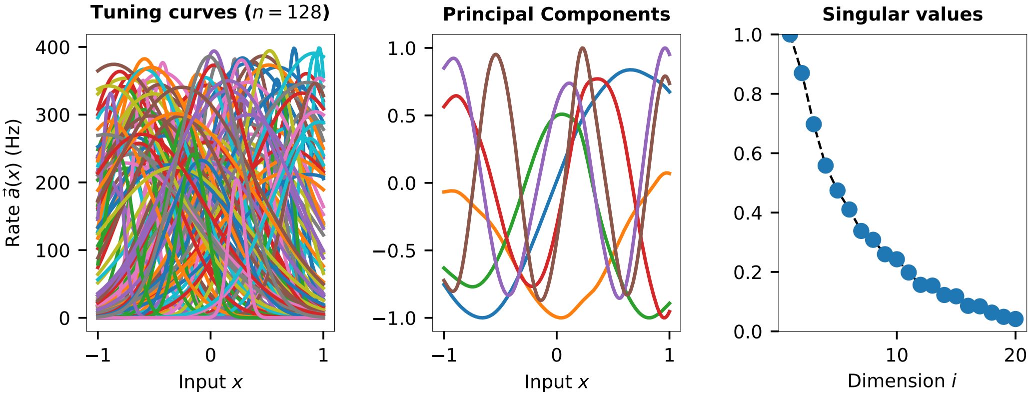
Modifying the Basis – Limited x-Intercepts (III)



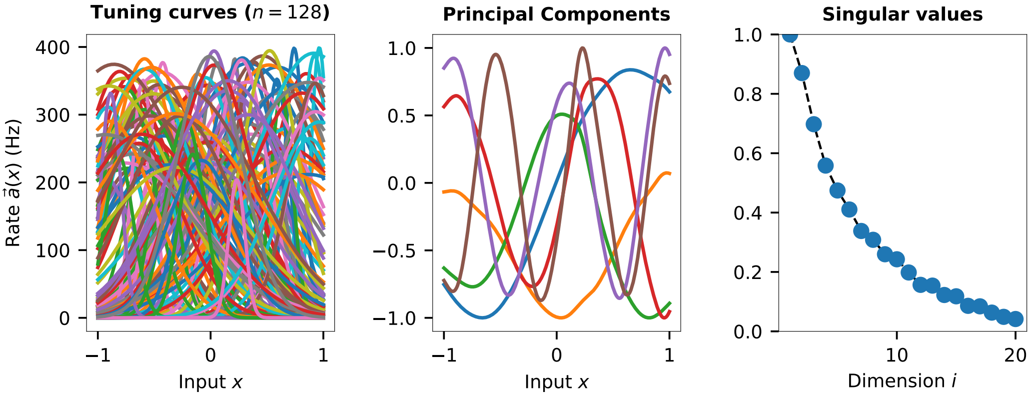
Modifying the Basis – Symmetric Tuning Curves (IV)



Gaussian Tuning Curve Principal Components

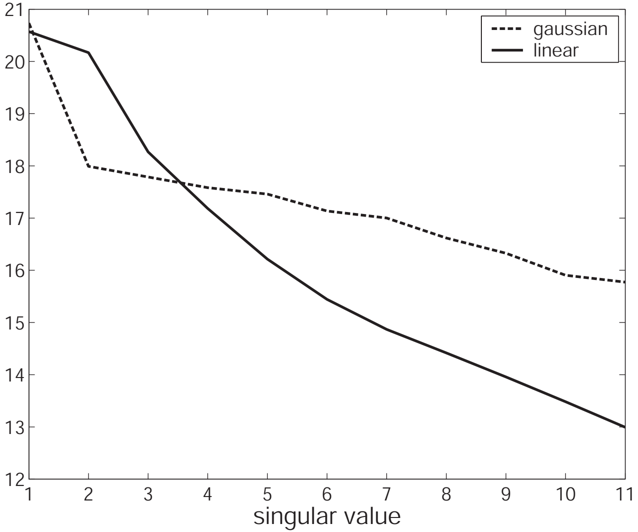


Gaussian Tuning Curve Principal Components

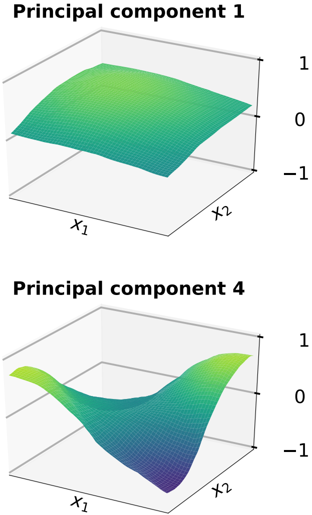


$\approx$ Fourier Basis

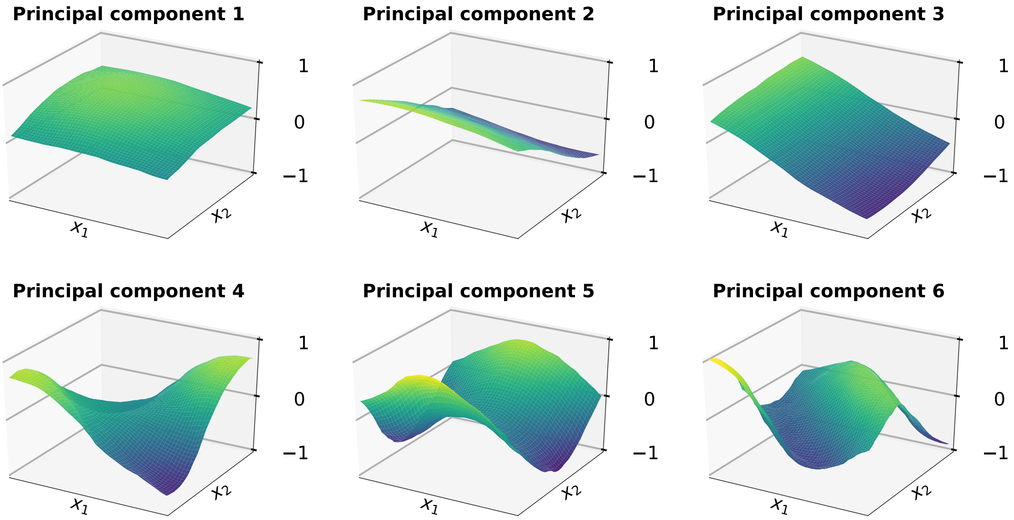
Gaussian vs LIF Singular Values



PCA of 2D Tuning Curves

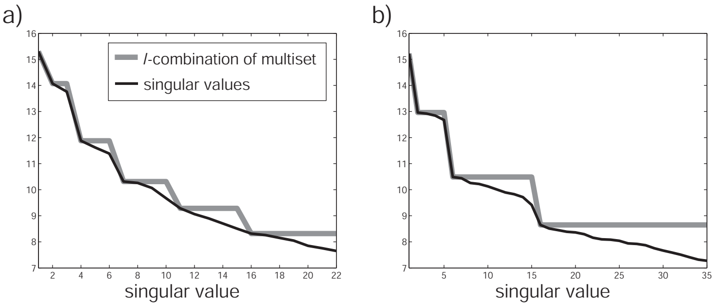


PCA of 2D Tuning Curves



Combination of 2D Polynomials

2D Singular Values



2D and 4D singular values compared to the prediction of the multiset (i.e., number of cross terms).

Conclusions

- Can use **PCA** to find the basis functions underlying neural representations
- **Singular values** inversely proportional to noise sensitivity
- **Basis function shape** depends on
  - max firing rate distribution (a bit)
  - x-intercept distribution
  - neuron response curve $G[J]$
- Finding optimal tuning curves for computing particular functions
  $\Rightarrow$ Full network optimization (must use gradient descent)

Image sources

Title slide
Maurice Denis: Homage to Cézanne, 1900
From Wikimedia.