

SYDE 556/750

Simulating Neurobiological Systems
Lecture 6: Recurrent Dynamics

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October 7 & 20, 2022

- ▶ Slide design: Andreas Stöckel
- ▶ Content: Terry Stewart, Andreas Stöckel, Chris Eliasmith

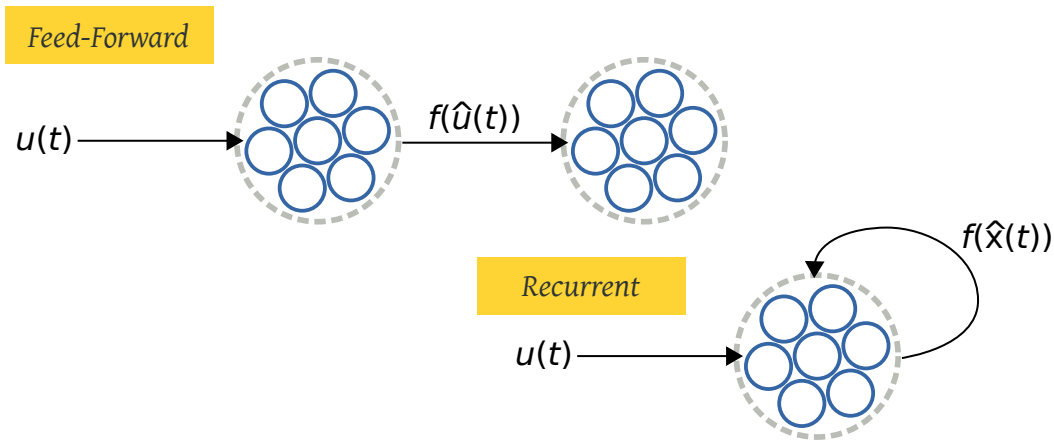


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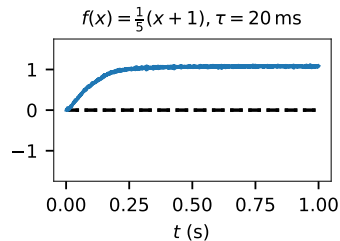
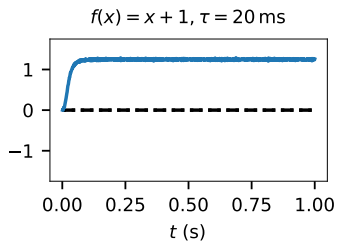
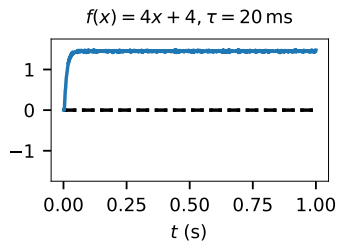
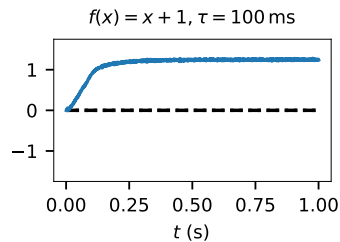
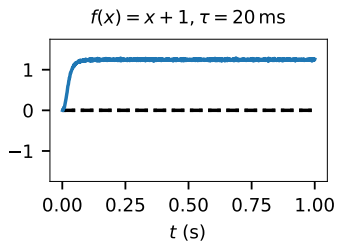
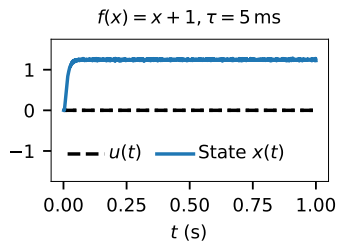
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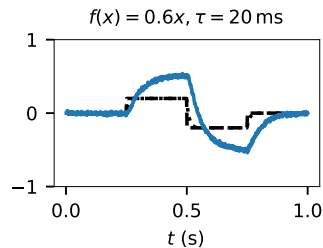
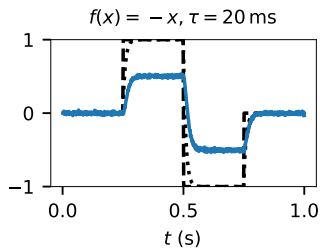
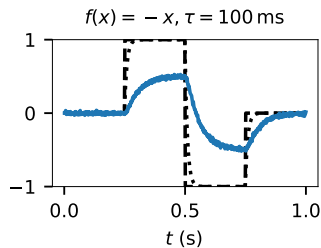
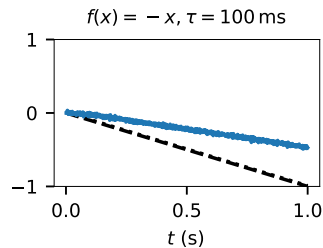
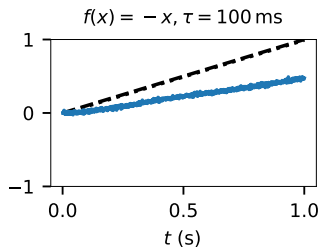
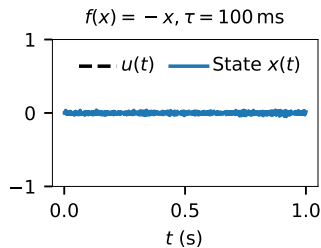
Feed Forward vs. Recurrent Connections



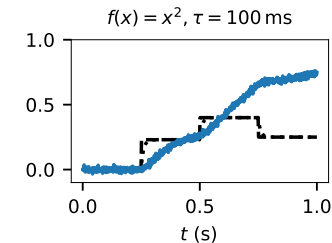
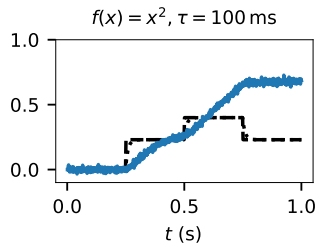
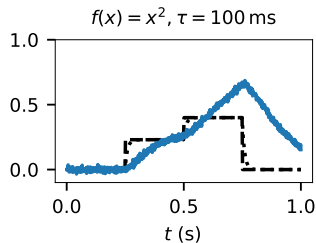
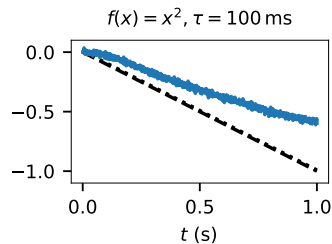
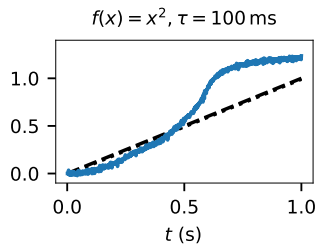
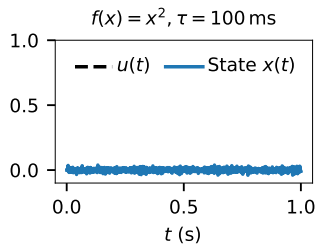
Recurrence Experiments (I)



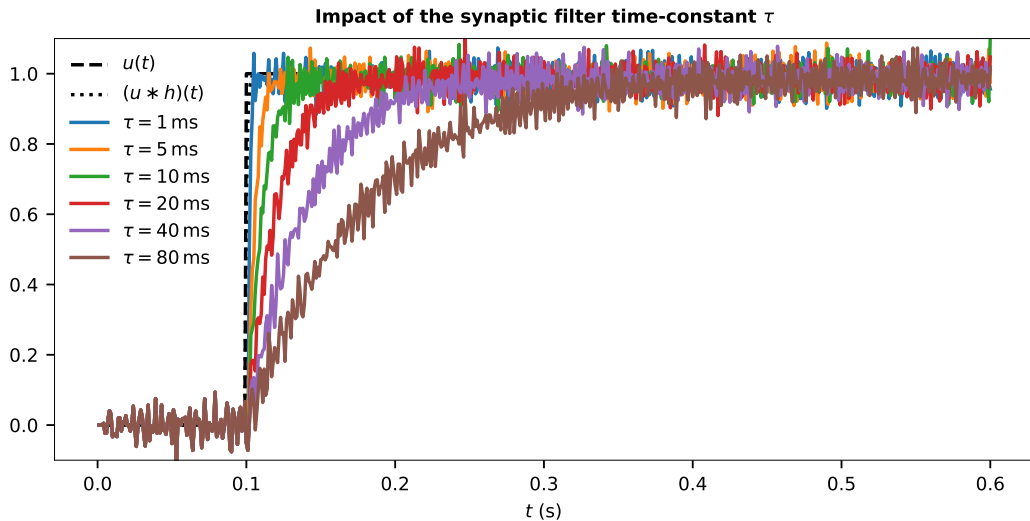
Recurrence Experiments (II)



Recurrence Experiments (III)



Making Sense of Dynamics



Behaviour of a Recurrent Connection

Feed-forward

$$x(t) = g(u(t)) * h(t)$$

Recurrent

$$x(t) = (g(u(t)) + f(x(t))) * h(t)$$

$$X(s) = (G(s) + F(s))H(s)$$

$$X(s) = (G(s) + F(s)) \frac{1}{1 + s\tau}$$

$$X(s) + s\tau X(s) = G(s) + F(s)$$

$$s\tau X(s) = G(s) + F(s) - X(s)$$

$$sX(s) = \frac{F(s) - X(s)}{\tau} + \frac{G(s)}{\tau}$$

$$\frac{dx}{dt} = \frac{f(x(t)) - x(t)}{\tau} + \frac{g(u(t))}{\tau}$$

Behaviour of a Recurrent Connection

$$\frac{dx}{dt} = \frac{f(x(t)) - x(t)}{\tau} + \frac{g(u(t))}{\tau}$$

If we want this

$$\frac{dx}{dt} = a(x) + b(u)$$

Then we set

$$f(x) = \tau a(x) + x$$

$$g(x) = \tau b(u)$$

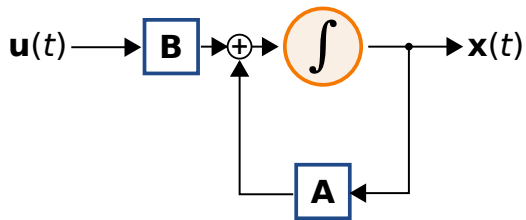
And we get

$$\frac{dx}{dt} = \frac{(\tau a(x) + x) - x}{\tau} + \frac{(\tau b(u))}{\tau}$$

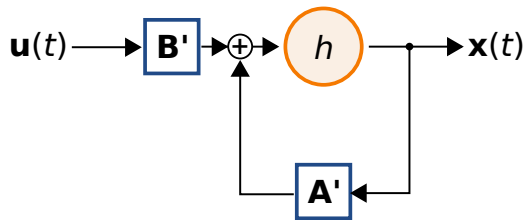
$$\frac{dx}{dt} = a(x) + b(u)$$

Implementing Dynamics using a Neural Ensemble

Evaluating an LTI
using an Integrator



Evaluating an LTI
using a Synaptic Filter



Implementing Dynamical Systems as a Neural Ensemble

LTI System

$$\varphi(\mathbf{u}, \mathbf{x}) = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$

$$\varphi'(\mathbf{u}, \mathbf{x}) = \mathbf{A}'\mathbf{x} + \mathbf{B}'\mathbf{u}$$

$$\mathbf{A}' = \tau\mathbf{A} + \mathbf{I}$$

$$\mathbf{B}' = \tau\mathbf{B}.$$

Additive Time-Invariant System

$$\varphi(\mathbf{u}, \mathbf{x}) = f(\mathbf{x}) + g(\mathbf{u})$$

$$\varphi'(\mathbf{u}, \mathbf{x}) = f'(\mathbf{x}) + g'(\mathbf{u})$$

$$f'(\mathbf{x}) = \tau f(\mathbf{x}) + \mathbf{x}$$

$$g'(\mathbf{u}) = \tau g(\mathbf{u})$$

“General” Recipe

Scale the original dynamics by τ , add feedback \mathbf{x}

NEF Principle 3: Dynamics

Time-Invariant Dynamical System

$$\frac{d\mathbf{x}(t)}{dt} = f(\mathbf{x}(t), \mathbf{u}(t))$$

Linear Time-Invariant (LTI) Dynamical System

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$

NEF Principle 3 – Dynamics

Neural dynamics are characterized by considering neural representations as control theoretic state variables. We can use control theory (and dynamical systems theory) to analyse and construct these systems.

Low-pass Filter Example

Desired behaviour

$$\frac{dx}{dt} = \frac{u - x}{\tau_{desired}}$$

Feed-forward input

$$g(u) = \tau_{synapse} \left(\frac{u}{\tau_{desired}} \right)$$

$$g(u) = \frac{\tau_{synapse}}{\tau_{desired}} u$$

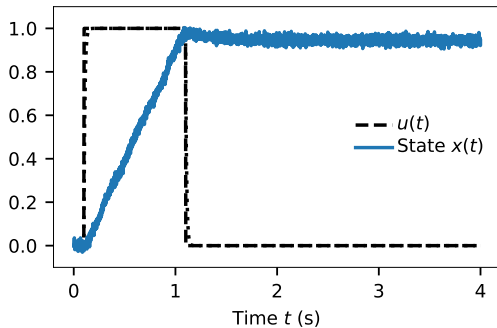
Recurrent

$$f(x) = \tau_{synapse} \left(\frac{-x}{\tau_{desired}} \right) + x$$

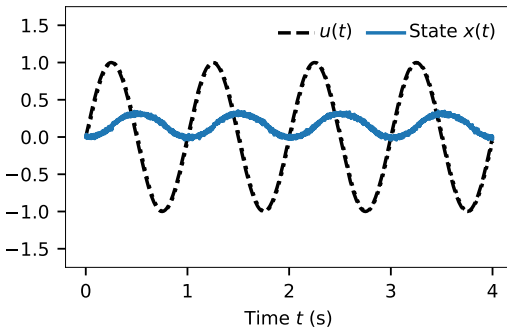
$$f(x) = \left(1 - \frac{\tau_{synapse}}{\tau_{desired}} \right) x$$

Integrator Example (I)

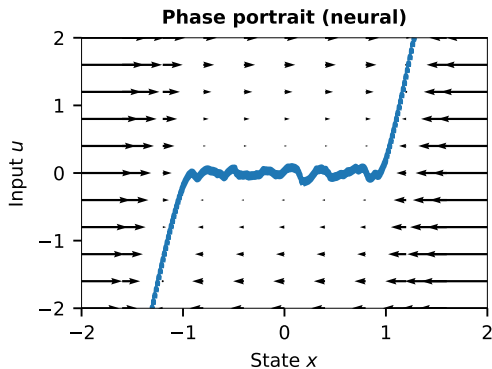
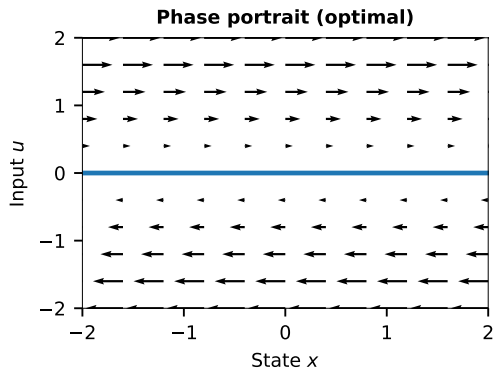
Step function input



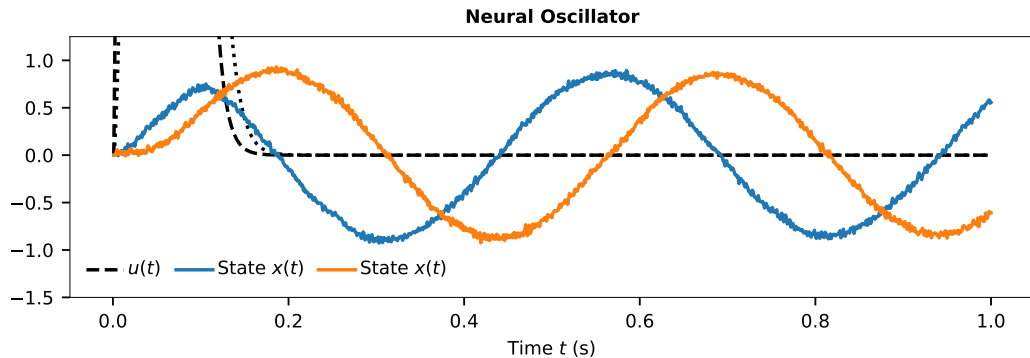
Sine input



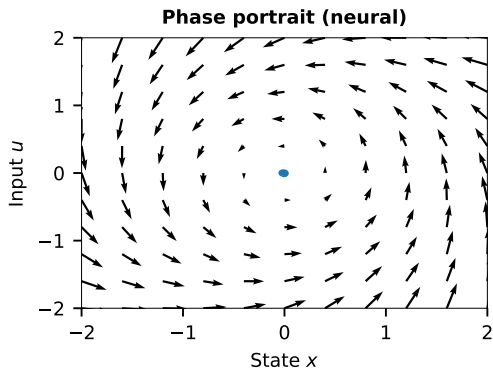
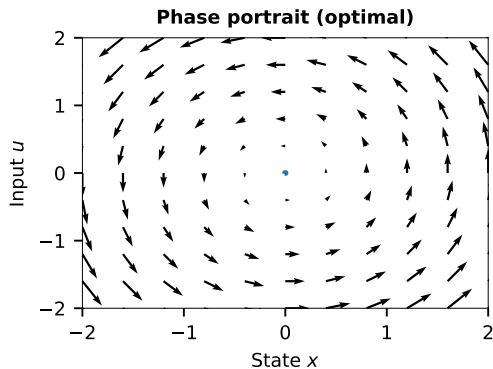
Integrator Example (II)



Oscillator Example (I)

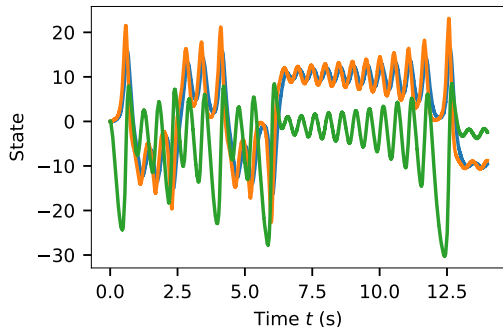


Oscillator Example (II)

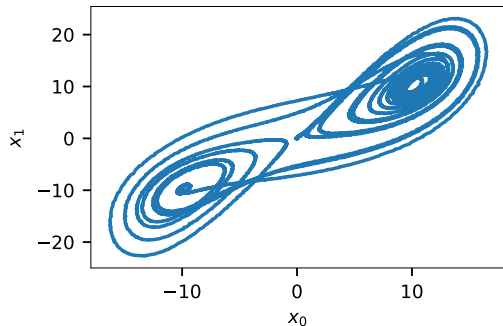


Lorentz Attractor

State over time



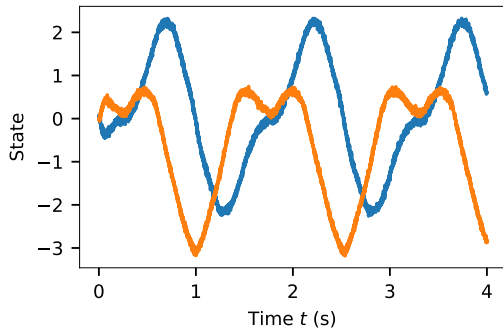
2D slice of the state space



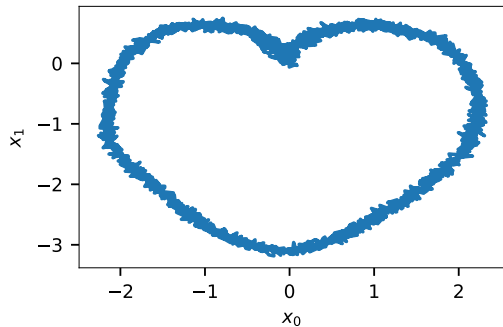
$$\frac{d\mathbf{x}(t)}{dt} = \begin{pmatrix} 10x_2(t) - 10x_1(t) \\ -x_1(t)x_3(t) - x_2(t) \\ x_1(t)x_2(t) - \frac{8}{3}(x_3(t) + 28) - 28 \end{pmatrix}$$

Heart Shape

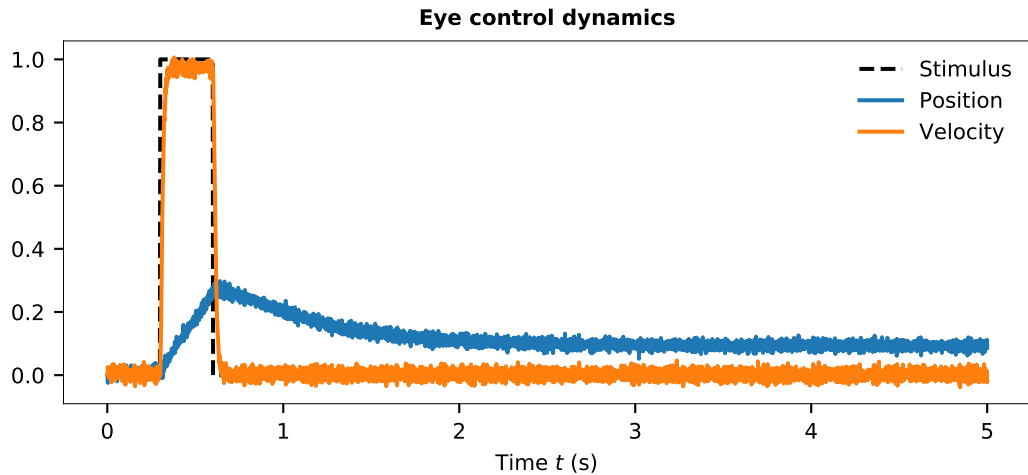
State over time



2D slice of the state space



Horizontal Eye Control



Administrative Remarks

- ▶ **Assignment 2** due Monday, October 25
- ▶ **Project proposals** due Wednesday, October 27

Image sources

Title slide

“The Canada 150 Mosaic Mural”

Author: Mosaic Canada Murals.

From Wikimedia.