

$$a_i(t) = \sum_{k=1}^{\infty} \delta(t - t_i^k)$$

$$D = (A^T A)^{-1} A^T X$$

Decoders: $X \quad A$

① Random $x(t) \leftarrow$ white noise,

② $X = [x_0 \ x_1 \ x_2 \ \dots]_{D \times N_t} \quad N_t = \frac{T}{\Delta t}$

③ Enc: $a_i(t) = \sum_k \delta(t - t_i^k) = G[x_i \otimes_i x(t) + J_i^b]$

④ $A = \begin{matrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_i \end{matrix} \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & \dots & \dots & \dots & \dots & \dots & \dots & \dots \end{bmatrix}$

$\xleftarrow{\Delta t} \quad \quad \quad \xrightarrow{\Delta t}$

$N \times N_t$

Random Signals

White Noise ① Δt

② sample $x \sim N(0, 1)$ at every st.

$x(t)$

or freq domain

① $X(\omega) = X(-\omega) = a + ib$

② a and $b \sim N(0, 1)$

Band limited

① Set all $X(\omega) = 0$ for $\omega > 2\pi F$

$\hat{=}$

do ②

③ $\mathcal{F}^{-1}(X(\omega)) = x(t)$

Decoders: $D = (AA^T)^{-1}A^T X$

Filtering:

① $F(t) * G(t) = F(\omega) \cdot G(\omega)$

② $\hat{x}(t) = \sum_i a_i(t) d_i$

$\hat{x}(t) * h(t) = \sum_i d_i (a_i(t) * h(t)) = \left(\sum_i d_i a_i(t) \right) * h(t)$

Optimal Filter

$$\hat{x}(t) = \arg \min_h \int_{-\infty}^{\infty} (x(t) - \hat{x}(t))^2 dt$$

$(\sum_i a_i d_i) * h(t)$

$$\hat{x}(t) = (a_1 d_1 + a_2 d_2) * h(t)$$



$$d_1 = -d_2$$

$$\hat{x}(t) = d_1 (a_1 - a_2) * h(t)$$

or $(r * h)(t)$

↑ response

time + "space"

$$E = \int_{-\infty}^{\infty} (x(t) - (r * h)(t))^2 dt$$

$$E(\omega) = \int_{-\infty}^{\infty} |X(\omega) - R(\omega) \cdot H(\omega)|^2 d\omega$$

$$\frac{dE}{dH} = 0$$

$$\begin{aligned} \frac{dE}{dH} &= 2 (X\omega - R(\omega)H(\omega)) (-\bar{R}) \\ &= -2X\bar{R} + 2HR\bar{R} \end{aligned}$$

$$2X\bar{R} = 2HR\bar{R}$$

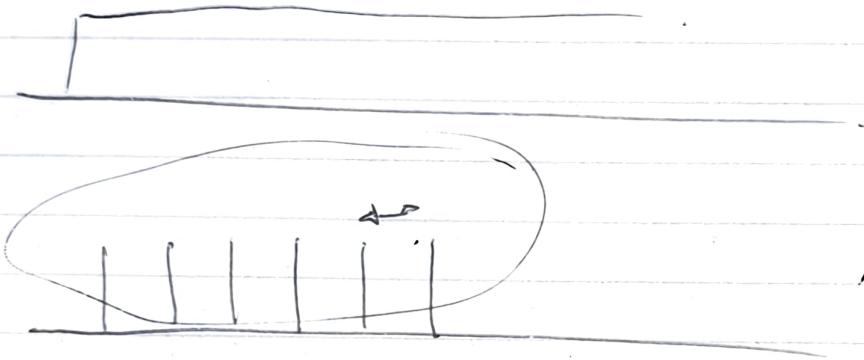
$$H = \frac{X\bar{R}}{|R|^2} \xrightarrow{\text{better}} \frac{(X\bar{R}) * W}{|R|^2 * W}$$

$$\int a(t) = I$$

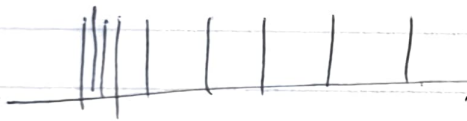
$$\hat{x} = \sum d_i a_i * u(t)$$

↑
"spatial"

"split" temporal
+
spatial
decode.



$$A = \begin{bmatrix} \text{---} & \text{---} \\ \text{---} & \text{---} \end{bmatrix}$$



$$A = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & \dots \end{bmatrix}$$

↑
temporally sensitive
decoders.