$$\hat{x} = \sum_{i=0}^{n} a_i d_i \Rightarrow Da$$

$$E = \frac{1}{2}\int_{-1}^{1} \left[x - \hat{x}\right]^2 dx$$

$$= \frac{1}{2}\int_{-1}^{1} \left[x - \sum_i a_i d_i\right]^2 dx$$

$$\frac{dE}{dd_i} = -\cancel{\frac{1}{2}}\int_{-1}^{1} \cancel{2}\left[x - \sum_j a_j d_j\right] a_i \, dx$$

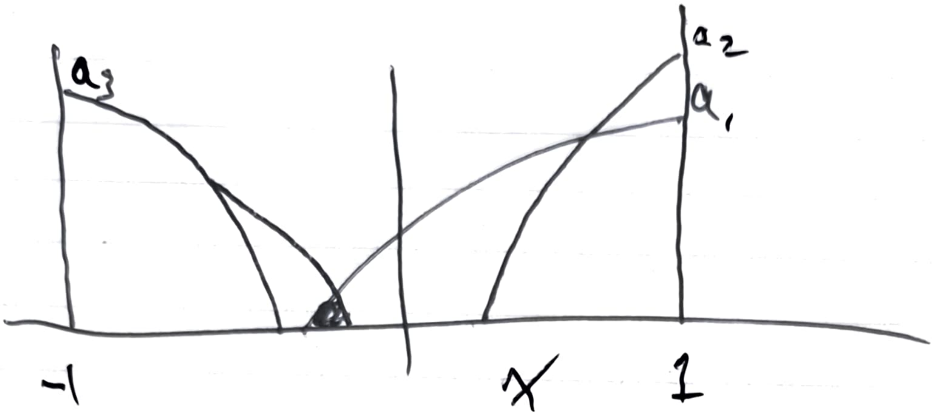
$$0 = -\int a_i x + \int \Big(\sum_j a_j d_j\Big) a_i$$

$$\int a_i x = \int \Big(\sum_j a_j d_j\Big) a_i$$

$$\underbrace{\int_{-1}^{1} a_i(x) x \, dx}_{Y} = \cancel{\sum_{j=0}^{n} \int_{-1}^{1} a_j a_i(x) d_j \, dx}$$

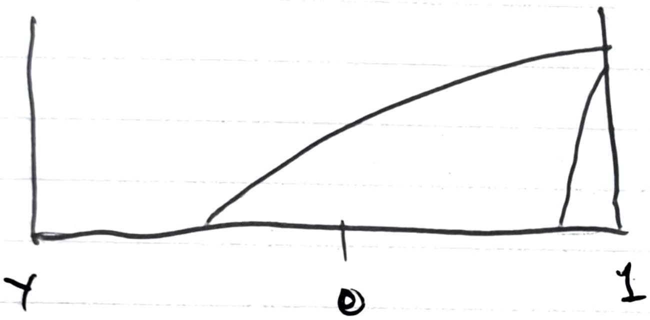
$$= \sum_{j=0}^{n} \underbrace{\int_{-1}^{1} a_j(x) a_i(x)}_{\Gamma} \underbrace{d_j}_{D} \, dx$$

$$\boxed{Y = \Gamma D} \Rightarrow \boxed{\Gamma^{-1} Y = D}$$



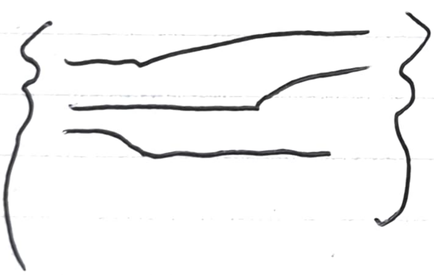
$\Gamma \Rightarrow$ correlation matrix.

Y.



$$\int_Y^1 x a_i(x)\,dx$$

$$D = \Gamma^{-1} Y$$

$A =$  $\Rightarrow \begin{bmatrix} a_1(x_1) & a_1(x_2) & a_1(x_3) & \cdots \\ a_2(x_1) & \cdots & & \\ & & & \\ & & & \end{bmatrix}_{N \times S}$

$$X = [x_1 \quad x_2 \quad \cdots]_{S\times 1} \qquad D = [d_1 \quad d_2 \quad \cdots]_{N \times 1}$$

$$X^T = A^T D^T$$

$$AX^T = (AA^T) D^T$$

$$\underbrace{(AA^T)^{-1} A X^T = D^T}$$

DECODERS UNDER NOISE

$\eta \sim N(0, \sigma^2)$

$$E = \mathbb{E}\left[\frac{1}{2} \int_{-1}^{1} \left(x - \sum_{i=1}^{N} (a_i(x) + \eta) d_i\right)^2 dx \right]_\eta$$

expand

cross term $\rightarrow \cancel{\mathbb{E}\left[\left(x - \sum_i a_i d_i\right)\left(\sum_i d_i \eta_i\right)\right]_\eta}$ 0 0

$$= \mathbb{E}\left[\frac{1}{2} \int_{-1}^{1} \left(\left(x - \sum_i a_i d_i\right) - \sum_i d_i \eta\right)^2 \right]_\eta$$

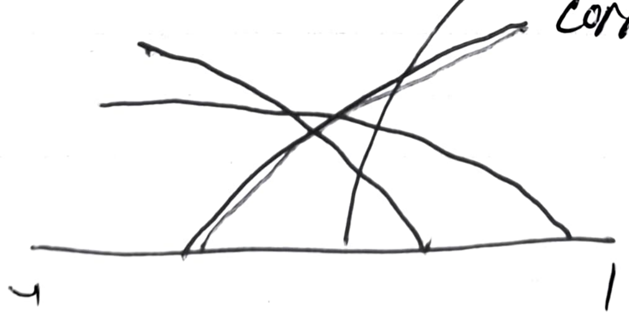
$$= \frac{1}{2} \int_{-1}^{1} \left(x - \sum_i a_i d_i\right)^2 dx + \frac{1}{2} \sum_i \sum_j d_i d_j \, \mathbb{E}(\eta_i \eta_j)_{\eta_i \eta_j}$$

$$= \frac{1}{2} \int_{-1}^{1} \left(x - \sum_i a_i d_i\right)^2 dx + \frac{1}{2} \sum_i d_i^2 \sigma^2 \rightarrow \mathbb{E}(\eta_i^2)$$

$$\frac{dE}{dd_i} = 0 = \qquad D = \Gamma_\eta^{-1} \Upsilon \qquad \Gamma_\eta = \int a_i a_j + \delta_{ij} \sigma^2$$

correlation

$i = j$



$$A = \begin{cases} a_1(x_1) + \eta_1, \quad a_2(x_2) + \eta_2, \quad \dots\dots \\ \\ \end{cases}$$

$$A = A_{GT} + E$$

$$A^T D^T = X^T$$

$$A A^T D^T = A X^T$$

$$\underline{D^T = (AA^T)^{-1} A X^T}$$

$$\cancel{AA^T = C_{GT}}$$

$$AA^T = (A_{GT} + E)(A_{GT} + E)^T$$

$$N\sigma^2 I$$

$$= A_{GT} A_{GT}^T + \underbrace{\cancel{A_{GT} E^T} + \cancel{E A_{GT}^T}} + EE^T$$

$$D^T = (A_{GT} A_{GT}^T + N\sigma^2 I)^{-1} A_{GT} X^T$$