

**SYDE 556/750**

## **Simulating Neurobiological Systems**

### **Lecture 4: Temporal Representations**

Chris Eliasmith

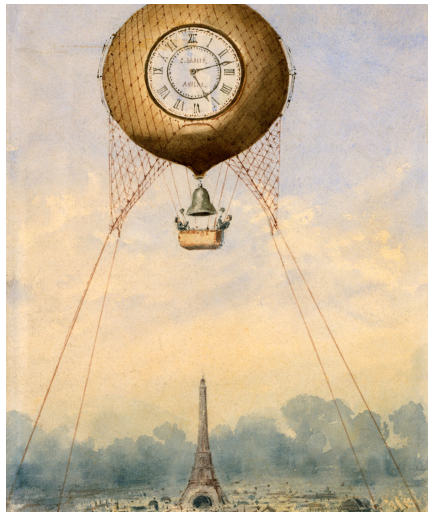
Sept 20 & 25, 2023

- ▶ Slide design: Andreas Stöckel
- ▶ Content: Terry Stewart, Andreas Stöckel, Chris Eliasmith

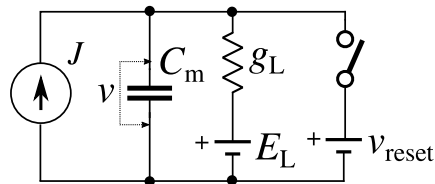
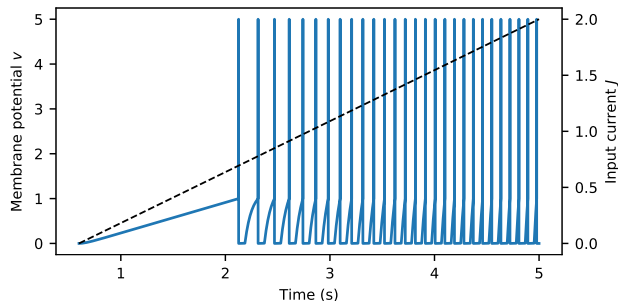


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ENGINEERING



## Reminder: The LIF Neuron



$$\frac{d}{dt}v(t) = -\frac{1}{\tau_{RC}}(v(t) - J),$$

$$v(t) \leftarrow \delta(t - t_{\text{th}}),$$

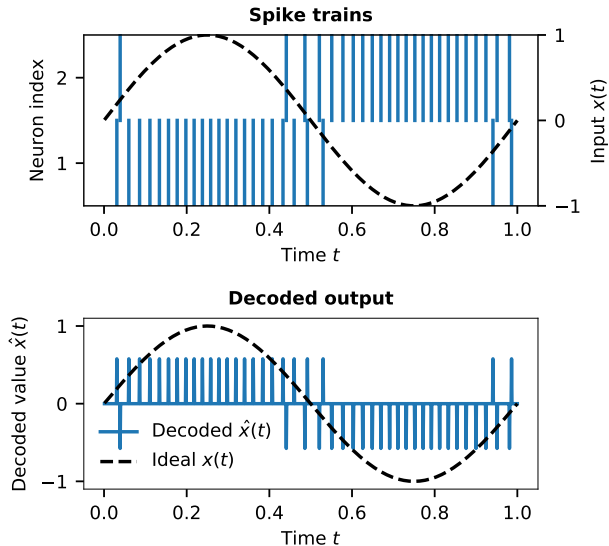
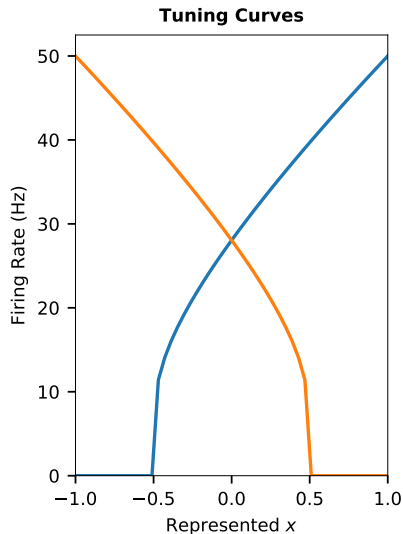
$$v(t) \leftarrow 0,$$

$$\text{if } v(t) < 1,$$

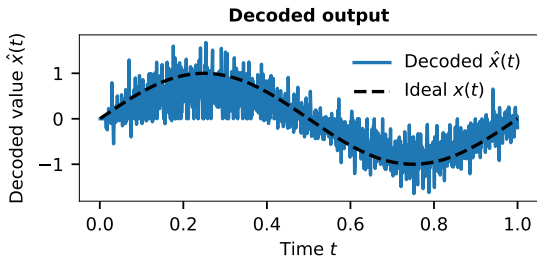
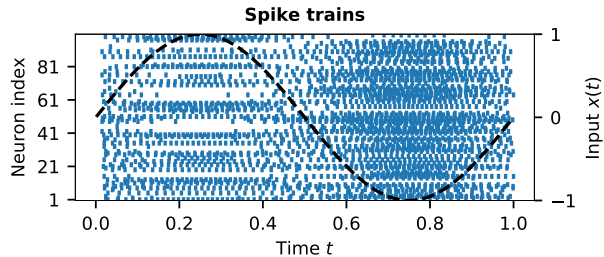
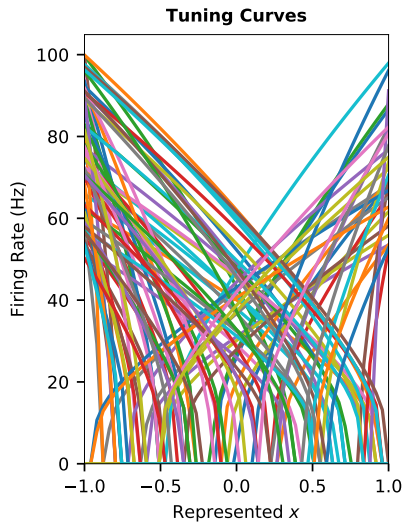
$$\text{if } t = t_{\text{th}},$$

$$\text{if } t > t_{\text{th}} \text{ and } t \geq t_{\text{th}} + \tau_{\text{ref}},$$

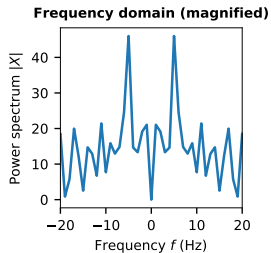
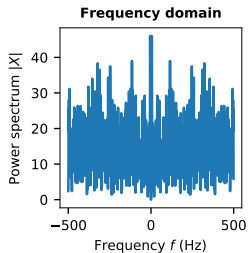
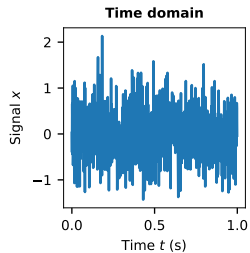
# Temporal Decoding of Two Neurons - Weighted Spikes



# Temporal Decoding of One Hundred Neurons - Weighted Spikes

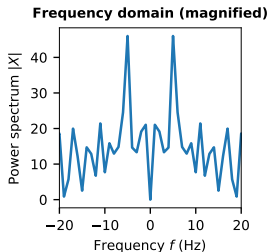
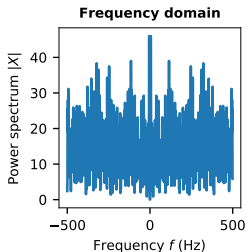
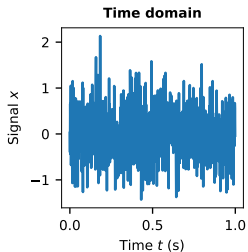


# Random Signals

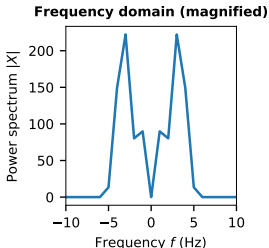
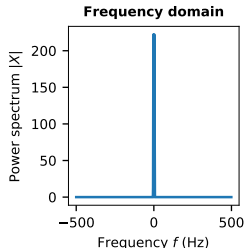
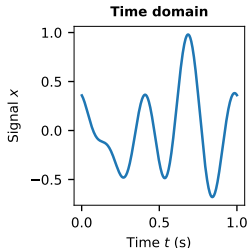


**White Noise**  
(zero mean)

# Random Signals

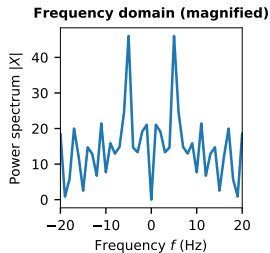
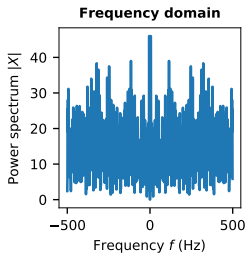
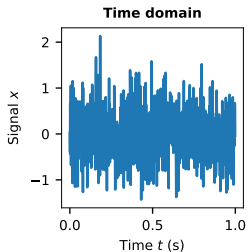


**White Noise**  
(zero mean)

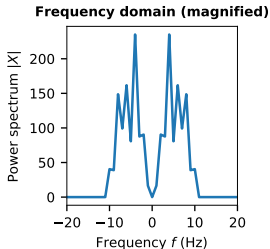
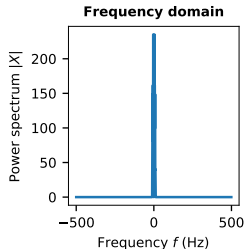
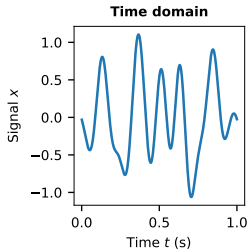


**Bandlimited**  
White Noise  
(zero mean,  
5 Hz bandwidth)

# Random Signals

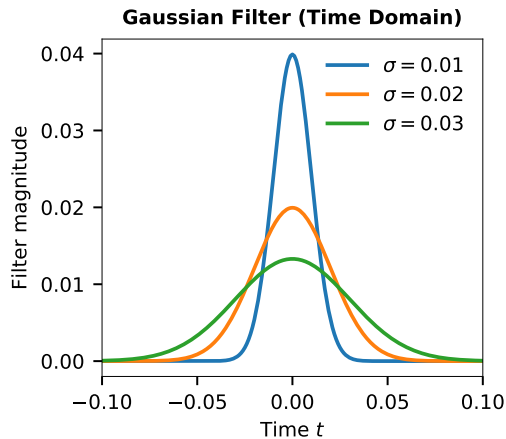


**White Noise**  
(zero mean)



**Bandlimited**  
White Noise  
(zero mean,  
10 Hz bandwidth)

# Filtering by Convolution



## Gaussian Filter

$$h(t) = c \exp\left(\frac{-t^2}{\sigma^2}\right)$$

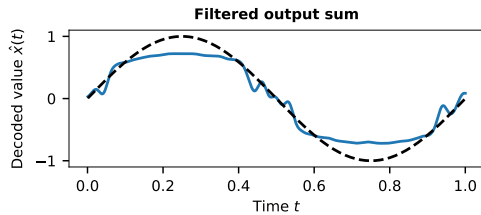
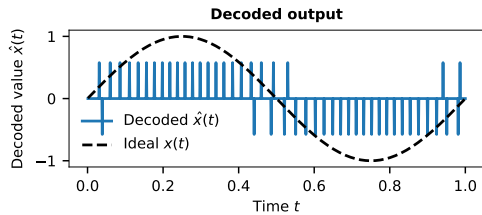
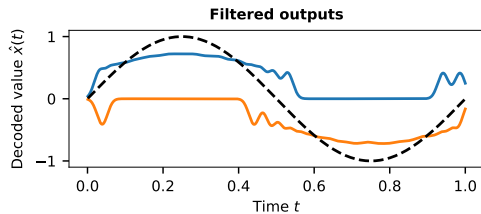
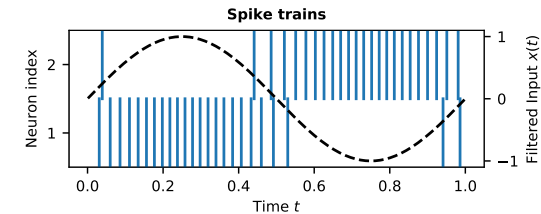
where  $c$  chosen s.t.  $\int_{-\infty}^{\infty} h(t) dt = 1$

## Convolution

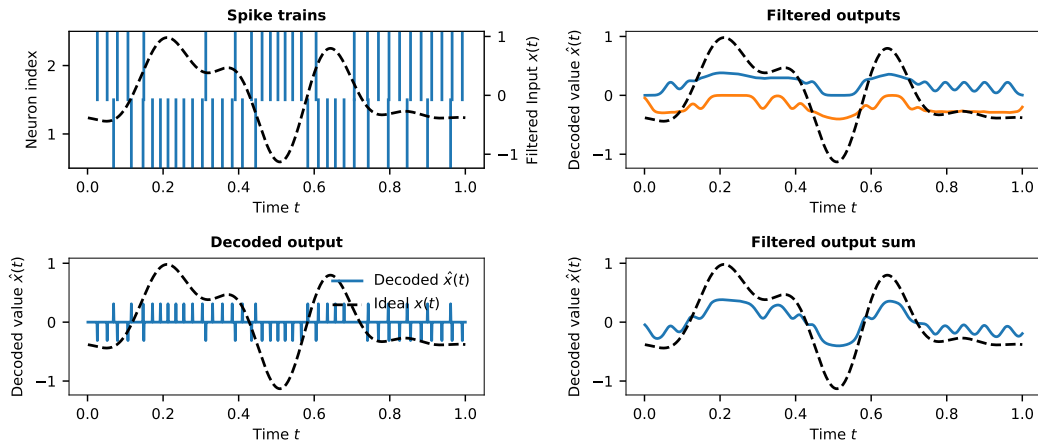
$$(f * h)(t) = \int_{-\infty}^{\infty} f(t - t') h(t') dt'$$



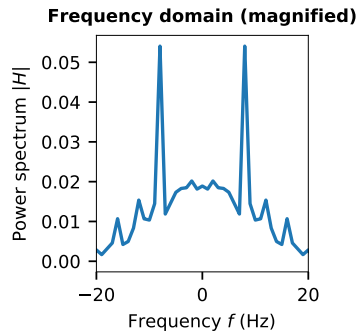
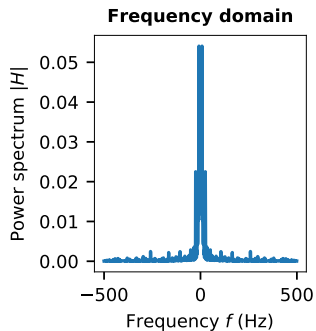
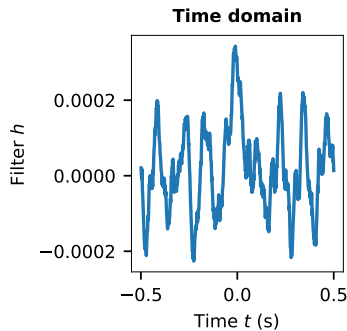
# Filtering a Spike Train



# Filtering a Spike Train for a Random Signal

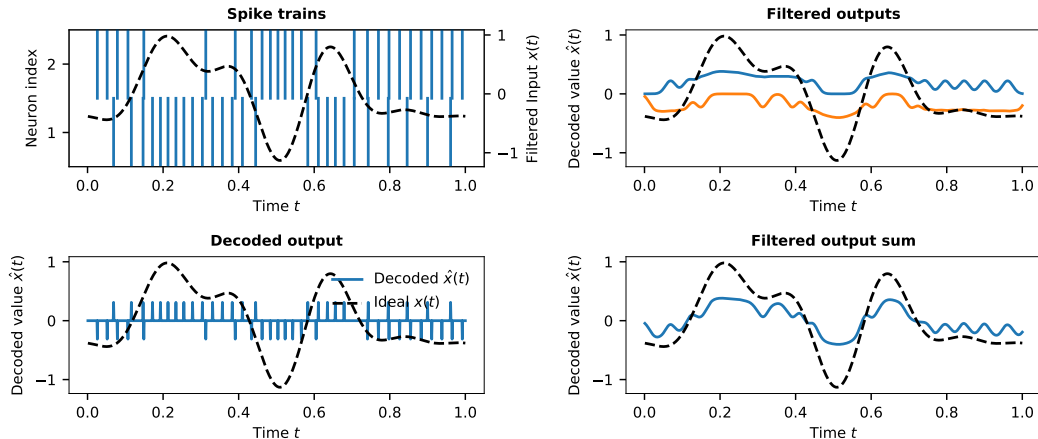


# Optimal Filter

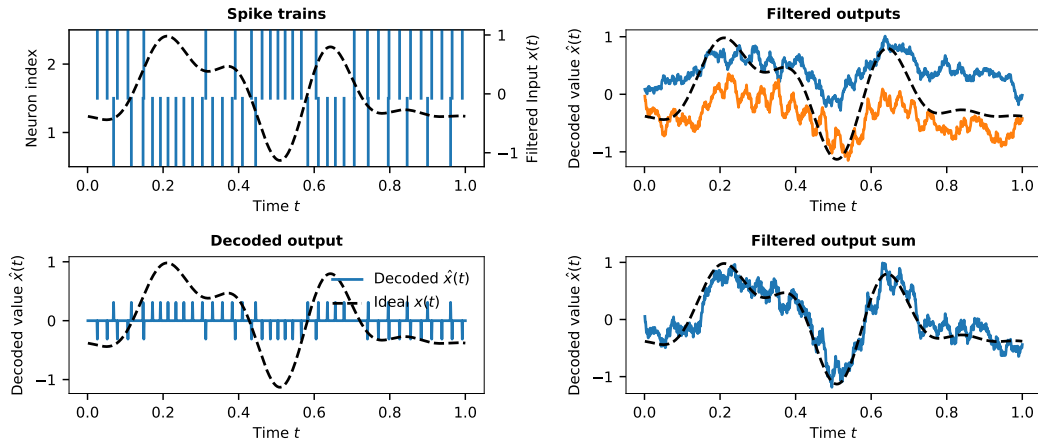


$$H(\omega) = \frac{X(\omega)\overline{R}(\omega)}{|R(\omega)|^2}$$

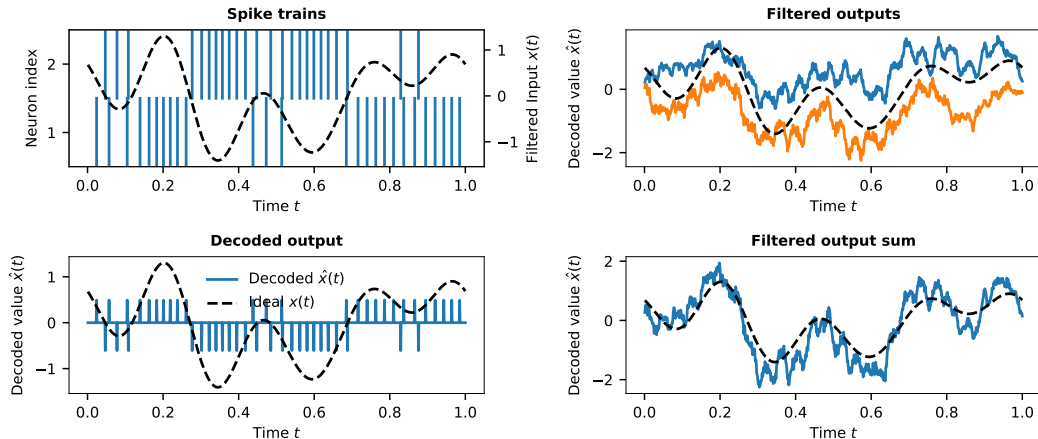
# Filtering a Spike Train for a Random Signal (Optimal Filter)



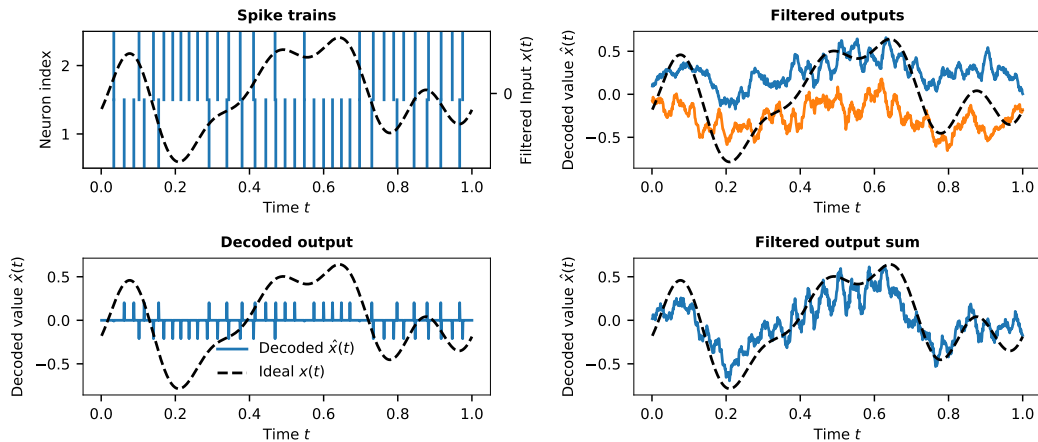
# Filtering a Spike Train for a Random Signal (Optimal Filter)



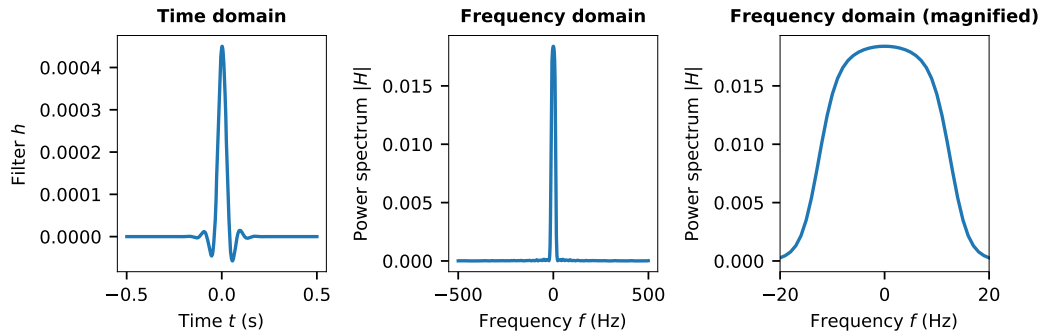
# Filtering a Spike Train for a Random Signal (Optimal Filter)



# Filtering a Spike Train for a Random Signal (Optimal Filter)



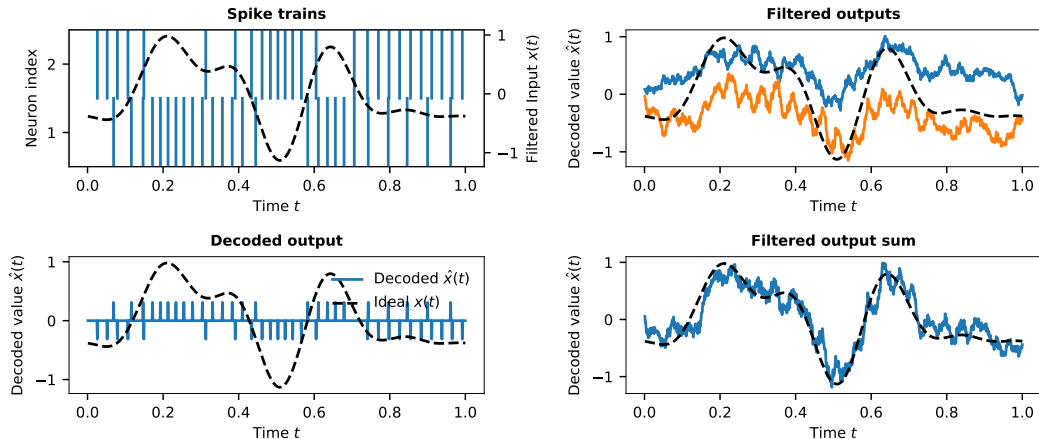
# Optimal Filter (Improved)



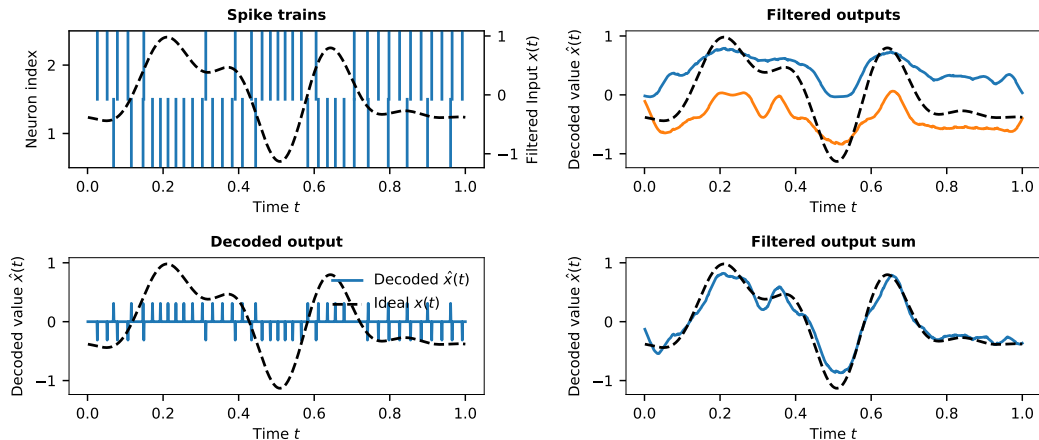
$$H(\omega) = \frac{X(\omega)\overline{R}(\omega) * W(\omega)}{|R(\omega)|^2 * W(\omega)}$$



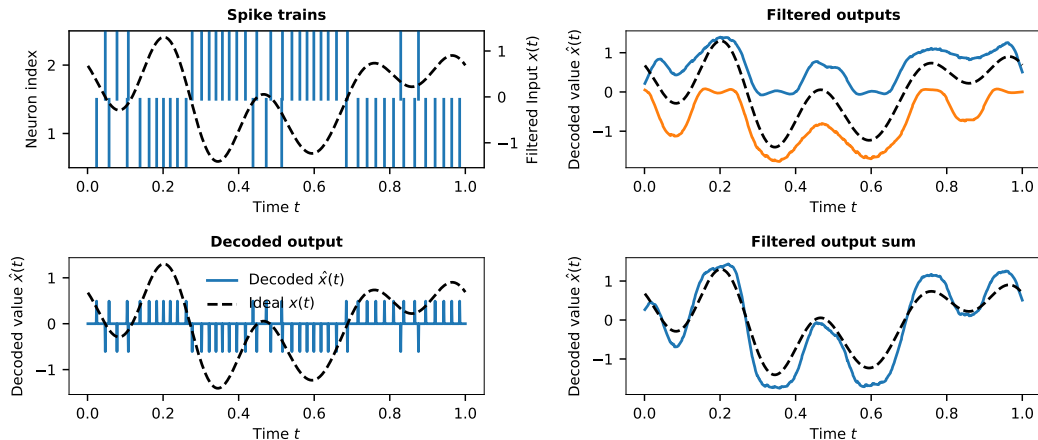
# Filtering a Spike Train for a Random Signal (Improved Optimal Filter)



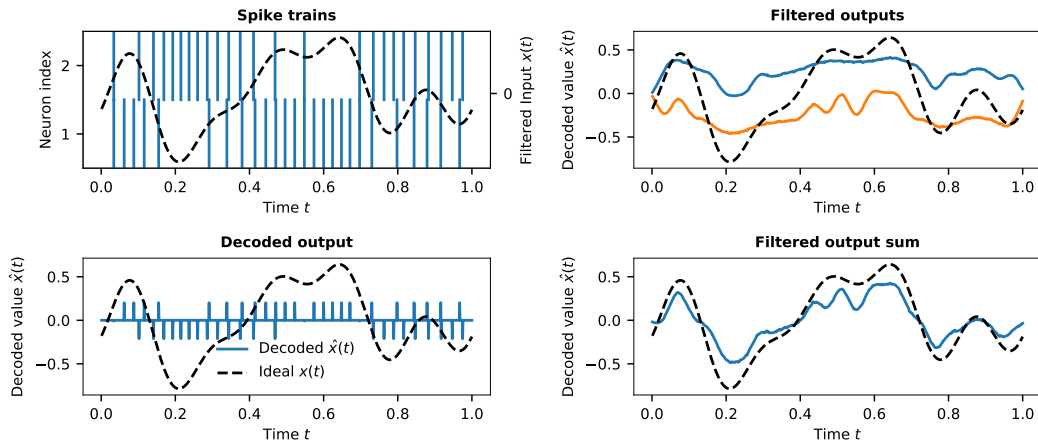
# Filtering a Spike Train for a Random Signal (Improved Optimal Filter)



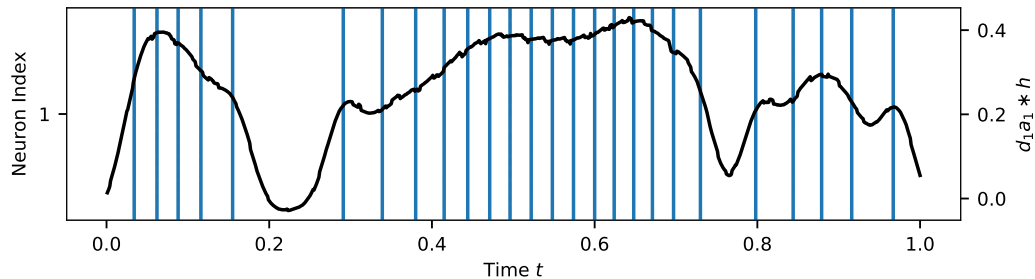
# Filtering a Spike Train for a Random Signal (Improved Optimal Filter)



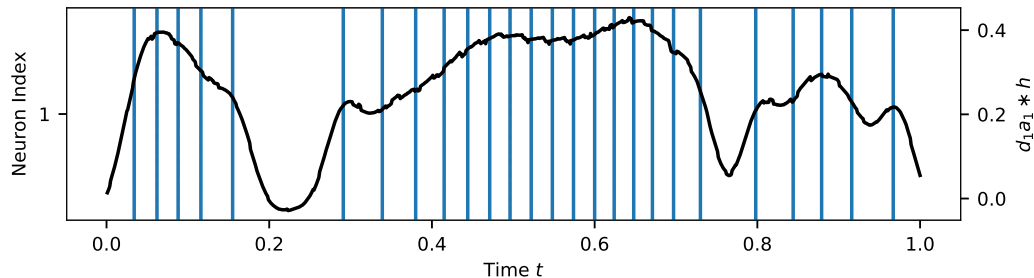
# Filtering a Spike Train for a Random Signal (Improved Optimal Filter)



## Pros and Cons of the Optimal Filter



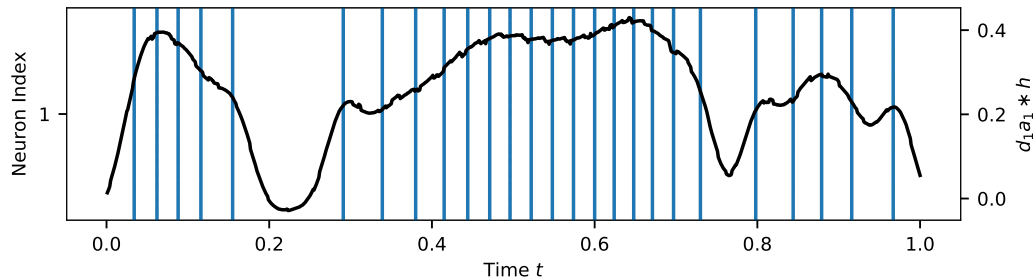
## Pros and Cons of the Optimal Filter



### + Precise

Good for analysing data after the fact

# Pros and Cons of the Optimal Filter



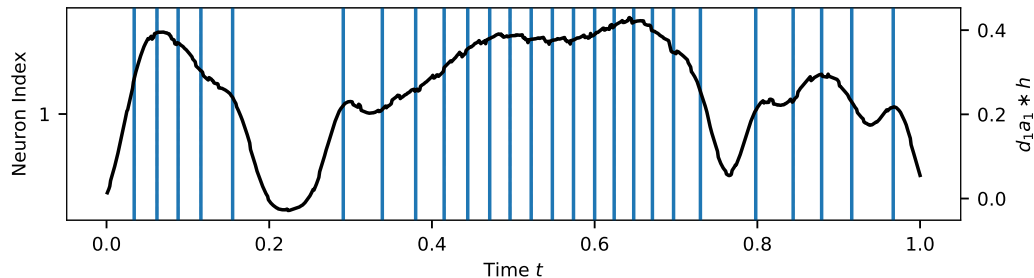
## + Precise

Good for analysing data after the fact

## - Non-causal

Does not describe a biological process

## Pros and Cons of the Optimal Filter



**+ Precise**

Good for analysing data after the fact

**-**

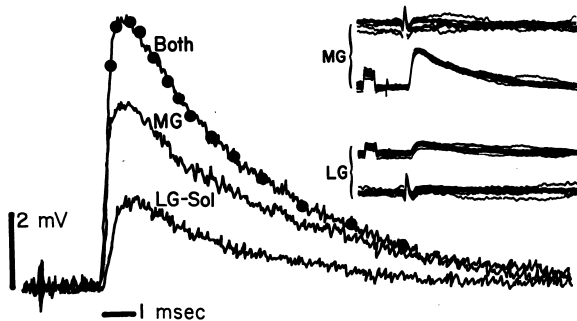
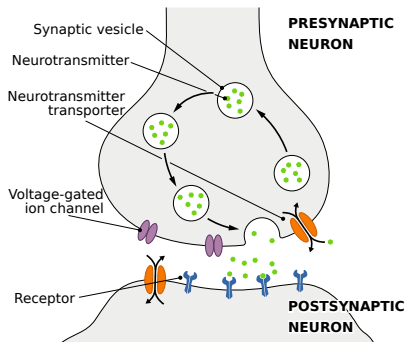
**Non-causal**

Does not describe a biological process

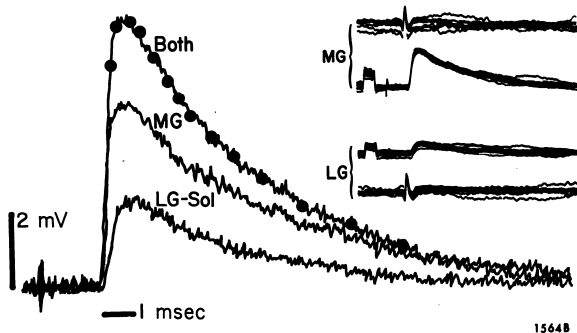
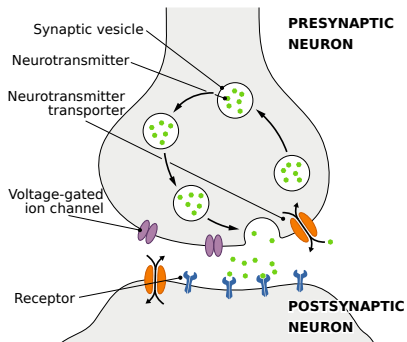
We need to find a mechanism that low-pass filters spikes over time!



# Synapses as Filters



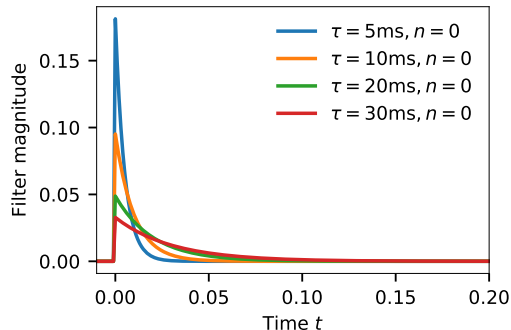
# Synapses as Filters



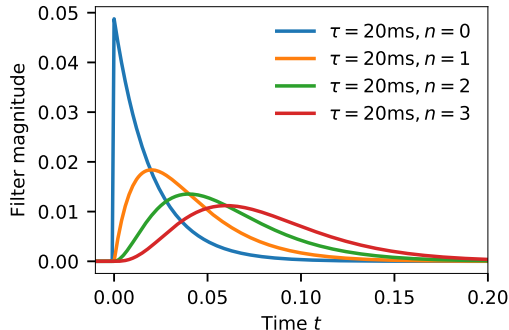
**Post-synaptic currents (EPSCs, IPSCs) are low-pass filtered spike trains!**

# Exponential Low-Pass Filter (I)

Synaptic Filter (Time Domain)



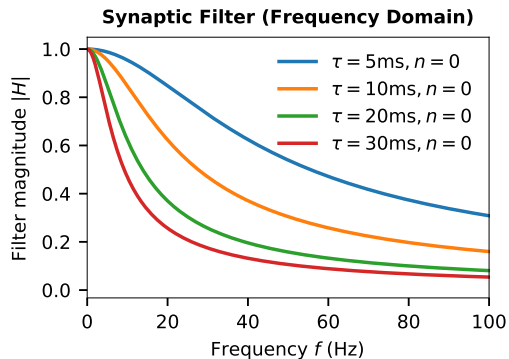
Synaptic Filter (Time Domain)



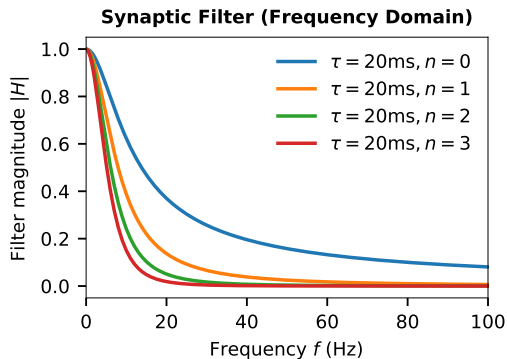
$$h(t) = \begin{cases} c^{-1} t^n \exp^{-t/\tau} & \text{if } t \geq 0, \\ 0 & \text{otherwise,} \end{cases}$$

$$\text{where } c = \int_0^{\infty} t^n \exp^{-t/\tau} dt.$$

# Exponential Low-Pass Filter (II)

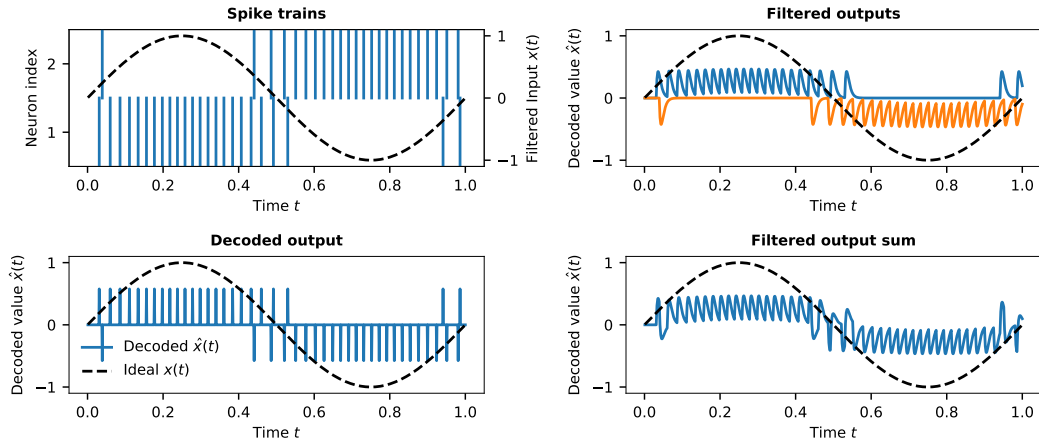


$$h(t) = \begin{cases} c^{-1} t^n \exp^{-t/\tau} & \text{if } t \geq 0, \\ 0 & \text{otherwise,} \end{cases}$$



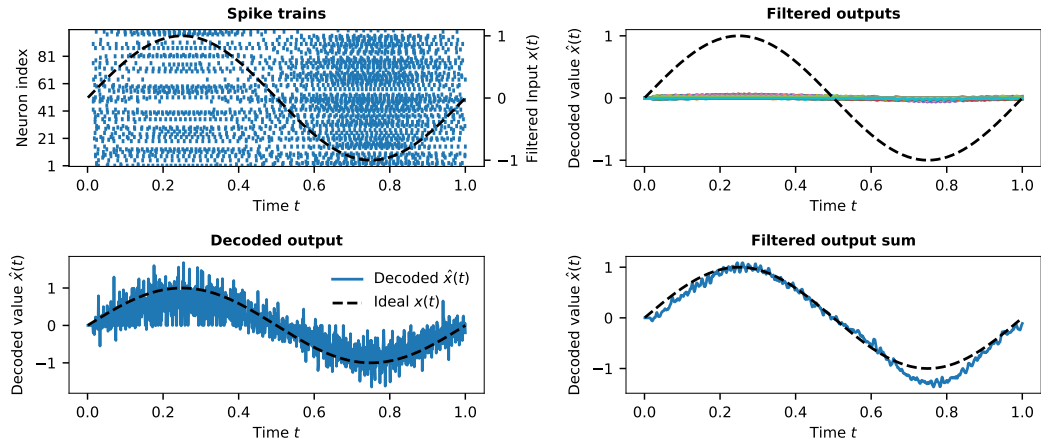
$$\text{where } c = \int_0^{\infty} t^n \exp^{-t/\tau} dt.$$

## Example: Synaptic Filter for Two Neurons



$$\tau = 5 \text{ ms}, n = 1$$

## Example: Synaptic Filter for One Hundred Neurons



$$\tau = 5 \text{ ms}, n = 1$$

# Image sources

## **Title slide**

“Captive balloon with clock face and bell, floating above the Eiffel Tower, Paris, France.”

Author: Camille Grávis, between 1889 and 1900.

From Wikimedia.