Computer Organization Homework 4

In this homework following three will be used.

- $\overline{A}$ = A' for all expression A.
- A.A = A = A + A Known as idempotent law.
  Proof:

A	A.A	A + A
0	0	0
1	1	1

  By the three expressions having the same truth table law is proven.
- (A')' = A Known as Double Negation Law

A	(A')'
0	0
1	1

  By the two expressions having the same truth table, the law is proven.

1) 0x33 = 51 = 32 + 16 + 2 + 1 and 0x55 = 85 = 64 + 16 + 4 + 1.

Therefore, the multiplication 0x55 * 0x33 can be done by shifting 0x55 5 place leftwards, then adding 0x55 shifted 4 and 1 places leftwards and then adding one more 0x55.

2) 63.25 can be written as 63 + 0.25. 63 = 111111 in binary and 0.25 = 0.01. Hence, 63.25 = 111111.01 in binary. = 1.1111101 * 2^5. The sign is positive and expn = 1028
   The double precision format is then 0x404FA00000000000.
3) (See the next page)

$$E = ((A \cdot B) + (A \cdot C) + (B \cdot C)) \cdot (\overline{A \cdot B \cdot C})$$

$$E = (A \cdot B \cdot \overline{C}) + (A \cdot C \cdot \overline{B}) + (B \cdot C \cdot \overline{A})$$

We will show the two expressions are equal.

$$E = ((A \cdot B) + (A \cdot C) + (B \cdot C)) \cdot (\overline{A \cdot B \cdot C})$$

$= ((A \cdot B) + (A \cdot C) + (B \cdot C)) \cdot (\overline{A \cdot B} + \overline{C})$ De Morgan's Law

$= ((A \cdot B) + (A \cdot C) + (B \cdot C)) \cdot (\overline{A} + \overline{B} + \overline{C})$ De Morgan's Law

$= ((A \cdot B) + (A \cdot C) + (B \cdot C)) \cdot \overline{A} +$
$((A \cdot B) + (A \cdot C) + (B \cdot C)) \cdot \overline{B} +$ Distributive Law
$((A \cdot B) + (A \cdot C) + (B \cdot C)) \cdot \overline{C}$

$= ((AB \cdot \overline{A}) + (AC\overline{A}) + (\overline{A}BC)) +$
$((A \cdot B \cdot \overline{B}) + (A \cdot C \cdot \overline{B}) + (BC \cdot \overline{B})) +$ Distributive Law
$((A \cdot B \cdot \overline{C}) + (A \cdot C \cdot \overline{C}) + (B \cdot C \cdot \overline{C}))$

$= ((\overline{A} A \cdot B) + (\overline{A} A \cdot C) + (\overline{A} BC)) +$ Commutative Law
$((A \cdot B \cdot \overline{B}) + (A C \overline{B}) + (\overline{B} B \cdot C)) +$
$((A \cdot B \overline{C}) + (A \cdot C \cdot \overline{C}) + (B \cdot C \overline{C}))$

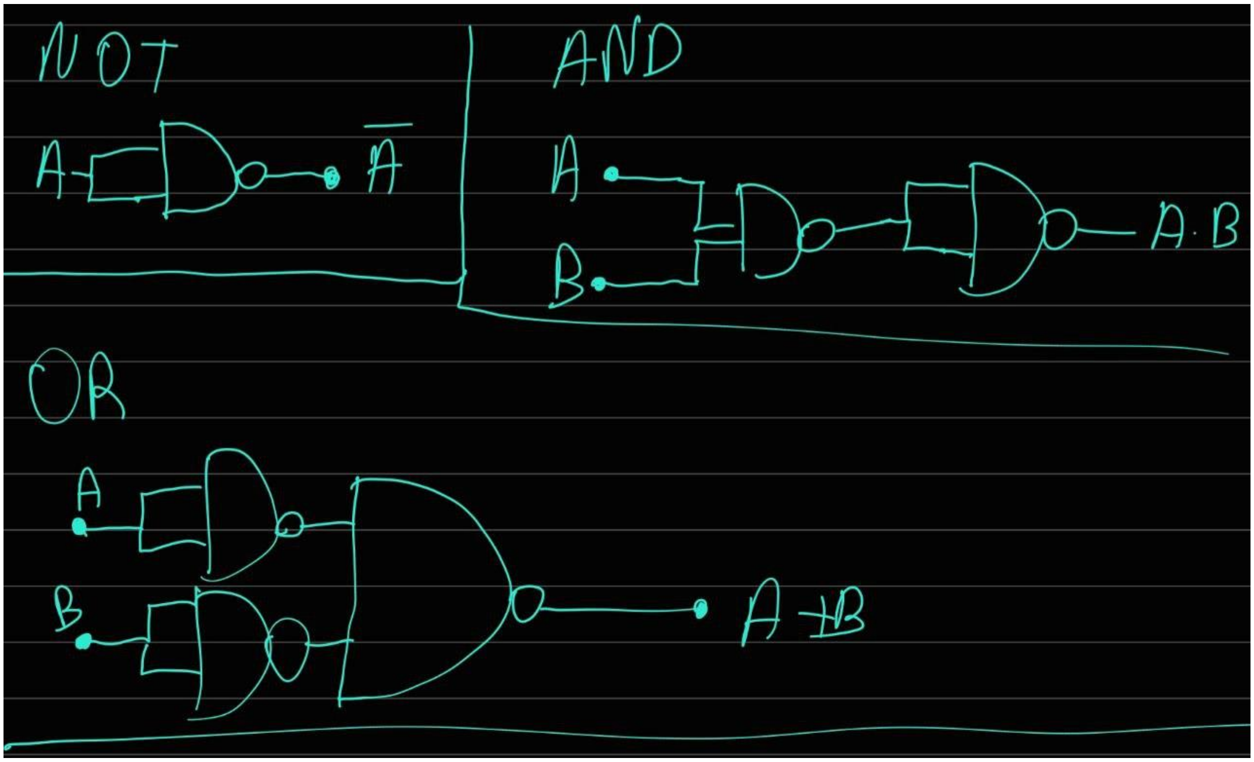
$= (0 B + 0 . C + (\overline{A} \cdot B \cdot C)) +$ Inverse Laws
$(A \cdot 0 + (A \cdot C \cdot \overline{B}) + 0 . C) +$
$((A \cdot B \cdot \overline{C}) + A \cdot 0 + B \cdot 0)$

$= (0 + 0 + (\overline{A} \cdot B \cdot C)) +$ Zero Laws
$(0 + (A \cdot C \cdot \overline{B}) + 0) +$
$((AB\overline{C}) + 0 + 0)$

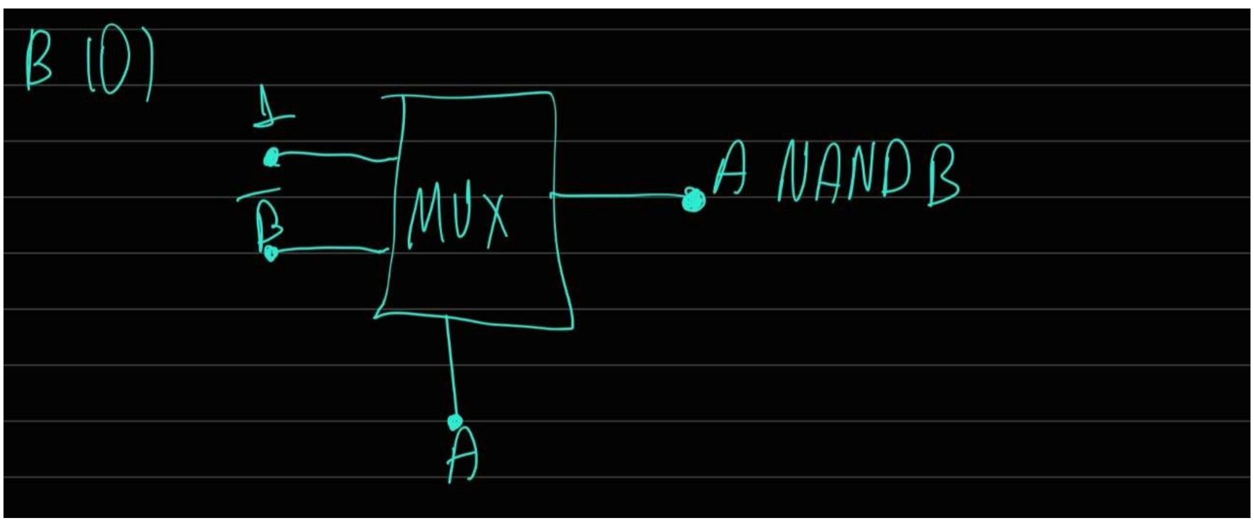
$= (\overline{A} \cdot B \cdot C) + (A \cdot C \cdot \overline{B}) + (A \cdot B \cdot \overline{C})$ Zero Laws

= (A.B.C') + (A.C.B') + (B.C.A') Commutative Laws

4)



5)



6)

$X < Y$ for unsigned

$$(Y_2 \cdot \overline{X_2}) + (Y_2 \cdot X_2 + \overline{Y_2}\,\overline{X_2}) \cdot$$

$$(Y_1 \cdot \overline{X_1} + (Y_1 \cdot X_1 + \overline{Y_1}\,\overline{X_1}) \cdot (Y_0 \cdot \overline{X_0}))$$

$X < Y$ for signed

$$(\overline{y_2} \cdot x_2) + (y_2 \cdot x_2 + \overline{y_2} \cdot \overline{x_2}) \cdot$$

$$((y_1 \cdot x_1 + \overline{y_1} \cdot \overline{x_1}) \cdot y_0 \cdot \overline{x_0} + y_1 \cdot \overline{x_1})$$

$X = \underline{y}$

$$(y_2 \cdot x_2 + \overline{y_2} \cdot \overline{x_2}) \cdot (y_1 \cdot x_1 + \overline{y_1} \cdot \overline{x_1}) \cdot$$

$$(y_0 \cdot x_0 + \overline{y_0} \cdot \overline{x_0})$$

For less than signs in the above answer, we only need to go from left to right and check the case where the y bit is greater than the x bit. And then, if the bits are equal, we keep comparing until the last bit is found. The difference between signed and unsigned is, for signed, we take the lesser digit for the most significant digit of the numbers (i.e. 2's component).
For equal sign, we check if all of the digits are equal to each other and connect them conjunctively as the numbers are equal if and only if all of their bits are equal correspondingly.
The hierarchical approach to the 6 bit case would be by grouping the first and second bit trios in each number, and then comparing the leftmost 3-bit group. If the leftmost group of y is greater, then y is greater. If they are equal, we compare the second 3-bit group. If the second group is greater, y is greater.

7)

f_a = A'.(A + B) + (A.A + B).(A + B')	
= A'.(A + B) + (A + B).(A + B')	Idempotent Law
= A'.A + A'.B + (A + B).(A + B')	Distributive Law
= 0 + A'.B + (A + B).(A + B')	Inverse Law
= A'.B + (A + B).(A + B')	Identity Law
= A'.B + A.A + A.B' + B.A + B.B'	Distributive Law
= A'.B + A + A.B' + B.A + B.B'	Idempotent Law
= A'.B + A + A.B' + BA + 0	Inverse Law
= A'.B + A + A.B' + BA	Identity Law
= A'.B + A.B' + BA + A	Commutative Law
= A'.B + A.B' + A.(B + 1)	Distributive Law
= A'.B + A.B' + A.1	Zeros and Ones Law
= A'.B + A.B' + A	Identity Law
= A'.B + A.(B' + 1)	Distributive Law
= A'.B + A.(1)	Zeros and Ones Law
= A'.B + A	Identity Law
= ((A'.B + A)')'	Double Negation Law
= ((A'.B)' . A')'	DeMorgan's Law
= (((A')' + B') . A')'	DeMorgan's Law
= ((A + B) . A')'	DeMorgan's Law
= (A.A' + A'B)'	Distributive Law
= (0 + A'B)'	Inverse Law
= (A'B)'	IdentityLaw
= A + B	DeMorgan's Law

f_b = A' + (A.C) + (A + B' + C)'

= A' + (A.C) + (A + B')' . C' DeMorgan's Law

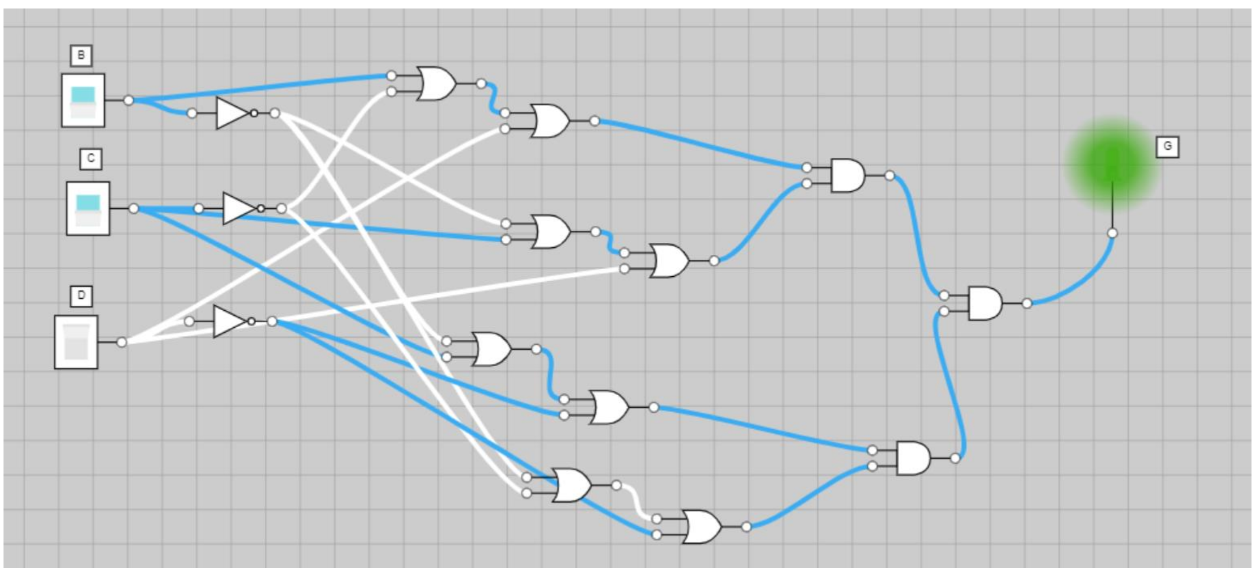
$= A' + (A.C) + A' . (B')' . C'$	DeMorgan’s Law
$= A' + (A.C) + A' . B . C'$	Double Negation Law
$= (A.C) + A' + A' . B . C'$	Commutative Law
$= (A.C) + A' . (1 + B.C')$	Distributive Law
$= (A.C) + A' . 1$	Zero and Ones Law
$= A.C + A'$	Identity Law
$= ((A.C + A')')'$	Double Negation Law
$= ((A.C)' . (A')')'$	DeMorgan’s Law
$= ((A.C)' . A)'$	Double Negation Law
$= ((A' + C') . A)'$	DeMorgan’s Law
$= (A'.A + C'.A)$	Distributive Law
$= (0 + C'.A)'$	Inverse Law
$= (C'.A)'$	Identity Law
$= (C')' + A'$	DeMorgan’s Law
$= C + A'$	Double Negation Law

$f_c = (B + C) . (A + B') . (A + C)$	
$= (B.A + B.B' + C.A + C.B').(A + C)$	Distributive Law
$= (B.A + 0 + C.A + C.B').(A + C)$	Inverse Law
$= (B.A + C.A + C.B').(A + C)$	Identity Law
$= (B.A.A + C.A.A + C.B'.A + B.A.C + C.A.C + C.B'.C)$	Distributive Law
$= (B.A.A + C.A.A + C.B'.A + B.A.C + C.C.A + C.C.B')$	Commutative Law
$= (B.A + C.A + C.B'.A + B.A.C + C.A + C.B')$	Idempotent Law
$= (B.A + B.A.C + C.A + C.A + C.A.B' + C.B')$	Commutative Law
$= (B.A.(1 + C) + C.A.(1 + 1 + B') + C.B')$	Distributive Law
$= (B.A.(1) + C.A.(1) + C.B')$	Zero and Ones Law
$= B.A + C.A + C.B'$	Identity Law
$= B.A + B'.C + C.A$	Commutative Law
$= B.A + B'.C + (1).C.A$	Identity Law
$= B.A + B'.C + (B + B').C.A$	Inverse Law

$= B.A + B'.C + B.C.A + B'.C.A$	Distributive Law
$= B.A + B'.C + B.C.A + B'.C.A$	Distributive Law
$= B.A + B.C.A + B'.C + B'C.A$	Commutative Law
$= B.A + B.A.C + B'.C + B'C.A$	Commutative Law
$= B.A\,(1 + C) + B'.C.\,(1 + A)$	Distributive Law
$= B.A\,(1) + B'.C.\,(1)$	Zero and One Law
$= B.A + B'.C$	Identity Law

8) The POS expression is $(B + C' + D) . (B' + C + D) . (B' + C + D') . (B' + C' + D')$

Following picture is the circuit version of the above expression:



9)

Truth Table

A	B	C	D	g
0	0	0	0	
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	1
1	1	1	1	0

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	1	0	1
$A\bar{B}$	0	1	0	1
AB	0	0	0	1
$\bar{A}B$	0	0	0	1

Expression: $\bar{B}\cdot\bar{C}D + C\bar{D}$

10)

$g = A'.B'.C'.D + A'.B'.C.D' + A.B'.C.D' + A.B'.C'.D + A'.B.C.D' + A.B.C.D'$	
$= A'.B'.C'.D + A.B'.C'.D + A'.B'.C.D' + A.B'.C.D' + A'.B.C.D' + A.B.C.D'$	Commutative Law
$= B'.C'.D.(A' + A) + B'.C.D'.(A' + A) + B.C.D'.(A' + A)$	Distributive Law
$= B'.C'.D.(1) + B'.C.D'.(1) + B.C.D'.(1)$	Inverse Law
$= B'.C'.D + B'.C.D' + B.C.D'$	Identity Law
$= B'.C'.D + C.D'.(B' + B)$	Distributive Law
$= B'.C'.D + C.D'.(1)$	Inverse Law
$= B'.C'.D + C.D'$	Identity Law
$= C.D' + B'.C'.D$	Commutative Law