

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

SEMESTER FINAL EXAMINATION

WINTER SEMESTER, 2017-2018

DURATION: 3 Hours

FULL MARKS: 200

Math 4141: Geometry and Differential Calculus

Programmable calculators are not allowed. Do not write anything on the question paper.

There are 8(eight) questions. Answer any 6(six) of them.

Figures in the right margin indicate marks.

1. a) Define even and odd functions. Test whether the followings are even, odd or neither: 15
 i. $f(x) = \frac{x}{x^2-1}$ ii. $f(x) = \sqrt[3]{2x^2+1}$, iii. $f(x) = k$, iv. $f(x) = \cot x$.
- b) Define exponential and logarithmic functions? Suppose you are interested to invest 5000 dollars in a bank in Bangladesh with 6.5 % compound interest annually. How long will it take the amount to reach 9500 dollars? Also calculate the rate of compound interest, if your amount may double itself in 6 years to invest it in another bank. 18.33
2. a) Prove that every differentiable function is continuous but the converse of the statement is not always true. Explain with example. 20
 b) Discuss the differentiability for the function $f(x)$ at the origin, where 13.33

$$f(x) = \begin{cases} 2x-1, & x \geq 0 \\ x^2+2x+7, & x < 0 \end{cases}$$
3. a) Find the values of h and k that makes the following function differentiable for all x: 13.33

$$f(x) = \begin{cases} hx+k, & x > -1 \\ kx^2-3, & x \leq -1 \end{cases}$$

 b) Suppose that a particle's position is given by $s = 3\cos t + 4\sin t$, where s is measured in meters and the time t is in seconds. Find the particle's velocity, speed, acceleration and jerk at time $t = 0$ and $t = \pi$. 10
 c) Find dy/dx for the following: 10
 i. $y = 4\sin\sqrt{1+\sqrt{x}}$, ii. $y = \sqrt{3x + \sqrt{2 + \sqrt{1-x}}}$
4. a) Find $\frac{d^2y}{dx^2}$ if 10
 i. $xy + y^2 = 1$, ii. $2\sqrt{y} = x - y$
 b) Show that the point $(1, \pi/2)$ lies on the curve $2xy + \pi \sin y = 2\pi$. Find the tangent and normal to the curve at that point. 10
 c) Find the local extreme values of $y = x^3(x-5)^2$, if any, and where they occur. 13.33
5. a) Write the statement of the Mean value theorem and verify it for the function $f(x) = x + \frac{1}{x}$ on the interval $[1/2, 2]$. 13.33
 b) Define increasing and decreasing functions. If $y = x - 4/x$ then answer the following: 20
 i. Find the critical points.
 ii. Find the intervals on which the function is increasing or decreasing.
 iii. Find the local maximum or minimum values, if any, using first derivative test.

6. a) Write the statement of second derivative test for local extrema of a function. Discuss the concavity of the function $f(x) = \frac{9}{14}x^{\frac{1}{3}}(x^2 - 7)$. Also find the local maximum or minimum values. 20
- b) A rectangle is to be inscribed in a semicircle of radius 2. What is the largest area the rectangle can have, and what are its dimensions? Also find the perimeter of that largest rectangle. 13.33
- ✓ 7. a) What is homogeneous equation? Find the angle between the lines represented by the equation $ax^2 + 2hxy + by^2 = 0$ and discuss the different cases. If $2x^2 - 4xy - y^2 = 0$ represents a pair of straight lines, then find the angle between them. 18
- b) Write the different conditions when the general equation of second degree represents a conic. Discuss the nature of the conic $2x^2 - 3y^2 + 8x + 30y - 27 = 0$. 15.33
- ✗ 8. a) Define the direction ratios and cosines of a line. What are the direction cosines of x , y and z axes? Test whether 0, -1, 1 are directions ratios or cosines of any line and then find the angles that makes with coordinate axes. 10
- b) Find the angle between the lines AB and CD, where the coordinates of the points A, B, C and D are (0, 1, -1), (1, 2, 3), (-1, 0, 1) and (2, -2, 3) respectively. Also find the projection of AB on CD and then verify it using any other method. 13.33
- c) If the edges of a rectangular parallelepiped are a , b , and c then show that the angles between the four diagonals are given by $\cos^{-1} \left(\frac{\pm a^2 \pm b^2 \pm c^2}{a^2 + b^2 + c^2} \right)$. 10