B.Sc. Engg. CSE 6th Semester

06 November 2017

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

Department of Computer Science and Engineering (CSE)

SEMESTER FINAL EXAMINATION **SUMMER SEMESTER, 2016-2017**

DURATION: 3 Hours **FULL MARKS: 150**

CSE 4645: Numerical Analysis

Programmable calculators are not allowed. Do not write anything on the question paper.

There are **8 (eight)** questions. Answer any **6 (six)** of them.

Figures in the right margin indicate marks.

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1. a) The following table gives the viscosity of sulfuric acid, in centipoises (unit of viscosity), as a function of concentration, in percentage (%): 15

Table 1: A table that shows the viscosity of sulfuric acid for increasing percentage of concentration

Concentration(C) [in percentage]	0	20	40	60	80	100
Viscosity (V) [in centipoises]	0.8	1.4	2.5	5.37	17.4	24.2

Using given data, construct a linear spline to approximate the viscosity V(C). Estimate the viscosity by when the concentration is 5% and 63% and 92%.

b) What is the motivation behind deriving Runge-Kutta methods despite Taylor series methods being sufficient to get better approximations? 4

c) Why does one point Gauss-Quadrature Rule and single segment Trapezoidal rule give same results? Which one is better computationally and why? 6

2. a) With proper explanation, show how Euler's method of solving ordinary differential equations can be used to solve a definite integral. 7

b) Derive the formula for Newton-Raphson method. 6

c) The following data of the velocity of a body is given as a function of time. 12

Table 2: Change of velocity with time

Time (in seconds)	12	15	18	21	24
Velocity(in meter/second)	22	24	37	25	123

Find the distance in meters, covered by the body from t = 12s to t =18s calculated using the 3 segment-trapezoidal rule.

3. a) Derive Richardson's Extrapolation formula from the approximation obtained from n-segment trapezoidal rule. Show the error of the estimate for Richardson's extrapolation formula is $O(h^4)$ 12

b) Perform polynomial regression to fit a second-order polynomial to the data in the first two columns of the following table: 13

Table 3: Data for polynomial regression

x	0	1	2	3	4	5
y	2.1	7.7	13.6	27.2	40.9	61.1

4. a) An 11-m beam is subjected to a load, and the shear force follows the equation: 15

$$V(x) = 5 + 0.25x^2$$

Here V is the shear force and x is length in distance along the beam. It is known that $V = \frac{dM}{dx}$, and M is the bending moment. Integration yields the relationship

$$M = M_0 + \int_0^x V dx$$

If M_0 is zero and x = 11, calculate M using two-point Gauss quadrature rule to approximate.

b) Write down the pseudo-code for Mixed Simpson 1/3 and 3/8 Rule for Integration. 6

c) Give your insights on how we can interpret the different assignments made to the constants in Heun's method, Midpoint method and Ralston's method. 4

5. a) Derive the Maclaurin series of $f(x) = e^x$. 7
Hint: Remember, that Maclaurin series is Taylor series with x= 0.
b) Find y(1.0) using RK method of order four by solving $y' = -2xy^2$, $y(0) = 1$ with step length of 0.25. Use the formulas derived by Runge and Kutta both to find two separate approximations. 18

6. a) A ball at 1200 K is allowed to cool down in air at an ambient temperature of 300K. Assuming heat is lost only due to radiation, the differential equation for the temperature of the ball is given by 17

$$\frac{d\theta}{dt} = -2.2067X10^{-12}(\theta^4 - 81X10^8)$$

where θ is in Kelvin and t is in seconds. Find the temperature at t= 480 seconds using Runge-Kutta 2nd order method. Assume a step size of h=240 seconds. Use Heun's method for the assignment of constants.
b) Compare the convergence of secant method with other methods that are used for finding roots of non-linear equations. 4
c) What should be the absolute relative approximate error of any estimate if you want 4 significant digits of your estimate to be true? 4

7. a) With the help of necessary figures, derive the formula for Euler's method of solving ordinary differential equations. 12
b) The population of Mississippi during three census periods was as follows: 8

Table 4: Data for polynomial regression

Year	1951	1961	1971
Population(in millions)	2.8	3.2	4.5

Interpolate the population at 1966 using quadratic Lagrange Interpolation.
c) What are the dangers of extrapolation? 5

8. a) The vertical distance in meters covered by a rocket from $t=8$ to $t=30$ seconds is given by 15

$$s = \int_8^{30} \left(2000 \ln\left[\frac{140000}{140000-2100t}\right] - 9.8t\right)dt$$

Use Simpson 3/8 rule to find the approximate value of the integral.
b) What are the ways of determining the adequacy of regression model? Provide details of those methods. 10

Taylor Series:

1. $\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$
2. $\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{7}{7!} + \cdots$
3. $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$
4. $f(x+h) = f(x) + f'(x)h + \frac{f''(x)^2}{2!}h^2 + \cdots$

Solving Non-Linear Equations

False Position Method:

1. $x_r = x_u - \frac{f(x_u)}{\left\{\frac{f(x_l)-f(x_u)}{x_l - x_u}\right\}}$
2. $x_r = x_l - \frac{f(x_l)}{\left\{\frac{f(x_u)-f(x_l)}{x_u - x_l}\right\}}$

Secant Method:

1. $x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$

Interpolation

Lagrange:

1. $f_n(x) = \sum_0^n L_i(x) f(x_i)$
   Where,
   i. $L_i(x) = \prod_{\substack{j=0 \\ j\neq i}}^{n} \frac{x - x_j}{x_i - x_j}$

Newton-Divided

1. $f_1(x) = f(x_0) + \frac{f(x_1)-f(x_0)}{x_1 - x_0}(x - x_0)$
2. Standard Error of Estimate $s_{\alpha/T} = \sqrt{\frac{S_r}{n-2}}$
   a. $S_r = \sum_{i=1}^{n}(a_i - a_0 - a_1 T_i)^2$

Numerical Differentiation

Divided Difference

1. Forward : $f'(x) = \frac{f(x+\Delta x) - f(x)}{\Delta x}$
2. Backward: $f'(x) = \frac{f(x) - f(x+\Delta x)}{\Delta x}$
3. Central: $f'(x) = \frac{f(x+\Delta x) - f(x+\Delta x)}{2\Delta x}$

4. 2^{nd} order : $\frac{f(x_{i+2})-2f(x_{i+1})+f(x_i)}{(\Delta x)^2}$

3 point formula:

1. $f'(x_0) = \frac{-3f(x_0)+4f(x_0+h)-f(x_0+2h)}{2h}$

5 point formula:

1. $f'(x_0) = \frac{f(x_0-2h)-8f(x_0-h)+8f(x_0+h)-f(x_0+2h)}{12h}$

Numerical Integration

Single- segment Trapezoidal:

1. $\int_a^b f(x)dx = (b-a)\left[\frac{f(a)+f(b)}{2}\right]$

Multiple- segment Trapezoidal:

1. $\int_a^b f(x)dx = \frac{(b-a)}{2n}[f(a) + 2\{\sum_{i=1}^{n-1} f(a+ih)\} + f(b)]$

Simpson's 1/3:

Single Segment

1. $\int_a^b f(x)dx = \frac{h}{3}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$

Multiple Segment

1. $\int_a^b f(x)dx = \frac{(b-a)}{3n}\left[f(x_0) + 4\left\{\sum_{\substack{i=1 \\ i=odd}}^{n-1} f(x_i)\right\} + 2\left\{\sum_{\substack{i=2 \\ i=even}}^{n-2} f(x_i)\right\} + f(x_n)\right]$

Romberg :

$$I_{k,j} = I_{k-1,j+1} + \frac{I_{k-1,j+1} - I_{k-1,j}}{4^{k-1}-1}\ , k \geq 2$$

Guass-Quadrature

2point:

1. $\int_a^b f(x)dx = \frac{b-a}{2} f\left(\frac{b-a}{2}\left(-\frac{1}{\sqrt{3}}\right) + \frac{b+a}{2}\right) + \frac{b-a}{2} f\left(\frac{b-a}{2}\left(\frac{1}{\sqrt{3}}\right) + \frac{b+a}{2}\right)$
2. $\int_a^b f(x)dx = \int_{-1}^{1} f\left(\frac{b-a}{2}x + \frac{b+a}{2}\right)\frac{b-a}{2}dx$

Simpson's 3/8:

$$\int_a^b f(x)dx = \frac{3h}{8}[f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3)]$$

Differential Equations

Euler's Method:

1. $y_{i+1} = y_i + f(x_i, y_i)h$

Runge-Kutta 2nd order

1. $y_{i+1} = y_i + (a_1 k_1 + a_2 k_2)h$
   a. $k_1 = f(x_i, y_i)$
   b. $k_2 = f(x_i + p_1 h, y_i + q_{11} k_1 h)$
      i. Heun's method:
         1. $a_2=1/2$
      ii. Midpoint method
         1. $a_2=1$
      iii. Ralston's method:
         1. $a_2=2/3$

Runge-Kutta 4th order

Runge's formulas

1. $y_{i+1} = y_i + (a_1 k_1 + a_2 k_2 + a_3 k_3 + a_4 k_4)h$
   i. $k_1 = f(x_i, y_i)$
   ii. $k_2 = f\left(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_1 h\right)$
   iii. $k_3 = f\left(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_2 h\right)$
   iv. $k_4 = f(x_i + h, y_i + k_3 h)$

Kutta's formulas

1. $y_{i+1} = y_i + \frac{1}{8}(k_1 + 3k_2 + 3k_3 + k_4)h$

   i. $k_1 = f(x_i, y_i)$

   ii. $k_2 = f\left(x_i + \frac{1}{3}h, y_i + \frac{1}{3}k_1 h\right)$

   iii. $k_3 = f\left(x_i + \frac{2}{3}h, y_i - \frac{1}{3}k_1 h + k_2 h\right)$

   iv. $k_4 = f(x_i + h, y_i + k_1 h - k_2 h + k_3 h)$