B.Sc. Engg. / HD CSE 1st Semester (108) 10 May 2017

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

SEMESTER FINAL EXAMINATION **WINTER SEMESTER, 2016-2017**

DURATION: 3 Hours **FULL MARKS: 200**

Math 4141: Geometry and Differential Calculus

Programmable calculators are not allowed. Do not write anything on the question paper.
There are 8 **(four)** questions. Answer any **6(three)** of them.
Figures in the right margin indicate marks.

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1. a) Define even and odd functions. Test whether the following functions are even, odd or neither. Give reason for your answer: i. f(x)=tanx, ii. $f(x)=x^4-10$, iii. $f(x)=x^{-5}$, iv. f(x) =k, v. $f(x)=x^2-x+5$ — 10

b) If $f(x)=\sqrt{x+1}$, $g(x)=\frac{1}{x+4}$ and, $h(x)=\frac{1}{x}$, find the following: — 10

i. $(f\circ g\circ h)(x)$, ii. $(f\circ g\circ h)\left(\frac{1}{2}\right)$

c) Define exponential and logarithm functions. It is estimated that the population of Gazipur city grows exponentially, if the population was 20 million in the year 1995 and 35 million in 2005. What will be the population in the year 2021? — 13.33

2. a) Give the basic (δ-ε) definition of limit of a function. Prove the limit statements for — 12

i. $\lim_{x\to 3} 3x-7=2$ ii. $\lim_{x\to 0}\sqrt{4-x}=2$

b) Let $f(x)=\begin{cases}\sqrt{1-x^2}, & 0\le x<1\\ 1, & 1\le x<2\\ x-1, & x\ge 2\end{cases}$ — 10

Show whether $\lim_{x\to 1} f(x)$ and $\lim_{x\to 2} f(x)$ exist or not? If not, why?

c) Define continuity of a function. A function is defined as follows: — 11.33

$$f(x)=\begin{cases}1+x, & -4\le x<-1\\ 4, & -1\le x\le 0\\ 1+x^2, & 0<x\le 4\end{cases}$$

Examine the continuity and discontinuity of the function at $x=0$ and -1.

3. a) Define derivative of a function at a point and write its various physical meanings. Find the equation for tangent to the curve $y=(x-1)^2+1$ at the point (1, 1). Sketch the curve and tangent together. — 17.33

b) Using the definition of derivative, differentiate the following functions: — 16

i. $y=\sqrt{x}$ ii. $y=cosx$

4. a) Prove that every differentiable function is continuous but the converse of the statement is not always true. Explain with example. — 20

b) Discuss the differentiability for the function f(x) at the origin, where — 13.33

$$f(x)=\begin{cases}2x-1, x\ge 0\\ x^2+2x+7, x<0\end{cases}$$

18.33

6. a) Find dy/dx for the following functions:

i. $y = 5\,sinx - 10e^x + 35x + 10$

ii. $y = x^2 cosx - \frac{3}{\sqrt{x}} + 5x^{-3} - 35x^{3/5}$

iii. $y = \frac{x^3+7}{x}$

iv. $y = x(2x+1)^4$ 15

b) Using chain rule, find $\frac{dy}{dx}$ if

i. $y = \sqrt{3x^2 - 4x + 6}$

ii. $y = cosu,\ u = sinx$

iii. $y = 6u - 9,\ u = \frac{x^4}{2}$

$\cos(\sin x)$

$-\sin x \cos \ldots$

$-\sin(\sin x)$

$f'(c) = \frac{f(a)-f(b)}{a-b}$

10

6. a) Find $\frac{d^2y}{dx^2}$ if i. $xy + y^2 = 1$, ii. $y^2 - 2x = 1 - 2y$ 10

b) Show that the point $(1, \pi/2)$ lies on the curve $2xy + \pi\ siny = 2\pi$. Also find the tangent and normal to the curve at that point. 13.33

c) State the Mean value theorem and verify it for the function $f(x) = x + \frac{1}{x}$ on the interval [1/2, 2]

7. a) What is monotonic function? Suppose the rate of change of any function is given by $(x-1)^2(x+2)$ Answer the following: i. Find the critical points. ii. Find the intervals, on which the function is increasing or decreasing and then draw the graph using the behavior of the function. 17

b) Write the necessary and sufficient conditions for local extrema of a function. Suppose $f(x) = \frac{1}{4}x^4 - 2x^2 + 4$, find the local maximum or minimum values using first derivative test and then verify it using second derivative test. 16.33

8. a) Determine the dimensions of the rectangle of largest area that can be inscribed in a semi-circle of radius 3. Also find the area and perimeter of that largest rectangle. 13.33

b) What do you mean by indeterminate forms? Write the statement of L' Hopitals's rule. Evaluate the following: i. $\lim_{x\to 1} x^{\frac{1}{1-x}}$ ii. $\lim_{x\to 1}\left(\frac{x}{x-1} - \frac{1}{lnx}\right)$, iii. $\lim_{x\to 0} \frac{(e^x-1)tan^2x}{x^3}$ 20