1854

M.Sc. Engg./PhD CSE 14 August 2018 (Afternoon)

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

MID SEMESTER EXAMINATION **SUMMER SEMESTER, 2017-2018**

DURATION: 1 Hour 30 Minutes **FULL MARKS: 75**

CSE 6261: Advanced Probability and Stochastic Process

Programmable calculators are not allowed. Do not write anything on the question paper.

There are **4 (four)** questions. Answer any **3 (three)** of them.

Figures in the right margin indicate marks.

---

1. a) A biased coin with probability of obtaining a head equal to p (where $0 < p < 1$) is tossed repeatedly and independently until the first head is observed. Compute the probability that the first head appears at an even numbered toss. 10

   b) Information is transmitted digitally as a binary sequence known as "bits". However, noise on the channel corrupts the signal, in that a digit transmitted as 0 is received as 1 with probability 0.1, with a similar random corruption when the digit 1 is transmitted. It has been observed that, across a large number of transmitted signals, the 0s and 1s are transmitted in the ratio 3:4. 15

   Given that the sequence 101 is received, what is the probability distribution over transmitted signals? Assume that the transmission and reception processes are independent.

2. a) A particular circuit works if all 10 of its component devices work. Each circuit is tested before leaving the factory. Each working circuit can be sold for k dollars, but each nonworking circuit is worthless and must be thrown away. Each circuit can be built with either ordinary devices or ultrareliable devices. An ordinary device has a failure probability of $q = 0.1$ while an ultrareliable device has a failure probability of $\frac{q}{2}$, independent of any other device. However, each ordinary device costs \$1 while an ultrareliable device costs \$3. Should you build your circuit with ordinary devices or ultrareliable devices in order to maximize your expected profit $E[R]$? Keep in mind that your answer will depend on k. 13

   b) An urn contains r red balls and b blue balls. Suppose n (where n less than r and b0 balls are randomly picked from the urn without replacement. Find the probability that more red balls are picked than blue balls. Note that you need to consider both the even and odd values of n. 12

3. a) Suppose that the IQ of a randomly selected student from a university is normal with mean 110 and standard deviation 20. Determine the interval of values that is centered at the mean and includes 50% of the IQ's of the students at that university. 10

   b) In data communication, messages are usually combinations of characters, and each character consists of a number of bits. A bit is the smallest unit of information and is either 1 or 0. Suppose that L, the length of a character (in bits) is a geometric random variable with parameter p. If a sender emits messages at the rate of 1000 bits per second, what is the distribution of T, the time it takes the sender to emit a character? 8

   c) A point is chosen at random on a line segment of length L. Find the probability that the ratio of the shorter to the longer segment is less than $\frac{1}{4}$. 7

185

4. a) Thieves stole four animals at random from a farm that had seven sheep, eight goats, and five cows. Calculate the joint probability mass function of the number of sheep and goats stolen. 10

b) Suppose random variable X and Y are jointly distributed with the following joint probability distribution function 15

$$f_{XY}(x, y) = \begin{cases} 2, & 0 \le x \le y \le 1 \\ 0, & \text{otherwise.} \end{cases}$$

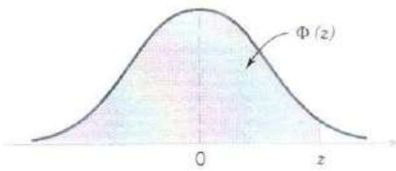
Find the joint cumulative distribution function of X and Y.

PMF/PDF and the expected values of some Random Variables

Distribution	PMF/PDF	Expected value	Variance
Bernoulli	$P_X(x) = \begin{cases} 1-p & x = 0 \\ p & x = 1 \\ 0 & otherwise \end{cases}$	$E[X] = p$	$Var[X] = p(1-p)$
Geometric	$P_X(x) = \begin{cases} p(1-p)^{x-1} & x \ge 1 \\ 0 & otherwise \end{cases}$	$E[X] = 1/p$	$Var[X] = (1-p)/p^2$
Binomial	$P_X(x) = \begin{cases} \binom{n}{x} p^x (1-p)^{n-x} & x = 1, \dots, n \\ 0 & otherwise \end{cases}$	$E[X] = np$	$Var[X] = np(1-p)$
Pascal	$P_X(x) = \begin{cases} \binom{x-1}{k-1} p^k (1-p)^{x-k} & x = k, k+1, \dots \\ 0 & otherwise \end{cases}$	$E[X] = k/p$	$Var[X] = k(1-p)/p^2$
Poisson	$P_X(x) = \begin{cases} \dfrac{(\lambda T)^x e^{-(\lambda T)}}{x!} & x \ge 0 \\ 0 & otherwise \end{cases}$	$E[X] = \alpha$ $\alpha = \lambda T$	$Var[X] = \alpha$
Uniform (discrete)	$P_X(x) = \begin{cases} \dfrac{1}{b-a+1}, & x = a, a+1, a+2, \dots, b \\ 0, & \text{otherwise} \end{cases}$	$E[X] = \dfrac{a+b}{2}$	$Var[X] = \dfrac{(b-a)(b-a+2)}{12}$
Exponential	$f_X(x) = \begin{cases} ae^{-ax} & x \ge 0 \\ 0 & otherwise \end{cases}$	$E[X] = 1/a$	$Var[X] = 1/a^2$
Gaussian	$f_X(x) = \begin{cases} \dfrac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(x-\mu)^2}{2\sigma^2}} & \sigma > 0 \\ 0 & otherwise \end{cases}$	$E[X] = \mu$	$Var[X] = \sigma^2$
Uniform (Continuous)	$f_X(x) = \begin{cases} \dfrac{1}{b-a}, & a \le x < b \\ 0, & otherwise \end{cases}$	$E[X] = \dfrac{a+b}{2}$	$Var[X] = \dfrac{(b-a)^2}{12}$

Appendix A: CDF of Standard Normal Distribution

$$\Phi(z) = P(Z \leq z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2}\, du$$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.500000	0.503989	0.507978	0.511967	0.515953	0.519939	0.532922	0.527903	0.531881	0.535856
0.1	0.539828	0.543795	0.547758	0.551717	0.555760	0.559618	0.563559	0.567495	0.571424	0.575345
0.2	0.579260	0.583166	0.587064	0.590954	0.594835	0.598706	0.602568	0.606420	0.610261	0.614092
0.3	0.617911	0.621719	0.625516	0.629300	0.633072	0.636831	0.640576	0.644309	0.648027	0.651732
0.4	0.655422	0.659097	0.662757	0.666402	0.670031	0.673645	0.677242	0.680822	0.684386	0.687933
0.5	0.691462	0.694974	0.698468	0.701944	0.705401	0.708840	0.712260	0.715661	0.719043	0.722405
0.6	0.725747	0.729069	0.732371	0.735653	0.738914	0.742154	0.745373	0.748571	0.751748	0.754903
0.7	0.758036	0.761148	0.764238	0.767305	0.770350	0.773373	0.776373	0.779350	0.782305	0.785236
0.8	0.788145	0.791030	0.793892	0.796731	0.799546	0.802338	0.805106	0.807850	0.810570	0.813267
0.9	0.815940	0.818589	0.821214	0.823815	0.826391	0.828944	0.831472	0.833977	0.836457	0.838913
1.0	0.841345	0.843752	0.846136	0.848495	0.850830	0.853141	0.855428	0.857690	0.859929	0.862143
1.1	0.864334	0.866500	0.868643	0.870762	0.872857	0.874928	0.876976	0.878999	0.881000	0.882977
1.2	0.884930	0.886860	0.888767	0.890651	0.892512	0.894350	0.896165	0.897958	0.899727	0.901475
1.3	0.903199	0.904902	0.906582	0.908241	0.909877	0.911492	0.913085	0.914657	0.916207	0.917736
1.4	0.919243	0.920730	0.922196	0.923641	0.925066	0.926471	0.927855	0.929219	0.930563	0.931888
1.5	0.933193	0.934478	0.935744	0.936992	0.938220	0.939429	0.940620	0.941792	0.942947	0.944083
1.6	0.945201	0.946301	0.947384	0.948449	0.949497	0.950529	0.951543	0.952540	0.953521	0.954486
1.7	0.955435	0.956367	0.957284	0.958185	0.959071	0.959941	0.960796	0.961636	0.962462	0.963273
1.8	0.964070	0.964852	0.965621	0.966375	0.967116	0.967843	0.968557	0.969258	0.969946	0.970621
1.9	0.971283	0.971933	0.972571	0.973197	0.973810	0.974412	0.975002	0.975581	0.976148	0.976705
2.0	0.977250	0.977784	0.978308	0.978822	0.979325	0.979818	0.980301	0.980774	0.981237	0.981691
2.1	0.982136	0.982571	0.982997	0.983414	0.983823	0.984222	0.984614	0.984997	0.985371	0.985738
2.2	0.986097	0.986447	0.986791	0.987126	0.987455	0.987776	0.988089	0.988396	0.988696	0.988989
2.3	0.989276	0.989556	0.989830	0.990097	0.990358	0.990613	0.990863	0.991106	0.991344	0.991576
2.4	0.991802	0.992024	0.992240	0.992451	0.992656	0.992857	0.993053	0.993244	0.993431	0.993613
2.5	0.993790	0.993963	0.994132	0.994297	0.994457	0.994614	0.994766	0.994915	0.995060	0.995201
2.6	0.995339	0.995473	0.995604	0.995731	0.995855	0.995975	0.996093	0.996207	0.996319	0.996427
2.7	0.996533	0.996636	0.996736	0.996833	0.996928	0.997020	0.997110	0.997197	0.997282	0.997365
2.8	0.997445	0.997523	0.997599	0.997673	0.997744	0.997814	0.997882	0.997948	0.998012	0.998074
2.9	0.998134	0.998193	0.998250	0.998305	0.998359	0.998411	0.998462	0.998511	0.998559	0.998605
3.0	0.998650	0.998694	0.998736	0.998777	0.998817	0.998856	0.998893	0.998930	0.998965	0.998999
3.1	0.999032	0.999065	0.999096	0.999126	0.999155	0.999184	0.999211	0.999238	0.999264	0.999289
3.2	0.999313	0.999336	0.999359	0.999381	0.999402	0.999423	0.999443	0.999462	0.999481	0.999499
3.3	0.999517	0.999533	0.999550	0.999566	0.999581	0.999596	0.999610	0.999624	0.999638	0.999650
3.4	0.999663	0.999675	0.999687	0.999698	0.999709	0.999720	0.999730	0.999740	0.999749	0.999758
3.5	0.999767	0.999776	0.999784	0.999792	0.999800	0.999807	0.999815	0.999821	0.999828	0.999835
3.6	0.999841	0.999847	0.999853	0.999858	0.999864	0.999869	0.999874	0.999879	0.999883	0.999888
3.7	0.999892	0.999896	0.999900	0.999904	0.999908	0.999912	0.999915	0.999918	0.999922	0.999925
3.8	0.999928	0.999931	0.999933	0.999936	0.999938	0.999941	0.999943	0.999946	0.999948	0.999950
3.9	0.999952	0.999954	0.999956	0.999958	0.999959	0.999961	0.999963	0.999964	0.999966	0.999967