B.Sc. Egg. (CEE)/2nd Sem. **31 October, 2019(Gr. A)**

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

DEPARTMENT OF CIVIL AND ENVIRONMENTAL ENGINEERING

FINAL SEMESTER EXAMINATION — SUMMER SEMESTER: 2018-2019

COURSE NO: **Math4253** — TIME : 3.0 Hours

COURSE TITLE: Vector Algebra, Vector Calculus and ODE — FULL MARKS:150

There are 4 (Four) questions. Answer any 3 (Three) questions. Programmable calculators are not allowed. Do not write on this question paper. The figures in the right margin indicate full marks. The Symbols have their usual meaning

1.(a) Define reciprocal system of vectors. Find a set of vectors reciprocal to the set of vectors $-i+j+k$, $i-j+k$ and $i+j-k$. (8)

(b) A particle moves along the curve $x=t^3+1,$, $y==t^2$, $z=2t+5$, where t is the time. Find the components of its velocity and acceleration at time $t=1$ in the direction $i+j+3k$. (8)

(c) If $\varphi(x,y,z)=xy^2z$ and $\overline{F}=xzi-xyj+yz^2k$, find $\dfrac{\partial^3(\varphi\overline{F})}{\partial x^2\partial z}$ at the point $(2,-1,1)$ (9)

2. (a) If $r=|\overline{r}|$, where $\overline{r}=xi+yj+zk$, prove that $\nabla r^n=nr^{n-2}\,\overline{r}$ (8)

(b) Taking $\overline{F}=x^2yi+xzj+2yzk$, verify that $div\,curl\ \overline{F}=0$ (9)

(c) Prove that $curl\,curl\ \overline{F}=grad\ div\overline{F}-\nabla^2\overline{F}$ (8)

3. (a) Define directional derivative. Find the directional derivative of the function $xy^2+yz^2+zx^2$ along the tangent to the curve $x=t,$, $y=t^2$, $z=t^3$ at the point $(1,1,1)$. (8)

(b) Find the constants a and bso that the surface $ax^2-byz=(a+z)x$ will be orthogonal to the surface $4x^2y+z^3=4$ at the point $(1,-1,2)$. (9)

(c) Give the physical significance of Curl of a vector point function. (8)

4. (a) The acceleration of a moving particle at any time t is given by $\dfrac{d^2\overline{r}}{dt^2}=2\cos 2t\,i-8\sin 2t\,j+16t\,k$ find the velocity $\overline{v}$ and displacement $\overline{r}$ any time t if at $t=0$,$\overline{v}$=0 and $\overline{r}$=0 (10)

(b) Evaluate $\iint_S \overline{F} \bullet \hat{n}\, dS$, where $\overline{F} = z\,i + x\,j - 3y^2 z\,k$ and S is the surface of the cylinder $x^2 + y^2 = 16$ included in the first octant between $x = 0$ and $z = 5$. (15)

5. State Gauss divergence theorem and verify for the vector function $\overline{F} = 2x^2 yi - y^2 j + 4xz^2 k$ taken over the region in the first octant bounded by $y^2 + z^2 = 9,\ x = 2$. (25)

6.(a) Find the integrating factor of differential equations $\sin x \dfrac{dy}{dx} + 3y = \cos x$ and hence solve it. (8)

(b) Solve: $\cos x \dfrac{dy}{dx} + \sin x \tan y = \cos^2 x \sec y$ (9)

(c) Solve: $x^2 p^3 + yp^2(1 + x^2 y) + y^3 p = 0$ (9)

7. Solve the following differential equations

(a) $x\dfrac{dy}{dx} - 3y = x^2,\quad y(1) = 1$ (5)

(b) $(D^2 - 2D + 1)y = xe^x \sin x$ (10)

(c) $(x^2 D^2 + xD + 1)y = \sin(\ln x^2)$ (10)

8 (a) Solve: $(D^2 + 16)y = \sec 4x$ (12)

(b) A circuit has in series an electromotive force given by $E = 10\sin 60t$, a resistor of 0.2Ω, an inductor of 0.01 H and a capacitor of $\dfrac{1}{26}$ farads. If the initial current and the initial charge on the capacitor are both zero, find the charge on the capacitor at any time $t > 0$. (13)