B.Sc. Engg. CSE 7th Semester 12 December 2022

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

SEMESTER FINAL EXAMINATION — WINTER SEMESTER, 2021-2022

DURATION: 3 HOURS — FULL MARKS: 150

CSE 4703: Theory of Computing

Programmable calculators are not allowed. Do not write anything on the question paper.
Answer **all** 6 (six) questions. Marks of each question and corresponding CO and PO are written in the right margin. Assume if any value is missing. Symbols have their usual meaning.

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1. a) With proper justification, determine if each of the following assertion is correct or incorrect. 2 × 5 (CO1) (PO1)

   i. It is known that the set $S = \{\, 0^n \mid n \text{ is a prime number}\}$ is not regular. But its complement is regular.

   ii. If a language is accepted by an NFA, then it is clearly context-free.

   iii. You cannot build a DFA to recognize $\{0^{500}1^{10000} \cup 1^{10000}0^{100}\}$

   iv. All languages are decidable.

   v. From a computability perspective, every multi-tape Turing machine has an equivalent single-tape TM.

   b) Prove that if L, M and N are any languages, then $L(M + N) = LM + LN$. 5 (CO1) (PO1)

   c) Design a Finite State Machine (FSM) that accepts the language L over the alphabet 10 (CO1) (PO2)

$$\sum = \{0, 1\}, \text{where}$$

$$L = \{w \mid w \text{ contains at least one } 1 \text{ and an even number of } 0s \text{ follow the last } 1\}.$$

2. a) Consider the NFA state diagram given in Figure 1 and answer the following questions. 2+2+4 (CO2) (PO2)

   i. Does this NFA accept the string *001001*?

   ii. What about the string *ba0011*?

   iii. Determine the regular expression that evaluates the same language that this NFA recognizes.

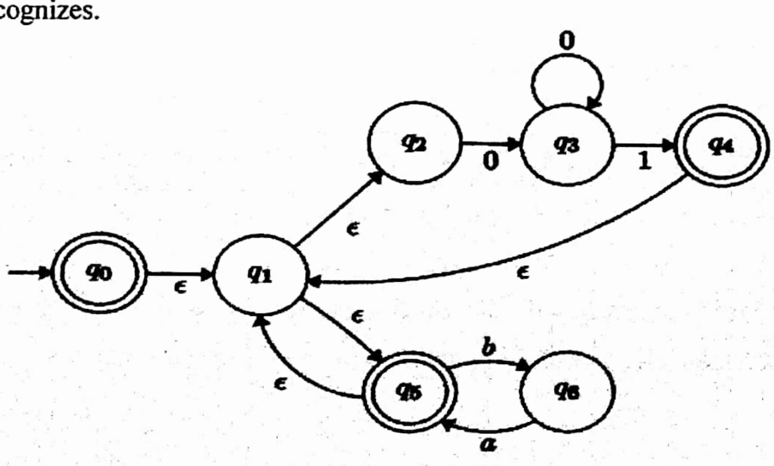

Figure 1: NFA diagram for question 2. (a)

b) Assume that $N1$ is a Nondeterministic Finite State Machine (NFA), where $N1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ recognize L_1. Construct an NFA, $N = (Q, \Sigma, \delta, q_0, F)$ that recognizes L_1^*. 9 (CO2) (PO2)

c) Consider the language, $L = \{1^n 2^{n+m} 1^m,\ m, n \geq 0\}$. Prove that, L is context-free language by creating a context-free grammar that generates L over the alphabet $\Sigma = \{1, 2\}$. 8 (CO2) (PO2)

3. a) What is Chomsky normal form of a grammar? Convert the following context free grammar (CFG) into Chomsky normal form. 2+8 (CO1) (PO1)

$$S \rightarrow ASA \mid aB$$
$$A \rightarrow B \mid S$$
$$B \rightarrow b \mid \varepsilon$$

b) Cocke-Younger-Kasami (CYK) algorithm is a membership algorithm of context free grammar. Determine if the string, $w = abab$ is the member of language generated by grammar given below using CYK algorithm. 10 (CO2) (PO2)

$$S \rightarrow AB \mid BC$$
$$A \rightarrow BA \mid a$$
$$B \rightarrow CC \mid b$$
$$C \rightarrow AB \mid a$$

c) Show the relationship among the classes of regular, context-free, decidable, turing-recognizable languages. 5 (CO2) (PO2)

4. a) Construct the Push-down Automata (PDA) that recognizes the language, $L = \{wcw^R \mid where\ w = (a+b)^*\ over\ the\ alphabet \sum \{a, b, c\}\ and\ w^R\ is\ reverse\ of w.\}$ 8 (CO2) (PO2)

b) The Turing Machine (TM) contains an infinite length of tape that stores the input string during TM simulation. We can determine the right end of the input string by scanning the blank symbol (B) at the right end of the input, but it is difficult to recognize the left end of the tape, shown in Figure 2. However, we can recognize the left end of the tape by inserting a special symbol ($) on the left end and shifting the input over one cell to the right. 9 (CO2) (PO2)

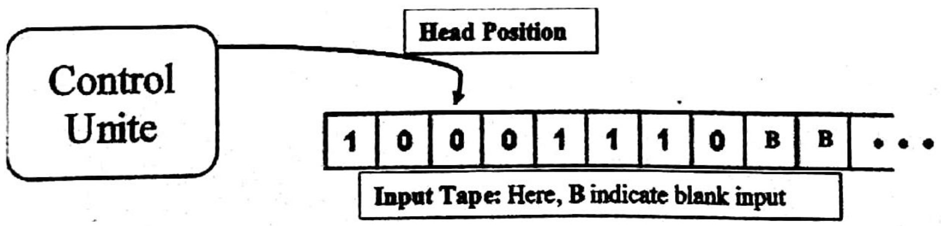

Figure 2: Tuning Machine

Now your task is to design a detail state diagram for Turing machine that inserts a special symbol ($) on the left-end and shifts the input over one cell to the right.

c) Give the formal definition of a Turing machine (TM). Based on your formal definition answer the following questions, and explain your reasoning. 2 × 4 (CO1) (PO1)

   i. Can a Turing machine ever write the blank symbol, ␣ on its tape?

   ii. Can the tape alphabet, Γ be the same as the input alphabet, Σ?

   iii. Can a Turing machine's head ever be in the same location in two successive steps?

   iv. Can a Turing machine contain just a single state?

5\. a) A Turing machine with *stay put* instead of *left* is similar to an ordinary Turing machine, but the transition function has the form as follows: 6+2 (CO2) (PO2)

$$\delta: Q \times \tau = Q \times \tau \times \{Right, Stay\}$$

At each point, the machine can move its head *Right*, or let it *Stay* in the same position.

   i. Show that this Turing machine variant is not equivalent to the usual version.

   ii. What class of languages do these machines recognize?

b) Language L_1 and L_2 are decidable languages. Show that $L_1 \circ L_2$ is also decidable language by constructing a Turning Machine, M that accept $L_1 \circ L_2$. Write a high-level description of a Turing machine that decides. 9 (CO3) (PO2)

c) Show that A_{REX} is decidable language, where 8 (CO3) (PO2)

$A_{REX} = \{\langle R, w\rangle \mid R \textit{ is a regular expression that generates string } w\}$.

6 a) Show that a language is Turing-recognizable if and only if some enumerator enumerates it. 9 (CO2) (PO2)

b) Show that A is decidable language, where 9 (CO3) (PO2)

$A = \{\langle M\rangle \mid M \textit{ is a DFA and doesn't accept any string containing an odd number of } 1s\}$

c) What does Church-Tuning thesis claim? Differentiate between enumerator and Turing Machine. 3+4 (CO1) (PO1)