B.Sc. Engg./HD CSE 5th Semester (45) 20 February 2012 (Morning)

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

MID SEMESTER EXAMINATION **WINTER SEMESTER, 2011-2012**

DURATION: 1 Hour 30 Minutes **FULL MARKS: 75**

CSE 4533: Graph Theory

Programmable calculators are not allowed. Do not write anything on the question paper.

There are **4 (four)** questions. Answer any **3 (three)** of them.

Figures in the right margin indicate marks.

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1. a) What is a graph? Describe two real-life scenarios where the knowledge of graphs and graph theory can be applied. 1+4

   b) Define **walk, path** and **circuit**. Draw the graph represented by the following adjacency matrix. 3+2

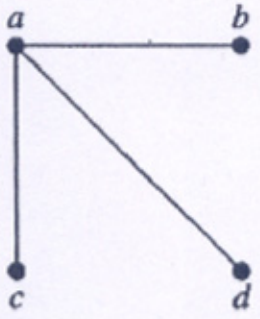
$$\begin{bmatrix} 1 & 2 & 0 & 1 \\ 2 & 0 & 3 & 0 \\ 0 & 3 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

   c) Define **complement of a graph**. Suppose a simple graph G has v vertices, e edges and the complement of graph G is denoted with $\overline{G}$. How many vertices and edges does $\overline{G}$ have? 2+3

   d) How many vertices and how many edges does the following graphs have? 5X2

   i. $K_{m,n}$ ii. C_n iii. W_n iv. Q_n v. K_n

2. a) Define **subgraph**. Draw all subgraphs of the following graph. 1+6



   b) Show that a simple graph with n vertices must be connected if it has more than $[(n-1)(n-2)]/2$ edges. 5

   c) Write short notes on the following: 3X3

   i. Arbitrarily traceable graph

   ii. Solving the travelling salesman problem efficiently

   iii. How the degree of a graph's vertices indicate where it is Eulerian or not?

d) How do you solve the **Chinese postman problem** for the following cases? 4

i. A graph which is Eulerian

ii. A graph which is semi-Eulerian

3. a) Define graph **isomorphism** and **degree sequence**. Classify the following statements as *true* or *false*: 4+2

i. Any two isomorphic graphs have the same degree sequence.

ii. Any two graphs with the same degree sequence are isomorphic.

b) Define **center, radius** and **diameter** of a tree. Draw a tree in which its diameter is not equal to twice the radius. Under what condition does this inequality hold? 3+4

c) What is a **rooted tree**? If the number of labeled trees with n vertices $(n \geq 2)$ is n^{n-2}, prove that the number of labeled rooted trees is n^{n-1}. 2+4

d) Write short notes on the following: 6

i. Cyclic interchange

ii. Application of shortest spanning tree

4. a) What is **weighted path length** of a graph? How are Huffman codes related to it? 2+3

b) Define **cut-sets, edge connectivity** and **cut-vertex.** Discuss an application of cut-sets. 3+2

c) For each of the following, give an example of a graph G with desired properties. If no such graph exists, explain why not. 3X3

i. G is connected, contains a cut-vertex and contains a cutest of cardinality 3

ii. Every edge of G is a bridge, but G is not a tree

iii. $G = K_5$ and the vertex connectivity of G is 5

d) Briefly describe the followings: 6

i. Height of a tree.

ii. Kruskal's algorithm.