ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANIZATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Information Technology (CIT)

SEMESTER FINAL EXAMINATION **SUMMER SEMESTER, 2010-2011**

DURATION: 3 Hours **FULL MARKS: 200**

Math 4405: Numerical Methods, Matrix Algebra & Fourier Series

Programmable calculators are not allowed. Do not write anything on the question paper.

There are 8 (eight) questions. Answer any 6 (six) of them.

Figures in the right margin indicate marks.

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1. a) Describe the column picture of these three equations. Solve by careful inspection of the columns(instead of eliminations) 10

$$\begin{aligned} x + 3y + 2z &= -3 \\ 2x + 2y + 2z &= -2 \\ 3x + 5y + 4z &= -5 \end{aligned}$$

What are the requirements of a systems of linear equations for having a unique solution?

b) In the process of photosynthesis solar energy is converted into forms that are used by living organisms. The chemical reaction that occurs in the leaves of plants converts carbon dioxide and water to carbohydrates with the release of oxygen. The chemical equation of the reaction takes the form 20

$$aCO_2 + bH_20 \rightarrow cO_2 + dC_6H_{12}0_6$$

where a, b, c, and d are some positive whole numbers. The law of conservation of mass states that the total mass of all substances present before and after a chemical reaction remains the same. That is, atoms are neither created nor destroyed in a chemical reaction, so chemical equations must be balanced. Now convert the chemical equation into linear systems and then solve the system to balance the chemical equation.

c) What does it means "inverse of matrix" ? Find the inverse of the matrix A. 10

$$A = \begin{bmatrix} 1 & 1 & -2 \\ -1 & 2 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

2. a) What is span of vector space? Let S be the subset of the vector space R^3 defined by 10

$$S = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}$$

Show that $v = \begin{bmatrix} -4 \\ 4 \\ -6 \end{bmatrix}$ is in span(S).

b) What does special solution mean for linear systems? Find the complete solution for the following linear systems: 20

$$\begin{aligned} x_1 + 2x_2 + x_3 &= 4 \\ 2x_1 + 4x_2 + 4x_3 + 8x_4 &= 2 \\ 4x_1 + 8x_2 + 6x_3 + 8x_4 &= 10 \end{aligned}$$

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c) Let $A = \begin{bmatrix} 1 & -1 & -2 \\ -1 & 2 & 3 \\ 2 & -2 & -2 \end{bmatrix}$ and $b = \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}$ 10

i. Determine whether b is in Col(A)
ii. Find N(A)

3. a) Find the projection of vector $b = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ onto the line through $a = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$. 15

b) Find the projection of $b = \begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix}$ onto the plane $A=\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}$. What is the projection error ? 15

c) Suppose the independent non orthogonal vectors a, b, c are

$$a = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, b = \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix}, c = \begin{bmatrix} 3 \\ -3 \\ 3 \end{bmatrix}$$

Find out the orthonormal vectors q_1, q_2, q_3 which are perpendicular to each other.(Use Gram Schmidt idea).

4. a) If Av is a scaling of the vector v, show that a non zero vector v is an Eigen vector of a matrix A. [Hints: Use geometric interpretation of Eigen value and Eigen vector] 13

b) Find the Eigen values and Eigen vectors of 15

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 5 & -10 \\ 1 & 0 & 2 & 0 \\ 1 & 0 & 0 & 3 \end{bmatrix}$$

c) Find the Eigen values of permutation matrix P. Which vectors are not changed by permutation? They are eigenvectors for λ=1 Can you find two more eigenvectors? 12

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

5. a) Diagonalize the matrix A and verify your answer. 15

$$A=\begin{bmatrix} 1 & 0 & 0 \\ 6 & -2 & 0 \\ 7 & -4 & 2 \end{bmatrix}$$

b) A group insurance plan allows three different options for participants: plan A, B,or C. Suppose that the percentages of the total number of participants enrolled in each plan are 25 percent, 30 percent, and 45 percent, respectively. From past experience assume that participants change plans as shown in the table. 25

	A	B	C
A	.75	.25	.2
B	.15	0.45	.4
C	.1	.3	.4

Find the percent of participants enrolled in each plan after 5 years. (Use matrix powers A^k)

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6\. a) Let A be the Symmetric matrix defined by 14

$$A = \begin{bmatrix} 2 & 0 & 2 \\ 0 & 0 & -2 \\ 2 & -2 & 1 \end{bmatrix}$$

Verify that the Eigenvalues and corresponding eigenvectors of A are real.

b) Describe the quick tests on a symmetric matrix that guarantee positive Eigenvalue with appropriate example. 13

c) For which values of c, $x^2 + 8xy + cy^2$ is always positive(or zero)? 13

7 a) Using the Vander monde method find the polynomial which interpolates the points (-2, -39), (0, 3), (1, 6), (3, 36). 13

b) What is Lagrange polynomial? Sketch the three Lagrange polynomials which are added to find the interpolating polynomial of the three points (2, 5), (6, 3), (7, 4). 14

c) Find the Newton polynomial which interpolates the points (2, 2), (3, 1), (5, 2). 13

8 a) What is singular value decomposition? 6

b) Find a singular value decomposition(SVD) of the matrix A given below: 14

$$A = \begin{bmatrix} -1 & 1 \\ -1 & 1 \\ 2 & -2 \end{bmatrix}$$

c) Define a linear transform $T : R^3 \to R^2$ by $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x \\ y \end{bmatrix}$ 20

i. Discuss the action of T on a vector in R^3 and give a geometric interpretation of the equation $T\left(\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}\right) = T\left(\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}\right) + T\left(\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}\right)$

ii. Find the image of the set $s_1 = \left\{ t\left(\begin{bmatrix} x \\ y \\ 3 \end{bmatrix} \middle| x, y \in R\right)\right\}$