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ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

SEMESTER FINAL EXAMINATION

SUMMER SEMESTER, 2016-2017

DURATION: 3 Hours

FULL MARKS:200

Math 4241: Integral Calculus and Differential Equations

Programmable calculators are not allowed. Do not write anything on the question paper.

There are 8(eight) questions. Answer any 6(six) of them.

Figures in the right margin indicate marks.

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1. a) Evaluate the following integrals and then graph the integrands. Finally find the area using appropriate formula from geometry: 20
 i. $\int_{-1}^1 (1 - |x|) dx$ ii. $\int_{-3}^3 \sqrt{9 - x^2} dx$
- b) Define average value of a function $f(x)$ on $[a, b]$. Graph the functions and find its average value over the given intervals: 13.33
 i. $f(x) = -3x^2 - 1$ on $[0, 1]$ ii. $f(x) = (x-1)^2$ on $[0, 3]$
2. a) Find dy/dx of $y = \int_0^{\tan x} \frac{dt}{1+t^2}$ i) by using Fundamental theorem, ii) by evaluating the integral and then differentiating the result. 12
- b) Evaluate the integral i) $\int_{-4}^4 |x| dx$, ii) $\int_0^{1/2} \frac{4}{\sqrt{1-x^2}} dx$ 10
- c) Find the total area between the region and the x -axis formed by the curve $y = -x^2 - 2x$, $-3 \leq x \leq 2$ 11.33
3. a) Define even and odd functions with examples. If f is a continuous function on the symmetric interval $[-a, a]$ then prove that: 17.33
 i. $\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$, for even function,
 ii. $\int_{-a}^a f(x) dx = 0$, for odd function.
- b) Find the area of the regions enclosed by the lines and the curves as follows: 16
 i. $y = x^2$ and $y = -x^2 + 4x$
 ii. $x - y^2 = 0$ and $x + 2y^2 = 3$
4. a) Find the volume of the given pyramid which has a square base of area 9 square meter and height 5 meters. 18.33
- b) If the solid is generated by revolving the regions bounded by the given curves $y = x^2$ and the lines $y = 0$, $x = 2$ about x -axis then find its volume. 15
- c) Find the length of the curve $y = (x/2)^{2/3}$ from $x = 0$ to $x = 2$.
5. a) Find the surface area generated by revolving the curve $y = \sqrt{x+1}$, $1 \leq x \leq 5$, about x -axis. 15
- b) Define proper and improper integrals with examples. Evaluate the following integrals and then state whether they are convergent or not: 18.33
 i. $\int_{-\infty}^{\infty} \frac{1}{e^x + e^{-x}} dx$,
 ii. $\int_0^{\ln 2} x^{-2} e^{-1/x} dx$

6. a) Define ordinary and partial differential equations. Explain with examples, order and degree of a differential equation, linear and non-linear ordinary differential equations. 13.33
- b) Form an ordinary differential equation from the curve $v = \frac{A}{r} + B$, where A and B are constants, and then identify it. 10
- c) Find the DE of the family of circles of variable radii r with center on x-axis. 10
7. a) What is exact differential equation and write its necessary condition. Determine whether the following differential equations are exact or not: 13.33
- i. $(3x + 2y)dx + (4x + y)dy = 0$
- ii. $(2y \sin x \cos x + y^2 \sin x)dx + (\sin^2 x + 2y \cos x)dy = 0$
- b) Determine the constant k such that the given equation is an exact DE and then solve it, 20
- $(x^2 + 3xy)dx + (kx^2 + 4y)dy = 0$
8. a) Solve the following differential equations: 16
- i. $(y + 2)dx + y(x + 4)dy = 0, y(-3) = -1$
- ii. $(x^2 + 3y^2)dx - 2xydy = 0, y(2) = 6$
- b) What is Bernoulli's differential equation? In what conditions, it becomes a first order linear DE, explain with examples. Is Bernoulli's DE linear or not? Finally, solve the IVP, 17.33
- $\frac{dy}{dx} + \frac{y}{2x} = \frac{x}{y^3}, y(1) = 2$

$$\int \frac{1}{x^{1/2}} dx + \int \frac{y}{y^{3/2}} dy = 0$$