B.Sc. Engg. SWE 1st Semester

30 May 2018

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

Department of Computer Science and Engineering (CSE)

SEMESTER FINAL EXAMINATION **WINTER SEMESTER, 2017-2018**

DURATION: 3 Hours **FULL MARKS: 200**

Math 4141: Geometry and Differential Calculus

Programmable calculators are not allowed. Do not write anything on the question paper.

There are **8 (eight)** questions. Answer any **6 (six)** of them.

Figures in the right margin indicate marks.

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1. a) Prove that if a function is differentiable then it is continuous. Give an example which shows that converse is not always true. 13.33

   b) Check the continuity and differentiability of the following function at $x = 0$ and $x = 1$ and draw the graph. 20

$$f(x) = \begin{cases} 1 + x^2 & x < 0 \\ x & 0 \le x \le 1 \\ \dfrac{1}{x} & x > 1 \end{cases}$$

2. a) The amount of water in a tank is $w(t) = 100(t-15)^2$ gal. t minutes after it has started to drain 15 then find 15

   i. At what rate is the water running out at the instant 5 minute?

   ii. What is the average rate at which water flows during 1 to 5 minute?

   b) If $f(x) = \begin{cases} \dfrac{1}{x+2} & x < -2 \\ x^2 - 5 & -2 < x \le 3 \\ \sqrt{x+13} & x > 3 \end{cases}$ Then find 8.33

   i. $\lim_{x\to -2} f(x)$ ii. $\lim_{x\to 3} f(x)$ iii. $\lim_{x\to 0} f(x)$

   c) Find all points where the following functions fail to be differentiable. 10

   i. $f(x) = x^{\frac{1}{3}}$ ii. $f(x) = |9 - x^2|$

3. a) A dynamic blast blows a heavy rock straight up with a launch velocity of 160 ft/sec. it reaches a height of $s = 160t - 16t^2$ ft after t second. 20

   i. How high does the rock go?

   ii. What is the acceleration of the rock?

   iii. When does the rock hit the ground again?

   b) Find all points on the curve $f(x) = \tan x$, $-\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$ where the tangent line is parallel to the line $y = 2x$ 8

   c) If $y = x + \sin x$, is there any horizontal tangent line? If so where is it? 5.33

4. a) A 13 ft ladder is leaning against a house when its base starts to slide away, by the time the base is 12 ft from the house, the base is moving at the rate of 5 ft/sec. 13.33

i. How fast the top of the ladder sliding down the wall?

ii. At what rate the area of triangle (formed by ladder, wall and ground) changing?

b) Verify that the following pair of curves meet orthogonally 20

i. $x^2+y^2=4$, $x^2=3y^2$ ii. $x=1-y^2$, $x=\frac{1}{3}y^2$

5. a) State and prove Rolle's theorem. 12

b) Does the function $f(x)=\begin{cases} x^2+1 & 0\le x\le 1 \\ 3-x & 1<x\le 2 \end{cases}$ satisfy the hypothesis of Rolle's theorem on the given interval [0,2]? Give reason for your answer. 12

c) Find $\frac{dy}{dx}$ from the followings 9.33

i. $x+\tan(xy)=0$ ii. $\frac{1}{\sqrt{x}}+\frac{1}{\sqrt{y}}=1$

6. a) Find the absolute maximum and minimum of $f(x)=2x^3-15x^2+36x$ on the interval [-1, 5] and determine where these values occur. 8

b) Find critical points and identify which critical points are stationary points for $f(x)=\frac{x+1}{x^2+3}$ 5.33

c) Sketch the graph of the function $f(x)=3x^4-4x^3+2$ by showing stationary point, relative extrema, inflection point and concavity. 20

7. a) Evaluate the following limits using L' Hospital rule 13.33

i. $\lim_{x\to 0}(1+\sin x)^x$ ii. $\lim_{x\to -\infty}(\sqrt{x^2+x}-x)$

b) Graph the rational function $f(x)=\frac{2x^2+x-1}{x^2-1}$ by showing all necessary steps. 20

8. a) Show that the equation of bisectors of angles between the line represented by 10

$ax^2+2hxy+by^2=0$ is $\frac{x^2-y^2}{a-b}=\frac{xy}{h}$.

b) Show that the equation $6x^2-5xy-6y^2+14x+5y+4=0$ represents a pair of straight lines. Find their equation, point of intersection, angle between them and equation of bisector. 12

c) If a straight line makes angle $\alpha, \beta, \gamma, \delta$ with the four diagonals of a cube then show that 11.33

$\cos^2\alpha+\cos^2\beta+\cos^2\gamma+\cos^2\delta=\frac{4}{3}$