

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

MID SEMESTER EXAMINATION**WINTER SEMESTER, 2011-2012****DURATION: 1 Hour 30 Minutes****FULL MARKS: 75****Math 4307: Linear Algebra**

Programmable calculators are not allowed. Do not write anything on the question paper.

There are **4 (four)** questions. Answer any **3 (three)** of them.

Figures in the right margin indicate marks.

1. a) Solve the following system by Gauss-Jordan elimination: 15

$$\begin{aligned} 3x_1 + 2x_2 - x_3 &= -15 \\ 5x_1 + 3x_2 + 2x_3 &= 0 \\ 3x_1 + x_2 + 3x_3 &= 11 \\ -6x_1 - 4x_2 + 2x_3 &= 30 \end{aligned}$$
- b) How many operations do we carry out during forward elimination on a matrix $[A|b]$, 5
 where A is a square matrix of size $m \times m$?
- c) Suppose the equations in 1.a) is multiplied by 3, 4, 5 and 6, respectively, and they 5
 become $Dx = f$.
 - i. Does the column space get changed after the multiplication? Explain your answer.
 - ii. What is the solution x of $Dx = f$?
2. a) Find the inverse of the following matrix A. Check your answer showing 15
 $AA^{-1} = I$ where the multiplication should be carried out with row-wise operations.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$$
- b) How many permutation matrices P shall you get for a 5×5 matrix? 5
 What differences do you find between the two set $\{P\}$ and $\{P^{-1}\}$? Explain your answer.
- c) Construct a 3 by 3 matrix whose column space contains (1,1,0) and (1,0,1) but not 5
 (1,1,1). Construct a 3 by 3 matrix whose column space is only a line.
3. a) Suppose A is the matrix as given below. Find all special solutions to $Ax = b$ and 12
 describe the whole nullspace of A. You can freely choose the vector b.

$$A = \begin{bmatrix} 1 & 2 & 2 & 2 \\ 0 & 3 & 8 & 7 \\ 0 & 0 & 4 & 2 \end{bmatrix}$$
- b) In order to find solutions to $Ax = b$, what conditions should the vector b fulfill? List 8
 them with examples.
- c) In the complete solution of $Ax = b$, can we multiply a constant c with the particular 5
 solution? Explain your answer with proof.

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4. a) Forward elimination changes $Ax = b$ to a row reduced $Rx = d$. The complete solution found is given below: 10

$$x = \begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix} + c_1 \begin{bmatrix} 5 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

- What is the 3 by 3 reduced row echelon matrix R and what is d ?
- If the process of elimination subtracted 3 times row 1 from row 2 and then 5 times row 1 from row 3, what matrix connects R and d to the original A and b ? Use this matrix to find A and b .

- b) For the matrix A given below: 9

$$A = \begin{bmatrix} 1 & 2 & 2 & 3 \\ 2 & 4 & 8 & 10 \\ 3 & 6 & 5 & 9 \end{bmatrix}$$

- The column space $C(A)$ is the combination of which basis vectors? Show how you found them?
 - What properties should a basis fulfill?
 - How many basis exists for the nullspace of A . Explain your answer.
- c) If r is the rank of a matrix A of size $m \times n$, what else can you say about this matrix? 6