Math 4241: Integral Calculus and Differential Equations

Programmable calculators are not allowed. Do not write anything on the question paper.
There are **4 (four)** questions. Answer any **3 (three)** of them.
Figures in the right margin indicate marks.

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1. a) Define integration, integrand and integral. Explain with examples, why it is necessary to add the integral constant to the result in indefinite integral. **8**

    b) An environmentalist finds that a certain type of tree grows in such a way that its height $h(t)$ after t years is changing at the rate of $0.2\, t^{2/3} + \sqrt{t}$ feet per year. If the tree was 2 feet tall when it was planted, how tall will it be in 27 years? **13.33**

    c) Evaluate the followings : **12**
    i. $\int 3x^5\sqrt{x^3+1}\; dx$, ii. $\int x\sin^{-1}x\; dx$, iii. $\int e^{-3x}\cos4x\; dx$,
    iv. $\int \cos^3x\, \sin^3x\; dx$

2. a) Obtain the reduction formula for $\int \sin^n x\; dx$ and then evaluate $\int \sin^4x\; dx$. **13.33**

    b) Express the integrand as a sum of partial fractions, using Heaviside cover up and any other method, and then evaluate the integral $\int \frac{2x+1}{x^2-7x+12}dx$. **20**

3. a) What is the physical meaning of $\int_a^b f(x)\, dx$. Show that the definite integral can be written as a limit of a finite sum. Also write the properties of definite integral. **13.33**

    b) Find the area under the graph of the function $f(x) = \frac{1}{x}$ between $x = 1$ to $x = 5$ using lower sum, upper sum and mid-point approximations considering four rectangles of equal width. Finally, find the percentage of error in each case and comments on your results. **20**

4. a) Suppose an industrial plant in Gazipur city spills pollutant into a river. The Pollutant spreads out as it carried downstream by the current of the river and three hours after the spill, it forms the following pattern as shown in Figure 1. Moreover, the same data are given in the table (all dimensions are in meter): **13.33**

Distance:	0	5	10	15	20	25	30	35	40
Height :	0	5	6	8	11	10	11	10	5

    Using appropriate approximation, find the polluted area of the river.
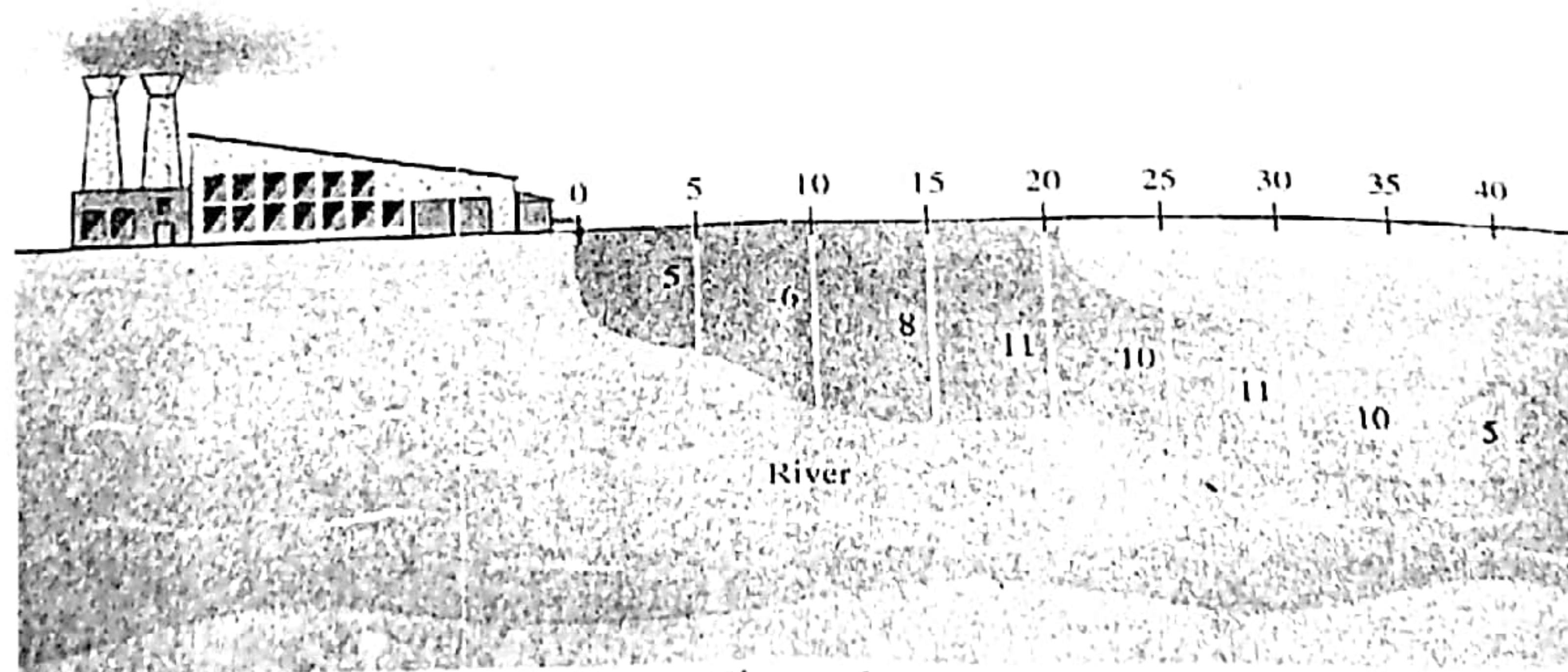

Figure 1.

    b) Evaluate $\int_{-1}^{1}(1+\sqrt{1-x^2})\, dx$. Graph the integrand and then compare your result using appropriate formula from geometry. **20**