262

B. Sc. Engg. CSE 5th Semester (45) — 29 April 2012 (Morning)

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

SEMESTER FINAL EXAMINATION — **WINTER SEMESTER, 2011-2012**

DURATION: 3 Hours — **FULL MARKS: 150**

CSE 4533: Graph Theory

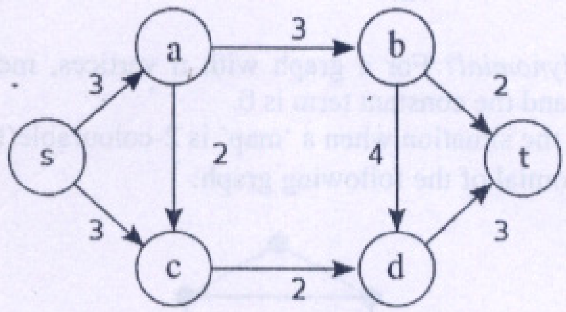
Programmable calculators are not allowed. Do not write anything on the question paper.

There are 8 (four) questions. Answer any 6 (three) of them.

Figures in the right margin indicate marks.

---

1 a) Define transport network and flow pattern. A transport network is shown below where the numbers written beside the edges are the edge capacities, and *s* and *t* denote the source and target of the network. Assign a flow pattern for the network by adjusting weights for all the edges. 4+4



b) State the *max-flow min-cut theorem*. Determine the maximum flow of the transport network you generated in 1.a) using the theorem. 2+7

c) Describe the possible extensions of the max-flow min-cut theorem for the following cases: 4+4

i. Multiple sources and sinks
ii. Vertices with specified capacity

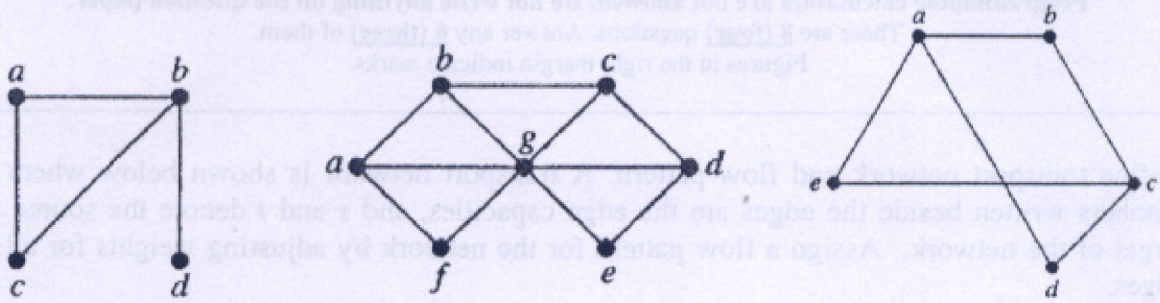
2. a) A lecture timetable is to be drawn up. Since some students wish to attend several lectures, certain lectures must not coincide. The asterisks in the following table show which pairs of lectures cannot coincide. How many periods are needed to prepare the timetable for all seven lectures? 7

	a	*b*	*c*	*d*	*e*	*f*	*g*
a	—	*	*	*	—	—	*
b	*	—	*	*	*	—	*
c	*	*	—	*	—	*	—
d	*	*	*	—	—	*	—
e	—	*	—	—	—	—	—
f	—	—	*	*	—	—	*
g	*	*	—	—	—	*	—

b) State Brook's theorem and define chromatic number. For each of the following graphs, what does Brook's theorem tell you about the chromatic number of the graph? 4+6

i. The complete graph K_{20}
ii. The bipartite graph $K_{10,20}$
iii. A cycle with 20 edges.

c) What is chromatic index? Find the chromatic index of the following graphs: 2+6

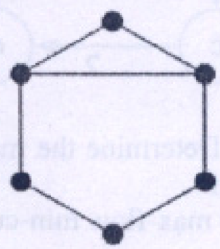


i. ii. iii.

3. a) What is a *chromatic polynomial*? For a graph with n vertices, mention why the chromatic polynomial's degree is n and the constant term is 0. 2+5

b) Define a 'map'. Describe the situation when a 'map' is 2-colourable(f). 2+6

c) Find the chromatic polynomial of the following graph: 10

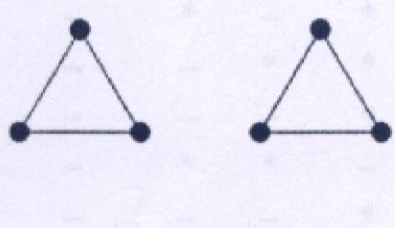


4. a) Define planar graph and embedding. How can you relate all nonplanar graphs to Kuratowski's two graphs? 4+6

b) What is 'infinite region'? Explain why the infinite region is not fundamentally different than the other regions. 2+5

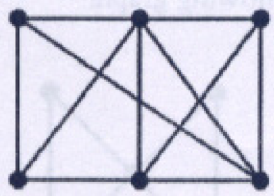
c) Mention the four steps of elementary reduction that is needed for the detection of planarity. 8

5. a) Let G be the disconnected planar graph shown below. Draw its dual G*, and the dual of the dual (G*)*. Show that if G is a disconnected planar graph, then G* is connected. 2+2+4



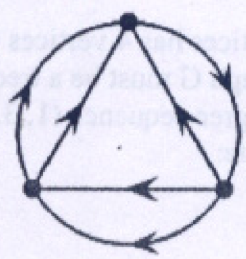
264

b) Show that the following graph is planar by drawing it in the plane without any edges crossing. Verify Euler's formula for this graph. 2+4



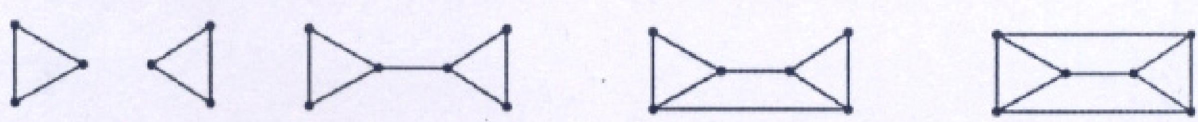
c) Define embedding, crossings, and thickness. Prove that if G is a connected planar bipartite graph, then its dual G* is Eulerian. *[Hint: Use the fact that each cycle of a bipartite graph is of even length]* 6+5

6\. a) Answer the following questions for the digraph shown below. 8



i. Is this digraph simple?
ii. Is it Eulerian?
iii. Is it Hamiltonian?
iv. Is it strongly connected?

b) What is an orientable graph? Which of the following four graphs are orientable? 3+4



i. ii. iii. iv.

c) A graph G has the following adjacency matrix: 2+4+4

$$\begin{pmatrix} 0 & 1 & 1 & 2 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{pmatrix}.$$

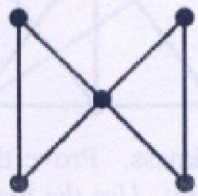
Answer the following questions:

i. Is G a simple graph?
ii. What is the degree sequence of G?
iii. How many edges does G have?

7. a) Prove that every tree is a bipartite graph. Are all bipartite graphs trees too? Show reasons for your answer. 3+3

b) Define fundamental cut-sets and fundamental circuits. How are they related? 4+4

c) Draw all the spanning trees of the following graph: 5



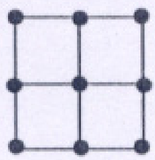
d) Define the following terms: pendant vertex, binary tree, bridge. 3X2

8. a) Write TRUE or FALSE for the following statements. Describe the reasoning behind your answer in brief. 3X3

i. A simple graph G with 13 vertices has 4 vertices of degree 3, 3 vertices of degree 4 and 6 vertices of degree 1. The graph G must be a tree.

ii. G is a simple graph having degree sequence (1, 3, 3, 3)

iii. The following graph is bipartite:



b) Define complement of a graph and disconnected graph. Can a simple graph G and its complement G` both be disconnected? Explain why or why not. 4+6

c) State the handshaking lemma. Prove that every graph has an even number of odd vertices. 2+4