ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

MID SEMESTER EXAMINATION **WINTER SEMESTER, 2012-2013**

DURATION: 1 Hour 30 Minutes **FULL MARKS: 75**

CSE 4703: Theory of Computing

Programmable calculators are not allowed. Do not write anything on the question paper.

There are **4 (four)** questions. Answer any **3 (three)** of them.

Figures in the right margin indicate marks.

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1. a) Define *proof* of a mathematical statement. Using the idea of *proof by induction*, prove that, "The sum of the squares of the first n natural numbers is: $\frac{n(n+1)(2n+1)}{6}$". 8

   b) Path between two vertices of a non-directed graph is an example of a binary relation. Show that this binary relation is an equivalence relation. 7

   c) Suppose you are asked to design a small part of a cricket game. This game is little bit different than usual cricket. In this game a for a boundary batsman gets 1 run, for a over-boundary batsman gets 2 runs otherwise batsman gets no run. When a batsman's total run is multiple of 3 (i.e. 3, 6, 9, 12…etc) it plays a sound of cheering crowd (Hurraaaaaaaaaaay). When batsman first enters into the crease crowd also cheers for him.
   Now simulate this feature of the game using a DFA. 10

2. a) Give the formal definition of an NFA given in figure 1 and convert the NFA into an equivalent DFA. 15

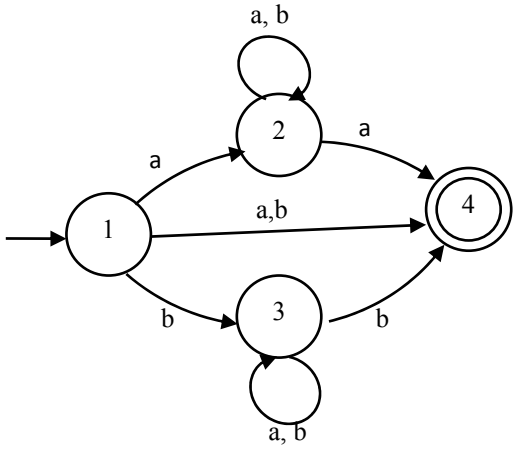

Figure 1: State diagram of an NFA

   b) Write formal description of the computation of an NFA. 5

   c) Give formal definition of *regular operation.* Illustrate with example. 5

3. a) Draw state diagram of DFAs recognizing the following languages. [Where $\Sigma = \{0,1\}$] 15
      i. {w | w has both 01 and 10 as a substring}
      ii. {w | w contains the substring 0101}

   b) Given an NFA N1 = (Q1, Σ, δ1, q1, F1) that accepts language A, and an NFA N2 = (Q2, Σ, δ2, q2, F2) that accepts language B. Show that there exists an NFA N that recognizes the language A **.** B. Give formal definition of that N. 10

4. a) Give formal definition of regular expression. If R is a regular expression then write outcome of each of the operation: 5

   i. $R \cup \varepsilon$

   ii. $R \quad \varepsilon$

   iii. $R \cup \emptyset$

   iv. $R \cdot \emptyset$

b) 10

Convert the regular expression ((a ∪ b) a)* to an NFA in a sequence of stages, starting from the smallest sub expression to larger sub expression.

c) What are the properties of a GNFA? Convert the following DFA into a regular expression. Construct GNFA as an intermediary step. 10

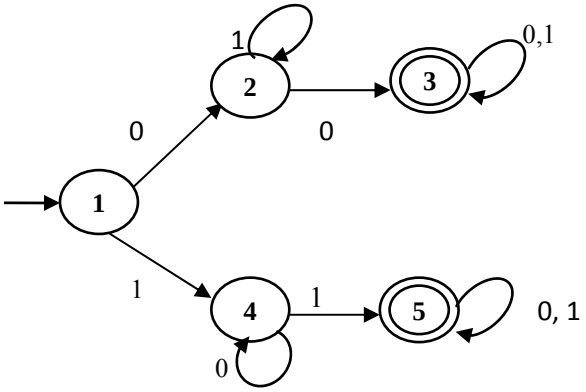

Figure 2: State diagram of a DFA