

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)

Department of Computer Science and Engineering (CSE)

MID SEMESTER EXAMINATION

WINTER SEMESTER, 2019-2020

DURATION: 1 Hour 30 Minutes

FULL MARKS: 75

CSE 4591: Discrete Mathematics

Programmable calculators are not allowed. Do not write anything on the question paper.

There are **4 (four)** questions. Answer any **3 (three)** of them.

Figures in the right margin indicate marks.

1. a) Let,
 $L(x)$:= x is a large bird.
 $H(x)$:= x lives on honey.
 $F(x)$:= x is female.
 $P(x)$:= x is a parent.
 $M(x, y)$:= x is the mother of y .
 $D(x)$:= x is diamond.
 $R(x)$:= x is Ruby.
 $B(x)$:= x is beautiful gem.
 $A(x)$:= x attends the class.
 $F(x, y)$:= x and y are friends.
 $Z(x, y)$:= y gives proxy for x .

Translate the following English sentences into logical expressions:

- i. No large birds live on honey. 8
 - ii. If a person is female and is a parent, then this person is someone's mother.
 - iii. Diamonds and Rubies are beautiful gems.
 - iv. Every student either attends the class or has a friend who gives his proxy.
- b) Using rules of inference and proper quantifiers, show that the premises "Somebody in this class loves football", and "Someone who loves football, watches football whenever he/she gets some free time" imply the conclusion "There is at least one person in this class who watches football whenever he/she gets some free time." 9
- c) Inhabitants of a remote island can be of two types- knights and knaves, where knights always tell the truth and knaves always lie. You encounter two people, A and B . Determine, what A and B are if they address you in the ways described. If you cannot determine what these two people are, can you draw any conclusion? 8
- i. A says "at least one of us is a knave" and B says nothing.
 - ii. A says "The two of us are both knights" and B says " A is a knave".
 - iii. A says "I am a knave or B is a knight" and B says nothing.
 - iv. Both A and B say "I am a knight".
2. a) Let m be a positive integer. If $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$, then prove that $a + c \equiv b + d \pmod{m}$ and $ac \equiv bd \pmod{m}$ 6+3
- Now, Suppose x and y are integers, $x \equiv 4 \pmod{13}$ and, $y \equiv 9 \pmod{13}$, Find integer z with $0 \leq z \leq 12$ such that
- i. $z \equiv (2x + 3y) \pmod{13}$.
 - ii. $z \equiv 9x \pmod{13}$.
 - iii. $z \equiv (x^2 + y^2) \pmod{13}$.

- b) Prove that, there are infinitely many primes. 8
- c) Find the greatest common divisor of 637 and 133 using the Euclidean algorithm and express the greatest common divisor as a linear combination of 637 and 133 . 8
3. a) A computer network consists of six computers. Each computer is directly connected to zero or more of the other computers. Show that there are at least two computers in the network that are directly connected to the same number of other computers. 10
- b) How many solutions are there to the equation, 9
- $$x_1 + x_2 + x_3 + x_4 + x_5 = 21$$
- Where, x_1, x_2, x_3, x_4, x_5 are nonnegative integers such that
- $x_1 > 2$
 - $x_1 \geq 2, x_2 \geq 3$
 - $0 \leq x_1 \leq 10$
- c) Programmers of IUT has created a new social media called IUTgram. Each user on IUTgram must have a unique username. Each username is six to nine characters long, where each character is an uppercase letter or a digit. Also each username must contain at least one digit if the length of the username is less than eight. 6
- What is the maximum number of users IUTgram can possibly have?
4. a) Each inhabitant of a remote village always tells the truth or always lies. A villager will give only a "Yes" or a "No" response to a question a tourist asks. Suppose you are a tourist visiting this area and come to a fork in the road. One branch leads to the ruins you want to visit, the other branch leads deep into the jungle. A villager is standing at the fork in the road. What one question can you ask the villager to determine which branch to take? 5
- b) Use the bubble sort to sort 6, 2, 3, 17, 9 , showing the list obtained at each step. 10
- c) What is the time complexity of *linear search algorithm* and *binary search algorithm*? Which one is better? Justify your answer. 5
- d) Give a Big-O estimate for $f(x) = 3x^2 + (x + 1) \log(x^2 + 1)$. 5