B.Sc. Eng. (CSE)/ 5th Sem.

13 March, 2018 (Afternoon)

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

DEPARTMENT OF MECHANICAL AND CHEMICAL ENGINEERING

TERM : Mid Semester Examination
Winter Semester: 2017-2018
COURSE NO. : Math-4541
TIME : 1½ Hours
COURSE TITLE: Multivariable Calculus and Complex Variables
FULL MARKS: 75

There are **4 (Four)** questions. Answer any **3 (Three)** of them. Programmable calculators are not allowed. Do not write anything on this question paper. The figures in the right margin indicate full marks. The Symbols have their usual meaning.

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1. a) Show that the function $f(z) = u + iv$, where **13**

$$f(z)=\begin{cases}\dfrac{x^3(1+i)-y^3(1-i)}{x^2+y^2}; & z\neq 0\\ 0 \qquad\qquad\qquad ; & z=0\end{cases}$$

satisfies the Cauchy-Riemann equations at $z = 0$. Is the function analytic at $z = 0$? Justify your answer.

b) Define a harmonic function and conjugate harmonic function. Are the following function harmonic? If your answer is yes, find a corresponding analytic function $f(z) = u(x,y) + iv(x,y)$. **12**

$$u=\frac{x}{x^2+y^2}$$

2. a) (i) Find the following functions in the form of $u + iv$ **8**

$$e^{2+3\pi i} \text{ and } \cosh(-1+2i)$$

(ii) Find all solutions and graph in the complex plane

$$e^{z=1} \text{ and } \sinh z = 0$$ **8**

b) Evaluate $\int_C (z-z^2)dz$ where C is the upper half of the circle $|z-2| = 3$ and z is the complex variable. What is the value of the integral if C is the lower half of the above given circle? **9**

3. a) State and verify Cauchy Integral Theorem by integrating e^{iz} along the boundary of the triangle with the vertices at the points $1+i$, $-1+i$ and $-1-i$. **13**

b) Evaluate, using Cauchy's integral formula, **12**

(*i*) $\int_c \frac{3z^2+z}{z^2-1}dz$, If c is circle $|z-1|=1$.

(*ii*) $\int_c \frac{z^2-2z}{(z+1)^2(z^2+4)}dz$, where c is the circle $|z|=10$.

4. a) **5**

(i) $z_n = \frac{n\pi}{4+2ni}$ is a sequence. Is it bounded? Convergent? Find its limit points.

(ii) What is radius of convergence? Write its role in complex series. Find the center and radius of converges of the following series: $\sum_{n=0}^{\infty} \frac{(2n)!}{4^n(n!)^2}(z-2i)^n$ **8**

b) Find the Taylor series with center z_o and its radius of convergence. **12**

(i) $\frac{1}{1-z}, z_o = i$, (*ii*) $\sin 2z^2, z_o = 0$

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