B.Sc. Eng. (CSE)/ 5th Sem. 24 May 2018

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

DEPARTMENT OF MECHANICAL AND CHEMICAL ENGINEERING

Term: Semester Final Examination | Winter Semester: 2017-2018
Course No.: Math-4541 | Time: 3 Hours
Course Title: Multivariable Calculus and Complex Variables | Full Marks: 150

There are **8 (Eight)** questions. Answer any **6 (Six)** of them. Programmable calculators are not allowed. Do not write anything on this question paper. The figures in the right margin indicate full marks. The Symbols have their usual meaning.

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1. a) Define a harmonic function and conjugate harmonic function. Compute the Laplacian, where it exists, of the following functions and indicate where a function is harmonic. **12**

   (i) $x^2 + y^2$ (ii) $e^{ax}\cos\beta y$ (iii) $\ln\sqrt{(x^2 + y^2)}$

   b) Prove that $u = x^2 - y^2 - 2xy - 2x + 3y$ is harmonic. Find a function v such that $f(z) = u + iv$ is analytic. Also express $f(z)$ in terms of z. **13**

2. a) (i) Find the following functions in the form of $u + iv$ **12**

   $$e^{2+3\pi i} \; and \; \cosh(-1+2i)$$

   (ii) Find all solutions and graph in the complex plane

   $$e^{z} = 1 \; and \; \sinh z = 0$$

   b) Evaluate the following integrals along the mentioned path. **13**

   (i) $\int_{0}^{1+i} (x^2 - iy)dz$, along the parabola $y = x^2$

   (ii) $\int_c (12z^2 - 4iz)dz$, along the curve C joining the points (1, 1) and (2, 3).

3. a) State Cauchy integral formula and Evaluate the following integral using Cauchy integral formula: **13**

   (i) $\int_c \frac{4-3z}{z(z-1)(z-2)}dz$, where c is the circle $|z| = \frac{3}{2}$.

   (ii) $\int_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)}dz$, where c is the circle $|z| = 3$.

b) Define the singularity of a function. Find out the zeros and discuss the nature of the singularities of 12

$$(i)\sin\frac{1}{z}\ (ii)\ \frac{e^{\frac{1}{z}}}{z^2}\ (iii)\frac{(z-2)}{z^2}\sin\left(\frac{1}{z-1}\right)$$

4\. a) Using Residue theorem, evaluate $\frac{1}{2\pi i}\int_C \frac{e^{zt}\,dz}{z^2(z^2+2z+2)}$. 12

b) Find the Laurent's series that converges for $0<|z-z_0|<R$ and determine the precise region of convergence of $\frac{\cos z}{(z-\pi)^2}, z_0=\pi$. 13

5\. a) (i) What is a graph of a function of two variables? How is it interpreted geometrically? Describe level curves. 13

If $T(x, y)$ is the temperature at a point (x, y) on a thin metal plate in the xy-plane, then the level curves of T are called *isothermal curves*. All points on such a curve are at the same temperature. Suppose that a plate occupies the first quadrant and

$$T(x, y) = xy.$$

(ii) Sketch the isothermal curves on which $T = 1$, $T = 2$, and $T = 3$.
(iii) An ant, initially at (1, 4), wants to walk on the plate so that the temperature along its path remains constant. What path should the ant take and what is the temperature along that path?

b) (i) State the definition of continuity of a function of two variables. 12

$$\text{Let, } f(x,y)=\begin{cases}-\dfrac{xy}{x^2+y^2} & ;(x,y)\neq(0,0)\\ 0 & ;(x,y)=(0,0)\end{cases}$$

(ii) Show that $f_x(x,y)$ and $f_y(x,y)$ exist at all points (x, y)
(iii) Explain why f is not continuous at (0, 0).

6\. a) (i) Define the total differentials of a function of two variables. 13
(ii) When using differentials, what is meant by the terms propagated error and relative error?

(iii) Use the differential dz to approximate the change in $z=\sqrt{4-x^2-y^2}$ as (x, y) moves from the point (1, 1) to the point (1.01, 0.97). Compare this approximation with the exact change in z.

(iv) The possible error involved in measuring each dimension of a rectangular box is ± 0.1 millimeter. The dimensions of the box are $x = 50$ centimeters, $y = 20$ centimeters and $z = 15$ centimeters as shown in Fig.: 01. Use dv to estimate the propagated error and the relative error in the calculated volume of the box.

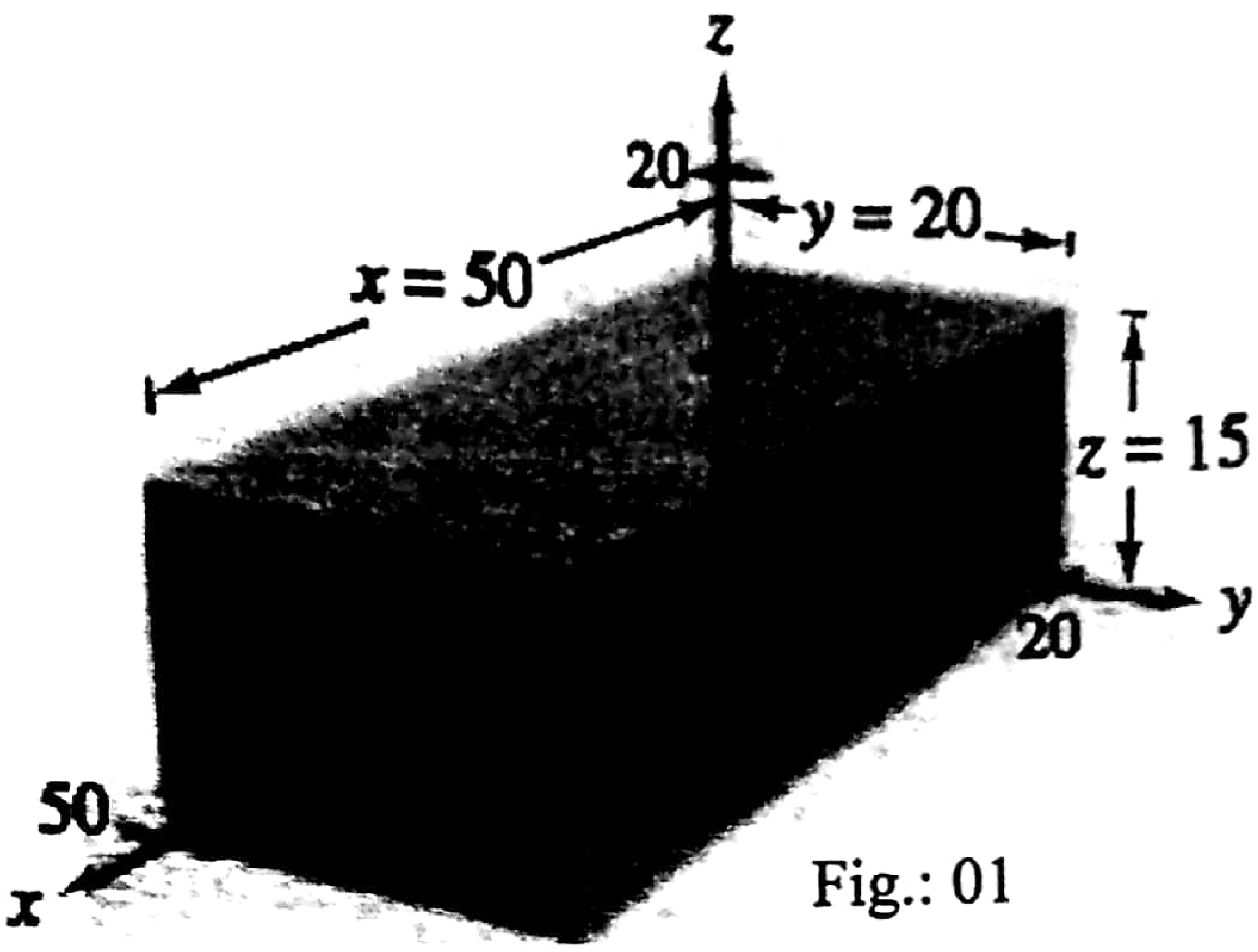

Fig.: 01

12

b) (i) If $f(x,y)=0$, give the rule for finding $\dfrac{dy}{dx}$ implicitly.

(ii) If $f(x,y,z)=0$, give the rule for finding $\partial z/\partial x$ and $\partial z/\partial y$ implicitly.

(iii) Find $\partial z/\partial x$ and $\partial z/\partial y$, given $3x^2z - x^2y^2 + 2z^3 + 3yz - 5 = 0$

(iv) Find the directional derivative of $f(x,y) = 4 - x^2 - \dfrac{1}{4}y^2$, at (1, 2) in the direction of $\overline{u} = \left(\cos\dfrac{\pi}{3}\right)\hat{i} + \left(\sin\dfrac{\pi}{3}\right)\hat{j}$.

7. a) (i) Write a paragraph describing the directional derivative of the function f in the direction $\overline{u} = \cos\theta\hat{i} + \sin\theta\hat{j}$, when $\theta = 0^\circ$ and $\theta = 90^\circ$. **12**

(ii) A heat–seeking particle is located at the point (2, -3) on a metal plate whose temperature at *(x, y)* is $T(x,y) = 20 - 4x^2 - y^2$. Find the path of the particle as it continuously moves in the direction of maximum temperature increase.

b) Consider the ellipsoid $x^2 + 4y^2 + z^2 = 18$. **13**

(i) Find an equation of the tangent plane to the ellipsoid at the point (1, 2, 1).

(ii) Find parametric equations of the line that is normal to the ellipsoid at the point (1, 2, 1).

(iii) Find the acute angle that the tangent plane at the point (1, 2, 1) makes with the *xy*-plane.

8\. a) A delivery company accepts only rectangular boxes, the sum of whose length and girth (perimeter of a cross-section) does not exceed 108 inches and shown in Fig.: 02. Find the dimensions of an acceptable box of largest volume. **12**

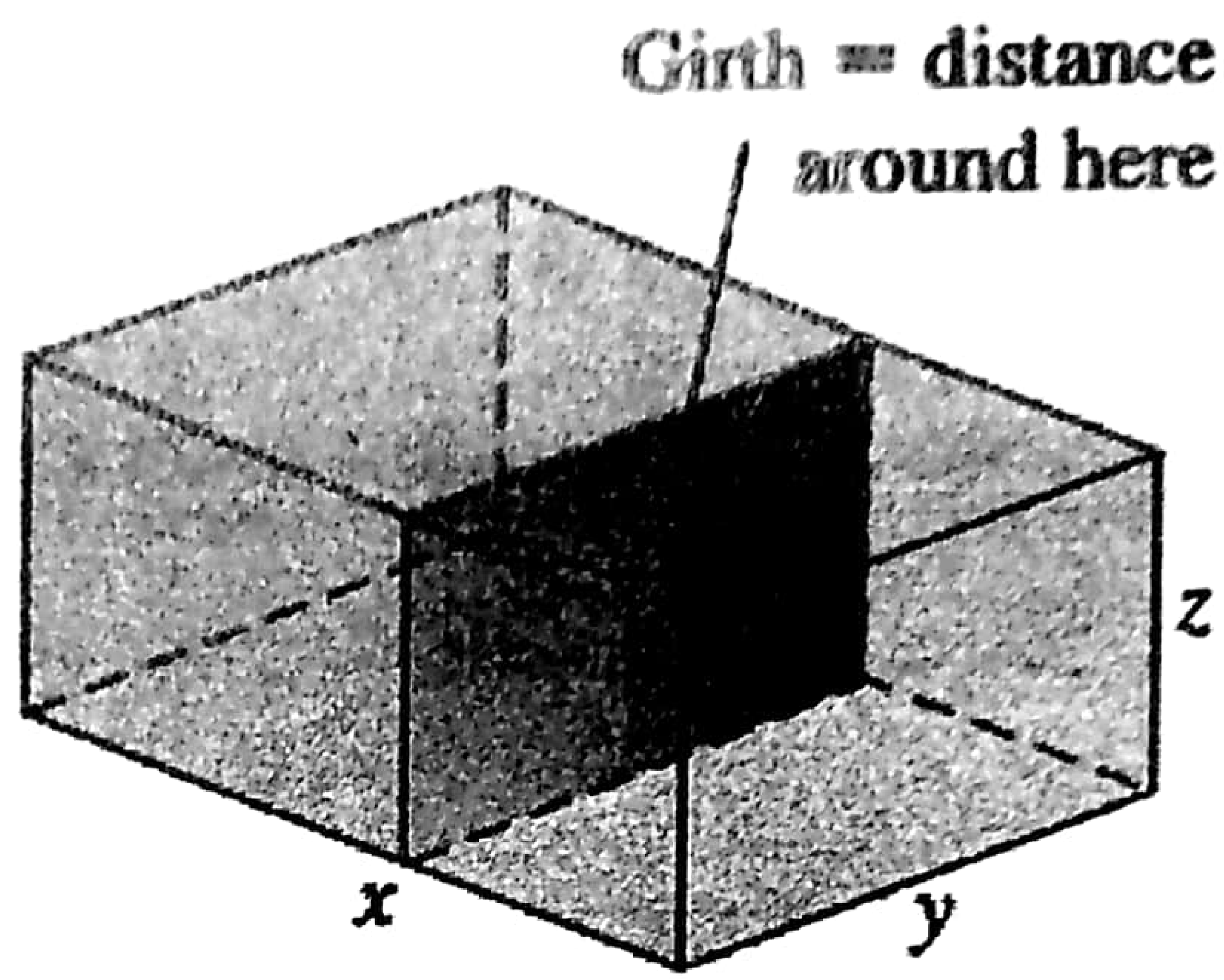

Fig.: 02

b) (i) Explain what is meant by constrained optimization problems. **13**

(ii) The operators of the Viking Princess, a luxury cruise liner are contemplating the addition of another swimming pool to the ship. The chief engineer has suggested that an area of the form of an ellipse located in the rear of the promenade deck would be suitable for this purpose. It has been determined that the shape of the ellipse may be described by the equation $x^2 + 4y^2 = 3600$ where x and y are measured in feet. Viking's operators would like to know the dimensions of the rectangular pool with the largest possible area that would meet these requirements.

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