

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
 THE ORGANIZATION OF THE ISLAMIC CONFERENCE (OIC)
Department of Computer Science and Information Technology (CIT)

SEMESTER FINAL EXAMINATION

WINTER SEMESTER, 2010-2011

DURATION: 3 Hours

FULL MARKS: 150

CIT 4529: Graph Theory

Programmable calculators are not allowed. Do not write anything on the question paper.

There are 8 (Eight) questions. Answer any 6 (Six) of them.

Figures in the right margin indicate marks.

1. a) In a precedence graph, each statement is represented by a vertex, and there is a directed edge from one vertex to another if the second statement cannot be executed before the first statement has been executed. Construct a precedence graph for the following program: 5

$$S_1 : x = 1$$

$$S_2 : x = x + 1$$

$$S_3 : y = 2$$

$$S_4 : z = y$$

$$S_5 : x = x + 2$$

$$S_6 : y = x + z$$

$$S_7 : z = 4$$

- b) Consider the following undirected graph H in fig 1, where each edge has a length of one. Count all possible paths of length four between any pair of vertices (v_i, v_j) in that graph. 20

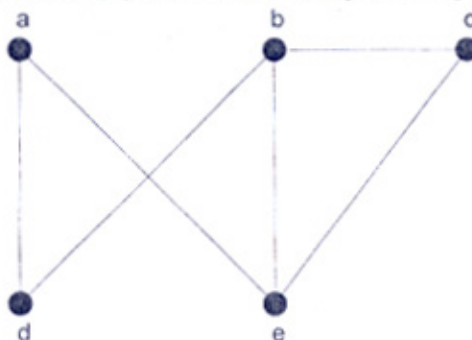


Fig 1. Graph H

2. a) Which type of graph can have both Euler circuit and Hamilton circuit? Explain your answer why only that class of graph can have both Euler and Hamilton circuit. 10
- b) Prove that a complete bipartite graph with an odd number of vertices does not have a Hamilton circuit. 15
3. a) If a connected planar simple graph has e edges and v vertices with $v \geq 3$ and no circuits of length 3, then prove that $e \leq 2v - 4$. 10
- b) Show that a connected simple planar graph where all of the vertices have degree at least 5 must have at least 12 vertices. 10
- c) If S is a set of vertices in a graph G , then prove that 5

$$[S, \bar{S}] = \sum_{v \in S} \deg(v) - 2e(G[S])$$

Where, $[S, \bar{S}]$ is the cut edge and $G[S]$ represents the induced subgraph.

4. a) An n -cube is a cube in n dimensions. A cube in one dimension is a line segment; in two dimensions, it's a square, in three, a normal cube, and so on. If we consider the cube to be composed of the vertices and edges only, show that every n -cube has a Hamiltonian circuit. 10
- b) Suppose, seven final exams has to be scheduled at a university. Label those courses from 1 to 7. Suppose that the following pairs of courses have common students: 1 and 2, 1 and 3, 1 and 4, 1 and 7, 2 and 3, 2 and 4, 2 and 7, 3 and 4, 3 and 6, 3 and 7, 4 and 5, 4 and 6, 5 and 6, 5 and 7, and 6 and 7. How many time slots are required to complete those exams? 15
5. a) Show that a graph with n vertices, $n-1$ edges and no circuits is minimally connected. 10
- b) Suppose that someone starts a chain mail. Each person who receives the mail is asked to send it on to four people. Some people do this, but others do not send any mails. How many people have seen the mail including the first person, if no one receives more than one mail and the chain mail ends after there have been 100 people who read it but did not send it out? How many people sent out the mail? 5
- c) Prove that if an m -ary tree of height h has l leaves, then $h \geq \lceil \log_m l \rceil$. How can you find the height of the tree if it is full and balanced? 10
6. a) How many weighings of a balance scale are needed to find a counterfeit coin among four coins if the counterfeit coin may be either heavier or lighter than the others? Illustrate your algorithm with help of a decision tree. 15
- b) Show that any tree with at least two vertices is bipartite. 5
- c) Evaluate the value of the following postfix expression, step by step, and draw the rooted tree representing the original expression. 5

$$521 - -314 + + *$$

7. a) Prove that a sorting algorithm based on binary comparisons requires at least $\lceil \log n! \rceil$ comparisons. Illustrate with an example case when sorting three values $\{a, b, c\}$. 10
- b) Given a graph G , if both Dijkstra's algorithm and Prim's algorithm produce a spanning tree for G . Then what are the differences between the spanning trees produced by them? 5
- c) The roads represented by the following graph in fig2 are all unpaved. The lengths of the roads between pairs of towns are represented by edge weights. Which roads should be paved so that there is a path of paved roads between each pair of towns so that a minimum road length is paved? 10

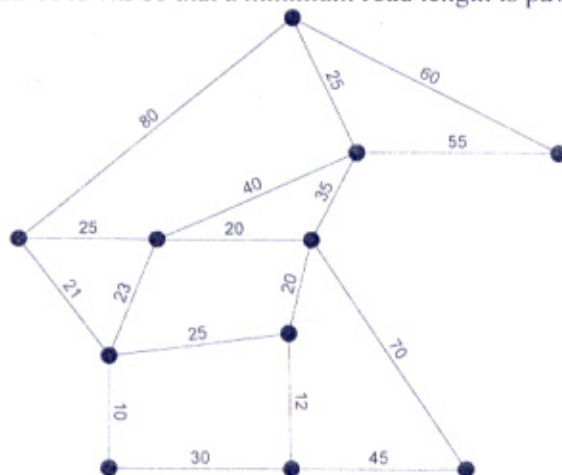


Fig 2. Graph representing towns and connecting roads.

8. a) Consider ten items with frequencies 1,1,3,3,4,5,5,6,7,7. Use Huffman's algorithm to construct an optimal binary code for these ten items. If tie occurs then consider those first with the least height. List the prefix codes for those items. 15
- b) What is eccentricity of a vertex in an unrooted tree? Prove that every tree has either one or two centers. 2+8