ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

MID SEMESTER EXAMINATION **WINTER SEMESTER, 2012-2013**

DURATION: 1 Hour 30 Minutes **FULL MARKS: 75**

CSE 4533: Graph Theory

Programmable calculators are not allowed. Do not write anything on the question paper.

There are **4 (four)** questions. Answer any **3 (three)** of them.

Figures in the right margin indicate marks.

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1. a) Describe applications of graphs in floor-planning and in map coloring. 6
   b) Show that a graph is bipartite if and only if it has no odd cycle. 7
   c) Give an efficient algorithm to determine whether a given graph is bipartite or not. Analyze the time complexity of your algorithm. 6
   d) Describe the following graph operations by illustrative examples. 6
      i. Deletion of a vertex
      ii. Subdividing an edge
      iii. Contraction of an edge

2. a) When do we call two graphs G_1 and G_2 isomorphic? Give an algorithm to check whether two graphs are isomorphic or not. Does the algorithm take polynomial time? Justify. 7
   b) What is a self-complementary graph? Give two examples of self-complementary graphs. 5
   c) Show that every u,v-walk contains an u,v-path. 7
   d) State the degree-sum formula. Show that the number of odd degree vertices of a graph is even. 6

3. a) Let G be a graph with the minimum degree 2. Then show that G contains a cycle. 4
   b) Is the following graph in figure 1 Eulerian? If yes, find out the Eulerian circuit in the graph. 10

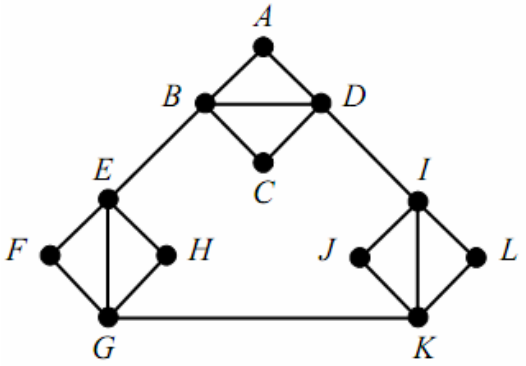

Figure 1: A graph

   c) Let G be a simple graph of n vertices and m edges. If G has k connected components, then show that the number m satisfies the constraint, $n\text{-}k \leq m \leq (n\text{-}k)(n\text{-}k+1)/2$. 7
   d) Construct the complements of C_5 and W_7. 4

4. a) Define a Hamiltonian graph. Show that $K_{n,n}$ is Hamiltonian. 6
   b) Let G be a simple graph of $n \geq 3$ vertices. Show that G is Hamiltonian if $d_G(u) + d_G(v) \geq n$ for every pair of nonadjacent vertices u and v. 10
   c) Describe the structure of the Petersen graph. Show that the Petersen graph has girth 5. 9