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B.Sc. Engg. / HD CSE 4th Semester (52) 29 September 2012 (Morning)

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

SEMESTER FINAL EXAMINATION **SUMMER SEMESTER, 2011-2012**

DURATION: 3 Hours **FULL MARKS: 150**

Math 4407: Statistical and Mathematical Analysis

Programmable calculators are not allowed. Do not write anything on the question paper.

There are **8 (eight)** questions. Answer any 6 **(six)** of them.

Figures in the right margin indicate marks.

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1. a) The least square regression line for a set of n data points is given by $y = ax + b$ where, 10+10

$$a = \frac{n\sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2} \text{ and } b = \frac{1}{n}\left(\sum_{i=1}^{n} y_i - a\sum_{i=1}^{n} x_i\right)$$

i. Find the least square regression line for the points: $\{(-2,1),(1,1),(3,2)\}$. Plot the given points and the regression line in the same rectangular system of axes.

ii. The sales of a company (in million dollars) for five years are shown in the table below

X (year)	2005	2006	2007	2008	2009
Y(sales)	12	19	29	37	45

Find the least square regression line $y = ax + b$. Use the least square regression line as a model to estimate the sales of the company in 2012.

b) Briefly describe three measures of dispersion. 5

2. a) Suppose you draw two cards from a standard deck without replacement. Given that the first card is an ace, what is the probability that the second card is a queen? 5

b) A new car salesperson knows from past experience that she will make a sale to about 20% of her customers. Find the probability that in five (randomly selected) attempts, she makes a sale to 15

i. Exactly three customer
ii. At most one customer
iii. At least one customer

c) Discuss on different types of skewness and kurtosis. 5

3. a) Briefly explain the function of a complex variable. 6

b) Define: 9

i. Polynomial function
ii. Rational function
iii. Harmonic function
iv. Power series

c) Derive Cauchy Riemann equation in polar form. 10

4. a) What is fundamental period and periodic function? 5

b) Discuss on the method to find the Fourier coefficients. 12

c) Find the Fourier series of the function $f(x) = x$ *where* $-\pi \le x \le \pi$. Assume a fundamental period of 2π for $f(x)$. 8

5. a) Discuss on the representation of $f(x)$ as Fourier Integral. 15

b) Find the Fourier sine transform and the Fourier cosine transform for the functions 10

$$f(x)=\begin{cases}1 & 0<x<1\\ -1 & 1<x<2\\ 0 & x\geq 2\end{cases}$$

6. a) If $a=\vec{i}+2\vec{j}+3\vec{k}$, $b=\vec{i}-3\vec{j}+4\vec{k}$, and $c=2\vec{i}+4\vec{j}-2\vec{k}$, then find $a\times(b\times c)$. 10

If $a=2\vec{i}+3\vec{j}-\vec{k}$, $b=\vec{i}-\vec{j}-2\vec{k}$, and $c=\vec{i}+2\vec{j}+2\vec{k}$, then find $(a\times b)\times c$.

b) What is Linear dependence and independence of vectors? Find out whether the following vectors are linearly dependent or independent. 15

i. $V_1=(2,5,3)$, $V_2=(1,1,1)$, *and* $V_3=(4,-2,0)$

ii. $V_1=(4,1,-2)$, $V_2=(-3,0,1)$, *and* $V_3=(1,-2,1)$

7. a) If $\vec{x}=3t\hat{i}-t^2\hat{j}$ and $\vec{y}=2t^2\hat{i}+3\hat{j}$, then verify 10

i. $\frac{d}{dt}(\vec{x}.\vec{y})=\vec{x}.\frac{d\vec{y}}{dt}+\frac{d\vec{x}}{dt}.\vec{y}$

ii. $\frac{d}{dt}(\vec{x}\times\vec{y})=\vec{x}\times\frac{d\vec{y}}{dt}+\frac{d\vec{x}}{dt}\times\vec{y}$

b) Let $\vec{a}$ and $\vec{b}$ be two three-dimensional vectors. Is the following result true? 10

$$\int_{t_1}^{t_2}\vec{a}\,dt\times\int_{t_1}^{t_2}\vec{b}\,dt=\int_{t_1}^{t_2}\vec{a}\times\vec{b}\,dt$$

Proof your opinion.

c) Briefly describe the differentiation and integration of a vector. 5

8. a) What is gradient of a function? Find the gradient of the following functions: 8

i. $f(x,y)=4x^3-3xy+2y^2$

ii. $f(x,y,z)=x^3y-xy^2z+yz^2$

b) Explain divergence and curl of a vector field. 5

c) Find the divergence of the following vector fields 6

i. $F_1=3x\hat{i}+4y\hat{j}$

ii. $F_2=6xy^2\hat{i}-3xy\hat{j}$

iii. $F_3=5x^3y\hat{i}+3xy\hat{j}-4y\hat{k}$

d) Find the curl of the following vector fields: 6

i. $F_1=4x\hat{i}-4y\hat{j}+2xy\hat{k}$

ii. $F_2=3xy\hat{i}+3y\hat{j}$

iii. $F_3=5x^2y\hat{i}+2xy\hat{j}$