B.Sc. Engg. / HD CSE 2nd Semester

07 November 2017

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

SEMESTER FINAL EXAMINATION **SUMMER SEMESTER, 2016-2017**

DURATION: 3 Hours **FULL MARKS: 150**

CSE 4203: Discrete Mathematics

Programmable calculators are not allowed. Do not write anything on the question paper.

There are **8 (eight)** questions. Answer any **6 (six)** of them.

Figures in the right margin indicate marks.

---

1. a) An iterative algorithm for computing Fibonacci numbers is given in Figure 1.

```
procedure iterative fibonacci(n: nonnegative integer)
if n = 0 then return 0
else
        x := 0
        y := 1
        for i := 1 to n - 1
                z := x + y
                x := y
                y := z
        return y
{output is the nth Fibonacci number}
```

Figure 1: Pseudocode for question 1(a).

i. Determine the number of comparisons used by the algorithm for any input n. 6

ii. Determine the complexity of the algorithm from the number of comparisons. Mention the witness values you have used for your calculation 4

b) Differentiate among Big-O, Big-Omega and Big-Theta notations. 9

c) Arrange the functions $\sqrt{n}$, 1000logn, nlogn, 2n!, 2^n, 3^n, $n^2/1000000$ in a list so that each function is big-O of the next function. 6

2. a) Chomp is a two-player game where cookies are laid out on a rectangular grid. The cookie in the top left position is poisoned, as shown in Figure 2(a). The two players take turns making moves; at each move, a player is required to eat a remaining cookie, together with all cookies to the right and/or below it (see Figure 2(b), for example). The loser is the player who has no choice but to eat the poisoned cookie

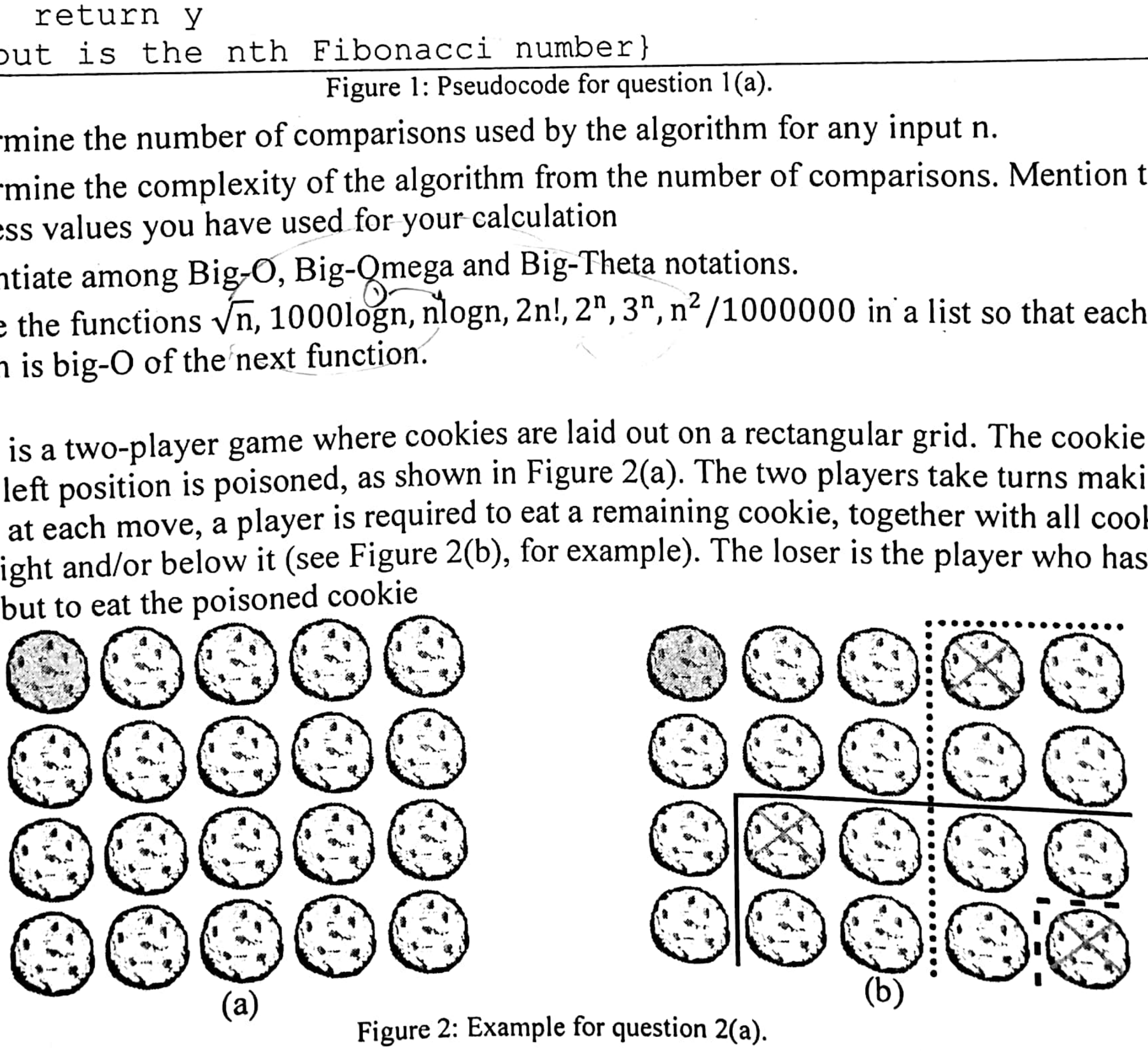

Figure 2: Example for question 2(a).

Using strong induction prove that the first player has a winning strategy if the initial board is two squares wide, that is, a $2 \times n$ board. [Hint: The first move of the first player should be to chomp the cookie in the bottom row at the far right.] 15

b) Prove that $3^n < n!$ if n is an integer greater than 6. 10

a) Use rules of inference to show that the hypotheses "If it does not rain or if it is not foggy, then the sailing race will be held and the lifesaving demonstration will go on," "If the sailing race is held, then the trophy will be awarded," and "The trophy was not awarded" imply the conclusion "It rained." 10

b) Four friends have been identified as suspects for an unauthorized access into a computer system. They have made the following statements to the investigating authorities: Alice said "Carlos did it." John said "I did not do it." Carlos said "Diana did it." Diana said "Carlos lied when he said I did it."
If the authorities also know that exactly one of the four suspects is telling the truth, who did it? Explain your reasoning. 8

c) Show that if n is an integer and $\mathbf{n^3 + 5}$ is odd, then n is even using contraposition. 7

a) Use the back-substitution method to find all solutions to the system of congruences $x \equiv 1 \pmod 2$, $x \equiv 2 \pmod 3$, $x \equiv 3 \pmod 5$, and $x \equiv 4 \pmod{11}$. 10

b) Show that if $a \equiv b \pmod m$ and $c \equiv d \pmod m$, where a, b, c, d, and m are integers with $m \geq 2$, then $a - c \equiv b - d \pmod m$. 8

c) Prove that if n is a composite integer, then n has a prime divisor less than or equal to $\sqrt{n}$. 7

a) Determine how many bit strings of length seven either begin with two 0s or end with three 1s with the help of a tree diagram. 10

b) Derive an equation for determining the number of r-combinations of a set with n elements, where n is a nonnegative integer and r is an integer with $0 \leq r \leq n$ from the equation of r-permutation. 8

c) For each course at a university, there may be one or more other courses that are its prerequisites. How can a graph be used to model these courses and which courses are prerequisites for which courses? Should edges be directed or undirected? Looking at the graph model, how can we find courses that do not have any prerequisites and how can we find courses that are not the prerequisite for any other courses? 7

6. a) Prove that if a and b are positive integers with $a \geq b$, then the number of divisions used by the Euclidean algorithm to find gcd(a, b) is less than or equal to five times the number of decimal digits in b. 12

b) Assume that in a group of six people, each pair of individuals consists of two friends or two enemies. Show that there are either three mutual friends or three mutual enemies in the group. 8

c) Use a merge sort to sort $b, d, a, f, g, h, z, p, o, k$ into alphabetic order. Show all the steps used by the algorithm. 5

a) Suppose that the ciphertext DVE CFMV KF NFEUVI, REU KYRK ZJ KYV JVVU FW JTZVETV was produced by encrypting a plaintext message using a shift cipher. What is the original plaintext? [Hint: the most common letters in English language are E, T, A, O, I, N, S, H, R] 10

b) Discuss the advantages and disadvantages of mathematical induction. 8

c) Represent the following graph with an adjacency list and also an adjacency matrix:

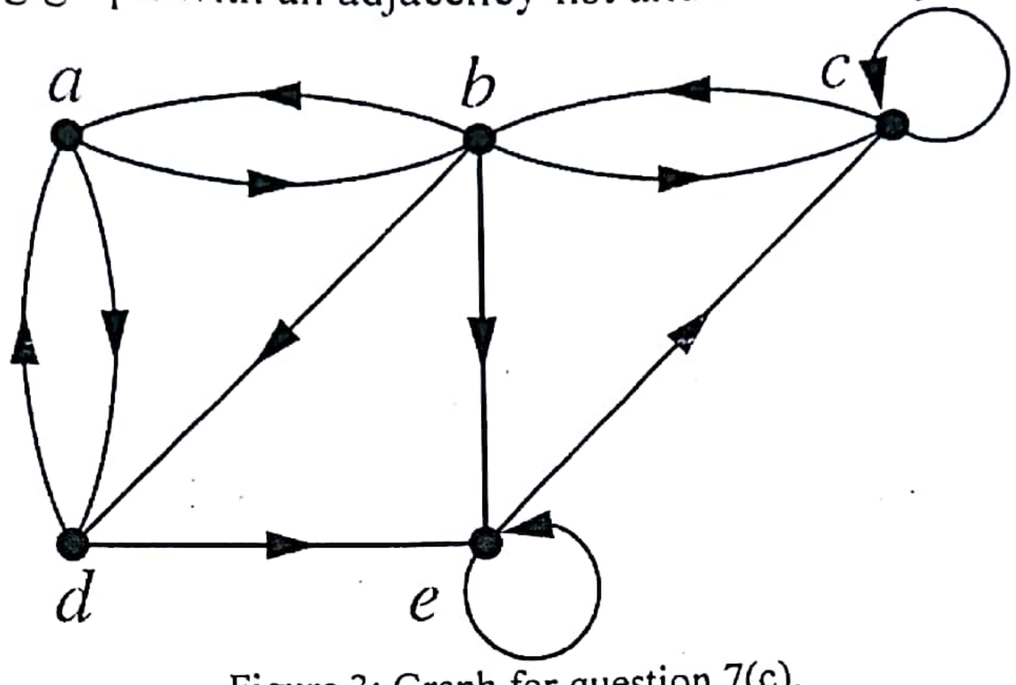

Figure 3: Graph for question 7(c).

8 a) Differentiate between the following: 4
   i. Vertex Cut and Cut Vertices 4
   ii. Edge Cut and Cut Edges
   b) Consider two sets A and B, where the elements of A are the id of 2nd semester students and elements of set B contains age of all students. If function from set A to set B $f(a)=b$ indicates that age of student a is b. 12
   For each of the case bellow whether these functions are one-to-one, onto or bijection. Justify your answer.
   i. More than one student having the same age.
   ii. Each student having unique age.
   iii. Set B contains age of the students of whole university, not only for 2nd year.
   Will this relation still be called a function if set A contains name of a student instead of id? Explain your answer.
   c) Give a recursive definition of a extended binary tree. 5