M2 Quantitative Finance

Pricing European floating-strike lookback options and computing their Greeks using the Monte Carlo method

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Abstract

Lookback options are path-dependent derivatives whose payoff depends on the extreme values attained by the underlying asset over the lifetime of the contract. Their valuation is significantly more challenging than that of standard European options, since the payoff depends on the entire price path rather than solely on the terminal asset value. For European floating–strike lookback options, a closed–form pricing formula is available under the Black–Scholes framework, as derived by Goldman et al. (1979), which provides an exact continuous–time benchmark. In this report, we study the pricing of European floating–strike lookback options using Monte Carlo simulation and compare the numerical results with the exact analytical pricing formula. The underlying asset is modeled as a geometric Brownian motion under the risk–neutral measure. Option sensitivities (Greeks) are computed within the Monte Carlo framework using: finite–difference estimators for Gamma, Theta and Rho and pathwise estimators for Delta and Vega. The numerical analysis highlights the presence of a systematic discretization bias arising from the discrete monitoring of the running extrema. As the time discretization is refined, Monte Carlo prices converge toward the exact analytical value, although at a relatively slow rate. To support the numerical study, the proposed methodology is implemented in a modular and reusable C++ library, complemented by an Excel–VBA interface for practical usage and result visualization.

Keywords: Lookback options, Monte Carlo, Geometric Brownian Motion, Greeks, Finite difference estimator, Pathwise estimator.

Index

1 Introduction

Lookback options are path–dependent exotic derivatives whose payoff depends not only on the terminal value of the underlying asset, but also on the extreme values (minimum or maximum) attained by the asset price over the life of the contract. At maturity, a lookback call grants the holder the right to purchase the underlying asset at the minimum price observed during the option's lifetime, while a lookback put allows the holder to sell the asset at the maximum recorded price.

In contrast to standard European options, whose payoff depends solely on the terminal price, lookback options incorporate information from the entire price trajectory between issuance and maturity. This path dependence implies a higher degree of optionality, which typically results in higher option premiums compared to vanilla contracts.

Closed–form pricing formulas for lookback options exist only under restrictive modeling assumptions and are often limited to idealized settings. In particular, under the Black–Scholes framework with continuous monitoring, exact pricing formulas derived by Goldman et al. (1979) are available and provide a natural benchmark for numerical methods. In addition to option prices, risk management and hedging applications require the accurate computation of option sensitivities, commonly referred to as Greeks, which measure the response of the option value to changes in key model parameters.

Given these challenges, numerical methods play a central role in the practical valuation of lookback options. In this project, we estimate European floating–strike lookback option prices using Monte Carlo simulation, which enables direct sampling of asset price paths under the assumed dynamics and naturally accommodates path–dependent payoffs. Alongside pricing, we compute the main Greeks using two complementary approaches: *finite difference estimators* employed for Gamma, Theta and Rho, while *pathwise estimators* are used for Delta and Vega.

Beyond the numerical results, a central contribution of this work is the development of a modular and reusable C++ library that implements flexible Monte Carlo pricing routines for lookback options. In addition, an Excel–based pricer implemented via VBA macros is developed to facilitate practical usage, parameter experimentation, and result visualization.

2 Risk Neutral Pricing

In this project, we focus on the problem of pricing and hedging derivative securities in a *risk–neutral world*. This setting is characterized by the absence of arbitrage opportunities and by market completeness, two fundamental assumptions that ensure the existence and uniqueness of arbitrage–free prices for financial derivatives. Under these conditions, derivative valuation does not depend on individual risk preferences, but rather on the probabilistic structure of the underlying asset prices.

The Black–Scholes model provides a canonical and mathematically tractable framework in which these ideas can be formalized. In this setting, the market is composed of two basic traded assets: a non–risky asset, representing a money market account, and a risky asset, representing the underlying security. The dynamics of these assets are specified below.

Definition 2.1 (Non-risky asset)**.** In the Black–Scholes market, the non–risky asset $(S_t^0)_{0\le t\le T}$ is given by:

$$S_t^0 = S_0^0 e^{rt}, \qquad \forall\, 0 \le t \le T, \tag{1}$$

where r denotes the instantaneous risk–free interest rate of the market. In particular, $(S_t^0)_{t\ge 0}$ satisfies the following differential equation:

$$\begin{cases} dS_t^0 = rS_t^0\, dt, \\ S_0^0 = 1. \end{cases} \tag{2}$$

Definition 2.2 (Risky-asset)**.** In the Black–Scholes model, the risky asset $(S_t)_{0\le t\le T}$ is given by:

$$S_t = S_0 \exp\left(\left(\mu - \tfrac{1}{2}\sigma^2\right) t + \sigma W_t\right), \qquad \forall\, 0 \le t \le T, \tag{3}$$

where μ denotes the drift, σ is the volatility, and W_t is a standard Brownian motion. In this framework, both μ and σ are assumed to be constant over time. In particular, S_t is the unique solution to the stochastic differential equation (SDE) given by the expression:

$$\begin{cases} dS_t = \mu S_t\, dt + \sigma S_t\, dW_t, \\ S_0 = S_0 \in \mathbb{R}. \end{cases} \tag{4}$$

Let $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\ge 0}, \mathbb{P})$ be a filtered probability space supporting the Brownian motion $(W_t)_{t\ge 0}$. Under the no arbitrage assumption and market completeness, there exists a unique *risk–neutral probability measure* under which the discounted risky asset is a martingale. *Girsanov's Theorem* allows us to construct this measure by performing a change of probability from the physical measure $\mathbb{P}$ to $\mathbb{Q}$ in such a way that the drift of the process $(S_t)_{t\ge 0}$ becomes equal to the risk–free rate r. This change of measure, the discounted price process $\widetilde{S}_t = S_t/S_t^0 = e^{-rt}S_t$ becomes a martingale, which is precisely the condition required to rule out arbitrage opportunities. Consequently, in the risk–neutral world we set $\mu = r$.

In this setting, the value of a derivative with payoff $f(St_0, \ldots, S_{t_m})$ at maturity time $T = t_m$ is given by:

$$V_t = e^{-r(T-t)}\mathbb{E}^{\mathbb{Q}}[\, f(S_{t_0}, \ldots, S_{t_m}) \,|\, \mathcal{F}_t] \,, \tag{5}$$

where the expectation is taken under the risk–neutral measure $\mathbb{Q}$, also called the *martingale measure*, and $\mathcal{F}_t$ denotes the information available up to time t.

3 Discretization scheme

The valuation framework introduced in Section 2 requires the computation of expectations under the risk–neutral measure, as expressed in equation (5). Since the underlying asset price is modeled by the stochastic differential equation (4), and closed–form solutions are generally unavailable for path–dependent payoffs, numerical approximation methods are required in order to simulate sample paths of the process.

A standard and widely used approach is based on time discretisation of the continuous–time dynamics. In this work, we adopt the *Euler–Maruyama scheme*, which provides a simple yet effective method for approximating solutions of stochastic differential equations. The time interval $[0, T]$ is partitioned into n subintervals according to the uniform grid:

$$0 = t_0 < t_1 < \cdots < t_m = T, \qquad t_i = \frac{iT}{n},\ 0 \leq i \leq n. \tag{6}$$

The underlying idea of the Euler–Maruyama method is to approximate the infinitesimal increments of the stochastic process over each small time interval by freezing the drift and diffusion coefficients at the beginning of the interval. Applied to the risk–neutral dynamics (3), this yields the following recursive approximation:

$$S_{i+1} = S_i + rS_i(t_{i+1} - t_i) + \sigma S_i \sqrt{t_{i+1} - t_i}\, Z_{i+1}, \tag{7}$$

where S_0 denotes the known initial asset price and $(Z_i)_{i\geq 1}$ is a sequence of independent standard normal random variables. Under a uniform discretisation ($\Delta t = t_{i+1} - t_i$), the previous scheme simplifies to:

$$S_{i+1} = S_i + rS_i\, \Delta t + \sigma S_i \sqrt{\Delta t}\, Z_{i+1}. \tag{8}$$

This discretisation allows us to generate Monte Carlo sample paths of the underlying asset under the risk–neutral measure, from which the expected discounted payoff of the derivative can be approximated. In particular, it provides a natural framework for pricing path–dependent derivatives such as lookback options, whose payoff depends on the extrema attained by the simulated paths.

Approximating a continuous–time stochastic process by a discrete–time scheme inevitably introduces a discretisation error. The quality of the approximation can be assessed through different notions of convergence, most notably *strong* and *weak* convergence, which capture distinct aspects of the numerical accuracy.

Strong convergence measures how close the simulated paths are to the true solution of the stochastic differential equation, path by path. The following classical result describes the strong convergence rate of the Euler–Maruyama scheme.

Theorem 3.1 (Strong convergence of the Euler–Maruyama scheme)**.** *Let the drift $b : \mathbb{R}^d \to \mathbb{R}^d$ and diffusion coefficient $\sigma : \mathbb{R}^d \to \mathbb{R}^{d\times m}$ be globally Lipschitz. Let $(X_t)_{0\le t\le T}$ be the strong solution of the SDE and $(X_t^{(n)})_{0\le t\le T}$ its Euler–Maruyama approximation with time step $\Delta t = T/n$. Then, for every $p \ge 2$, there exists a constant $C_{p,T} > 0$ such that*

$$\left(\mathbb{E}\left[\,|X_T - X_T^{(n)}|^p\right]\right)^{1/p} \le C_{p,T}\,(\Delta t)^{1/2}. \tag{9}$$

*Thus, the Euler–Maruyama method has **strong order of convergence** 1/2.*

On the other hand, *weak convergence* measures how well the expected payoff is approximated, which is exactly what determines an option's price.

Theorem 3.2 (Weak convergence of the Euler–Maruyama scheme)**.** *Assume that the drift b and diffusion coefficient σ are C^4 with bounded derivatives, and let $f : \mathbb{R}^d \to \mathbb{R}$ be a smooth function with polynomial growth. Let (X_T) denote the solution of the SDE at time T and $(X_T^{(n)})$ the Euler–Maruyama approximation with time step Δt. Then there exists a constant $C > 0$ such that*

$$\Big|\mathbb{E}\big[f(X_T^{(n)})\big] - \mathbb{E}\big[f(X_T)\big]\Big| \le C\,(\Delta t). \tag{10}$$

*Hence, under these regularity assumptions, the Euler–Maruyama method has **weak order of convergence** 1.*

In practice, *strong convergence* is relevant when the goal is to obtain pathwise accurate simulations. In contrast, *weak convergence* plays a central role in derivative pricing: the Monte Carlo estimator of the discounted payoff depends only on the distribution of S_T, not on the precise shape of the simulated paths.

For the Euler–Maruyama scheme, the classical result is that the strong order is 1/2, while the weak order is typically 1, provided that the drift, diffusion and payoff function are sufficiently smooth. However, in many derivative pricing problems, such as lookback options, the payoff involves maxima or minima of the underlying path, which introduces non-smoothness. In such cases, the weak order of Euler may deteriorate, causing a slower decay of the discretization error unless additional correction techniques are employed.

4 Monte Carlo Simulation

Monte Carlo methods are used to value complex financial derivatives, that is, those for which it is very difficult or even impossible to apply an analytical formula such as the one introduced by Black and Scholes (1973). In particular, Glasserman (2003) explains that these methods are typically employed for derivatives whose payoff depends on the path of the underlying asset, or in cases involving multiple underlying assets. This is precisely the case of lookback options, which are the focus of this work.

Monte Carlo simulation allows us to estimate integrals in one or more dimensions easyly. Let us consider the problem of estimating the integral of a Lebesgue integrable function, $f \in L^2(0,1)$, over the unit interval $[0,1]$. Then, we can express this integral as an expectation:

$$\mathbb{E}[f(X)] = \int_{[0,1]} f(x)\,dx, \tag{11}$$

with X uniformly distributed on $[0,1]$. This can be extended to the unit cube $[0,1]^d$ in d dimensions as follows:

$$\mathbb{E}[f(\mathbf{X})] = \int_{[0,1]^d} f(x)\,dx, \tag{12}$$

where $\mathbf{X}$ is a vector random variable uniformly distributed on $[0,1]^d$.

Let us focus on the one-dimensional case. If we sample points X_i independently, uniformly, and at random from $[0,1]$, then we obtain that the classical Monte Carlo estimator associated with the integral (11) is:

$$\widehat{\mathbb{E}}[f(X)] = \frac{1}{n}\sum_{i=1}^{n} f(X_i). \tag{13}$$

Then, by the *Strong Law of Large Numbers*, we have that with probability 1:

$$\frac{1}{n}\sum_{i=1}^{n} f(X_i) \xrightarrow[n\to\infty]{} \int_{[0,1]} f(x)\,dx. \tag{14}$$

Moreover, if we define the variance of f by:

$$\sigma_f^2 = \int_{[0,1]} \left(f(x) - \mathbb{E}[f(x)]\right)^2 dx, \tag{15}$$

then, the Monte Carlo approximation introduces an error term $e_n(f) = \widehat{\mathbb{E}}[f(X)] - \mathbb{E}[f(X)]$, which is normally distributed, with mean 0 and standard deviation $\sigma_f/\sqrt{n}$, i.e. $e_n \approx \mathcal{N}\left(0, \sigma_f^2/n\right)$.

In practice, the unknown parameter σ_f^2 is estimated by the empirical variance of the sample, which yields the following unbiased estimator:

$$s_f = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(f(X_i) - \widehat{\mathbb{E}}[f(X)]\right)^2}. \tag{16}$$

Then, by the *Central Limit Theorem* (CLT), for a sequence of independent and identically distributed random variables with finite variance, the properly normalized sample mean converges in distribution to a normal random variable. In particular,

$$\sqrt{n}\,(\widehat{a}_n - a) \xrightarrow[n\to\infty]{d} \mathcal{N}(0, \sigma_f^2), \tag{17}$$

where $a = \mathbb{E}[f(X)]$ and $\sigma_f^2 = \mathrm{Var}(f(X))$.

Therefore, we can conclude that the standard error of the Monte Carlo method has the following form:

$$e_n(f) = \left| \frac{1}{n} \sum_{i=1}^{n} f(X_i) - \int_{[0,1]^d} f(x)\, du \right| \leq \frac{\sigma_f}{\sqrt{n}} \xi, \tag{18}$$

where $\xi \sim \mathcal{N}(0,1)$. The latter expression shows that Monte Carlo method converges at the rate $\mathcal{O}(n^{-1/2})$, independently of the dimension of the problem. This is the main advantage of Monte Carlo methods and makes them particularly well suited for high–dimensional problems, such as the valuation of lookback options.

The path-dependent nature of lookback options makes Monte Carlo simulation a natural and flexible tool for numerical valuation. The payoff of a lookback option depends on the running minimum or maximum of the underlying process, then the expectation cannot be expressed in closed form in most settings. Monte Carlo simulation approximates this expectation by generating many sample paths of the underlying asset price and averaging the corresponding discounted payoffs. In summary, we use the following simple algorithm, presented in Table 1, to price a derivative security in risk neutral world.

Table 1: Derivative security pricing in risk-neutral world by Monte Carlo.

Pricing algorithm by Monte Carlo
1. Simulate the dynamics of the underlying asset using the Euler scheme.
2. Calculate the payoff of the derivative security on each path.
3. Discount the payoff at the risk–free rate.
4. Compute the average over all simulated paths.

5 Pricing lookback Options

Lookback options are a class of *path-dependent* exotic derivatives that can be of several types, depending on whether the strike is **fixed** or **floating** and whether the payoff is based on the running **minimum** or **maximum** of the underlying asset. Let S_t denote the price of the underlying asset for $t \in [0,T]$, and let:

$$S_{\min} = \min_{0 \leq t \leq T} S_t, \qquad S_{\max} = \max_{0 \leq t \leq T} S_t,$$

denote the running minimum and maximum of the asset price over the lifetime of the option.

We first introduce *fixed-strike lookback options*, in which the strike price is specified at inception and remains constant throughout the contract's life.

Definition 5.1 (Fixed-Strike lookback Options)**.** A **fixed-strike lookback option** is a derivative whose payoff depends on the best historical underlying's price relative to a fixed strike K.

- **Fixed-strike lookback call:**

$$\text{payoff} = \max(S_{\max} - K,\, 0)\,. \tag{19}$$

The holder benefits from the highest price reached during the option's life.

- **Fixed-strike lookback put:**

$$\text{payoff} = \max(K - S_{\min},\, 0)\,. \tag{20}$$

The holder benefits from the lowest price observed over $[0, T]$.

An alternative specification is provided by *floating-strike lookback options*, where the strike price is not fixed in advance but is instead determined endogenously by the extreme value of the underlying asset price.

Definition 5.2 (Floating-Strike lookback Options)**.** A **floating-strike lookback option** is a derivative whose strike is determined by the extremum of the underlying price during the life of the contract.

- **Floating-strike lookback call:**

$$\text{payoff} = S_T - S_{\min}. \tag{21}$$

The holder buys at the lowest price observed over the interval $[0, T]$.

- **Floating-strike lookback put:**

$$\text{payoff} = S_{\max} - S_T. \tag{22}$$

The holder sells at the highest price reached during the life of the option.

These contracts automatically incorporate the most favourable historical price, removing the uncertainty associated with choosing an optimal exercise time. As the payoff depends on the entire trajectory of the underlying, lookback options are inherently path-dependent, making closed-form analytical pricing expressions difficult or impossible to obtain under most models. This path dependence motivates the use of *Monte Carlo simulation* as a natural and flexible numerical tool for pricing lookback options within the Black–Scholes framework.

In this project, we focus on European floating-strike lookback options and price them using Monte Carlo simulation. The numerical results are then compared with the closed-form solution derived by Goldman et al. (1979) for European floating-strike lookback options under the continuous-time Black–Scholes model.

5.1 Discretization error reduction

In the case of lookback options, a fundamental challenge arises from the fact that the payoff depends on the running minimum $S_{\min}$ or the running maximum $S_{\max}$ of the underlying asset over the entire interval $[0, T]$.

In practice, Monte Carlo methods simulate the price process on a discrete time grid:

$$0 = t_0 < t_1 < \cdots < t_m = T, \qquad t_i = i\,\Delta t, \quad \Delta t = \frac{T}{m}, \tag{23}$$

and generate the sequence of approximated values $S_0, S_1, \ldots, S_m$, where each S_i denotes either the exact or Euler–Maruyama approximation of S_{t_i}. A natural discrete–time estimator of the running extrema is then defined as:

$$\widehat{S}_{\max} = \max\{S_0, S_1, \ldots, S_m\}, \qquad \widehat{S}_{\min} = \min\{S_0, S_1, \ldots, S_m\}. \tag{24}$$

However, even if the simulation produced exact values of the asset price at the grid points, this procedure would *not* recover the true running extremum. Since the asset price evolves continuously between two consecutive points t_i and t_{i+1}, it is entirely possible that the true maximum (or minimum) is attained at some time $t \in (t_i, t_{i+1})$. In that case,

$$\widehat{S}_{\max} < S_{\max}, \qquad \widehat{S}_{\min} > S_{\min}, \tag{25}$$

which introduces a systematic bias in the Monte Carlo estimator of the payoff.

In the case of a floating-strike lookback call with payoff $S_T - S_{\min}$, the discrete-time approximation overestimates $S_{\min}$ and therefore *underestimates* the option price. Conversely, for a floating-strike lookback put with payoff $S_{\max} - S_T$, the discretization procedure underestimates $S_{\max}$, again leading to an *underestimation* of the true option value. This "discretization bias" is an inherent consequence of monitoring the running extremum only at discrete time points.

In order to reduce this bias, one must approximate the maximum and minimum attained *within* each interval $[t_i, t_{i+1}]$, rather than only at the grid points. Andersen (1996) proposed a a method to reduce this error. However, the implementation of such bias–reduction techniques lies beyond the scope of the present project.

5.2 Monte Carlo

The valuation of lookback options within the risk–neutral framework generally requires numerical methods, due to the path–dependent nature of their payoffs. Among the available numerical techniques, Monte Carlo simulation is particularly well suited to this setting, as it allows for the direct generation of sample paths of the underlying asset price and the evaluation of payoffs that depend on the entire trajectory of the process.

Using the crude Monte Carlo method, the price of the floating-strike lookback put at time t may be estimated as:

$$\widehat{P}_{\text{float}}(t) = e^{-r(T-t)}\Big(\max\{S_i : i = 0, 1, \dots, m\} - S_m\Big), \tag{26}$$

where $\{S_i\}_{i=0}^{m}$ denotes the simulated trajectory of the asset price on a discrete time grid. This approach relies solely on the discrete observations of the process and therefore suffers from the discretization bias discussed in the previous section. For each simulated path, the algorithm proceeds as shown in Table 2.

Table 2: Monte Carlo estimation of the price of a floating-strike lookback option under the risk-neutral measure.

1. Initialise the asset price at S_0 and set

$$S_{\min} = S_{\max} = S_0.$$

2. Evolve the price over n time steps using a discretisation of the Black–Scholes stochastic differential equation.
3. Update the running minimum or maximum at each step:

$$S_{\min} \leftarrow \min(S_{\min}, S_t), \qquad S_{\max} \leftarrow \max(S_{\max}, S_t).$$

4. Compute the payoff at maturity according to the option type.
5. Discount the payoff by e^{-rT}.

Repeating this procedure over a large number of simulated paths yields a consistent estimator of the option's fair value.

5.3 Exact Pricing Formula

Goldman et al. (1979) proposed a closed–form pricing formulas for European floating–strike lookback options in the Black–Scholes model. We first present the pricing formula for a *floating–strike lookback call*, given by:

$$c_{\text{fl}} = S_0\left[N(a_1) - \frac{\sigma^2}{2r}N(-a_1)\right] - S_{\min}e^{-rT}\left[N(a_2) - \frac{\sigma^2}{2r}e^{Y_1}N(-a_3)\right], \tag{27}$$

where

$$a_1 = \frac{\ln\left(\frac{S_0}{S_{\min}}\right) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}, \tag{28}$$

$$a_2 = a_1 - \sigma\sqrt{T}, \tag{29}$$

$$a_3 = \frac{\ln\left(\frac{S_0}{S_{\min}}\right) + \left(-r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}, \tag{30}$$

$$Y_1 = -\frac{2\left(r - \frac{1}{2}\sigma^2\right)\ln\left(\frac{S_0}{S_{\min}}\right)}{\sigma^2}. \tag{31}$$

Here, $S_{\min}$ is the minimum asset price observed up to date (if the lookback is newly issued, then $S_{\min} = S_0$).

We now turn to the pricing formula for a *floating–strike lookback put*, which is the following:

$$p_{\text{fl}} = -S_{\max}e^{-rT}\left[N(b_1) - \frac{\sigma^2}{2r}e^{Y_2}N(-b_3)\right] + S_0\left[\frac{\sigma^2}{2r}N(-b_2) - N(b_2)\right]. \tag{32}$$

where the parameters b_1, b_2 and b_3 are defined as:

$$b_1 = \frac{\ln\left(\frac{S_{\max}}{S_0}\right) + \left(-r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}, \tag{33}$$

$$b_2 = b_1 - \sigma\sqrt{T}, \tag{34}$$

$$b_3 = \frac{\ln\left(\frac{S_{\max}}{S_0}\right) + \left(r - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}, \tag{35}$$

$$Y_2 = \frac{2\left(r - \frac{1}{2}\sigma^2\right)\ln\left(\frac{S_0}{S_{\max}}\right)}{\sigma^2}. \tag{36}$$

Here, $S_{\max}$ is the maximum asset price observed up to date (if the lookback is newly issued, then $S_{\max} = S_0$).

6 Computation of Greeks

After pricing a derivative security, it is essential to manage the risks associated with the resulting position. One way to eliminate such risks is to construct a dynamic trading strategy that replicates the payoff of the derivative. For instance, the exposure of a short position in a derivative with price V can be hedged by holding Δ units of the underlying asset, where Δ denotes the partial derivative of the option price with respect to the current value of the underlying. This procedure is known as *delta-hedging*.

Therefore, the construction of hedging strategies relies on the computation of sensitivities of the derivative price with respect to the underlying asset and to the relevant model parameters. These sensitivities are collectively known as the *Greeks*. In this context, the Greeks play a fundamental role in describing how the value of an option responds to small changes in the variables on which it depends, making them indispensable tools for risk analysis and risk management.

Greeks measure the sensitivity of the option price with respect to key model parameters. Table 3 reports the main option Greeks, their analytical expressions, and their typical values.

Table 3: Main option Greeks and their possibble values.The option price is denoted by V, S represents the underlying asset price, t is the time to maturity, σ the volatility parameter, and r the (continuously compounded) risk-free interest rate. Source: Own elaboration.

Greek	Expression	Values
Delta (Δ_t)	$\frac{\partial V}{\partial S}$	Call: $0 \leq \Delta \leq 1$ Put: $-1 \leq \Delta \leq 0$
Gamma (Γ_t)	$\frac{\partial^2 V}{\partial S^2}$	$\Gamma \geq 0$
Theta (Θ_t)	$\frac{\partial V}{\partial t}$	$\Theta \leq 0$
Vega (ν_t)	$\frac{\partial V}{\partial \sigma}$	$\nu \geq 0$
Rho (ρ_t)	$\frac{\partial V}{\partial r}$	Call: $\rho \geq 0$ Put: $\rho \leq 0$

6.1 Finite-difference method

The *finite-difference methods* constitute a standard numerical approach for approximating these derivatives, since the pathwise method cannot be applied in the case of lookback options due to the nonsmooth nature of their payoff, which depends on the running maximum or minimum of the underlying asset.

Let $Y(\Theta)$ be a random variable with respect to the parameter of interest $\Theta \in \mathbb{R}$. We define the expectation of $Y(\Theta)$ as:

$$\alpha(\Theta) = \mathbb{E}[Y(\Theta)]. \tag{37}$$

The goal of the finite-difference method is to estimate the derivative of $Y'(\Theta)$ with respect to Θ, that is:

$$Y'(\Theta) = \frac{d}{d\Theta}\mathbb{E}[Y(\Theta)]. \tag{38}$$

In the context of option pricing, $Y(\Theta)$ represents the **discounted payoff** of an option, $Y'(\Theta)$ corresponds to the **option price**, and Θ denotes a **parameter** influencing that price. Depending on the choice of Θ, the derivative $\alpha'(\Theta)$ yields a different Greek.

The *forward difference* estimator approximates the derivative by using simulations at two nearby points, Θ and $\Theta + h$, where $h > 0$ is a small increment. To construct this estimator, we first generate n independent simulations of the model at the parameter value Θ, denoted by $Y_1(\Theta), \ldots, Y_n(\Theta)$. Similarly, we generate another n simulations at the parameter value $\Theta + h$, denoted by $Y_1(\Theta + h), \ldots, Y_n(\Theta + h)$. From these simulations, we compute the corresponding sample averages:

$$\bar{Y}_n(\Theta) = \frac{1}{n}\sum_{i=1}^{n} Y_i(\Theta) \quad \text{and} \quad \bar{Y}_n(\Theta + h) = \frac{1}{n}\sum_{i=1}^{n} Y_i(\Theta + h).$$

The *forward difference* estimator is then defined as:

$$Y'_F(\Theta, h) = \frac{\bar{Y}(\Theta + h) - \bar{Y}(\Theta)}{h}. \tag{39}$$

If the function α is twice differentiable at Θ, then the Taylor expansion of $\alpha(\Theta + h)$ is given by:

$$\alpha(\Theta + h) = \alpha(\Theta) + \alpha'(\Theta)\, h + \tfrac{1}{2}\alpha''(\Theta)\, h^2 + o(h^2). \tag{40}$$

Substituting the Taylor expansion (40) into the expression for the forward-difference estimator (39), we obtain that the **expected value** of the forward-difference estimator satisfies:

$$\mathbb{E}[Y'_F(\Theta, h)] = \frac{\alpha(\Theta + h) - \alpha(\Theta)}{h} = \alpha'(\Theta) + \tfrac{1}{2}\alpha''(\Theta)\, h + o(h). \tag{41}$$

Consequently, the **bias** of the forward-difference estimator is given by:

$$\mathrm{Bias}\big(Y'_F(\Theta, h)\big) = \mathbb{E}[Y'_F(\Theta, h) - \alpha'(\Theta)] = \tfrac{1}{2}\alpha''(\Theta)\, h + o(h), \tag{42}$$

which shows that the forward-difference estimator is biased of order $O(h)$.

The overall accuracy of the forward-difference estimator depends not only on its bias but also on its variance. Since the estimator involves the difference of two Monte Carlo sample means divided by h, its variance behaves as:

$$\mathrm{Var}\big(Y'_F(\Theta, h)\big) = O\left(\frac{1}{nh^2}\right), \tag{43}$$

while the squared bias is of order $O(h^2)$. Therefore, the mean squared error (MSE) of the estimator takes the form:

$$\mathrm{MSE}(h, n) = O(h^2) + O\left(\frac{1}{nh^2}\right). \tag{44}$$

Minimising the MSE with respect to h yields the optimal choice $h \sim n^{-1/2}$, which balances the truncation error and the Monte Carlo variance. Substituting this optimal step size into the expression for the MSE gives the convergence rate:

$$\mathrm{RMSE}(Y'_F) = O(n^{-1/4}), \tag{45}$$

which is the standard rate of convergence for forward finite-difference estimators in Monte Carlo sensitivity analysis.

The *central difference* estimator. In this case, the function is evaluated at two symmetric points around Θ, namely $\Theta - h$ and $\Theta + h$, where $h > 0$ denotes a small perturbation. To apply this method, we perform n simulations at each of the two perturbed values, obtaining two independent sets of outcomes:

$$Y_1(\Theta - h), \ldots, Y_n(\Theta - h) \quad \text{and} \quad Y_1(\Theta + h), \ldots, Y_n(\Theta + h). \tag{46}$$

The sample means of both sets are then computed in the same manner as in the forward-difference case. The resulting *central difference* estimator is given by:

$$Y_C'(\Theta, h) = \frac{\bar{Y}(\Theta + h) - \bar{Y}(\Theta - h)}{2h}. \tag{47}$$

The *central difference* estimator can be extended to compute second-order derivatives. In this case, the estimator takes the following form:

$$Y_C''(\Theta, h) = \frac{\bar{Y}_n(\Theta + h) - 2\bar{Y}_n(\Theta) + \bar{Y}_n(\Theta - h)}{h^2}, \tag{48}$$

If the function α is at least twice differentiable at Θ, then a Taylor expansion can be obtained around this point. The Taylor series for $Y(\Theta + h)$ was already given in (40), then, similarly, the expansion for $Y(\Theta - h)$ is:

$$Y(\Theta - h) = Y(\Theta) - Y'(\Theta)\, h + \tfrac{1}{2} Y''(\Theta)\, h^2 + o(h^2). \tag{49}$$

Substituting the Taylor expansions (40) and (49) into the expression for the central-difference estimator (47), the **expected value** of the estimator is:

$$\mathbb{E}\big[Y_C'(\Theta, h)\big] = \frac{Y(\Theta + h) - Y(\Theta - h)}{2h} = \frac{2Y'(\Theta)\, h + o(h^2)}{2h} = Y'(\Theta) + o(h^2). \tag{50}$$

Consequently, the **bias** of the central-difference estimator satisfies:

$$\mathrm{Bias}\big(Y_C'(\Theta, h)\big) = \mathbb{E}\big[Y_C'(\Theta, h) - Y'(\Theta)\big] = o(h^2), \tag{51}$$

showing that the central-difference estimator has a smaller truncation error and is asymptotically unbiased of order $o(h)$.

The overall accuracy of the cetral-difference estimator depends not only on its bias but also on its variance. Since the estimator involves the difference of two Monte Carlo sample means divided by h, its variance behaves as:

$$\mathrm{Var}\big(Y_F'(\Theta, h)\big) = O\left(\frac{1}{nh^2}\right), \tag{52}$$

while the squared bias is of order $O(h^4)$.

Therefore, the mean squared error (MSE) of the estimator is given by:

$$\mathrm{MSE}(h, n) = O(h^4) + O\left(\frac{1}{nh^2}\right). \tag{53}$$

Minimising the MSE with respect to h leads to the optimal choice $h \sim n^{-1/6}$, which balances the truncation error and the Monte Carlo variance. Substituting this optimal value into the expression for the MSE yields the convergence rate:

$$\mathrm{RMSE}(Y_F') = O(n^{-1/3}), \tag{54}$$

which is the standard rate associated with finite-difference estimators of option sensitivities in Monte Carlo simulation.

In this project, we adopt the central difference estimator for computing Greeks, as it provides a more accurate approximation by reducing the bias inherent in forward-difference methods. Let $V(\cdot)$ denote the Monte Carlo estimator of the option price. The Greeks are computed as shown in Table 4.

Table 4: Finite-difference estimators for the Greeks of a derivative price V.

Greek	**Central-difference estimator**
Delta (Δ)	$\Delta \approx \dfrac{V(S_0+h) - V(S_0-h)}{2h}$
Gamma (Γ)	$\Gamma \approx \dfrac{V(S_0+h) - 2V(S_0) + V(S_0-h)}{h^2}$
Theta (Θ)	$\Theta \approx \dfrac{V(T+h_T) - V(T-h_T)}{2h_T}$
Vega (ν)	$\nu \approx \dfrac{V(\sigma+h_\sigma) - V(\sigma-h_\sigma)}{2h_\sigma}$
Rho (ρ)	$\rho \approx \dfrac{V(r+h_r) - V(r-h_r)}{2h_r}$

6.2 Pathwise method

The *pathwise method* is presented as an alternative to the finite–difference approach described previously, as it allows the estimation of derivatives directly, without the need to simulate multiple values of the parameter. The main difference with respect to the previous method is that the derivative of $\alpha(\Theta) = \mathbb{E}[Y(\Theta)]$ is estimated directly using the definition:

$$Y'(\Theta) = \lim_{h \to 0} \frac{Y(\Theta+h) - Y(\Theta)}{h}. \tag{55}$$

This estimator has expectation $\mathbb{E}[Y'(\Theta)]$ and is an **unbiased** estimator of $\alpha'(\Theta)$ provided that

$$\mathbb{E}\left[\frac{dY(\Theta)}{d\Theta}\right] = \frac{d}{d\Theta}\mathbb{E}\left[Y(\Theta)\right], \tag{56}$$

that is, given that a set of conditions ensuring the interchangeability of the differentiation and expectation operators are satisfied.

Now, we derive the pathwise estimators for each Greek by relying on the interchangeability condition in (56). Let S_t, $t \in [0, T]$, denote the price of the underlying asset, and define the running extrema:

$$S_{\min} = \min_{0\le t\le T} S_t, \qquad S_{\max} = \max_{0\le t\le T} S_t. \tag{57}$$

We consider floating–strike lookback options with payoff functional Φ given by:

$$\Phi^{\text{call}} = S_t - S_{\min}, \qquad \Phi^{\text{put}} = S_{\max} - S_t. \tag{58}$$

Let $Y(\Theta) = e^{-rT}\,\Phi(\Theta)$ denote the discounted payoff, where the dependence on the parameter Θ is induced through the asset price process S_t. Whenever the regularity assumptions required by (56) are satisfied, we may write:

$$\frac{\partial V}{\partial \Theta} = \frac{\partial}{\partial \Theta}\,\mathbb{E}[Y(\Theta)] = \mathbb{E}\left[\frac{\partial Y(\Theta)}{\partial \Theta}\right], \tag{59}$$

so that a Monte Carlo estimator of $\partial_\Theta V$ is obtained by averaging the pathwise derivatives $\partial_\Theta Y(\Theta)$ along the simulated trajectories.

6.2.1 Delta

Let S_0 denote the initial asset price, which follows a GBM. From the explicit solution of the GBM dynamics, it follows that:

$$\frac{\partial S(t)}{\partial S_0} = \frac{S(t)}{S_0}. \tag{60}$$

Since the running minimum and maximum are almost surely attained at a unique time under a continuous diffusion, we have:

$$\frac{\partial S_{\min}}{\partial S_0} = \frac{S_{\min}}{S_0}, \qquad \frac{\partial S_{\max}}{\partial S_0} = \frac{S_{\max}}{S_0}. \tag{61}$$

Therefore, for both the floating–strike call and put, the pathwise derivative of the payoff satisfies:

$$\frac{\partial \Phi}{\partial S_0} = \frac{\Phi}{S_0}, \tag{62}$$

and the Delta estimator is given by:

$$\Delta = \frac{\partial V}{\partial S_0} = \mathbb{E}\left[e^{-rT}\,\frac{\Phi}{S_0}\right]. \tag{63}$$

6.2.2 Vega

In order to derive the pathwise estimator for Vega, we differentiate the asset price with respect to the volatility parameter σ. Define:

$$D_t := \frac{\partial S_t}{\partial \sigma}. \tag{64}$$

Differentiating the explicit solution of the GBM dynamics yields:

$$D_t = S_t \left(W(t) - \sigma t\right). \tag{65}$$

Using again the almost sure uniqueness of the time at which the extrema are attained, we obtain

$$\frac{\partial S_{\min}}{\partial \sigma} = D_{t_{\min}}, \qquad \frac{\partial S_{\max}}{\partial \sigma} = D_{t_{\max}}, \tag{66}$$

where $t_{\min}$ and $t_{\max}$ denote the times at which $S_{\min}$ and $S_{\max}$ are reached, respectively.

Consequently, the pathwise Vega of the floating–strike call is given by:

$$\nu_{\text{call}} = \frac{\partial V^{\text{call}}}{\partial \sigma} = \mathbb{E}\big[e^{-rT}\left(D_T - D_{t_{\min}}\right)\big]\,, \tag{67}$$

while for the floating–strike put we obtain:

$$\nu_{\text{put}} = \frac{\partial V^{\text{put}}}{\partial \sigma} = \mathbb{E}\big[e^{-rT}\left(D_{t_{\max}} - D_T\right)\big]\,. \tag{68}$$

6.2.3 Gamma

The pathwise method is not well suited for the estimation of second–order derivatives. While the payoff of a floating–strike lookback option is almost surely differentiable with respect to model parameters, it is typically not *twice* differentiable in the classical sense once the payoff involves running extrema computed from the path. Intuitively, the following map:

$$(S(t))_{t\in[0,T]} \longmapsto S_{\min} = \min_{0\le t\le T} S(t) \qquad \text{or} \qquad (S(t))_{t\in[0,T]} \longmapsto S_{\max} = \max_{0\le t\le T} S(t) \tag{69}$$

behaves like a piecewise function: small perturbations of the path can change the time at which the extremum is attained, which prevents the existence of stable second derivatives. Therefore, the interchangeability condition

$$\frac{d^2}{dS_0^2}\mathbb{E}[\Phi] = \mathbb{E}\left[\frac{d^2\Phi}{dS_0^2}\right] \tag{70}$$

does not hold, preventing the use of pathwise differentiation for Gamma. For this reason, Gamma is computed using a finite–difference estimator.

6.2.4 Theta and Rho

The computation of Θ and ρ is also performed via finite differences. The reasons are: first, the pathwise approach requires differentiability of the discounted payoff with respect to the parameter of interest; and second, even when a formal pathwise derivative exists, the resulting estimator may exhibit high variance.

Theta is defined as the sensitivity of the option value with respect to the maturity T,

$$\Theta := \frac{\partial V}{\partial T}, \qquad V_T = \mathbb{E}\big[e^{-rT}\,\Phi_T\big]\,, \tag{71}$$

where Φ_T denotes the payoff functional evaluated over the horizon $[0,T]$. For lookback options, changing T modifies the time interval over which the running extrema are computed, i.e.,

$$S_{\min}(T) = \min_{0\le t\le T} S(t), \qquad S_{\max}(T) = \max_{0\le t\le T} S(t), \tag{72}$$

so that an infinitesimal variation of T may alter the attained minimum or maximum with strictly positive probability. This makes the pathwise method unsuitable for Theta. Therefore, we estimate Theta using a central finite–difference approximation.

Rho measures the sensitivity of the option value with respect to the risk–free rate r:

$$\rho := \frac{\partial V_0}{\partial r}, \qquad V_T = \mathbb{E}\big[e^{-rT}\,\Phi(r)\big]\,. \tag{73}$$

Under the GBM assumption, the pathwise derivative of the asset price with respect to r is:

$$\frac{\partial S(t)}{\partial r} = t\,S(t). \tag{74}$$

For a floating–strike lookback option, whose payoff can be written as:

$$\Phi^{\text{call}} = S(T) - S_{\min}, \qquad \Phi^{\text{put}} = S_{\max} - S(T), \tag{75}$$

the pathwise derivative of the payoff with respect to r involves contributions from the terminal value as well as from the running extremum. Denoting by $t_{\min}$ and $t_{\max}$ the (random) times at which the minimum and maximum are attained, respectively, we obtain:

$$\frac{\partial \Phi^{\text{call}}}{\partial r} = TS(T) - t_{\min}S_{\min}, \qquad \frac{\partial \Phi^{\text{put}}}{\partial r} = t_{\max}S_{\max} - TS(T). \tag{76}$$

In addition, differentiating the discount factor yields:

$$\frac{\partial}{\partial r}e^{-rT} = -Te^{-rT}. \tag{77}$$

Combining these terms, the pathwise derivative of the discounted payoff is given by:

$$\frac{\partial Y}{\partial r} = e^{-rT}\Big(\frac{\partial \Phi}{\partial r} - T\Phi\Big). \tag{78}$$

Although this expression provides an unbiased pathwise estimator for ρ, it involves several terms that scale linearly with the maturity T and depend on the entire trajectory of the underlying asset. In Monte Carlo simulations, these features typically result in a high–variance estimator and slow convergence. For this reason, Rho is computed using a finite–difference approach.

Table 5: Pathwise estimators for the Greeks of an European floating-strike lookback option.

Greek	**Pathwise estimator**
Delta (Δ)	$\mathbb{E}\left[e^{-rT}\frac{\Phi}{S_0}\right]$
Vega Call (ν^{call})	$\mathbb{E}\left[e^{-rT}\left(D_T - D_{t_{\min}}\right)\right]$
Vega Put (ν^{put})	$\mathbb{E}\left[e^{-rT}\left(D_{t_{\max}} - D_T\right)\right]$

7 Results

In this section, we present the numerical results obtained by applying the Monte Carlo method to the pricing of floating-strike lookback options.

First, Table 6 reports the results of a crude Monte Carlo estimation of European floating–strike lookback call and put options for increasing levels of time discretization. The parameters are fixed at $S_0 = 100$, $r = 0.05$, $\sigma = 0.25$, and $T = 1$, and all estimates are based on 10^6 Monte Carlo simulations. The exact prices are computed using the closed–form formulas of Goldman et al. (1979) and are used as benchmarks.

Table 6 clearly illustrates the impact of time discretization on pricing accuracy. For both call and put options, the Monte Carlo estimates systematically underestimate the exact prices when the running extrema are monitored only at discrete time points. As the number of time steps increases, the discretization grid becomes finer, leading to a progressive reduction in the absolute pricing error and convergence toward the exact continuous–time price.

Table 6: Crude Monte Carlo estimation of European floating-strike lookback options for increasing time discretization.

	CALL			**PUT**		
Time-steps	**Exact Price**	**MC Price**	**Abs. Error**	**Exact Price**	**MC Price**	**Abs. Error**
20	20.5522	18.1489	2.403330	18.7233	15.1868	3.536440
40	20.5522	18.8145	1.737710	18.7233	16.1539	2.569420
80	20.5522	19.3197	1.232440	18.7233	16.8712	1.852120
160	20.5522	19.6604	0.891809	18.7233	17.3839	1.320420
320	20.5522	19.9248	0.627416	18.7233	17.7782	0.945078

Table 7 reports the estimated Greeks for European floating–strike lookback call options as a function of the number of time steps used in the Monte Carlo discretization. Several patterns emerge from the results. First, all sensitivities exhibit a clear and monotonic convergence as the temporal resolution increases, which confirms the numerical stability and consistency of the estimation procedure. In particular, **Delta** gradually increases and stabilizes around 0.20, indicating that coarse time discretizations tend to underestimate the true sensitivity of the option price with respect to the underlying asset. Similarly, **Theta** converges towards more negative values, approaching approximately 10, which is consistent with the time decay typically observed for path-dependent options. The behavior of **Rho** suggests a mild downward adjustment as the number of time steps grows, implying that interest-rate sensitivity is slightly overestimated when using rough temporal grids. In contrast, **Vega** displays a systematic increase and stabilizes, highlighting the importance of fine discretization for accurately capturing the effect of volatility in exotic derivatives. Finally, **Gamma** remains effectively zero across all discretization levels, which is coherent with the payoff structure of floating–strike lookback options, whose dependence on the underlying price is largely linear, resulting in negligible convexity.

Table 7: Estimated Greeks for the European floating-strike lookback call for different numbers of time-steps per path.

Time-steps	**Delta**	**Gamma**	**Theta**	**Rho**	**Vega**
20	0.181489	0	-9.50412	45.2833	57.986
40	0.188145	0	-9.77305	44.8745	60.1427
80	0.193197	0	-9.95179	44.5349	61.8034
160	0.196604	0	-10.0857	44.3458	62.8880
320	0.199248	0	-10.164	44.1773	63.7175

Table 8 reports the estimated Greeks for European floating–strike lookback put options as a function of the number of time steps used in the Monte Carlo discretization. The results show that first–order sensitivities, such as **Delta** and **Vega**, converge smoothly as the number of time steps increases, indicating that these Greeks are relatively robust to discretization effects. The magnitudes of **Theta** and **Rho** also converge more gradually, requiring finer discretizations to achieve stable estimates.

Table 8: Estimated Greeks for the European floating-strike lookback put for different numbers of time-steps per path.

Time-steps	**Delta**	**Gamma**	**Theta**	**Rho**	**Vega**
20	0.151868	0	-6.63431	-51.0335	73.4314
40	0.161539	0	-7.16868	-51.5597	77.9921
80	0.168712	0	-7.57713	-51.9811	81.3380
160	0.174029	0	-7.86353	-52.1710	83.8418
320	0.177820	0	-8.08683	-52.4082	85.6198

The behavior of **Delta** and **Gamma** can be explained by the structural properties of the floating–strike lookback payoff. Under the geometric Brownian motion model, the underlying price process admits the multiplicative representation:

$$S_t = S_0 X_t, \qquad X_t = \exp\big((r - \tfrac{1}{2}\sigma^2)t + \sigma W_t\big)\,, \tag{79}$$

where the stochastic factor X_t is independent of the initial spot price S_0. As a consequence, the entire price path scales linearly with S_0.

For a floating–strike lookback put option, the payoff at maturity is given by:

$$\Phi(S_0) = \max_{0\le t\le T} S_t - S_T. \tag{80}$$

Substituting the multiplicative form of S_t yields:

$$\Phi(S_0) = S_0\left(\max_{0\le t\le T} X_t - X_T\right) = S_0\,\Phi(1), \tag{81}$$

which shows that the payoff is homogeneous of degree one with respect to the initial spot price.

The option price under the risk–neutral measure therefore satisfies:

$$V(S_0) = e^{-rT}\,\mathbb{E}[\Phi(S_0)] = S_0\, e^{-rT}\,\mathbb{E}[\Phi(1)]\,, \tag{82}$$

and is thus linear in S_0. Differentiating with respect to the spot price immediately gives:

$$\Delta = \frac{\partial V}{\partial S_0} = e^{-rT}\,\mathbb{E}[\Phi(1)]\,, \qquad \Gamma = \frac{\partial^2 V}{\partial S_0^2} = 0. \tag{83}$$

These analytical properties are fully confirmed by the numerical results reported in Tables 7 and 8. The estimated Gamma is identically zero across all discretization levels, while Delta remains stable as the number of time steps increases and does not depend on the spot level.

Overall, the results confirm that while Monte Carlo estimators for prices and first–order Greeks converge reliably with increasing time resolution, higher–order sensitivities require finer discretizations to achieve stable and accurate estimates. This highlights the importance of carefully balancing computational cost and discretization accuracy when computing Greeks for path–dependent options, such as lookback options.

8 Conclusions

In this project, European floating–strike lookback options were priced under the Black–Scholes framework using Monte Carlo simulation. Option sensitivities (Greeks) are computed within the Monte Carlo framework using: finite–difference estimators for Gamma, Theta and Rho and pathwise estimators for Delta and Vega.

An exact continuous–time pricing formula derived by Goldman et al. (1979) was used as a benchmark throughout the analysis. The numerical experiments clearly confirm the presence of a systematic discretization bias when the running extrema of the underlying asset are monitored only at discrete time points. As expected, refining the time grid progressively reduces this bias and leads to convergence toward the exact analytical price. However, the convergence rate is relatively slow, highlighting the intrinsic difficulty of pricing lookback options using naive discretization schemes.

The computed Greeks exhibit stable behavior and increasing consistency as the time discretization is refined, supporting the practical reliability of the implemented numerical methodology. While first–order sensitivities converge smoothly, higher–order Greeks require finer discretizations to achieve stable estimates.

Beyond the computation of prices and sensitivities, a central contribution of this work is the development of a reusable and modular C++ codebase. In addition, an Excel–based pricer implemented via VBA macros was developed, enabling practical usage, parameter interaction, and result visualization.

Overall, the Monte Carlo framework developed in this project provides a robust and extensible tool for the valuation of path–dependent options. While closed–form solutions exist under idealized assumptions, the numerical and software infrastructure presented here is well suited for more complex stochastic models and derivative payoffs for which analytical pricing formulas are not available. As future work, the framework could be extended by incorporating variance reduction techniques to improve numerical efficiency.

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