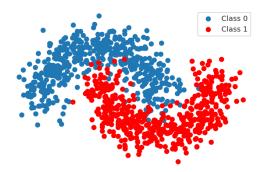




Predicting Smart Grid Stability with Probabilistic Machine Learning Models.

Baurice Nafack Fatma Moustafa July 18, 2022



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 - 'stab': if positive, the system is linearly unstable; if negative, linearly stable.
 - 'stabf': a categorical (binary) label ('stable' or 'unstable').

Gaussian processes for classification: Regression recap

• A Gaussian process (GP) for regression is a random process where any point $x \in \mathbb{R}^d$ is assigned a random variable f(x) and where the joint distribution of a finite number of these variables $p(f(x_1), ..., f(x_N))$ is itself Gaussian:

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- A GP is a prior over functions whose shape (smoothness, ...) is defined by $K = \kappa(X, X)$ where κ is a parametric kernel function.
- Given observed noisy function values y at points X we want to predict a noise-free function value f** at point x**. The joint distribution of observed values y and prediction f** is also a Gaussian:

Gaussian processes for classification: Regression recap

$$p(y, f_* \mid X, x_*) = \mathcal{N}\left(\begin{pmatrix} y \\ f_* \end{pmatrix} \middle| 0, \begin{pmatrix} K_y & k_* \\ k_*^T & k_{**} \end{pmatrix}\right)$$
(2)

where $K_y = K + \sigma_y^2 I$, $k_* = \kappa(X, x_*)$ and $k_{**} = \kappa(x_*, x_*)$. σ_y^2 models noise in the observed function values y.

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• Turning the joint distribution (2) into a conditional distribution we obtain a predictive distribution:

$$p(f_* \mid \mathsf{x}_*, \mathsf{X}, \mathsf{y}) = \mathcal{N}(f_* \mid \boldsymbol{\mu}_*, \boldsymbol{\Sigma}_*) \tag{3}$$

with

$$\mu_* = k_*^T K_y^{-1} y$$

$$\Sigma_* = k_{**} - k_*^T K_y^{-1} k_*$$
(4)

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- The predictive distribution can be defined as:

$$p(t_* = 1 \mid t, x_*, X) = \int p(t_* = 1 \mid f_*) p(f_* \mid t, x_*, X) df_* \quad (5)$$

$$p(f_* \mid t, x_*, X) = \int p(f_* \mid f, x_*, t) p(f \mid t, X) df$$
 (6)

Where p(f | t, X) can be approximated with a Gaussian distribution q(f) using the Laplace approximation, minimising the KL divergence.

$$q(f) = \mathcal{N}(f \mid \hat{f}, H^{-1}) \tag{7}$$

where $H = W + K_f^{-1}$. W is a diagonal matrix with elements $\sigma(f_n)(1 - \sigma(f_n))$ with f_n being the elements of f. Written in vector notation the diagonal is $\sigma(1 - \sigma)$.

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• The mean \hat{f} can be obtained iteratively with the following update equation:

$$f^{\text{new}} = K_f (I + WK_f)^{-1} (t - \sigma + Wf)$$
 (8)

At convergence $\hat{f} = f^{\text{new}}$.

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• The Gaussian approximation of $p(f_* \mid t)$ is therefore given by

$$p(f_* \mid \mathbf{t}) \approx \mathcal{N}(f_* \mid \mu_{f_*}, \sigma_{f_*}^2)$$
 (9)

with

$$\mu_{f_*} = \mathsf{k}_*^{\mathsf{T}} (\mathsf{t} - \boldsymbol{\sigma}) \sigma_{f_*}^2 = \mathsf{k}_{**} - \mathsf{k}_*^{\mathsf{T}} (\mathsf{W}^{-1} + \mathsf{K}_f)^{-1} \mathsf{k}_*$$
 (10)

Finally, $p(t_* = 1 \mid f_*)$ in Equation (4) is approximate with the inverse probit function $\Phi(f_*)$ so that the predictive distribution can be approximated with:

$$p(t_* = 1 \mid t) \approx \sigma(\mu_{f_*}(1 + \pi \sigma_{f_*}^2/8)^{-1/2})$$
 (11)

Gaussian Naive Bayes

• It is a Naive Bayes classifier with the likelihood of the features is assumed to be Gaussian:

..................

$$P(x_i \mid C) = \frac{1}{\sqrt{2\pi\sigma_C^2}} \exp\left(-\frac{(x_i - \mu_C)^2}{2\sigma_C^2}\right)$$
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• The parameters $\sigma_{\it C}$ and $\mu_{\it C}$ are estimated using maximum likelihood



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- Family of distributions
- Find an approximate posterior distribution from this family that has the lowest KL divergence from the true posterior.

Stochastic Variational Inference

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• KL divergence: Similarity between the posterior distribution and approximate posterior distribution

Stochastic Variational Inference

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- KL divergence: Similarity between the posterior distribution and approximate posterior distribution
- Inference \rightarrow Optimization problem
- Objective: minimize the KL-divergence from the approximate posterior to the true posterior



Variational Inference

$$KL[q_{\phi}(z)||p(z|D)] = \int q_{\phi}(z) \log \frac{q_{\phi}(z)}{p(z|D)}$$

$$= \mathbb{E}_{q_{\phi}(z)} [\log q_{\phi}(z) - \log p(z|D)]$$
(13)

Stochastic Variational Inference

 We can't directly optimize (minimize) the KL divergence, instead we optimize (maximize) the ELBO: Evidence Lower Bound with respect to the variational parameters

$$\begin{aligned} \text{ELBO} &\equiv \mathbb{E}_{q_{\phi}(\mathsf{z})} \left[\log p_{\theta}(\mathsf{z}, \mathsf{D}) - \log q_{\phi}(\mathsf{z}) \right] \end{aligned} \tag{14}$$

$$\mathbb{E}_{q_{\phi}(\mathsf{z})} \left[\log q_{\phi}(\mathsf{z}) - \log p(\mathsf{z}|\mathsf{D}) \right] = \mathbb{E}_{q_{\phi}(\mathsf{z})} \left[\log p_{\theta}(\mathsf{z}, \mathsf{D}) - \log q_{\phi}(\mathsf{z}) \right] + \\ \left[\log p(\mathsf{D}) \right] \end{aligned}$$

(15)

$$KL ext{ divergence} = -ELBO + Evidence$$
 (16)

Bayesian Neural Networks

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Artificial Neural Network: train network parameters as point estimates

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Model uncertainty in network parameters

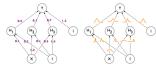
Bayesian Neural Networks

- Artificial Neural Network: train network parameters as point estimates
- Bayesian Neural Network:
 - Model uncertainty in network parameters

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Prior over parameters represented with probability distribution

- Artificial Neural Network: train network parameters as point estimates
- Bayesian Neural Network:
 - Model uncertainty in network parameters
 - Prior over parameters represented with probability distribution
 - Posterior of parameters given dataset is approximated using the stochastic variational inference



Weights in ANN Vs. BNN

Experimental Methods

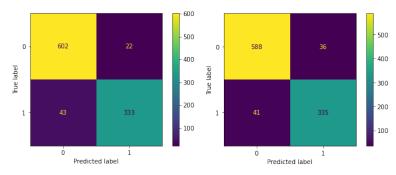
 we split the data into test data, training data, and validation data to train the BNN method and others. The validation data will be used to track the bias-variance tradeoff in our models.

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- we split the data into test data, training data, and validation data to train the BNN method and others. The validation data will be used to track the bias-variance tradeoff in our models.
- To train the Gaussian Process classifier, we select the first 1000 samples and the last 1000 samples for the test. the same test data will be used to evaluate all of our models.
- We perform Cross-Validation to choose the best kernel for the Gaussian process classification.

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a) RBF kernel b) Rational Quadratic kernel

Figure: Confusion matrix

Gaussian Process Performance Metrics

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		precision	recall	f1-score	support
	unstable	0.93	0.96	0.95	624
Best Mean Accuracy: 0.937 Best Config: {'kernel': 1**2 * RationalOuadratic(alpha=1, length scale=1)}	stable	0.94	0.89	0.91	376
>0.628 with: {'kernel': 1**2 * R8F(length_scale=1)} >nan with: {'kernel': 1**2 * DotProduct(sigma 0=1)}	accuracy			0.94	1000
>0.905 with: {'kernel': 1**2 * Matern(length_scale=1, nu=1.5)}	macro avg	0.94	0.93	0.93	1000
>0.937 with: {'kernel': 1**2 * RationalQuadratic(alpha=1, length_scale=1)} >0.628 with: {'kernel': 1**2 * WhiteKernel(noise level=1)}	weighted avg	0.94	0.94	0.93	1000

a) Cross-Validation b)Classification report using RBF kernel

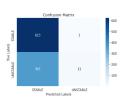
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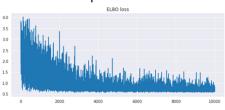
c)Classification report using RQ kernel

Bayesian Logistic Regression Result

Classificatio	n Report: precision	recall	f 1-score	support
1 0	0.9167 0.6306	0.0293 0.9984	0.0567 0.7730	376 624
accuracy macro avg weighted avg	0.7736 0.7381	0.5138 0.6340	0.6340 0.4148 0.5036	1000 1000 1000

a) Classification Report





b)Confusion Matrix

c) ELBO loss

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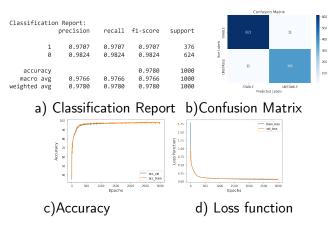
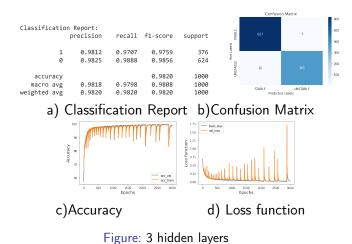


Figure: 1 hidden layer



Parameters of Bayesian Neural Network

```
model = nn.Sequential(
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                                                                                                           bnn.BayesLinear(prior mu=0, prior signa=0.1, in features=12, out features=198),
    bnn.BayesLinear(prior mu=0, prior sigma=0.1, in features=12, out features=100),
                                                                                                           bnn.Bayeslinear(prior mu=0, prior sigma=0.1, in features=100, out features=100).
    bnn.BayesLinear(prior mu=0, prior sigma=0.1, in features=100, out features=2),
                                                                                                            bon.BayesLinear(prior mu=0, prior sigma=0.1, in features=100, out features=50),
                                                                                                           bnn.BavesLinear(prior mu+8, prior sigma+8.1, in features=58, out features=2),
ce loss = nn.CrossEntropyLoss()
kl_loss = bnn.BKLLoss(reduction='mean', last_layer_only=False)
                                                                                                        ce_loss = nn.CrossEntropyLoss()
kl weight = 0.01
                                                                                                        kl_loss = bnn.EKLloss(reductions'mean', last_layer_only:False)
                                                                                                        kl weight = 0.01
optimizer = optim.Adam(model.parameters(), lr=0.005)
                                                                                                        optimizer = optim.Adam(model.parameters(), lr=0.005)
```

a) 1 hidden layer

b) 3 hidden layers

Figure: BNN parameters

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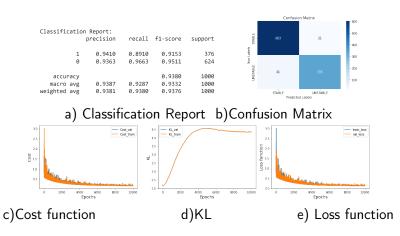
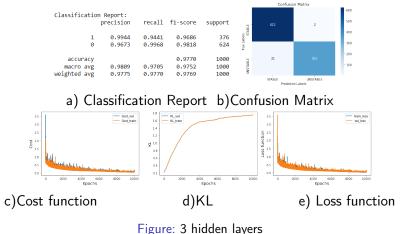


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Comparison of Predictive Accuracy using Different Techniques

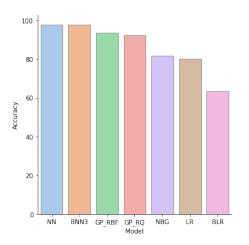


Figure: Comparison of Predictive Accuracy using Different Techniques

Comparison of Predictive Accuracy using Different Techniques

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Table: Comparison of Predictive Accuracy using Different Techniques

Prediction model	Test accuracy %
GP RBF	93.5
GP RQ	92.3
NN with one hidden layer	97.8
NN with 3 hidden layer	98.2
BNN with one hidden layer	(90.5, 95.3)
BNN with 3 hidden layer	(95.2, 98.6)
NB Gaussian	81.7
LR	80
BLR	63.4

Comparison of Predictive Accuracy using Different KL weight

Table: Comparison of predictive accuracy for different KL weight (inspire from Bowman et al. 2016 work) using BNN with 1 hidden layers.

KL weight	Test accuracy %	epoch
1	(37.6, 74.7)	3000
0.5	(58.3, 77.5)	3000
0.05	(85.5, 93.0)	3000
0.01	(82.8, 92.4)	3000
0.01	(90.5, 95.3)	10000

 For higher values of KL weight, we obtained a decreasing KL value. However, we get a low accuracy and the model tends to overfit.

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- For higher values of KL weight, we obtained a decreasing KL value. However, we get a low accuracy and the model tends to overfit
- As we decrease the KL weight, we get an increasing KL value.
 The accuracy gets better, and our model learns from the data.

Comparison of Predictive Accuracy to previous work

The following table done by Paulo Breviglieri 2021 present the comparison of prediction accuracy using different techniques.

Prediction method	Accuracy (%)	Used system
XGBoost [23]	97.8	IEEE 39-bus test system
Bayesian rate (BR) [24]	91.6	IEEE 39-bus test system
DT [25]	90.3	IEEE 39-bus test system
CNN [26]	89.22	IEEE 118-bus and IEEE 145-bus systems
Long Short Term Memory (LSTM) [27]	99.98	IEEE 39-bus test system and 162 and 145 bus systems
FNN-based transient stability predictor [28]	99.2	IEEE 39-bus test system
CART [4]	80.0	4-node star architecture
Proposed	99.62	4-node star architecture

Conclusion and future work

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 The comparative analysis shows that the Bayesian Neural Networks performs well on the test data compared to other probabilistic machine learning models. The comparative analysis shows that the Bayesian Neural Networks performs well on the test data compared to other probabilistic machine learning models.

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- The Gaussian processes method is not as computationally expensive as other learning methods, however, it does not scale well with large datasets.
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- The Gaussian processes method is not as computationally expensive as other learning methods, however, it does not scale well with large datasets.
- The hyperparameters in the Bayesian Neural Networks plays an important role on how the model learn the data. Analysis of hyperparameters tuning would give more insight on the power of Bayesian Neural Networks.
- Implementation could be done on an HPC system which could help in obtaining reliable and more accurate results.



Bowman, Samuel R. et al. (2016). "Generating Sentences from a Continuous Space". In.



Paulo Breviglieri Türkücan Erdem, Süleyman Eken (2021). "Predicting Smart Grid Stability with Optimized Deep Models".

In: SN Computer Science, pp. 293-298. DOI:

https://doi.org/10.1007/s42979-021-00463-5.

