Path Integrals Homework

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Submit your homework here by June 15. Credit will be given for a good-faith attempt to solve these problems.

1 Path Integral Derivation of Euclidean Propagator

In the path integral formalism the propagator for a free non-relativistic particle in imaginary time is

$$K(q_f, \beta\hbar, q_i, 0) = \lim_{N\to\infty} \left(\frac{m}{2\pi\Delta\tau\hbar}\right)^{N/2} \int dq_1 dq_2 \cdots dq_{N-1} \exp\left[-\Delta\tau\hbar \sum_{j=1}^{N} \frac{m}{2}\left(\frac{q_j - q_{j-1}}{\Delta\tau\hbar}\right)^2\right] . \tag{1}$$

(a) Show that the propagator can be rewritten as

$$K(q_f, \beta\hbar, q_i, 0) = \lim_{N\to\infty} \left(\frac{m}{2\pi\Delta\tau\hbar}\right)^{N/2} \left(\frac{2\Delta\tau\hbar}{m}\right)^{\frac{N-1}{2}} \int dy_1 dy_2 \cdots dy_{N-1} \exp\left[-\sum_{j=1}^{N} (y_j - y_{j-1})^2\right] . \tag{2}$$

(b) Use induction to show that

$$\int dy_1 dy_2 \cdots dy_{N-1} \exp\left[-\sum_{j=1}^{N} (y_j - y_{j-1})^2\right] = \left(\frac{\pi^{N-1}}{N}\right)^{1/2} e^{-(y_N - y_0)^2/N} . \tag{3}$$

(c) Take the continuum limit $N \to \infty$ to show

$$K(q_f, \beta\hbar, q_i, 0) = \sqrt{\frac{m}{2\pi\beta\hbar^2}} \exp\left(-\frac{m(q_f - q_i)^2}{2\beta\hbar^2}\right) . \tag{4}$$

(d) What is/are the classical path(s) $\bar{q}(\tau)$?

(e) Compute the Euclidean action of each of the classical paths you found in the previous part.

(f) Comment on your results.

(g) Who did you collaborate with on this assignment? (Colloboration is encouraged, but you must acknowledge your collaborators.)