KAUST

CEMSE151 - Linear Algebra

Problem Set 3

To be returned by October, 5th, 2023, 5:00pm

September 20, 2023

Part I

From the book of Strang, Gilbert. Introduction to Linear Algebra. 4th ed. Wellesley, MA: Wellesley-Cambridge Press, February 2009. ISBN: 9780980232714.

1. from section 3.1: problems 18, 20 and 23 (same numbers in the 5th edition);
2. from section 3.2: problems 2, 16, 18 and 24 (problems 2, 11, 13 and 18 in the 5th edition);
3. from section 3.3: problems 7, 19 and 25 (problems 38, 50 and 56 in section 3.2 of the 5th edition);
4. from section 3.4: problems 13, 18 and 25 (same numbers in section 3.3 of the 5th edition).

Part II

1. Describe geometrically the column space and the nullspace of the following matrices:
$$A_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}.$$
2. Give an example of a matrix A such that its $N(A)$ is $a[2 \quad 0 \quad 1]^T, a \in \mathbb{R}$.
3. Consider the following four matrices:

a) $A = \begin{bmatrix} 3 & 0 & -6 & 0 \\ 1 & 0 & -2 & 0 \end{bmatrix}$;

b) $A = \begin{bmatrix} -1 & 3 & 0 & 2 \\ 0 & 2 & 2 & 0 \\ -1 & 3 & 0 & 2 \end{bmatrix}$;

c) $A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$;

d) $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$.

For each of these matrices, determine if the system $Ax = 0$ has no solution, has infinitely many solutions or has a unique solution. Also, determine if $Ax = b$ has at least one solution for an arbitrary b or if $Ax = b$ has no more than one solution for an arbitrary b.