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Spring 2018

AMCS 255: Advanced Computational Physics 2018 @ KAUST
Exercise Sheet 1: Variational Principles

Exercise 1 *(Double Pendulum with Lagrange, 5 + 5 + 2 + 2 = 14 Points)*

Consider a double pendulum in a gravitational field/frictionless environment as illustrated in Fig. 1.

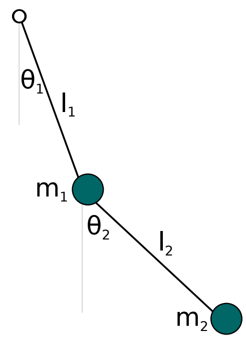

Figure 1: Double pendulum with particle masses m_1, m_2 and stiff rods with lengths l_1, l_2.

1. Derive the second order equations of motion using appropriate generalized coordinates and the Euler-Lagrange formalism. (Hint: A solution sketch can be found on the lecture slides.)
2. Implement a simple forward Euler scheme to solve the previously derived equations of motion numerically.
3. Plot phase space diagrams and visualize the motion dynamically.
4. Run different experiments with varying temporal step sizes and discuss your results.

Exercise 2 *(*Fundamental Lemma of Calculus of Variations, 5 Bonus Points)*

Prove that for a continuous function $f : \mathbb{R} \supseteq [t_i, t_f] \ni t \mapsto f(t) \in \mathbb{R}$ with

$$\int_{t_i}^{t_f} f(t)\alpha(t)\,\mathrm{d}t = 0$$

for all twice continuously differentiable functions $\alpha : \mathbb{R} \supseteq [t_i, t_f] \ni t \mapsto a(t) \in \mathbb{R}$ with $\alpha(t_i) = \alpha(t_f) = 0$, holds $f(t) = 0$ for all $t \in [t_i, t_f]$.

Notes

- For submissions and in case you have any questions about the assignments, please contact Dr. Dmitry A. Lyakhov <dmitry.lyakhov@kaust.edu.sa> or Professor Dominik L. Michels <dominik.michels@kaust.edu.sa> directly via email.
- Office hours are directly after the lecture or by appointment.
- KAUST expects both instructors and students to respect and follow the policies of the university.