KAUST

CEMSE151 - LINEAR ALGEBRA

PROBLEM SET 4

To be returned by October, 19th, 2023, 5:00pm

October 5, 2023

PART I

The following problems are taken from the book of Strang, Gilbert. Introduction to Linear Algebra. 4th ed. Wellesley, MA: Wellesley-Cambridge Press, February 2009. ISBN: 9780980232714.

1. from section 3.5: problems 2, 3, 8 and 20; (these correspond to the problems with the same number in section 3.4, from the 5th edition);

2. from section 3.6: problems 6, 11, 24, 28 and 30; (these correspond to the problems with the same number in section 3.5, from the 5th edition, except that problem 28 (4th) corresponds to 27 (5th) and problem 30 (4th) corresponds to 29 (5th));

3. from section 4.1: problems 6, 7, 9, 11, 14, 21, 32 and 33; (these correspond to the problems with the same number in section 4.1, from the 5th edition).

PART II

1. Find if the following sets of vectors are linearly independent or dependent:

   (a) in $\mathbb{R}^3$, $\left\{ [1 \quad 1 \quad 2]^T, [1 \quad 2 \quad 1]^T, [3 \quad 1 \quad 1]^T \right\}$;

   (b) in $\mathbb{R}^3$, $\left\{ [1 \quad 1 \quad 0]^T, [1 \quad 0 \quad 0]^T, [0 \quad 1 \quad 1]^T, [x \quad y \quad z]^T \right\}$, with x, y and z arbitrary real numbers;

   (c) in $\mathbb{R}^n$, $\left\{ [1 \quad 0 \quad 0 \quad \dots \quad 0]^T, [0 \quad 1 \quad 0 \quad \dots \quad 0]^T, \dots, [0 \quad 0 \quad 0 \quad \dots \quad 1]^T \right\}$.

2. Determine the dimension and basis for $C(A)$ and $C(A^T)$ for the next four A matrices (try to guess if possible):

(a)
$$A = \begin{bmatrix} 3 & 0 & -6 & 0 \\ 1 & 0 & -2 & 0 \end{bmatrix};$$

(b)
$$A = \begin{bmatrix} -1 & 3 & 0 & 2 \\ 0 & 2 & 2 & 0 \\ -1 & 3 & 0 & 2 \end{bmatrix};$$

(c)
$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix};$$

(d)
$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}.$$