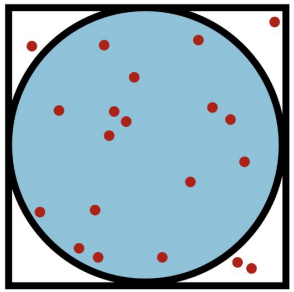
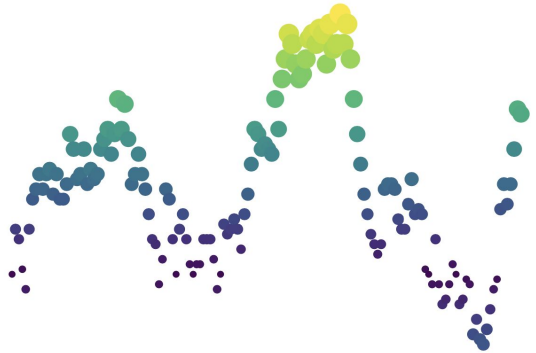
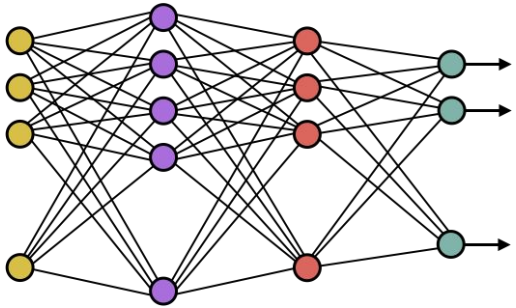
Numerical Methods



Lauren Hayward
PSI START Online School





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Territorial land acknowledgement and Indigenous education

Perimeter Institute acknowledges that it is situated on the traditional territory of the Anishinaabe, Haudenosaunee, and Neutral peoples. Perimeter Institute is located on the Haldimand Tract. After the American Revolution, the tract was granted by the British to the Six Nations of the Grand River and the Mississaugas of the Credit First Nation as compensation for their role in the war and for the loss of their traditional lands in upstate New York. Of the 950,000 acres granted to the Haudenosaunee, less than 5 percent remains Six Nations land. Only 6,100 acres remain Mississaugas of the Credit land. We thank the Anishinaabe, Haudenosaunee, and Neutral peoples for hosting us on their land.

We encourage students to spend time deepening their knowledge of Indigenous history and lived realities through resources such as those curated by the University of Waterloo Office of Indigenous Relations (https://uwaterloo.ca/indigenous/engagement-knowledge-building).

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Course summary:

This course is designed to introduce students to fundamental programming skills for use within theoretical physics. The course will also discuss selected research areas that rely on numerical methods. Students will become familiar with the programming language Python.

Learning objectives:

By the end of this course, students should be able to:

- Write code in Python using if statements, for loops, while loops, and functions
- Import and use Python libraries such as NumPy
- Write code to make plots and perform curve fitting
- Describe Monte Carlo methods
- Describe machine learning, its general categories, and the architecture of a neural network

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Outline of topics:

- Introduction to Programming in Python
- Data visualization and curve fitting
- Monte Carlo methods
- Machine learning
- Guest lectures on numerical methods within modern theoretical physics research

Course requirements:

To receive official recognition of participation in this course, students must attend all sessions of the course on Zoom (excused absences are possible with written permission from the instructor).

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Accommodations:

If accommodations would be helpful within this course, please don't hesitate to contact Lauren or People and Culture (peopleandculture@perimeterinstitute.ca).

Additional information:

See the course outline for useful references as well as statements regarding what is expected of students regarding Academic Integrity and Equity, Diversity & Inclusion.

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https://colab.research.google.com/drive/1qhML2ajO3WaJSkm5Bz-otlhcBbhVhVns?usp=sharing

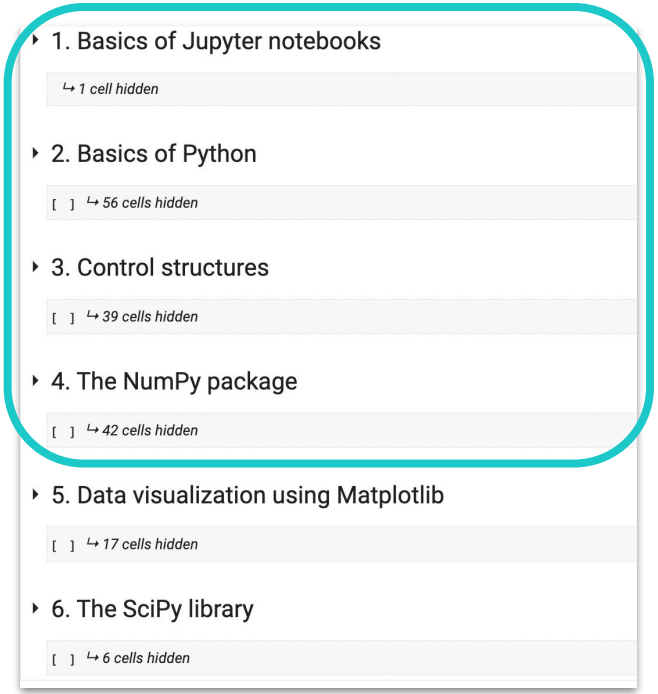
Numerical Methods

Introduction to Programming in Python

PSI Start Online School 2023

Lauren Hayward

This notebook introduces how to use Jupyter notebooks, basics of Pythons, and some libraries and packages that will be useful throughout the course. You will find a total of 8 exercises (with solutions) in the sections below.



Today

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Goals for today

- To become more familiar with writing code in Python and using the Numpy library
- To understand situations where random sampling can help us to numerically solve difficult problems
- To study how various parameters affect the precision of numerical results

Monte Carlo methods

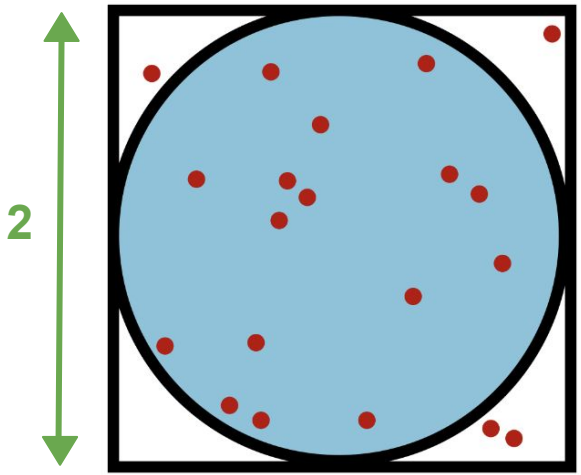
- Rely on random sampling to numerically solve difficult problems.

- Require access to some kind of random-number generator:
  - **True random number generators** output random numbers based on some kind of a physical process, such as the noise generated from electrical circuits.
  - **Pseudorandom number generators** use algorithms to output sequences of numbers that have similar properties to true random numbers. The output is completely deterministic when given a seed, which may or may not come from a truly random source.
  - Pseudorandom number generators are usually used in practice within Monte Carlo methods because they are much faster than true random number generators.

Estimating π

Today, we will study how to estimate π using Monte Carlo methods with random numbers sampled from a uniform distribution.

Consider a set-up where we randomly throw darts at a 2×2 square with a unit circle inscribed inside of it.



Let N_{darts} be the total number of darts thrown and let N_{circle} be the number of darts that land inside the circle.

How can we use N_{darts} and N_{circle} to estimate π?

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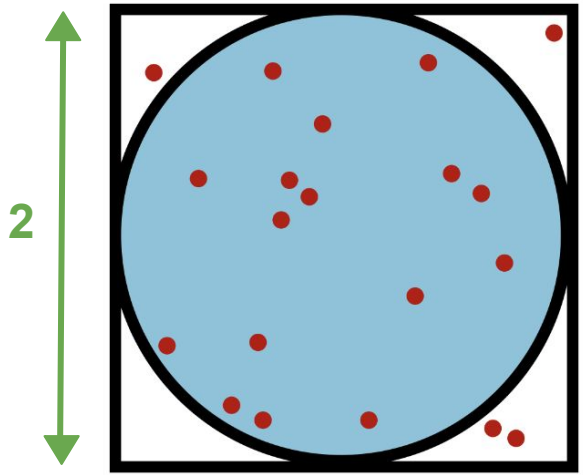
Estimating π



Notation:

- $A_{\text{circle}} = \pi$ is the area of the circle
- $A_{\text{square}} = 4$ is the area of the square

Then $$\frac{N_{\text{circle}}}{N_{\text{darts}}} \approx \frac{A_{\text{circle}}}{A_{\text{square}}} = \frac{\pi}{4} \quad \Longrightarrow \quad \boxed{\pi \approx 4\frac{N_{\text{circle}}}{N_{\text{darts}}}}$$

So for the example above, we would estimate that $\pi \approx 4\frac{16}{20} = 3.2$

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https://colab.research.google.com/drive/1O10XjEPyh2ii588SBRmqt2JCfpbI-Yp1?usp=sharing

Open this view-only notebook in Google Colaboratory. To do so, you may need to first choose, ‘Open with’, then ‘Connect more apps’, then search for ‘Google Colaboratory’.

Then, once you have opened the notebook, to save a copy of your changes to your own account, choose the option “Copy to Drive”.

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