

Homework #2, due on Wednesday, September 20, 2023

1. Solve the following differential equations (please show essential steps):

a. $y'' - \frac{1+x}{x} y' + \frac{1}{x} y = 0$

b. $y''' - 3y'' + 3y' - y = 0$

c. $x^2 y'' - 4xy' + 6y = x^4$

2. One solution of the 2nd order differential equation $(x + 2)y'' - (2x + 5)y' + 2y = 0$ is $y_1 = e^{2x}$. Try to find the other solution by using

- the reduction of order
- the Wronskian in conjunction with Abel's formula

3. Are the following sets of functions linearly dependent or not on the specified interval? If they are linearly dependent, please write down the constants c_1 and c_2 , such that $c_1 f(x) + c_2 g(x) = 0$ for every x on the interval.

a. $f(x) = 2\cos(x)$; $g(x) = 3\sin(x)$; $x \in (0, 2\pi)$

b. $f(x) = 2\cos(2x)$; $g(x) = 3\sin(x)$; $x \in (0, 2\pi)$

c. $f(x) = 2\cos(2x)$; $g(x) = 2 - 4\sin^2(x)$; $x \in (0, 2\pi)$

d. $f(x) = x$; $g(x) = x^2$; $x \in (0, \infty)$

e. $f(x) = xe^x$; $g(x) = e^x$; $x \in (0, \infty)$