Lecture 1 Exercises

Quantum Information, PSI START Summer 2023

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Exercise #1:

Given operator $\mathcal{O} = \sum_{ij=0}^{1} c_{ij}|i\rangle\langle j|$ and state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ with matrix/vector representations $\mathcal{O}_M$ and ψ_M, show that

$$(\mathcal{O}|\psi\rangle)_M = \mathcal{O}_M\psi_M$$

Exercise #2:

Show that σ^x is also a reflection. What is the angle of the reflecting line?

Exercise #3:

Show that $|+\rangle \equiv (|0\rangle + |1\rangle)/\sqrt{2}$ and $|-\rangle \equiv (|0\rangle - |1\rangle)/\sqrt{2}$ are eigenstates of σ^x. [Hint: you can show it geometrically using the result of the previous exercise.]

Exercise #4:

Then find the geometric representation of the operator that maps $|0\rangle \mapsto |+\rangle$ and $|1\rangle \mapsto |-\rangle$. Find the bra-ket and matrix representations of this operator, which is called the "Hadamard gate."

Exercise #5:

Find the matrix representation of the operator U that rotates both basis vectors $|0\rangle$ and $|1\rangle$ by angle θ. Show that $U_M U_M^T = U_M^T U_M = \mathrm{Id}$