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1. $\vec{u}_1 = A\vec{u}_0$

$$= \begin{bmatrix} .8 & .3 \\ .2 & .7 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} .8 \\ .2 \end{bmatrix}$$

$\vec{u}_2 = A\vec{u}_1$

$$= \begin{bmatrix} .8 & .3 \\ .2 & .7 \end{bmatrix} \begin{bmatrix} .8 \\ .2 \end{bmatrix} = \begin{bmatrix} 0.7 \\ .3 \end{bmatrix}$$

$\vec{u}_3 = A\vec{u}_2$

$$= \begin{bmatrix} .8 & .3 \\ .2 & .7 \end{bmatrix} \begin{bmatrix} .7 \\ .3 \end{bmatrix} = \begin{bmatrix} .65 \\ .35 \end{bmatrix}$$

* The components of each new vector add to 1, and so is the case for $\vec{u}_0$.

2. $k = 0, -3$, and 3

- $k=0 \Rightarrow \begin{bmatrix} 0 & 3 \\ 3 & 0 \end{bmatrix}$ for which elimination can be fixed by exchanging the rows.

$$\left[\begin{array}{cc|c} 0 & 3 & 6 \\ 3 & 0 & -6 \end{array}\right] \longrightarrow \left[\begin{array}{cc|c} 3 & 0 & 6 \\ 0 & 3 & -6 \end{array}\right] \longrightarrow \begin{array}{l} x = 2 \\ y = -2 \end{array}$$

- $k=-3 \Rightarrow \begin{bmatrix} -3 & 3 \\ 3 & -3 \end{bmatrix}$ elimination will give us a row of zeros, which implies that there're infinitely many sols.

$$\left[\begin{array}{cc|c} -3 & 3 & 6 \\ 3 & -3 & -6 \end{array}\right] \xrightarrow{R_2 - R_1} \left[\begin{array}{cc|c} -3 & 3 & 6 \\ 0 & 0 & 0 \end{array}\right]$$

▷ $k = 3 \Rightarrow \begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix}$ no solution

$$\left[\begin{array}{cc|c} 3 & 3 & 6 \\ 3 & 3 & -6 \end{array}\right] \xrightarrow{R_2 - R_1} \left[\begin{array}{cc|c} 3 & 3 & 6 \\ 0 & 0 & -12 \end{array}\right]$$

3. row1 + row2 = row3 → singular system

and thus row3 cannot be solved w/ r1 & r2.

This means row3: $2x - y \quad = 7$

→ no planes are parallel & no sol.

4. $$\begin{bmatrix} 1 & 1 & 0 \\ 4 & 6 & 1 \\ -2 & 2 & 0 \end{bmatrix} \xrightarrow[R_3 + 2R_1]{R_2 - 4R_1} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 4 & 0 \end{bmatrix} \xrightarrow{R_3 - 2R_2} \underbrace{\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & -2 \end{bmatrix}}_{U}$$

$$E_{21} = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad E_{31} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \quad E_{32} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

$$\Rightarrow \underbrace{E_{32}E_{31}E_{21}}A = U$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 10 & -2 & 1 \end{bmatrix} \underbrace{\begin{bmatrix} 1 & 1 & 0 \\ 4 & 6 & 1 \\ -2 & 2 & 0 \end{bmatrix}}_{A} = \underbrace{\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & -2 \end{bmatrix}}_{U}$$

5.

a) $\sum a_{3j} x_j$

b) $a_{21} - a_{11}$

c) $a_{21} - 2a_{11}$

d) $\sum a_{1j} x_j$

6.

a) $A \begin{bmatrix} b_{13} \\ b_{23} \\ \vdots \\ b_{n3} \end{bmatrix}$ where $A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{23} & \cdots & a_{2n} \\ \vdots & & & \\ a_{n1} & \cdots & \cdots & a_{nn} \end{bmatrix}$

b) $(a_{11} \; a_{12} \; \cdots \; a_{1n}) B$ where $B =$

$$\begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{23} & \cdots & b_{2n} \\ \vdots & & & \\ b_{n1} & \cdots & \cdots & b_{nn} \end{bmatrix}$$

c) 3^{rd} row of A times 4^{th} coulmn of B

d) 1^{st} row of A times D times 1^{st} column of E

7.

$$A = \begin{bmatrix} | & | & | \\ a_1 & a_2 & a_3 \\ | & | & | \end{bmatrix} \qquad X = \begin{bmatrix} | & | & | \\ x_1 & x_2 & x_3 \\ | & | & | \end{bmatrix}$$

$$AX = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} = I$$

8.

$r1 + r2 = r3$

a) $\left[\begin{array}{c|c} - r_1 - & 1 \\ - r_2 - & 0 \\ - r_3 - & 0 \end{array}\right] \xrightarrow{r_1 + r_2 - r_3} \left[\begin{array}{ccc|c} 0 & 0 & 0 & 1 \\ & - r_2 - & & 0 \\ & - r_3 - & & 0 \end{array}\right]$

b) we must have $b = (0, b_2, b_3) \Rightarrow b_1 = 0$

c) It gets eliminated, since it is a lin. comb. of r_1 & r_2

9.

24.

9.

24.

$$\left[\begin{array}{ccc|ccc} 1 & a & b & 1 & 0 & 0 \\ 0 & 1 & c & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_1 - aR_2} \left[\begin{array}{ccc|ccc} 1 & 0 & b-ac & 1 & -a & 0 \\ 0 & 1 & c & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_1 - (b-ac)R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -a & ac-b \\ 0 & 1 & c & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_2 - cR_3}$$

$$\left[\begin{array}{ccc|c} 1 & & & \\ & 1 & & \underbrace{\begin{matrix} 1 & -a & ac-b \\ 0 & 1 & -c \\ 0 & 0 & 1 \end{matrix}}_{U^{-1}} \\ & & 1 & \end{array}\right]$$

30. $$\left[\begin{array}{ccc|ccc} a & b & b & 1 & 0 & 0 \\ a & a & b & 0 & 1 & 0 \\ a & a & a & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_3 - R_1} \left[\begin{array}{ccc|ccc} a & b & b & 1 & 0 & 0 \\ a & a & b & 0 & 1 & 0 \\ 0 & a-b & a-b & -1 & 0 & 1 \end{array}\right] \xrightarrow{\frac{1}{a-b}R_3}$$

$$\left[\begin{array}{ccc|ccc} a & b & b & 1 & 0 & 0 \\ a & a & b & 0 & 1 & 0 \\ 0 & 1 & 1 & -\frac{1}{a-b} & 0 & \frac{1}{a-b} \end{array}\right] \xrightarrow[\frac{1}{a-b}R_2]{R_2 - R_1} \left[\begin{array}{ccc|ccc} a & b & b & 1 & 0 & 0 \\ 0 & 1 & 0 & -\frac{1}{a-b} & \frac{1}{a-b} & 0 \\ 0 & 1 & 1 & -\frac{1}{a-b} & 0 & \frac{1}{a-b} \end{array}\right]$$

$$\xrightarrow{R_3 - R_2} \left[\begin{array}{ccc|ccc} a & b & b & 1 & 0 & 0 \\ 0 & 1 & 0 & -1/a-b & 1/a-b & 0 \\ 0 & 0 & 1 & 0 & -1/a-b & 1/a-b \end{array}\right] \xrightarrow{R_1 - bR_2} \left[\begin{array}{ccc|ccc} a & 0 & b & \frac{a}{a-b} & -\frac{b}{a-b} & 0 \\ 0 & 1 & 0 & -\frac{1}{a-b} & \frac{1}{a-b} & 0 \\ 0 & 0 & 1 & 0 & -\frac{1}{a-b} & \frac{1}{a-b} \end{array}\right]$$

$$\xrightarrow{R_1 - bR_3} \left[\begin{array}{ccc|ccc} a & 0 & 0 & \frac{a+b}{a-b} & -\frac{2b}{a-b} & -\frac{b}{a-b} \\ 0 & 1 & 0 & -\frac{1}{a-b} & \frac{1}{a-b} & 0 \\ 0 & 0 & 1 & 0 & -\frac{1}{a-b} & \frac{1}{a-b} \end{array}\right] \xrightarrow{\frac{1}{a} R_1}$$

$$\left[\begin{array}{ccc|ccc} 1 & & & \frac{a+b}{a(a-b)} & -\frac{2b}{a(a-b)} & -\frac{b}{a(a-b)} \\ & 1 & & -\frac{1}{a-b} & \frac{1}{a-b} & 0 \\ & & 1 & -\frac{1}{a-b} & 0 & \frac{1}{a-b} \end{array}\right]$$

$$\underbrace{\hphantom{\frac{a+b}{a(a-b)} \quad -\frac{2b}{a(a-b)} \quad -\frac{b}{a(a-b)}}}_{A^{-1}}$$

10.

13) $$E_{41}E_{31}E_{21}A = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ -1 & & & 1 \end{bmatrix}\begin{bmatrix} 1 & & & \\ & 1 & & \\ -1 & & 1 & \\ & & & 1 \end{bmatrix}\begin{bmatrix} 1 & & & \\ -1 & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix} A$$

$$= \underbrace{\begin{bmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & b-a & c-a & c-a \\ 0 & b-a & c-a & d-a \end{bmatrix}}_{A'}$$

$$E_{42}E_{32}A' = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ 0 & -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & & & \\ & 1 & & \\ 0 & -1 & 1 & 0 \\ & & & 1 \end{bmatrix} A'$$

$$= \underbrace{\begin{bmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & 0 & c-b & c-b \\ 0 & 0 & c-b & d-b \end{bmatrix}}_{A''}$$

$$E_{43}A'' = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ 0 & 0 & -1 & 1 \end{bmatrix} A'' = \begin{bmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & 0 & c-b & c-b \\ 0 & 0 & 0 & d-c \end{bmatrix} = U$$

$L = (E_{43}E_{42}E_{32}E_{41}E_{31}E_{21})^{-1}$

$$= \begin{bmatrix} 1 & & & \\ 1 & 1 & & \\ 1 & 1 & 1 & \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

* empty spaces are zeros

∴ Conditions: $a \neq 0$
for L & U to exist
$b \neq a$
$c \neq b$
$d \neq c$

18) $LDU = L_1D_1U_1$

$L_1^{-1}LDU = L_1^{-1}L_1D_1U_1$

$= D_1U_1$

$L_1^{-1}LDUU^{-1} = D_1U_1U^{-1}$

$L_1^{-1}LDI = D_1U_1U^{-1}$

$L_1^{-1}LD = D_1U_1U^{-1}$

low triangular matrix

upper

$L = L_1$ and $U = U_1$ since they all have diagonals 1's

$\Rightarrow L_1^{-1}L = I$ and $U_1U^{-1} = I$

$\Rightarrow L_1^{-1}LD = D_1U_1U^{-1} \Rightarrow D = D_1$

23) $A = \begin{bmatrix} 5 & a_2 & a_3 \\ a_4 & 9 & a_6 \\ a_7 & a_8 & 3 \end{bmatrix}$ Upper left submatrix:

$A' = \begin{bmatrix} 5 & a_2 \\ a_4 & 9 \end{bmatrix}$ Ans.: 5 & 9

11.

13)

a. consider $P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

$\Rightarrow PP = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} = P^2$

$\Rightarrow P^2P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\therefore P^3 = I$

b. consider $\hat{P} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

$\Rightarrow \hat{P}^3 = \begin{bmatrix} 1 & & & \\ & & & 1 \\ & 1 & & \\ & & 1 & \end{bmatrix} \begin{bmatrix} 1 & & & \\ & & & 1 \\ & 1 & & \\ & & 1 & \end{bmatrix} \begin{bmatrix} 1 & & & \\ & & & 1 \\ & 1 & & \\ & & 1 & \end{bmatrix} = I$

$\hat{P}^4 = \hat{P}^3\hat{P} = I\hat{P} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \neq I$

36. G: A group

- A group of lower triangular matrices includes L_1 & L_2 as elements. Thus, L_1^{-1} also belongs to the group, as well as L_1L_2. Therefore, this set is a group.

- Symmetric matrices S:

$S_1, S_2 \in G$

$\Rightarrow S_1^T, S_2^T \in G$.

$\Rightarrow S_1 S_2 \in G$

$[S_1 S_2]^T = S_2 S_1 \notin G$ ∴ This set is not G

- Positive matrices M

Let $M_1 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \in G$ $\quad M^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \notin G$ ∴ This set is not G

- Diagonal Invertible Matrices D

Let D_1 & $D_2 \in G \Rightarrow D_1^{-1}, D_2^{-1} \in G$ (the inverse is also diagonal)

$\Rightarrow D_1 D_2 \in G$ (product also diagonal)

∴ This set is G

- Permutation Matrices P

Let P_1 & $P_2 \in G \Rightarrow P_1^{-1}$ & $P_2^{-1} \in G$

$\Rightarrow P_1 P_2 \in G$ ∴ is G

• $Q^T = Q^{-1}$

Let Q_1 & $Q_2 \in G \Rightarrow Q_1^{-1} = Q_{-1}^T$ & $Q_2^{-1} = Q_2^T$

$(Q_1^{-1})^T = (Q_1^T)^{-1} = Q_1^{-1} \Rightarrow$ the inverse $\in G$

Now, consider $Q_1 Q_2$

$$
\begin{aligned}
(Q_1 Q_2)^T &= Q_2^T Q_1^T \\
&= Q_2^{-1} Q_1^{-1} \\
&= (Q_1 Q_2)^T \quad \Rightarrow Q_1 Q_2 \in G
\end{aligned}
$$

∴ This set is G

12) on the left

$$\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & I \end{bmatrix} = \begin{bmatrix} a & b & c \\ -2a+d & -2b+e & -2c+f \\ g & h & I \end{bmatrix}$$

$\therefore E_{21}A \equiv R_2 - 2R_1$; substract 1st row of (time 2) from 2nd row

on the right

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & I \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} a-2b & b & c \\ d-2e & e & f \\ g-2h & h & I \end{bmatrix}$$

$\therefore AE_{21} \equiv C_1 - 2C_2$; substract 2nd column (x2) from 1st column

13) All components of AB are 4;

$$A_{2x2} \cdot B_{2x2} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix}$$

14)

a) $$\left[\begin{array}{ccc|c} 2 & 1 & 4 & 2 \\ 6 & 1 & 0 & -10 \\ -1 & 2 & -10 & -4 \end{array}\right] \xrightarrow{R_2 - 3R_1} \left[\begin{array}{ccc|c} 2 & 1 & 4 & 2 \\ 0 & -2 & -12 & -16 \\ -1 & 2 & -10 & -4 \end{array}\right] \xrightarrow{R_3 + \frac{1}{2}R_1}$$

0   -2   -12   -16

$0 \quad \frac{1}{2} \quad -32$

$$\left[\begin{array}{ccc|c} 2 & 1 & 4 & 2 \\ 0 & -2 & -12 & -16 \\ 0 & \frac{1}{2} & -8 & -3 \end{array}\right] \xrightarrow{4R_3 + R_2} \left[\begin{array}{ccc|c} 2 & 1 & 4 & 2 \\ 0 & -2 & -12 & -16 \\ 0 & 0 & -92 & -92 \end{array}\right]$$

$-92z = -92 \Rightarrow z = 1$

$-2y - 12 = -16 \Rightarrow -2y = -4 \Rightarrow y = 2$

$2x + 2 + 4 = 2 \Rightarrow 2x = -4 \Rightarrow x = -2$

b) $$\left[\begin{array}{ccc|c} 0 & 1 & 1 & 3 \\ 1 & 2 & -1 & 1 \\ 1 & 1 & 1 & 4 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & 1 & 1 & 3 \\ 1 & 1 & 1 & 4 \end{array}\right] \xrightarrow{R_3 - R_1} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & 1 & 1 & 3 \\ 0 & -1 & 2 & 3 \end{array}\right] \xrightarrow{R_3 + R_2}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 3 & 6 \end{array}\right]$$

- $3z = 6 \Rightarrow z = \frac{6}{3} \Rightarrow z = 2$
- $y + 2 = 3 \Rightarrow y = 1$
- $x + 2(1) - 1(2) = 1 \Rightarrow x = 1$

c) $$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 0 \\ 0 & 0 & 5 & 6 & 0 \\ 0 & 0 & a & 6 & 0 \\ 0 & 1 & 7 & 8 & 1 \end{array}\right] \xrightarrow{R_4 \leftrightarrow R_2} \left[\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 0 \\ 0 & 1 & 7 & 8 & 1 \\ 0 & 0 & a & 6 & 0 \\ 0 & 0 & 5 & 6 & 0 \end{array}\right]$$

if $a=5$:

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 0 \\ 0 & 1 & 7 & 8 & 1 \\ 0 & 0 & 5 & 6 & 0 \\ 0 & 0 & 5 & 6 & 0 \end{array}\right] \xrightarrow{R_4 - R_5} \left[\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 0 \\ 0 & 1 & 7 & 8 & 1 \\ 0 & 0 & 5 & 6 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Then, we have *infinitely many solutions*

let $w = g$

- $5z + 6g = 0 \Rightarrow z = \frac{-6g}{5}$
- $y + 7\left(-\frac{6}{5}g\right) + 8g = 1 \Rightarrow y - \frac{42}{5}g + 8g = 1$

$$\Rightarrow y = 1 - \frac{2}{5}g$$

- $x + 2\left(1 - \frac{2}{5}g\right) + 3\left(-\frac{6}{5}g\right) + 4g = 0$

  $x = \frac{8}{5}g$

if $a \neq 5$: $w=0$, $z=0$, $y=1$, and $x=-2$

$x + 2(1) = 0$

15.

a) $E_{32}E_{31}E_{21}A = U$

$$\begin{bmatrix} 2 & 1 & 4 \\ 6 & 1 & 0 \\ -1 & 2 & 10 \end{bmatrix} \xrightarrow[R_3 + \frac{1}{2}R_1]{R_2 - 3R_1} \begin{bmatrix} 2 & 1 & 4 \\ 0 & -2 & -12 \\ 0 & \frac{5}{2} & 12 \end{bmatrix} \xrightarrow{R_3 + \frac{5}{4}R_2} \begin{bmatrix} 2 & 1 & 4 \\ 0 & -2 & -12 \\ 0 & 0 & -3 \end{bmatrix} \quad U$$

$$E_{21} = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad E_{31} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1/2 & 0 & 1 \end{bmatrix} \qquad E_{32} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 5/4 & 1 \end{bmatrix}$$

b)

$$E_{21}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad E_{31}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1/2 & 0 & 1 \end{bmatrix} \qquad E_{32}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -5/4 & 1 \end{bmatrix}$$

$$A = LU$$

$$I\,(E_{32}E_{31}E_{21})^{-1}(E_{32}E_{31}E_{21})\,A = \overbrace{(E_{32}E_{31}E_{21})^{-1}}^{L}\,U$$

$$L = (E_{32}E_{31}E_{21})^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -1/2 & -5/4 & 1 \end{bmatrix}$$

c) $A = LDU^*$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -1/2 & -5/4 & 1 \end{bmatrix} \underbrace{\overbrace{\begin{bmatrix} 2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}}^{\substack{D \\ \text{(given)}}} \overbrace{\begin{bmatrix} 1 & 1/2 & 2 \\ 0 & 1 & 6 \\ 0 & 0 & 1 \end{bmatrix}}^{U^*}}_{\text{supposed to equal } U}$$

16.

$A = LU$ solve $A\vec{x} = \vec{b}$:

$(\underbrace{U\vec{x}}_{\vec{y}}L) = \vec{b}$

1. find $L\vec{y} = \vec{b}$

2. find $U\vec{x} = \vec{y}$

$$\begin{matrix} -1 & 1 & 0 & 2 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 3 \end{matrix}$$

1. $\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \vec{y} = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$:

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ -1 & 1 & 0 & 2 \\ 0 & 2 & 1 & 4 \end{array}\right] \xrightarrow{R_2 + R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 2 & 1 & 4 \end{array}\right] \xrightarrow{R_3 - 2R_2}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -2 \end{array}\right] \qquad \vec{y} = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$$

2. $\begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \vec{x} = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$:

$$\left[\begin{array}{ccc|c} 1 & 3 & 0 & 1 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 1 & -2 \end{array}\right] \xrightarrow{R_2 - R_3} \left[\begin{array}{ccc|c} 1 & 3 & 0 & 1 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & -2 \end{array}\right] \xrightarrow{R_1 - 3R_2}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -14 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$\vec{x}$

17. In Problem 15, we found $(E_{32}E_{31}E_{21})A = U$

Now we have to find D: $E_{12}E_{13}E_{23}\underbrace{\overset{\text{known}}{E_{32}E_{31}E_{21}A}}_{U} = D$

$$\begin{bmatrix} 2 & 1 & 4 \\ 0 & -2 & -12 \\ 0 & 0 & -3 \end{bmatrix} \xrightarrow{R_1+\frac{4}{3}R_2} \begin{bmatrix} 2 & 1 & 0 \\ 0 & -2 & -12 \\ 0 & 0 & -3 \end{bmatrix} \xrightarrow{R_2-4R_3} \begin{bmatrix} 2 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} \xrightarrow{R_1+\frac{1}{2}R_2} \begin{bmatrix} 2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}$$

$$E_{13} = \begin{bmatrix} 1 & 0 & 4/3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad E_{23} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix} \qquad E_{12} = \begin{bmatrix} 1 & 1/2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$E_{12}E_{13}E_{23}U = D$

$$\begin{bmatrix} 1 & 1/2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 4/3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 4 \\ 0 & -2 & -12 \\ 0 & 0 & -3 \end{bmatrix}} = D$$

$$\begin{bmatrix} 1 & 1/2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \underbrace{\begin{bmatrix} 1 & 0 & 4/3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 4 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}} = D$$

$$\underbrace{\begin{bmatrix} 1 & 1/2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}} = D$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} = D$$

b) $2, -2, -3$

c) $E_{12}E_{13}E_{23}E_{32}E_{31}E_{21}A = D$

$$D^{-1} = \begin{bmatrix} 1/2 & & \\ & -1/2 & \\ & & -\frac{1}{3} \end{bmatrix} \quad ; D^{-1}D = I$$

$$\Rightarrow D^{-1}E_{12}E_{13}E_{23}E_{32}E_{31}E_{21}A = \underbrace{D^{-1}D}_{I}$$

$$\Rightarrow D^{-1}E_{12}E_{13}E_{23}E_{32}E_{31}E_{21} = A^{-1}$$

18.

a. Matrix 1: $\begin{bmatrix} 0 & 1 & 2 & 0 \\ 0 & 2 & 4 & 0 \end{bmatrix} \xrightarrow{R_2-2R_1} \underbrace{\begin{bmatrix} 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{U}$

$E = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} \Rightarrow E^{-1} = L = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

Matrix2: $\begin{bmatrix} 0 & 0 \\ 1 & 2 \\ 2 & 4 \\ 0 & 0 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & 2 \\ 0 & 0 \\ 2 & 4 \\ 0 & 0 \end{bmatrix} \xrightarrow{R_3 \leftrightarrow R_2} \begin{bmatrix} 1 & 2 \\ 2 & 4 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \xrightarrow{R_2-2R_1} \underbrace{\begin{bmatrix} 1 & 2 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}}_{U}$

$$P = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

b. Matrix 1: $b_2 = 2b_1$

Matrix 2: $b_1 = 0$, $b_3 = 2b_2$, and $b_4 = 0$