Lecture 2 Exercises

Quantum Information, PSI START Summer 2023

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Exercise #1:

Let

$$|\Psi\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

Show that there do not exist $|\psi_1\rangle = \alpha|0\rangle + \beta|1\rangle$ and $|\psi_2\rangle = \gamma|0\rangle + \delta|1\rangle$ such that $|\Psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle$.

Exercise #2:

Let H be the operator

$$H = \frac{|0\rangle + |1\rangle}{\sqrt{2}}\langle 0| + \frac{|0\rangle - |1\rangle}{\sqrt{2}}\langle 1|$$

Find $(H \otimes \mathrm{Id})|00\rangle$

Exercise #3:

Prove that $a|00\rangle + b|11\rangle$ is equivalent to $a|11\rangle + b|00\rangle$.

In other words, explicitly find U_A and U_B, valid one-qubit state transformations (unitaries) such that

$$(U_A \otimes U_B)(a|00\rangle + b|11\rangle) = b|00\rangle + a|11\rangle$$

Exercise #4:

Prove the following:

- Suppose $|\Psi\rangle$ and $|\Phi\rangle$ have the same (a, b). In other words, suppose $|\Psi\rangle = (U_A \otimes U_B)(a|00\rangle + b|11\rangle)$ and $|\Phi\rangle = (U'_A \otimes U'_B)(a|00\rangle + b|11\rangle)$, where U_A, U_B, U'_A, and U'_B are one-qubit unitaries. Prove that $|\Psi\rangle$ and $|\Phi\rangle$ are equivalent.
- Conversely, suppose $|\Psi\rangle$ and $|\Phi\rangle$ are equivalent. Prove that they have the same (a, b).

Exercise #5:

We measure $a|00\rangle + b|11\rangle$ in the ZZ-basis, $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$.

(a) What is the probability of getting $|0\rangle$ for the first qubit? This is the sum of the probability of getting $|00\rangle$, and $|01\rangle$, and we denote the probability as $P(0)_1$. Also find $P(1)_1$, $P(0)_2$, and $P(1)_2$.

(b) If the probabilities for qubits 1 and 2 were independent, what would be the probability that the measurements agree?

(c) What is the actual probability that the measurements agree?

(d) If (b) and (c) have different answers, what does that say about entanglement as a function of a and b?

Exercise #6:

We repeat the previous exercise, but measuring $a|00\rangle + b|11\rangle$ in the XX-basis, $\{|++\rangle, |+-\rangle, |-+\rangle, |--\rangle\}$ where for example $|++\rangle$ is shorthand for $|+\rangle \otimes |+\rangle$, and $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$, $|-\rangle = (|0\rangle - |1\rangle)/\sqrt{2}$.

(a) Find the probabilities of the outcomes $|+\rangle$ and $|-\rangle$ on each qubit, $P(+)_1$, $P(+)_2$, $P(-)_1$, $P(-)_2$.

(b) If the probabilities were independent, what would be the probability that the measurements agree?

(c) What is the actual probability that the measurements agree?

(d) If (b) and (c) have different answers, what does that say about entanglement as a function of a and b?