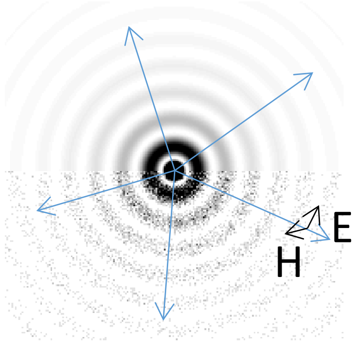
Optics and Nanophotonics

Interfaces

- Reflection & Refraction
- Conservation principles
- Special angles
- Intensity
- Interface conditions
- Fresnel equations
- Phase
- Thin films



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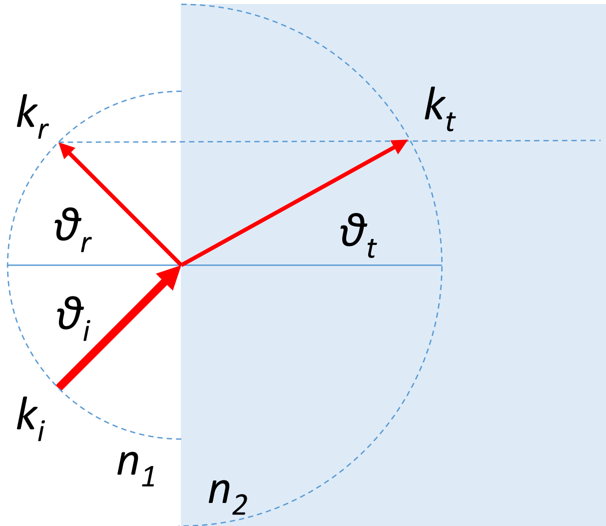
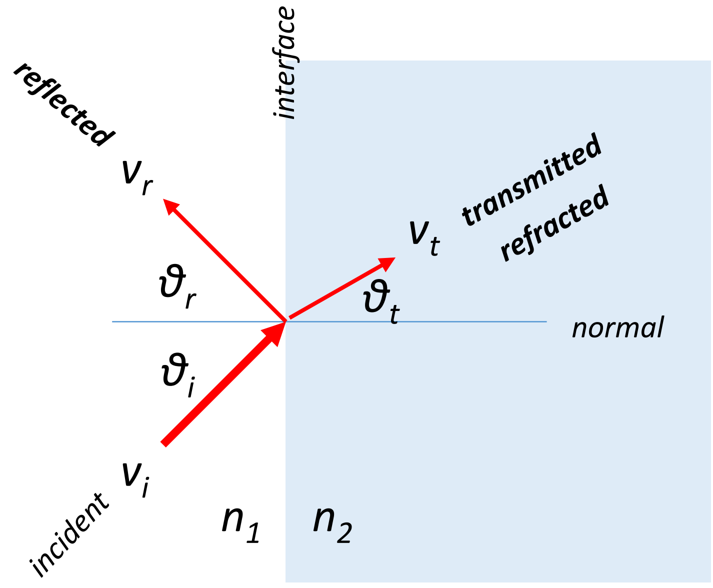


Interfaces

- All optical devices have interfaces
- **Energy is split between reflection and refracted beams**
- Engineering Q: *what angle, how much energy, (what phase)?*



Plane of incidence (page)

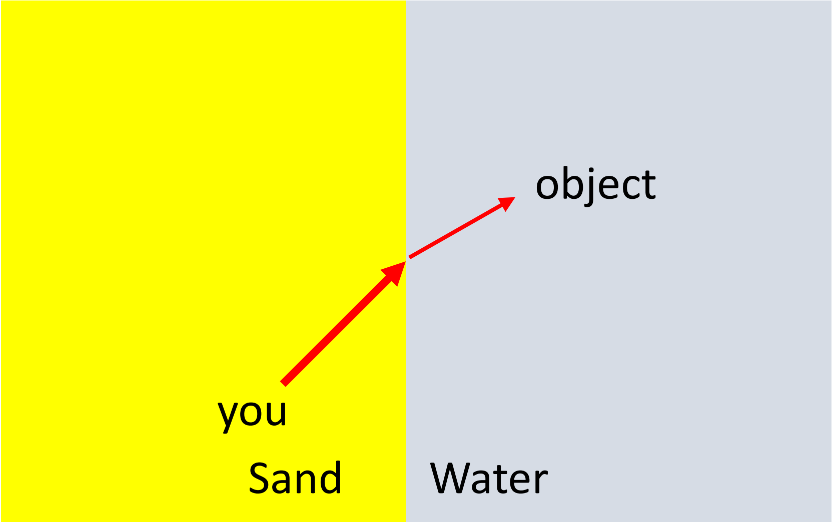
http://en.wikipedia.org/wiki/Fermat's_principle

What angle?

Fermat's principle: *ray traces path of least time*

Wave/particle justification:
- *all* paths taken and interfere
- *only shortest* interferes *constructively*

Thought experiment:
Imagine you're at the beach...
want to reach something in water along beach.
You swim slower than you walk: fastest path?



Solution: (not straight-line!)
run further, swim shorter
"Refract" into water

Can be solved to give angles,
but doesn't give flux division
Need conservation + EM

What angle?
Phase-matching/conservation

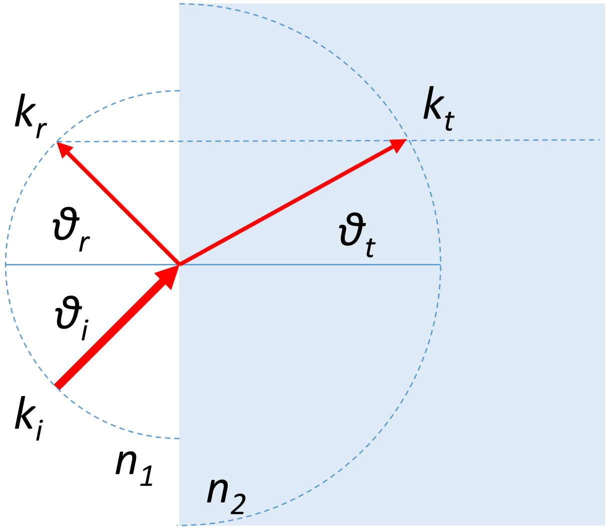
- Incident flux is split between transmission & reflection (conservation of flux) $1 = R + T$
- Speed *changes* $c = v_r n_1 = v_i n_1 = v_t n_2$
- Waves phase match (equal *e*$^{-i\omega t}$) @ all time

***ω* same** (conservation of energy) $\omega_r = \omega_i = \omega_t$

- |*k*| *changes* $k_0 = k_r / n_1 = k_i / n_1 = k_t / n_2$
- Waves phase match (equal *e*^{ikr}) @ interface

***k* same along interface** (conservation of p)

$$k_r \sin\theta_r = k_i \sin\theta_i = k_t \sin\theta_t$$



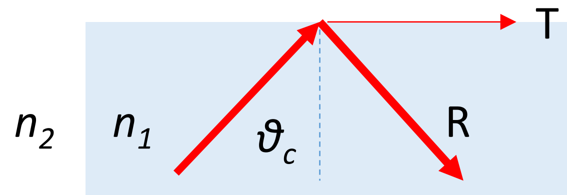
Law of reflection	Refraction (Snell/Descarte)
$\theta_r = \theta_i$	$n_1 \sin\theta_i = n_2 \sin\theta_t$

To get fluxes we need electromagnetism

Saleh & Teich Ch2.4 http://en.wikipedia.org/wiki/Snell’s_law

Total internal reflection

$$\theta_c = \sin^{-1}(n_2 / n_1)$$



If $n_1 < n_2$, refraction angle is undefined beyond critical angle ϑ_c

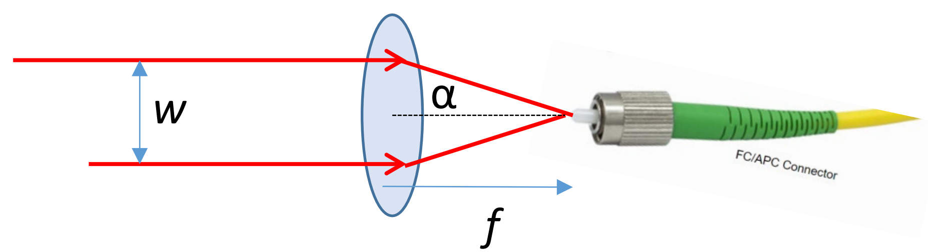
•All energy goes into reflection (in absence of another interface)

•Transmitted wave evanescent (travels along interface, decays away from it)

e.g. (into air)
water n=1.3, ϑ_c=...
Si n=3.4, ϑ_c=...

This has important consequences for devices

- Acceptance angle of fiber
  (indirect: $\alpha \neq \theta_c$ & n core:clad
  more later)



- Emission efficiency of LED
  n of semiconductors high → critical angle small → most photons trapped

previously

Electromagnetic wave roadmap

$$\nabla \bullet \mathbf{D} = \rho_{free}$$
$$\nabla \bullet \mathbf{B} = 0$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$
$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

Maxwell's equations

Constitutive equations

$$\mathbf{D} = \boldsymbol{\varepsilon}\mathbf{E}$$
$$\mathbf{B} = \boldsymbol{\mu}\mathbf{H}$$
$$\mathbf{J} = \boldsymbol{\sigma}\mathbf{E}$$

$$\nabla^2 \mathbf{E} = \varepsilon\mu \frac{\partial^2 \mathbf{E}}{\partial t^2}$$
$$\nabla^2 \mathbf{H} = \varepsilon\mu \frac{\partial^2 \mathbf{H}}{\partial t^2}$$

Helmholtz (wave) equation

$$E = E_0 \exp[i\{kr - \omega t\}]$$
$$H = H_0 \exp[i\{kr - \omega t\}]$$

Sinusoidal waves

$$i\mathbf{k.D} = \rho_{free}$$
$$\mathbf{k.B} = 0$$
$$\mathbf{k} \times \mathbf{H} = \mathbf{J} - \omega\mathbf{D}$$
$$\mathbf{k} \times \mathbf{E} = \omega\mathbf{B}$$

Harmonic Maxwell

$$S_{av} = \mathrm{Re}[\mathbf{E} \times \mathbf{H}^*]/2$$

Properties of time-harmonic EM wave in simple media

$\mathbf{k} \perp \mathbf{E} \perp \mathbf{H}$ $\quad \boxed{n \equiv \frac{c}{v} = \sqrt{\frac{\varepsilon}{\varepsilon_0}\frac{\mu}{\mu_0}}}$ $\quad \boxed{\eta \equiv \frac{E}{H} = \sqrt{\frac{\mu}{\varepsilon}} = 1/(v\varepsilon)}$ $\quad v = \frac{\omega}{k}$ $\quad \boxed{S_{av} \propto \mathrm{Re}[n]|E|^2}$

Saleh & Teich Ch5

Fresnel equation road-map

Properties of time-harmonic EM wave in simple media

$$\mathbf{k} \perp \mathbf{E} \perp \mathbf{H} \qquad n \equiv \frac{c}{v} = \sqrt{\frac{\varepsilon}{\varepsilon_0}\frac{\mu}{\mu_0}} \qquad \eta \equiv \frac{E}{H} = \sqrt{\frac{\mu}{\varepsilon}} = 1/(v\varepsilon) \qquad v = \frac{\omega}{k} \qquad S_{av} \propto \mathrm{Re}[n]|E|^2$$

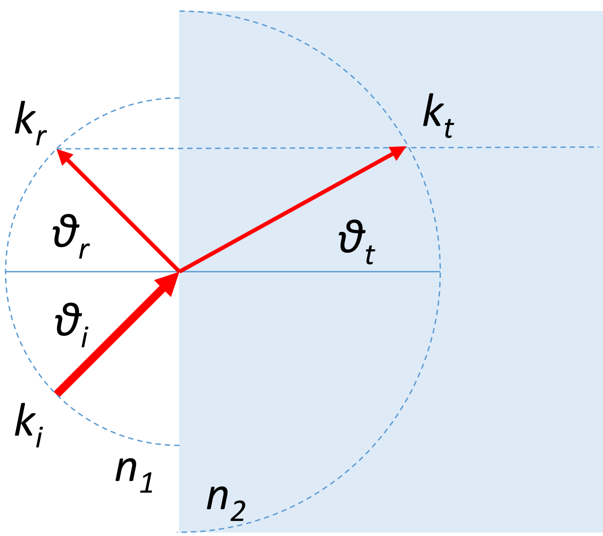
+ Conservation

$$k_0 = k_r / n_1 = \quad k_i / n_1 \quad = k_t / n_2$$

$$k_r \sin\theta_r = k_i \sin\theta_i = k_t \sin\theta_t$$

+ Interface conditions

$$\Delta E_{=} = 0$$
$$\Delta H_{=} = J_s$$
$$\Delta D_{\perp} = \rho_s$$
$$\Delta B_{\perp} = 0$$



Fresnel's equations

$$r_{\perp} \equiv \left(\frac{E_{0r}}{E_{0i}}\right)_{\perp} = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t}$$

$$r_{\parallel} \equiv \left(\frac{E_{0r}}{E_{0i}}\right)_{\parallel} = \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_i \cos\theta_t + n_t \cos\theta_i}$$

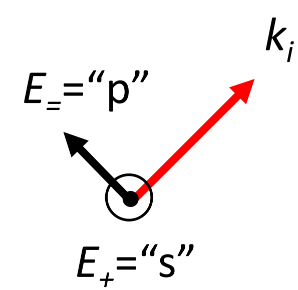
$$t_{\perp} \equiv \left(\frac{E_{0t}}{E_{0i}}\right)_{\perp} = \frac{2n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t}$$

$$t_{\parallel} \equiv \left(\frac{E_{0t}}{E_{0i}}\right)_{\parallel} = \frac{2n_i \cos\theta_i}{n_i \cos\theta_t + n_t \cos\theta_i}$$

$$R = r^2 \qquad \& \qquad T = \frac{n_t \cos\theta_t}{n_i \cos\theta_i} t^2$$

+ Polarization

(more next week)



http://en.wikipedia.org/wiki/Fresnel_equations

Fresnel's equations

- Give the **ratio** of the **electric field magnitudes** *at the interface*
- Electric field ratios can be *complex* numbers
- **Note: the apparent sign depends on the derivation**
- Fresnel's equations must be interpreted *with reference to derivation diagram*
- r & t are **not** magnetic field ratios – need extra impedance factors
- **Intensities are unambiguously defined**

Fresnel's equations

$$r_\perp \equiv \left(\frac{E_{0r}}{E_{0i}}\right)_\perp = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t}$$

$$t_\perp \equiv \left(\frac{E_{0t}}{E_{0i}}\right)_\perp = \frac{2n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t}$$

$$r_\parallel \equiv \left(\frac{E_{0r}}{E_{0i}}\right)_\parallel = \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_i \cos\theta_t + n_t \cos\theta_i}$$

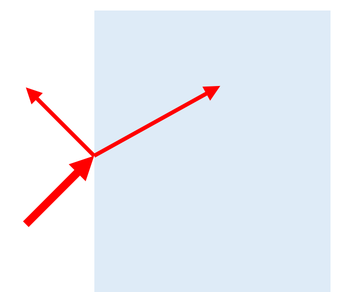
$$t_\parallel \equiv \left(\frac{E_{0t}}{E_{0i}}\right)_\parallel = \frac{2n_i \cos\theta_i}{n_i \cos\theta_t + n_t \cos\theta_i}$$

$$R = r^2 \qquad \& \qquad T = \frac{n_t \cos\theta_t}{n_i \cos\theta_i} t^2$$

Two "polarizations"
direction of E wrt
"Plane of Incidence" (PoInc)

PoInc
contains all rays
= page

"s" all E perpendicular
to PoInc



"p" all H perpendicular
to PoInc
(so E is *in* PoInc)

http://en.wikipedia.org/wiki/Fresnel_equations

We will do the full derivation later – let's first see how intensity is calculated

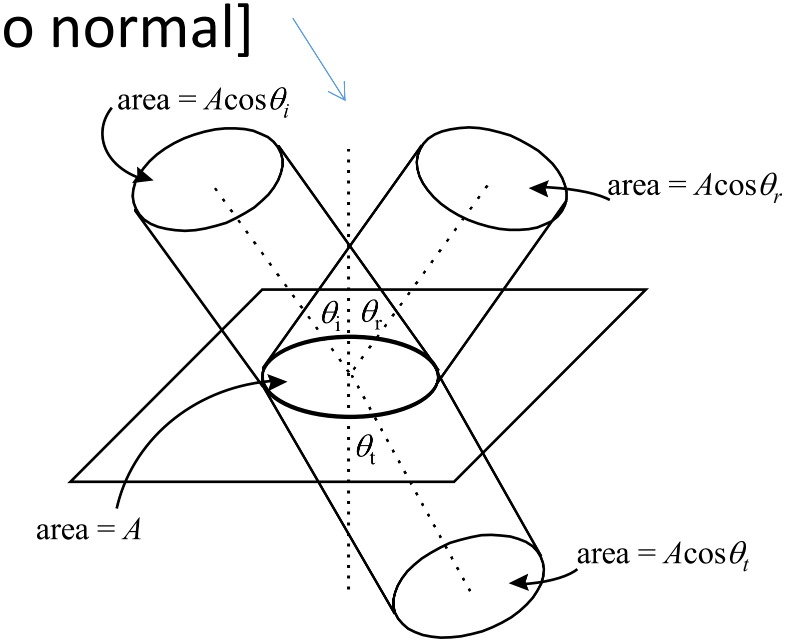
Fresnel Equations for intensity

- Transform from E → power
  - **|E|²**
  - Factor of *n*
  - Factor of cos(θ) [flux projection to normal]

$$S_{av} \equiv \mathrm{Re}[\mathbf{E}\times\mathbf{H}^*]/2 \propto \mathrm{Re}[n]|E|^2$$

$$\boxed{R = r^2 \qquad \& \qquad T = \frac{n_t \cos\theta_t}{n_i \cos\theta_i} t^2}$$

R: same medium, factors cancel



Some implications of Fresnel equations

- Interfaces are polarizing (more next week)
- (usually) small reflection at normal
- (always) large reflection at glancing (see TIR)
- Minimum "p" reflection at Brewster's angle

$$\theta_B = \tan^{-1}(n_2 / n_1)$$

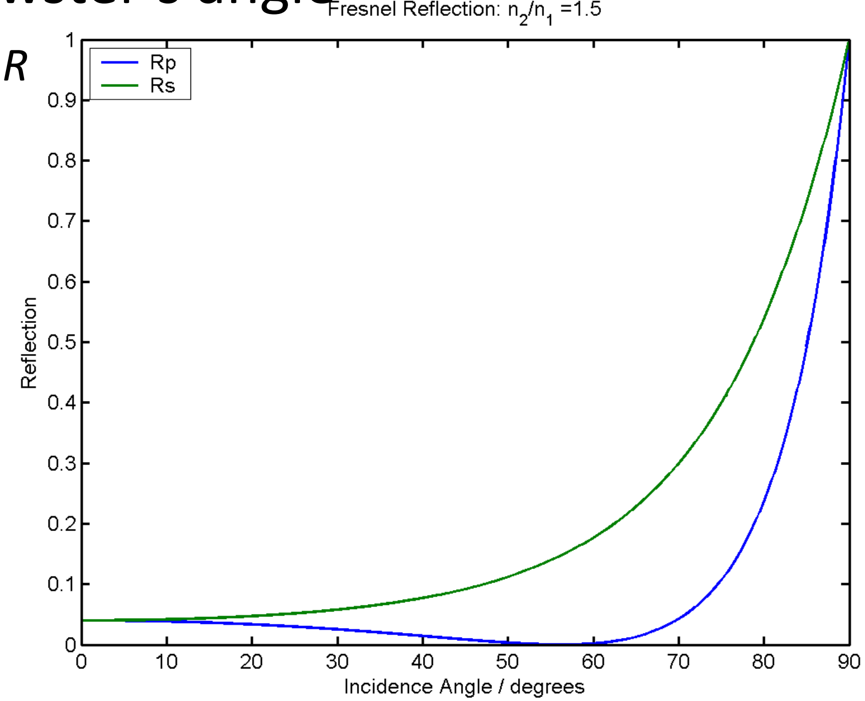
- Phase changes (later)

e.g. (from air)

Water: n=1.3, ϑ_B=...

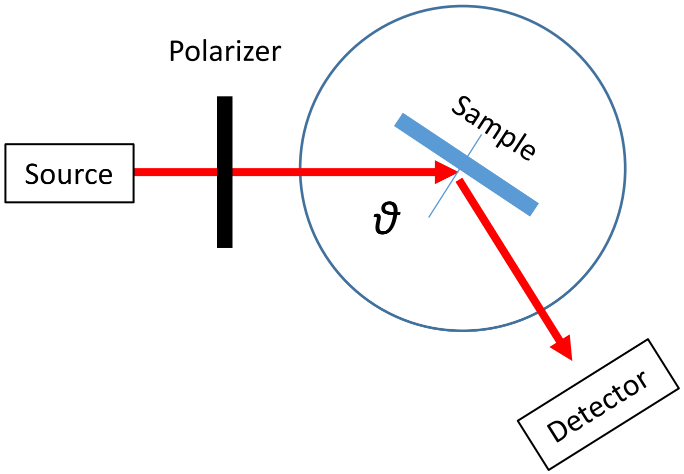
Si: n=3.4, ϑ_B=...

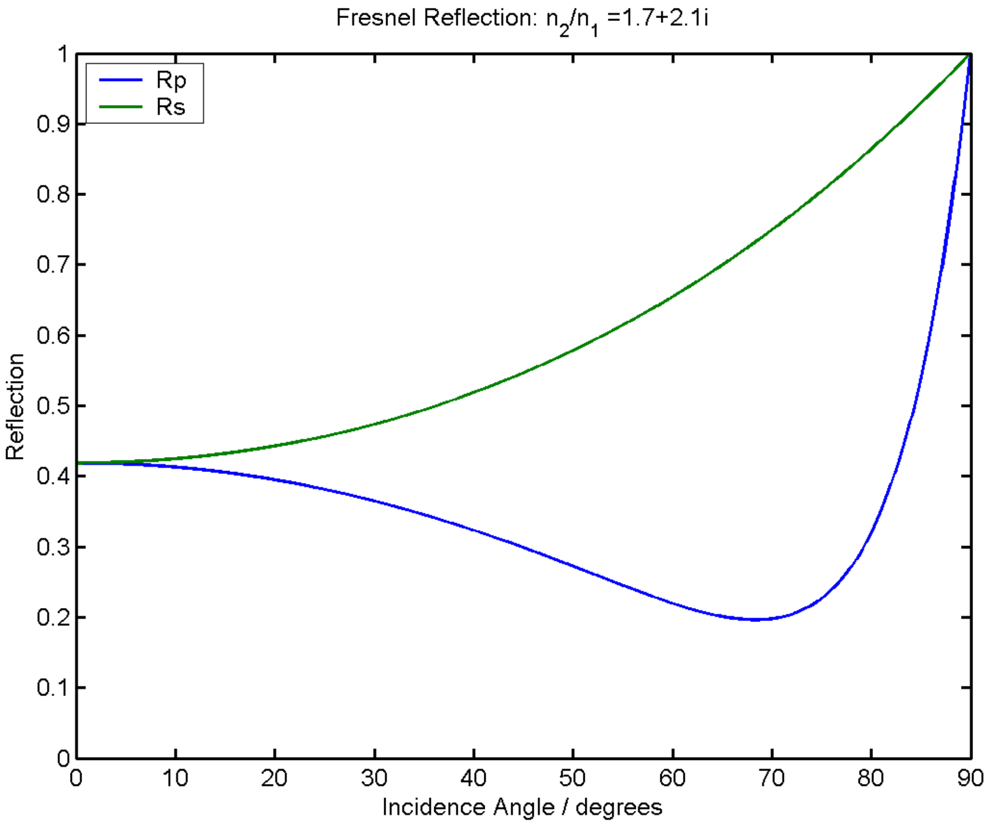
Ta @ 633nm: n=1.7 + 2.1i?



Applications of Fresnel's equations

- Interference filters
- Polarizers
- Photonic engineering (solar cells, LED)
- Measuring refractive index (fitting) →electronic structure
- …





Fresnel equation summary

Properties of time-harmonic EM wave in simple media

$$\mathbf{k} \perp \mathbf{E} \perp \mathbf{H} \qquad n \equiv \frac{c}{v} = \sqrt{\frac{\varepsilon}{\varepsilon_0}\frac{\mu}{\mu_0}} \qquad \eta \equiv \frac{E}{H} = \sqrt{\frac{\mu}{\varepsilon}} = 1/(v\varepsilon) \qquad v = \frac{\omega}{k} \qquad S_{av} \propto \mathrm{Re}[n]|E|^2$$

Wave-vector solution

$$k_0 = k_r / n_1 = \quad k_i / n_1 \quad = k_t / n_2$$

$$k_r \sin\theta_r = k_i \sin\theta_i = k_t \sin\theta_t$$

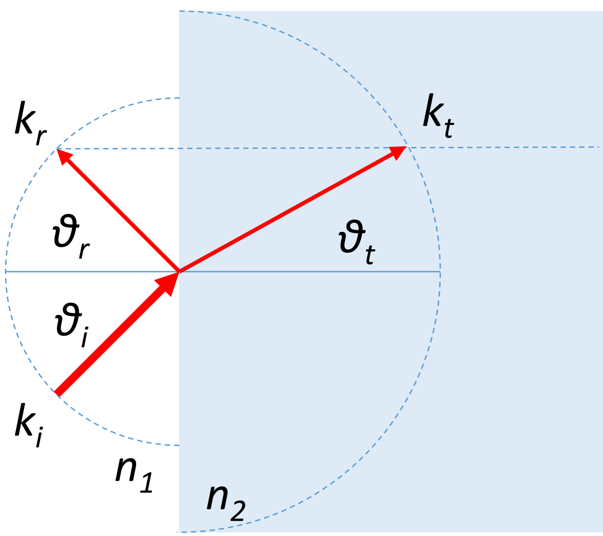
Boundary conditions

$$\Delta E_{=} = 0$$

$$\Delta H_{=} = J_s$$

$$\Delta D_{\perp} = \rho_s$$

$$\Delta B_{\perp} = 0$$



Fresnel's equations

$$r_{\perp} \equiv \left(\frac{E_{0r}}{E_{0i}}\right)_{\perp} = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t}$$

$$r_{\parallel} \equiv \left(\frac{E_{0r}}{E_{0i}}\right)_{\parallel} = \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_i \cos\theta_t + n_t \cos\theta_i}$$

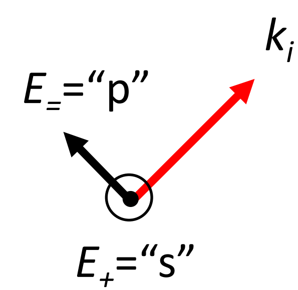
$$t_{\perp} \equiv \left(\frac{E_{0t}}{E_{0i}}\right)_{\perp} = \frac{2n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t}$$

$$t_{\parallel} \equiv \left(\frac{E_{0t}}{E_{0i}}\right)_{\parallel} = \frac{2n_i \cos\theta_i}{n_i \cos\theta_t + n_t \cos\theta_i}$$

$$R = r^2 \qquad \& \qquad T = \frac{n_t \cos\theta_t}{n_i \cos\theta_i} t^2$$

Polarization

(more next week)



http://en.wikipedia.org/wiki/Fresnel_equations

Maxwell's equations

Rules governing EM fields in space and time

- Differential form

$$\nabla \cdot \boldsymbol{D} = \rho_{free}$$

$$\nabla \cdot \boldsymbol{B} = 0$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}$$

- Integral form

$$\boxed{\begin{aligned} &\oint \boldsymbol{D} \cdot dA = \int \rho_{free} dV \\ &\oint \boldsymbol{B} \cdot dA = 0 \\ &\oint \boldsymbol{E} \cdot dl = -\frac{\partial}{\partial t} \int \boldsymbol{B} \cdot dA \\ &\oint \boldsymbol{H} \cdot dl = \int \left(\boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t} \right) \cdot dA \end{aligned}}$$

$\nabla \cdot$ "divergence"

$\nabla \times$ "rotation", "curl"

E electric field
H magnetic field
D electric flux
B magnetic flux
J electric current density
ρ electric charge
t time

l distance along path
A area of surface
V volume in volume

Interface conditions

https://en.wikipedia.org/wiki/Interface_conditions_for_electromagnetic_fields

$$\oint \boldsymbol{E} \cdot d\boldsymbol{l} = -\frac{\partial}{\partial t} \int \boldsymbol{B} \cdot d\boldsymbol{A}$$

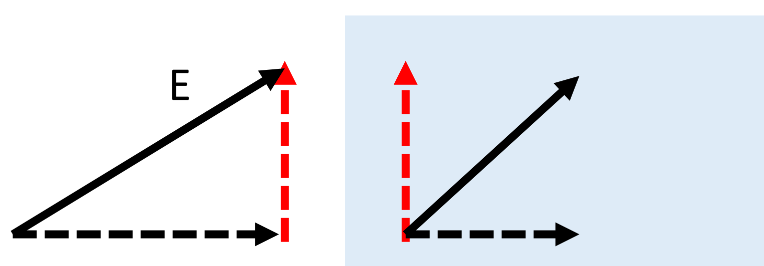
$$\oint \boldsymbol{H} \cdot d\boldsymbol{l} = \int \left(\boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t} \right) \cdot d\boldsymbol{A}$$

$$\oint \boldsymbol{D} \cdot d\boldsymbol{A} = \int \rho_{free} dV$$

$$\oint \boldsymbol{B} \cdot d\boldsymbol{A} = 0$$

$$\boxed{\begin{aligned} \Delta E_{=} &= 0 \\ \Delta H_{=} &= J_s \\ \Delta D_{\perp} &= \rho_s \\ \Delta B_{\perp} &= 0 \end{aligned}}$$

Usually ignore surface sources

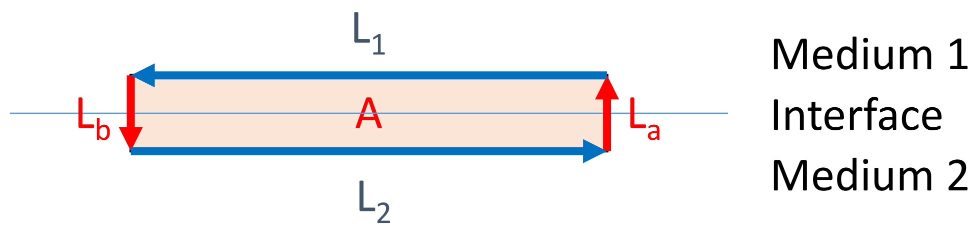


Tangential component of E (and H) continuous across interface

Use this to formulate transfer of fields across interface

Tangential component continuity

$$\oint \boldsymbol{E}\cdot d\boldsymbol{l} = -\frac{\partial}{\partial t}\int \boldsymbol{B}\cdot d\boldsymbol{A} \qquad\qquad \oint \boldsymbol{H}\cdot d\boldsymbol{l} = \int\left(\boldsymbol{J}+\frac{\partial \boldsymbol{D}}{\partial t}\right)\cdot d\boldsymbol{A}$$



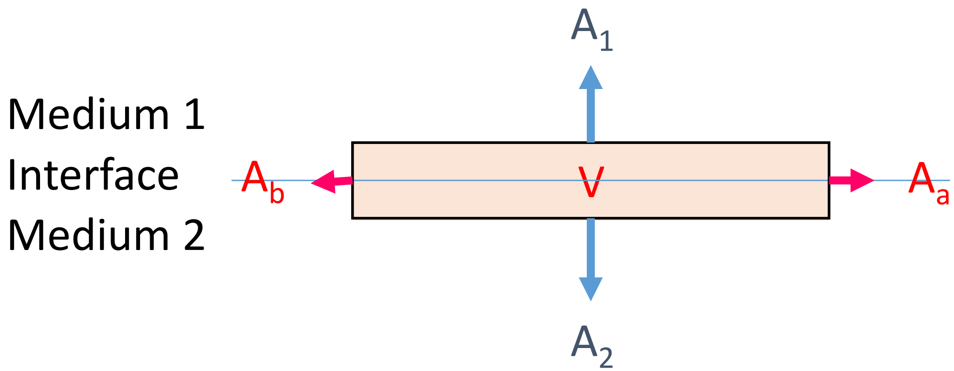
1. Make a loop around interface
2. Collapse loop perpendicular to interface
3. A→0 (so rhs→0 *except for surface current*), L_a→0, L_b→0
4. Final line integral has only tangential fields e.g. $E_{2t}\, L_2$ - $E_{1t}\, L_1 = 0$
5. Notice $L_1 = -L_2$ so that factor disappears

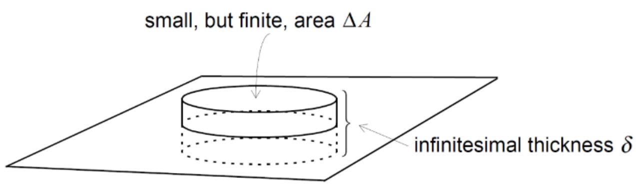
$$\Delta E_{=} = 0 \qquad\qquad \Delta H_{=} = J_s$$

Normal component continuity

$$\oint \boldsymbol{D} \cdot d\boldsymbol{A} = \int \rho_{free} dV$$

$$\oint \boldsymbol{B} \cdot d\boldsymbol{A} = 0$$





1. Make a cylinder across interface
2. Collapse cylinder perpendicular to interface
3. V → 0 (so rhs → 0 *except for surface charge*), A_a → 0, A_b → 0
4. Final surface integral only has normal components e.g. $D_{1n} A_1 - D_{2n} A_2 = 0$
5. Notice $A_1 = -A_2$, so that factor disappears

$$\Delta D_{\perp} = \rho_s$$

$$\Delta B_{\perp} = 0$$

Consequences of the interface conditions

Typically in optics (e.g. no external sources)

$$\boxed{\begin{aligned} \Delta E_{=} &= 0 \\ \Delta H_{=} &= J_s \\ \Delta D_{\perp} &= \rho_s \\ \Delta B_{\perp} &= 0 \end{aligned}} \qquad \boxed{\begin{aligned} \Delta E_{=} &= 0 \\ \Delta H_{=} &= \cancel{J_s} \\ \Delta D_{\perp} &= \cancel{\rho_s} \\ \Delta B_{\perp} &= 0 \end{aligned}}$$

- **Tangential E & H continuous**
- Normal D & B continuous

Normal E↓ as ε↑

What happens (at low ω) at a dielectric : metal interface?

Perfect electric conductor (PEC) approximation

- “E=0 inside PEC (otherwise induced current would be infinite)”
- “Interface condition implies no tangential E”
- “E outside must be normal to the surface”

PEC approximation useful for determining metal waveguide modes (later)

Don’t need it for Fresnel’s equations

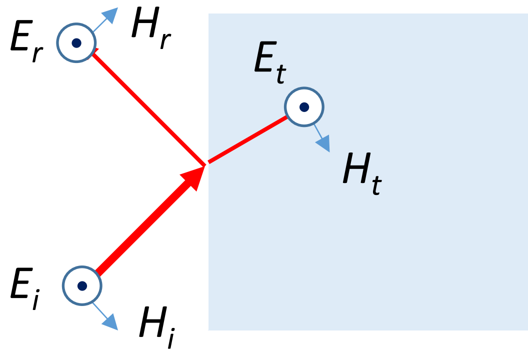
EM field interface examples

The next two pages show EM component fields in red & blue:

- ❖ *Why are the other components not shown?*
- ❖ *Which are the tangential/normal components?*
- ❖ *Which components are continuous across the interface?*
- ❖ *Why do the magnetic fields behave differently than the electric fields?*
- ❖ *Why does the wave on the left look different than the wave on the right?*
- ❖ *How does the angle of the transmitted wave compare to incidence?*

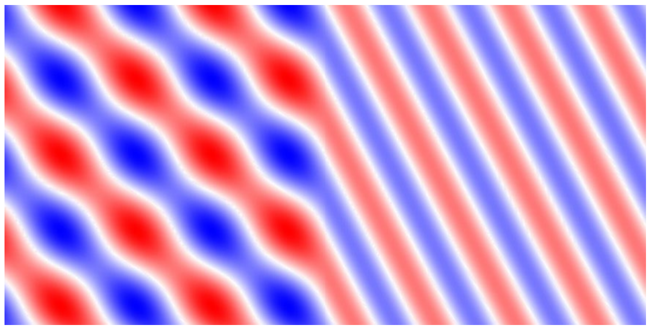
“s” polarization

$$\Delta E_{=} = 0$$
$$\Delta H_{=} = J_s$$
$$\Delta D_{\perp} = \rho_s$$
$$\Delta B_{\perp} = 0$$

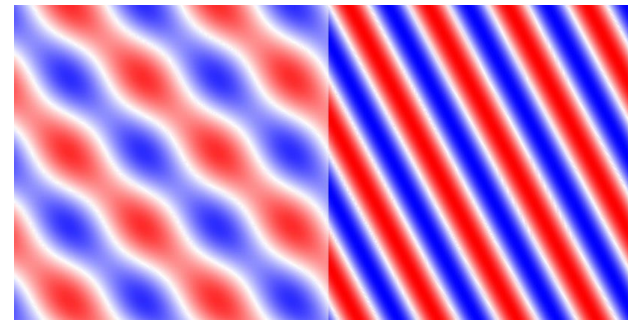


n=1	n=1.5
ε_r=1	ε_r=2.25
μ_r=1	μ_r=1
η_r=1	η_r=0.67

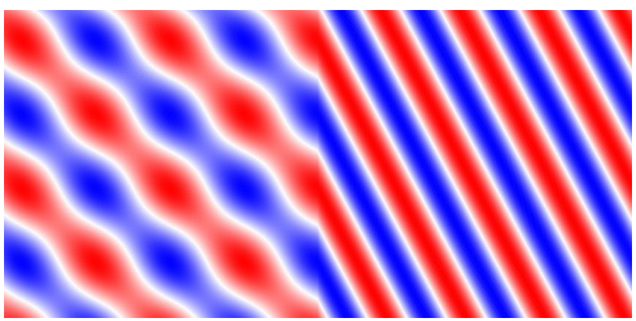
E in/out



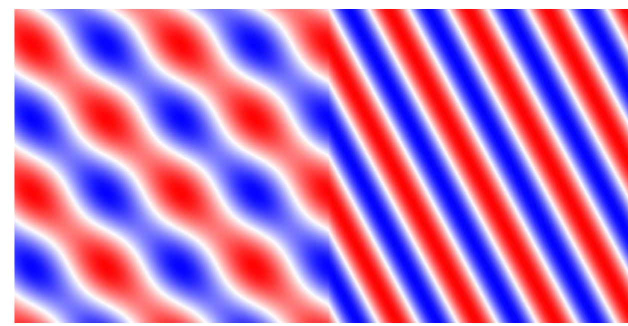
D in/out



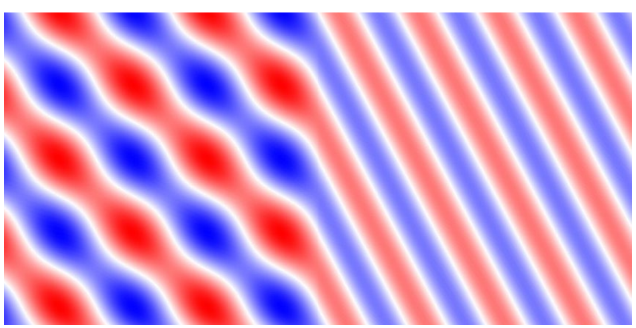
H ↕



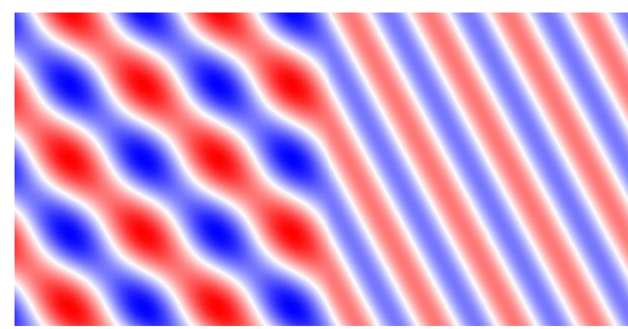
B ↕



H ↔

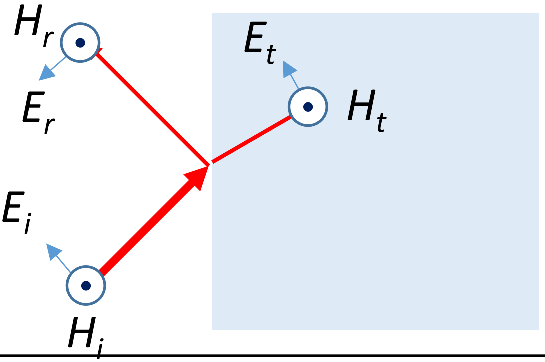


B ↔



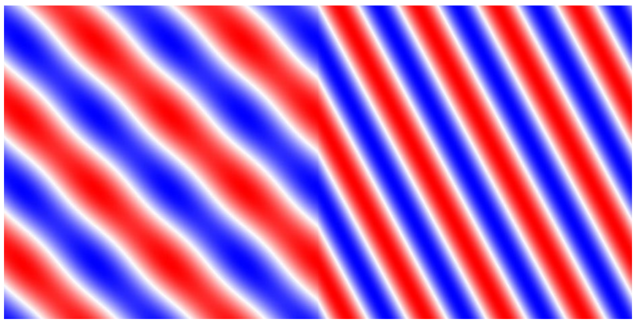
"p" polarization

$$\Delta E_{=} = 0$$
$$\Delta H_{=} = J_s$$
$$\Delta D_{\perp} = \rho_s$$
$$\Delta B_{\perp} = 0$$

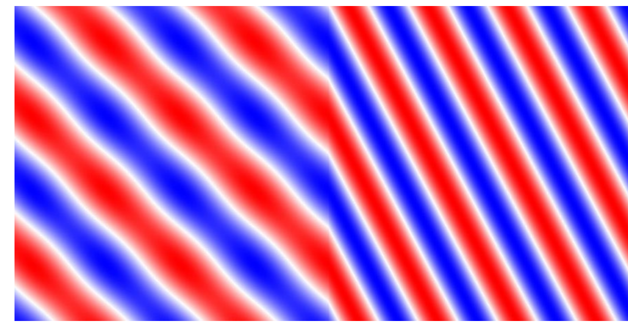


n=1	n=1.5
ε_r=1	ε_r=2.25
μ_r=1	μ_r=1
η_r=1	η_r=0.67

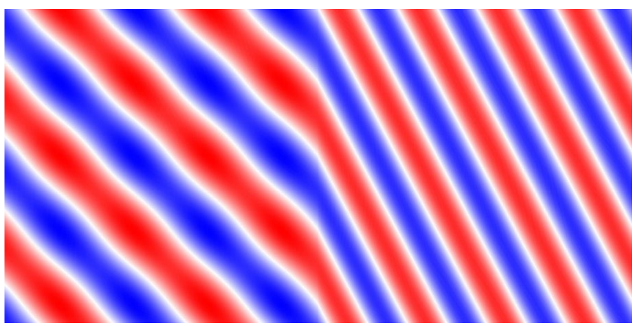
H in/out



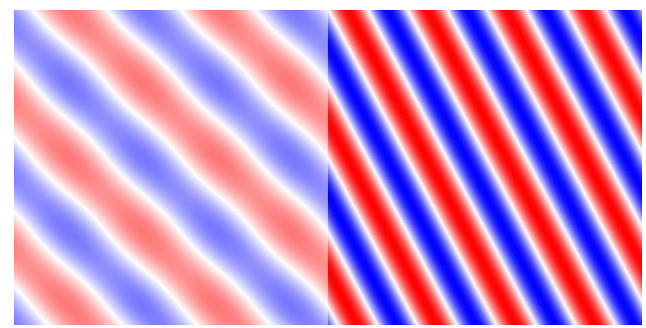
B in/out



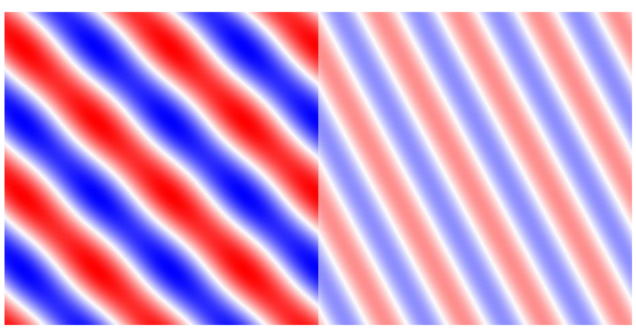
E ↕



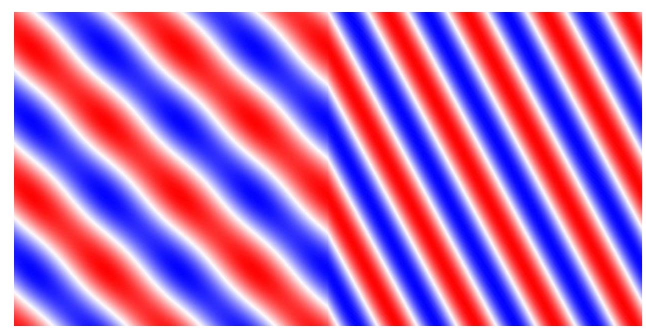
D ↕



E ↔



D ↔



Recap for Fresnel's equations

What we know so far:

✓ **Properties of time-harmonic EM wave in simple media**

$$\mathbf{k} \perp \mathbf{E} \perp \mathbf{H} \qquad n \equiv \frac{c}{v} = \sqrt{\frac{\varepsilon}{\varepsilon_0}\frac{\mu}{\mu_0}} \qquad \eta \equiv \frac{E}{H} = \sqrt{\frac{\mu}{\varepsilon}} = 1/(v\varepsilon) \qquad v = \frac{\omega}{k} \qquad S_{av} \propto \mathrm{Re}[n]|E|^2$$

✓ **Conservation**

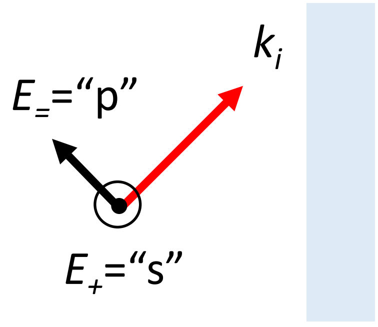
$$k_0 = k_r / n_1 = k_i / n_1 = k_t / n_2$$

$$k_r \sin\theta_r = k_i \sin\theta_i = k_t \sin\theta_t$$

✓ **Continuity of fields**

$$\boxed{\begin{aligned} \Delta E_= &= 0 \\ \Delta H_= &= J_s \\ \Delta D_\perp &= \rho_s \\ \Delta B_\perp &= 0 \end{aligned}}$$

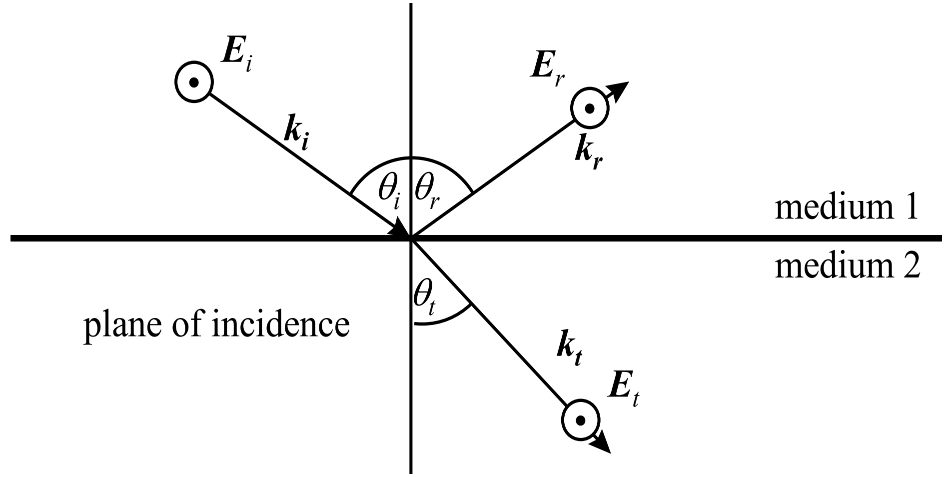
+ Two polarizations "s" & "p"



+ Geometry → Fresnel's equations

Fresnel Equations for E

e.g. "**s**" polarization



We will need E --> H
Use impedance = E/H
Assuming non-mag
$H = En/\eta_0$
η_0 is common factor, will cancel

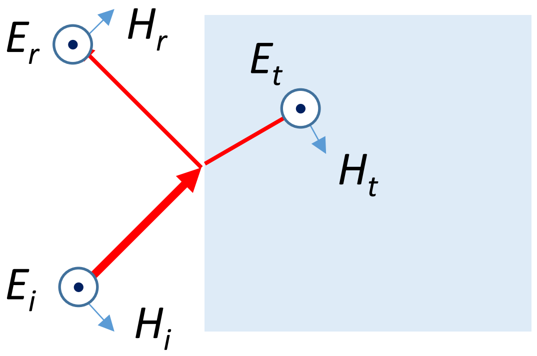
- Draw on unspecified field H (use k || S ~ E x H & RHR)
- Drop complex exponentials – they're matched already by conservation
- Note total tangential E fields are continuous:
- Write down total tangential E on each side:
- Divide all by incident field:
- Replace with $r=E_r/E_i$ etc. Eq (1):
- Note total tangential H fields are continuous:
- Projecting total tangential H on each side, -left, +right…
- Also noting $\theta_r=\theta_i$
- Convert H->E using field ratio:
- Divide by incident and use $r=E_r/E_i$ etc. Eq (2):
- Solve simultaneous equation for Eqs (1) & (2):

$$r_\perp \equiv \left(\frac{E_{0r}}{E_{0i}}\right)_\perp = \frac{n_i\cos\theta_i - n_t\cos\theta_t}{n_i\cos\theta_i + n_t\cos\theta_t} \qquad t_\perp \equiv \left(\frac{E_{0t}}{E_{0i}}\right)_\perp = \frac{2n_i\cos\theta_i}{n_i\cos\theta_i + n_t\cos\theta_t}$$

Phase: important for thin film devices

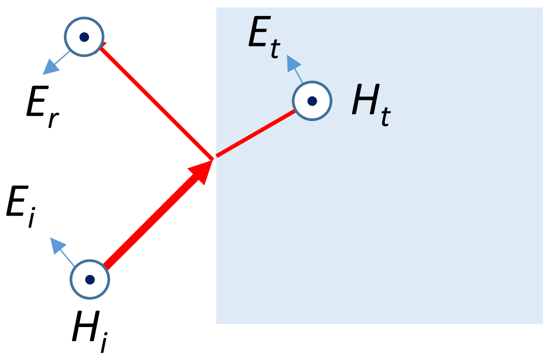
Does negative coefficient actually = negative phase?
Need to check diagram

"s" polarization



All E same direction
-ve coefficient is –ve phase

"p" polarization

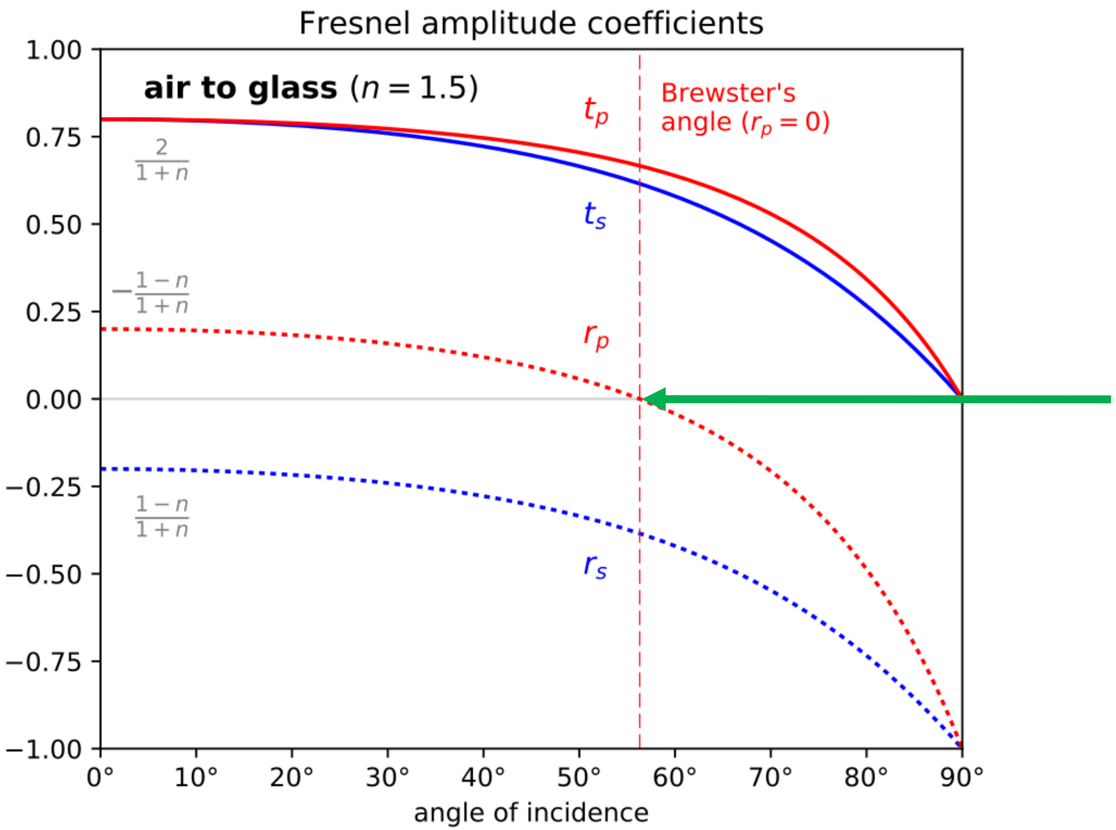


Less obvious
H consistent so E "consistent"
(rotate all so k aligned to see)
-ve coefficient is –ve phase

Our sign convention means –ve coefficient = -ve phase
Not true for other Fresnel equations

https://en.wikipedia.org/wiki/Fresnel_equations

Phase near Brewster angle



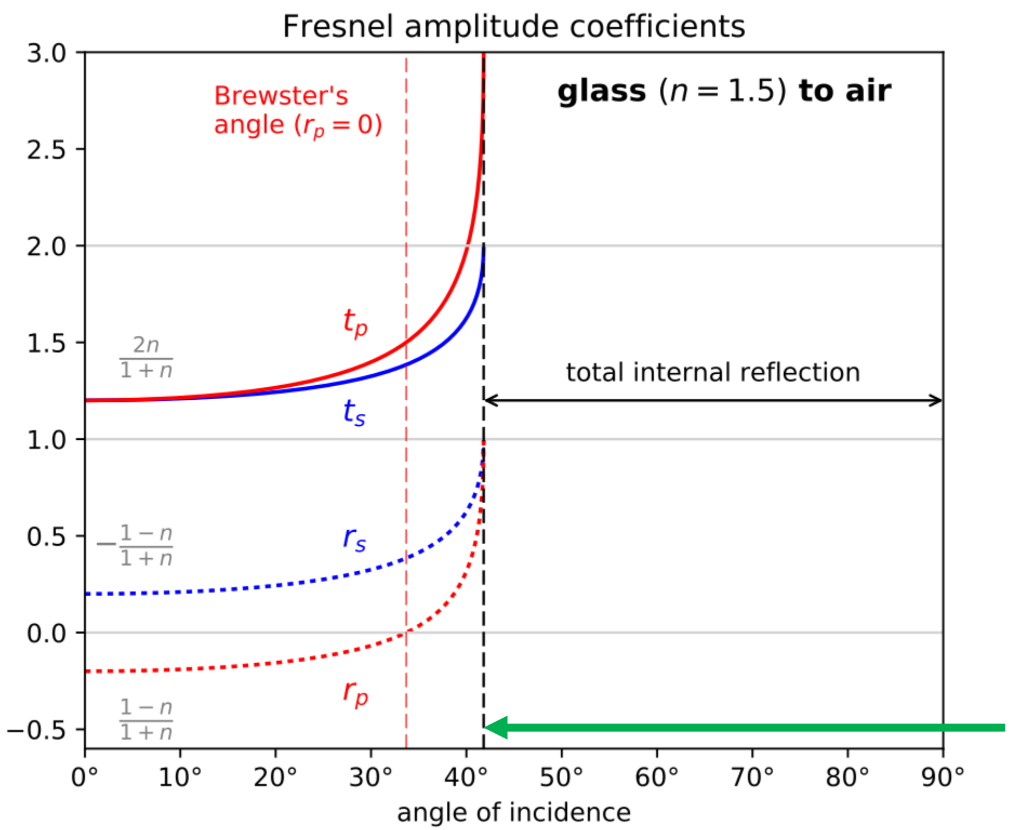
Obvious phase shift of r_p @ ϑ_B

Phase-shifted above ϑ_B

r_s phase-shifted @ all angles

https://en.wikipedia.org/wiki/Fresnel_equations

Phase near critical angle



(*Aside: t* is greater than one. How is this possible?)

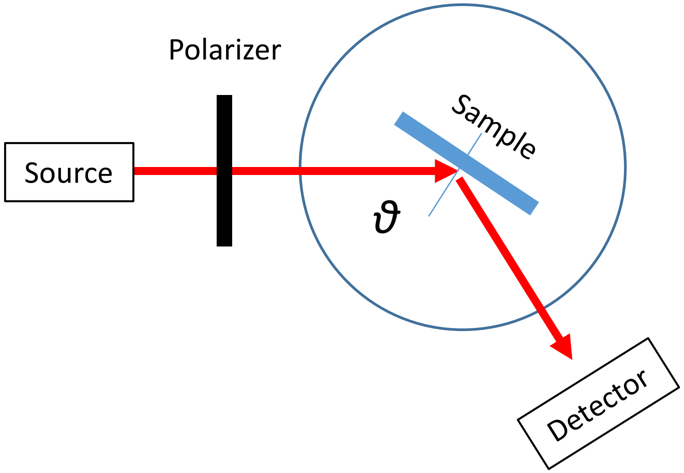
r phase reversed compared to external reflection

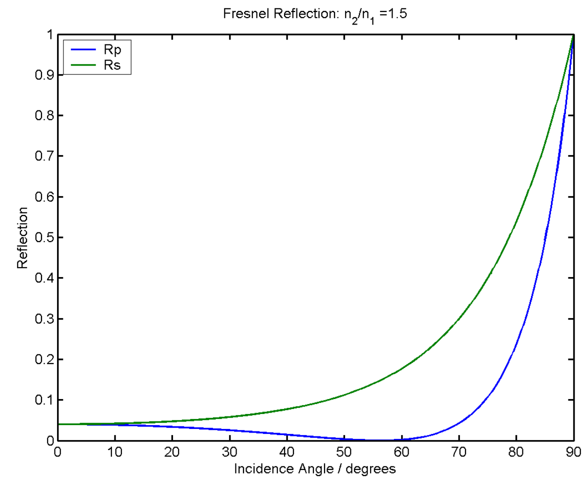
Beyond ϑ_c the coeffs are complex
Phase lies between 0 & π

https://en.wikipedia.org/wiki/Fresnel_equations

Summary

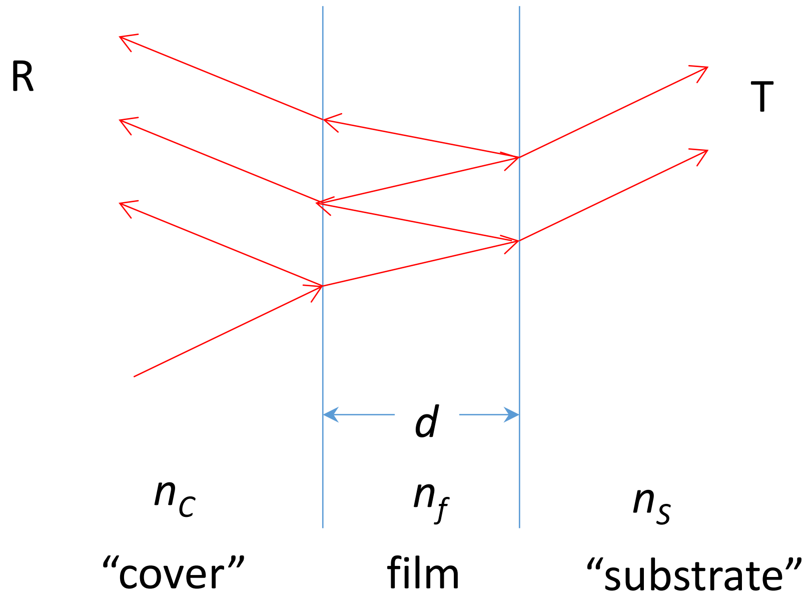
- **Fresnel's equations** give angles, power & phase at interfaces
- Come from wave & EM specific properties
- Total internal reflection beyond critical angle $\theta_c = \sin^{-1}(n_2 / n_1)$
- Minimum R, maximum pol @ Brewster angle $\theta_B = \tan^{-1}(n_2 / n_1)$
- Useful for (e.g.) measuring refractive index
- FE = k conservation + EH continuity+ impedance + geometry





Thin films

- **Important practical application of optics**
- Additional interfaces create reflection backwards & forwards
- Full modelling needs both **Fresnel** & **interference**
- Possible to write down answer for single film
- Film stacks need matrices – use software (e.g. OpenFilters)

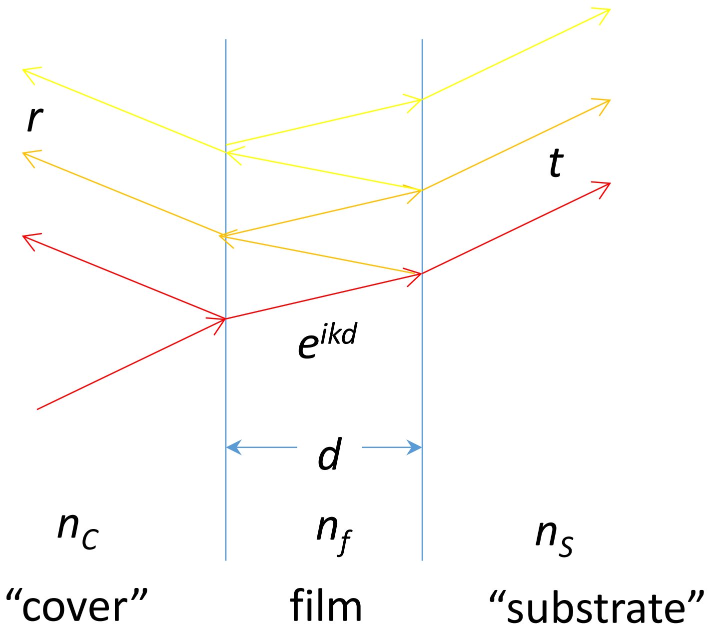


Important examples:
Quarter wave (anti-reflection)
Quarter wave high-low stack (mirror)

http://en.wikipedia.org/wiki/Thin-film_interference

Multibeam interference model

<u>Add up all beams...</u>



$$t = \boxed{t_{cf}e^{ikd}[1} + \boxed{r_{fsf}e^{ikd}r_{fcf}e^{ikd}(1} + \boxed{r_{fsf}e^{ikd}r_{fcf}e^{ikd}(1} + \ldots]\boxed{t_{fs}}$$

$$= t_{cf}t_{fs}e^{ikd}\sum_{b=0}^{\infty}[r_{fsf}r_{fcf}e^{i2kd}]^b$$

$$= \frac{t_{cf}t_{fs}e^{ikd}}{1 - r_{fsf}r_{fcf}e^{i2kd}}$$

$$r = \boxed{r_{cfc}} + \boxed{t_{cf}e^{ikd}r_{fsf}e^{ikd}[1} + \boxed{r_{fcf}e^{ikd}r_{fsf}e^{ikd}(1} + \ldots]\boxed{t_{fc}}$$

$$= r_{cfc} + t_{cf}t_{fc}e^{i2kd}r_{fsf}\sum_{b=0}^{\infty}[r_{fcf}r_{fsf}e^{i2kd}]^b$$

$$= r_{cfc} + \frac{t_{cf}t_{fc}e^{i2kd}r_{fsf}}{1 - r_{fsf}r_{fcf}e^{i2kd}}$$

Problem: even more complicated for more than two interfaces!

(A sketch of a) matrix model

- Transfer of continuous field components (tangential E & H)
- Relate to each other: $-H_{=} = \gamma E_{=} = (n\cos\theta/\eta_0)E_{=}$ “s”

  $E_{=} = \gamma H_{=} = (\eta_0 \cos\theta/n)H_{=}$ “p”

  [different γ]

- Observe the effect of propagation within a layer $e^{ik_{\perp}d} = e^{i\phi}$

  (actually use reverse travel direction for numerical stability)

- Thin film matrix $\begin{bmatrix} E' \\ H' \end{bmatrix} = M \begin{bmatrix} E \\ H \end{bmatrix}$ $M = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} = \begin{bmatrix} \cos\phi & -(i/\gamma)\sin\phi \\ -(i\gamma)\sin\phi & \cos\phi \end{bmatrix}$

- Thin film stack (→C 1 2 ... S) $M = M_1 M_2 \ldots$
- Solve fields in cover (1+r) & substrate (t)

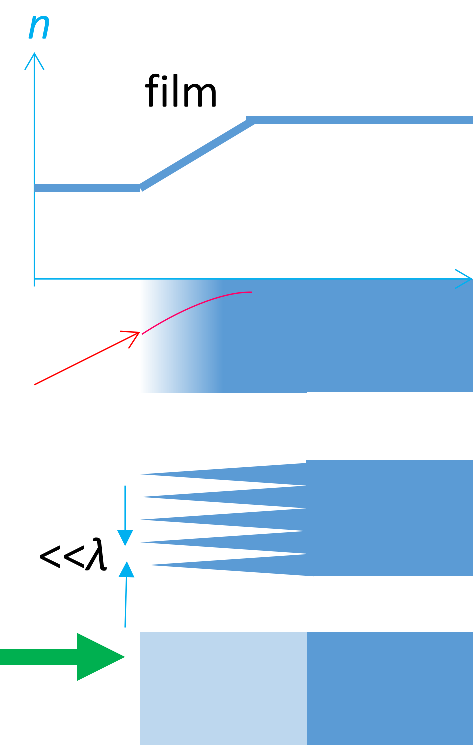
$$r = \frac{\gamma_c m_{11} + \gamma_c \gamma_s m_{12} - m_{21} - \gamma_s m_{22}}{\gamma_c m_{11} + \gamma_c \gamma_s m_{12} + m_{21} + \gamma_s m_{22}} \qquad t = \frac{2\gamma_c}{\gamma_c m_{11} + \gamma_c \gamma_s m_{12} + m_{21} + \gamma_s m_{22}}$$

$$R = |r|^2 \qquad T = \mathrm{Re}[\gamma_s/\gamma_c]|t|^2$$

Hecht Ch 9

Antireflection (AR) coatings

- Reflection reduces efficiency of transmissive components
- Best AR coating has graded index (GRIN)
  - e.g. from air n=1 to glass n=1.5
  - **Challenge: normal materials restricted to n>1.3**
  - Research & natural solution: nanostructure
  - Commercial solution:
    approximate with solid layers ($\lambda/4$ MgF_2, BBAR)



http://en.wikipedia.org/wiki/Anti-reflective_coating

Quarter-wave AR

- Single quarter-wave optical thickness

e.g. for λ_0=550nm & MgF_2 (n=1.38), $d=\lambda_0/(4n)$=...

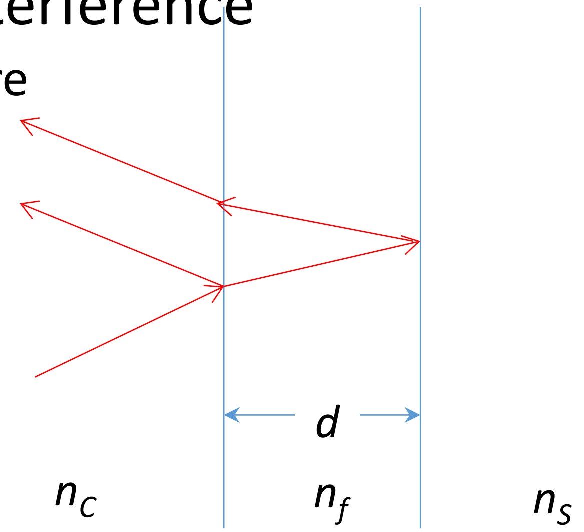
- Round-trip adds 180° phase = destructive interference
  - Works well near design λ (and ϑ), poorly elsewhere
- Ideally $n_f^2 = n_c n_s$

e.g. air-to-glass, ideal n_f=...

No solid materials are suitable for glass, usually use MgF_2.

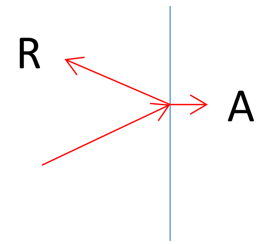
e.g. air-to-silicon (n_c=3.4), ideal n_f=...

- Single layer does not perform well
  - Use multiple layers
  - Can get broadband AR (but terrible outside band, poor tilt tolerance)



https://www.cvimellesgriot.com/Products/Documents/GeneralInfo/AntireflectionCoatings.pdf

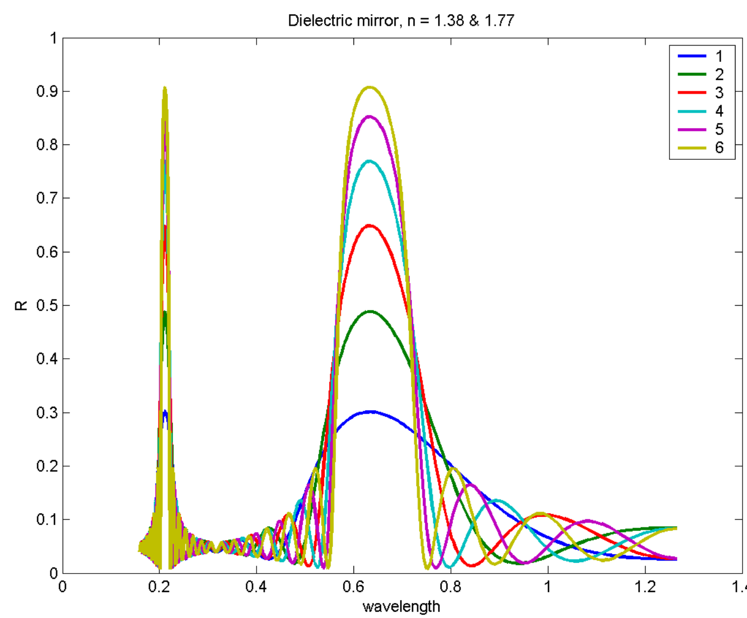
Dielectric mirrors



- Metals simple reflectors, BUT
  - Reflection < 100%
  - Absorption (heating, damage, inefficiency)
  - Not always practical to use (bio-organisms, poisons fab processes)

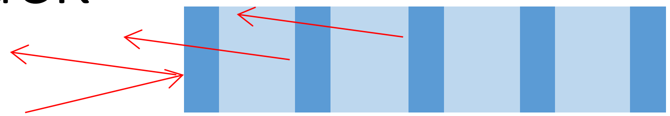
e.g. Al @ 633nm, $n+i\kappa=1.4+7.6i$, normal $R=[(n-1)^2+\kappa^2]/[(n+1)^2+\kappa^2]=\ldots$
where has the rest of the energy gone?

- Dielectric mirrors
  - Minimal heating (no A)
  - Beam splitting: R & T (no A)
  - Spectral filtering (interference)



http://en.wikipedia.org/wiki/Dielectric_mirror

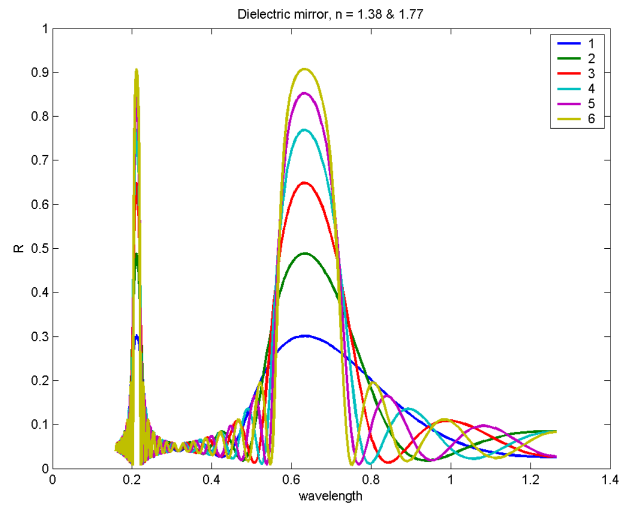
High-low stack



(N=4)

- Alternating λ/4 layers of high-low index
- Bragg diffraction (type of photonic crystal)
- Field decays rapidly with depth

  (waves in the reflection band are effectively evanescent)
- More layers (N) = better *R*, sharper edges



$$R_{\max} = \left[\frac{1 - n_H^2 (n_H / n_L)^{2N} / n_C n_S}{1 + n_H^2 (n_H / n_L)^{2N} / n_C n_S} \right]^2$$

- Better contrast (n_H/n_L) = as above + bigger band

$$\Delta\lambda / \lambda = (4/\pi)\sin^{-1}[(n_H - n_L)/(n_H + n_L)]$$

e.g. Bandwidth of silicon (3.48) & silica (1.53) @ 1.55μm=...

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