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1. The datasets: MNIST

1.1. Description

The MNIST database (Modified National Institute of Standards and Technology database) is a very popular Machine Learning dataset comosed of large collection of handwritten digits. It has 70 000 black-and-white 28x28 pixels images with a handwritten digit ('0' to '9'). The goal is to classify the black-and-white 28x28 pixels images among these 10 classes.

1.2. Load the dataset

In [1]:
```python
import tensorflow as tf
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt

X = np.load('MNIST_X_28x28.npy')
y = np.load('MNIST_y.npy')
```

1.4. Works / Questions

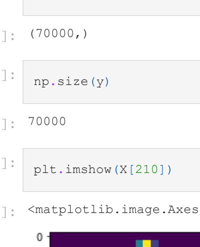
Q1/ What are the shape of the data? Display samples from the dataset

In [2]:
```python
np.shape(X)
```
Out[2]: (70000, 28, 28)

In [3]:
```python
np.shape(y)
```
Out[3]: (70000,)

In [4]:
```python
np.size(y)
```
Out[4]: 70000

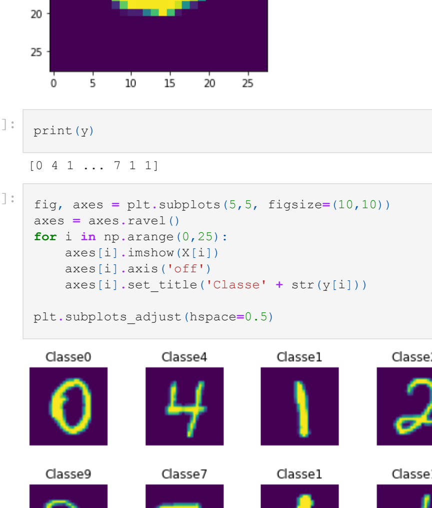
In [5]:
```python
plt.imshow(X[210])
```
Out[5]: <matplotlib.image.AxesImage at 0x1b8410b1310>



In [6]:
```python
print(y)
```
[0 4 1 ... 7 1 1]

In [7]:
```python
fig, axes = plt.subplots(5,5, figsize=(10,10))
axes = axes.ravel()
for i in np.arange(0,25):
    axes[i].imshow(X[i])
    axes[i].axis('off')
    axes[i].set_title('Classe' + str(y[i]))

plt.subplots_adjust(hspace=0.5)
```



Q2/ Use the sklearn method train_test_split to split the dataset in one train set and one test set. Why this split is important in Machine Learning?

In [8]:
```python
from sklearn.model_selection import train_test_split
(X_train, X_test, y_train, y_test) = train_test_split(X, y,  test_size = 0.2)
```

In [9]:
```python
np.shape(X_train)
```
Out[9]: (56000, 28, 28)

In [10]:
```python
np.shape(y_train)
```
Out[10]: (56000,)

In [11]:
```python
np.shape(X_test)
```
Out[11]: (14000, 28, 28)

In [12]:
```python
np.shape(y_test)
```
Out[12]: (14000,)

We split the data to evaluate the performance of our Machine Learning model and to identify overfitting.

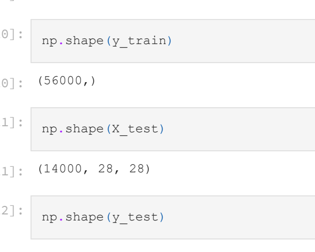
We notice that our training data represents 80% of the MNIST Dataset whilst our test data represents 20% as we set test_size to 0.2 (20%).

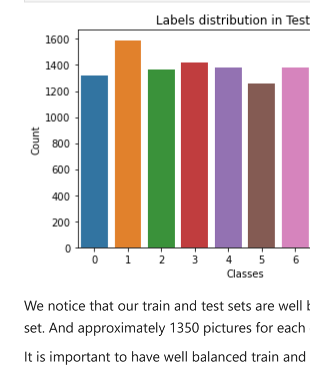
.

Q3/ Are the train and test sets well balanced (distribution of labels)? Why is it important for supervised Machine Learning?

In [13]:
```python
from collections import Counter
class_frequencies = Counter (y_train)
print (class_frequencies)
```
Counter({1: 6287, 7: 5813, 3: 5722, 2: 5622, 0: 5586, 9: 5547, 6: 5499, 4: 5447, 8: 5418, 5: 5059})

In [14]:
```python
import seaborn as sns
fig = sns.countplot(x=y_train)
plt.xlabel("Classes")
plt.ylabel("Count")
plt.title("Labels distribution in Train Set")
plt.show(fig)
```





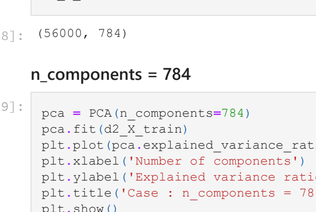
2. Unsupervised Machine Learning

2.1. Dimensionality reduction

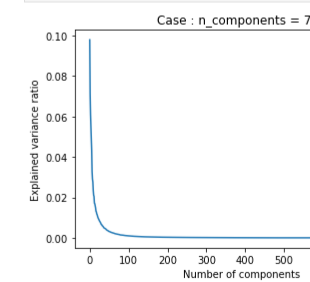
Works / Questions

Q1/ Perform a Principal Component Analysis (PCA) with sklearn-Try to keep different n_components

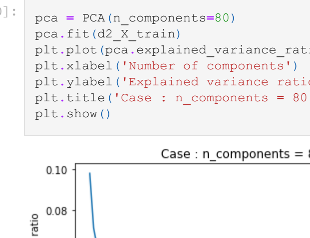
n_components = 784



n_components = 80



n_components = 220



Q2/ An interesting feature is PCA.explained_varianceratio. Explain these values according to your understanding of PCA and use these values to fit a relevant value for n_components

```
2.19782144e-11 1.87783080e-11 1.85734224e-11 1.73592757e-11
1.49796301e-11 1.48034890e-11 1.18987479e-11 1.14049965e-11
1.03784706e-11 8.51991657e-12 8.31313178e-12 8.31313178e-12
8.31313178e-12 8.31313178e-12 8.31313178e-12 8.31313178e-12
8.31313178e-12 8.31313178e-12 8.31313178e-12 8.31313178e-12
8.31313178e-12 8.31313178e-12 8.31313178e-12 8.31313178e-12
8.31313178e-12 8.31313178e-12 8.31313178e-12 8.31313178e-12
8.31313178e-12 8.31313178e-12 8.31313178e-12 8.31313178e-12
8.31313178e-12 8.31313178e-12 8.31313178e-12 8.31313178e-12
8.31313178e-12 8.31313178e-12 8.31313178e-12 8.31313178e-12
8.31313178e-12 8.31313178e-12 8.31313178e-12 8.31313178e-12
8.31313178e-12 8.31313178e-12 8.31313178e-12 8.31313178e-12
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8.31313178e-12 8.31313178e-12 8.31313178e-12 8.31313178e-12
8.31313178e-12 8.31313178e-12 8.31313178e-12 8.31313178e-12
8.31313178e-12 8.31313178e-12 8.31313178e-12 8.31313178e-12
8.31313178e-12 8.31313178e-12 8.26238443e-12 7.25195395e-12
5.89863788e-12 5.46074232e-12 2.82532465e-12 1.88227097e-12]
```
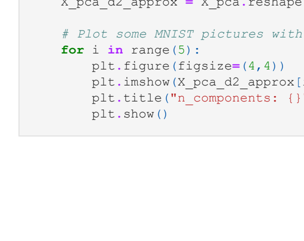

PCA.explained_variance*ratio* is used to get the ration of variance (eigenvalue / total eigenvalues). If we sum up the array's values it will be equal to 1 which means that if we gather all the principal components it will indeed explain the total of the variance of the data. Let's try to verify that using the following line of code :

In [24]:
```python
print(np.sum(pca.explained_variance_ratio_))
```
0.9999999999999998

In [25]:
```python
cumulative_explained_variance = np.cumsum(pca.explained_variance_ratio_)
n_components = np.where(cumulative_explained_variance >= 0.95)[0][0] + 1
print(n_components)
```
154

This means that having 154 components will enable us to explain 95% of the variance of the data. Furthermore, We can also find it approximately by plotting the graph, using the following code :

In [26]:
```python
plt.plot(cumulative_explained_variance)
plt.xlabel('Number of components')
plt.ylabel('Cumulative explained variance %')
plt.title('Cumulative explained variance Graph')
plt.grid()
plt.show()
```



Indeed, we notice that having 154 components give us a cumulative explained variance of 95%

Q3/ Display some MNIST pictures with different values of n_components.

In [27]:
```python
# Initialize the PCA model
n_components = [25, 64, 100, 169, 225]

for n in n_components:
    pca = PCA(n_components=n)

    reduced_d2_X = pca.fit_transform(d2_X_train)

    X_pca = pca.inverse_transform(reduced_d2_X)
    X_pca_d2_approx = X_pca.reshape(nsamples, nx, ny)

    # Plot some MNIST pictures with different values of n_components
    for i in range(5):
        plt.figure(figsize=(4,4))
        plt.imshow(X_pca_d2_approx[i])
        plt.title("n_components: {}".format(n))
        plt.show()
```

2.2. Data clustering

Questions / Works

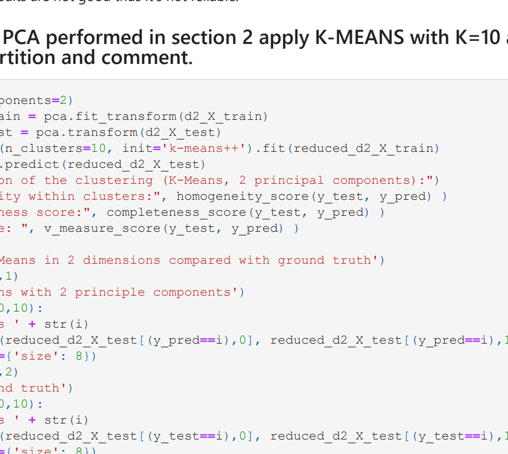
Q1/ You already split X (and y) in a train and test sets with the dataset method: split train test. With sklearn, perform K-MEANS. Play with the parameter K as well as the initialization (KMEANS++, random or fixed array).

K-means++ initialization

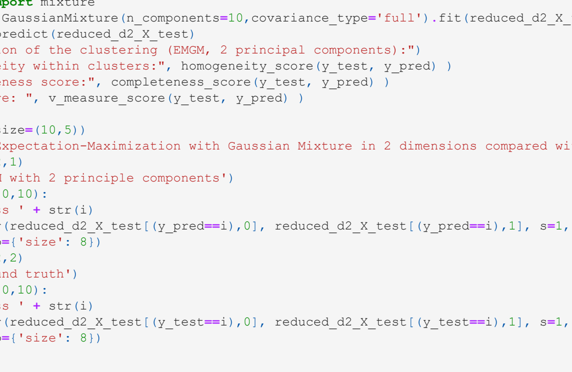
Random initialization

Q2/ For the values K (K=10), evaluate how good is the partition (with the knowledge of y).

Q3/ Using the PCA performed in section 2 apply K-MEANS with K=10 and n_components = 2. Display the partition and comment.



Q4/ Do the same job with the EM clustering using the good K parameter (10 for MNIST). Comment your results.



3. Supervised Machine Learning

3.1. Decision Tree, SVM and Logistic Regression

Questions / Works

Q1/ What is the major difference between Naive Bayes Classifier and Support Vector Machine (or Logistic Regression)? (if you forgot the course n2, a clue: what are we trying to predict?)

Q2/ With sklearn, perform a classification using your favorite methods. With the documentation, check how to modify the parameters and comment how it influences the results. For example, if you chose SVM, change the kernel between 'linear' and 'rbf' (Gaussian kernel). You can also play with 'C' parameter to switch from hard-margin SVM to soft-margin SVM.

Q2.1 Decision Tree :

Q2.2 SVM :

Q2.3 Naive Bayes :

Q2.4 Logistic Regression :

```
C:\Users\ayael\anaconda3\lib\site-packages\sklearn\linear_model\_logistic.py:444: ConvergenceWarning: lbfgs fai
led to converge (status=1):
STOP: TOTAL NO. of ITERATIONS REACHED LIMIT.

Increase the number of iterations (max_iter) or scale the data as shown in:
    https://scikit-learn.org/stable/modules/preprocessing.html
Please also refer to the documentation for alternative solver options:
    https://scikit-learn.org/stable/modules/linear_model.html#logistic-regression
  n_iter_i = _check_optimize_result(
Logistic regression (penalty L1):
Accuracy on train set:  0.9335178571428572 Accuracy on test set:  0.9169285714285714
Logistic regression (penalty L2):
Accuracy on train set:  0.9424642857142858 Accuracy on test set:  0.9175714285714286
```

We get a warning suggesting to try to make the solver converge since lbfgs has a better convergence on small datasets, it only stores few vectors that represent the gradients approximation in an implicit manner.

Q3/ With the score method, compute the accuracy of the model on the training and the test datasets. Why do we need to analyze the performance of the model at training and testing time?

For Decision tree we got:

- Model tree classifier: unrestricted model
- Depth: 50 , Number of leaves: 3742
- Accuracy on the training set: 1.0
- Accuracy on the test set: 0.87

Interpretation : First of all, "unrestricted model" means that the model don't have any restrictions on its complexity or size. We notice that the deeper the tree (number of nodes/depths) the more accurate the training set is. Having an Accuracy on the training set of 1.0 means that the model is capable of correctly classifying all the data in the training set which can make us worried that we may have an overfitting model. But luckily the Accuracy on the test set is about 0.87 which means that there's no overfitting indeed, the model is able to accurately classify 87% of the data points in the test set. The decision tree model is for sure more accurate than clustering models that we previously used.

For SVM, it took so much time to run the model along with PCA and K-Cross in order to find the best kernel to use.

For Naive Bayes we got:

- Naive Bayes Classifier:
- Accuracy on train set: 0.56
- Accuracy on test set: 0.56

Interpretation : The model correctly predictes approximately 56% of the data points in the train set and again 56% on the test set. The Naives Bayes model is not really appropriate to classify our dataset's images.

For Logistic Regression we got:

- Logistic regression (penalty L1):
- Accuracy on train set: 0.93
- Accuracy on test set: 0.92
- Logistic regression (penalty L2):
- Accuracy on train set: 0.94
- Accuracy on test set: 0.92

Interpretation : Whether we use L1 or L2 norm, we get quite the same result. Although, the lbfgs didn't converge, we still have a great accuracy on the test sets because convergence is not a sufficient condition for a good model, however, it is a necessary condition for getting the optimal solution.

Q4/ In section 2.1. you applied a PCA to X so that the projected set – hereafter Xred –lies on a “reduced” space. Among the supervised methods you chose, select one method and apply your code to Xred. Does the PCA influence the performance of the classification (according to the intensity of the reduction)?

In [47]:

```
#!pip install prettytable
```

In [48]:

```
from prettytable import PrettyTable

info = [0.66, 0.8, 0.9, 0.95]
depths = len(info)*[None]
nleaves = len(info)*[None]
accuracies_train = len(info)*[None]
accuracies_test = len(info)*[None]

for index in range(0, len(info)):
    elem = info[index]
    pca = PCA(elem)
    reduced_d2_X_train = pca.fit_transform(d2_X_train)
    reduced_d2_X_test = pca.transform(d2_X_test)
    tree = DecisionTreeClassifier()
    model = tree.fit(reduced_d2_X_train, y_train)
    depths[index] = tree.get_depth()
    nleaves[index] = tree.get_n_leaves()
    accuracies_train[index] = tree.score(reduced_d2_X_train, y_train)
    accuracies_test[index] = tree.score(reduced_d2_X_test, y_test)

t = PrettyTable()
t.add_column("% information", info)
t.add_column("Tree depth", depths)
t.add_column("number of leaves", nleaves)
t.add_column("Accuracy on train set", accuracies_train)
t.add_column("Accuracy on test set", accuracies_test)
print(t)

res_model.append("Decision tree")
res_param.append("Unrestricted max depth \n66% of information")
res_train_acc.append(accuracies_train[0])
res_valid_acc.append("-")
res_test_acc.append(accuracies_test[0])
```

% information	Tree depth	number of leaves	Accuracy on train set	Accuracy on test set
0.66	40	4865	1.0	0.8479285714285715
0.8	47	4658	1.0	0.8401428571428572
0.9	51	4519	1.0	0.8349285714285715
0.95	57	4458	1.0	0.8299285714285715

- Interpretation: The decision tree model overfits more when it has more tree depth. Indeed, overfitting occurs when the model is too complex.

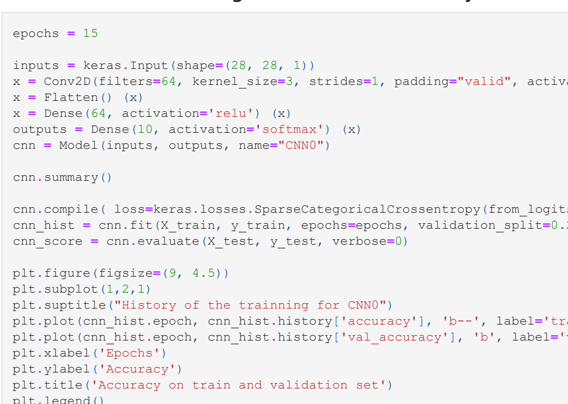
[illegible]

```
x = Dense(128, activation='relu') (inputs)
x = Dense(64, activation='relu') (x)
x = Dense(32, activation='relu') (x)
outputs = Dense(10, activation='softmax') (x)
mlp4 = Model(inputs, outputs, name="MLP4")

inputs = keras.Input(shape=(d2_X_train.shape[1], ))
x = Dense(128, activation='relu') (inputs)
x = Dropout(0.15) (x)
x = Dense(64, activation='relu') (x)
x = Dropout(0.15) (x)
x = Dense(32, activation='relu') (x)
outputs = Dense(10, activation='softmax') (x)
mlp5 = Model(inputs, outputs, name="MLP5")

mlp4.compile( loss=keras.losses.SparseCategoricalCrossentropy(from_logits=True), optimizer=keras.optimizers.Ada
mlp4_hist = mlp4.fit(d2_X_train, y_train, epochs=epochs, validation_split=0.2, verbose=0)
mlp4_score = mlp4.evaluate(d2_X_test, y_test, verbose=0)
mlp5.compile( loss=keras.losses.SparseCategoricalCrossentropy(from_logits=True), optimizer=keras.optimizers.Ada
mlp5_hist = mlp5.fit(d2_X_train, y_train, epochs=epochs, validation_split=0.2, verbose=0)
mlp5_score = mlp5.evaluate(d2_X_test, y_test, verbose=0)

mlp4.summary()

plt.figure(figsize=(9, 4.5))
plt.subplot(1,2,1)
plt.suptitle("History of the trainning for MLP4 and MLP5")
plt.plot(mlp4_hist.epoch, mlp4_hist.history['accuracy'], 'b--', label='MLP4 Train')
plt.plot(mlp4_hist.epoch, mlp4_hist.history['val_accuracy'], 'b', label='MLP4 Validation')
plt.plot(mlp5_hist.epoch, mlp5_hist.history['accuracy'], 'r--', label='MLP5 Train')
plt.plot(mlp5_hist.epoch, mlp5_hist.history['val_accuracy'], 'r', label='MLP5 Validation')
plt.xlabel('Epochs')
plt.ylabel('Accuracy')
plt.title('Accuracy on train and validation set')
plt.legend()
plt.subplot(1,2,2)
plt.plot(mlp4_hist.epoch, mlp4_hist.history['loss'], 'b--', label='MLP4 Train')
plt.plot(mlp4_hist.epoch, mlp4_hist.history['val_loss'], 'b', label='MLP4 Validation')
plt.plot(mlp5_hist.epoch, mlp5_hist.history['loss'], 'r--', label='MLP5 Train')
plt.plot(mlp5_hist.epoch, mlp5_hist.history['val_loss'], 'r', label='MLP5 Validation')
plt.xlabel('Epochs')
plt.ylabel('Loss')
plt.title('Loss on train and validation set')
plt.legend()
plt.show()
print(mlp4.name,": Test loss:", mlp4_score[0], "Test accuracy:", mlp4_score[1])
print(mlp5.name,": Test loss:", mlp5_score[0], "Test accuracy:", mlp5_score[1])

res_model.append("MLP")
res_param_string = "3 hidden layers," + str(mlp4.count_params()) + " parameters \n" + str(mlp4_hist.epoch[-1]+1
res_param.append(res_param_string)
res_train_acc.append(mlp4_hist.history['accuracy'][-1])
res_valid_acc.append(mlp4_hist.history['val_accuracy'][-1])
res_test_acc.append(mlp4_score[1])
res_model.append("MLP")
res_param_string = "3 hidden layers," + str(mlp5.count_params()) + " parameters \n" + str(mlp5_hist.epoch[-1]+1
res_param.append(res_param_string)
res_train_acc.append(mlp5_hist.history['accuracy'][-1])
res_valid_acc.append(mlp5_hist.history['val_accuracy'][-1])
res_test_acc.append(mlp5_score[1])
```

Model: "MLP4"

Layer (type)	Output Shape	Param #
input_4 (InputLayer)	[(None, 784)]	0
dense_12 (Dense)	(None, 128)	100480
dense_13 (Dense)	(None, 64)	8256
dense_14 (Dense)	(None, 32)	2080
dense_15 (Dense)	(None, 10)	330

```
Total params: 111,146
Trainable params: 111,146
Non-trainable params: 0
```




```
MLP4 : Test loss: 0.20590750873088837 Test accuracy: 0.9652857184410095
MLP5 : Test loss: 0.14148634612861633 Test accuracy: 0.9690714478492737
```

MLP4 is clearly overfitting since its training loss << validation loss and its validation loss keeps increasing while the train loss keeps decreasing. MLP5 is performing well since its training loss and its validation loss are approximately the same. The dropout method helped to reduce overfitting indeed.

Convolutional Neural Network (CNN)

Q1/ What is the size of the input tensor? Why it is not the same as for your previous MLP model?

The images are grayscale so the size of the input tensor is (samples_number × 28 × 28 × 1). We know that CNN is more relevant than MLP when we deal with an image classification problem.

Q5/ Comment the training results. Is it better than your MLP? Faster? Lighter?

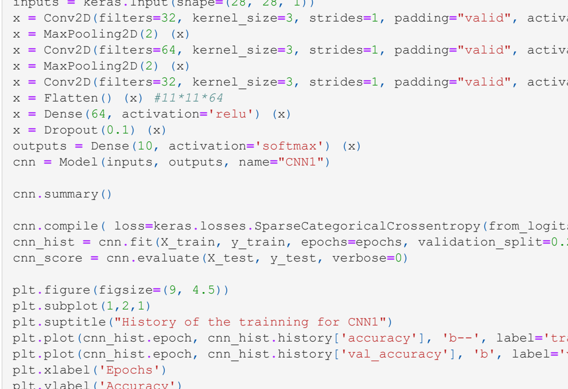
In [61]:

```
epochs = 15

inputs = keras.Input(shape=(28, 28, 1))
x = Conv2D(filters=64, kernel_size=3, strides=1, padding="valid", activation='relu', input_shape=(32,28,28,1))
x = Flatten() (x)
x = Dense(64, activation='relu') (x)
outputs = Dense(10, activation='softmax') (x)
cnn = Model(inputs, outputs, name="CNN0")

cnn.summary()

cnn.compile( loss=keras.losses.SparseCategoricalCrossentropy(from_logits=True), optimizer=keras.optimizers.Adam
cnn_hist = cnn.fit(X_train, y_train, epochs=epochs, validation_split=0.2, verbose=0)
cnn_score = cnn.evaluate(X_test, y_test, verbose=0)

plt.figure(figsize=(9, 4.5))
plt.subplot(1,2,1)
plt.suptitle("History of the trainning for CNN0")
plt.plot(cnn_hist.epoch, cnn_hist.history['accuracy'], 'b--', label='train')
plt.plot(cnn_hist.epoch, cnn_hist.history['val_accuracy'], 'b', label='validation')
plt.xlabel('Epochs')
plt.ylabel('Accuracy')
plt.title('Accuracy on train and validation set')
plt.legend()
plt.subplot(1,2,2)
plt.plot(cnn_hist.epoch, cnn_hist.history['loss'], 'b--', label='train')
plt.plot(cnn_hist.epoch, cnn_hist.history['val_loss'], 'b', label='validation')
plt.xlabel('Epochs')
plt.ylabel('Loss')
plt.title('Loss on train and validation set')
plt.legend()
plt.show()
print("CNN: Test loss:", cnn_score[0], "Test accuracy:", cnn_score[1])

res_model.append("CNN")
res_param_string = "1 convolutionnal layer," + str(cnn.count_params()) + " parameters \n" + str(cnn_hist.epoch[
res_param.append(res_param_string)
res_train_acc.append(cnn_hist.history['accuracy'][-1])
res_valid_acc.append(cnn_hist.history['val_accuracy'][-1])
res_test_acc.append(cnn_score[1])
```

Model: "CNN0"

Layer (type)	Output Shape	Param #
input_6 (InputLayer)	[(None, 28, 28, 1)]	0
conv2d (Conv2D)	(None, 26, 26, 64)	640
flatten (Flatten)	(None, 43264)	0
dense_20 (Dense)	(None, 64)	2768960
dense_21 (Dense)	(None, 10)	650

```
Total params: 2,770,250
Trainable params: 2,770,250
Non-trainable params: 0
```




```
CNN: Test loss: 0.24335673451423645 Test accuracy: 0.9739285707473755
```

Using CNN on our dataset is more computationally expensive than using MLP thus it takes longer to run and it's not lighter than MLP in terms of memory and computational requirements.

Q6/ Is there any overfitting? Why? If yes, what could be the causes? How to fix this issue?If you do not observe overfitting, how can you make your model overfit? Try and demonstrate the overfitting.

Yes the model is clearly overfitting! The train loss of the model is below the validation loss and this validation loss keeps on increasing whilst the train loss keeps on decreasing.

Q7/ According to this first performance, change the architecture of the CNN(add/remove kernels, add/remove layers...) as well as hyper-parameters, explain why, what are the influence on the results...?

In [63]:

```
epochs = 15

inputs = keras.Input(shape=(28, 28, 1))
x = Conv2D(filters=32, kernel_size=3, strides=1, padding="valid", activation='relu', input_shape=(32,28,28,1))
x = MaxPooling2D(2) (x)
x = Conv2D(filters=64, kernel_size=3, strides=1, padding="valid", activation='relu', input_shape=(32,28,28,1))
x = MaxPooling2D(2) (x)
x = Conv2D(filters=32, kernel_size=3, strides=1, padding="valid", activation='relu', input_shape=(32,28,28,1))
x = Flatten() (x) #11*11*64
x = Dense(64, activation='relu') (x)
x = Dropout(0.1) (x)
outputs = Dense(10, activation='softmax') (x)
cnn = Model(inputs, outputs, name="CNN1")

cnn.summary()

cnn.compile( loss=keras.losses.SparseCategoricalCrossentropy(from_logits=True), optimizer=keras.optimizers.Adam
cnn_hist = cnn.fit(X_train, y_train, epochs=epochs, validation_split=0.2, verbose=0)
cnn_score = cnn.evaluate(X_test, y_test, verbose=0)

plt.figure(figsize=(9, 4.5))
plt.subplot(1,2,1)
plt.suptitle("History of the trainning for CNN1")
plt.plot(cnn_hist.epoch, cnn_hist.history['accuracy'], 'b--', label='train')
plt.plot(cnn_hist.epoch, cnn_hist.history['val_accuracy'], 'b', label='validation')
plt.xlabel('Epochs')
plt.ylabel('Accuracy')
plt.title('Accuracy on train and validation set')
plt.legend()
plt.subplot(1,2,2)
plt.plot(cnn_hist.epoch, cnn_hist.history['loss'], 'b--', label='train')
plt.plot(cnn_hist.epoch, cnn_hist.history['val_loss'], 'b', label='validation')
plt.xlabel('Epochs')
plt.ylabel('Loss')
plt.title('Loss on train and validation set')
plt.legend()
plt.show()
print("CNN: Test loss:", cnn_score[0], "Test accuracy:", cnn_score[1])

res_model.append("CNN")
res_param_string = "3 convolutionnal layers," + str(cnn.count_params()) + " parameters \n" + str(cnn_hist.epoch
res_param.append(res_param_string)
res_train_acc.append(cnn_hist.history['accuracy'][-1])
res_valid_acc.append(cnn_hist.history['val_accuracy'][-1])
res_test_acc.append(cnn_score[1])
```

Model: "CNN1"

Layer (type)	Output Shape	Param #
input_7 (InputLayer)	[(None, 28, 28, 1)]	0
conv2d_1 (Conv2D)	(None, 26, 26, 32)	320
max_pooling2d (MaxPooling2D)	(None, 13, 13, 32)	0
conv2d_2 (Conv2D)	(None, 11, 11, 64)	18496
max_pooling2d_1 (MaxPooling2D)	(None, 5, 5, 64)	0
conv2d_3 (Conv2D)	(None, 3, 3, 32)	18464
flatten_1 (Flatten)	(None, 288)	0
dense_22 (Dense)	(None, 64)	18496
dropout_2 (Dropout)	(None, 64)	0
dense_23 (Dense)	(None, 10)	650

```
Total params: 56,426
Trainable params: 56,426
Non-trainable params: 0
```



```
CNN: Test loss: 0.0737231969833374 Test accuracy: 0.9861428737640381
```

The model is still overfitting, we should consider decreasing the number of epochs and we need to accentuate more on the dropout method and weight decay.

Conclusion

We went through numerous models to classify handwritten digits images of the MNIST dataset. Each one of them had its cons and pros, some of them were overfitting but it only takes to change parameters and not neglect the power of regularization technics. We are so grateful to make it to the end of this project report, it was a great introductory adventure in the dazzling world of Machine Learning!

THE END