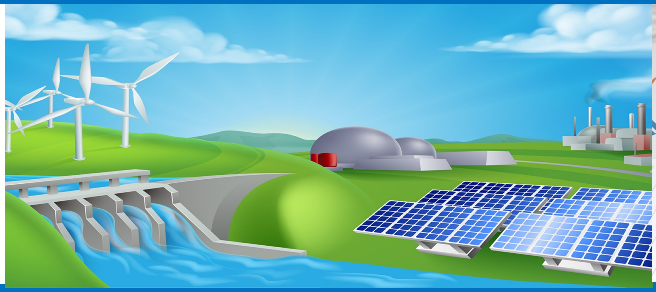



TIME SERIES ANALYSIS FOR ENERGY DATA

M8 - Model Diagnostics, Selection and Performance

Prof. Luana Medeiros Marangon Lima, Ph.D.

Nicholas School of the Environment - Duke University

Learning Goals

- Forecast fit vs forecast error
- Model Selection
  - Residual Analysis
  - AIC, AICc and BIC
- Performance measures
  - MAD, MSE, MAPE

Forecast fit vs forecast error

- Forecast fit
  - Backward-looking assessment
  - Residual Analysis: describes the difference between actual historical data and the fitted values generated by a statistical model
  - How well the model represents historical data
  - Help choose the model that will be further used to forecast unobserved values (Model Selection/Diagnostics)

- Forecast error
  - Forward-looking assessment
  - Difference between actual and forecasted values

Model Selection/Diagnostics

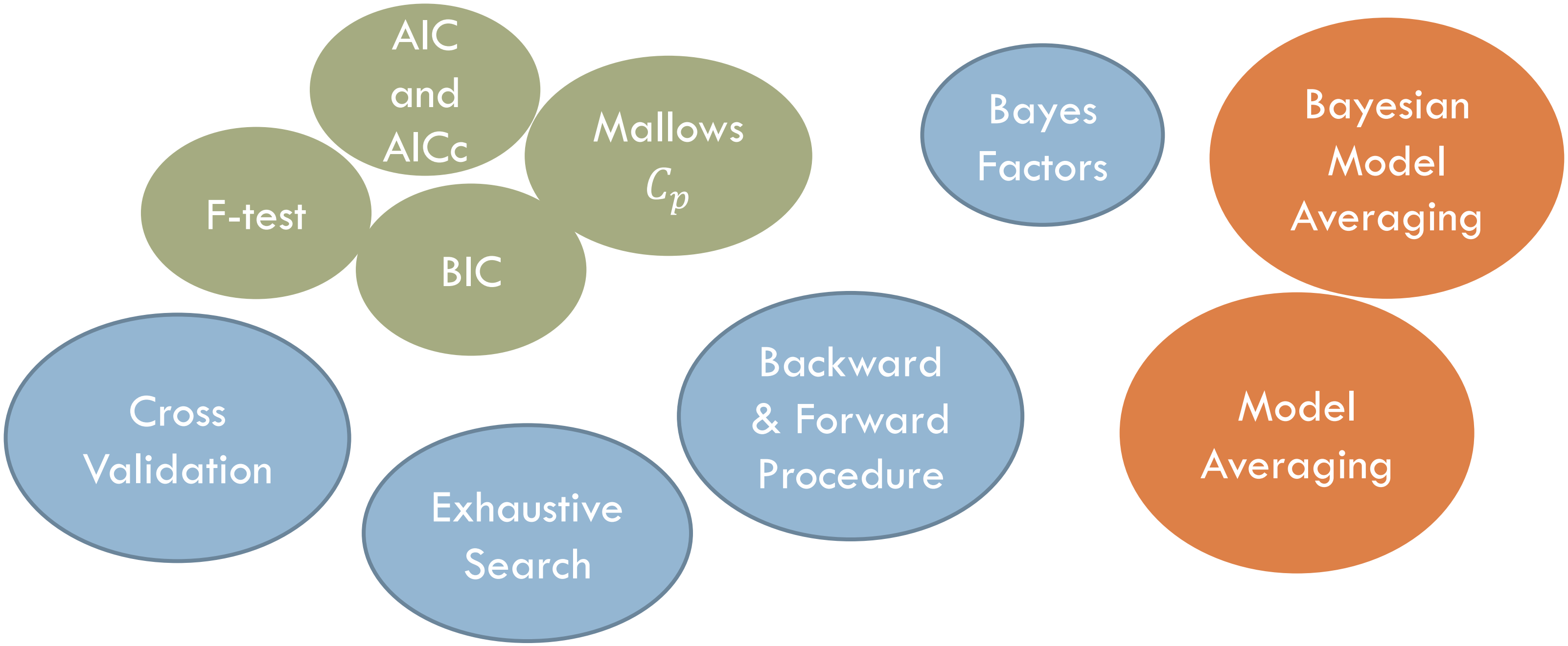
Model Selection

"Unsolved" problem in statistics: there are no magic procedures to get you the "best model" (Kadane and Lazar)

- With a limited number of predictors, it is possible to search all possible models
- But when we have many predictors, it can be difficult to find a good model (many possibilities)
- How do we select models?
  - We need a criteria or benchmark to compare two models
    AIC can serve as a model
  - We need a search strategy

Model Selection Criteria

- Some popular and well-known methods



- Some criteria work well for some types of data, others for different data

Standard criteria is the AIC

Model Selection Criteria (cont'd)

- We will focus on the ones that R prints after fitting an ARIMA model

```
auto.arima(deseasonal_cnt, seasonal=FALSE)

Series: deseasonal_cnt
ARIMA(1,1,1)

Coefficients:
         ar1      ma1
      0.5510  -0.2496
s.e.  0.0751   0.0849

sigma^2 estimated as 26180:  log likelihood=-4708.91
AIC=9423.82   AICc=9423.85   BIC=9437.57
```

- And the residual analysis

Akaike Information Criterion (AIC)

- Estimator of the **quality of statistical models**
- Select the model with **lowest AIC** For negative numbers, the bigger the better
- Let k be the number of estimated parameters and $\hat{L}$ be the maximum value of the likelihood function

$$AIC = 2k - 2\ln(\hat{L})$$

Penalty for increasing number of parameters

Reward based on the likelihood

- Trade-off between the **goodness of fit** and the **simplicity** of the model
- The AICc is used when sample size (n) is small

$$AICc = AIC + \frac{2k^2 + 2k}{n - k - 1}$$

corrects the AIC for small sample size.

Bayesian Information Criterion (BIC)

- Closely Related to AIC
- Also an estimator of **quality of model**
- Select the model with **lowest BIC**
- Let k be the number of estimated parameters, $\hat{L}$ be the maximum value of the likelihood function and n the number of observations (sample size)

$$BIC = k * \ln(n) - 2\ln(\hat{L})$$

BIC is an optional comparative parameter.

- Sample size should be much larger than number of parameters

Recall Electricity Prices Example

```
Series: deseasonal_price
ARIMA(1,1,0)

Coefficients:
          ar1
      -0.0311
s.e.   0.0707

sigma^2 estimated as 0.007868:  log likelihood=203.22
AIC=-402.43   AICc=-402.37   BIC=-395.82
```

```
Series: deseasonal_price
ARIMA(2,1,0)

Coefficients:
          ar1     ar2
      -0.0288  0.0755
s.e.   0.0705  0.0710

sigma^2 estimated as 0.007863:  log likelihood=203.78
AIC=-401.56   AICc=-401.44   BIC=-391.64
```

```
Series: deseasonal_price
ARIMA(2,1,2) with drift

Coefficients:
         ar1      ar2      ma1     ma2   drift
      0.5275  -0.7416  -0.5714  0.9283  0.0184
s.e.  0.1039   0.0782   0.0680  0.0479  0.0066

sigma^2 estimated as 0.007162:  log likelihood=214.3
AIC=-416.59   AICc=-416.16   BIC=-396.74
```

Drift means the mean difference to with zero.

Recall Electricity Prices Example

```
Series: price
ARIMA(1,1,1)(1,1,0)[12]

Coefficients:
         ar1      ma1     sar1
      0.6735  -0.6051  -0.4545
s.e.  0.3308   0.3540   0.0640

sigma^2 estimated as 0.008488:  log likelihood=183.63
AIC=-359.25   AICc=-359.04   BIC=-346.26
```

```
Series: price
ARIMA(0,1,0)(0,1,1)[12]

Coefficients:
         sma1
      -0.6371
s.e.   0.0615

sigma^2 estimated as 0.007602:  log likelihood=191.35
AIC=-378.71   AICc=-378.64   BIC=-372.21
```

Residual Analysis

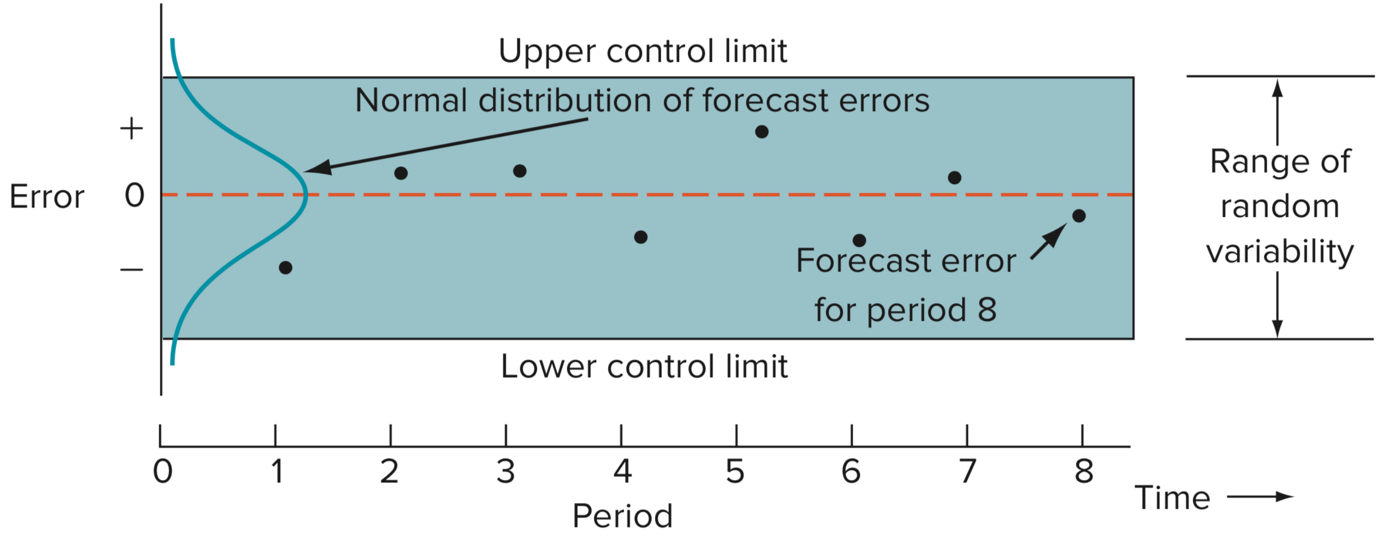
These patterns seem to show that there is something wrong,
Maybe we need to see whether the data were detrended.

Monitoring the Forecast

- **Tracking forecast errors** and analyzing them can provide useful insight into whether forecasts are performing satisfactorily
- **Sources of forecast errors**
  - The model may be inadequate
  - Irregular variations may have occurred
  - The forecasting technique has been incorrectly applied
  - Random variation
- **Residual analysis are useful for identifying the presence of non-random error in forecasts**

Residuals Analysis

- Errors are plotted on a chart in the order that they occur

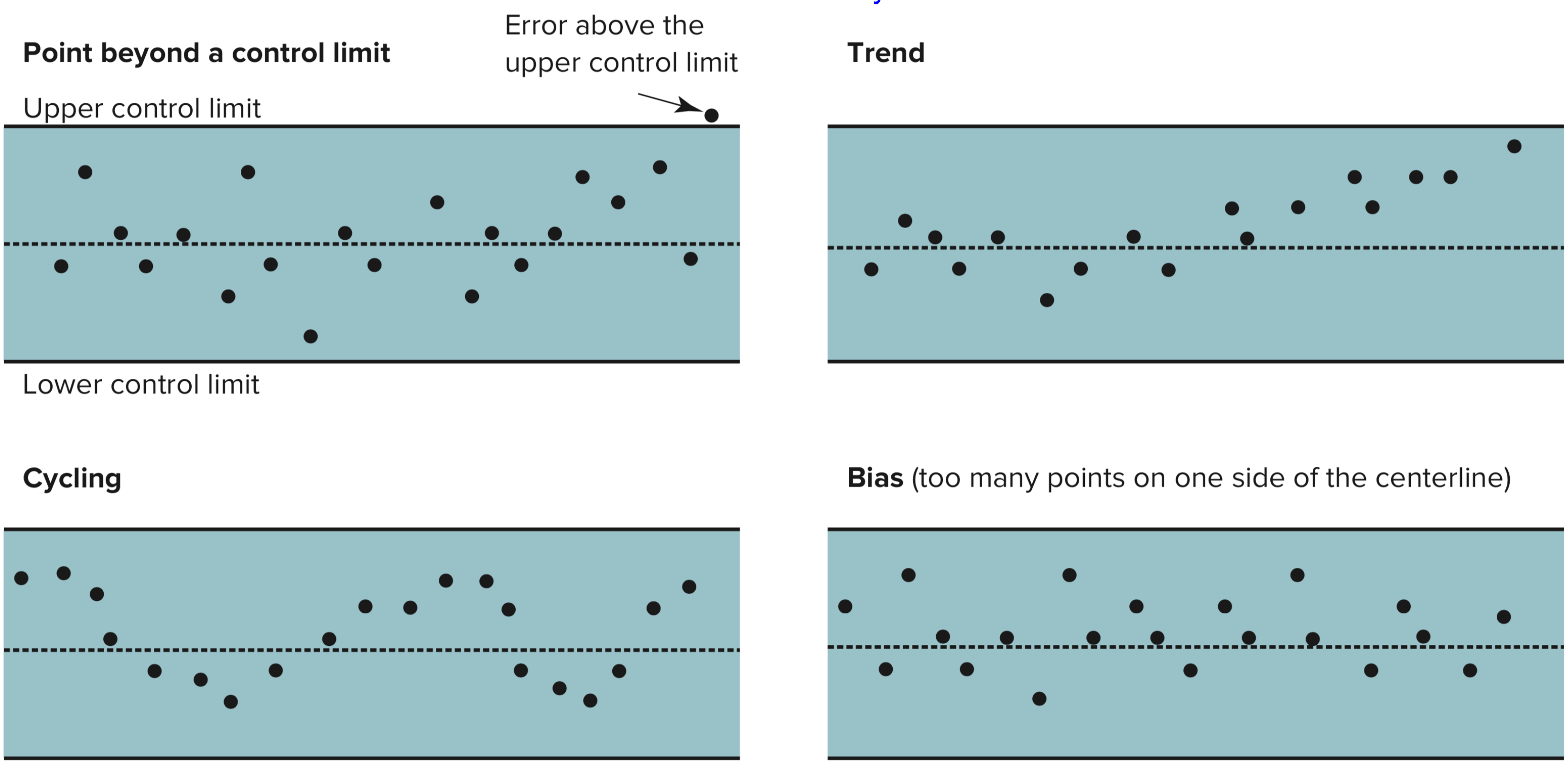


- Forecasts are in control when:
  - All errors within control limits
  - No patterns are present (e.g. seasonality, cycles, non-centered data)

Examples of Nonrandomness

FIGURE 3.12 Examples of nonrandomness

These patterns seem to show that there is something wrong,
Maybe we need to see whether the data were detrended.



Constructing a Control Chart

- Compute the mean square error (MSE)
- The **square root of the MSE** is used in practice as an estimate of the **standard deviation** of the distribution of errors → $s = \sqrt{\text{MSE}}$
- **Errors are random**, therefore, they will be distributed according to a **normal distribution** around a mean of zero
- For a normal distribution:
  - +/- 95.5 % of the values (errors in this case) can be expected to fall within limits of $0 \pm 2S$ (i.e., 0 ± 2 standard deviations)
  - +/- 99.7 % of the values can be expected to fall within $\pm 3s$ of zero
- Compute the limits as: →

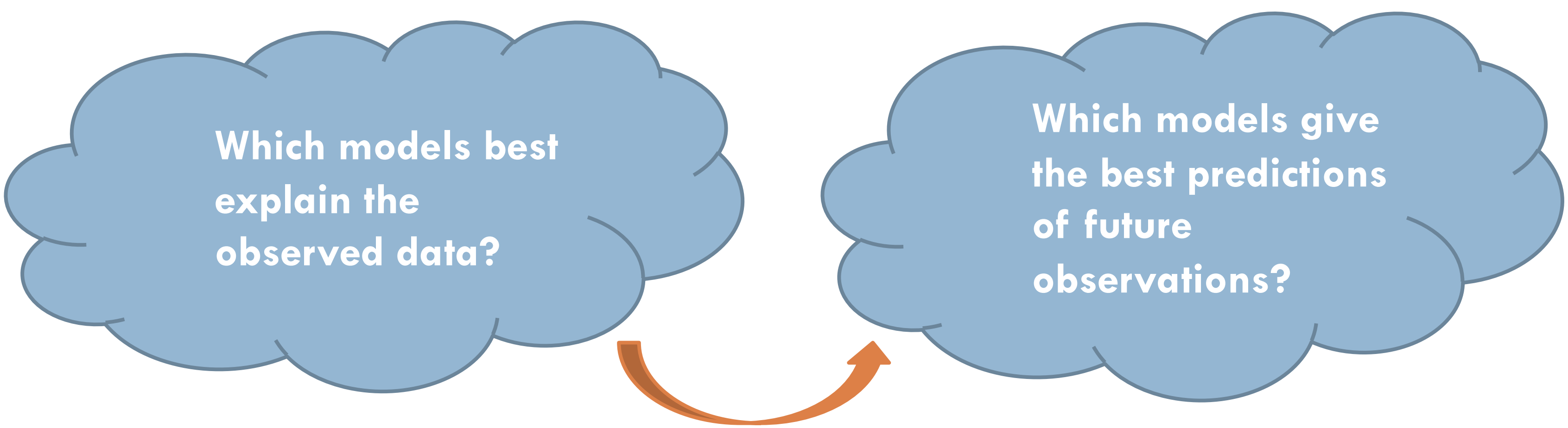
$$\text{UCL: } 0 + z\sqrt{\text{MSE}}$$
$$\text{LCL: } 0 - z\sqrt{\text{MSE}}$$

Number of standard deviations

Model Evaluation/Performance

Model Performance

- ***Keep in mind that these criteria are not measures of predictive power, they just represent how good the model fit the observed data***
- It's possible to look at the predictions from the various models
- In this case we shift the question



Model Performance (cc'ed)

- Model Performance measures the **forecast accuracy**
- Forecasters want to **minimize forecast errors**
  - It is nearly **impossible to correctly forecast real-world variable** values on a regular basis
  - So, it is important to **provide an indication of the extent to which the forecast might deviate** from the value of the variable that actually occurs
- **Forecast accuracy** should be an important forecasting technique selection criterion
  - Error = Actual – Forecast

    Observed value
  - If errors fall beyond acceptable bounds, corrective action may be necessary

Common Performance Measures

- Mean Error (ME)
- **Mean Squared Error (MSE)**
- **Root Mean Squared Error (RMSE) or** Standard Error (SE)
- Coefficient of Determination or R-Squared (R2)
- **Mean Absolute Deviation (MAD)** or Mean Absolute Error (MAE)
- **Mean Absolute Percentage Error (MAPE)**

Forecast Accuracy Metrics

Mean-absolute Deviation

$$\text{MAD} = \frac{\sum \left|\text{Actual}_t - \text{Forecast}_t\right|}{n}$$

MAD weights all errors evenly

Mean-squared Error

$$MSE = \frac{\sum (Actual_t - Forecast_t)^2}{n}$$

MSE weights errors according to their squared values

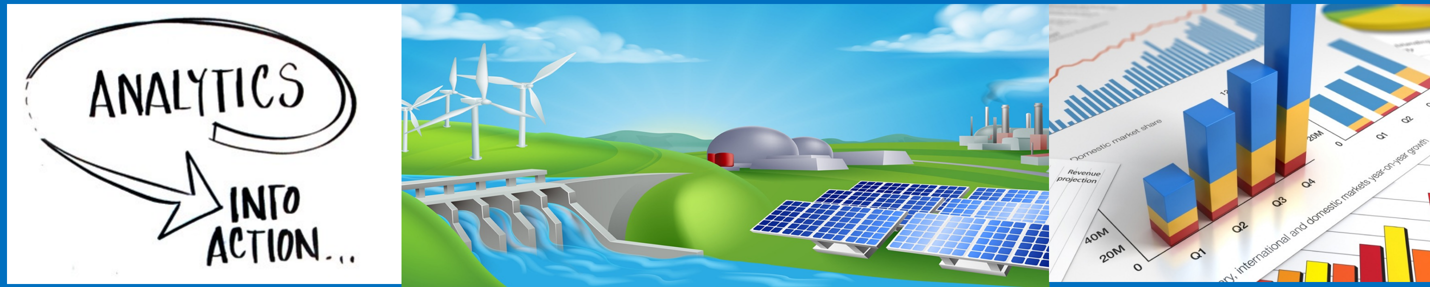
Mean-absolute Percent Error

$$\text{MAPE} = \frac{\sum \frac{\left|\text{Actual}_t - \text{Forecast}_t\right|}{\text{Actual}_t} \times 100}{n}$$

MAPE weights errors according to relative error

Forecast Error Calculation

Period	Actual (A)	Forecast (F)	(A-F) Error	|Error|	Error^2	[|Error|/Actual]x100
1	107	110	-3	3	9	2.80%
2	125	121	4	4	16	3.20%
3	115	112	3	3	9	2.61%
4	118	120	-2	2	4	1.69%
5	108	109	1	1	1	0.93%
			Sum	13	39	11.23%
				$n = 5$	$n = 5$	$n = 5$
				MAD	**MSE**	**MAPE**
				= 2.6	= 7.8	= 2.25%



THANK YOU !

luana.marangon.lima@duke.edu