# ENV797 - TIME SERIES ANALYSIS FOR ENERGY AND ENVIRONEMNET APPLICATIONS

## M7 - Introduction to Forecasting

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# Learning Goals

- Intro to Forecasting

- Simple Averaging techniques

- Forecasting with ARIMA models

- Intro to Forecasting in R

# Intro to Forecasting

# Intro to Time Series Forecasting

- Assume that future values of the time-series can be estimated from past values of the time-series

- Simple Forecasting techniques
  - Naïve Forecast
  - Simple Average
  - Moving average
  - Weighted moving average
  - Exponential smoothing

# Introduction to Forecasting

□ **Forecast** ➡ **statement about the future value** of a **variable of interest**

- Forecasts are often used for weather, demand, and resource availability

- Important element in making informed decisions

# Forecasts affect decisions

| Accounting | Cost/profit estimates |
|---|---|
| Finance | Cash flow and funding |
| Human Resources | Hiring/recruiting/training |
| Marketing | Pricing, promotion, strategy |
| Operations | Schedules, workloads |
| Product/service design | New products and services |

# Two Important Aspects of Forecasts

- Expected <u>level</u> of demand <span style="color:blue">or any other variable of interest.</span>
  - The level of demand may be a function of some <u>structural variation</u> such as trend or seasonal variation
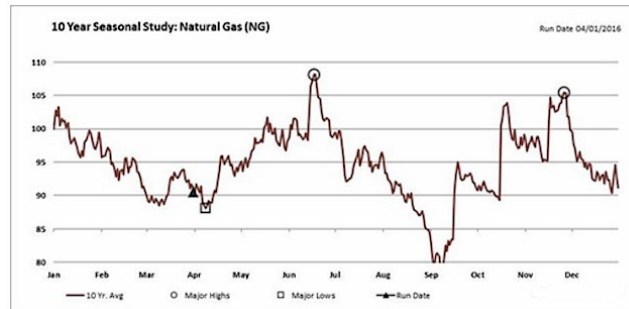


- Accuracy
  - Related to the potential size of forecast error

**Calculating Forecast Error**

# Features Common to All Forecasts

1. Techniques **assume some underlying causal system that existed in the past will persist** into the future

2. **Forecasts are not perfect**

3. **Forecasts for groups of items are more accurate** than those for individual items

4. **Forecast accuracy decreases** as the forecasting **horizon increases**

How many times you want for forcast.

# Elements of a Good Forecast

The forecast:

- ☐ Should be *timely*
- ☐ Should be *accurate*
- ☐ Should be *reliable*
- ☐ Should be **expressed in** *meaningful units*
- ☐ Technique should be *simple to understand and use*
- ☐ Should be *cost effective*

# Forecasting Process Steps



1. Determine the purpose of the forecast

2. Establish a time horizon

3. Obtain, clean, and analyze appropriate data

4. Select a forecasting technique

5. Make the forecast

6. Monitor the forecast
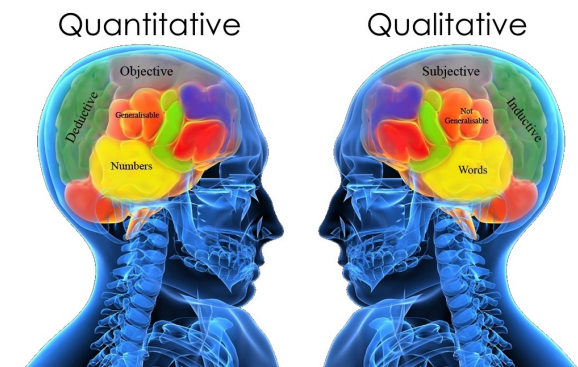
# Forecasting Approaches

Qualitative
- Delphi Method
- Nominal Group Technique Method
- Market Survey Method
- Historical Analogy Mathod

☐ **Qualitative Forecasting**

- ◻ Qualitative techniques permit the inclusion of *soft* information such as:
  - ▪ Human factors
  - ▪ Personal opinions
  - ▪ Hunches
- ◻ These factors are difficult, or impossible, to quantify

☐ **Quantitative Forecasting**

- ◻ Involve either the projection of historical data or the development of associative methods that attempt to use *causal variables*
- ◻ These techniques rely on *hard* data

Quantitative
- Causal Method
- Time Series Method
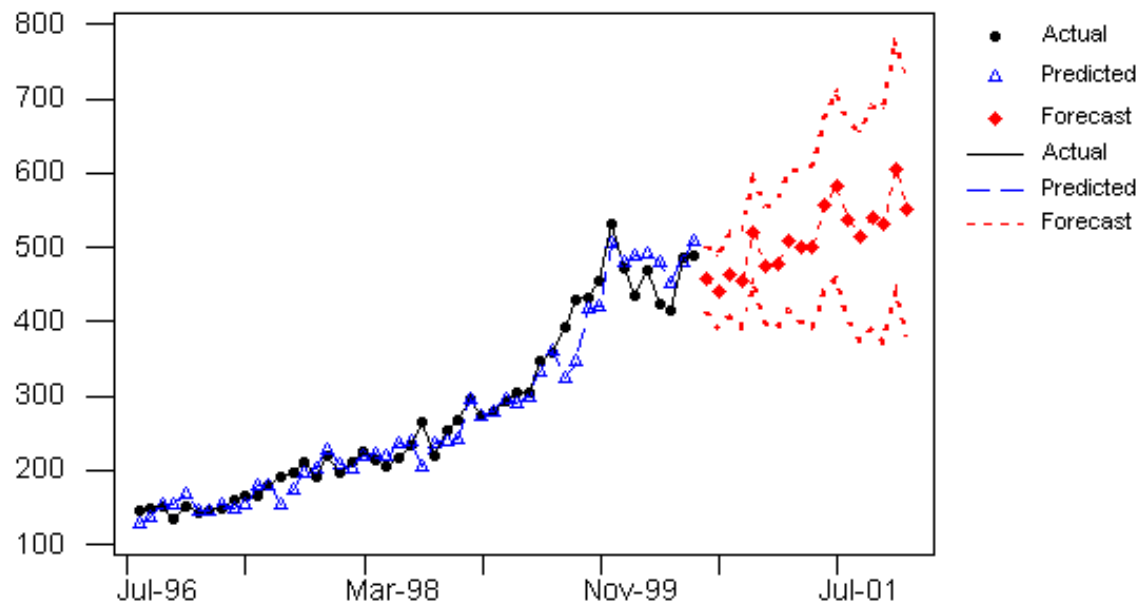  - Moving Average Method
  - Simple Exponential Smoothing
  - Holt's Method
  - Winter's Method

# Quantitative Forecasting

- Forecasts that project patterns identified in recent time-series observations

- Assume that future values of the time-series can be estimated from past values of the time-series

# Simple Averaging Forecasting Methods

# Time Series Forecasting - Naïve Forecast

□ Naïve Forecast

- Uses a **single previous value** of a time series as the basis for a forecast

- The forecast for a time period is equal to the previous time period value

- Can be used with
  - a stable time series
  - seasonal variations
  - trend

**Long Range Weather Forecast** by Eric Per1in

Summer will be hot.
Winter will be cold.

www.funnytimes.com
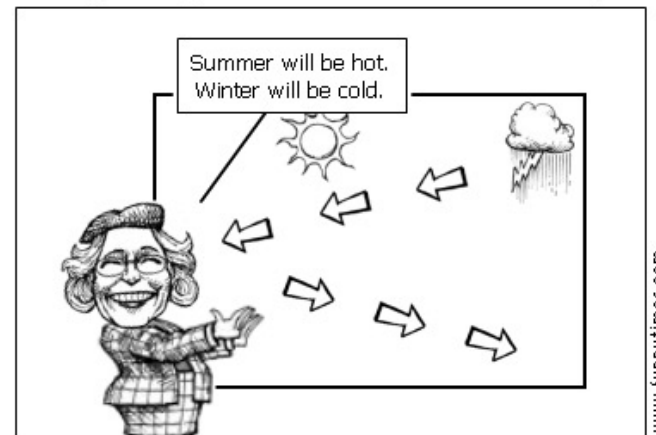
# Naïve Forecasts

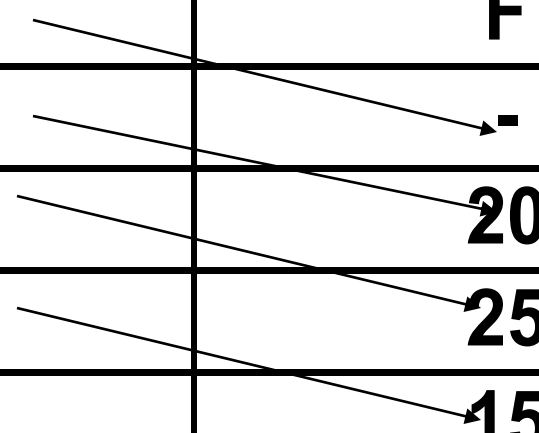- Forecast for any period = previous period's actual value

$$F_t = A_{t-1}$$

Can say Yt=Yt-1 meaning tomorrow depends on yesterday.

F: forecast    A: Actual    t: time period

Type text here

# Naïve Forecast Example

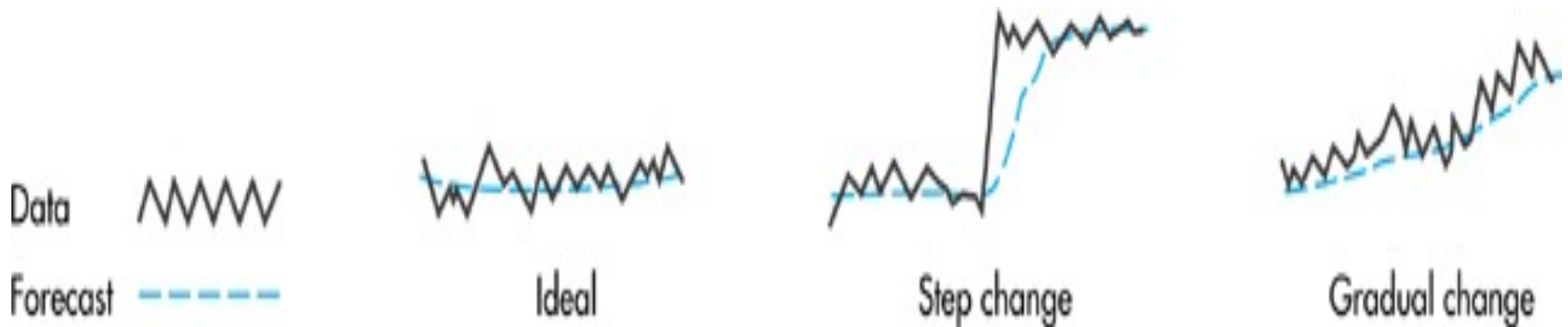| Week | Sales (actual) | Sales (forecast) | Error |
|------|----------------|------------------|-------|
| t | A | F | A - F |
| 1 | 20 | - | |
| 2 | 25 | 20 | 5 |
| 3 | 15 | 25 | -10 |
| 4 | 30 | 15 | 15 |
| 5 | 27 | 30 | -3 |

# Naïve Forecasts

- Simple to use

- Virtually no cost

- Quick and easy to prepare

- Data analysis is nonexistent

- Easily understandable

- Cannot provide high accuracy

# Uses for Naïve Forecasts



Data

Forecast − − − − −

Ideal

Step change

Gradual change

Seasonal naive forecasting : repeating the values from the seasonal lag.

# Time Series Forecasting - Averaging

- These techniques work best when a series tends to vary about an average
- Averaging techniques **smooth variations** in the data
- They can handle **step changes or gradual changes** in the level of a series
- Techniques
  1. Moving average
  2. Weighted moving average
  3. Exponential smoothing

# Moving Average

□ Technique that averages a number of the most recent actual values in generating a forecast

$$F_t = \text{MA}_n = \frac{\sum_{i=1}^{n} A_{t-i}}{n}$$

where

$F_t = $ Forecast for time period $t$

$\text{MA}_n = n$ period moving average

$A_{t-1} = $ Actual value in period $t-1$

$n = $ Number of periods in the moving average

# Moving Average

- As **new data become available**, the **forecast is updated** by adding the newest value and dropping the oldest and then re-computing the average

- The number of data points included in the average determines the model's sensitivity
  - **Fewer data points used-- more responsive**
  - **More data points used-- less responsive**

# Moving Average Example

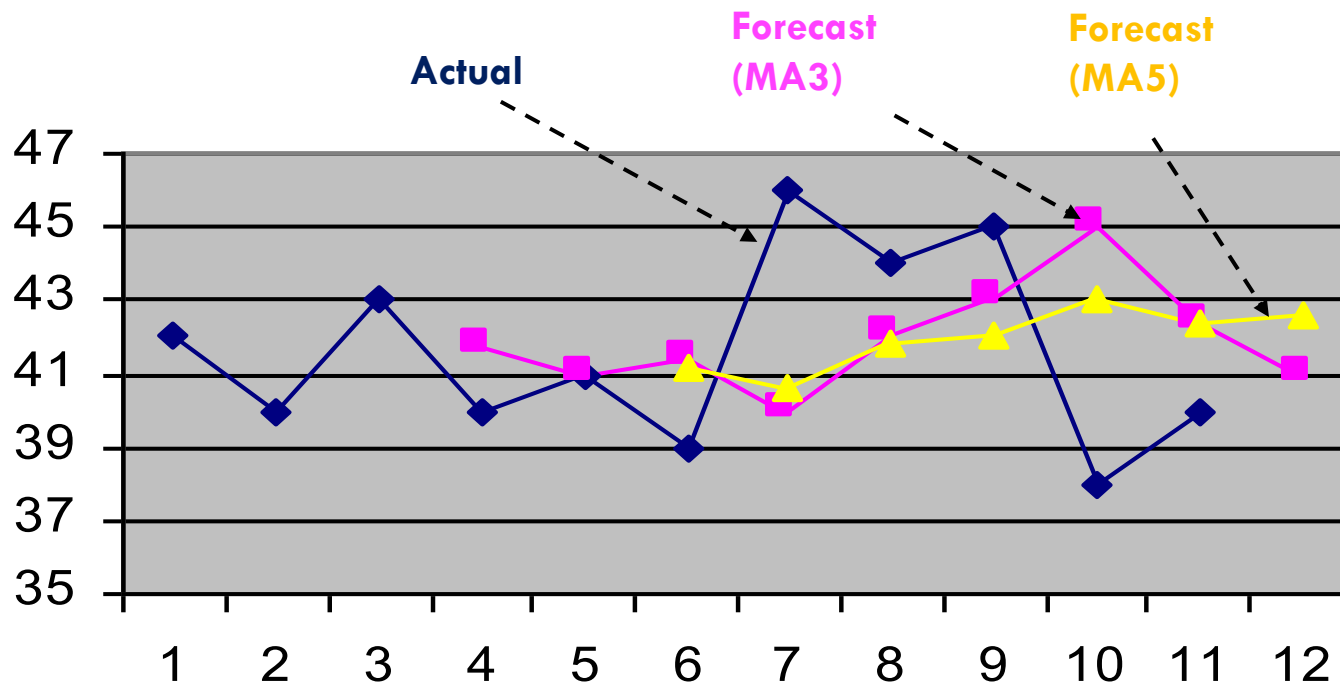| Week | Sales (actual) | Sales (forecast) | Error |
|:---:|:---:|:---:|:---:|
| t | A | F = MA3 | A - F |
| 1 | 20 | - | |
| 2 | 25 | - | |
| 3 | 15 | - | |
| 4 | 30 | 20 | 10 |
| 5 | 27 | 23.3333 | 3.66667 |
| 6 | | 24 | |

# Simple Moving Average
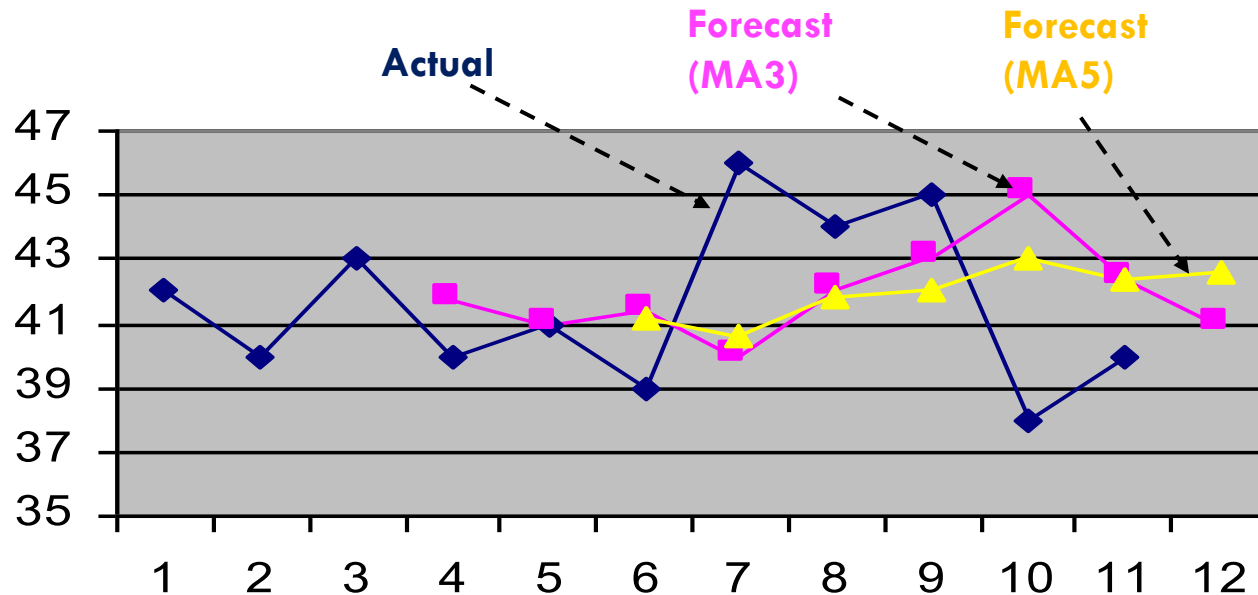


Questions:

- Why is MA3 longer than MA5?
- Which curve fluctuate the most?
- Which curve is the smoothest?

# Simple Moving Average

Responsiveness vs. Stability

- Smaller m, responsiveness ↑, stability ↓
- Larger m, responsiveness ↓, stability ↑
- Must maintain stability when fluctuations are high

# Weighted Moving Average

□ The most recent values in a time series are given more weight in computing a forecast

  ▫ The choice of weights, *w*, is somewhat arbitrary and involves some trial and error

$$F_{t+1} = w_t A_t + w_{t-1} A_{t-1} + w_{t-2} A_{t-2} + \cdots + w_{t-n} A_{t-n}$$

where

$w_t$ = weight for period $t$, $w_{t-1}$ = weight for period $t-1$, etc.

$A_t$ = the actual value for period $t$, $A_{t-1}$ = the actual value for period $t-1$, etc.

# Weighted Moving Average Example

| Week | Sales (actual) | Sales (forecast) | Error |
|------|----------------|------------------|-------|
| t | A | F = MA3 | A - F |
| 1 | 20 | - | |
| 2 | 25 | - | |
| 3 | 15 | - | |
| 4 | 30 | 19 | 11 |
| 5 | 27 | 24.5 | 2.5 |
| 6 | | 25.5 | |

$$w_{t-1} = 0.5, \qquad w_{t-2} = 0.3, \qquad w_{t-3} = 0.2,$$

# Exponential Smoothing

- A weighted averaging method that is based on the previous forecast plus a percentage of the forecast error

$$F_t = F_{t-1} + \alpha(A_{t-1} - F_{t-1})$$

$F_t$: forecast for period t

$F_{t-1}$: forecast for previous period t-1

$\alpha$: smoothing constant

$A_{t-1}$: actual value from previous period

$A_{t-1} - F_{t-1}$ is the error term

# Example 3 - Exponential Smoothing

$$F_t = F_{t-1} + \alpha(A_{t-1} - F_{t-1})$$

| Period | Actual | Alpha = 0.1 | Error | Alpha = 0.4 | Error |
|---|---|---|---|---|---|
| 1 | 42 | | | | |
| 2 | 40 | 42 | -2.00 | 42 | -2 |
| 3 | 43 | 41.8 | 1.20 | 41.2 | 1.8 |
| 4 | 40 | 41.92 | -1.92 | 41.92 | -1.92 |
| 5 | 41 | 41.73 | -0.73 | 41.15 | -0.15 |
| 6 | 39 | 41.66 | -2.66 | 41.09 | -2.09 |
| 7 | 46 | 41.39 | 4.61 | 40.25 | 5.75 |
| 8 | 44 | 41.85 | 2.15 | 42.55 | 1.45 |
| 9 | 45 | 42.07 | 2.93 | 43.13 | 1.87 |
| 10 | 38 | 42.36 | -4.36 | 43.88 | -5.88 |
| 11 | 40 | 41.92 | -1.92 | 41.53 | -1.53 |
| 12 | | 41.73 | | 40.92 | |

$$F_3 = 42 + 0.1(40 - 42) = 41.8$$

# Picking a Smoothing Constant
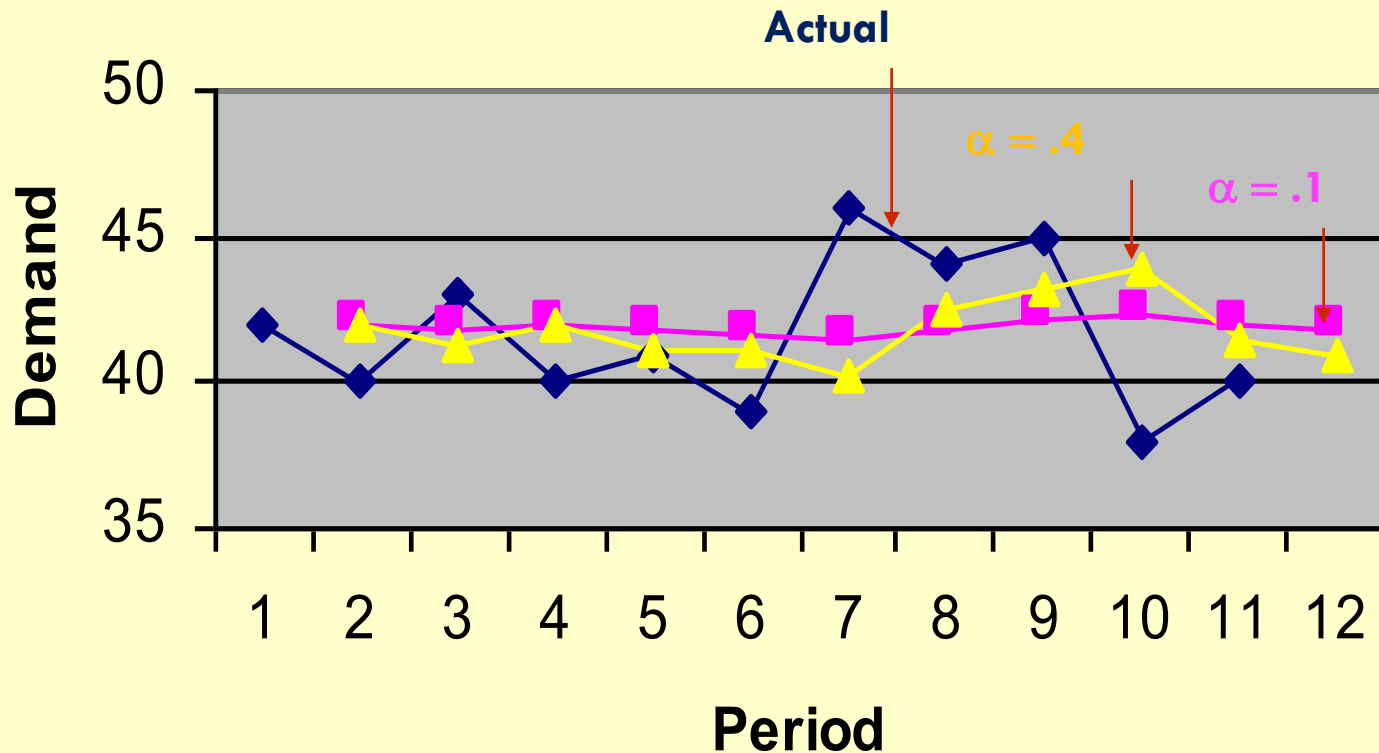


The smaller alpha represents the smoother series.

# Associative Forecasting Techniques

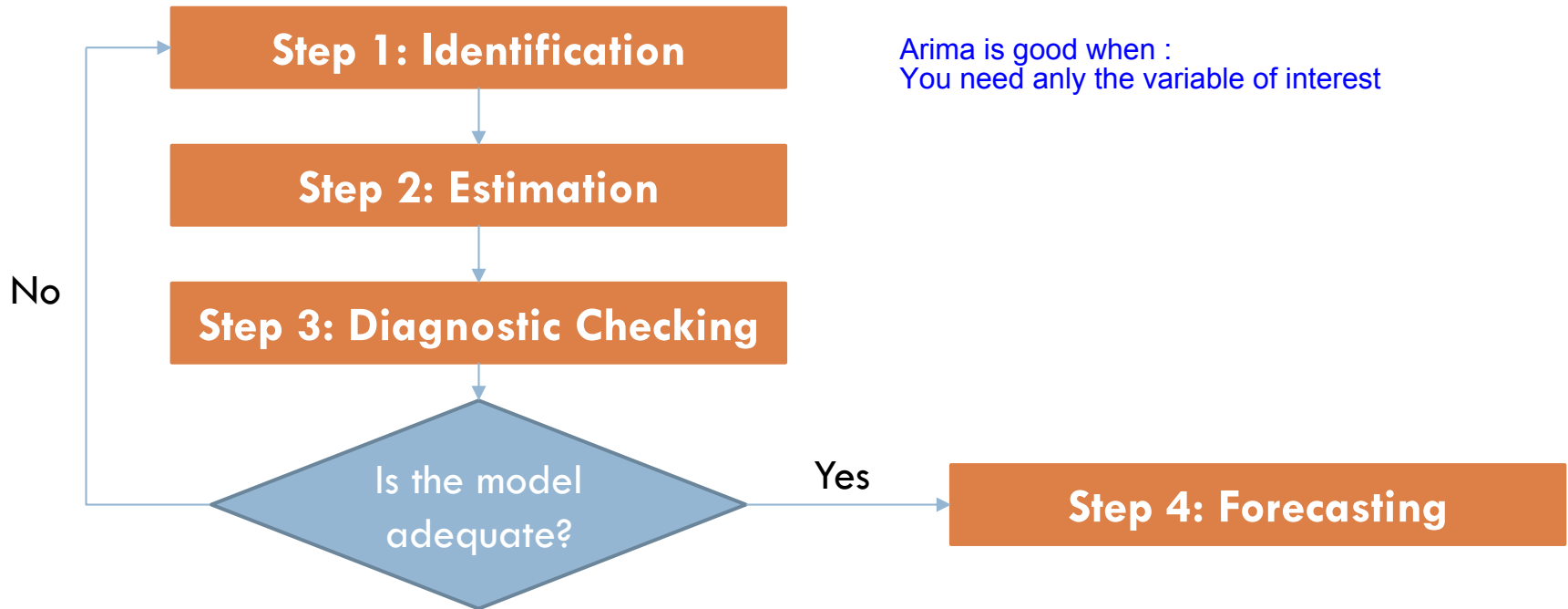□ Associative techniques are **based on the development of an equation that summarizes the effects of predictor variables** A linear regression is also a forecasting technic

- ◻ **Predictor variables** - variables that can be used to predict values of the variable of interest

- ◻ **Home values may be related** to: home and **property size, location, number of bedrooms**, and number of bathrooms
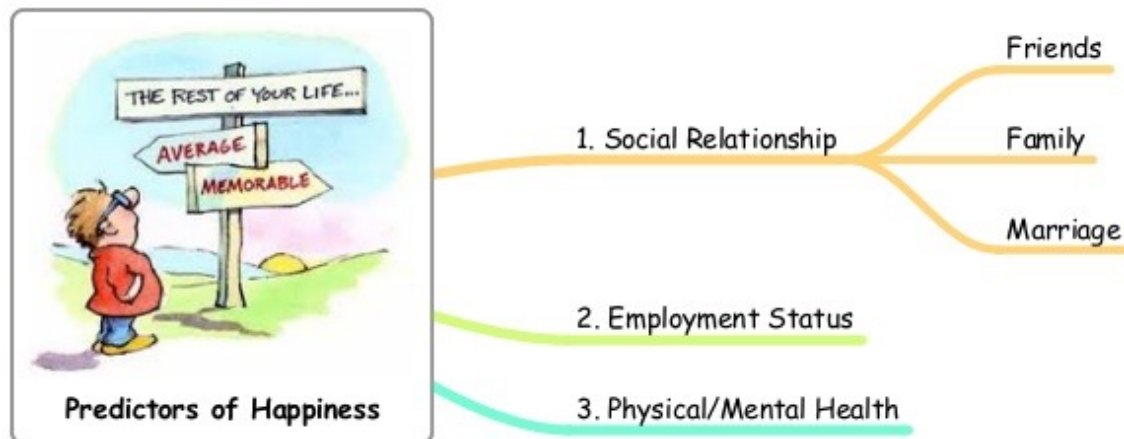
# Forecasting with ARIMA Models

# ARIMA Modeling Process

```
   ┌──────────────────────────────────────┐
   │         Step 1: Identification       │        Arima is good when :
   │                                      │        You need anly the variable of interest
   └──────────────────────────────────────┘
                    │
   ┌──────────────────────────────────────┐
   │           Step 2: Estimation         │
   └──────────────────────────────────────┘
                    │
   ┌──────────────────────────────────────┐
   │     Step 3: Diagnostic Checking      │
   └──────────────────────────────────────┘
                    │
No            ◇ Is the model adequate? ◇──── Yes ──── ┌─────────────────────────┐
                                                       │   Step 4: Forecasting   │
                                                       └─────────────────────────┘
```

□ The use of ARIMA is appropriate when

- □ little or nothing is known about the dependent variable being forecasted,

- □ the independent variables known to be important cannot be forecasted effectively

- □ all that is needed is a one or two-period forecast

# ARIMA Forecasting

- Recall the ARMA(1,1) model equation

$$Y_t = \phi_1 Y_{t-1} + a_t - \theta_1 a_{t-1} \qquad\qquad for\ t = 1,2,\ldots,n$$

$$a_t \sim i.i.d.\ N(0,\sigma^2)$$

- What would be the value of the next Y, say $Y_{t+1} = ?$

- From the estimation step you have $\phi = \left(\phi_1\ \phi_2\ \ldots \phi_p\right)'$ and $\sigma^2$

- All you need to do is plug in the parameters and past Y values

- Same principle is extended for the more general class of ARIMA Models

# Forecasting with MA(2)

- MA(2) model $\quad Y_t = \mu + a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2}$

- Forecasting

  - 1-step
    $$Y_{n+1} = \mu + a_{n+1} - \theta_1 a_n - \theta_2 a_{n-1}$$
    $$\hat{Y}_{n+1} = \mu + \qquad -\theta_1 a_n - \theta_2 a_{n-1}$$
    $$E[a_{n+1}] = 0$$

    $$Y_{n+1} - \hat{Y}_{n+1} = a_{n+1} \qquad Var\left(Y_{n+1} - \hat{Y}_{n+1}\right) = Var(a_{n+1}) = \sigma^2$$

  - 2-step
    $$Y_{n+2} = \mu + a_{n+2} - \theta_1 a_{n+1} - \theta_2 a_n$$
    $$\hat{Y}_{n+2} = \mu \qquad\qquad -\theta_2 a_n$$
    $$E[a_{n+1}] = 0$$
    $$E[a_{n+2}] = 0$$

    $$Y_{n+2} - \hat{Y}_{n+2} = a_{n+2} - \theta_1 a_{n+1} \qquad Var\left(Y_{n+2} - \hat{Y}_{n+2}\right) = \sigma^2(1 + \theta_1^2)$$

  - 3-step
    $$Y_{n+3} = \mu + a_{n+3} - \theta_1 a_{n+2} - \theta_2 a_{n+1}$$
    $$\hat{Y}_{n+3} = \mu$$
    $$E[a_{n+1}] = 0$$
    $$E[a_{n+2}] = 0$$
    $$E[a_{n+3}] = 0$$

    $$Y_{n+3} - \hat{Y}_{n+3} = a_{n+3} - \theta_1 a_{n+2} - \theta_2 a_{n+1} \qquad Var\left(Y_{n+3} - \hat{Y}_{n+3}\right) = \sigma^2(1 + \theta_1^2 + \theta_2^2)$$

# Forecasting with AR(2)

- AR(2) model $\qquad Y_t = \delta + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + a_t$

- Forecasting

  - 1-step

    $Y_{n+1} = \delta + \phi_1 Y_n + \phi_2 Y_{n-1} + a_{n+1}$

    $\hat{Y}_{n+1} = \delta + \phi_1 Y_n + \phi_2 Y_{n-1}$

    $E[a_{n+1}] = 0$

    $Y_{n+1} - \hat{Y}_{n+1} = a_{n+1} \qquad Var(Y_{n+1} - \hat{Y}_{n+1}) = Var(a_{n+1}) = \sigma^2$

  - 2-step

    $Y_{n+2} = \delta + \phi_1 Y_{n+1} + \phi_2 Y_n + a_{n+2}$

    $\hat{Y}_{n+2} = \delta + \phi_1 \hat{Y}_{n+1} + \phi_2 Y_n$

    $E[a_{n+2}] = 0$

    $Y_{n+2} - \hat{Y}_{n+2} = a_{n+2} + \phi_1 a_{n+1} \qquad Var(Y_{n+2} - \hat{Y}_{n+2}) = \sigma^2(1 + \phi_1^2)$

  - 3-step

    $Y_{n+3} = \delta + \phi_1 Y_{n+2} + \phi_2 Y_{n+1} + a_{n+3}$

    $\hat{Y}_{n+3} = \delta + \phi_1 \hat{Y}_{n+2} + \phi_2 \hat{Y}_{n+1}$
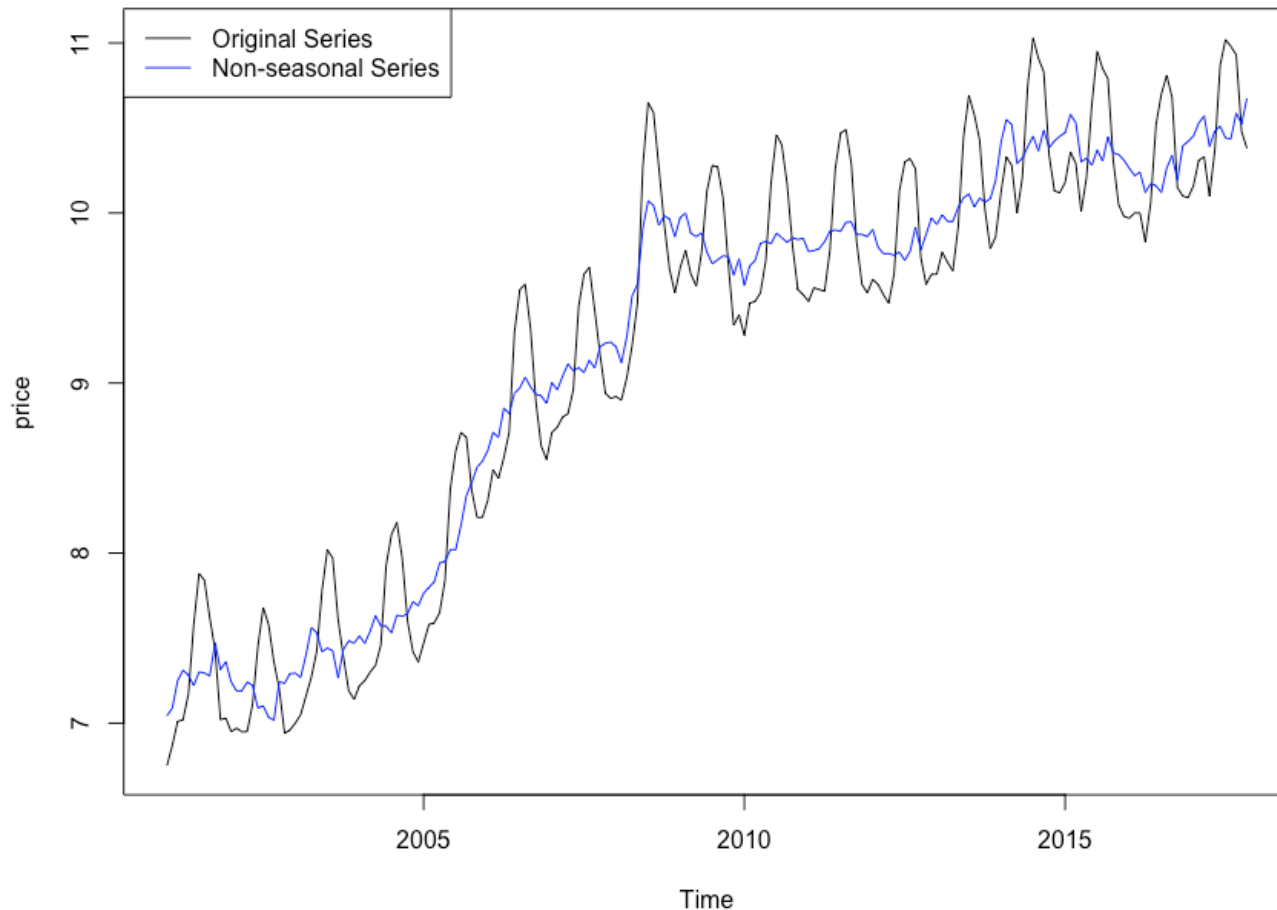
    $E[a_{n+3}] = 0$

    $Y_{n+3} - \hat{Y}_{n+3} = a_{n+3} + \phi_1 a_{n+2} + (\phi_1^2 + \phi_2) a_{n+1} \qquad Var(Y_{n+3} - \hat{Y}_{n+3}) = \sigma^2(1 + \phi_1^2 + (\phi_1^2 + \phi_2)^2)$

*Let's revisit the electricity price data example…*
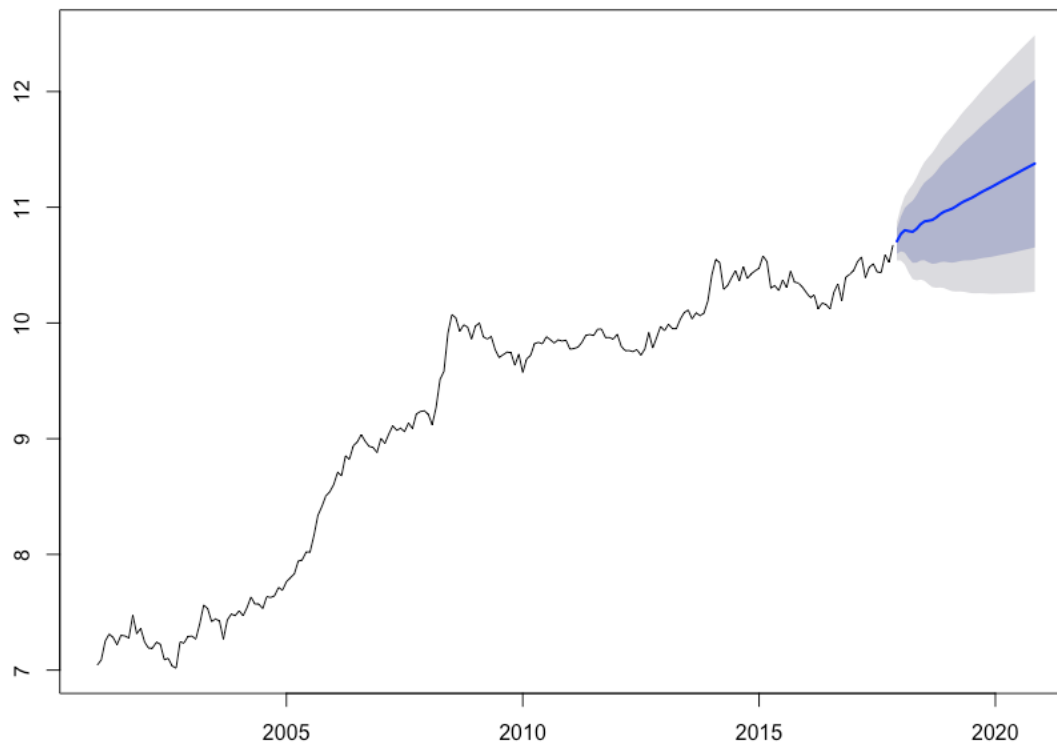
# From previous analysis

❑ Data exhibits trend and seasonality

# Example for Energy Price Data

☐ Taking seasonality out before fitting the ARIMA model

Forecasts from ARIMA(2,1,2) with drift



Forecast estimates are provided with confidence bounds

　　80% in darker blue, and 95% in lighter blue

　Longer term forecasts will have more uncertainty, as the model will regress future Y on previously predicted values

As a result the shape of the confidence bounds start to widen as we increase horizon

# Example for Energy Price Data

☐ Note what happens if you don't take seasonality and run the non-seasonal ARIMA
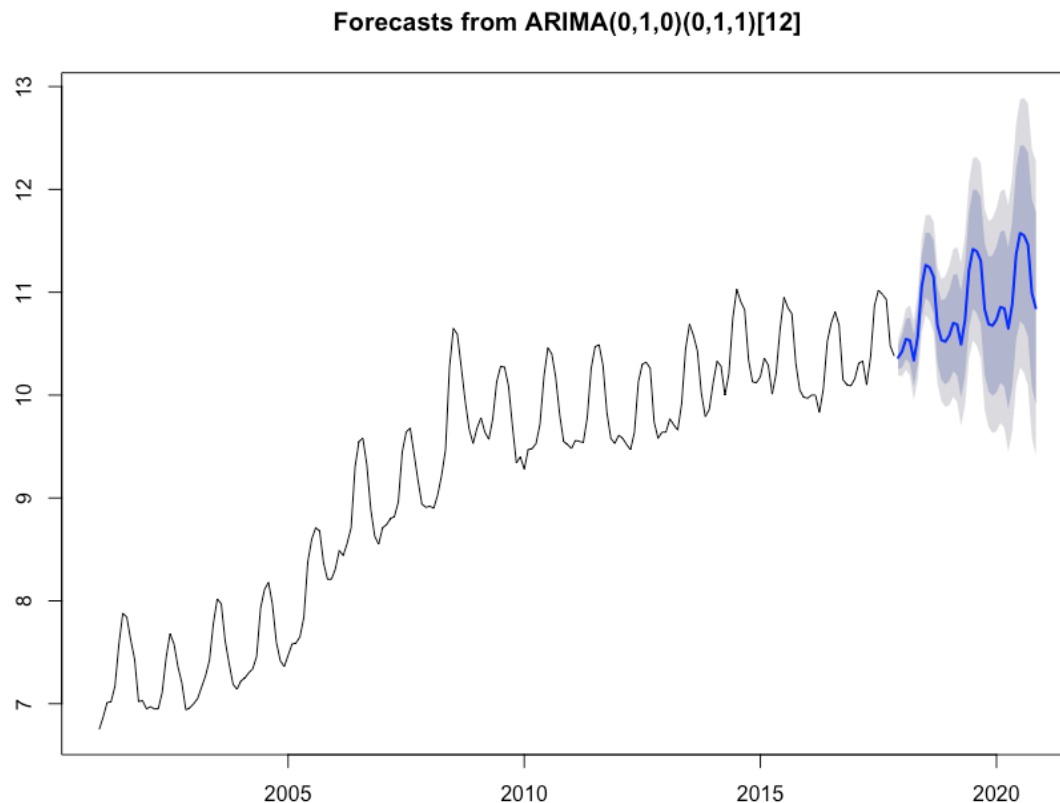
**Forecasts from ARIMA(2,1,1) with drift**
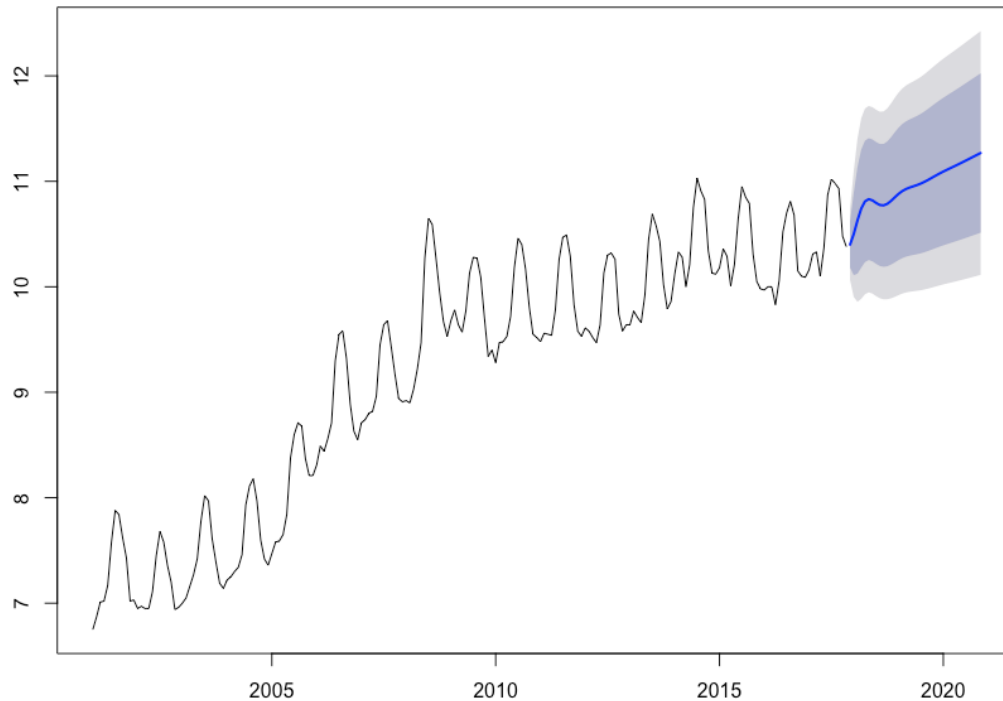


The results will be similar to the naïve forecasting, you are just modeling the trend, but there is no seasonal variation on the forecasts.

# Example for Energy Price Data

□ Now consider the SARIMA on original data



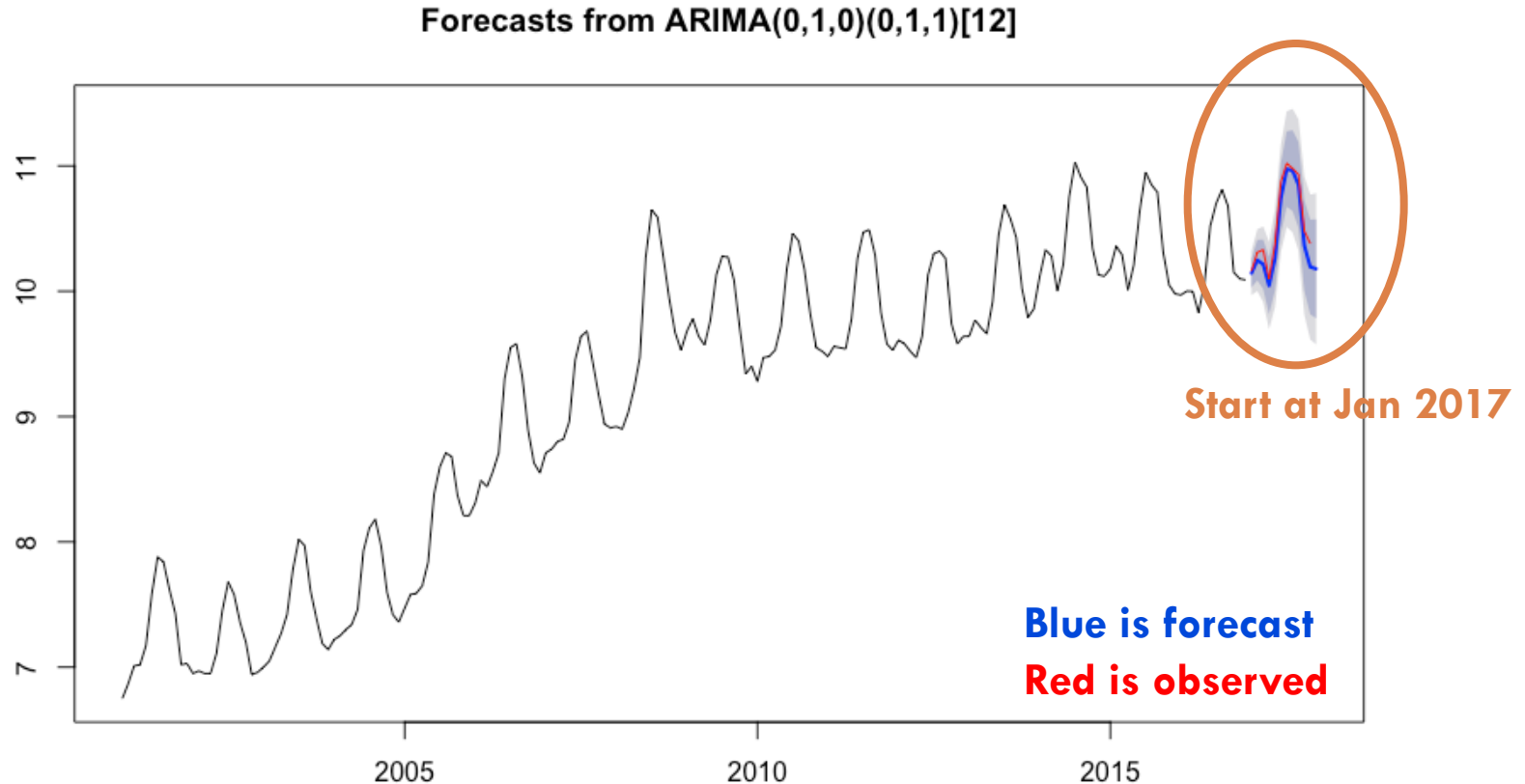Forecasts from ARIMA(0,1,0)(0,1,1)[12]

Note that now the forecasts reproduce both trend and seasonal component

# How do you check model performance?

- Suppose you want to compute forecast error

- Forecasts can be either in-sample or out-of-sample

- In general the out-of-sample forecasts are a better test of how well the model works, as the forecast uses data not included in the estimation of the model

  - If you want to get a sense of how the model will perform in the future, reserve a portion of the data as a "hold-out" set, fit the model, and then compare the forecast to the actual observed values

# ARIMA Model Performance – Short-term

□ To check your model performance over a year, leave one year of data out of the analysis



Forecasts from ARIMA(0,1,0)(0,1,1)[12]

Start at Jan 2017

Blue is forecast
Red is observed

# ARIMA Model Performance – Long-term

□ To check your model performance over three years, leave three years of data out of the analysis



Forecasts from ARIMA(1,0,0)(0,1,1)[12] with drift

Start at Jan 2015

Blue is forecast
Red is observed
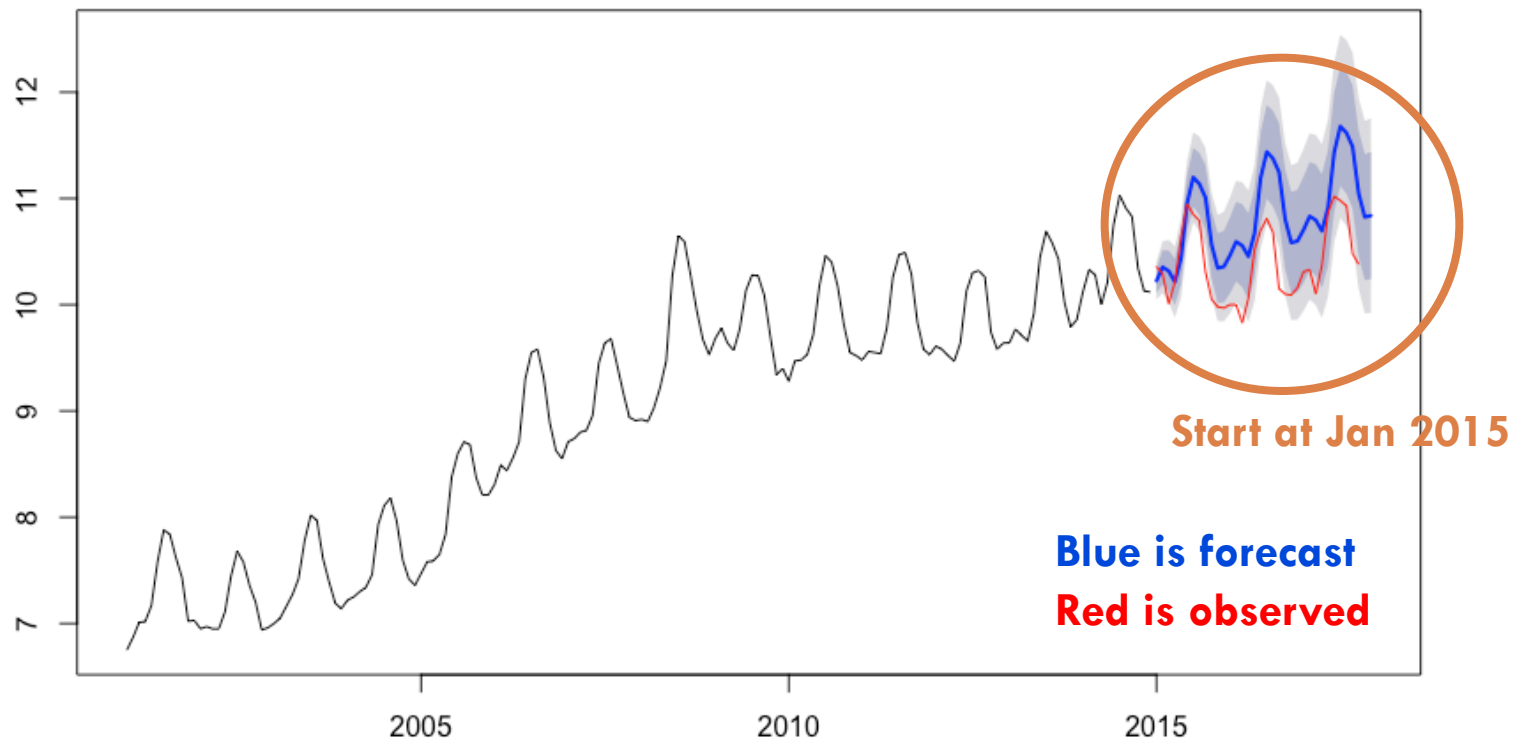
# Let's see this in R

# Forecasting in R

- Forecasting with ARIMA model
    1. *Fit ARIMA using **auto.arima() or Arima()***
    2. ***forecast(object, ...)***

- Forecasting with Moving Average Model
*sma(y, order = NULL, h = 10, holdout = FALSE, level = 0.95, silent = c("all", "graph", "legend", "output", "none"), ...)*

- Forecasting with Exponential Smoothing
*ses(y, h = 10, level = c(80, 95), alpha = NULL, ...)*

# More Simple Forecasting Methods in R

☐ Arithmetic Mean

*package "forecast"*

*meanf(y, h = 10, level = c(80, 95),…)*

☐ Naïve and Seasonal Naïve Method

*naive(y, h = 10, level = c(80, 95), …)*

*snaive(y, h = 10, level = c(80, 95), …)\**

*need to specify frequency when defining ts() object

# More Simple Forecasting Methods in R

☐ All these functions output a forecast object similar to the *forecast()* function

☐ The forecast object contains

Original series

Prediction Interval

Residuals and in-sample one-step forecasts

Point forecasts

Forecasting Method

# Measuring Forecast Accuracy in R

*accuracy(f, x, test = NULL, d = NULL, D = NULL, ...)*

**package "forecast"**

- *f* needs to be an object of class "forecast"
- *x* numerical vector containing actual values of the same length as object
- The measures calculated are:
  - ME: Mean Error
  - RMSE: Root Mean Squared Error $\Longrightarrow$ MSE
  - **MAE: Mean Absolute Error** $\Longrightarrow$ MAD
  - MPE: Mean Percentage Error
  - **MAPE: Mean Absolute Percentage Error**
  - MASE: Mean Absolute Scaled Error
  - ACF1: Autocorrelation of errors at lag 1.

# THANK YOU !

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