

# I dont know yet



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# Introduction

One of Prof. Rauschenbeutel projects uses a novel type of whispering-gallery-mode (WGM) resonator interfaced via nanowaveguides and coupled to single Rubidium atoms to carry out experiments in the realm of Cavity Quantum Electrodynamics. The WGM resonator is a so-called bottle-microresonator (BMR) manufactured from a standard optical glass fiber in a heat and pull process. The light is radially confined inside the resonator by total internal reflection and propagates along the circumference of the resonator. In such a structure, a significant fraction of the light field propagates in the evanescent field. By overlapping this field with the evanescent field of an optical nanofiber, light can be coupled into and out of the resonator very efficiently. Due to the extremely low absorption of silica (and low surface roughness) we can produce bottle-resonators with ultra-high optical Q-factor exceeding  $10^8$ . Rubidium atoms are delivered to the resonator using an atomic fountain. For the moment the atoms are only flying by the resonator and when they enter the evanescent field of the BMR, they are coupled to the cavity light field. But only for  $\sim 2\text{ }\mu\text{s}$  and moreover the distance between the resonator and the atom is not controlled. This prevents the realization from more complicated experiments. For that reason one needs to trap the atom.



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# Chapter 1

## Theory of laser trapping of atoms

The strategy pursued to trap is a optical dipole trap. For that the laser light needs to be detuned from a resonance of the atom. Thereafter the atoms are trapped to the maxima of intensity for a red detuned laser. The beam will be reflected from the resonator surface and creates thereby a standing wave. The 1<sup>st</sup> maxima is at  $\frac{1}{4} \lambda_{trap}$ .

So how to choose  $\lambda_{trap}$ ?

Because of the interaction with the BMR evanescent field the atoms need to be trapped really close:  $\lambda/2\pi \approx 130$  nm

Most common resonance of rubidium is  $5S_{1/2} \rightarrow 5P_{3/2}$  @ 780.24 nm. If we use a laser red-detuned from  $\lambda = 780.24$  nm then our first maxima would be at 195 nm  $\Rightarrow$  Not close enough!

But rubidium has another transition from  $5S_{1/2} \rightarrow 6P_{3/2}$  @ 420.29 nm, which leads to a distance of 105 nm from the BMR to the 1<sup>st</sup> maxima. But in the formula [1] of the trap potential ( $U_{dip}$ ) arises the transition strength ( $\Gamma_\omega$ ) of this specific transition:

$$U_{dip}(\mathbf{r}) = -\frac{\pi c^2}{\hbar \omega_0^3} \left( \frac{\Gamma_\omega}{\Delta} \right) I(\mathbf{r}) \quad (1.1)$$

We have to compare this potential to the kinetic energy of our rubidium atoms. The atoms

fall approximately 60 ms and the corresponding kinetic energy would be  $E_{kin} = \frac{1}{2} m_{Rb} v^2$ . In terms of temperature we would get:  $E_{kin}/k_B = 1.77$  mK. This is quite huge for a dipole trap. For that reason one needs to know  $\Gamma_{\omega, 420nm-Line}$  and one also needs to have a trap with a small detuning. This requires to see the transitions to have a reference to lock the laser afterwards. To determine  $\Gamma_{\omega, 420nm-Line}$  we can use the relation with the intensity saturation:

$$I_{s,420} = \frac{\Gamma_{\omega,tot,420} \cdot \omega_{420}^3 \cdot I_{s,780}}{\Gamma_{\omega,420} \cdot \Gamma_{\omega,780} \cdot \omega_{780}^3} \quad (1.2)$$

$$\text{with } \Gamma_{\omega,tot,420} = \frac{1}{\text{total lifetime of } 6P_{3/2} \text{ state}}$$

$\Rightarrow$  We want to measure  $I_s$  for the blue 420.29 nm-line.

## Chapter 2

# Absorption of photon by an atom

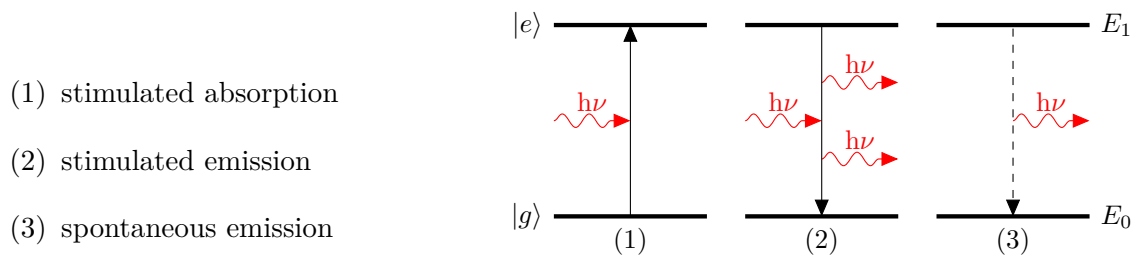
The purpose of this section is to outline the basic features observed in saturated absorption spectroscopy and relate them to simple atomic and laser physics principles. For this we will follow the guidance of [2] and [3].

### 2.1 Laser interactions - Two-level atom

We begin with the interaction between a laser field and a sample of stationary atoms having only two possible energy levels. Aspects of thermal motion will be treated subsequently. The difference  $\Delta E = E_1 - E_0$  between the excited state  $|e\rangle$  energy  $E_1$  and ground state  $|g\rangle$  energy  $E_0$  is used with Planck's law to determine the photon frequency  $\nu$  associated with transitions between the two states:

$$\Delta E = h\nu_0 \quad (2.1)$$

There are three transition processes involving atoms and laser fields:



**Fig. 2.1** Two-level atom model

We consider spontaneous emission first – a process characterized by a transition rate or probability per unit time for an atom in the excited state to decay to the ground state. This transition rate will be denoted  $\Gamma_\omega$  and is about  $2\pi \cdot 1.3$  MHz for the rubidium levels studied here.

In the absence of an external field, any initial population of excited state atoms would decay exponentially to the ground state with a mean life time  $\Delta t = 1/\Gamma_\omega \approx 122$  ns. In the rest frame of the atom, spontaneous photons are emitted in all directions with an energy spectrum having a mean  $E = h\nu_0$  and a full width at half maximum (FWHM)  $\Delta E$  given by the Heisenberg uncertainty principle  $\Delta E \Delta t = \hbar$  or  $\Delta E = \Gamma_\omega \hbar$ . Expressed in frequency units, the FWHM is called the *natural linewidth* and given the symbol  $\Gamma_\nu$ . Thus

$$\Gamma_\nu = \frac{\Gamma_\omega}{2\pi} \quad (2.2)$$

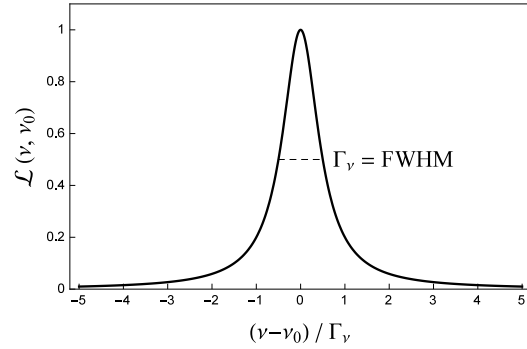
For our rubidium levels,  $\Delta E \approx 5.4 \cdot 10^{-9}$  eV or  $\Gamma_\nu \approx 1.3$  MHz.

The stimulated emission and absorption processes are also described by a transition rate – a single rate giving the probability per unit time for a ground state atom to absorb a laser photon or for an excited state atom to emit a laser photon. The stimulated transition rate is proportional to the laser intensity  $I$  (SI units of  $\text{W m}^{-2}$ ) and is only significantly different from zero when the laser frequency  $\nu$  is near the resonance frequency  $\nu_0$ . This transition rate will be denoted  $\alpha I$ , where

$$\alpha = \alpha_0 \mathcal{L}(\nu, \nu_0) \quad (2.3)$$

and

$$\mathcal{L}(\nu, \nu_0) = \frac{1}{1 + 4(\nu - \nu_0)^2 / \Gamma_\nu^2} \quad (2.4)$$



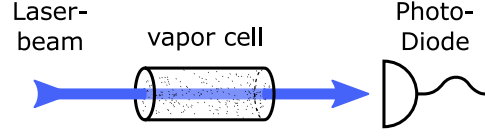
**Fig. 2.2** The Lorentzian line shape profile for resonance absorption

gives the *Lorentzian* frequency dependence as shown in Fig. 2.2.  $\mathcal{L}(\nu, \nu_0)$  also describes the spectrum of radiation from spontaneous emission and the width  $\Gamma_\nu$  is the same for both cases. The maximum transition rate  $\alpha_0 I$  occurs right on resonance ( $\nu = \nu_0$ ).

The value of  $\Gamma_\omega / \alpha_0$  defines the saturation intensity  $I_s$  of the atoms. Its significance is that when the laser intensity is equal to the saturation intensity, excited state atoms are equally likely to decay by stimulated emission or by spontaneous emission.

## 2.2 Basic laser absorption spectroscopy

The basic arrangement for ordinary laser absorption spectroscopy through a gaseous sample is shown in Fig. 2.3. A laser beam passes through the vapor cell and its intensity is measured by a photodiode detector as the laser frequency  $\nu$  is scanned through the natural resonance frequency.



**Fig. 2.3** Basic arrangement for ordinary laser absorption spectroscopy.

When a laser beam propagates through a gaseous sample, the two stimulated transition processes change the intensity of the laser beam and affect the density of atoms in the ground and excited states. To get a better understanding of absorption spectroscopy we begin with the basic equation describing how the laser intensity changes as it propagates through the sample and then continue with the effects of Doppler shifts and population changes.

## 2.3 Laser absorption

Because of stimulated emission and absorption, the laser intensity  $I(x)$  varies as it propagates from  $x$  to  $x + dx$  in the medium. The following equation describes this process:

$$I(x + dx) - I(x) = -I(x)h\nu\alpha n_0(P_0 - P_1)dx \quad (2.5)$$

where:

$n_0$  ... atom density

$n_0 P_0$  ... proportion of atoms in  $|g\rangle$

$n_0 P_1$  ... proportion of atoms in  $|e\rangle$

Equation 2.5 leads to

$$\frac{dI}{dx} = -\kappa I \quad (2.6)$$

where the *absorption coefficient* (fractional absorption per unit length)

$$\kappa = h\nu n_0 \alpha (P_0 - P_1) \quad (2.7)$$

The proportionality to  $P_0 - P_1$  arises from the competition between stimulated emission and absorption and it is important to appreciate the consequences. If there are equal numbers of atoms in the ground and excited state ( $P_0 - P_1 = 0$ ), laser photons are as likely to be

emitted by an atom in the excited state as they are to be absorbed by an atom in the ground state and there will be no attenuation of the incident beam. The attenuation maximizes when all atoms are in the ground state ( $P_0 - P_1 = 1$ ) because only upward transitions would be possible. And the attenuation can even reverse sign (become an amplification as it does in laser gain media) if there are more atoms in the excited state ( $P_0 > P_1$ ).

## 2.4 Doppler shifts

Atoms in a vapor cell move randomly in all directions with each component of velocity having a distribution of values. Only the component of velocity parallel to the laser beam direction will be important when taking into account Doppler shifts and it is this component we refer to with the symbol  $v$ . The density of atoms  $dn$  in the velocity group between  $v$  and  $v + dv$  is given by the Boltzmann velocity distribution:

$$dn = n_0 \sqrt{\frac{m}{2\pi k_B T}} e^{-mv^2/2k_B T} dv \quad (2.8)$$

With a standard deviation given by:

$$\sigma_v = \sqrt{\frac{k_B T}{m}} \quad (2.9)$$

This is just a standard Gaussian distribution

$$dn = n_0 \frac{1}{\sqrt{2\pi}\sigma_v} e^{-v^2/2\sigma_v^2} dv \quad (2.10)$$

with a mean of zero – indicating the atoms are equally likely to be going in either direction. It is properly normalized so that the integral over all velocities ( $-\infty \rightarrow \infty$ ) is  $n_0$ , the overall atom density.

Atoms moving with a velocity  $v$  see the laser beam Doppler shifted by the amount  $\nu(v/c)$ . We will take an equivalent, alternate view that atoms moving with a velocity  $v$  have a Doppler shifted resonance frequency

$$\nu'_0 = \nu_0 \left(1 + \frac{v}{c}\right) \quad (2.11)$$

in the lab frame. The sign has been chosen to be correct for a laser beam propagating in the positive direction so that the resonance frequency is blue shifted to higher frequencies if  $v$  is positive and red shifted if  $v$  is negative.

The absorption coefficient  $d\kappa$  from a velocity group  $dn$  at a laser frequency  $\nu$  is then obtained from Eq. 2.7 by substituting  $dn$  for  $n_0$  and by adjusting the Lorentzian dependence of  $\alpha$  so



that it is centered on the Doppler shifted resonance frequency  $\nu'_0$  (Eq. 2.11).

$$d\kappa = h\nu\alpha_0(P_0 - P_1)\mathcal{L}(\nu, \nu'_0)dn \quad (2.12)$$

The absorption coefficient from all atoms is then found by integrating over all velocity groups.

## 2.5 Population

Before we can determine the absorption coefficient we need an expression for  $P_0 - P_1$ . For that we have to take into account the changes to the ground and excited state populations arising from a laser beam propagating through the cell. The rate equations for the ground and excited state probabilities or fractions become:

$$\begin{aligned} \frac{dP_0}{dt} &= \Gamma_\omega P_1 - \alpha I(P_0 - P_1) \\ \frac{dP_1}{dt} &= -\Gamma_\omega P_1 + \alpha I(P_0 - P_1) \end{aligned} \quad (2.13)$$

where the first term on the right in each equation arises from spontaneous emission and the second term arises from stimulated absorption and emission.

Considering  $P_0 + P_1 = 1$  and the steady state condition

$$\frac{dP_0}{dt} = \frac{dP_1}{dt} = 0 \quad (2.14)$$

we get for the populations

$$P_0 = \frac{\Gamma_\omega + \alpha I}{\Gamma_\omega + 2\alpha I} ; \quad P_1 = \frac{\alpha I}{\Gamma_\omega + 2\alpha I} \quad (2.15)$$

which leads to

$$(P_0 - P_1) = \frac{\Gamma_\omega}{\Gamma_\omega + 2\alpha I} \quad (2.16)$$

As we can see the population difference is dependent on  $\Gamma_\omega$  (spontaneous decay rate), which is related to the *Lorentzian width parameter*  $\Gamma_\nu$ . To combine the Lorentzians in  $\alpha$  and  $P_0 - P_1$  we rewrite both expressions with Eq. 2.2 and  $\Delta\nu = 2(\nu - \nu_0)$ :

$$\alpha = \alpha_0 \frac{1}{1 + \Delta\nu^2/\Gamma_\nu^2} = \alpha_0 \frac{\Gamma_\nu^2}{\Gamma_\nu^2 + \Delta\nu^2} ; \quad (P_0 - P_1) = \frac{2\pi\Gamma_\nu}{2\pi\Gamma_\nu + 2\alpha I} \quad (2.17)$$

and get

$$(P_0 - P_1)\alpha = \frac{2\pi\Gamma_\nu}{2\pi\Gamma_\nu + 2I\alpha_0\frac{\Gamma_\nu^2}{\Gamma_\nu^2 + \Delta\nu^2}} \alpha_0 \frac{\Gamma_\nu^2}{\Gamma_\nu^2 + \Delta\nu^2} = \frac{\alpha_0\pi\Gamma_\nu^2}{\pi(\Gamma_\nu^2 + \Delta\nu^2) + I\alpha_0\Gamma_\nu} \quad (2.18)$$

dividing with  $\pi\Gamma_\nu$  and substitute in the denominator  $\alpha_0$  with the definition of  $I_s = 2\pi\Gamma_\nu/\alpha_0$  leads to

$$\alpha_0 \frac{1}{1 + \frac{\Delta\nu^2}{\Gamma_\nu^2} + \frac{2I}{I_s}} = \frac{\alpha_0}{(1 + \frac{2I}{I_s})} \frac{1}{1 + \frac{\Delta\nu^2}{\Gamma_\nu^2(1 + \frac{2I}{I_s})}} \quad (2.19)$$

with the definition of the power-broadened *width parameter*

$$\Gamma'_\nu = \Gamma_\nu \sqrt{1 + 2I/I_s} \quad (2.20)$$

we obtain

$$(P_0 - P_1)\alpha = \frac{\alpha_0}{(1 + \frac{2I}{I_s})} \mathcal{L}'(\nu, \nu_0) \quad \text{and} \quad \mathcal{L}'(\nu, \nu_0) = \frac{1}{1 + \frac{(\nu - \nu_0)^2}{\Gamma'^2_\nu}} \quad (2.21)$$

## 2.6 Behavior of absorption coefficient

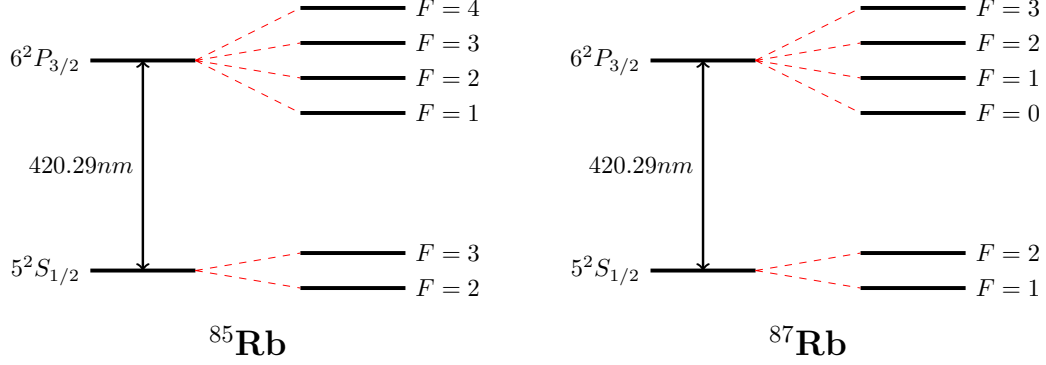
## 2.7 Non-linear differential equation

## 2.8 Relevant data

	Rubidium	
Isotope	85	87
Atomic mass	84.911794	86.909187
in $10^{-25}\text{kg}$	1.40999	1.44316
Abundance	72.17%	27.83%
Spin I	$5/2$	$3/2$
lifetime $6^2P_{3/2}$	112 ns	
Natural linewidth	$2\pi \times 1.421\text{ MHz}$	

**Table 2.1** Properties of rubidium isotopes

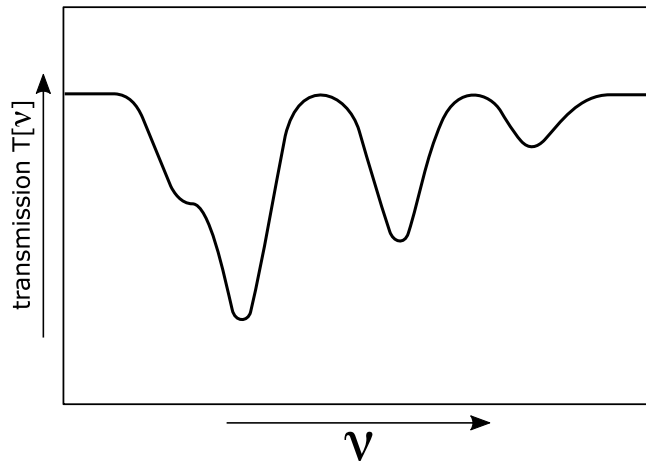
## 2.9 D2 line



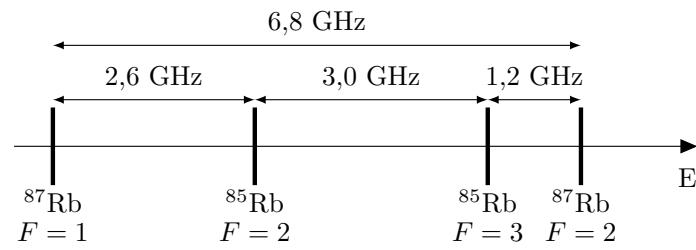
**Fig. 2.4**  $5^2S_{1/2} \rightarrow 6^2P_{3/2}$  transition of  $^{85}\text{Rb}$  and  $^{87}\text{Rb}$  with corresponding hyperfine structure

The transition of interest is, as we have discussed before, the  $5^2S_{1/2} \rightarrow 6^2P_{3/2}$  of rubidium. As known rubidium occurs in two isotopes,  $^{85}\text{Rb}$  and  $^{87}\text{Rb}$ . As we can see both isotopes have the same transition energy, but due to the different spin  $I$  (see table: 2.1) we get different spin  $I$  energy levels for the groundstate [4]. This is the reason why we witness four Doppler peaks in our spectrum.

**Caution:** Both figures below show the correct correlation between energy and isotopes. The reason for this is that the spectrum shows transition energy and the other one the specific energy levels.



**Fig. 2.5** Doppler spectrum of D2 line



**Fig. 2.6** Relative energy gaps of the groundstates between both isotopes





## Chapter 3

# Experiment

3.1 Setup & Tools

3.2 Laser diameter measurement

3.3 Power / intensity measurement

3.4 Doppler-free measurement



## Chapter 4

# Evaluation

4.1 Data processing

4.2 Temperature & saturation intensity

4.3 Comparison with theory

4.4 Compare Doppler-free measurement with theoretical values



# References

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Appendix A

Theory





Appendix B

Experiment



Appendix C

**Evaluation**

