Artificial Intelligent

Sem. Ganjil 2024/2025

06–Adversarial Search

Lionov

- Game Search

- Minimax & $\alpha - \beta$ Pruning

- Monte Carlo Tree Search
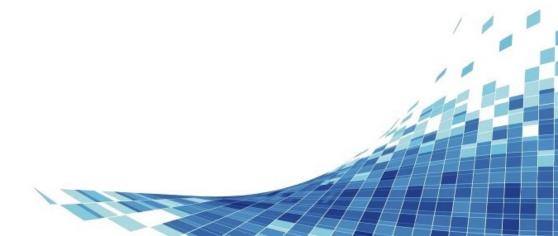
- Stochastic Game

- Partially Observable Games

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Adversarial
involving people opposing or disagreeing with each other

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Games are modeled as search problems and use heuristic evaluation functions.

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- Deterministic
- Perfect information (fully observable)
- Sequential (taking turn)
- Static
- Multi agents

We use term **move** for "action" and **position** for "state"

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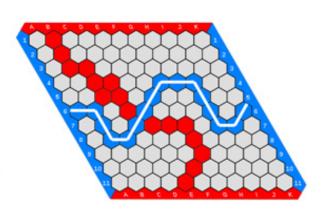
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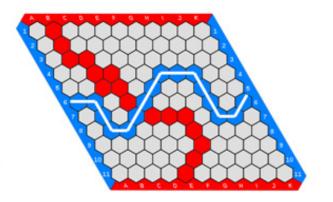- Tic-Tac-Toe
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Othello: human champions *refuse* to compete :D

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- **Is-Terminal(s)** : true when the game is over (at terminal states)
- **Utility(s, p)**: A utility function (objective/ payoff function) that defines the final numeric value to players p when the game ends in terminal state s

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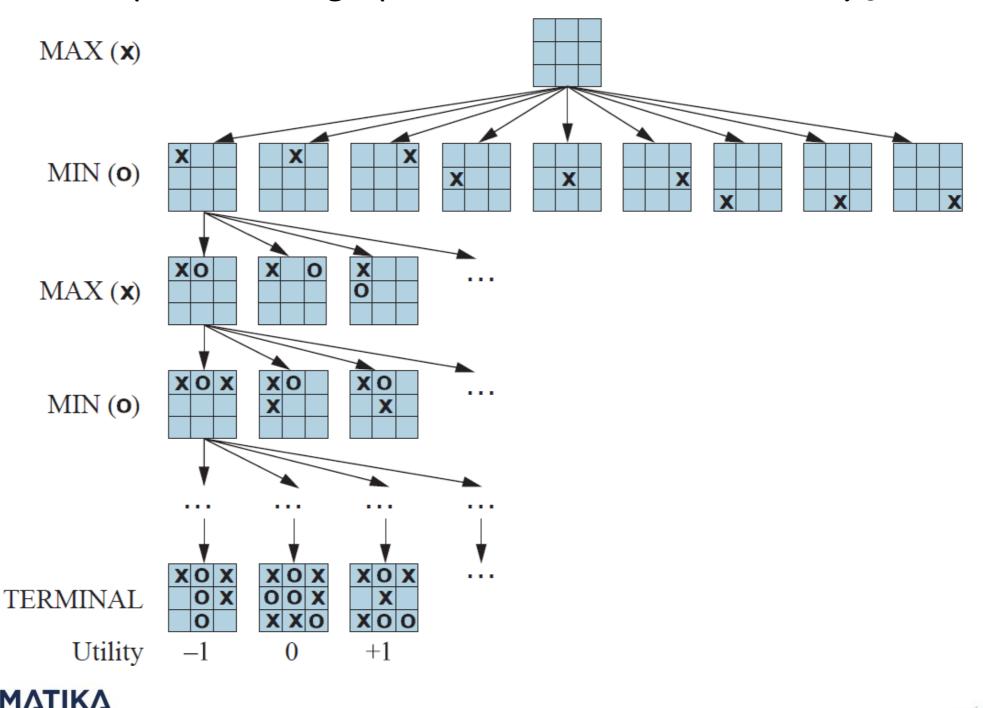The intial state $\mathcal{S}_0$, ACTIONS, and RESULT define the **State-space Graph**

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Minimax Search

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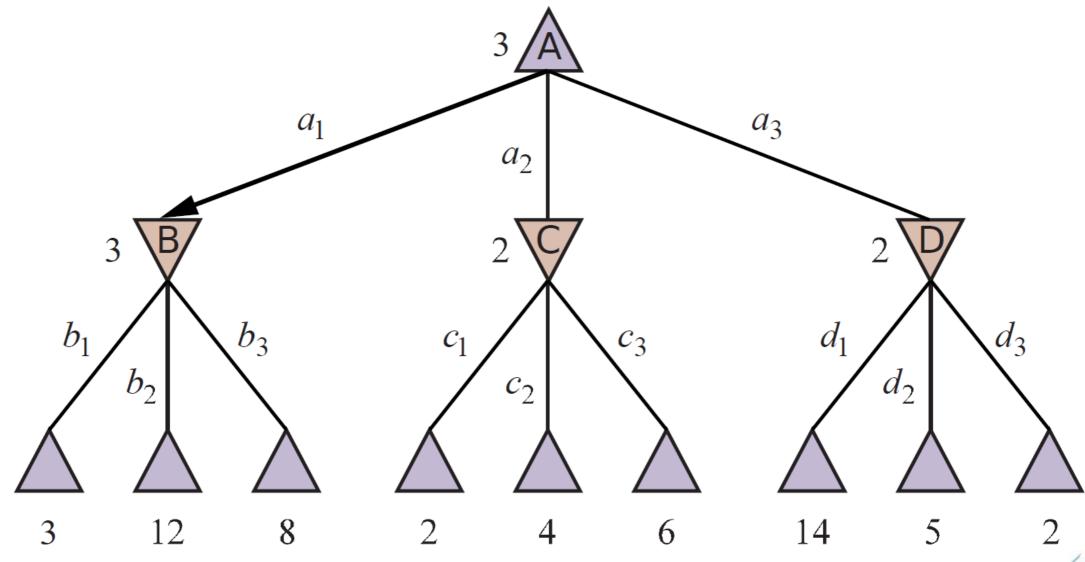
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$$
\text{MINIMAX}(s) =
\begin{cases}
\text{UTILITY}(s, \text{MAX}) & \text{if IS-TERMINAL}(s) \\
\max_{a \in Actions(s)} \text{MINIMAX}(\text{RESULT}(s, a)) & \text{if TO-MOVE}(s) = \text{MAX} \\
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We can identify the **minimax decision** at the root

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```
function MINIMAX-SEARCH(game, state) returns an action
    player ← game.TO-MOVE(state)
    value, move ← MAX-VALUE(game, state)
    return move


function MAX-VALUE(game, state) returns a (utility, move) pair
    if game.IS-TERMINAL(state) then return game.UTILITY(state, player), null
    v ← −∞
    for each a in game.ACTIONS(state) do
        v2, a2 ← MIN-VALUE(game, game.RESULT(state, a))
        if v2 > v then
            v, move ← v2, a
    return v, move


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```

Time complexity is $O(b^m)$ and the space complexity is $O(bm)$

Where m is the maximum depth of the tree and there are b legal moves at each point

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Minimax in Multiplayer Games

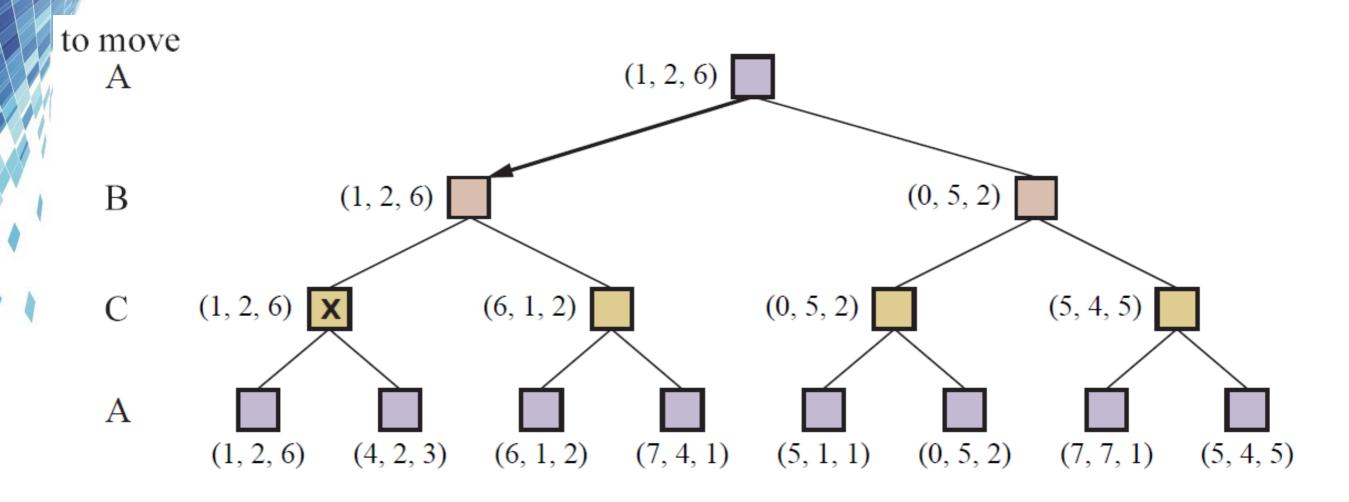Returns a vector of n values (n = the number of players)

Natural alliance may emerge

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Pruning: cut off parts of the tree that make no difference to the outcome

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α: the value of the best (highest-value) choice found so far for **MAX**("at least")

β: the value of the best (lowest-value) choice found so far for **MIN** ("at most")

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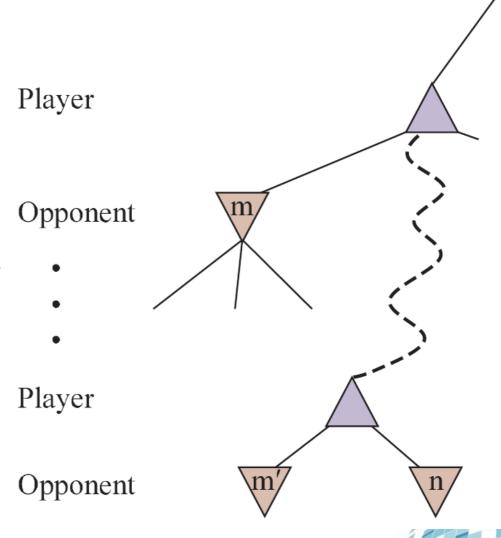
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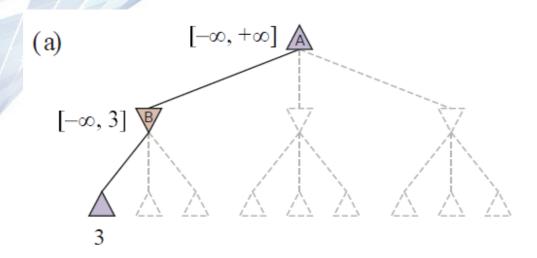
The general principle:
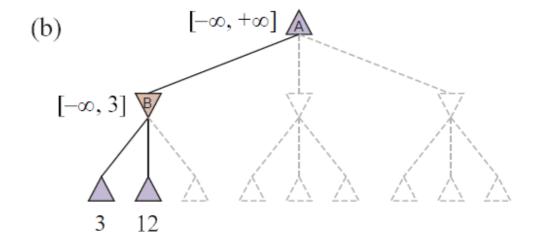Consider a node n such that a player has a choice of moving to n.

If the player has **a better choice** either at the same level (m') or at any point higher up in the tree (m), then the player will never move to n.
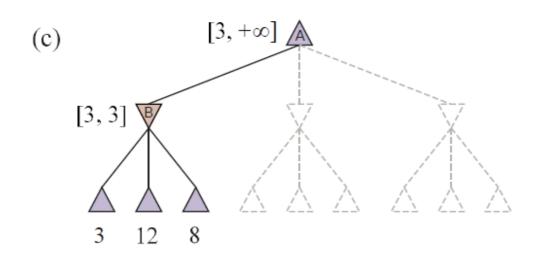
Once we have found out enough about n (by examining some of its descendants) to reach this conclusion, we can prune it.
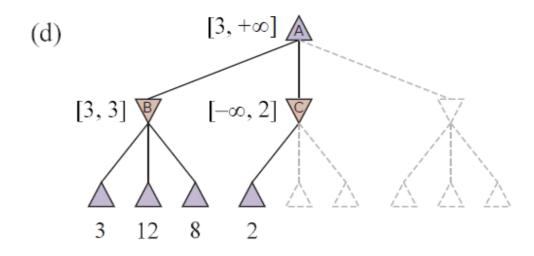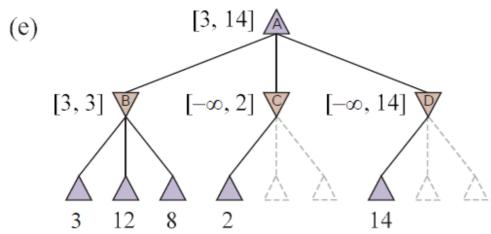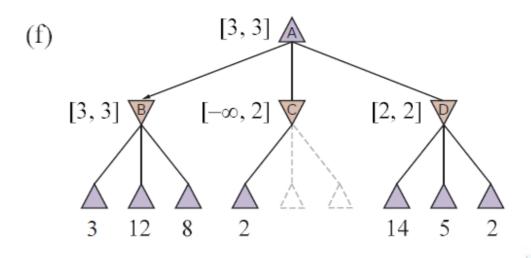
Player

Opponent   m

Player

Opponent   m'   n

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$\alpha - \beta$ Pruning Algorithm

function ALPHA-BETA-SEARCH($game$, $state$) **returns** an action
  player $\leftarrow$ $game$.TO-MOVE($state$)
  $value$, $move$ $\leftarrow$ MAX-VALUE($game$, $state$, $-\infty$, $+\infty$)
  return $move$

function MAX-VALUE($game$, $state$, α, β) **returns** a ($utility$, $move$) pair
  if $game$.IS-TERMINAL($state$) **then return** $game$.UTILITY($state$, $player$), $null$
  $v \leftarrow -\infty$
  for each a **in** $game$.ACTIONS($state$) **do**
    $v2$, $a2$ $\leftarrow$ MIN-VALUE($game$, $game$.RESULT($state$, a), α, β)
    if $v2 > v$ **then**
      v, $move$ $\leftarrow$ $v2$, a
      $\alpha \leftarrow$ MAX(α, v)
    if $v \geq \beta$ **then return** v, $move$
  return v, $move$

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    if $v2 < v$ **then**
      v, $move$ $\leftarrow$ $v2$, a
      $\beta \leftarrow$ MIN(β, v)
    if $v \leq \alpha$ **then return** v, $move$
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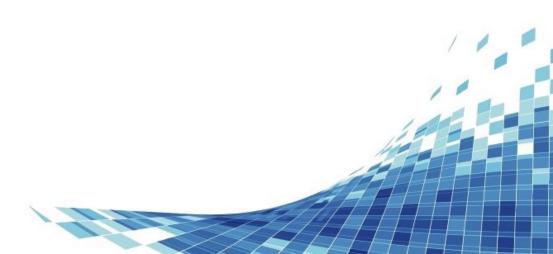
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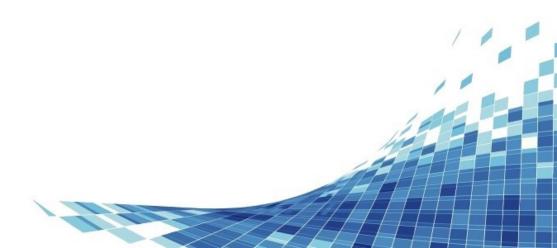Performance of the Alpha-Beta Pruning depends on the order of the moves

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- **Type A strategy**: considers all possible moves to a certain depth in the search tree, and then uses a heuristic evaluation function to estimate the utility of states at that depth.
- **Type B strategy**: ignores moves that look bad, and follows promising lines "as far as possible".

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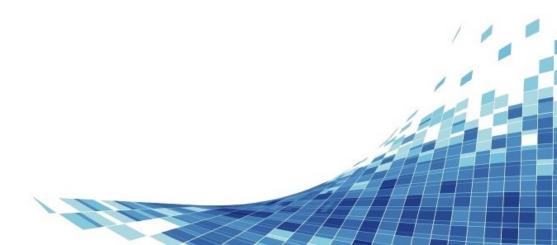# Heuristic for $\alpha - \beta$ Search

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Weighted combination of features of a game state:

$$\textsc{Eval}(s) = \omega f_1(s) + \omega f_2(s) + \ldots + \omega f_n(s) = \Sigma_i^n \omega f_i(s)$$

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Properties of a good evaluation function:

- the computation must be fast
- should be strongly correlated with the actual chances of winning

Most evaluation functions work by calculating various features of the state
Weighted combination of features of a game state:

$$\textsc{Eval}(s) = \omega f_1(s) + \omega f_2(s) + \ldots + \omega f_n(s) = \Sigma_i^n \omega f_i(s)$$

The features and their weights are results of years of experience or estimated by machine learning techniques

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Evaluation Function

The heuristic $\text{EVAL}(s, p)$ returns an *estimate* of the expected utility of state s

- For terminal states: $\text{EVAL}(s, p) = \text{UTILITY}(s, p)$.
- For nonterminal states: $\text{UTILITY}(loss, p) \leqslant \text{EVAL}(s, p) \leqslant \text{UTILITY}(win, p)$

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if game. IS-CUTOFF(STATE, DEPTH) **then return** GAME.EVAL(STATE, PLAYER), NULL

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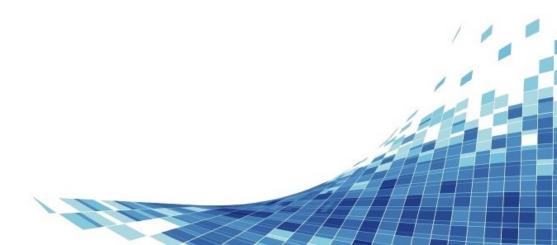
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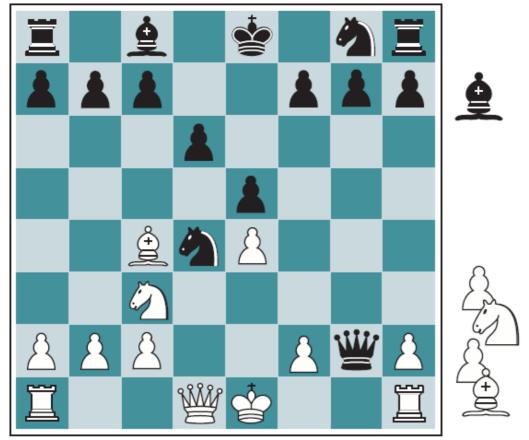
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This strategies may lead to error:

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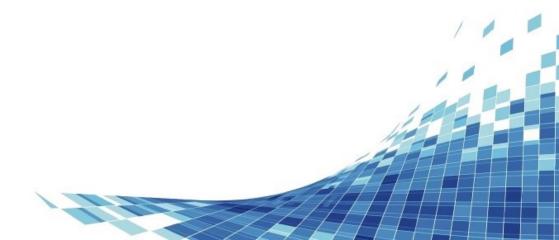
Search vs Lookup

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  Map every possible state to the best move in that state. The computer can play perfectly by looking up the right move in this table.

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Monte Carlo Tree Search

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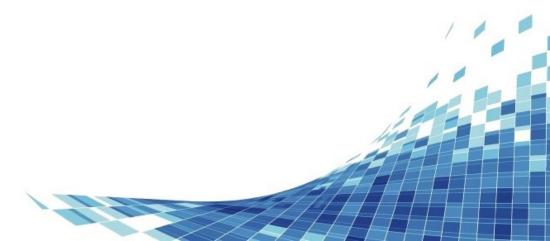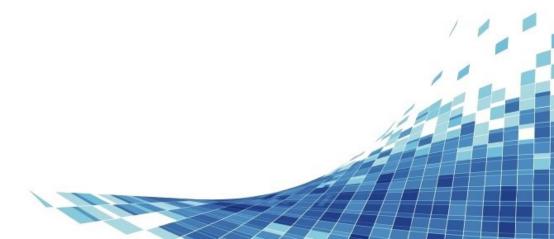
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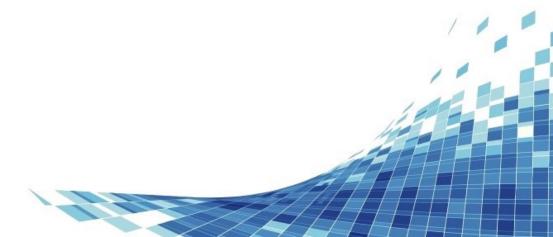Heuristic of Alpha-Beta Tree Search have two major weaknesses:
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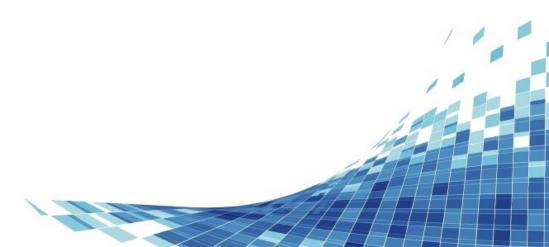- Difficult to define a "perfect" evaluation functions

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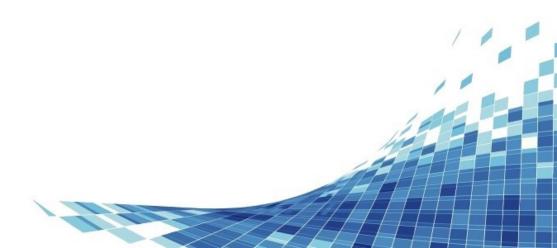
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Pure Monte Carlo search: do N simulations starting from the current state and track which of the possible moves has the highest win percentage

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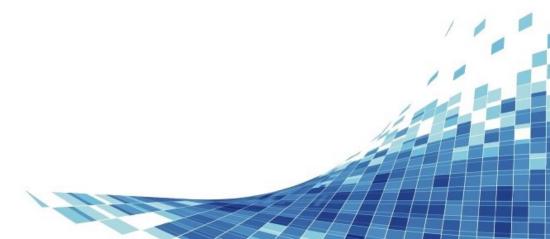
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Pure MCTS is not enough, we need a selection policy that focuses the computational resources on the important parts of the game tree and balace:

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MCTS maintains a search tree and grows it on each iteration with this four steps:

1. SELECTION
2. EXPANSION
3. SIMULATION
4. BACK-PROPAGATION

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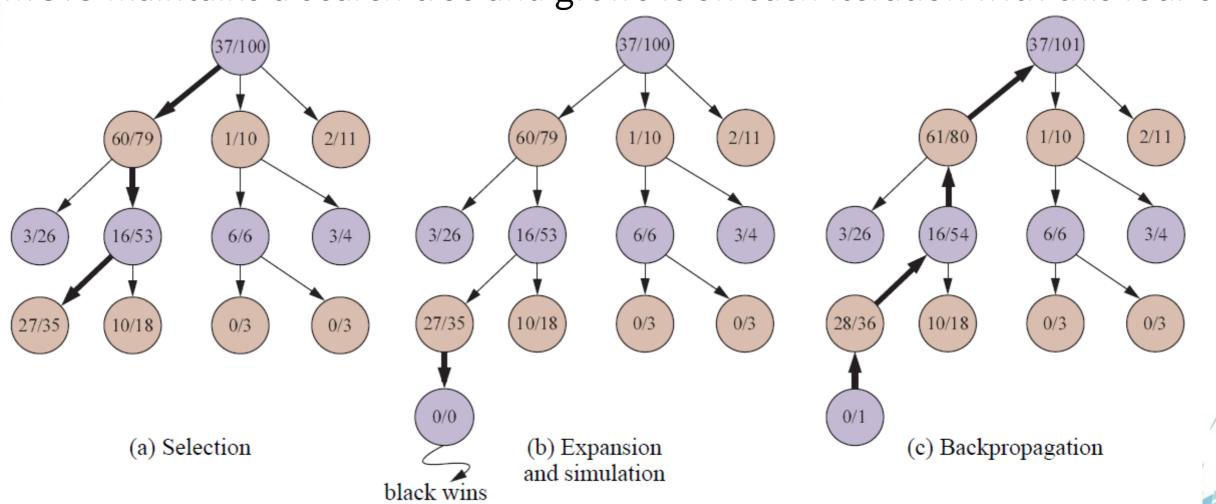UNPAR

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black wins

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Exp. of a selection policy: "upper confidence bounds applied to trees" or UCT. UCT ranks possible move based on an upper confidence bound formula UCB1

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```
function MONTE-CARLO-TREE-SEARCH(state) returns an action
    tree ← NODE(state)
    while IS-TIME-REMAINING() do
        leaf ← SELECT(tree)
        child ← EXPAND(leaf)
        result ← SIMULATE(child)
        BACK-PROPAGATE(result, child)
    return the move in ACTIONS(state) whose node has highest number of playouts
```

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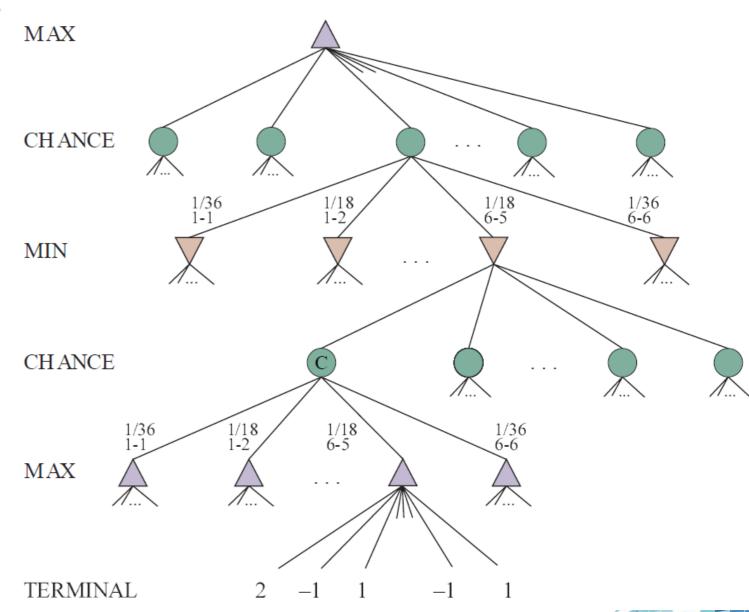
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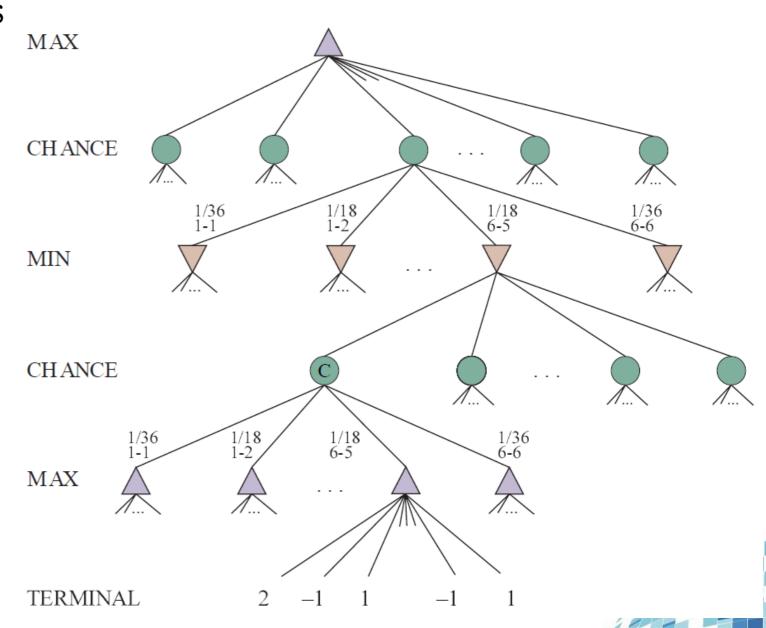
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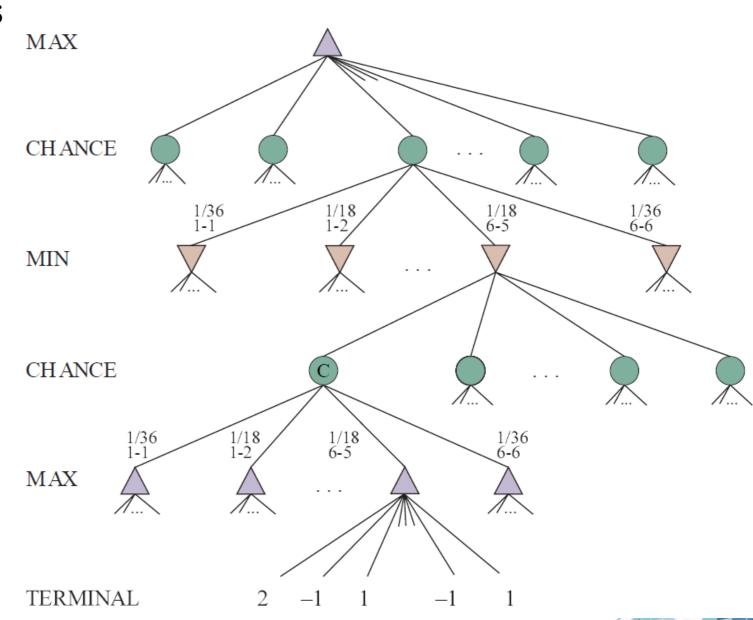
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Expectiminimax value for games with chance nodes, a generalization of the minimax value for deterministic games

Partially Observable Games

In games of imperfect information, such as Kriegspiel and poker, optimal play requires reasoning about the current and future belief states of each player. A simple approximation can be obtained by averaging the value of an action over each possible configuration of missing information.

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Limitations of Game Search

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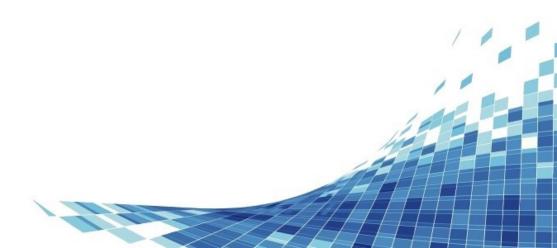
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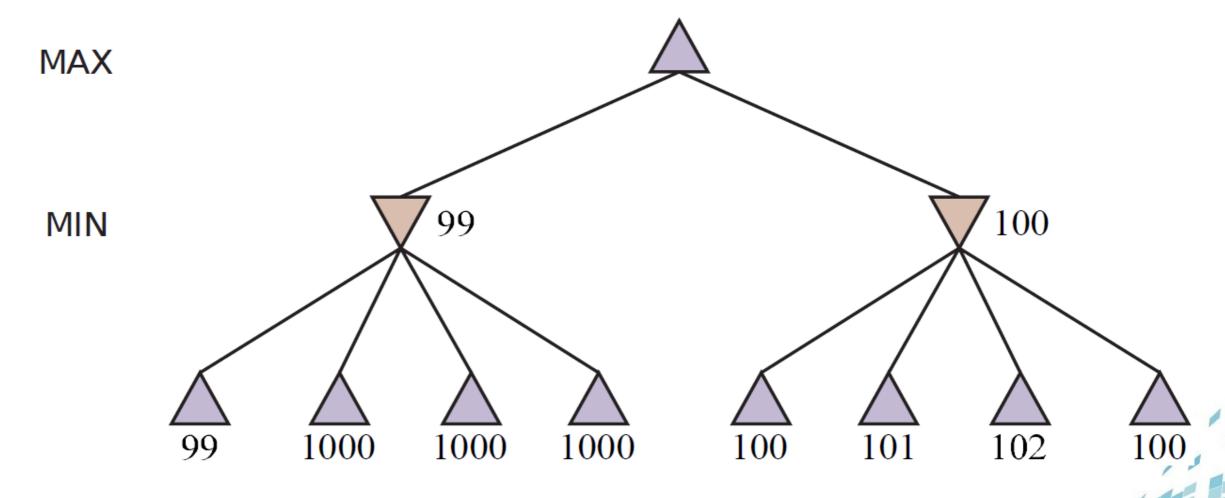
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- Reasoning are done at the level of individual moves. Humans play games differently: reasoning at the abstract level
- No ability to incorporate machine learning into the game search process

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