Recursive Functions Ex

Question #01:

Find $F(8, 12)$

$$f(x,y) = \begin{cases} f(x+1),(y-2) + 3 & \text{if } x<y \\ x^2+y^2 & \text{if } x=y \\ F(y-1,x)-1 & \text{if } x>y \end{cases}$$

We start it with $F(8,12)$. since $x<y$ we apply the first condition:

$F(8+1, 12-2) + 3$

$F(9, 10) + 3$

Now $x<y$ still hold true for $F(9,10)$ So we apply the first condition

$F(9+1, 10-2) + 3$

$F(10, 8) + 3$

Now $x>y$ for $F(10, 8)$ So we apply third condition

$F(8-1, 10) - 1$

$F(7, 10) - 1$

Now $x<y$ still hold true for $F(7,10)$

$F(7+1, 10-2)+3$

$F(8,8)+3$

Now $x=y$ For $F(8,8)$ So we apply second condition.

$$F(8,8) = 8^2+8^2$$
$$= 64+64$$
$$= 128$$

Back Substitution

$F(7,10) = 128+3 = 131$

$F(10,8) = 131-1 = 130$

$F(9,10) = 130+3 = 133$

$F(8,12) = 133+3 = 136.$

So, our Answer is

$F(8,12) = 136.$

Question #02:

Find $f(f(f(f(f(10)))))$

$$f(x) = \begin{cases} f(x-5)-3 & \text{if } x>7 \\ f(x+2)+2 & \text{if } 3<x\leq 7 \\ x+5 & \text{if } x\leq 3 \end{cases}$$

let's start by $f(10)$ since $x > 7$ we use first condition

$$f(10) = f(10-5) - 3$$
$$= f(5) - 3$$

Now we have to find $f(5)$ we use second condition

$$f(5) = f(5+2) + 2$$
$$= f(7) + 2$$

Now we have to find $f(7)$ we use 2nd condition

$$f(7) = f(7+2) + 2$$
$$= f(9) + 2.$$

Now we have to find $f(9)$ we use 1st condition

$$f(9) = f(9-5) - 3$$
$$f(4) - 3$$

Now we have to find $f(4)$ we use 2nd condition

$$f(4) = f(4+2) + 2$$
$$f(6) + 2$$

Now we have to find $f(6)$ we use 2nd condition

$F(6) = f(6+2)+2$

$= f(8)+2$

Now we have to find $f(8)$ we use 1st condition

$f(8) = f(8-5)-3$

$= f(3)-3$

Now we have to find $f(3)$ we use third condition

$f(3) = 3+5 = 8$

by Back Substitution

$f(3) = 8$

$f(8) = f(3)-3 = 8-3 = 5$

$f(6) = f(8)+2 = 5+2 = 7$

$f(4) = f(6)+2 = 7+2 = 9$

$f(9) = f(4)-3 = 9-3 = 6$

$F(7) = f(9)+2 = 6+2 = 8$

$F(5) = f(7)+2 = 8+2 = 10$

$f(10) = f(5)-3 = 10-3 = 7$

So,

$f(f(f(f(f(10))))) = f(f(f(f(7)))) =$

$f(f(f(8))) = f(f(5)) = f(10) = 7$.

Ans.

$f(f(f(f(f(10))))) = 7$. Ans.

Question #03:

Find f(30,12) where max is the bigger result of the 2 arguments.

$$f(x,y)=\begin{cases}\max(f(x-4,y+3),f(y,x)) & \text{if } x>y \\ x*y & \text{otherwise}\end{cases}$$

We have to evaluate f(30,12) using the given function.

Since $x>y$ $(30>12)$ we will use the first condition

$$\max(f(30-4,12+3),f(12,30))$$

$$\max(f(26,15),f(12,30))$$

Again $x>y$ $(26>15)$ we use first Case.

$$\max(f(26-4,15+3),f(15,26))$$

$$\max(f(22,18),f(15,26))$$

Again $x>y$ $(22>18)$ we use first condition.

$F(22,18) = \max(f(22-4, 18+3), f(18,22))$

$= \max f(18,21), f(18,22)$

Now comparing $f(18,21)$ and $f(18,22)$

$f(18,21) = 18 \times 21 = 378$

$f(18,22) = 18 \times 22 = 396$

So, $f(22,18) = \max(378, 396) = 396$.

Now going back to $f(26,15)$

$f(26,15) = \max(396, f(15,26))$

$f(15,26) = 15 \times 26 = 390$

So, $f(26,15) = \max(396, 390) = 396$

Now going back to $f(30,12)$

$f(30,12) = \max(f(26,15), f(12,30))$

$= \max(396, f(12,30))$

$f(12,30) = 12 \times 30 = 360$

So, $f(30,12) = \max(396, 360) = 396$

Therefore, $f(30,12) = 396$. Ans.

Question #04:

Find $F(32)$, given

$$f(x)=\begin{cases} f(x/2-1)+3 & \text{if } x \text{ is even} \\ 2f(x-3)-5 & \text{if } x \text{ is odd positive} \\ x^2-3 & \text{if } x \text{ is odd negative} \end{cases}$$

Starting with $x=32$, it is even. So we apply the first condition

$$f(32) = f\left(\frac{32}{2}-1\right)+3$$

$$F(15)+3$$

15 is odd and positive So, we apply the second condition

$$F(15) = 2F(15-3)-5$$

$$2F(12)-5$$

Now 12 is even, so we apply first condition

$$F(12) = f\left(\frac{12}{2}-1\right)+3$$

$$= F(5)+3$$

5 is odd and positive so apply the second condition

$$F(5) = 2F(5-3)-5$$

$$= 2F(2)-5$$

Now, 2 is even so apply the First condition.

$$F(2) = F\left(\frac{2}{2} - 1\right) + 3$$

$$= F(0) + 3$$

0 is even so apply the first condition

$$F(0) = F\left(\frac{0}{2} - 1\right) + 3$$

$$= F(-1) + 3$$

Now -1 is the odd and negative So apply the third condition

$$F(-1) = (-1)^2 - 3$$

$$= 1 - 3$$

$$= -2$$

Now back Substitution

$$F(0) = -2 + 3 = 1$$

$$F(2) = 1 + 3 = 4$$

$$F(5) = 2(4) - 5 = 3$$

$$F(12) = 3 + 3 = 6$$

$$F(15) = 2(6) - 5 = 7$$

$F(32) = 7 + 3 = 10$

So our Answer is $F(32) = 10$

Question #05:

if $f(1) = 8$ and $F(n+1) = 2f(n) - 4$

Find $f(5)$

Given, $F(1) = 8$

$F(n+1) = 2f(n) - 4$ ——— (1)

$F(5) = ?$

Put $n = 1$ in equ (1)

$F(1+1) = 2f(1) - 4$

$F(2) = 2(8) - 4$

$= 16 - 4$

$= 12$

Put $n = 2$ in equ in equ (1)

$F(2+1) = 2f(2) - 4$

$F(3) = 2(12) - 4$

$= 24 - 4$

$= 20$

put $n = 3$ in equ (1)

$F(3+1) = 2F(3) - 4$

$= 2(20) - 4$

$= 40 - 4$

$= 36$

Put $n=4$ in eqn (i)

$$F(4+1) = 2F(4) - 4$$
$$F(5) = 2(36) - 4$$
$$= 72 - 4$$
$$= 68$$

So, $F(5) = 68$ is our Answer.

Question # 06:

Find $F(35, 8)$ given

$$f(x,y) = \begin{cases} f(x-y, y+2) + y & \text{if } x \geq y \\ x^2 - y & \text{if } x < y \end{cases}$$

We start with $f(35, 8)$ since $x \geq y$ so we use first Condition

$$f(35,8) = f(35-8, 8+2) + 8$$
$$f(27, 10) + 8$$

Again $F(27, 10)$ x is greater than 10. So we use first condition

$$F(27,10) = f(27-10, 10+2) + 10$$
$$f(17, 12) + 10$$

Now $x \geq y$ So we use 1st Condition

$$F(17, 12) = F(17-12, 12+2) + 12$$

$$= F(5, 14) + 12$$

Now $x < y$ so we apply the second condition

$$f(5,14) = 5^2 - 14 = 25 - 14 = 11$$

Now back Substitution

$$F(17,12) = 11 + 12 = 23$$

$$F(27,10) = 23 + 10 = 33$$

$$F(35, 8) = 33 + 8 = 41$$

So,

$F(35, 8) = 41$ Answer.

Question # 07:

Find the Sum of

$f(1) + f(2) + f(3) \cdots\cdots + f(10)$

given $f(1) = 1$ and

$F(n) = f(n-1) + 5$

Given that $f(1) = 1$

$$f(n) = f(n-1) + 5 \quad -(1)$$

Put $n = 2$ we get

$$f(2) = f(2-1) + 5$$
$$= f(1) + 5$$
$$= 1 + 5 = 6$$

Put $n = 3$ we get

$$f(3) = f(3-1) + 5$$
$$= f(2) + 5$$
$$6 + 5 = 11$$

Put $n = 4$ we get

$$f(4) = f(4-1) + 5$$
$$= f(3) + 5$$
$$11 + 5 = 16$$

Put $n = 5$ we get

$$f(5) = f(5-1) + 5$$
$$= f(4) + 5$$
$$= 16 + 5 = 21$$

Put $n = 6$ we get

$$F(6) = f(6-1) + 5$$
$$F(5) + 5$$
$$21 + 5 = 26$$

Put $n=7$ we get

$$F(7) = F(7-1) + 5$$
$$= F(6) + 5$$
$$= 26 + 5 = 31$$

Put $n=8$ we get

$$F(8) = F(8-1) + 5$$
$$= F(7) + 5$$
$$31 + 5 = 36$$

Put $n=9$ we get

$$F(9) = F(9-1) + 5$$
$$F(8) + 5$$
$$36 + 5 = 41$$

Put $n=10$ we get

$$F(10) = F(10-1) + 5$$
$$= F(9) + 5$$
$$= 41 + 5 = 46.$$

Now we can sum these values

$$F(1) + F(2) \cdots F(10) = 1 + 6 + 11 + 16 + 21 + 26 + 31 + 36 + 41 + 46$$

$$= 235$$ Answer

Question #08:

Find $f(f(f(f(18))))$

$$f(x) = \begin{cases} f(x-5)-2 & \text{if } x \geq 10 \\ f(x+4)+6 & \text{if } 7 \leq x < 10 \\ x-4 & \text{if } x < 7 \end{cases}$$

We start by $f(18)$ since $18 \geq 10$ so we apply first condition

$$f(18) = f(18-5)-2$$
$$= f(13)-2$$

Since $13 \geq 10$ we use the first condition

$$f(13) = f(13-5)-2$$
$$= f(8)-2$$

To find $f(8)$, since $7 \leq 8 < 10$ we use 2nd condition

$$f(8) = f(8+4)+6$$
$$= f(12)+6$$

Now we have to find $f(12)$ we use 2st condition

$$f(12) = f(12-5)-2$$
$$= f(7)-2$$

Now we have to find F(7) since $7 \leq 7 < 10$ we use 2nd condition

$$f(7) = f(7+4) + 6$$
$$f(11) + 6$$

Now we have to find f(11) since $11 \geq 10$ we use 1st condition.

$$f(11) = f(11-5) - 2$$
$$= f(6) - 2$$

Now we have to find f(6) since $6 < 7$ we use third case

$$f(6) = 6 - 4 = 2.$$

by Back substitution.

$$f(11) = 2 - 2 = 0$$

$$F(7) = 0 + 6 = 6$$

$$f(12) = 6 - 2 = 4$$

$$f(8) = 4 + 6 = 10$$

$$f(13) = 10 - 2 = 8$$

$$f(18) = 8 - 2 = 6$$

Finally, we have to evaluate

$f(f(f(f(18))))$.

First $f(f(18) = f(6) = 2$

$f(f(f(18) = f(2) = 2-4 = -2$

$f(f(f(f(18)))) = f(-2) = -6$

So our Answer is -6.