Control Systems Project

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Introduction to Project

Consider the following state-space model:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 3 \\ 3 & 2 & 4 \\ 1 & 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 4 \\ 0 \\ 3 \end{bmatrix} u \tag{1}$$

$$y(t) = \begin{bmatrix} 1 & 1 & 0.5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \tag{2}$$

a. Check the stability of the system using all methods that you know.

b. Compute the controllability and observability for the system.If the system is unstable, design a suitable controller for it.

Introduction to project

c. Simulate the system using the controller(that you design) and show all the responses.

d. Design a PID Controller and show the response of the system using PID Controller. Compare the results obtained in parts c and d.

e. Compute the steady state errors before and after designing controller.

f. Design a tracking controller for step tracking of amplitude 2u(t) and ramp tracking of tu(t).

Stability Analysis of the System

- The eigenvalues of the system are:

$$\boldsymbol{\lambda_1 = 6.8023, \lambda_2 = 1.3099, \lambda_3 = -0.1122} \tag{3}$$

- The poles of the system are:

$$\boldsymbol{p_1 = 6.8023, p_2 = 1.3099, p_3 = -0.1122} \tag{4}$$

- As we can see two of the eigenvalues and poles is positive, which indicates the system is unstable.

Stability Analysis of the System

- Stability Check using Routh-Hurwitz Criterion:

$$G(s) = \frac{5.5s^2+7.5s-13}{s^3-8s^2+8s+1}$$

s^3	1	8
s^2	-8	1

- As there are sign changes in the first column, the system is unstable.

Stability Analysis of the System

- The step response of the system is as below. The system is unbounded, which means it's unstable.

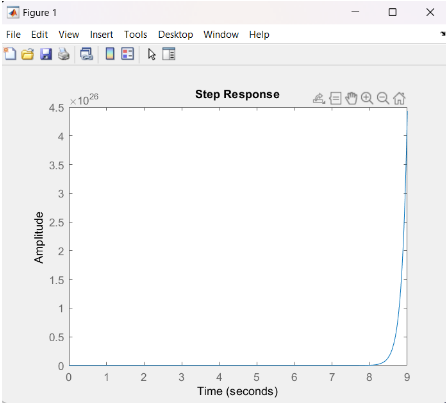

Figure: step response.

Stability Analysis of the System

- The pole-zero map of the system is:

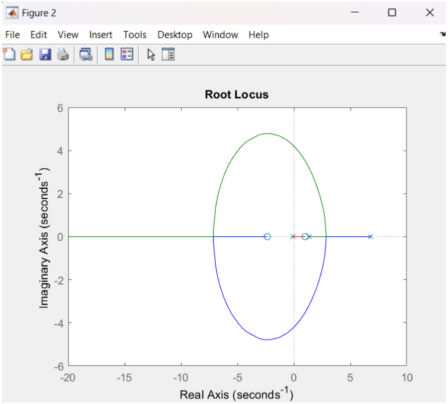

Figure: pole zero-map.

Controllability Analysis

- For Controllability, We Check the Rank(P)

$$\boldsymbol{P = \begin{bmatrix} B & AB & A^2B \end{bmatrix}} \tag{5}$$

$$\boldsymbol{P = \begin{bmatrix} 4 & 21 & 150 \\ 0 & 24 & 163 \\ 3 & 13 & 84 \end{bmatrix}} \tag{6}$$

- As, Rank(P) = 3 = Order of matrix A, n = 3, Which means the system is Controllable.
- Thus the system pass controllability test.

Observability Analysis

- For Observability, We Check the Rank(Q)

$$\boldsymbol{Q} = \begin{bmatrix} \boldsymbol{C} \\ \boldsymbol{CA} \\ \boldsymbol{CA^2} \end{bmatrix} \tag{7}$$

$$\boldsymbol{Q} = \begin{bmatrix} \mathbf{1} & \mathbf{1} & \mathbf{0.5} \\ \mathbf{6} & \mathbf{4.5} & \mathbf{8.5} \\ \mathbf{41} & \mathbf{30} & \mathbf{63} \end{bmatrix} \tag{8}$$

- As, Rank(Q) = 3 = Order of matrix A,n = 3, Which means the system is Observable.
- Thus the system pass observability test.

Controller Design

$$\boldsymbol{C} = \begin{bmatrix} \mathbf{1} & \mathbf{1} & \mathbf{0.5} \end{bmatrix} \neq \boldsymbol{I} \tag{9}$$

- not equal to the identity
- This means that the Observer-based state feedback Controller can be designed.
- Desired Controller eigenvalues (-16,-8, -10).

$$\mathsf{K} = \begin{bmatrix} \mathbf{14.7996} & \mathbf{-0.0944} & \mathbf{-173.7328} \end{bmatrix}$$

- Desired Observer eigenvalues (-80,-40, -50).

$$\mathsf{L} = 1.0\mathrm{e}{+}04 \begin{bmatrix} \mathbf{-3.8105} \\ \mathbf{3.1401} \\ \mathbf{1.3764} \end{bmatrix}$$

System without controller

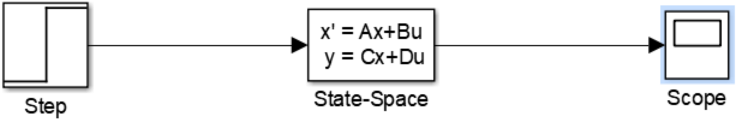

Figure: System without controller.

Step response without controller

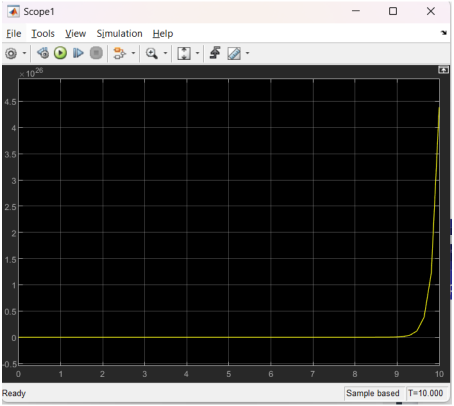

Figure: Step response without controller.

Unstable system with observer-based state feedback Controller

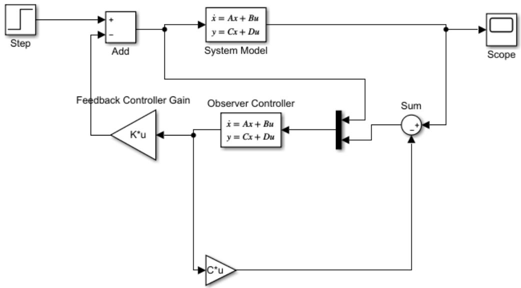

Figure: Schematic of Observer-based State Feedback Controller.

Step Response

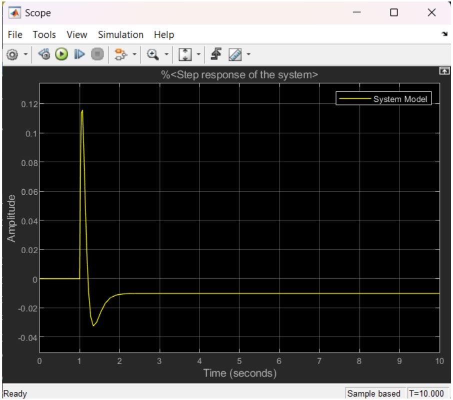

Figure: Plot of Observer-based State Feedback Controller.

PID controller With Controlled system

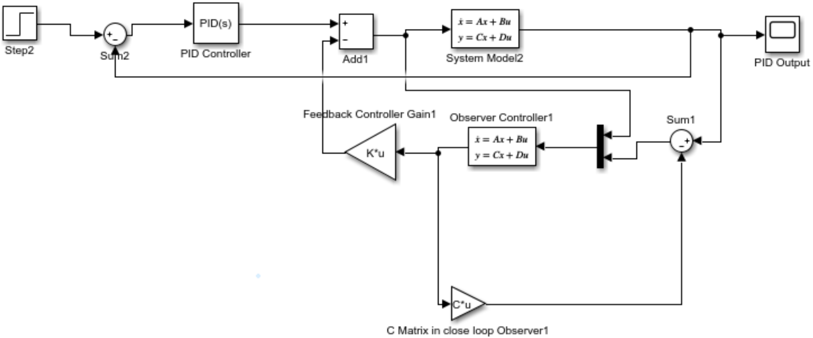

Figure: Schematic of PID Controller with controlled system.

Step Response

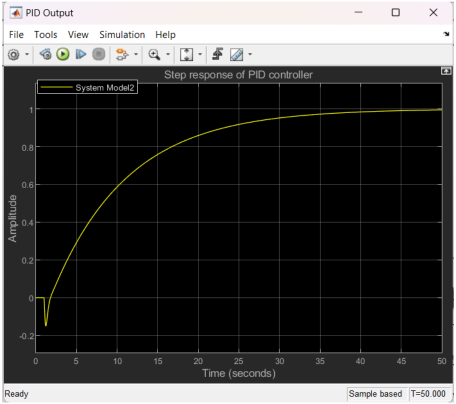

Figure: Plot of step response of PID controller.

Comparison of PID Controller with Controlled system

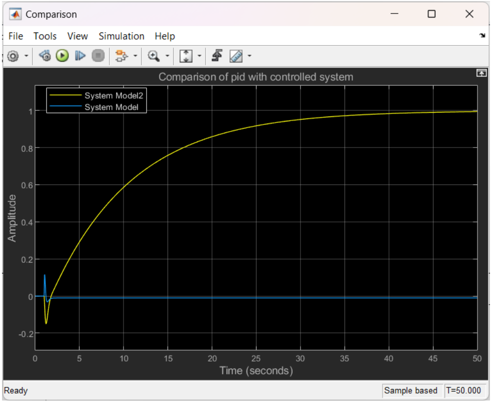

Figure: Comparison of step responses of observer-based feedback controller with PID controller.

Steady State Errors

- Steady state before the controller.
  Undefined because the system is unstable.
- steady state error for step input, after the controller.
- steady state error = input - output
- steady state error = 1 - (-0.01)= 1.01

Steady State Errors

After PID controller

- Steady state error for step input, after PID controller.
- Steady state error = 1 - 1 = 0

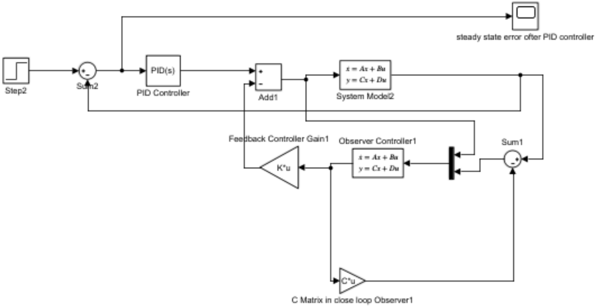

Figure: Schematic of Steady State Errors ofter PID controller.

Steady State Errors

After PID controller

- Steady state error for step input, after PID controller.

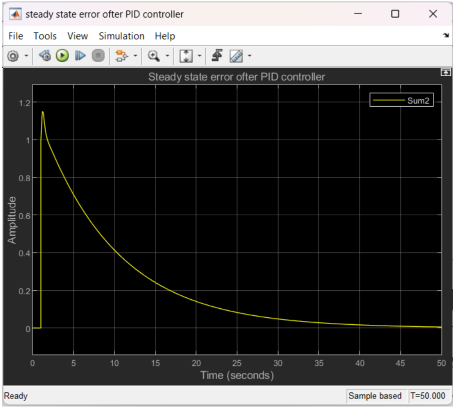

Figure: Steady state error after PID controller.

Tracking Controller

Tracking Controller for step tracking

- Step tracking of 2tu(t)

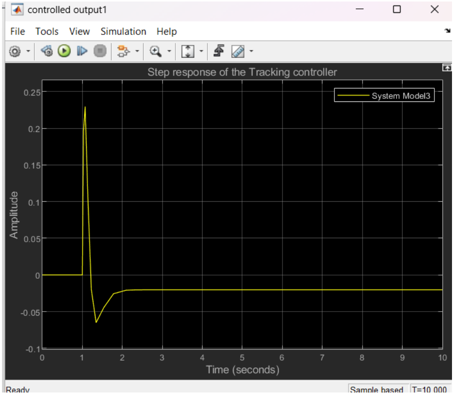

Figure: Plot of Step tracking of amplitude 2u(t).

Tracking Controller

Tracking Controller for ramp tracking

- Ramp tracking of tu(t)

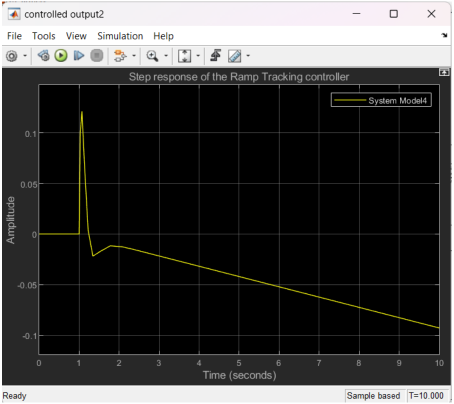

Figure: Plot of Ramp tracking of amplitude tu(t).

Thank You!

Thank You

Thanks for your attention