University of Engineering and Technology, Peshawar

CONTROL SYSTEMS

PROJECT REPORT

Fall 2022

CSE310L Control Systems Lab

Submitted by: FAWAD ALI

Registration No: 19PWCSE1845

Class Section: C

"On my honor, as student of University of Engineering and Technology, I have neither given nor received unauthorized assistance on this academic work."

Student Signature: _______________

Submitted to:

Dr. Salam Ahmed

Date (Jan, 11, 2023)

Department of Computer Systems Engineering

University of Engineering and Technology, Peshawar

Project Report

Problem which is considered

Consider the following state-space model:

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \end{bmatrix} = \begin{bmatrix} 3 & 2 & 3 \\ 3 & 2 & 4 \\ 1 & 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 4 \\ 0 \\ 3 \end{bmatrix} u \tag{1}$$

$$y(t) = \begin{bmatrix} 1 & 1 & 0.5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \tag{2}$$

a. Check the stability of the system using all methods that you know.
b. Compute the controllability and observability for the system.If the system isunstable, design a suitable controller for it.
c. Simulate the system using the controller(that you design) and show all the responses.
d. Design a PID Controller and show the response of the system using PID Controller.Compare the results obtained in part c and d.
e. Compute the steady state errors before and after designing controller.
f. Design a tracking controller for step tracking of amplitude 7u(t) and ramp tracking of 2tu(t).

Guide for choosing desired location of controller eigen values: Consider regis- tration number 20PWCSE1234, then f = 1, g = 2, h = 3, i = 4.choose your controller eigen values as (-f x 2, -g x 2,-h x 2, -i x 2) and observer eigen valuesas (-f x 10, -g x 10,-h x 10, -i x 10). Use your own registration number instead of 20PWCSE1234.

Solution:

In this report, we address the above problem and explain each sub problem in detail.

Stability analysis of the system:

In this section, we analyze the stability of the system. The stability can be checked using different ways namely eigenvalues, step response, poles, pole-zero map and RH-stability criteria. For our case, the system is of 3rd-order and therefore there will be three eigenvalues. Let λ_1, λ_2 and λ_3 denote the eigenvalues of the system. The eigenvalues can be written as follows:

$\lambda_1 = 6.8023$, $\lambda_2 = 1.3099$, $\lambda_3 = -0.1122$

As we can see two of the eigenvalues is positive, which indicates the system is unstable. Next, we verify the same fact by observing the poles of the system. Let p_1, p_2 and p_3 denote the poles of the system. The values for poles are as follows:

$p_1 = 6.8023$, $p_2 = 1.3099$, $p_3 = -0.1122$

We observe here again that two of the poles is positive, which indicates the system is unstable. Next, we verify the same fact by seeing the step-response of the system. The step-response of the open-loop system is shown in Figure 1.

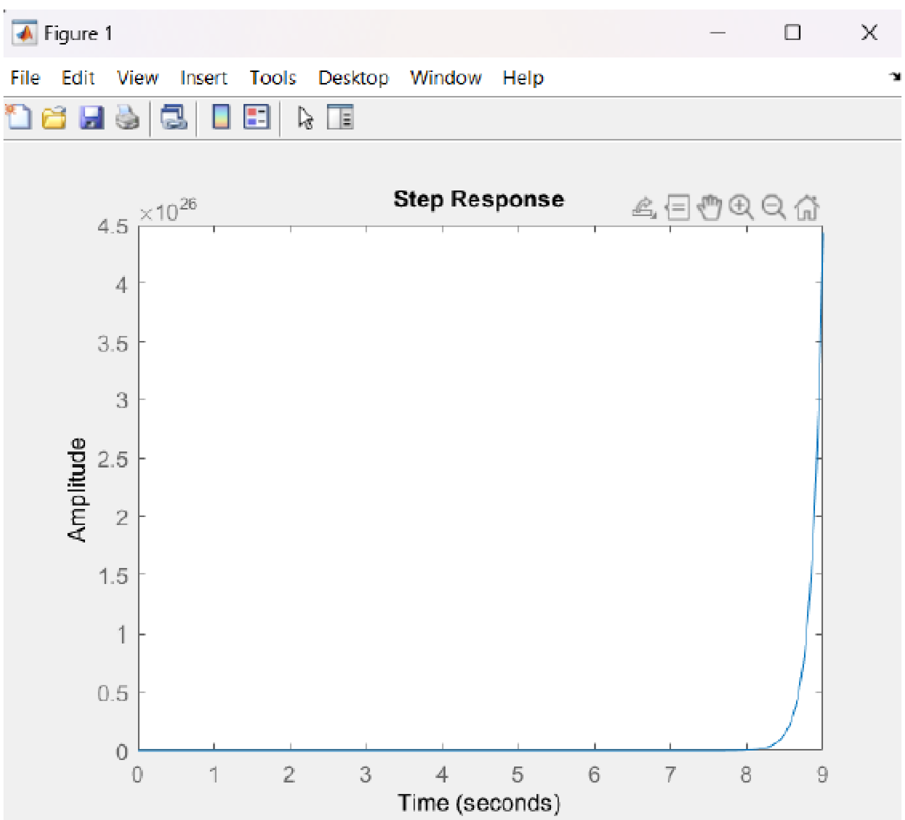

Figure 1: Plot of step response

From Figure 1, we can do the following analysis:

$$\%OS = \infty$$

$$T_r = Undefined$$

$$T_s = Undefined$$

$$PeakValue = \infty$$

$$FinalValue = \infty$$

Next, we construct a Routh-Hurwitz table to check the stability of the system. My equation is:

$$G(s) = \frac{5.5s^2+7.5s-13}{s^3-8s^2+8s+1}$$

s^3	1	8
s^2	-8	1

As there are sign changes in the first column, the system is unstable. we can also find the stability of the system by pole-zero map'

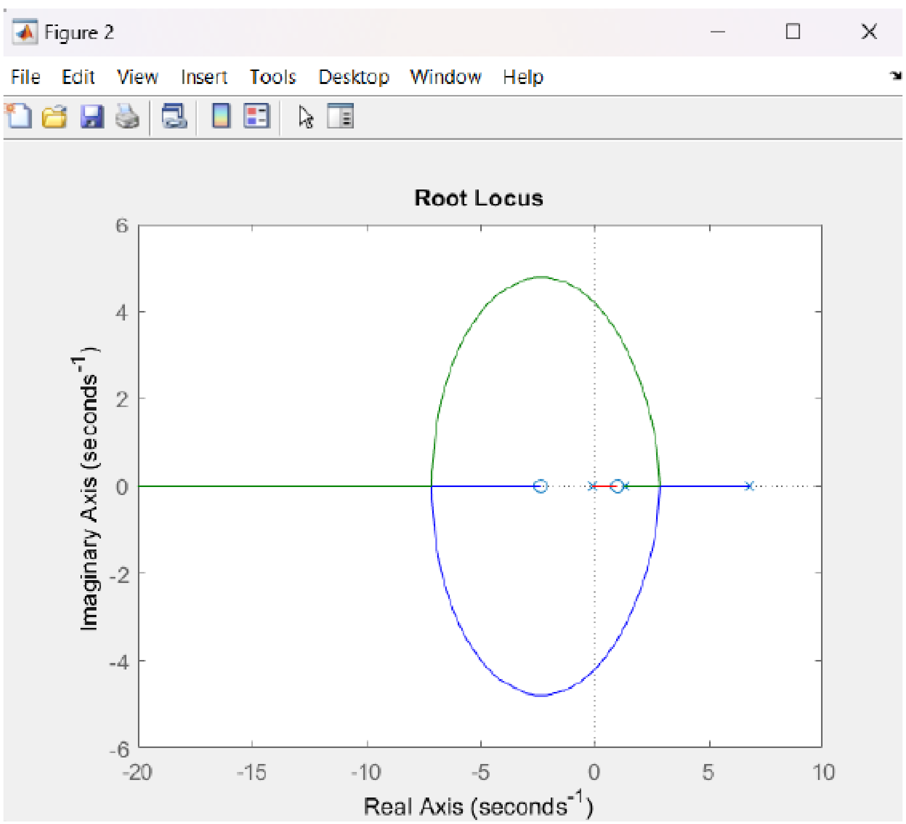

Figure 2: Plot of pole-zero map.

As we can see in figure 2, one of the poles is on the positive side which indicates the system is unstable.

Controllability analysis of the system:

For Controllability, We Check the Rank(P)

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$$P = \begin{bmatrix} B & AB & A^2B \end{bmatrix} \tag{5}$$

$$P = \begin{bmatrix} 4 & 21 & 150 \\ 0 & 24 & 163 \\ 3 & 13 & 84 \end{bmatrix} \tag{6}$$

- As, Rank(P) = 3 = Order of matrix A, n = 3, Which means the system is Controllable.
- Thus the system pass controllability test.

Observability analysis of the system:

For Observability, We Check the Rank(Q)

For Observability, We Check the Rank(Q)

$$Q = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix} \tag{7}$$

$$Q = \begin{bmatrix} 1 & 1 & 0.5 \\ 6 & 4.5 & 8.5 \\ 41 & 30 & 63 \end{bmatrix} \tag{8}$$

As, Rank(Q) = 3 = Order of matrix A,n = 3, Which means the system is Observable.

Thus the system pass observability test.

Controller Design for the system:

$$C = \begin{bmatrix} 1 & 1 & 0.5 \end{bmatrix} \neq I \tag{9}$$

- not equal to identity
- This means that the Observer-based state feedback Controller can be designed.
- Desired Controller eigenvalues (-16,-8, -10).

$$\mathsf{K} = \begin{bmatrix} 14.7996 & -0.0944 & -173.7328 \end{bmatrix}$$

- Desired Observer eigenvalues (-80,-40, -50).

$$\mathsf{L} = 1.0e{+}04 \begin{bmatrix} -3.8105 \\ 3.1401 \\ 1.3764 \end{bmatrix}$$

The observer-based state feedback controller Schematic is given below in figure 3

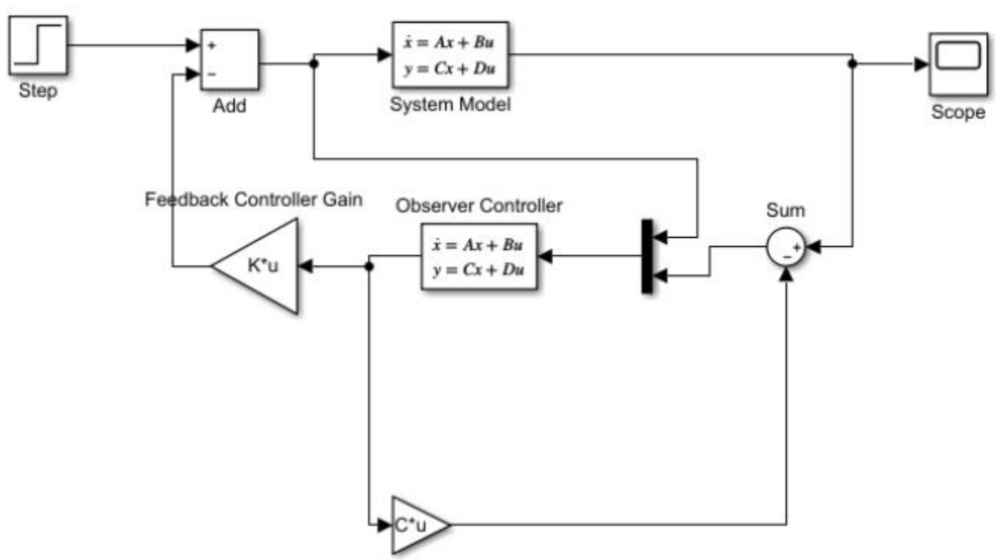


Figure 3: Schematic of Observer-based State Feedback Controller.

Step response of the observer-based state feedback controller is given belowin figure 4

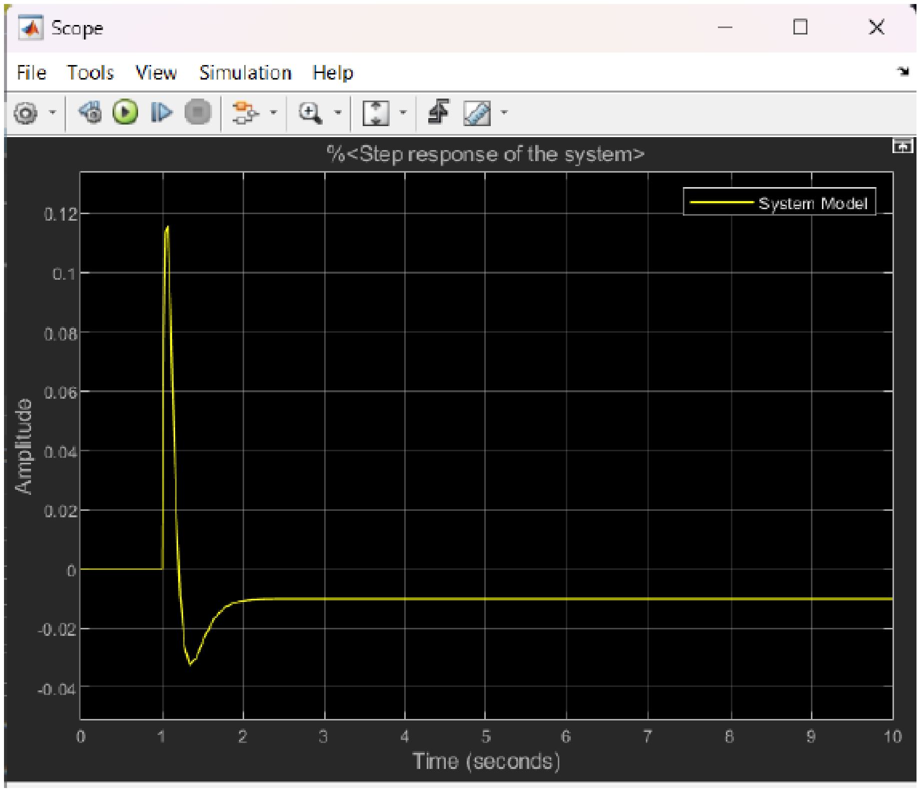


Figure 4: Plot of Observer-based State Feedback Controller.

Model of the PID controller with observer-based state feedback controller is given below

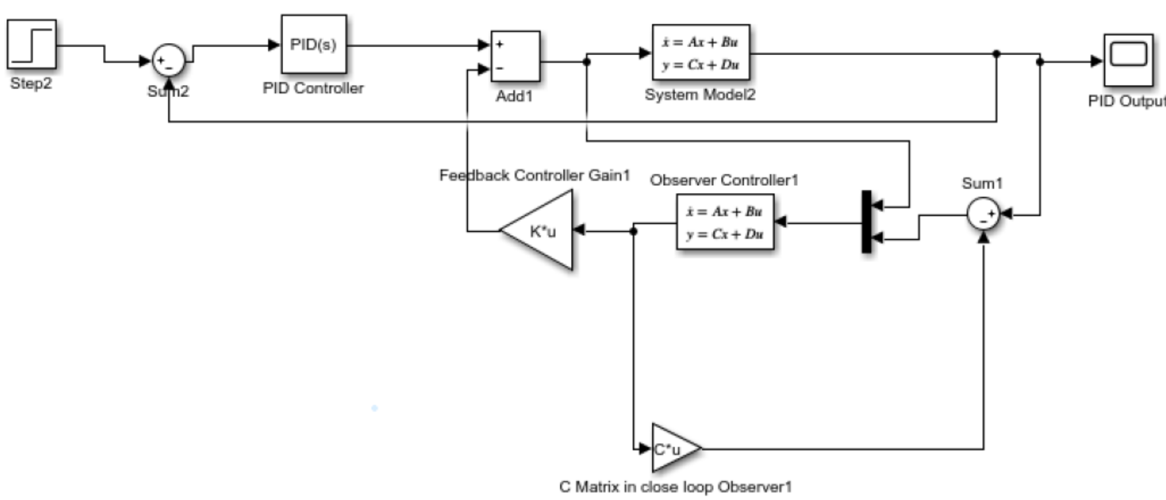


Figure 5: Schematic of PID Controller with controlled system.

The response of the system after the design of PID controller is given below

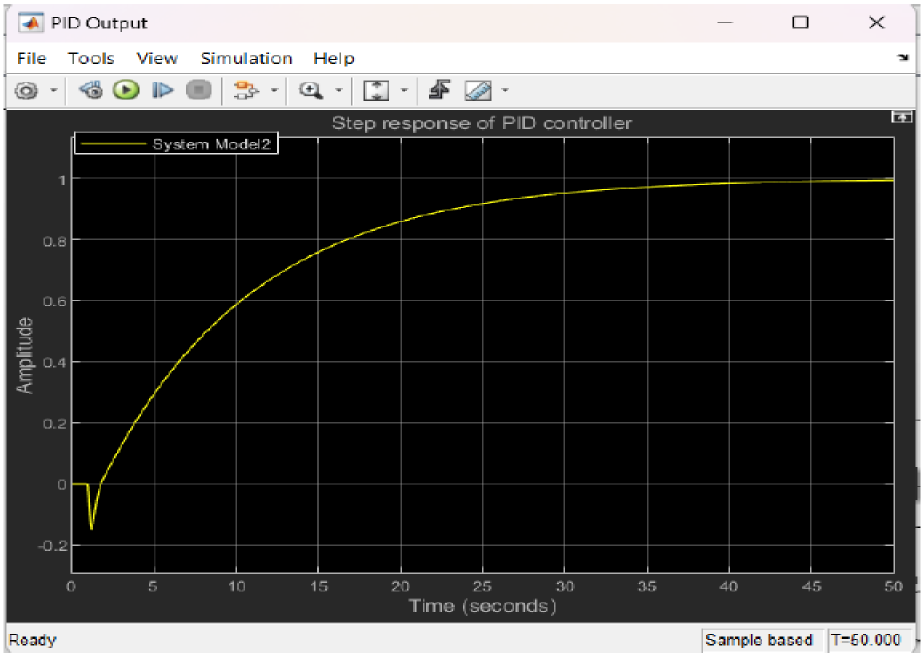


Figure 6: Plot of step response of PID controller.

Steady State Errors:

Steady State error before controller is undefined because the system is unstable.
Steady state error for step input, after the controller is.
Steady state error = input - output Steady state
error Steady state error = 1 - (-0.01) = 1.01

Steady state error of the controlled system is given below
Step response of output show to minus in input.

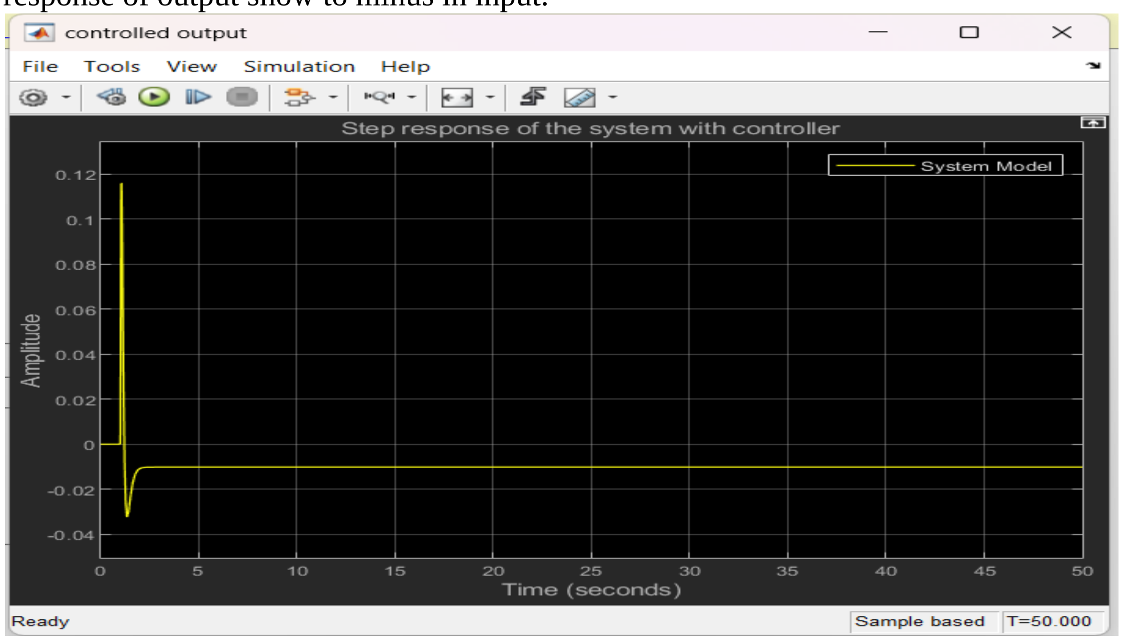


Figure 7: Steady state error after observer-based state feedback controller

Steady state error after PID controller is zero because for unit step intput the output of the PID controller is also one.
Steady state error = input - output Steady
state error = 1 - 1 = 0
Step response after PID controller is given below

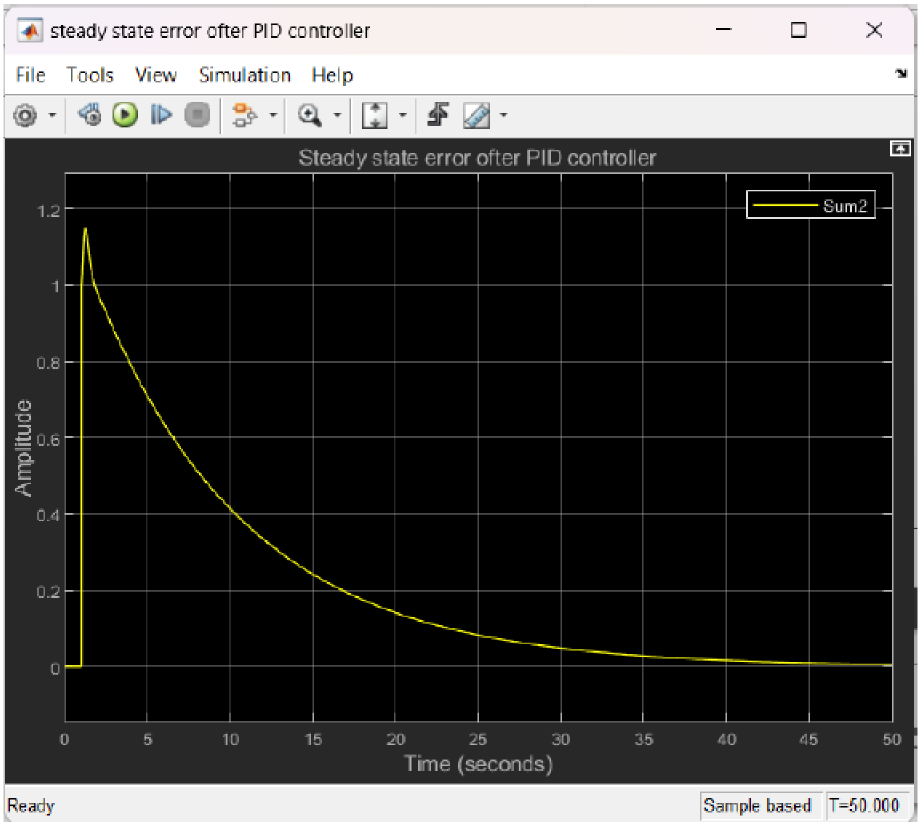


Figure 8: steady state error after PID controller.

Steady state error after step of 2u(t) is

Steady state error = input - output
steady state error = 2 - (-0.0277) = 2.0277
step response after step of 2u(t) is given below

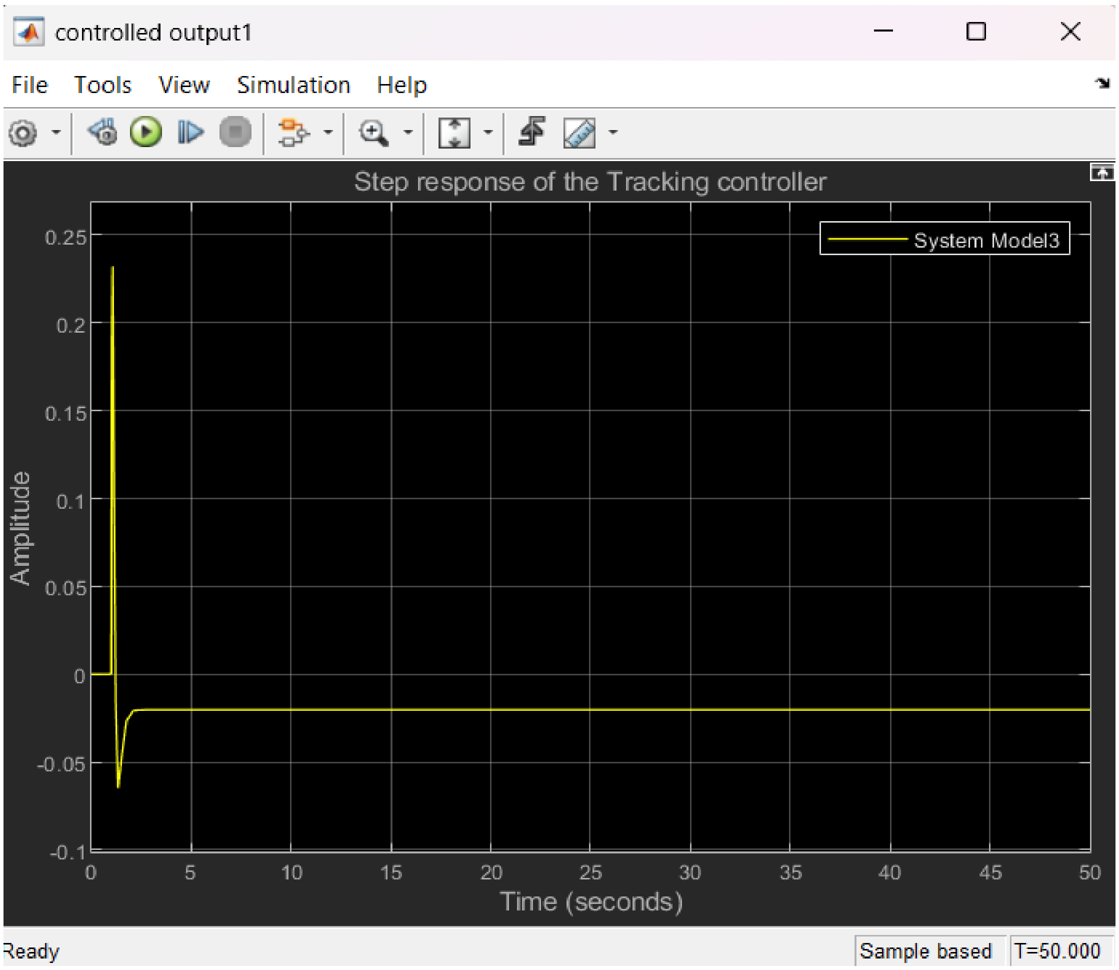


Figure 9: steady state error after tracking controller.

Steady state error for ramp input, after the controller is infinite because my controller is not tracking the ramp input.
The response for the ramp input is given below

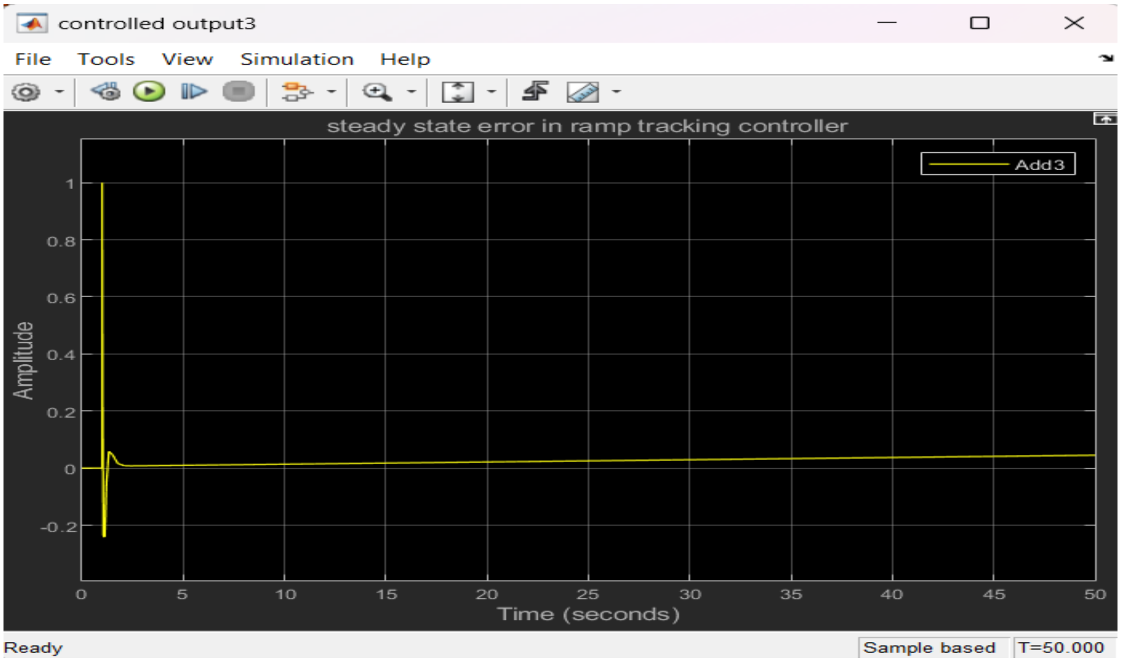


Figure 10: Ramp step response.

Tracking Controller:

Figure 11: Plot of step tracking of amplitude 2u(t).

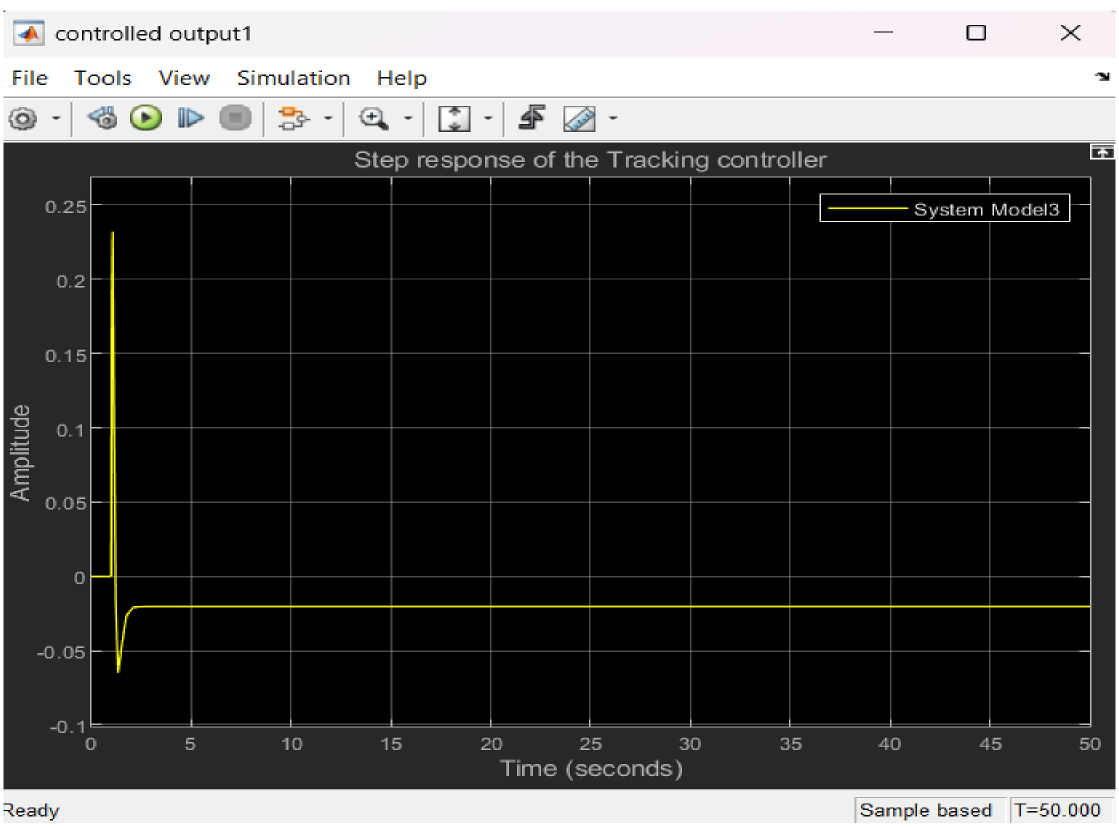
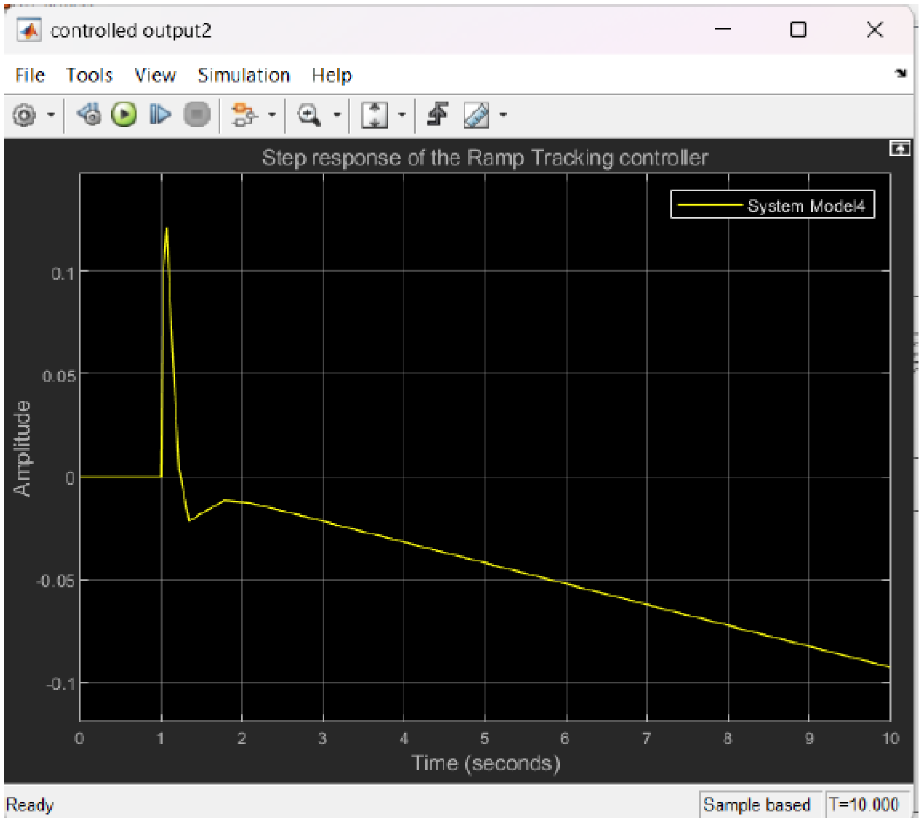


Figure 12: Plot of Ramp tracking of amplitude tu(t).



Results and Discussions:

We simulated the above system. The schematic for simulation and all the responses are given one by one as above.