CS310
Data Structures

K. Raven Russell
krusselc@gmu.edu
George Mason University

Today

- **Last Lecture**
  - Hashing
- **Today**
  - Introduction to Trees
  - Recursion Review

Review

Warm-up: Open Addressing

Perform the following operations for Table 1:

```
add(10.1)
add(4.6)
add(9.3)
add(2.5)
add(0.1)
remove(10.1)
rehash(10) //rehash to Table 2 (size 10)
```

Both Tables:
Open Addressing
with Quadradic Probing

Hash Function:
$h(x) = \lfloor x \rfloor$

Remember to indicate any special states!

Summary (with words):

1. Hash all numbers into Table 1 (5 slots) using open addressing with quadradic probing and $h(x) = \lfloor x \rfloor$.
2. Remove the first inserted value.
3. Rehash remaining numbers into Table 2 (10 slots).

Table 1:

0	1	2	3	4

Table 2:

0	1	2	3	4	5	6	7	8	9

Sets and Maps So Far

Implementation	Worse-Case		Average-Case		Order Ops	Remarks
	Search	Insert	Search	Insert		
Unordered List	N	[1]N	N	[1]N	No	
Ordered Array	lg N	N	lg N	N	Yes	
HT Chaining	?	?	?	?	No	
HT Probing	?	?	?	?	No	

Sets and Maps So Far

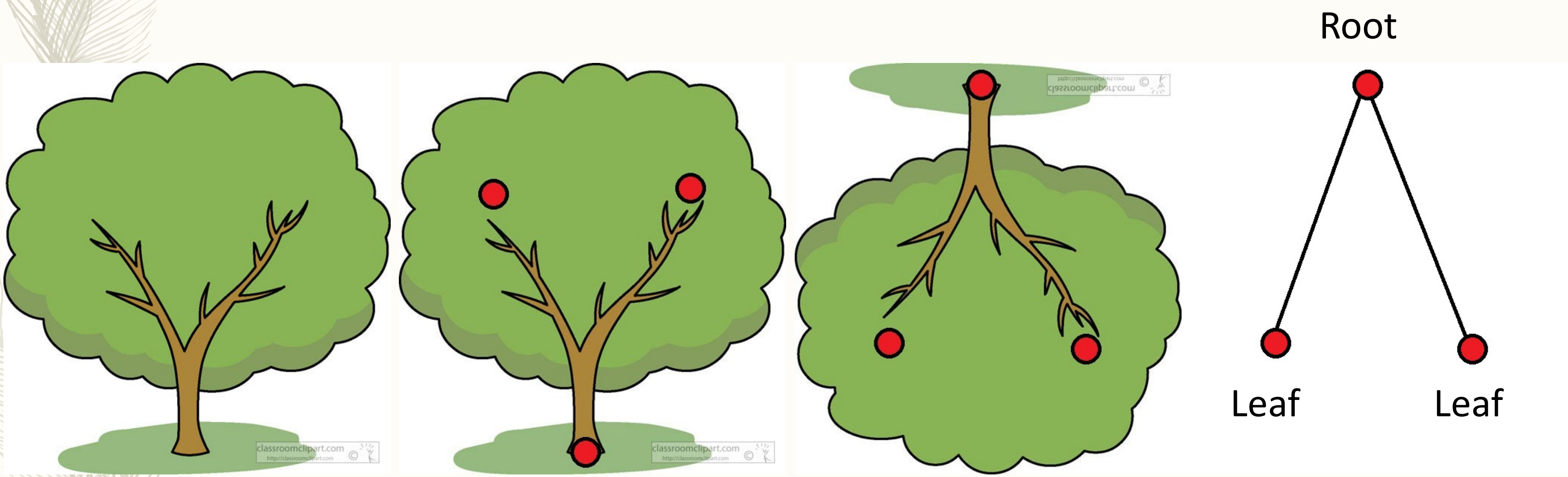
Implementation	Worse-Case		Average-Case		Order Ops	Remarks
	Search	Insert	Search	Insert		
Unordered List	N	[1]N	N	[1]N	No	
Ordered Array	lg N	N	lg N	N	Yes	
HT Chaining	N	[1]N	[2]N / M	[2]N / M	No	Often Used[3]
HT Probing	N	N	1	1	No	

1. O(n) to check for duplicates, O(1) if this is not needed for some reason
2. M is the size of the table. If N/M (load) is < a constant, then O(1)
3. Good constants and relatively easy to implement, used in many libraries

New Material

Trees

- Data structure which looks like an upside down tree (or the root system of a tree)
  - Nodes have parents and children
  - No loops



Trees

- Collection of **nodes**
  - Any shape, but can't have a loop
  - acyclic = “no cycles” = no loops
- Nodes have:
  - **data**
  - (possibly) a “key” to sort/search by
  - (possibly) pointer to children
  - (possibly) pointer to parent
- Common **tree operations**
  - Searching for an item
  - Adding items
  - Deleting items (synonymous with removing)
  - Balancing
  - Iterating = mention things one by one
    - *all the items (in some order)*
    - *a section of a tree*

Tree Definitions

Note: Unfortunately, due to the manner in which the field of computer science grew out of many other fields (math, engineering, etc.)… there are no completely-agreed-upon definitions of some of these terms.

We will use the definitions given in these slides for CS310.

Tree Definitions 1/3

- The **root** node is the top most node in the tree
- The **descendants** of a node are all the nodes below it (and sometimes includes the node itself)
- The **ancestors** of a node are the nodes on the path from the node to the root (and sometimes includes the node itself)
- Nodes are **siblings** if they have the same parent

Tree Definitions Practice 1/3

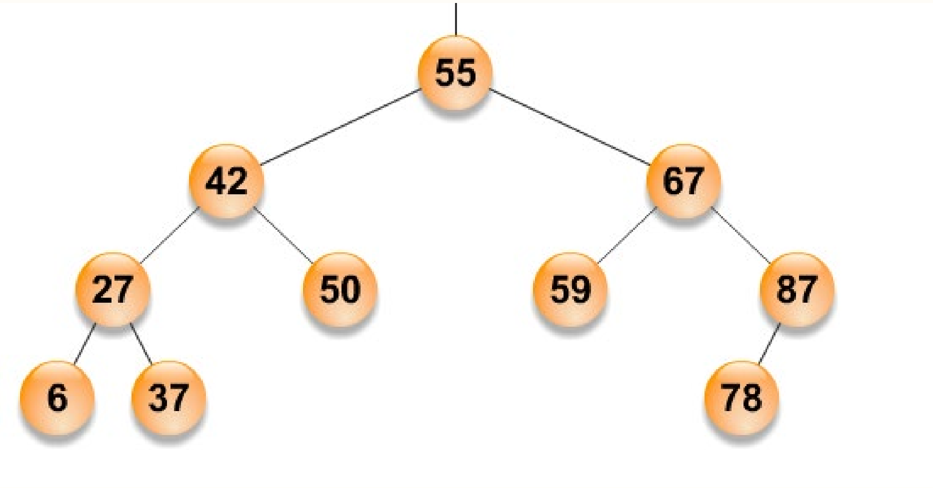
– What is the root?

– What is the parent of 27?

– What are the children of 67?

– What are the ancestors of 59?

– What are the descendants of 55?



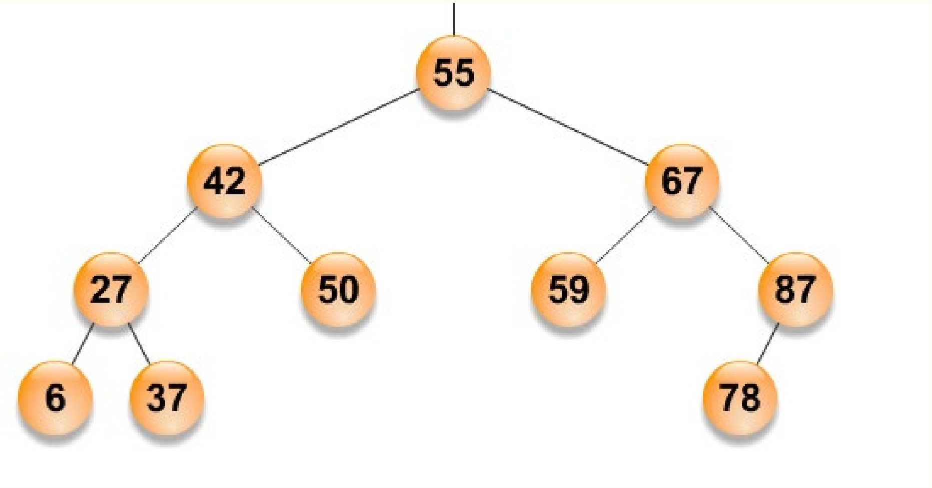
Tree Definitions 2/3

- The **leaf** nodes have no children, the root node has no parent
- An **inner** node has at least one child (i.e. not a leaf)
- A **null link** describes an empty link, typically child link

- The **depth** of a node is the length (number of edges) of the path from the node to the root
- The **node height** is the length of the path from the node to the deepest leaf.
- The **tree height** is the maximum depth of any node in the tree

Tree Definitions Practice 2/3

- Which nodes are leaf nodes?
- Which nodes are inner nodes?
- Where are the null links?
- What is the tree height?
- What is the depth of node 59?



Tree Definitions 3/3

- **Full tree**
  - every node other than the leaves has the max number of children
- **Perfect tree** (sometimes called "complete")
  - all leaves have the same depth
  - every node other than the leaves has the max number of children
- **Nearly complete tree** (sometimes called "complete")
  - last level is not completely filled
- **Balanced tree**
  - height of the left and right sub trees of every node differ by 1 or less (as used by AVL tree)
- **Degenerate tree**
  - each parent node has only one associated child node
  - equiv. to linked list, maximum height?

Tree Definitions Practice 3/3

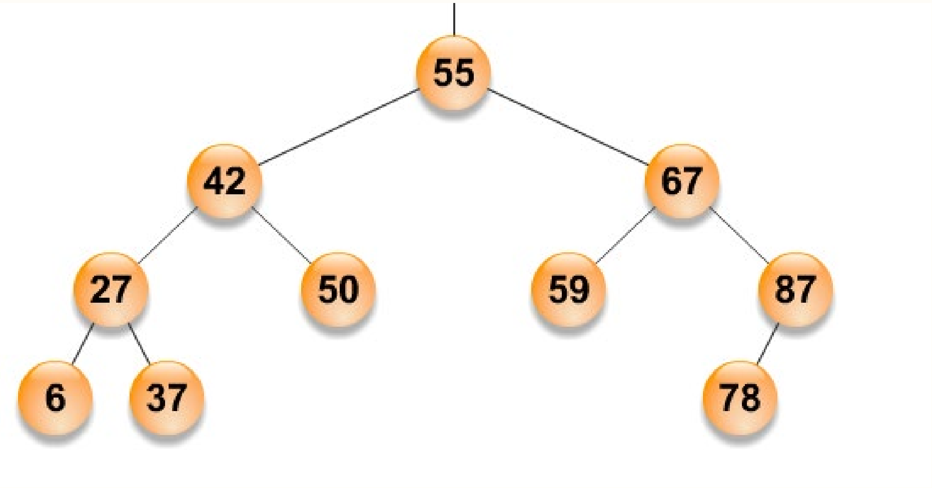
– Is this tree full?

– Is this tree perfect or nearly complete?

– Is this tree balanced?

– Is this tree degenerate?

  – what nodes could we remove to make it degenerate?



Questions?

k-ary Trees

- aka. n-ary and m-ary trees
- each parent can have only k children
  - k is the “branching factor”
- number of nodes in a perfect k-ary tree
  - $(k^{h+1}-1)/(k-1)$
    - *h is the height of the tree*
- number of leaves in a perfect k-ary tree
  - first level k^0, second k^1, third k^2... k^h
- height of a perfect k-ary tree of with n nodes
  - $\log_k((k-1)*(n)+1)-1$

Binary Trees (k-ary where k=2)

- each parent can have only two children
- number of nodes in a perfect binary tree
  - min: 2^h+1 max: $2^{h+1}-1$
    - *max from k-ary tree formula:* $(k^{h+1}-1)/(k-1)$, $k=2$ $\therefore$ $2^{h+1}-1/(2-1) = 2^{h+1}-1$
- number of leaves in perfect binary tree
  - first level 2^0, second 2^1, third 2^2... 2^h
- number of internal nodes in perfect binary tree of n nodes
  - $\lfloor n/2 \rfloor$
- height of a balanced binary tree of n nodes
  - $\lceil \lg(n+1) \rceil$

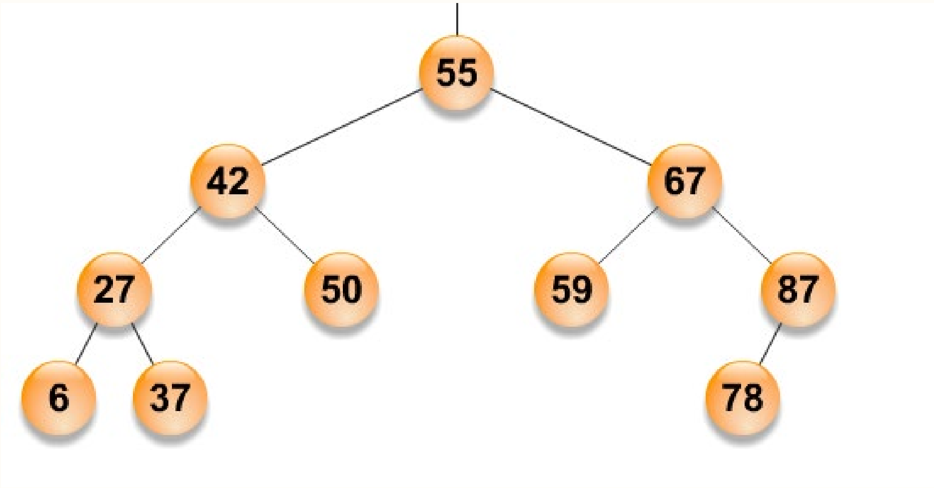
Binary Tree Storage: Arrays

- Root at index 0
- Children at index:
  - parentIndex*2+1
  - parentIndex*2+2
  - e.g. root at 0, children of root at index 1 and 2
- Parent at index
  - $\lfloor (childIndex-1)/2 \rfloor$
  - e.g. parent of item at index 2 = $\lfloor 1/2 \rfloor = 0$
- Common variant is to start root at index 1

Array Storage Example

– Draw the binary tree for:

[55, 42, 67, 27, 50, 59, 87, 6, 37, null, null, null, null, 78]



Binary Tree Storage: Links

- (optional) key
- value
- link to child 1
- link to child 2

- Do we need a link from child to the parent?

```
class Node<V> {
      V value;
      Node<V> left;
      Node<V> right;
}

class Node {
      int value;
      Node[] children;
}

class Node<K,V> {
      K key;
      V value;
      Node<K,V> left;
      Node<K,V> right;
}
```

Other Tree Storage

K-ary Tree Storage

- node structure
  - array/list of children
- array structure
  - root, children of root, grandchildren of root, etc.
  - same as binary tree, but different math

Arbitrary Tree Storage

- node structure
  - list of children (dynamic or linked)
- array structure
  - first-child-next-sibling storage (see textbook 18.1.2)

Tree Storage: Arrays vs. Linked

- **Arrays**
  - Need to know where each item is
  - How? Need to limit number of children and/or use other methods of storage (see first-child-next-sibling storage)
  - Most common for balanced trees (very little wasted space)
  - Not good for degenerate trees (lots of wasted space)
  - Fast memory access (compared to linked list)
- **Linked Data Structures**
  - easy to add, delete, and swap around parts of the tree