Time Value of Money[1]

Pay me Rs. 5000 every month for next 30 years (360 months), I pay you Rs. 15000 per month for the next 50 years (600 months)

This is the advertisement that you saw in the first page of a newspaper. The advertiser is one of the nationalized banks with a good financial health. You are 25 years old and started earning a decent salary. You can definitely save Rs. 5000 for this scheme, if you decide to sign up. You read the details of scheme again. You find you will be getting three times of your monthly savings as pension and it is available to your entire life in the earth. You start seriously thinking about your retired life, which you were ignoring till this point of time. You plan to sign up the scheme in the next few days and wondering how much to invest.

Next day morning, you are in a big surprise. Another government bank advertised as follows:

Pay me Rs. 5000 every month for next 30 years (360 months), I pay you or your nominee Rs. 20000 per month for the next 100 years (1200 months).

You picked up the yesterday's paper and start comparing the two schemes. The second one looks better because it gives Rs. 5000 more and also doubled the period from 50 years to 100 years. You thought of putting your daughter, who is 1 years-old as nominee so that she will get the pension around 65 years of her age.

Next day morning, you could not believe what you read in the paper. You have one more highly rated private sector bank advertising as follows:

Pay me Rs. 5000 every month for next 30 years (360 months), I pay you or your nominee or your nominee's nominee Rs. 25000 per month for the next 150 years (1800 months).

It looks like five times of your savings as pension for yourself, your daughter and your granddaughter. You realize the third one is better because on both the pension amount and period of pension, it outperforms the other two schemes. However, you were at loss to understand the mathematics behind the computation. This chapter helps you to understand the mathematics behind these offers.

Money and Time

We all know money has value. With money, we can buy many things. Money also has "time" value. A Rupee today is worth more than a Rupee tomorrow. There are several reasons for this statement. One simple reason is risk associated with the money. There is an old adage - A bird in hand is better

[1] This note is prepared by Prof. M S Narasimhan as a background reading material on Time Value of Money. You can send your feedback to msn@iimb.ernet.in

than two in the bush. It means a bird in hand is certain but two in the bush is an uncertain. Similarly, a Rupee today in hand is certain whereas a Rupee of tomorrow is uncertain. If you are asked to choose between cash bonus and stock bonus (which will be vested 5 years from now), many of you will opt for cash bonus. Another reason for your preference of today's Rupee over tomorrow's Rupee is interest difference. With one day, interest difference is not visible. Let us consider a slightly different situation. Suppose you won a lottery and got a prize of Rs. 1 million. You were asked to choose between taking Rs. 1 million today and received the prize amount at the end of 1 year. You would definitely choose the first option, receiving the prize today. The reason is you can invest the amount for 1 year and you will have more than Rs. 1 million at the end of one year. The difference is 'time' value of money. In simple terms, it is interest amount.

Compound Value of Rupee

In July 1996, Industrial Finance Corporation of India (IFCI) has issued a bond branded as "Millionaire Bond". In this an investment of Rs. 10000 in 1996 will mature as Rs. 1 million (Rs. 10 lakhs) at the end of 2026 (i.e. 30 years after the issue date). It was one of the most successful bond issues at that point of time because it gives an opportunity to be a millionaire at the end of 30 years by just depositing Rs. 10000. Many elderly people invested Rs. 10000 and gifted the same to their granddaughters and grandsons so that they become millionaire at the end of 30 years. IFCI collected about Rs. 1200 cr. in those days. You may be wondering what interest rate an investment of Rs. 10000 will mature Rs. 10 lakhs. A simple equation will help you to answer this question.

Compound or Future Value of Rupee = $\mathbf{P\ (1+r)^n}$

Where: P = Present Value or Amount invested today
r = interest rate per period
n = period, which can be year or month or day

Example: If Rs. 1000 invested for 5 years at 8% in State Bank of India, how much the depositor will get at the end of 5 years? The answer is $1000 \times 1.08^5 = 1469.33$. That is, the maturity value of Rs. 1000 deposited today at 8% for 5 year is Rs. 1469.33.

In the above example, the interest is added with the principal at the end of every year. Suppose the interest is added at the end of every 6 months, what is the maturity value of the deposit? We now need to find out what is the interest rate for 6 months period. Suppose SBI is willing to give 4% interest rate for every six months, the deposit will be with SBI for 10 six-month period. That is, r = 4% and n = 10. The maturity value will be $1000 \times 1.04^{10} = 1480.24$. The maturity value under semi-annual compounding is more than annual compounding. Similarly, the maturity value under quarterly compounding will be more than semi-annual compounding. The maturity value under monthly compounding will be more than quarterly compounding. Let us summarize the maturity value of Rs. 1000 at an interest rate of 8% per annum for 5 year period if the compounding is done annually, semi-annually, quarterly and monthly.

Compounding	Investment (P)	Interest	Period	Maturity Value
Annually	1000	8%	5	1469.33
Semi-annually	1000	4%	10	1480.24
Quarterly	1000	2%	20	1485.95
Monthly	1000	0.67%	60	1489.85

What would be the maturity value of Rs. 1000 compounded continuously (i.e. interest is added at the end of every fractions of seconds) at 8% at the end of 5 years? This is given by $\mathbf{P \times e^{rt}}$. The e value is equal to 2.718281828. The continuous compound value of Rs. 1000 at 8% for 5 years is Rs. 1491.82 $\left[\mathbf{1000 \times 2.718281828^{0.08 \times 5}}\right]$

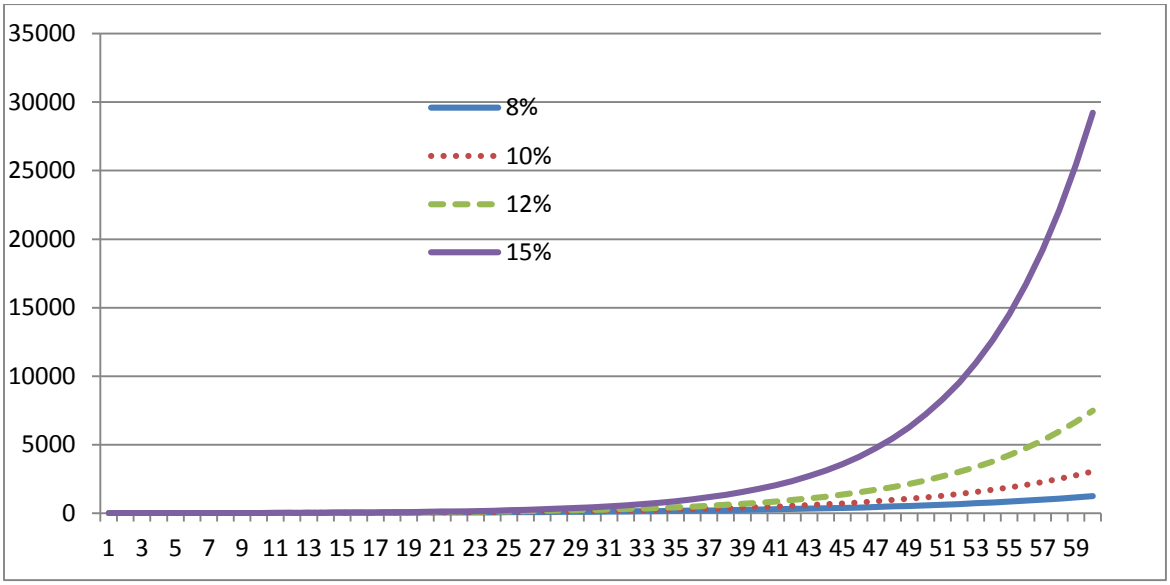
The following table shows the compound value of Rupee 1 at different interest rates for different periods. Appendix 1 expands this table for additional interest rates and periods.

		Interest Rate (r)								
		1%	3%	5%	7%	8%	9%	10%	12%	15%
Period (n)	1	1.0100	1.0300	1.0500	1.0700	1.0800	1.0900	1.1000	1.1200	1.1500
	2	1.0201	1.0609	1.1025	1.1449	1.1664	1.1881	1.2100	1.2544	1.3225
	3	1.0303	1.0927	1.1576	1.2250	1.2597	1.2950	1.3310	1.4049	1.5209
	4	1.0406	1.1255	1.2155	1.3108	1.3605	1.4116	1.4641	1.5735	1.7490
	5	1.0510	1.1593	1.2763	1.4026	1.4693	1.5386	1.6105	1.7623	2.0114
	10	1.1046	1.3439	1.6289	1.9672	2.1589	2.3674	2.5937	3.1058	4.0456
	15	1.1610	1.5580	2.0789	2.7590	3.1722	3.6425	4.1772	5.4736	8.1371
	20	1.2202	1.8061	2.6533	3.8697	4.6610	5.6044	6.7275	9.6463	16.3665
	30	1.3478	2.4273	4.3219	7.6123	10.0627	13.2677	17.4494	29.9599	66.2118

The table shows a Rupee invested today at 10% for 15 years will have a maturity value of Rs. 6.73. At 15% interest rate for 30 years, the maturity value of Rupee 1 is Rs. 66.21. It means Rs. 10000 maturing to Rs. 10 lakhs at the end of 30 years should have an interest rate above 15%. IFCI used 15.96% interest rate and compounded semi-annually to derive the value of Rs. 10000 [10000 x 1.0798^{60} = 10,01,381]. Figure 1 shows how money grows at different interest rates for different periods.

The figure shows the gap between the maturity values widens between interest rates when the period increases. If you examine the table again, you will see this difference. For example, for 10 years period, the maturity value of Re. 1 at 15% is Rs. 4.05 against the maturity value of Rs. 3.11 at 12%. The gap is less than Rupee 1. The gap between the maturity value of Rupee 1 at the end of 30 years for an interest rate of 12% and 15% is about Rs. 36. The magic of growth shows up as the period increases.

Figure 1: Compound or Future Value of Rupee 1



Compound Value of Annuity

Instead of making one time investment, if an investor invests the amount at the end of every period, it is called annuity. Recurring deposit is an example of annuity. Annuity refers to constant cash flow ***at the end*** of every period. If the investment is made at the beginning of the period (typically in recurring deposit), it is called ***annuity due***. Thus, recurring deposit is actually annuity due. The equation for compound value of annuity is bit complex.

$$FV\ of\ Annuity = R\left[\frac{(1+r)^n - 1}{r}\right]$$

Where, R = Periodic constant cash flow
r = interest rate
n = period

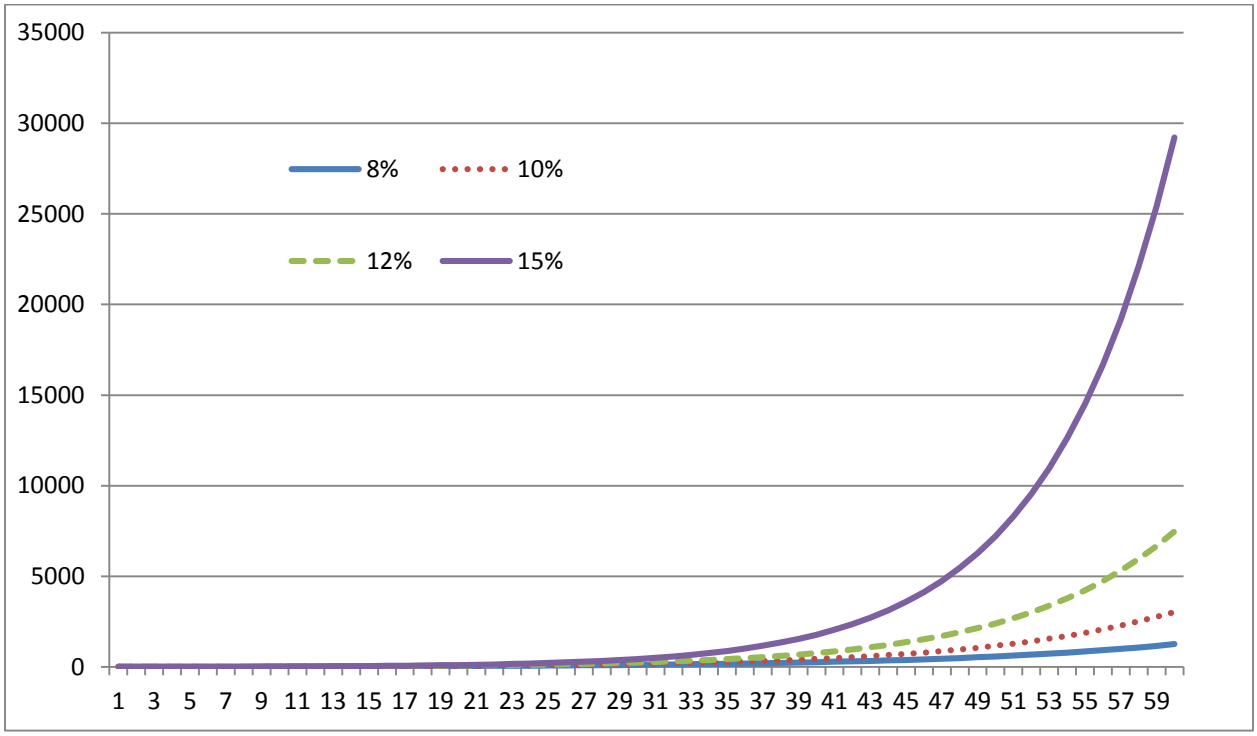
Future value of Annuity Due = Future Value of Annuity x (1+r)

The following table shows the compound or future value of Rupee 1 invested at the end of every year (R = 1) for "n" number of years at "r" interest rate. Appendix 2 expands this table for additional interest rates and periods.

		Interest Rate (r)								
		1%	3%	5%	7%	8%	9%	10%	12%	15%
Period (n)	1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	2	2.010	2.030	2.050	2.070	2.080	2.090	2.100	2.120	2.150
	3	3.030	3.091	3.153	3.215	3.246	3.278	3.310	3.374	3.473
	4	4.060	4.184	4.310	4.440	4.506	4.573	4.641	4.779	4.993
	5	5.101	5.309	5.526	5.751	5.867	5.985	6.105	6.353	6.742
	10	10.462	11.464	12.578	13.816	14.487	15.193	15.937	17.549	20.304
	15	16.097	18.599	21.579	25.129	27.152	29.361	31.772	37.280	47.580
	20	22.019	26.870	33.066	40.995	45.762	51.160	57.275	72.052	102.444
	30	34.785	47.575	66.439	94.461	113.283	136.308	164.494	241.333	434.745

Figure 2 shows the magic of time in the compound value of Annuity.

Figure 2: Compound or Future Value of Annuity of Rupee 1.



Present Value of Rupee

Earlier in discussing compound or future value of Rupee, we defined future value is equal to present value multiplied (1+r) raised to power time [$FV = PV \times (1+r)^n$]. From this equation, we can derive present of Rupee received at the end of 1 year is equal to

$$PV = FV/(1+r)^n$$

Where PV = Present Value r = Interest rate or discounting rate
FV = Future Value n = Time (normally years or months)

Suppose you won a lottery that will give two options. In the first option, you will get the prize of Rs. 1 million today. In the second option, you will get the prize in three installments as Rs. 200,000 today, Rs. 400000 at the end of one year and Rs. 800,000 at the end of two years. Assume deposit interest rate prevailing in the market 8% per year. Which one you will choose? This issue can be resolved by finding the present value of the two options. The present value of the option one is simple and it is equal to Rs. 1 million $[1{,}000{,}000/(1+8\%)^0]$. For option two, we need to find the present value of three payments and add them up to get the total present value.

Today's payment	:	Rs. $200{,}000/1.08^0$ = Rs. 200,000
Payment at the end of 1 year	:	Rs. $400{,}000/1.08^1$ = Rs. 370,370
Payment at the end of 2 year	:	Rs. $800{,}000/1.08^1$ = Rs. 685,871
Total Present Value of three payment		Rs. 1,256,241

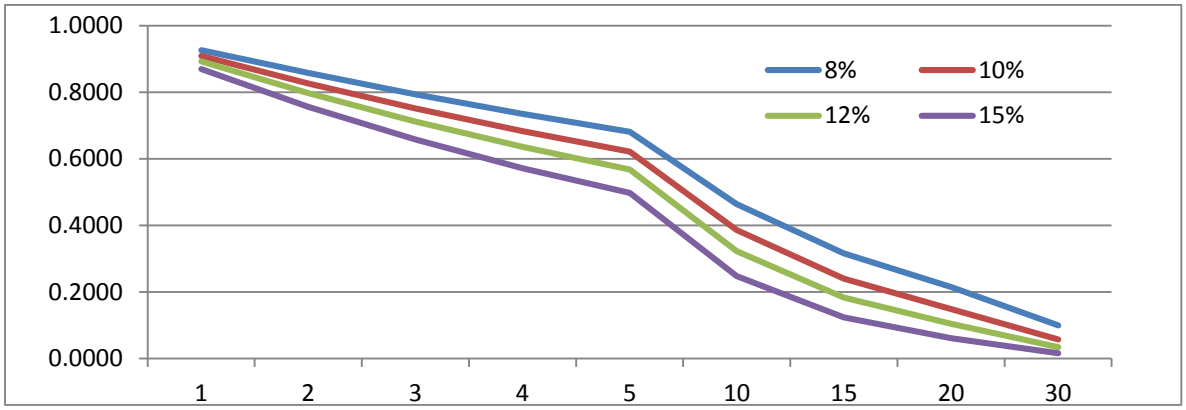
When you compare the two streams of cash flows in terms of present value, you will find the option two is better. Suppose the deposit interest rate is 15% in the market, which one of the two options is better. You can now find the present value of 200,000; 400,000 and 800,000 discounting at 15% and check whether the sum is more than Rs. 1 million. If it is, then option 2 is better else option 1 is desirable.

The present value of cash received in the future declines as we move forward and approach zero if the cash received time is far away from today (say 50 years). Similarly, the present value will decline when the interest rate (or discounting rate) increases. The following table gives the present value of Rupee at different interest rate and for different periods.

		Interest Rate (r)								
		1%	3%	5%	7%	8%	9%	10%	12%	15%
Period (n)	1	0.9901	0.9709	0.9524	0.9346	0.9259	0.9174	0.9091	0.8929	0.8696
	2	0.9803	0.9426	0.9070	0.8734	0.8573	0.8417	0.8264	0.7972	0.7561
	3	0.9706	0.9151	0.8638	0.8163	0.7938	0.7722	0.7513	0.7118	0.6575
	4	0.9610	0.8885	0.8227	0.7629	0.7350	0.7084	0.6830	0.6355	0.5718
	5	0.9515	0.8626	0.7835	0.7130	0.6806	0.6499	0.6209	0.5674	0.4972
	10	0.9053	0.7441	0.6139	0.5083	0.4632	0.4224	0.3855	0.3220	0.2472
	15	0.8613	0.6419	0.4810	0.3624	0.3152	0.2745	0.2394	0.1827	0.1229
	20	0.8195	0.5537	0.3769	0.2584	0.2145	0.1784	0.1486	0.1037	0.0611
	30	0.7419	0.4120	0.2314	0.1314	0.0994	0.0754	0.0573	0.0334	0.0151

The figure below (Figure 3) shows how fast the Rupee loses its value and approach to zero. In about five year time, Rupee loses nearly 50% of its value and hits a value close to zero by 30 years. This figure highlights the saying "a Rupee today is better than a Rupee of tomorrow".

Figure 3: Present Value of Rupee



Here is another example to understand the concept of present value. Suppose there are two Government of India bonds issues few years back available for investment and you need to choose one of them. The details are as follows:

	Bond 1	Bond 2
Face Value	1000	1000
Time to Maturity (years)	3	3
Interest Rate	8%	0%

The holder of the Bond 1 will get the repayment of Rs. 1000 at the end of 3 years. In addition, the bondholder will get an interest of Rs. 80 at the end of year 1, year 2 and year 3. The holders of Bond 2 will get Rs. 400 every year starting from end of the first year through third year. There is no separate interest payment. Assume prevailing interest rate is 8%. Which of the two bonds you will select? Find the present value of cash flows and choose the one whose present value is high. The workings are as follows:

Year	Bond 1		Bond 2	
	Cash Flow	PV of CF	Cash Flow	PV of CF
1	80	74.07	400	370.37
2	80	68.59	400	342.94
3	1080	857.34	400	317.53
Total	1240	1000.00	1200	1030.84

Though the absolute value of cash flows of Bond 2 (Rs. 1200) is lower than the cash flows of Bond 1 (Rs. 1240), the present value of Bond 2's cash flow is higher than present value of Bond 1's cash flow. Hence it makes sense to buy Bond 2 instead of Bond 1. Basically, we are examining which one

of the two bonds makes us rich in today's value after adjusting the time value differences in the cash flow. Bond 2 scores better because it start paying the investment faster than Bond 1. If you have any difficulty in understanding the concept, you can take the decision by using future value concept. Let us assume we reinvest all cash flows such that we will receive all cash flow at the end of three years. We will select the bond whose maturity value of all cash flow is higher at the end of year 3.

Year	Bond 1		Bond 2	
	Cash Flow	Future Value at t=3 at 8%	Cash Flow	Future Value at t=3 at 8%
1	80	93.31	400	466.56
2	80	86.40	400	432.00
3	1080	1080.00	400	400.00
Total		1259.71		1298.56

Note: $80 \times 1.08^2 + 80 \times 1.08^1 + 1080 \times 1.08^0 = 1259.71$

$400 \times 1.08^2 + 400 \times 1.08^1 + 400 \times 1.08^0 = 1298.56$

The decision of investing in Bond 2 remains same whether we use present value concept or future value concept. The common practice in finance is using present value for such decisions.

Exercise: You recently sold an ancestral agriculture land for Rs. 60 lakhs. You are considering two investment options. In the first option, you will deposit the amount in State Bank of India at 8% for 5 years under cumulative investment scheme. That is, you will be getting interest along with principal at the end of five years. In the second option, you can buy an apartment for an all-inclusive cost of Rs. 60 lakhs. You will get an annual rental income of Rs. 250,000 per year for the next five years. You also expect the apartment will appreciate at an average compound rate of 6% per year. You plan to sell the apartment at the end of 5 years. What is your decision – deposit in State Bank of India or investing in real estate? Ignore taxes and other factors.
Tips: The maturity value of SBI deposit is 60×1.08^5; Sale value of the flat at year 5 is $60 * 1.06^5$

Present Value of Annuity

If you have subscribed any pension scheme, the insurance company before commencing the pension will ask you whether you would like to commute a part of the pension. The commutation means lump sum instead of pension. Normally pension is paid on monthly basis but here we will use yearly pension to make the computation simple. Suppose your normal pension per year is Rs. 1000000 and the first pension starts 1 year from now. You can either receive a pension of Rs. 10 lakhs per year for the next 30 years or Rs 5 lakh per year (starting from at the end of year 1) plus a lump sum of Rs. 50 lakhs today. Assume interest rate is 8% per year. Will you opt for a pension of Rs. 10 lakhs per year or Rs. 50 lakhs today plus Rs. 5 lakhs per year? We can use present value of Rupee concept but it will be lengthy. Annuity formula is a short-cut for present value or future value formula if the cash flows are constant. Present of value of annuity is as follows:

$$PV\ of\ Annuity = C\left[\frac{1}{r} - \frac{1}{r(1+r)^n}\right]$$

Where, C = Periodic constant cash flow

r = interest rate; n = period

If the pension starts at the beginning of the period instead of end of period, PV of Annuity equation requires a slight modification.

Present Value of Annuity Due = PV of Annuity * (1+r)

Here is a comparison of two options.

	Option 1	Option 2
Pension	10,00,000	5,00,000
Period	30	30
Interest	8%	8%
Present Value of Pension	112,57,783	56,28,892
Lump Sum	0	50,00,000
Present Value of Pension and lump sum	112,57,783	106,28,892

The conclusion is commutation of pension is not worth if the current interest rate is 8%. Suppose the interest rate is 10%. It means you can invest the lump sum (commuted value of pension) at 10%. You will find commutation is beneficial.

	Option 1	Option 2
Pension	10,00,000	5,00,000
Period	30	30
Interest	10%	10%
Present Value of Pension	94,26,914	47,13,457
Lump Sum	0	50,00,000
Total	94,26,914	97,13,457

The table below shows the present value of annuity.

		Interest Rate (r)								
		1%	3%	5%	7%	8%	9%	10%	12%	15%
Period (n)	1	0.9901	0.9709	0.9524	0.9346	0.9259	0.9174	0.9091	0.8929	0.8696
	2	1.9704	1.9135	1.8594	1.8080	1.7833	1.7591	1.7355	1.6901	1.6257
	3	2.9410	2.8286	2.7232	2.6243	2.5771	2.5313	2.4869	2.4018	2.2832
	4	3.9020	3.7171	3.5460	3.3872	3.3121	3.2397	3.1699	3.0373	2.8550
	5	4.8534	4.5797	4.3295	4.1002	3.9927	3.8897	3.7908	3.6048	3.3522
	10	9.4713	8.5302	7.7217	7.0236	6.7101	6.4177	6.1446	5.6502	5.0188
	15	13.8651	11.9379	10.3797	9.1079	8.5595	8.0607	7.6061	6.8109	5.8474
	20	18.0456	14.8775	12.4622	10.5940	9.8181	9.1285	8.5136	7.4694	6.2593
	30	25.8077	19.6004	15.3725	12.4090	11.2578	10.2737	9.4269	8.0552	6.5660

The incremental present value of Rupee 1 after certain point of time is very low. For example, at 15% the present value of annuity for 20 years if 6.2593. If the period is increased to 30 years, the present value is 6.5660. The increase in value for 10 years additional annuity is 0.3066.

Perpetuity

Perpetuity is an extension of annuity in a sense the period is indefinite. That is, it is annuity forever. The present value perpetuity is as follows:

Present Value of Perpetuity = C/r where C = constant cash flow and 'r' is interest rate.

The derivation of the formula is fairly simple. In the present value of annuity formula, if you sent the 'n' is equal to infinity, the second term of the equation is reduced to zero. It leaves C * 1/r or C/r. A few European government have issued perpetual bonds during world war times. Recently many PSU Banks have issued perpetual bonds to shore up their capital base to comply Basel III norms (see the news item "Banks line up perpetual bond issues")

Banks line up perpetual bond issues

The last quarter of 2014 is set to witness quite a few issuances of perpetual bonds from banks, according to some arrangers

The last quarter of 2014 is set to witness quite a few issuances of perpetual bonds from banks, according to some arrangers. Perpetual bonds have no maturity date. Bank of Baroda (BoB) is expected to raise Rs. 1,500 crore through Basel-III-compliant additional tier-I (AT-I) perpetual bonds, according to an industry expert. A source said BoB wished to price the bond at 9.25-9.30%, but investors had bid 9.45% last week. The 10-year government bond yield was at 7.88% whereas the average bond yield for 10-year AAA-rated Indian corporate bonds was at 8.55%. The yield for perpetual bonds is typically higher than that of its limited-tenure peers as a way of offering premium for holding non-redeemable bonds. Bank of Maharashtra may also tap the bond markets to raise Rs. 1,000 crore via Basel-III-compliant additional Tier-I (AT-I) perpetual bonds at a coupon rate of 9.45%, according to a leading bond arranger. Vijaya Bank, Allahabad Bank and Syndicate bank are also expected to raise between Rs. 500 crore and Rs. 1,000 crore via the same category of bonds. Public sector banks tap the markets for meeting their capital adequacy towards the end of the quarter. Moreover, with corporate bond yields at one of the lowest points over the last one year, it becomes an attractive time to raise funds through the bond markets. "People are very confident that the Reserve Bank will cut interest rates this policy," said Ashish Jalan, assistant vice-president, SPA Securities.

Source: The Financial Express, January 2, 2015

Growing Perpetuity

In the above example, the cash flows are constant. If the cash flows are growing at a constant rate, then such cash flows are growing perpetuity. The present value of growing perpetuity is as follows:

Present Value of Growing Perpetuity = $C_1/(r-g)$
where C_1 = expected cash flow at time 1
r = interest rate
g = growth rate in the cash flows.

Suppose an issuer of bond gives an interest amount of Rs. 100 at the end of year 1, Rs. 103 at the end of year 2, etc., such that interest of period t is equal to interest of period t-1 x 1.03. If the interest rate is 8%, the present value of this growing perpetual cash flows is equal to 100/(8%-3%) = 2000. The market value of the bond will be Rs. 2000/- In the absence of growth, the value of the bond is Rs. 1250 [100/.08]. The difference of Rs. 750 is on account of growth.

Exercise

A matured company with stable profit is expected to pay a dividend of Rs. 15 per share next year. An analysis of historical dividends shows the dividend is growing at an average rate of 4% per year. The cost of equity (discounting rate) is 12%. What is the fair value of the share?

Since business has perpetual life and the dividend is growing at the rate of 4%, the present value of future dividend is as follows:

Present Value of Future Dividends = 15/(0.12 – 0.04) = Rs. 187.50

Flat Rate and Implicit Rate

Often automobile dealers advertise that you can pay Rupee 1 and drive the car, pay the balance at an interest rate of 6%. You may find this is a great deal because banks charge around 12% for car loan. If you visit the dealer and ask for the details, the dealer will say the cost of vehicle is Rs. 7 lakhs, will add Rs. 294000 towards interest and then divide the loan plus interest by 84 months (7 year loan period) to get monthly instalment of Rs. 11833.

Loan Period (years)	7	
Car Value	Rs. 700000	
Interest	294000	= 700000*6%*7
Loan + Interest	994000	
Number of Months	84	
Instalment	11833	= 994000/84
Interest Rate per month	0.88%	=RATE(84,-11833,700000,0,0)
Annualized Interest Rate	10.58%	=0.88 x 12

It means the dealer asks you to pay Rs. 11833 per month for 84 months (7 years) to repay the loan of Rs. 7 lakhs. The effective interest rate (using RATE function in Excel) is 0.88% per month or 10.58% per annum. The effective or implicit interest rate in this great deal is 10.58%. The 6% interest rate offered by the automobile dealer is called FLAT rate and is aimed to mislead the customer. It is more of marketing gimmick.

Nominal and Real Rate

The interest rate that banks and other bond issuers offer is called nominal interest rate. You may feel that a substantial part of the interest you will be receiving will go away due to inflation. You may want to know the inflation-adjusted interest rate. It is call real rate. The relationship between nominal rate and real rate is as follows:

(1+Nominal Rate) = (1+Real Rate) x (1+Inflation Rate)
Real Rate = [(1+Nominal Rate)/(1+Inflation Rate)] – 1

While the above definition is precise one, a back of envelope computation is Nominal Rate less Inflation Rate is equal to Real Rate.

Exercise

You have Rs. 100 with you and you can invest at 8% in a bank for one year. The price of sugar per KG is Rs. 25 and today you can buy 4 KGs of sugar today. The inflation during the year is expected to be 5% and it means 1 KG of sugar will cost Rs. 26.25 at the end of one year. If you deposit the amount for one year in a bank, you will get Rs. 108 at the one year. You can buy 108/26.25 = 4.1142 KG of sugar. In real terms, you are able to consume 0.1142 KG of sugar additionally at the end of 1 year. The real return to you is 0.1142/4 = 2.8751%. Let us apply the real rate equation.

$$\text{Real Rate} = (1+8\%)/(1+5\%) - 1 = 1.08/1.05 - 1 = 2.8751\%$$

Financial Planning Exercise

We discussed a few time value of money concepts. Let us apply them to work out financial planning. The following assumptions will be useful.

Current Age	: 25 years
Retirement Age	: 60 years (or 35 years of service)
Post Retirement Life	: 40 years including spouse life
Interest Rate	: 10%
Inflation	: 4%
Pension at today's cost	: Rs. 6 lakhs per year

How much savings are required per year for the next 35 years so that the corpus will give inflation adjusted pension of Rs. 6 lakhs per year for 40 years starting from the age of 61 years?

Solution

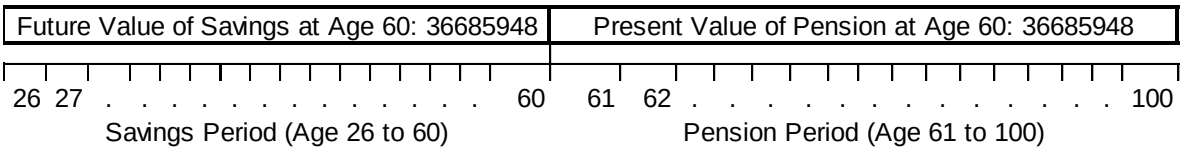
First pension	: $600000*1.04^{36}$	= Rs. 24,62,360
Last Pension	: $2462360 * 1.04^{39}$	= Rs. 113,67,155

Present value of Pension from year 61 to 100 (nominal value) as on Age 60 = 366,85,948

Now you need to find out how much you need to save every year such that the maturity value of the corpus is equal to Rs. 366,85,948.

Suppose you save Re. 1 every year starting from your age 26 for the next 35 years. The compound value of annuity of Re. 1 every year for 35 years at 10% interest rate is equal to Rs. 271.

To build a corpus of Rs. 366.86 lakhs over next 35 years, you need to save Rs. 135,360 every year from next year [366,85,948/271]. It means you need to save Rs. 11280/- per month to get a pension of Rs. 600,000 adjusted for inflation.



	Savings	FV of Savings
26	135360	3458141
27	135360	3143765
28	135360	2857968
29	135360	2598153
57	135360	180164
58	135360	163786
59	135360	148896
60	135360	135360
	Total	36685948

	Pension	PV of Pension
61	2462360	2238509
62	2540854	2116408
63	2663288	2000968
64	2769820	1891824
97	10105357	297182
98	10509572	280972
99	10929955	265646
100	11367153	251156
	Total	36685948

Summary

Money and time have both value. An understanding of time value of money will be useful to handle most issues related to finance. Future or compound value of Rupee, Future value of annuity, Present value of Rupee and Present value of annuity are four important concepts of time value of money. Present value of perpetuity and growing perpetuity are extension of these concepts. These concepts are used in valuation of investments including investing in bonds, equity, real estate, projects and personal finance decisions. A few important formulas used in time value of money are summarized below.

$$FV\ of\ Rupee = P*(1+r)^n$$

$$PV\ of\ Rupee = \frac{FV}{(1+r)^n}$$

$$FV\ of\ Annuity = R\left[\frac{(1+r)^n - 1}{r}\right]$$

$$PV\ of\ Annuity = C\left[\frac{1}{r} - \frac{1}{r(1+r)^n}\right]$$

$$PV\ of\ Perpetuity = \left[\frac{C}{r}\right]$$

$$PV\ of\ Growing Perpetuity = \left[\frac{C_1}{r-g}\right]$$

Excel Function for Present Value and Present Value of Annuity: = PV(Rate, Period, Amount)

Excel Function for Future Value and Future Value of Annuity: = FV(Rate, Period, Amount)

Appendix - 1: Compound or Future Value of Rupee 1																	
	Interest Rates																
Time	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	18%	20%
1	1.0100	1.0200	1.0300	1.0400	1.0500	1.0600	1.0700	1.0800	1.0900	1.1000	1.1100	1.1200	1.1300	1.1400	1.1500	1.1800	1.2000
2	1.0201	1.0404	1.0609	1.0816	1.1025	1.1236	1.1449	1.1664	1.1881	1.2100	1.2321	1.2544	1.2769	1.2996	1.3225	1.3924	1.4400
3	1.0303	1.0612	1.0927	1.1249	1.1576	1.1910	1.2250	1.2597	1.2950	1.3310	1.3676	1.4049	1.4429	1.4815	1.5209	1.6430	1.7280
4	1.0406	1.0824	1.1255	1.1699	1.2155	1.2625	1.3108	1.3605	1.4116	1.4641	1.5181	1.5735	1.6305	1.6890	1.7490	1.9388	2.0736
5	1.0510	1.1041	1.1593	1.2167	1.2763	1.3382	1.4026	1.4693	1.5386	1.6105	1.6851	1.7623	1.8424	1.9254	2.0114	2.2878	2.4883
6	1.0615	1.1262	1.1941	1.2653	1.3401	1.4185	1.5007	1.5869	1.6771	1.7716	1.8704	1.9738	2.0820	2.1950	2.3131	2.6996	2.9860
7	1.0721	1.1487	1.2299	1.3159	1.4071	1.5036	1.6058	1.7138	1.8280	1.9487	2.0762	2.2107	2.3526	2.5023	2.6600	3.1855	3.5832
8	1.0829	1.1717	1.2668	1.3686	1.4775	1.5938	1.7182	1.8509	1.9926	2.1436	2.3045	2.4760	2.6584	2.8526	3.0590	3.7589	4.2998
9	1.0937	1.1951	1.3048	1.4233	1.5513	1.6895	1.8385	1.9990	2.1719	2.3579	2.5580	2.7731	3.0040	3.2519	3.5179	4.4355	5.1598
10	1.1046	1.2190	1.3439	1.4802	1.6289	1.7908	1.9672	2.1589	2.3674	2.5937	2.8394	3.1058	3.3946	3.7072	4.0456	5.2338	6.1917
11	1.1157	1.2434	1.3842	1.5395	1.7103	1.8983	2.1049	2.3316	2.5804	2.8531	3.1518	3.4785	3.8359	4.2262	4.6524	6.1759	7.4301
12	1.1268	1.2682	1.4258	1.6010	1.7959	2.0122	2.2522	2.5182	2.8127	3.1384	3.4985	3.8960	4.3345	4.8179	5.3503	7.2876	8.9161
13	1.1381	1.2936	1.4685	1.6651	1.8856	2.1329	2.4098	2.7196	3.0658	3.4523	3.8833	4.3635	4.8980	5.4924	6.1528	8.5994	10.6993
14	1.1495	1.3195	1.5126	1.7317	1.9799	2.2609	2.5785	2.9372	3.3417	3.7975	4.3104	4.8871	5.5348	6.2613	7.0757	10.1472	12.8392
15	1.1610	1.3459	1.5580	1.8009	2.0789	2.3966	2.7590	3.1722	3.6425	4.1772	4.7846	5.4736	6.2543	7.1379	8.1371	11.9737	15.4070
16	1.1726	1.3728	1.6047	1.8730	2.1829	2.5404	2.9522	3.4259	3.9703	4.5950	5.3109	6.1304	7.0673	8.1372	9.3576	14.1290	18.4884
17	1.1843	1.4002	1.6528	1.9479	2.2920	2.6928	3.1588	3.7000	4.3276	5.0545	5.8951	6.8660	7.9861	9.2765	10.7613	16.6722	22.1861
18	1.1961	1.4282	1.7024	2.0258	2.4066	2.8543	3.3799	3.9960	4.7171	5.5599	6.5436	7.6900	9.0243	10.5752	12.3755	19.6733	26.6233
19	1.2081	1.4568	1.7535	2.1068	2.5270	3.0256	3.6165	4.3157	5.1417	6.1159	7.2633	8.6128	10.1974	12.0557	14.2318	23.2144	31.9480
20	1.2202	1.4859	1.8061	2.1911	2.6533	3.2071	3.8697	4.6610	5.6044	6.7275	8.0623	9.6463	11.5231	13.7435	16.3665	27.3930	38.3376
21	1.2324	1.5157	1.8603	2.2788	2.7860	3.3996	4.1406	5.0338	6.1088	7.4002	8.9492	10.8038	13.0211	15.6676	18.8215	32.3238	46.0051
22	1.2447	1.5460	1.9161	2.3699	2.9253	3.6035	4.4304	5.4365	6.6586	8.1403	9.9336	12.1003	14.7138	17.8610	21.6447	38.1421	55.2061
23	1.2572	1.5769	1.9736	2.4647	3.0715	3.8197	4.7405	5.8715	7.2579	8.9543	11.0263	13.5523	16.6266	20.3616	24.8915	45.0076	66.2474
24	1.2697	1.6084	2.0328	2.5633	3.2251	4.0489	5.0724	6.3412	7.9111	9.8497	12.2392	15.1786	18.7881	23.2122	28.6252	53.1090	79.4968
25	1.2824	1.6406	2.0938	2.6658	3.3864	4.2919	5.4274	6.8485	8.6231	10.8347	13.5855	17.0001	21.2305	26.4619	32.9190	62.6686	95.3962
26	1.2953	1.6734	2.1566	2.7725	3.5557	4.5494	5.8074	7.3964	9.3992	11.9182	15.0799	19.0401	23.9905	30.1666	37.8568	73.9490	114.4755
27	1.3082	1.7069	2.2213	2.8834	3.7335	4.8223	6.2139	7.9881	10.2451	13.1100	16.7386	21.3249	27.1093	34.3899	43.5353	87.2598	137.3706
28	1.3213	1.7410	2.2879	2.9987	3.9201	5.1117	6.6488	8.6271	11.1671	14.4210	18.5799	23.8839	30.6335	39.2045	50.0656	102.9666	164.8447
29	1.3345	1.7758	2.3566	3.1187	4.1161	5.4184	7.1143	9.3173	12.1722	15.8631	20.6237	26.7499	34.6158	44.6931	57.5755	121.5005	197.8136
30	1.3478	1.8114	2.4273	3.2434	4.3219	5.7435	7.6123	10.0627	13.2677	17.4494	22.8923	29.9599	39.1159	50.9502	66.2118	143.3706	237.3763
31	1.3613	1.8476	2.5001	3.3731	4.5380	6.0881	8.1451	10.8677	14.4618	19.1943	25.4104	33.5551	44.2010	58.0832	76.1435	169.1774	284.8516
32	1.3749	1.8845	2.5751	3.5081	4.7649	6.4534	8.7153	11.7371	15.7633	21.1138	28.2056	37.5817	49.9471	66.2148	87.5651	199.6293	341.8219
33	1.3887	1.9222	2.6523	3.6484	5.0032	6.8406	9.3253	12.6760	17.1820	23.2252	31.3082	42.0915	56.4402	75.4849	100.6998	235.5625	410.1863
34	1.4026	1.9607	2.7319	3.7943	5.2533	7.2510	9.9781	13.6901	18.7284	25.5477	34.7521	47.1425	63.7774	86.0528	115.8048	277.9638	492.2235
35	1.4166	1.9999	2.8139	3.9461	5.5160	7.6861	10.6766	14.7853	20.4140	28.1024	38.5749	52.7996	72.0685	98.1002	133.1755	327.9973	590.6682
36	1.4308	2.0399	2.8983	4.1039	5.7918	8.1473	11.4239	15.9682	22.2512	30.9127	42.8181	59.1356	81.4374	111.8342	153.1519	387.0368	708.8019
37	1.4451	2.0807	2.9852	4.2681	6.0814	8.6361	12.2236	17.2456	24.2538	34.0039	47.5281	66.2318	92.0243	127.4910	176.1246	456.7034	850.5622
38	1.4595	2.1223	3.0748	4.4388	6.3855	9.1543	13.0793	18.6253	26.4367	37.4043	52.7562	74.1797	103.9874	145.3397	202.5433	538.9100	1020.6747
39	1.4741	2.1647	3.1670	4.6164	6.7048	9.7035	13.9948	20.1153	28.8160	41.1448	58.5593	83.0812	117.5058	165.6873	232.9248	635.9139	1224.8096
40	1.4889	2.2080	3.2620	4.8010	7.0400	10.2857	14.9745	21.7245	31.4094	45.2593	65.0009	93.0510	132.7816	188.8835	267.8635	750.3783	1469.7716

Appendix - 2: Compound or Future Value of Annuity of Rupee 1																	
	Interest Rates																
Time	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	18%	20%
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0200	2.0300	2.0400	2.0500	2.0600	2.0700	2.0800	2.0900	2.1000	2.1100	2.1200	2.1300	2.1400	2.1500	2.1800	2.2000
3	3.0301	3.0604	3.0909	3.1216	3.1525	3.1836	3.2149	3.2464	3.2781	3.3100	3.3421	3.3744	3.4069	3.4396	3.4725	3.5724	3.6400
4	4.0604	4.1216	4.1836	4.2465	4.3101	4.3746	4.4399	4.5061	4.5731	4.6410	4.7097	4.7793	4.8498	4.9211	4.9934	5.2154	5.3680
5	5.1010	5.2040	5.3091	5.4163	5.5256	5.6371	5.7507	5.8666	5.9847	6.1051	6.2278	6.3528	6.4803	6.6101	6.7424	7.1542	7.4416
6	6.1520	6.3081	6.4684	6.6330	6.8019	6.9753	7.1533	7.3359	7.5233	7.7156	7.9129	8.1152	8.3227	8.5355	8.7537	9.4420	9.9299
7	7.2135	7.4343	7.6625	7.8983	8.1420	8.3938	8.6540	8.9228	9.2004	9.4872	9.7833	10.0890	10.4047	10.7305	11.0668	12.1415	12.9159
8	8.2857	8.5830	8.8923	9.2142	9.5491	9.8975	10.2598	10.6366	11.0285	11.4359	11.8594	12.2997	12.7573	13.2328	13.7268	15.3270	16.4991
9	9.3685	9.7546	10.1591	10.5828	11.0266	11.4913	11.9780	12.4876	13.0210	13.5795	14.1640	14.7757	15.4157	16.0853	16.7858	19.0859	20.7989
10	10.4622	10.9497	11.4639	12.0061	12.5779	13.1808	13.8164	14.4866	15.1929	15.9374	16.7220	17.5487	18.4197	19.3373	20.3037	23.5213	25.9587
11	11.5668	12.1687	12.8078	13.4864	14.2068	14.9716	15.7836	16.6455	17.5603	18.5312	19.5614	20.6546	21.8143	23.0445	24.3493	28.7551	32.1504
12	12.6825	13.4121	14.1920	15.0258	15.9171	16.8699	17.8885	18.9771	20.1407	21.3843	22.7132	24.1331	25.6502	27.2707	29.0017	34.9311	39.5805
13	13.8093	14.6803	15.6178	16.6268	17.7130	18.8821	20.1406	21.4953	22.9534	24.5227	26.2116	28.0291	29.9847	32.0887	34.3519	42.2187	48.4966
14	14.9474	15.9739	17.0863	18.2919	19.5986	21.0151	22.5505	24.2149	26.0192	27.9750	30.0949	32.3926	34.8827	37.5811	40.5047	50.8180	59.1959
15	16.0969	17.2934	18.5989	20.0236	21.5786	23.2760	25.1290	27.1521	29.3609	31.7725	34.4054	37.2797	40.4175	43.8424	47.5804	60.9653	72.0351
16	17.2579	18.6393	20.1569	21.8245	23.6575	25.6725	27.8881	30.3243	33.0034	35.9497	39.1899	42.7533	46.6717	50.9804	55.7175	72.9390	87.4421
17	18.4304	20.0121	21.7616	23.6975	25.8404	28.2129	30.8402	33.7502	36.9737	40.5447	44.5008	48.8837	53.7391	59.1176	65.0751	87.0680	105.9306
18	19.6147	21.4123	23.4144	25.6454	28.1324	30.9057	33.9990	37.4502	41.3013	45.5992	50.3959	55.7497	61.7251	68.3941	75.8364	103.7403	128.1167
19	20.8109	22.8406	25.1169	27.6712	30.5390	33.7600	37.3790	41.4463	46.0185	51.1591	56.9395	63.4397	70.7494	78.9692	88.2118	123.4135	154.7400
20	22.0190	24.2974	26.8704	29.7781	33.0660	36.7856	40.9955	45.7620	51.1601	57.2750	64.2028	72.0524	80.9468	91.0249	102.4436	146.6280	186.6880
21	23.2392	25.7833	28.6765	31.9692	35.7193	39.9927	44.8652	50.4229	56.7645	64.0025	72.2651	81.6987	92.4699	104.7684	118.8101	174.0210	225.0256
22	24.4716	27.2990	30.5368	34.2480	38.5052	43.3923	49.0057	55.4568	62.8733	71.4027	81.2143	92.5026	105.4910	120.4360	137.6316	206.3448	271.0307
23	25.7163	28.8450	32.4529	36.6179	41.4305	46.9958	53.4361	60.8933	69.5319	79.5430	91.1479	104.6029	120.2048	138.2970	159.2764	244.4868	326.2369
24	26.9735	30.4219	34.4265	39.0826	44.5020	50.8156	58.1767	66.7648	76.7898	88.4973	102.1742	118.1552	136.8315	158.6586	184.1678	289.4945	392.4842
25	28.2432	32.0303	36.4593	41.6459	47.7271	54.8645	63.2490	73.1059	84.7009	98.3471	114.4133	133.3339	155.6196	181.8708	212.7930	342.6035	471.9811
26	29.5256	33.6709	38.5530	44.3117	51.1135	59.1564	68.6765	79.9544	93.3240	109.1818	127.9988	150.3339	176.8501	208.3327	245.7120	405.2721	567.3773
27	30.8209	35.3443	40.7096	47.0842	54.6691	63.7058	74.4838	87.3508	102.7231	121.0999	143.0786	169.3740	200.8406	238.4993	283.5688	479.2211	681.8528
28	32.1291	37.0512	42.9309	49.9676	58.4026	68.5281	80.6977	95.3388	112.9682	134.2099	159.8173	190.6989	227.9499	272.8892	327.1041	566.4809	819.2233
29	33.4504	38.7922	45.2189	52.9663	62.3227	73.6398	87.3465	103.9659	124.1354	148.6309	178.3972	214.5828	258.583	312.094	377.170	669.447	984.068
30	34.7849	40.5681	47.5754	56.0849	66.4388	79.0582	94.4608	113.2832	136.3075	164.4940	199.0209	241.3327	293.199	356.787	434.745	790.948	1181.882
31	36.1327	42.3794	50.0027	59.3283	70.7608	84.8017	102.0730	123.3459	149.5752	181.9434	221.9132	271.2926	332.315	407.737	500.957	934.319	1419.258
32	37.4941	44.2270	52.5028	62.7015	75.2988	90.8898	110.2182	134.2135	164.0370	201.1378	247.3236	304.8477	376.516	465.820	577.100	1103.496	1704.109
33	38.8690	46.1116	55.0778	66.2095	80.0638	97.3432	118.9334	145.9506	179.8003	222.2515	275.5292	342.4294	426.463	532.035	664.666	1303.125	2045.931
34	40.2577	48.0338	57.7302	69.8579	85.0670	104.1838	128.2588	158.6267	196.9823	245.4767	306.8374	384.5210	482.903	607.520	765.365	1538.688	2456.118
35	41.6603	49.9945	60.4621	73.6522	90.3203	111.4348	138.2369	172.3168	215.7108	271.0244	341.5896	431.6635	546.681	693.573	881.170	1816.652	2948.341
36	43.0769	51.9944	63.2759	77.5983	95.8363	119.1209	148.9135	187.1021	236.1247	299.1268	380.1644	484.4631	618.749	791.673	1014.346	2144.649	3539.009
37	44.5076	54.0343	66.1742	81.7022	101.6281	127.2681	160.3374	203.0703	258.3759	330.0395	422.9825	543.5987	700.187	903.507	1167.498	2531.686	4247.811
38	45.9527	56.1149	69.1594	85.9703	107.7095	135.9042	172.5610	220.3159	282.6298	364.0434	470.5106	609.8305	792.211	1030.998	1343.622	2988.389	5098.373
39	47.4123	58.2372	72.2342	90.4091	114.0950	145.0585	185.6403	238.9412	309.0665	401.4478	523.2667	684.0102	896.198	1176.338	1546.165	3527.299	6119.048
40	48.8864	60.4020	75.4013	95.0255	120.7998	154.7620	199.6351	259.0565	337.8824	442.5926	581.8261	767.0914	1013.704	1342.025	1779.090	4163.213	7343.858

Appendix - 3: Present Value of Rupee 1																	
	Interest Rates																
Time	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	18%	20%
1	0.9901	0.9804	0.9709	0.9615	0.9524	0.9434	0.9346	0.9259	0.9174	0.9091	0.9009	0.8929	0.8850	0.8772	0.8696	0.8475	0.8333
2	0.9803	0.9612	0.9426	0.9246	0.9070	0.8900	0.8734	0.8573	0.8417	0.8264	0.8116	0.7972	0.7831	0.7695	0.7561	0.7182	0.6944
3	0.9706	0.9423	0.9151	0.8890	0.8638	0.8396	0.8163	0.7938	0.7722	0.7513	0.7312	0.7118	0.6931	0.6750	0.6575	0.6086	0.5787
4	0.9610	0.9238	0.8885	0.8548	0.8227	0.7921	0.7629	0.7350	0.7084	0.6830	0.6587	0.6355	0.6133	0.5921	0.5718	0.5158	0.4823
5	0.9515	0.9057	0.8626	0.8219	0.7835	0.7473	0.7130	0.6806	0.6499	0.6209	0.5935	0.5674	0.5428	0.5194	0.4972	0.4371	0.4019
6	0.9420	0.8880	0.8375	0.7903	0.7462	0.7050	0.6663	0.6302	0.5963	0.5645	0.5346	0.5066	0.4803	0.4556	0.4323	0.3704	0.3349
7	0.9327	0.8706	0.8131	0.7599	0.7107	0.6651	0.6227	0.5835	0.5470	0.5132	0.4817	0.4523	0.4251	0.3996	0.3759	0.3139	0.2791
8	0.9235	0.8535	0.7894	0.7307	0.6768	0.6274	0.5820	0.5403	0.5019	0.4665	0.4339	0.4039	0.3762	0.3506	0.3269	0.2660	0.2326
9	0.9143	0.8368	0.7664	0.7026	0.6446	0.5919	0.5439	0.5002	0.4604	0.4241	0.3909	0.3606	0.3329	0.3075	0.2843	0.2255	0.1938
10	0.9053	0.8203	0.7441	0.6756	0.6139	0.5584	0.5083	0.4632	0.4224	0.3855	0.3522	0.3220	0.2946	0.2697	0.2472	0.1911	0.1615
11	0.8963	0.8043	0.7224	0.6496	0.5847	0.5268	0.4751	0.4289	0.3875	0.3505	0.3173	0.2875	0.2607	0.2366	0.2149	0.1619	0.1346
12	0.8874	0.7885	0.7014	0.6246	0.5568	0.4970	0.4440	0.3971	0.3555	0.3186	0.2858	0.2567	0.2307	0.2076	0.1869	0.1372	0.1122
13	0.8787	0.7730	0.6810	0.6006	0.5303	0.4688	0.4150	0.3677	0.3262	0.2897	0.2575	0.2292	0.2042	0.1821	0.1625	0.1163	0.0935
14	0.8700	0.7579	0.6611	0.5775	0.5051	0.4423	0.3878	0.3405	0.2992	0.2633	0.2320	0.2046	0.1807	0.1597	0.1413	0.0985	0.0779
15	0.8613	0.7430	0.6419	0.5553	0.4810	0.4173	0.3624	0.3152	0.2745	0.2394	0.2090	0.1827	0.1599	0.1401	0.1229	0.0835	0.0649
16	0.8528	0.7284	0.6232	0.5339	0.4581	0.3936	0.3387	0.2919	0.2519	0.2176	0.1883	0.1631	0.1415	0.1229	0.1069	0.0708	0.0541
17	0.8444	0.7142	0.6050	0.5134	0.4363	0.3714	0.3166	0.2703	0.2311	0.1978	0.1696	0.1456	0.1252	0.1078	0.0929	0.0600	0.0451
18	0.8360	0.7002	0.5874	0.4936	0.4155	0.3503	0.2959	0.2502	0.2120	0.1799	0.1528	0.1300	0.1108	0.0946	0.0808	0.0508	0.0376
19	0.8277	0.6864	0.5703	0.4746	0.3957	0.3305	0.2765	0.2317	0.1945	0.1635	0.1377	0.1161	0.0981	0.0829	0.0703	0.0431	0.0313
20	0.8195	0.6730	0.5537	0.4564	0.3769	0.3118	0.2584	0.2145	0.1784	0.1486	0.1240	0.1037	0.0868	0.0728	0.0611	0.0365	0.0261
21	0.8114	0.6598	0.5375	0.4388	0.3589	0.2942	0.2415	0.1987	0.1637	0.1351	0.1117	0.0926	0.0768	0.0638	0.0531	0.0309	0.0217
22	0.8034	0.6468	0.5219	0.4220	0.3418	0.2775	0.2257	0.1839	0.1502	0.1228	0.1007	0.0826	0.0680	0.0560	0.0462	0.0262	0.0181
23	0.7954	0.6342	0.5067	0.4057	0.3256	0.2618	0.2109	0.1703	0.1378	0.1117	0.0907	0.0738	0.0601	0.0491	0.0402	0.0222	0.0151
24	0.7876	0.6217	0.4919	0.3901	0.3101	0.2470	0.1971	0.1577	0.1264	0.1015	0.0817	0.0659	0.0532	0.0431	0.0349	0.0188	0.0126
25	0.7798	0.6095	0.4776	0.3751	0.2953	0.2330	0.1842	0.1460	0.1160	0.0923	0.0736	0.0588	0.0471	0.0378	0.0304	0.0160	0.0105
26	0.7720	0.5976	0.4637	0.3607	0.2812	0.2198	0.1722	0.1352	0.1064	0.0839	0.0663	0.0525	0.0417	0.0331	0.0264	0.0135	0.0087
27	0.7644	0.5859	0.4502	0.3468	0.2678	0.2074	0.1609	0.1252	0.0976	0.0763	0.0597	0.0469	0.0369	0.0291	0.0230	0.0115	0.0073
28	0.7568	0.5744	0.4371	0.3335	0.2551	0.1956	0.1504	0.1159	0.0895	0.0693	0.0538	0.0419	0.0326	0.0255	0.0200	0.0097	0.0061
29	0.7493	0.5631	0.4243	0.3207	0.2429	0.1846	0.1406	0.1073	0.0822	0.0630	0.0485	0.0374	0.0289	0.0224	0.0174	0.0082	0.0051
30	0.7419	0.5521	0.4120	0.3083	0.2314	0.1741	0.1314	0.0994	0.0754	0.0573	0.0437	0.0334	0.0256	0.0196	0.0151	0.0070	0.0042
31	0.7346	0.5412	0.4000	0.2965	0.2204	0.1643	0.1228	0.0920	0.0691	0.0521	0.0394	0.0298	0.0226	0.0172	0.0131	0.0059	0.0035
32	0.7273	0.5306	0.3883	0.2851	0.2099	0.1550	0.1147	0.0852	0.0634	0.0474	0.0355	0.0266	0.0200	0.0151	0.0114	0.0050	0.0029
33	0.7201	0.5202	0.3770	0.2741	0.1999	0.1462	0.1072	0.0789	0.0582	0.0431	0.0319	0.0238	0.0177	0.0132	0.0099	0.0042	0.0024
34	0.7130	0.5100	0.3660	0.2636	0.1904	0.1379	0.1002	0.0730	0.0534	0.0391	0.0288	0.0212	0.0157	0.0116	0.0086	0.0036	0.0020
35	0.7059	0.5000	0.3554	0.2534	0.1813	0.1301	0.0937	0.0676	0.0490	0.0356	0.0259	0.0189	0.0139	0.0102	0.0075	0.0030	0.0017
36	0.6989	0.4902	0.3450	0.2437	0.1727	0.1227	0.0875	0.0626	0.0449	0.0323	0.0234	0.0169	0.0123	0.0089	0.0065	0.0026	0.0014
37	0.6920	0.4806	0.3350	0.2343	0.1644	0.1158	0.0818	0.0580	0.0412	0.0294	0.0210	0.0151	0.0109	0.0078	0.0057	0.0022	0.0012
38	0.6852	0.4712	0.3252	0.2253	0.1566	0.1092	0.0765	0.0537	0.0378	0.0267	0.0190	0.0135	0.0096	0.0069	0.0049	0.0019	0.0010
39	0.6784	0.4619	0.3158	0.2166	0.1491	0.1031	0.0715	0.0497	0.0347	0.0243	0.0171	0.0120	0.0085	0.0060	0.0043	0.0016	0.0008
40	0.6717	0.4529	0.3066	0.2083	0.1420	0.0972	0.0668	0.0460	0.0318	0.0221	0.0154	0.0107	0.0075	0.0053	0.0037	0.0013	0.0007

Appendix - 4: Present Value of Annuity of Rupee 1																	
	Interest Rates																
Time	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	18%	20%
1	0.9901	0.9804	0.9709	0.9615	0.9524	0.9434	0.9346	0.9259	0.9174	0.9091	0.9009	0.8929	0.8850	0.8772	0.8696	0.8475	0.8333
2	1.9704	1.9416	1.9135	1.8861	1.8594	1.8334	1.8080	1.7833	1.7591	1.7355	1.7125	1.6901	1.6681	1.6467	1.6257	1.5656	1.5278
3	2.9410	2.8839	2.8286	2.7751	2.7232	2.6730	2.6243	2.5771	2.5313	2.4869	2.4437	2.4018	2.3612	2.3216	2.2832	2.1743	2.1065
4	3.9020	3.8077	3.7171	3.6299	3.5460	3.4651	3.3872	3.3121	3.2397	3.1699	3.1024	3.0373	2.9745	2.9137	2.8550	2.6901	2.5887
5	4.8534	4.7135	4.5797	4.4518	4.3295	4.2124	4.1002	3.9927	3.8897	3.7908	3.6959	3.6048	3.5172	3.4331	3.3522	3.1272	2.9906
6	5.7955	5.6014	5.4172	5.2421	5.0757	4.9173	4.7665	4.6229	4.4859	4.3553	4.2305	4.1114	3.9975	3.8887	3.7845	3.4976	3.3255
7	6.7282	6.4720	6.2303	6.0021	5.7864	5.5824	5.3893	5.2064	5.0330	4.8684	4.7122	4.5638	4.4226	4.2883	4.1604	3.8115	3.6046
8	7.6517	7.3255	7.0197	6.7327	6.4632	6.2098	5.9713	5.7466	5.5348	5.3349	5.1461	4.9676	4.7988	4.6389	4.4873	4.0776	3.8372
9	8.5660	8.1622	7.7861	7.4353	7.1078	6.8017	6.5152	6.2469	5.9952	5.7590	5.5370	5.3282	5.1317	4.9464	4.7716	4.3030	4.0310
10	9.4713	8.9826	8.5302	8.1109	7.7217	7.3601	7.0236	6.7101	6.4177	6.1446	5.8892	5.6502	5.4262	5.2161	5.0188	4.4941	4.1925
11	10.3676	9.7868	9.2526	8.7605	8.3064	7.8869	7.4987	7.1390	6.8052	6.4951	6.2065	5.9377	5.6869	5.4527	5.2337	4.6560	4.3271
12	11.2551	10.5753	9.9540	9.3851	8.8633	8.3838	7.9427	7.5361	7.1607	6.8137	6.4924	6.1944	5.9176	5.6603	5.4206	4.7932	4.4392
13	12.1337	11.3484	10.6350	9.9856	9.3936	8.8527	8.3577	7.9038	7.4869	7.1034	6.7499	6.4235	6.1218	5.8424	5.5831	4.9095	4.5327
14	13.0037	12.1062	11.2961	10.5631	9.8986	9.2950	8.7455	8.2442	7.7862	7.3667	6.9819	6.6282	6.3025	6.0021	5.7245	5.0081	4.6106
15	13.8651	12.8493	11.9379	11.1184	10.3797	9.7122	9.1079	8.5595	8.0607	7.6061	7.1909	6.8109	6.4624	6.1422	5.8474	5.0916	4.6755
16	14.7179	13.5777	12.5611	11.6523	10.8378	10.1059	9.4466	8.8514	8.3126	7.8237	7.3792	6.9740	6.6039	6.2651	5.9542	5.1624	4.7296
17	15.5623	14.2919	13.1661	12.1657	11.2741	10.4773	9.7632	9.1216	8.5436	8.0216	7.5488	7.1196	6.7291	6.3729	6.0472	5.2223	4.7746
18	16.3983	14.9920	13.7535	12.6593	11.6896	10.8276	10.0591	9.3719	8.7556	8.2014	7.7016	7.2497	6.8399	6.4674	6.1280	5.2732	4.8122
19	17.2260	15.6785	14.3238	13.1339	12.0853	11.1581	10.3356	9.6036	8.9501	8.3649	7.8393	7.3658	6.9380	6.5504	6.1982	5.3162	4.8435
20	18.0456	16.3514	14.8775	13.5903	12.4622	11.4699	10.5940	9.8181	9.1285	8.5136	7.9633	7.4694	7.0248	6.6231	6.2593	5.3527	4.8696
21	18.8570	17.0112	15.4150	14.0292	12.8212	11.7641	10.8355	10.0168	9.2922	8.6487	8.0751	7.5620	7.1016	6.6870	6.3125	5.3837	4.8913
22	19.6604	17.6580	15.9369	14.4511	13.1630	12.0416	11.0612	10.2007	9.4424	8.7715	8.1757	7.6446	7.1695	6.7429	6.3587	5.4099	4.9094
23	20.4558	18.2922	16.4436	14.8568	13.4886	12.3034	11.2722	10.3711	9.5802	8.8832	8.2664	7.7184	7.2297	6.7921	6.3988	5.4321	4.9245
24	21.2434	18.9139	16.9355	15.2470	13.7986	12.5504	11.4693	10.5288	9.7066	8.9847	8.3481	7.7843	7.2829	6.8351	6.4338	5.4509	4.9371
25	22.0232	19.5235	17.4131	15.6221	14.0939	12.7834	11.6536	10.6748	9.8226	9.0770	8.4217	7.8431	7.3300	6.8729	6.4641	5.4669	4.9476
26	22.7952	20.1210	17.8768	15.9828	14.3752	13.0032	11.8258	10.8100	9.9290	9.1609	8.4881	7.8957	7.3717	6.9061	6.4906	5.4804	4.9563
27	23.5596	20.7069	18.3270	16.3296	14.6430	13.2105	11.9867	10.9352	10.0266	9.2372	8.5478	7.9426	7.4086	6.9352	6.5135	5.4919	4.9636
28	24.3164	21.2813	18.7641	16.6631	14.8981	13.4062	12.1371	11.0511	10.1161	9.3066	8.6016	7.9844	7.4412	6.9607	6.5335	5.5016	4.9697
29	25.0658	21.8444	19.1885	16.9837	15.1411	13.5907	12.2777	11.1584	10.1983	9.3696	8.6501	8.0218	7.4701	6.9830	6.5509	5.5098	4.9747
30	25.8077	22.3965	19.6004	17.2920	15.3725	13.7648	12.4090	11.2578	10.2737	9.4269	8.6938	8.0552	7.4957	7.0027	6.5660	5.5168	4.9789
31	26.5423	22.9377	20.0004	17.5885	15.5928	13.9291	12.5318	11.3498	10.3428	9.4790	8.7331	8.0850	7.5183	7.0199	6.5791	5.5227	4.9824
32	27.2696	23.4683	20.3888	17.8736	15.8027	14.0840	12.6466	11.4350	10.4062	9.5264	8.7686	8.1116	7.5383	7.0350	6.5905	5.5277	4.9854
33	27.9897	23.9886	20.7658	18.1476	16.0025	14.2302	12.7538	11.5139	10.4644	9.5694	8.8005	8.1354	7.5560	7.0482	6.6005	5.5320	4.9878
34	28.7027	24.4986	21.1318	18.4112	16.1929	14.3681	12.8540	11.5869	10.5178	9.6086	8.8293	8.1566	7.5717	7.0599	6.6091	5.5356	4.9898
35	29.4086	24.9986	21.4872	18.6646	16.3742	14.4982	12.9477	11.6546	10.5668	9.6442	8.8552	8.1755	7.5856	7.0700	6.6166	5.5386	4.9915
36	30.1075	25.4888	21.8323	18.9083	16.5469	14.6210	13.0352	11.7172	10.6118	9.6765	8.8786	8.1924	7.5979	7.0790	6.6231	5.5412	4.9929
37	30.7995	25.9695	22.1672	19.1426	16.7113	14.7368	13.1170	11.7752	10.6530	9.7059	8.8996	8.2075	7.6087	7.0868	6.6288	5.5434	4.9941
38	31.4847	26.4406	22.4925	19.3679	16.8679	14.8460	13.1935	11.8289	10.6908	9.7327	8.9186	8.2210	7.6183	7.0937	6.6338	5.5452	4.9951
39	32.1630	26.9026	22.8082	19.5845	17.0170	14.9491	13.2649	11.8786	10.7255	9.7570	8.9357	8.2330	7.6268	7.0997	6.6380	5.5468	4.9959
40	32.8347	27.3555	23.1148	19.7928	17.1591	15.0463	13.3317	11.9246	10.7574	9.7791	8.9511	8.2438	7.6344	7.1050	6.6418	5.5482	4.9966