CS-218
DATA STRUCTURE

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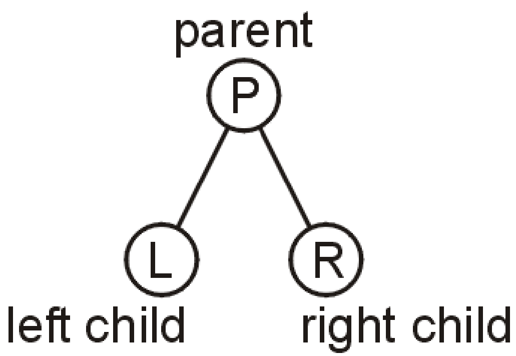
BINARY TREE

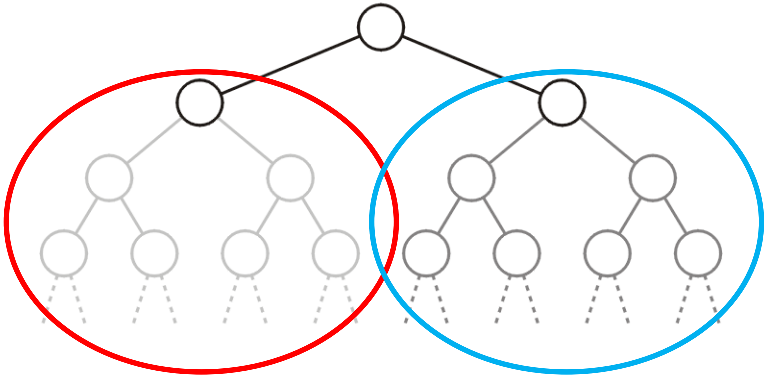
Binary Trees

Binary Tree

- In a binary tree, each node has at most two children
  - Allows to label the children as left and right



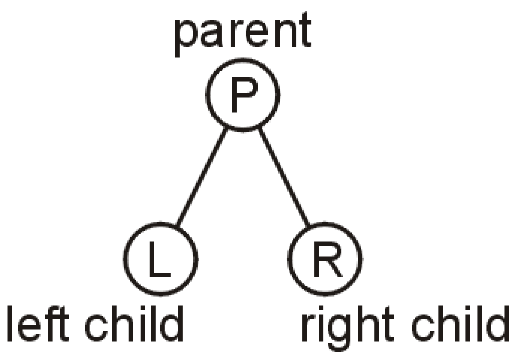
- Likewise, the two sub-trees are referred as
  - Left sub-tree
  - Right sub-tree



Binary Trees

Binary Tree

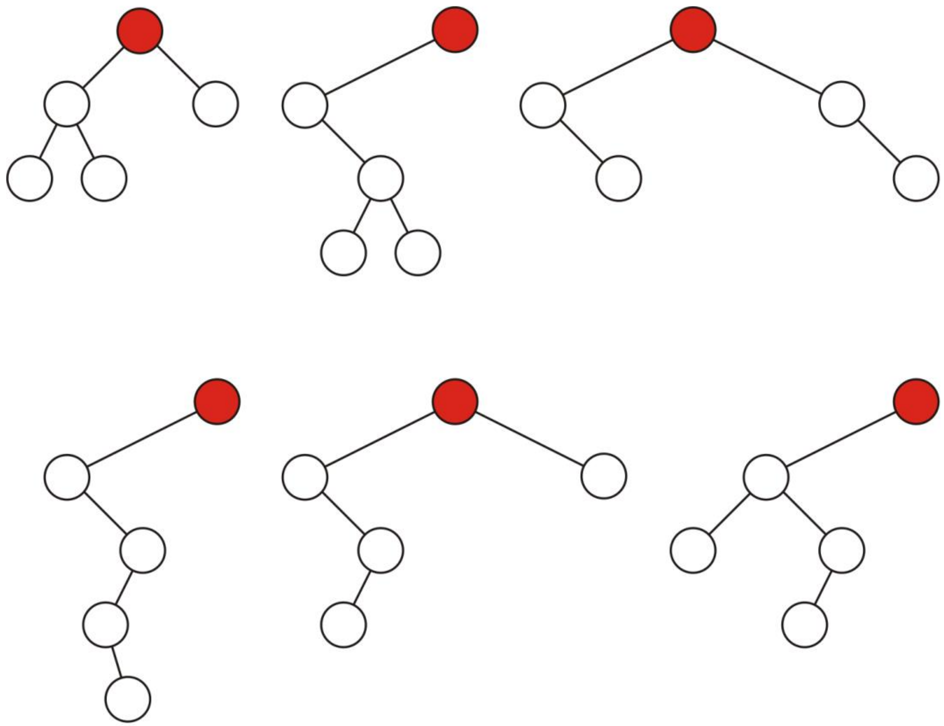
The mathematical definition of a binary tree is
"A binary tree is a finite set of elements that is either empty or is partitioned into three disjoint subsets. The first subset contains a single element called the root of the tree. The other two subsets are themselves binary trees called the left and right sub-trees".
Each element of a binary tree is called a node of the tree.
Following figure shows a binary tree.



Binary Trees

Binary Tree

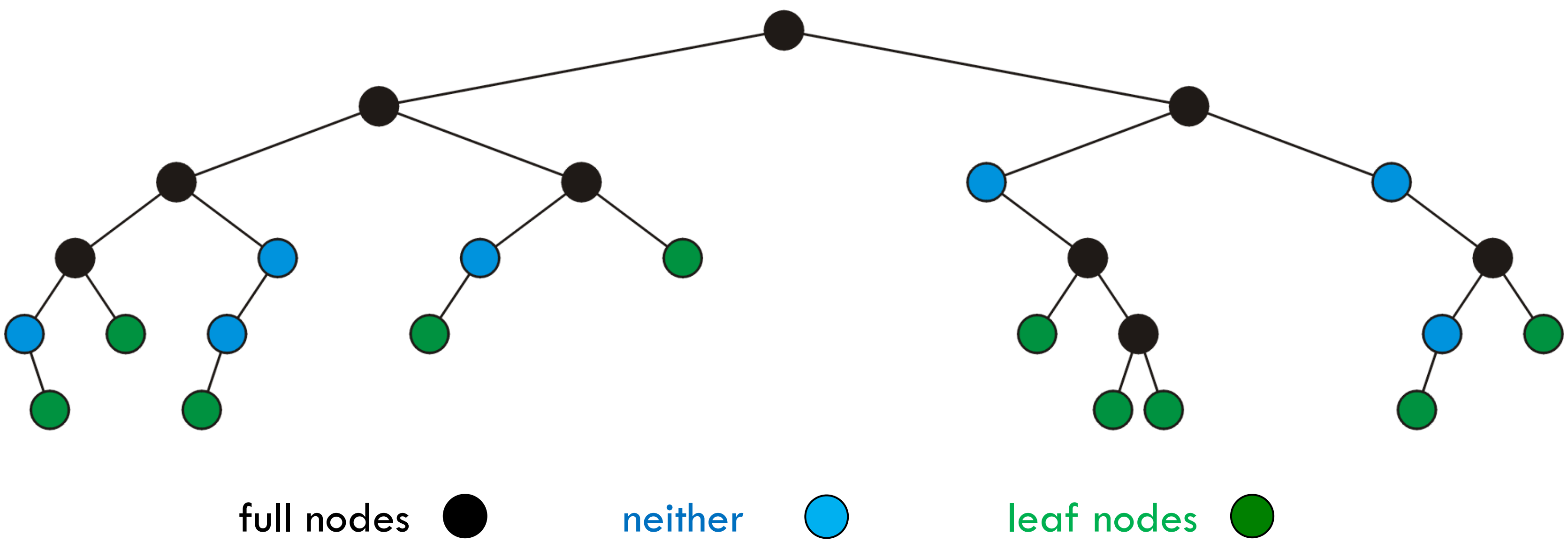
Some variations on binary trees with five nodes



Binary Trees

Full Node

A **full node** is a node where both the left and right sub-trees are non-empty trees

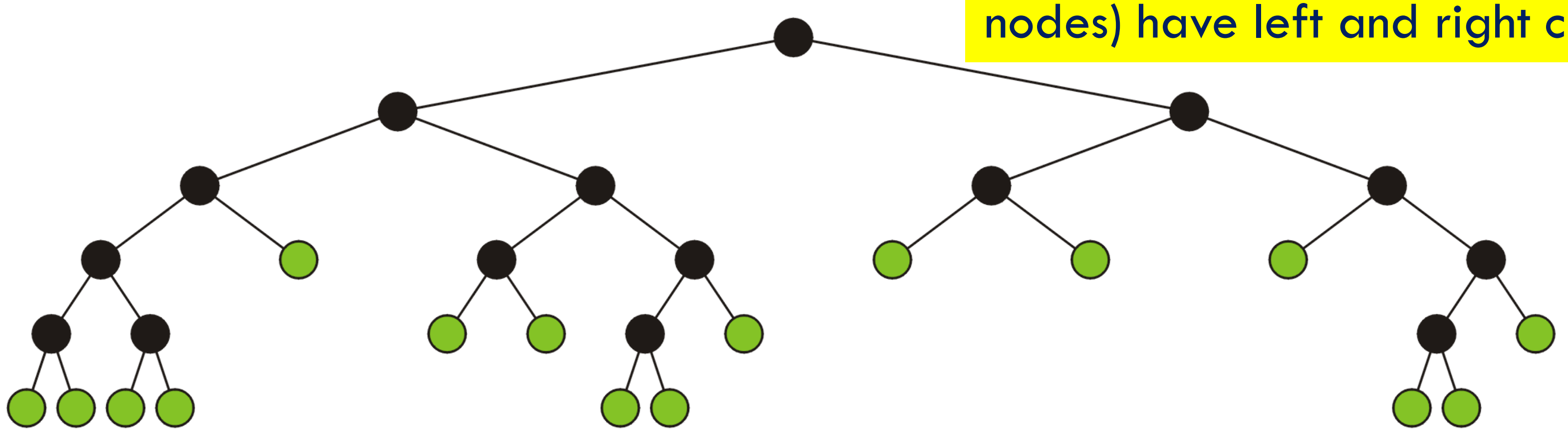


Full Binary Trees

Full Binary Tree

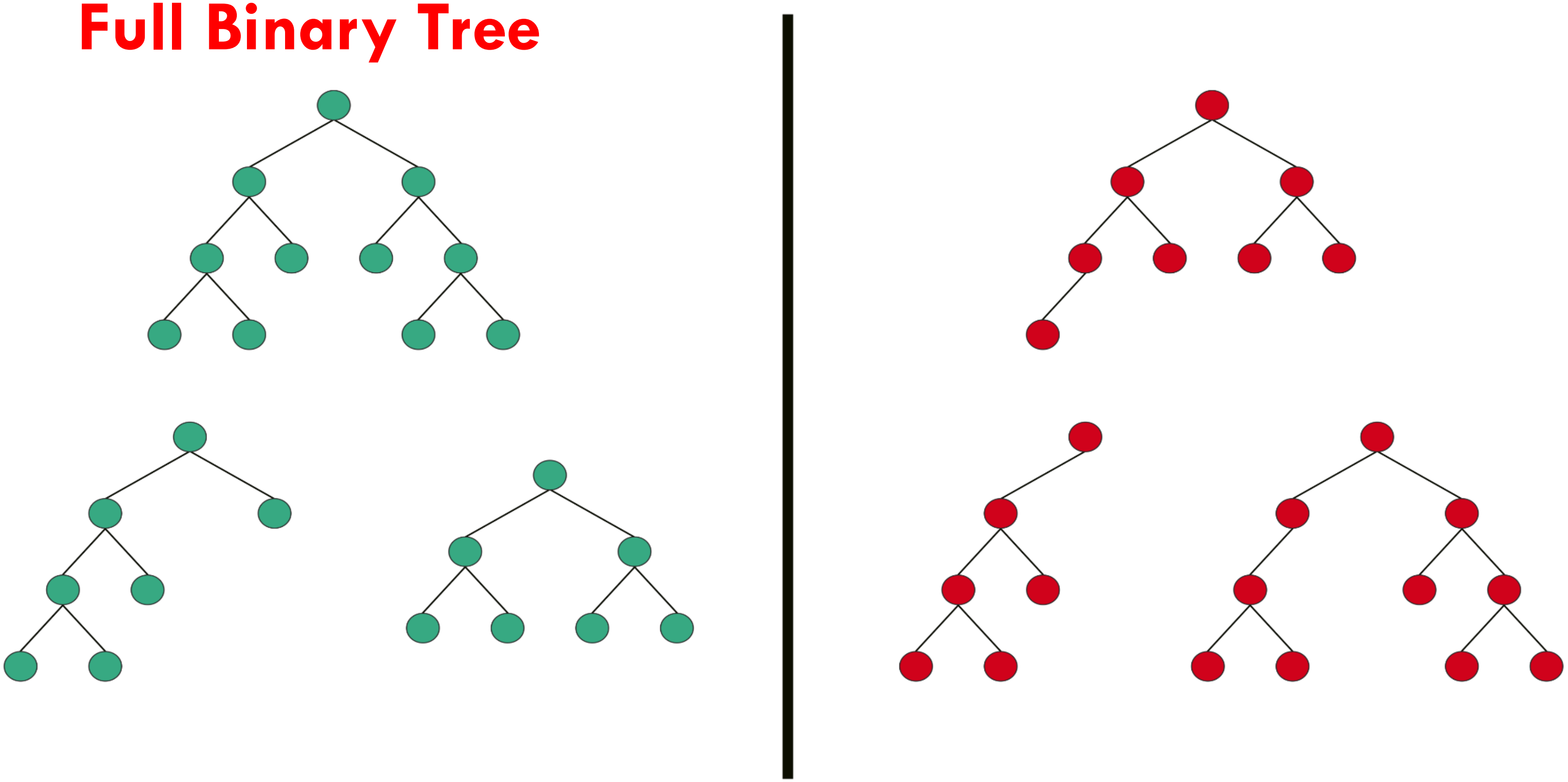
- A full binary tree is where each node is:
  - a full node, or
  - a leaf node.
- Full binary tree is also called **proper binary tree**, **strictly binary tree** or **2-tree**

All the nodes (other than leaf nodes) have left and right child



Full Binary Trees

Full Binary Tree

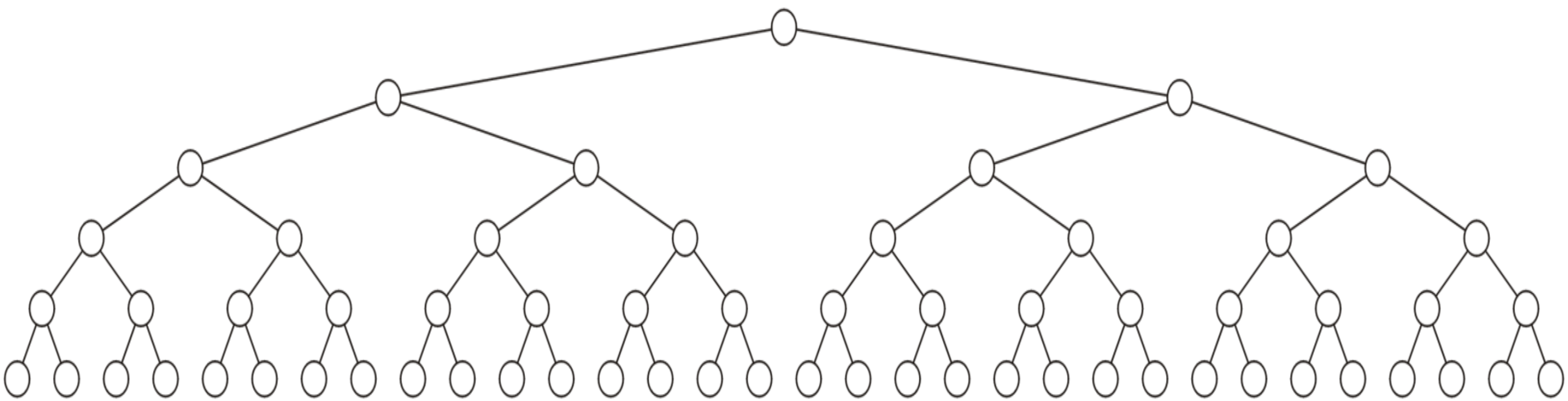


Valid and Invalid Structure of Full Binary Tree

Complete Binary Trees

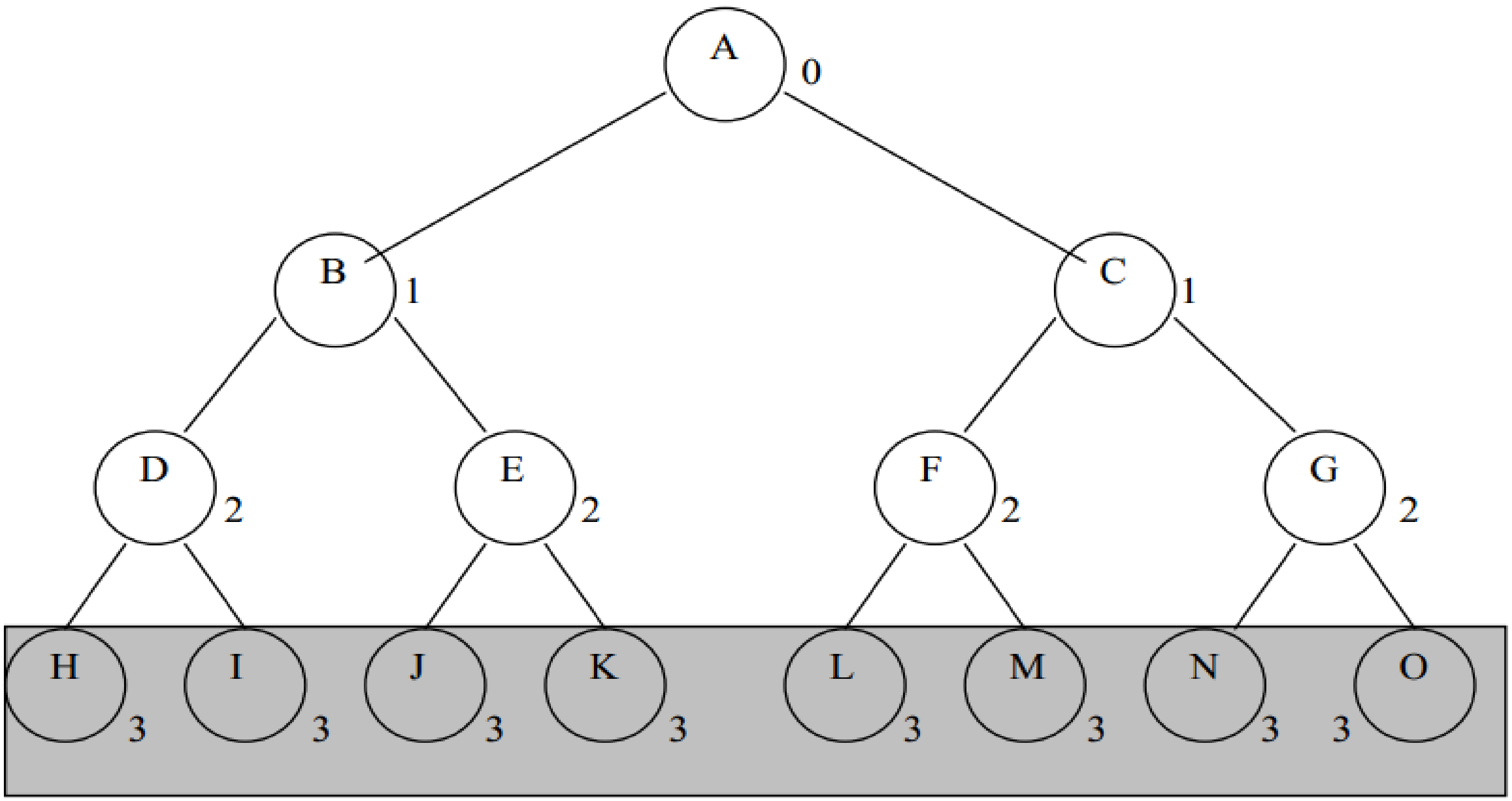
Complete (Perfect) Binary Tree

- A complete binary tree of height h is a binary tree, where,
  - All leaf nodes have the same depth h
  - All other nodes are **full** nodes



Complete Binary Trees

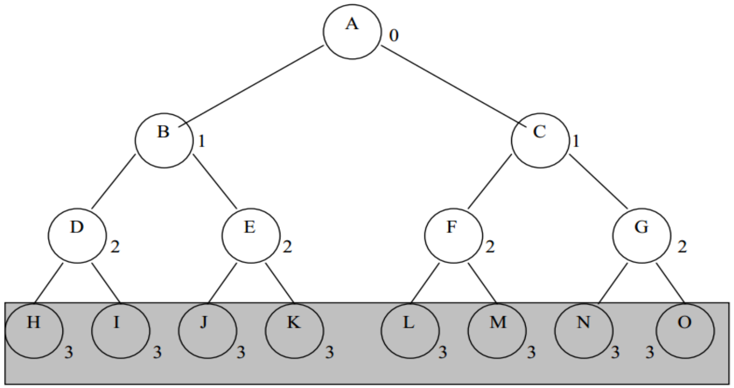
Complete (Perfect) Binary Tree … Examples



Complete Binary Trees

Complete (Perfect) Binary Tree … Examples

The definition of the complete binary tree is
"A complete binary tree of depth d is the strictly binary tree all of whose leaves are at level d".
Now look at the tree, the leaf nodes of the tree are at level 3 and are H, I, J, K, L, M, N and O. There is no such a leaf node that is at some level other than the depth level *d* i.e. 3. All the leaf nodes of this tree are at level 3 (which is the depth of the tree i.e. d). So this is a complete binary tree. In the figure, all the nodes at level 3 are highlighted.



Complete Binary Trees

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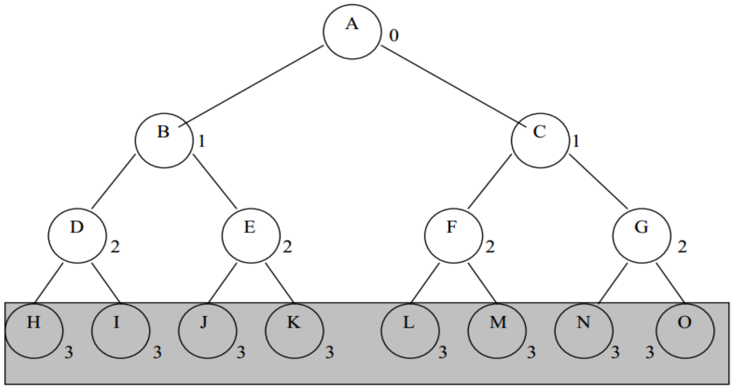
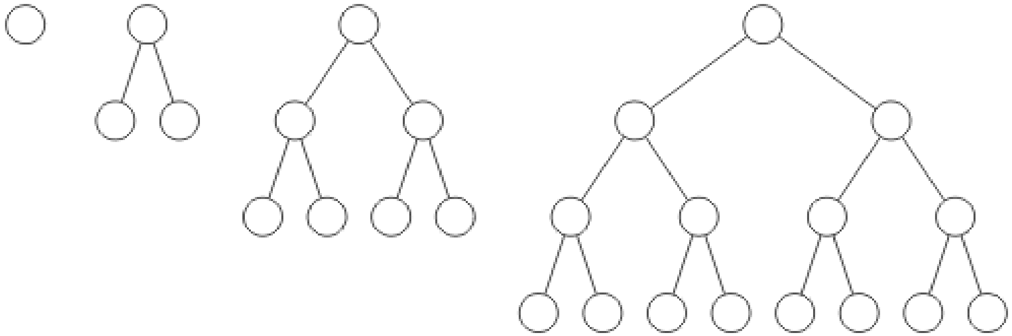
We can say every Strictly Binary Tree may or may not be a complete binary tree but every complete or perfect binary tree will always Strictly as well

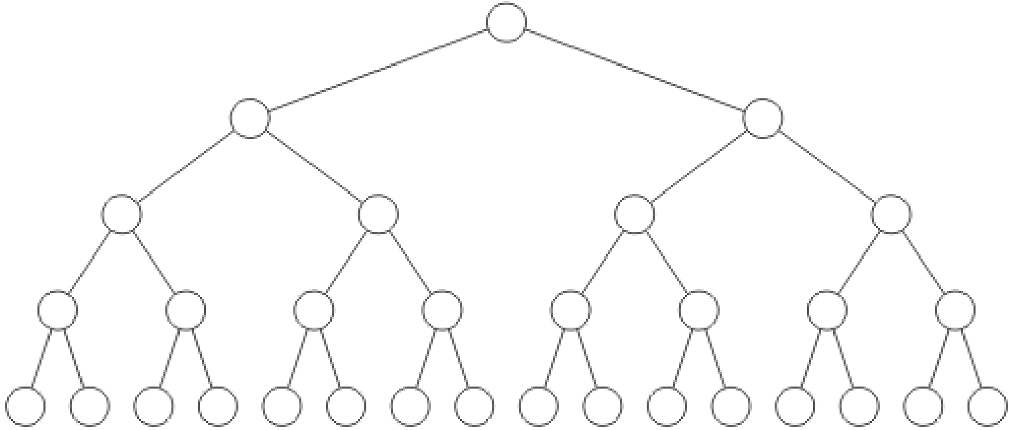


Complete Binary Trees

Complete (Perfect) Binary Tree … Examples



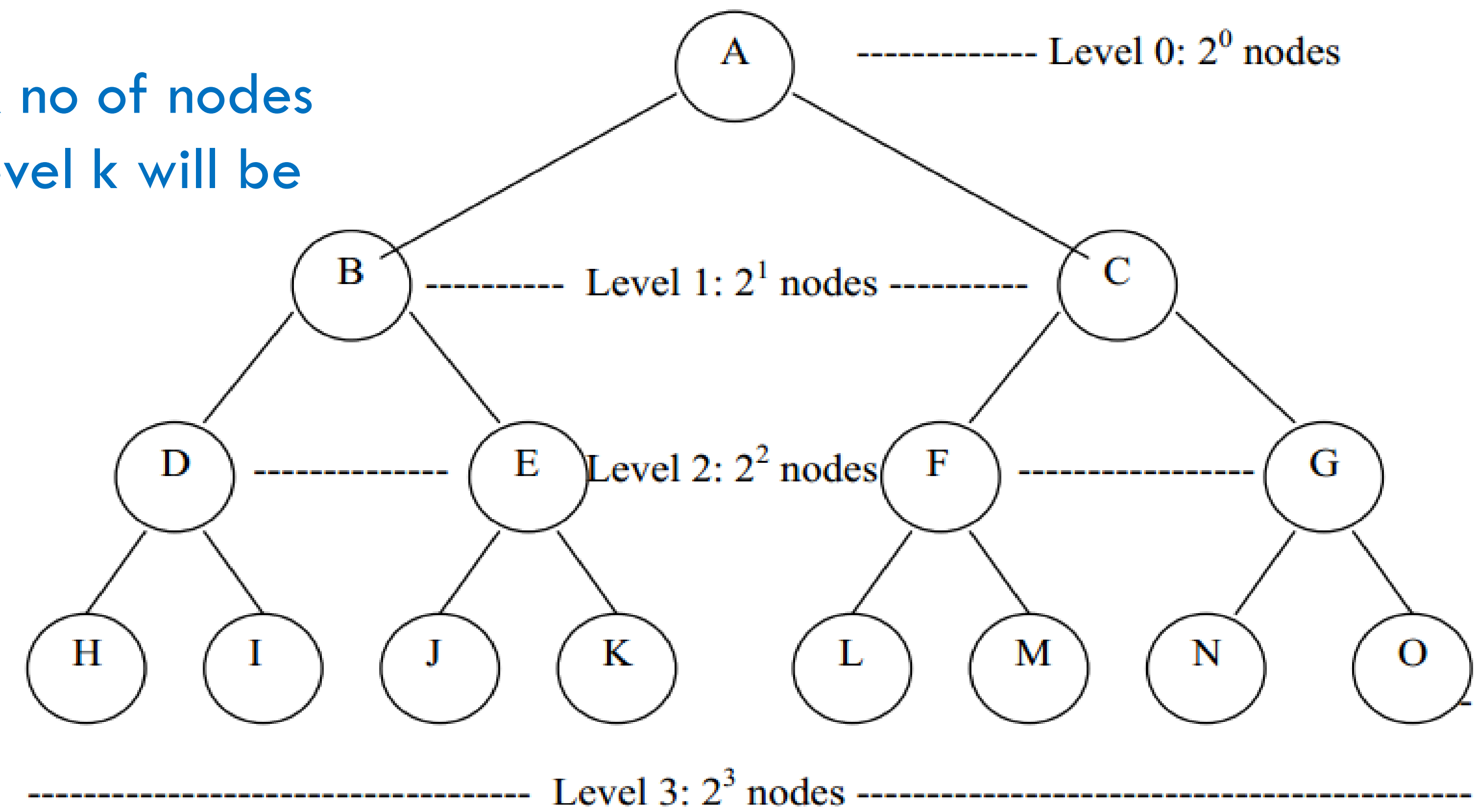
Complete binary trees of height $h = 0, 1, 2, 3$ *and* 4



Complete Binary Trees

Complete (Perfect) Binary Tree … Examples

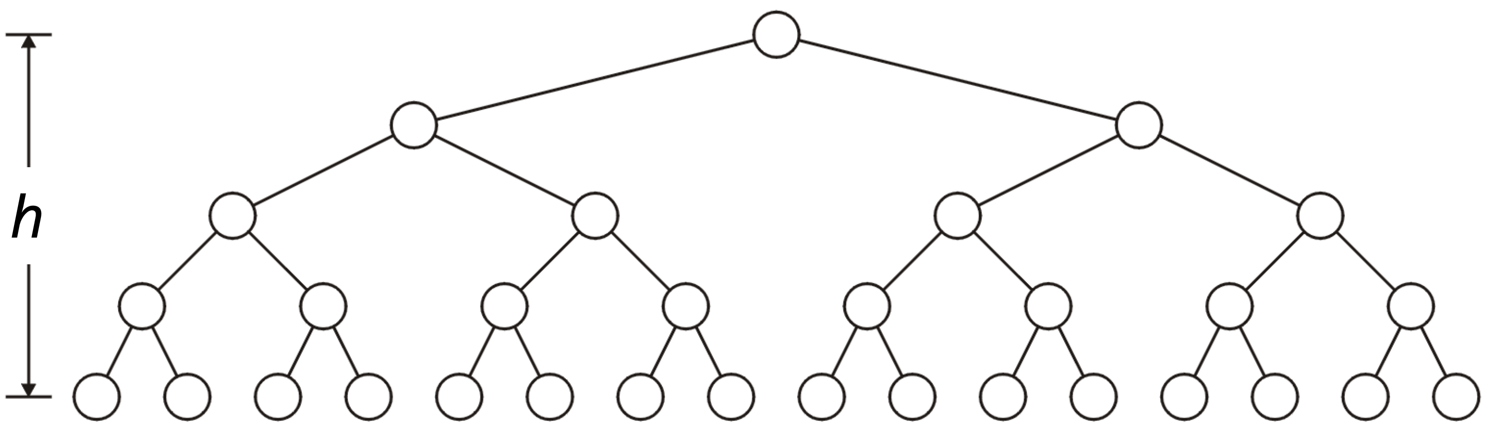
Max no of nodes at level k will be 2^k



Complete Binary Trees

Complete (Perfect) Binary Tree … Properties

- A complete binary tree with height h has 2^h leaf nodes



Complete Binary Trees

Complete (Perfect) Binary Tree … Properties

Depth d	# nodes at depth d	# of child nodes
0	$1 = 2^0$	2 (each node has 2 children)
1	$2 = 2^1$	4 (each node has 2 children)
2	$4 = 2^2$	8 (each node has 2 children)
...		

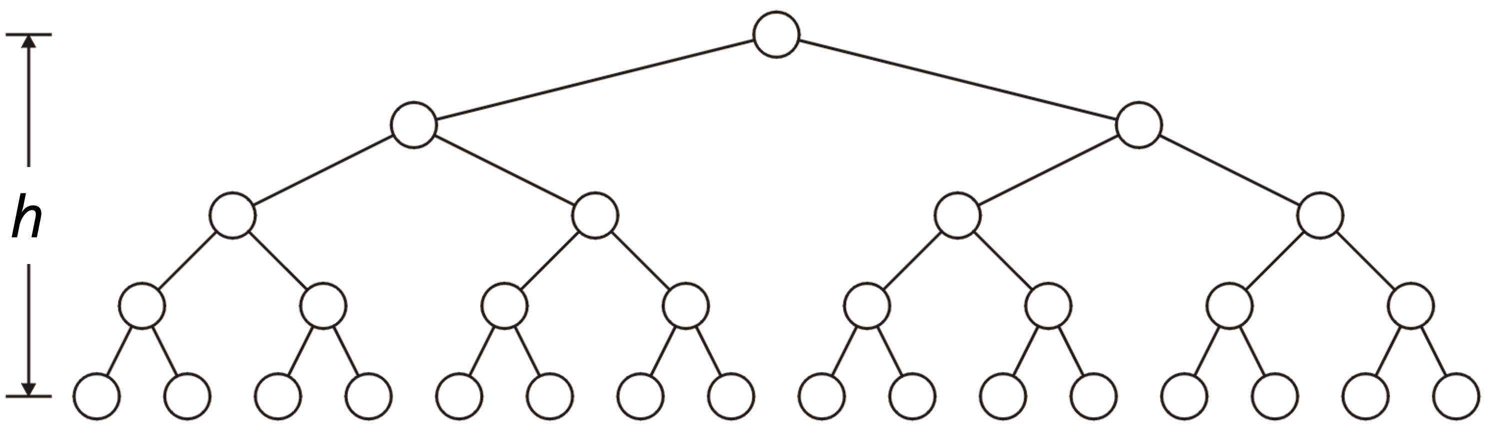
- The **number** of nodes **doubles** every time the **depth** increases by **1** !

nodes at depth d = 2^d

Complete Binary Trees

Complete (Perfect) Binary Tree … Properties

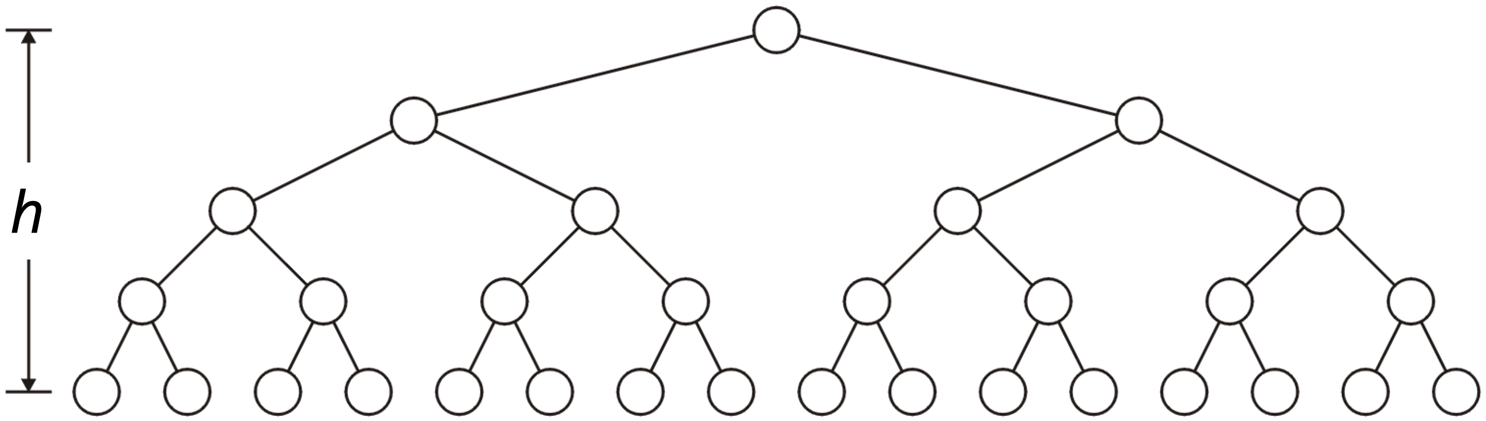
- A complete binary tree with height h has 2^h leaf nodes
- A complete binary tree of height h has $2^{h+1} - 1$ nodes



Complete Binary Trees

Complete (Perfect) Binary Tree … Properties

- A complete binary tree with height h has 2^h leaf nodes
- A complete binary tree of height h has $2^{h+1} - 1$ nodes
  - Number of leaf nodes: $L = 2^h$
  - Number of internal nodes: $2^h - 1$
  - Total number of nodes: $2^{h+1} - 1$



Complete Binary Trees

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- A complete binary tree with n nodes has height $\log_2(n + 1) - 1$

$$n = 2^{h+1} - 1$$

$$2^{h+1} = n + 1$$

$$h + 1 = \log_2(n + 1)$$

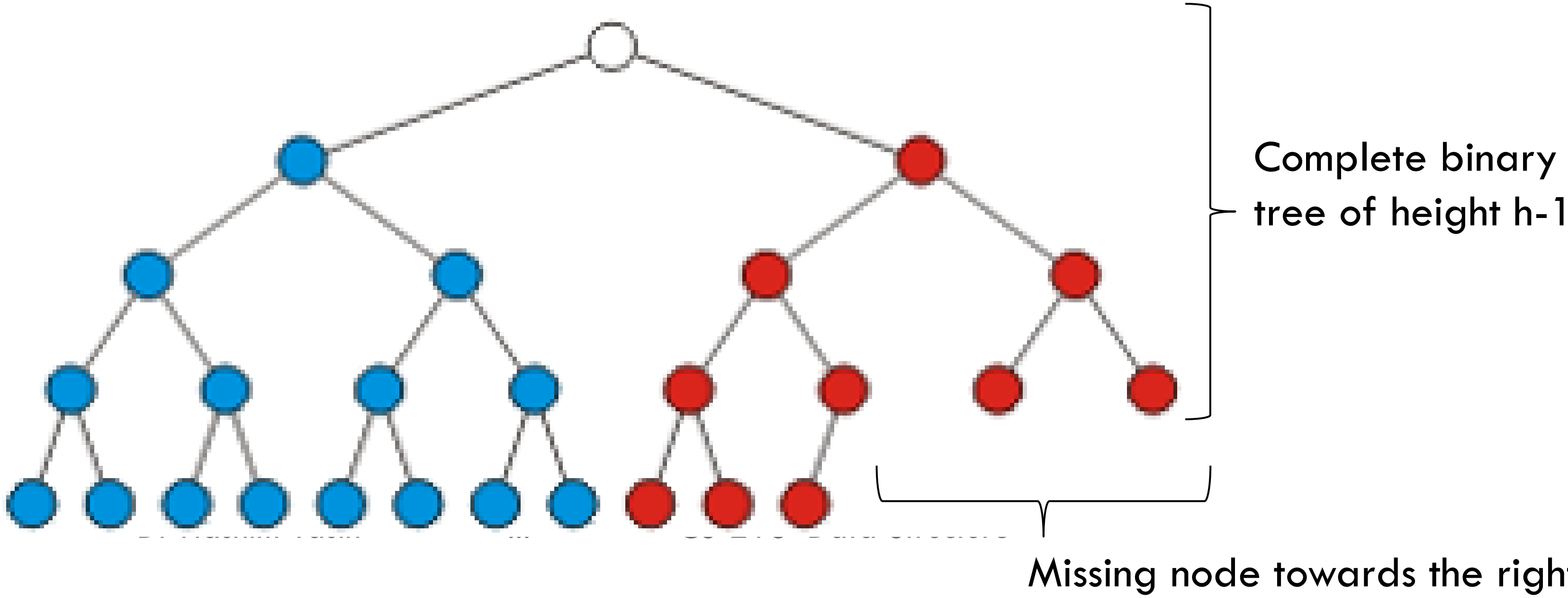
$$\Rightarrow h = \log_2(n + 1) - 1$$

Almost Complete Binary Trees

Almost Complete Binary Tree

Almost complete binary tree of height h is a binary tree in which

1. There are 2^d nodes at depth d for d = 1,2,...,h−1.
   Each leaf in the tree is either at level h or at level h−1.
2. ***The nodes at depth h are as far left as possible***

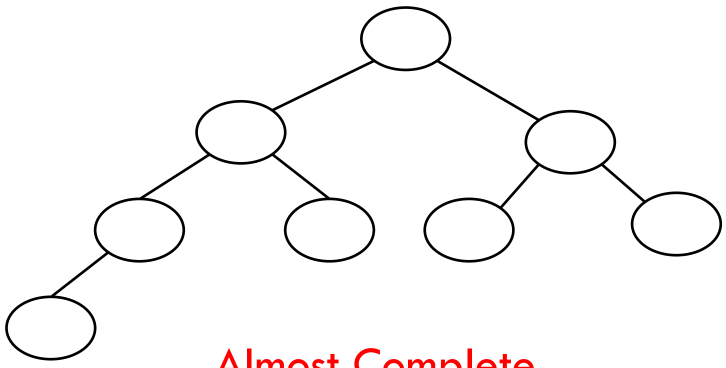


Almost Complete Binary Trees

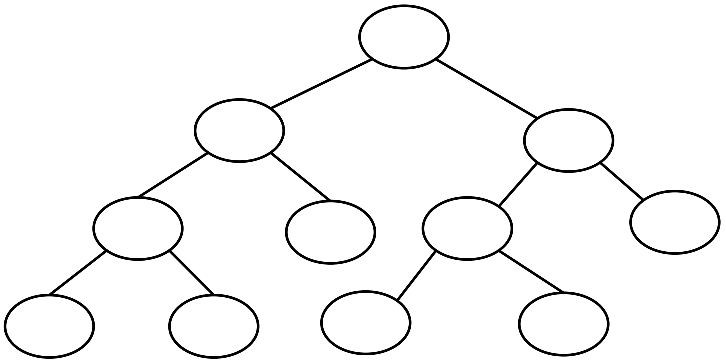
Almost Complete Binary Tree

The nodes at depth h are as far left as possible

- If a node p at depth h−1 has a left child
  - Every node at depth h−1 to the left of p has 2 children
- If a node at depth h−1 has a right child
  - It also has a left child



Almost Complete binary tree



Not Almost Complete binary tree (condition 2 violated)

Almost Complete Binary Trees

Almost Complete Binary Tree … Properties

Total number of nodes n are between

Complete binary tree of height $h-1$, i.e., 2^h nodes

Complete binary tree of height h, i.e., $2^{h+1} - 1$ nodes

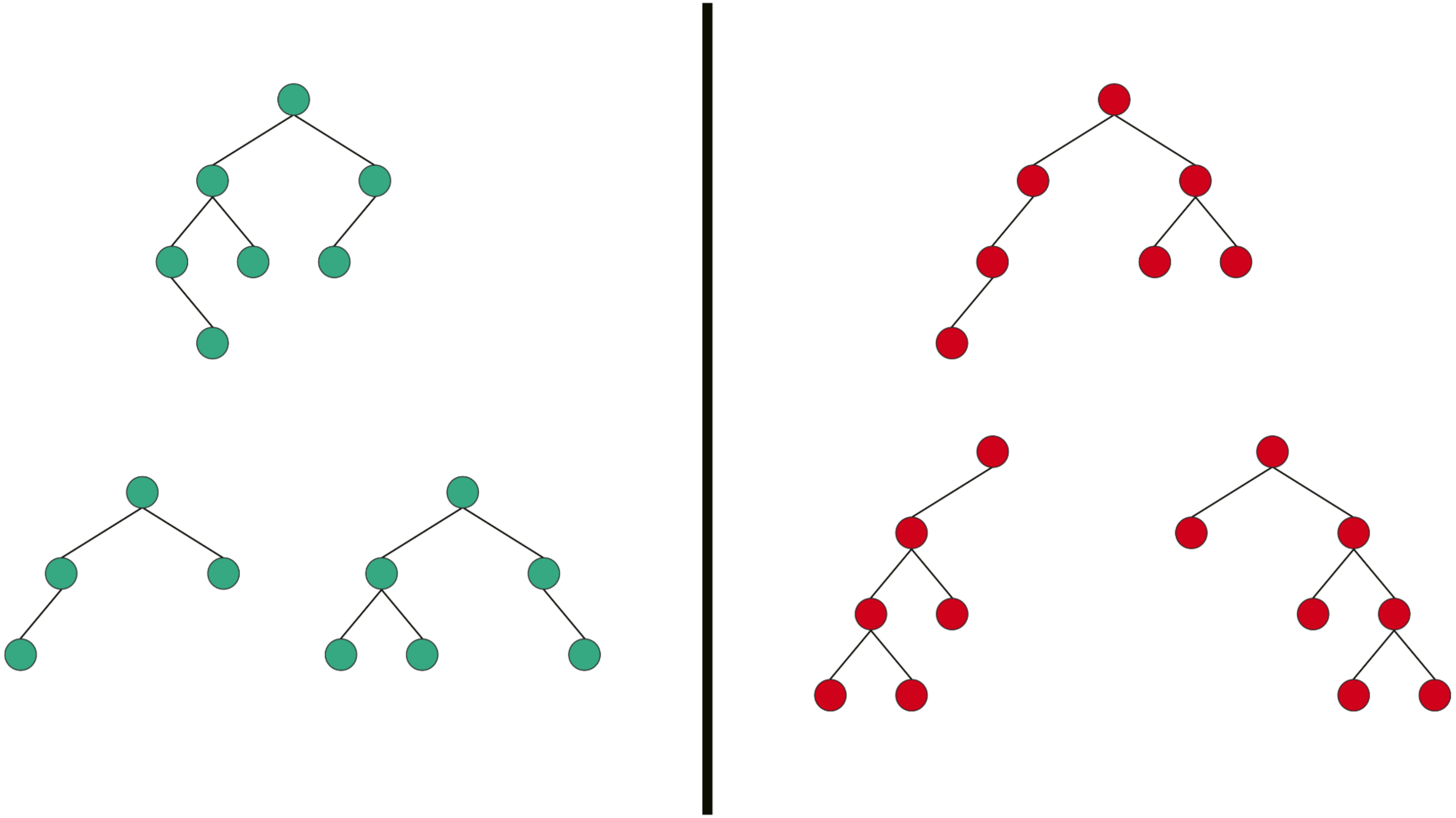
Height h is the largest integer less than or equal to $\log_2(n)$

Balanced Binary Trees

Balanced Binary tree

- is a Binary tree in which height of the left and the right sub-trees of every node may differ by at most 1

Balanced Binary Trees



Valid and Invalid Structure of Balanced Binary Tree

Reading Materials

- Nell Dale Chapter#8
- Schaum's Outlines Chapter#7
- D. S. Malik Chapter#11