



# **CS-218**

## **DATA STRUCTURE**

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STACKS

# Stack

3

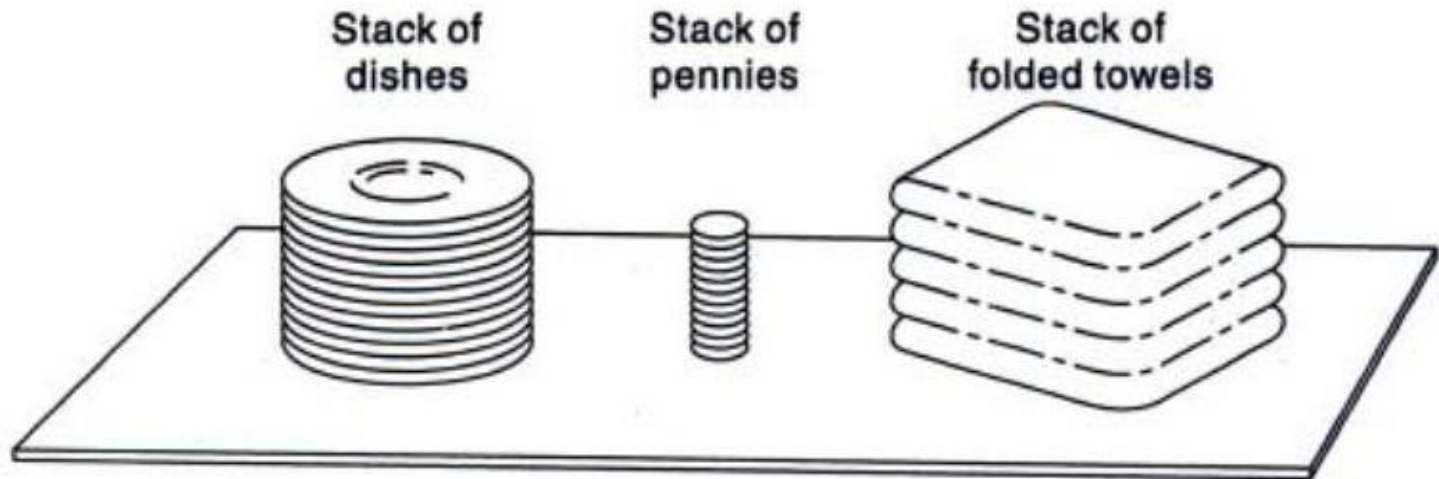
- “A ***Stack*** is a special kind of list in which all insertions and deletions take place at one end, called the ***Top***”
- Other Names
  - Pushdown List
  - Last In First Out (LIFO)

# Stack

4

Examples:

- Folded towels on shelf
- Dishes on a shelf
- Pennies on shelf



# Common Operations

5

1. **MAKENULL(S)**: Make Stack  $S$  be an empty stack.
2. **TOP(S)**: Return the element at the top of stack  $S$ .
3. **POP(S)**: Remove the top element of the stack.
4. **PUSH(S)**: Insert the element  $x$  at the top of the stack.
5. **ISEMPTY(S)**: Return true if  $S$  is an empty stack; return false otherwise.

# Static and Dynamic Stacks

6

- There are two kinds of stack data structure,
  - a) **Static**, i.e. they have a **fixed size**, and are *implemented as arrays*.
  - b) **Dynamic**, i.e. they **grow in size** as needed, and *implemented as linked lists*

# APPLICATION OF STACKS

# Application of Stacks

8

- Tower of Hanoi
- Expressions
  - ▣ Infix:  $A+B-C$
  - ▣ Postfix:  $AB+C-$
  - ▣ Prefix:  $-+ABC$
- Recursion



# Tower of Hanoi

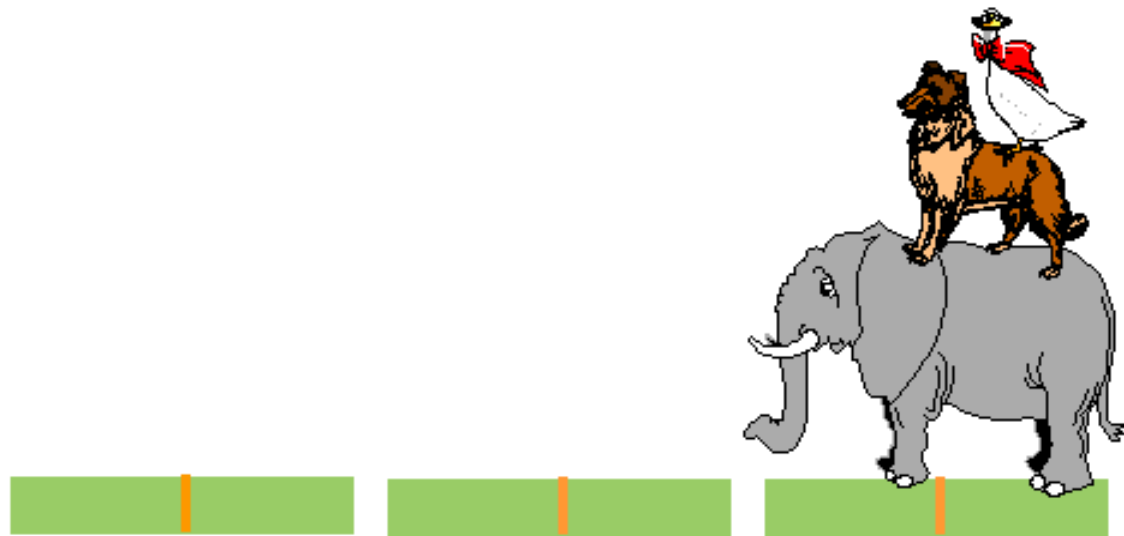
9

- GIVEN: Three poles
  - a set of discs on the first pole,
  - discs of different sizes,
  - the smallest discs at the top
- GOAL: move all the discs from the left pole to the right one.
- CONDITIONS: only one disc may be moved at a time.
  - A disc can be placed either on an empty pole or on top of a larger disc.



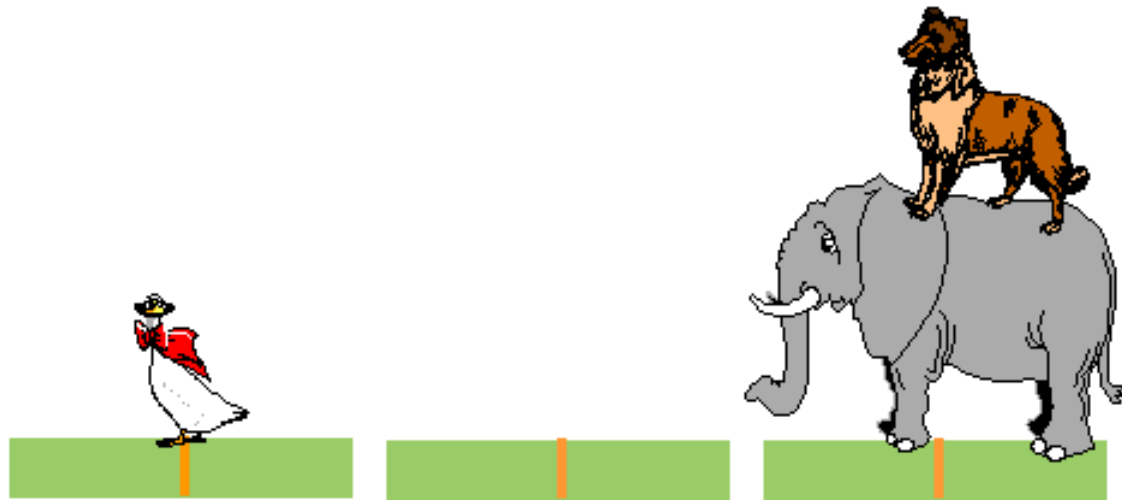
# Tower of Hanoi

10



# Tower of Hanoi

11



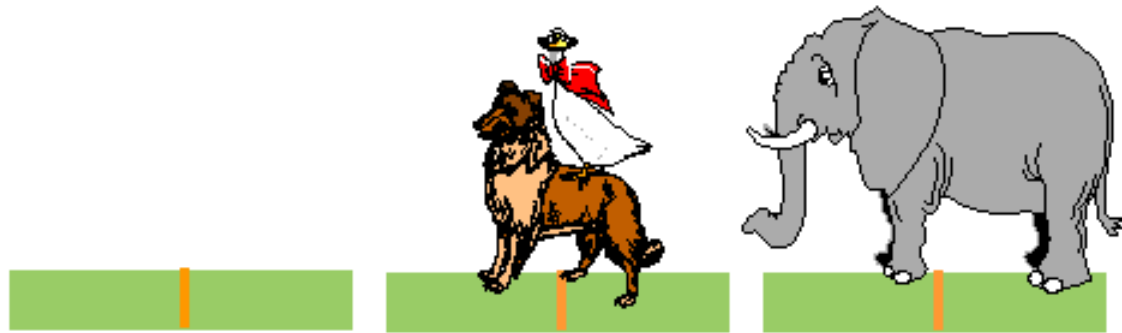
# Tower of Hanoi

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# Tower of Hanoi

13



# Tower of Hanoi

14



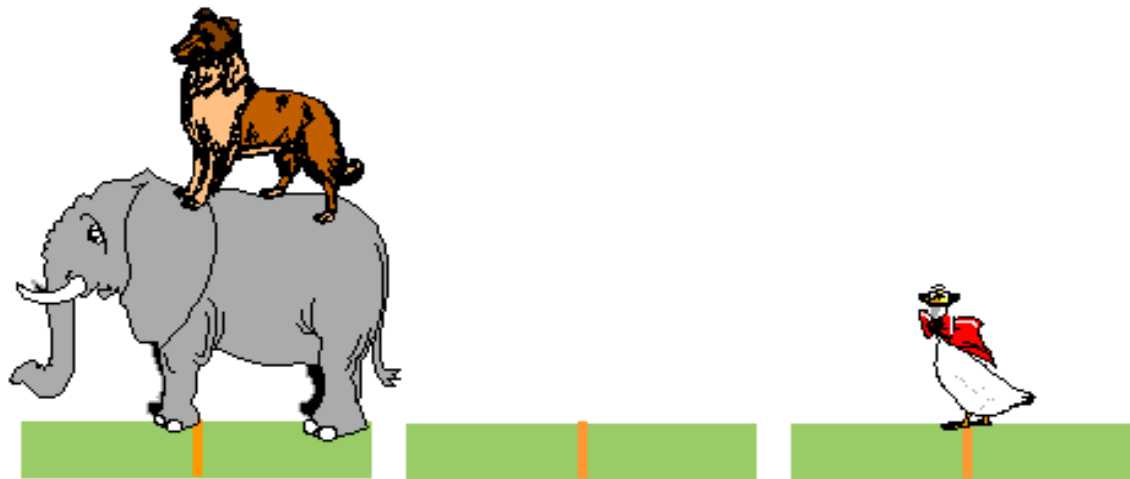
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15



# Tower of Hanoi

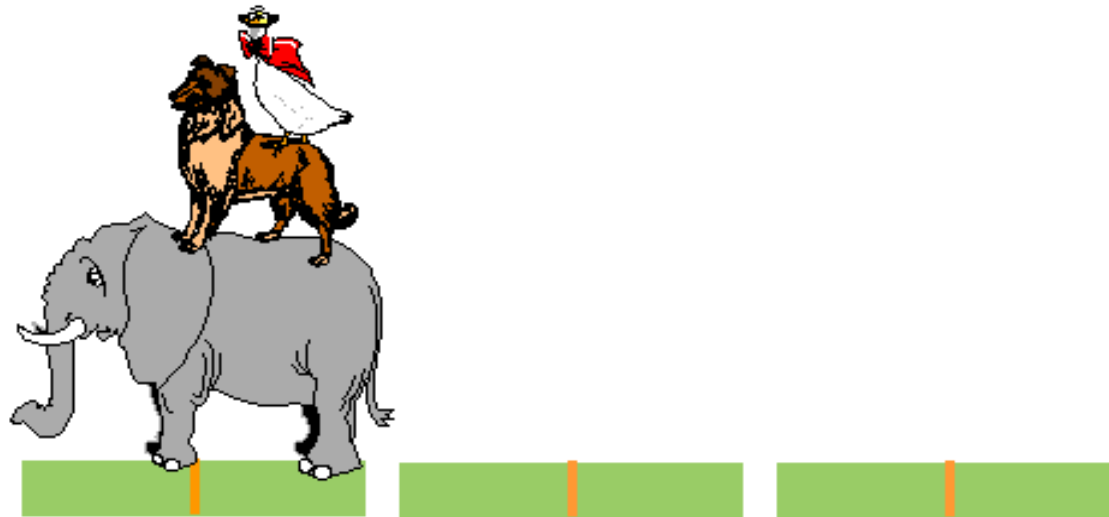
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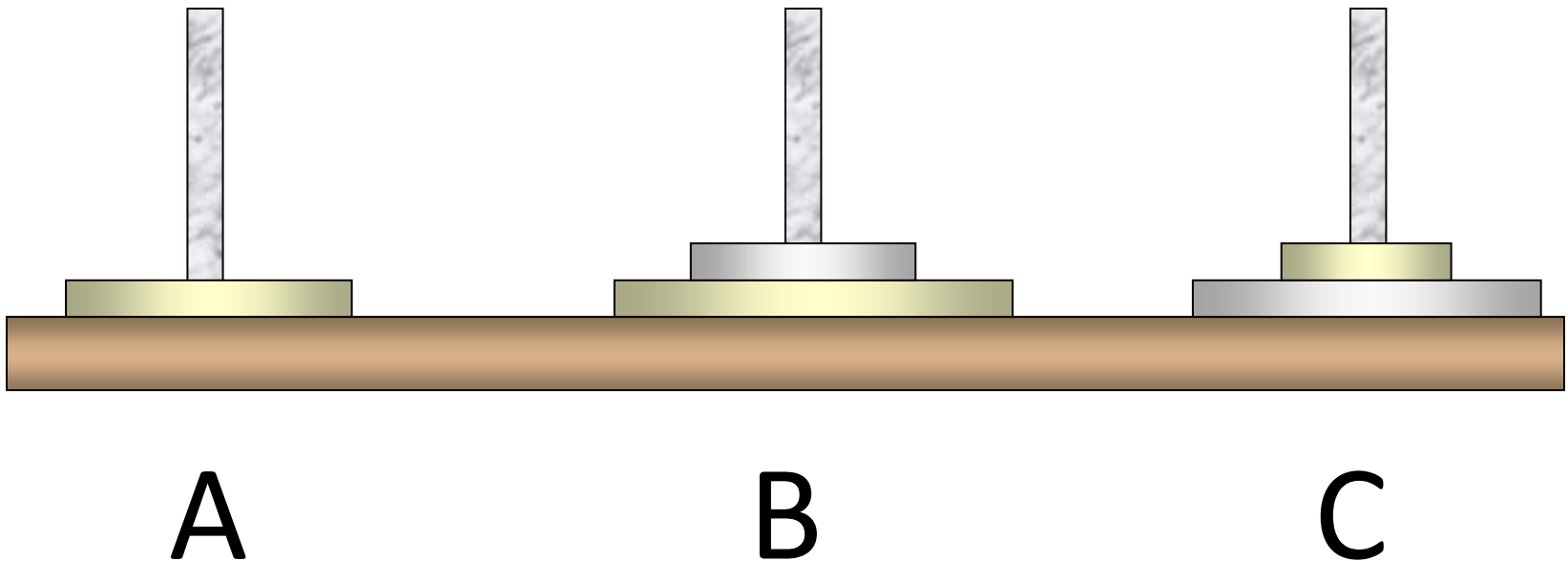
# Tower of Hanoi

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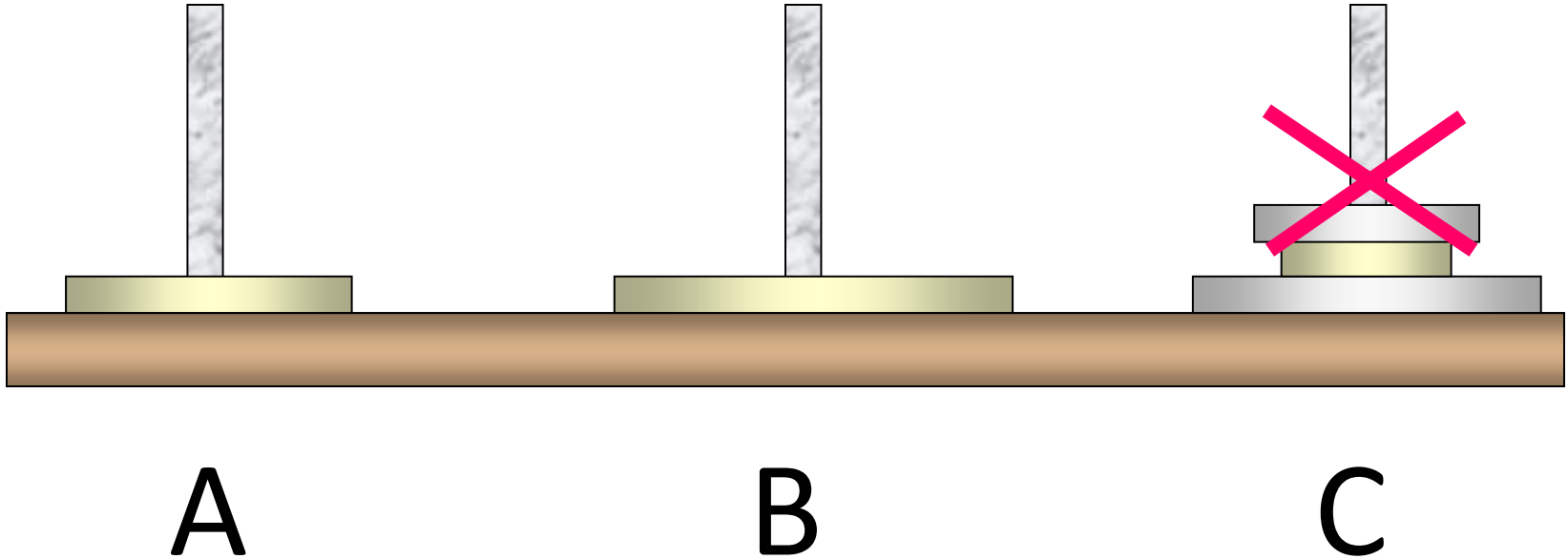
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5 Tower

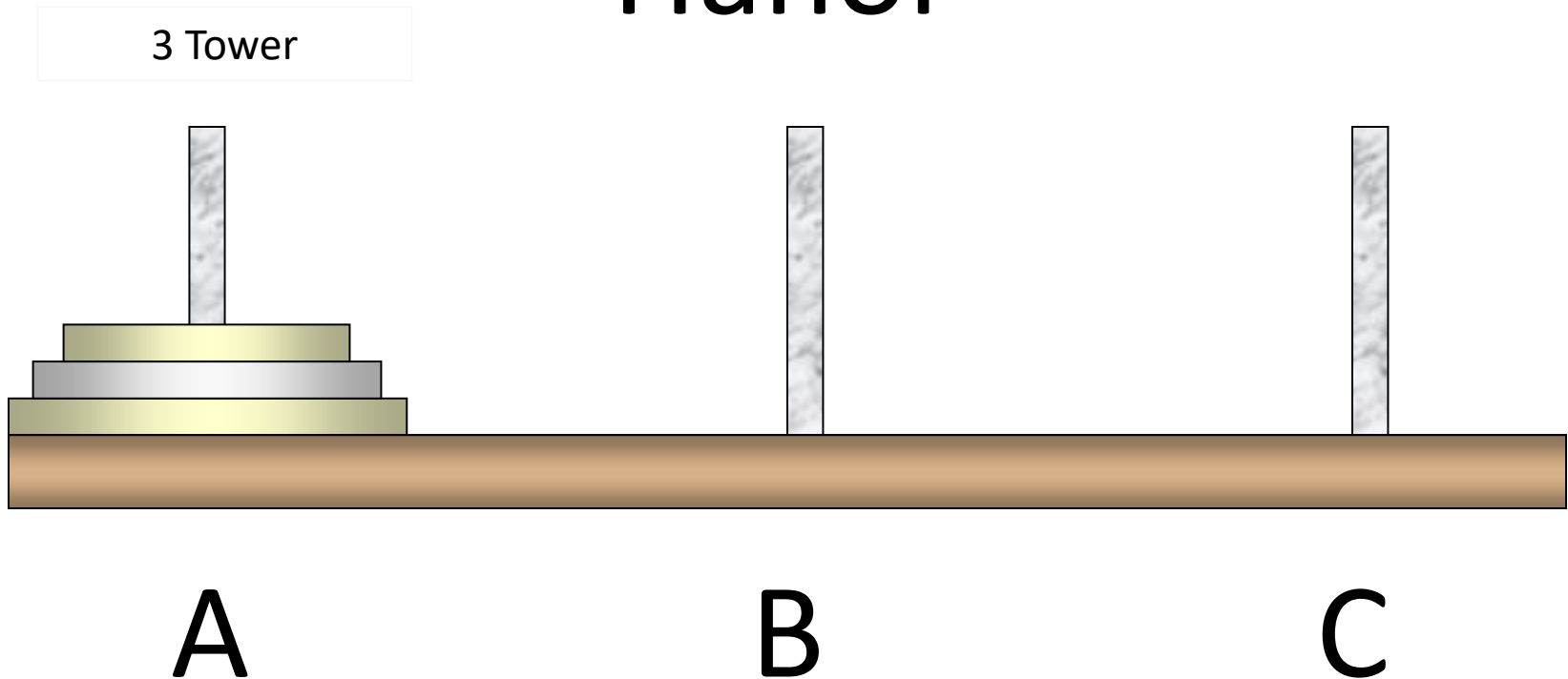


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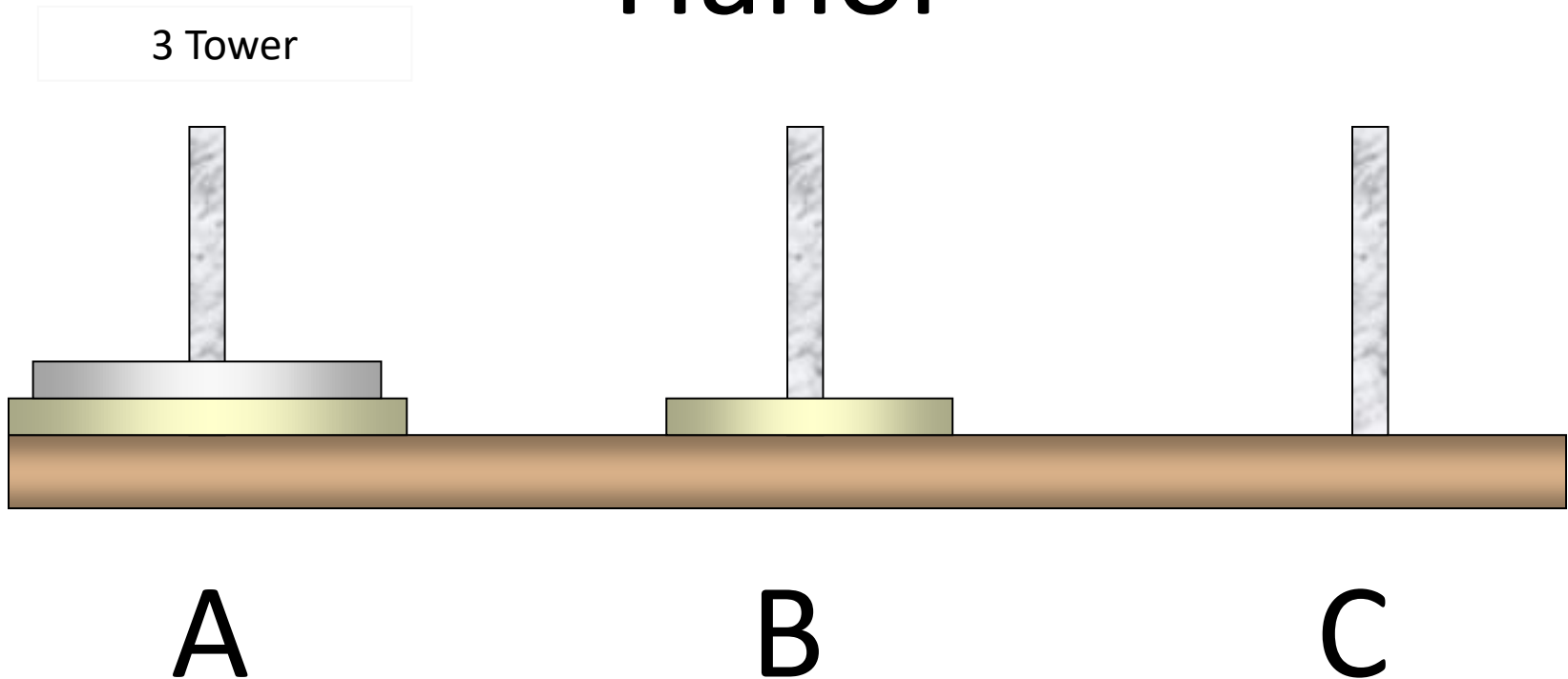
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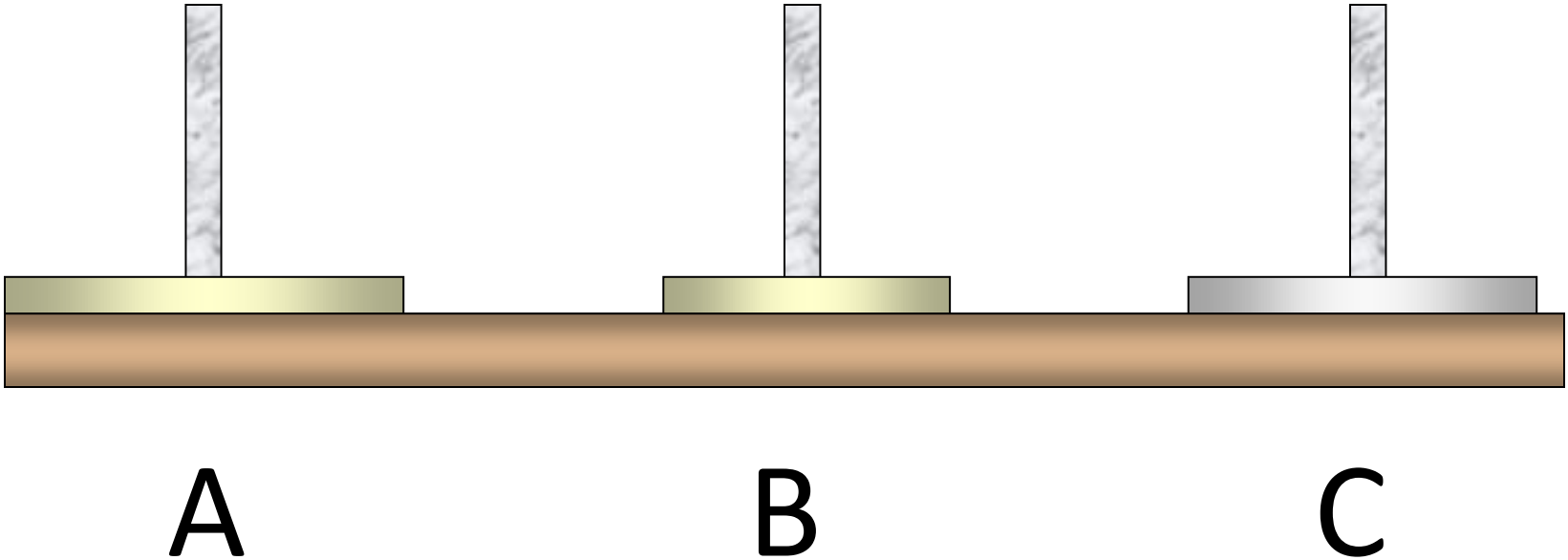


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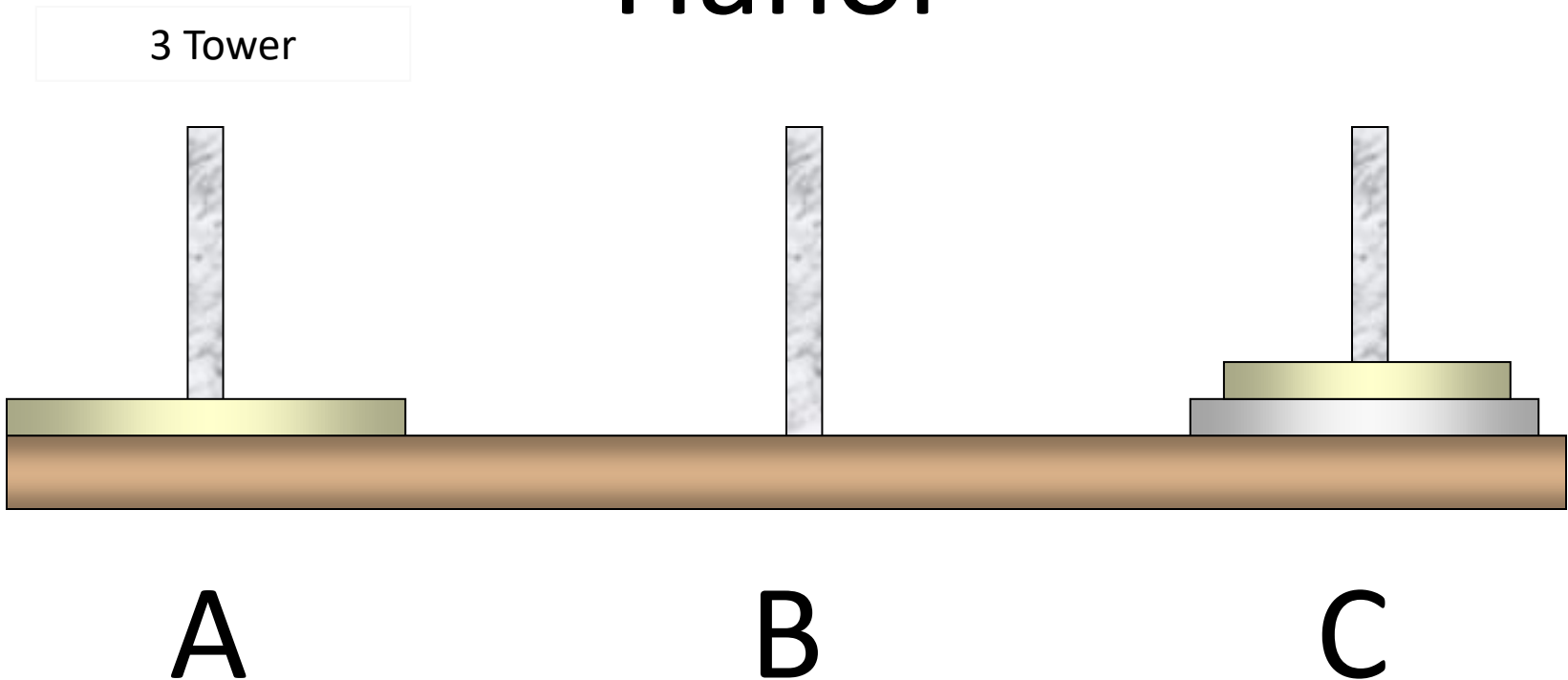


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3 Tower

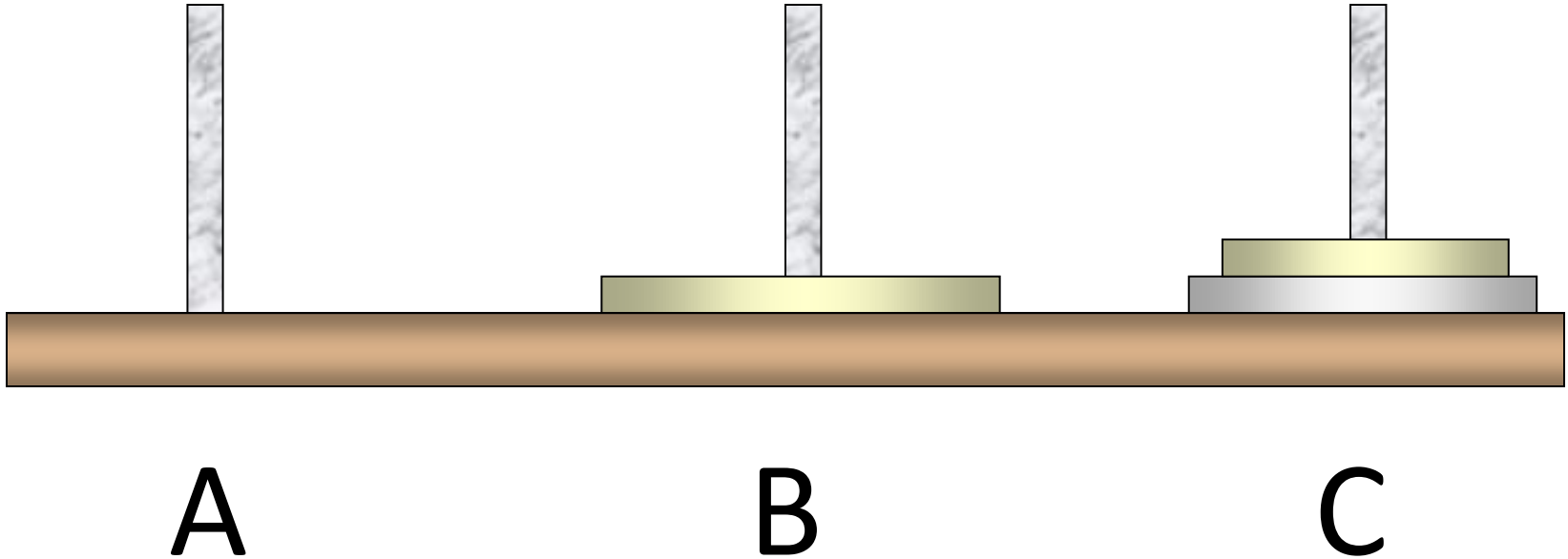


# The Tower of Hanoi



# The Tower of Hanoi

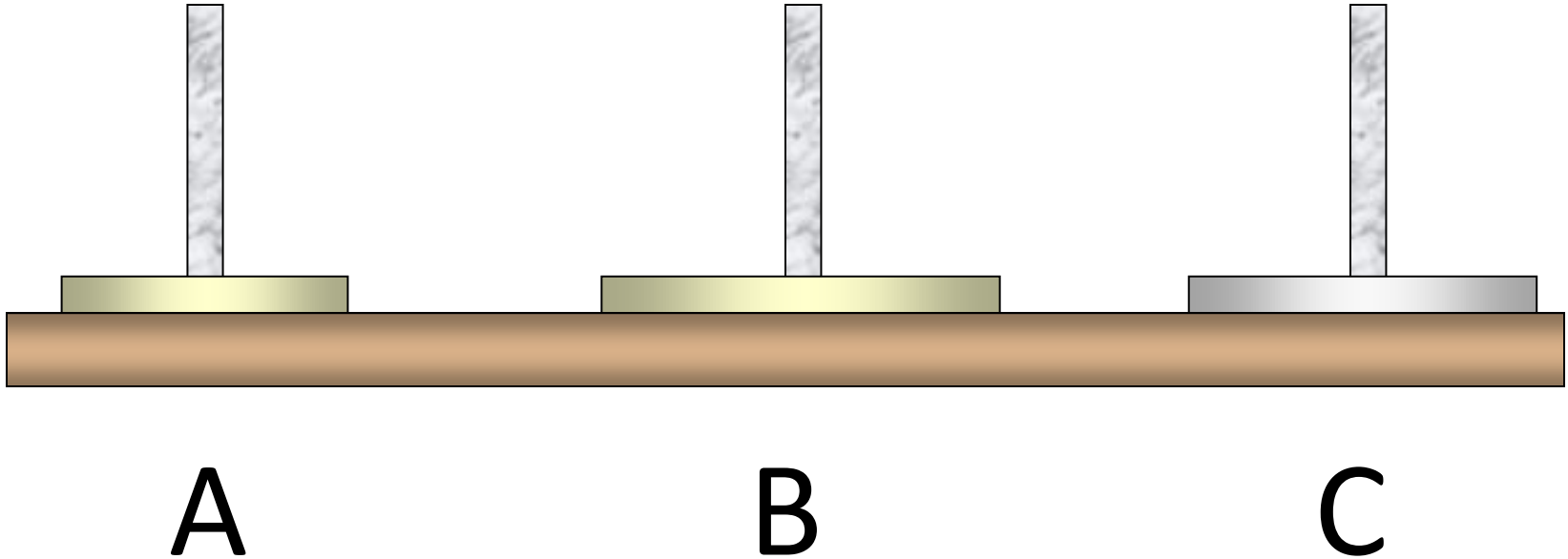
3 Tower





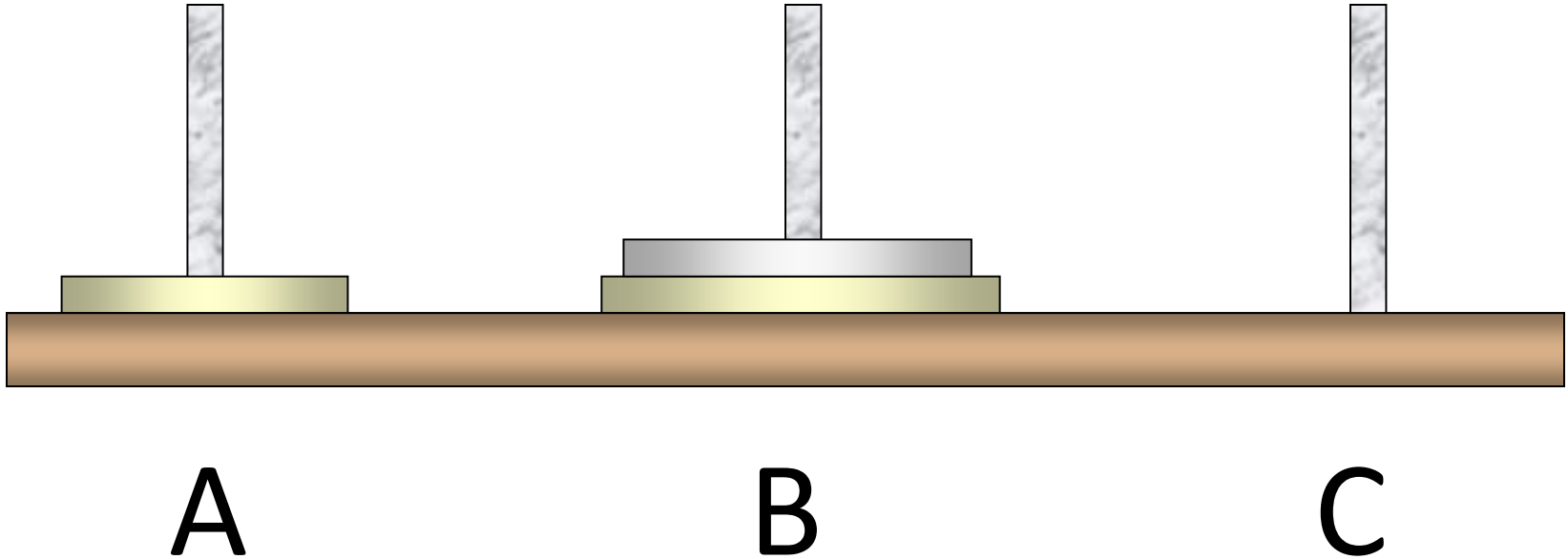
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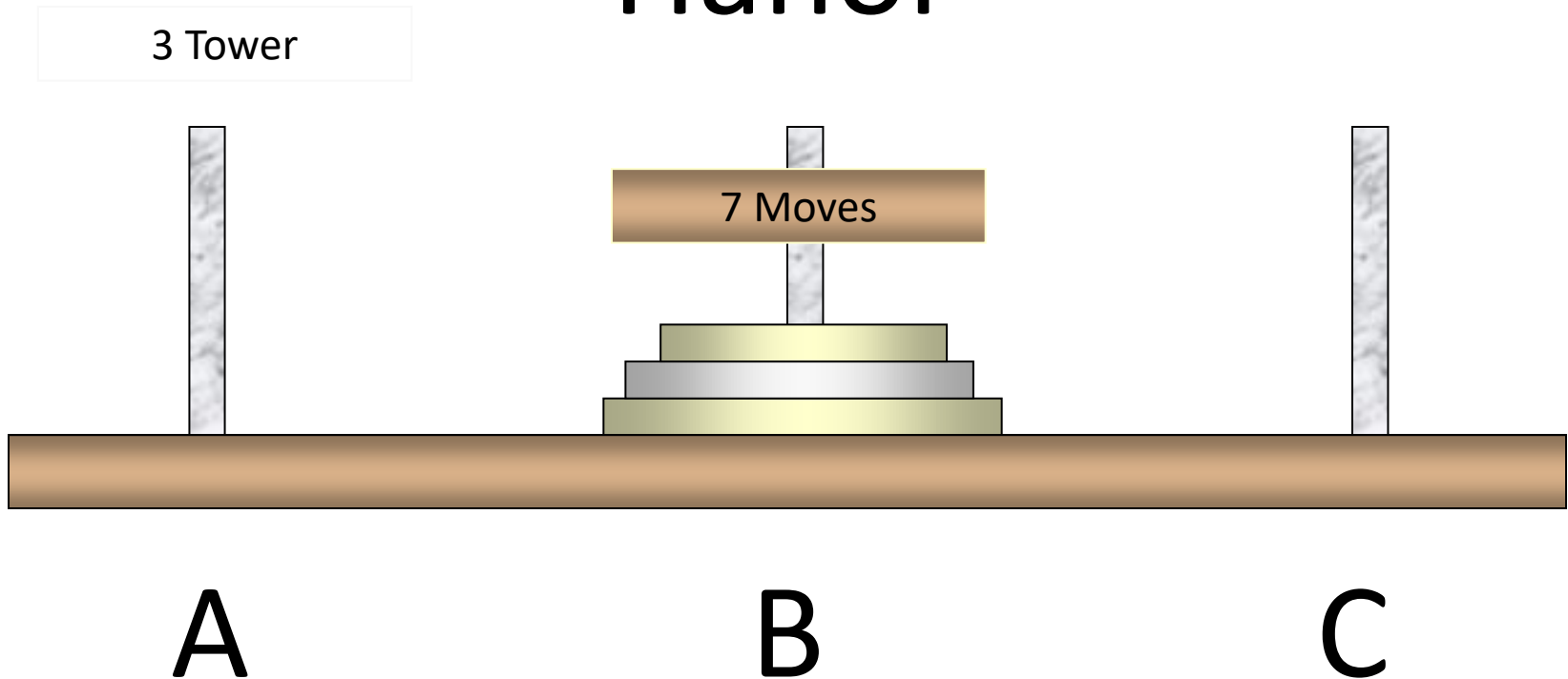


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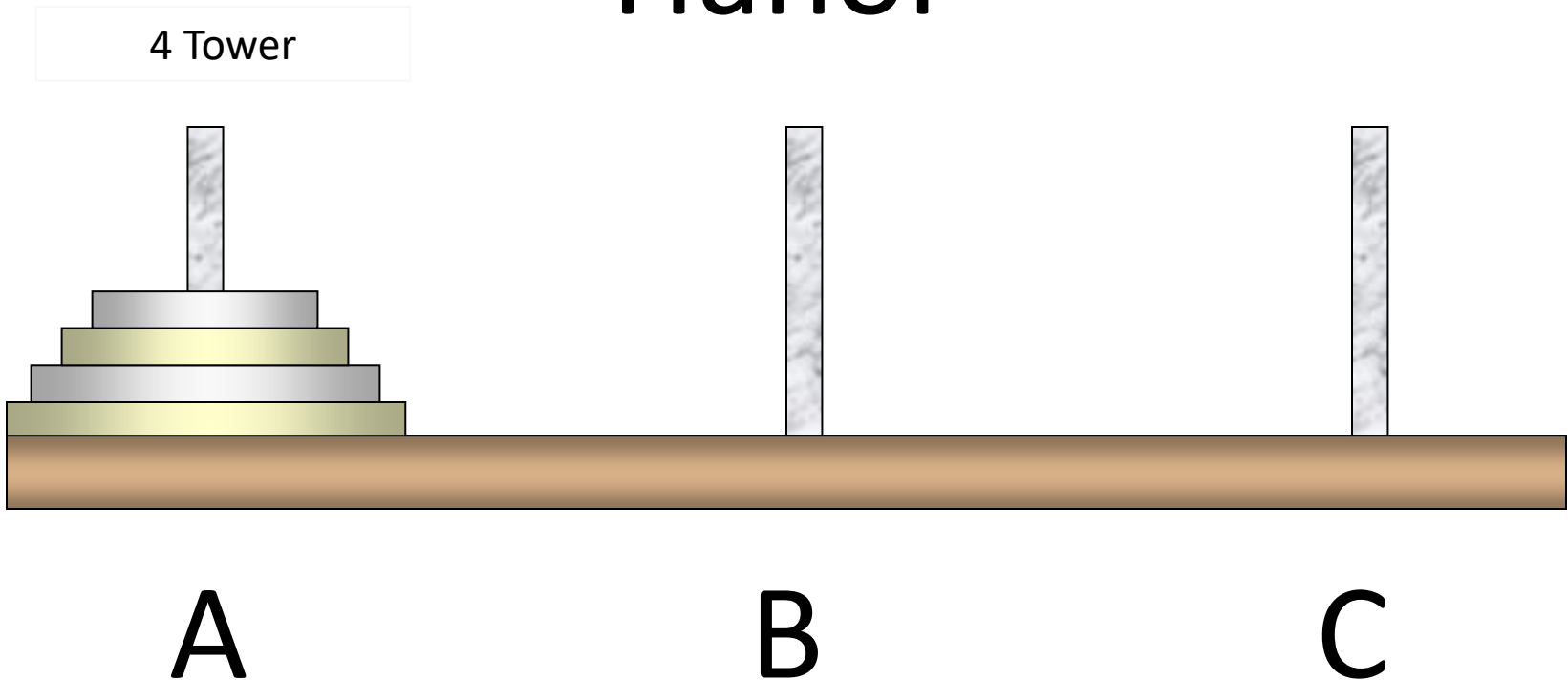
3 Tower



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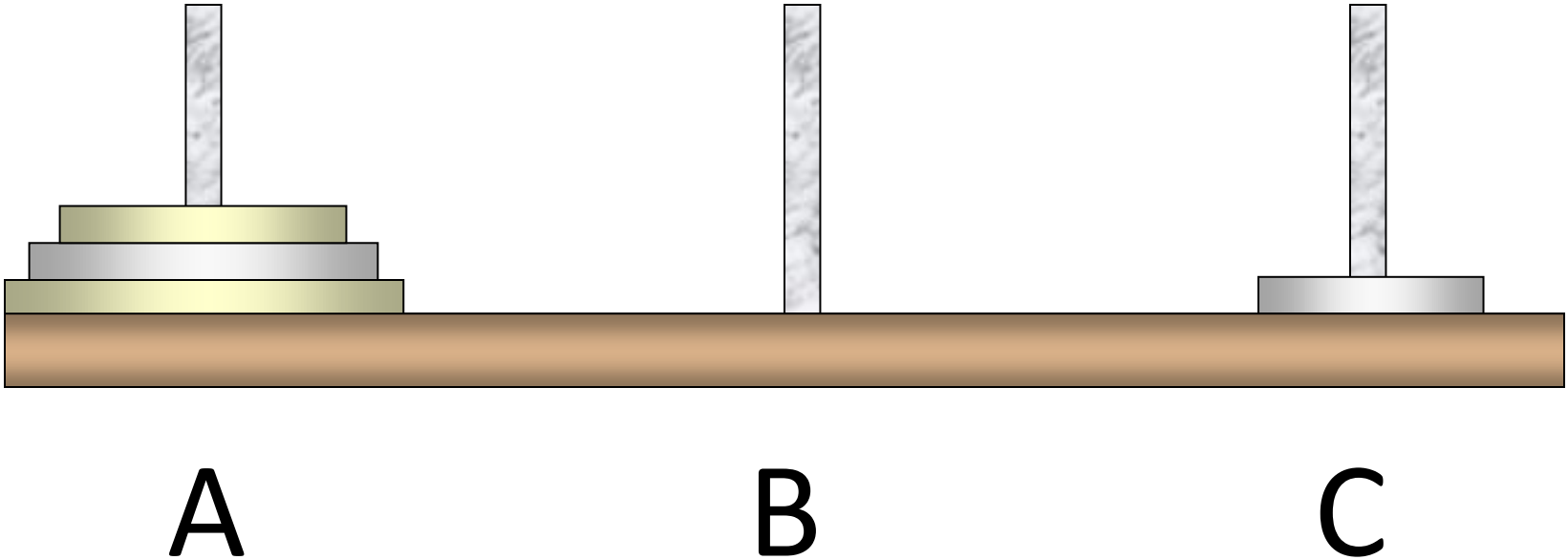


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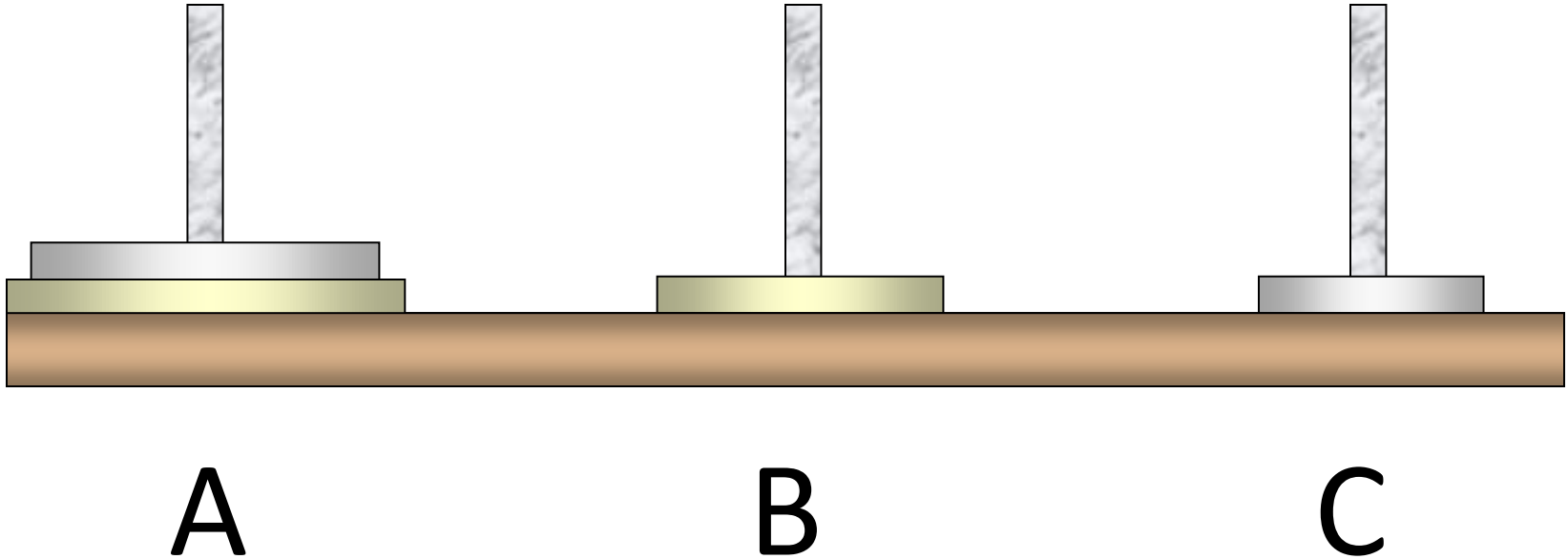
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4 Tower

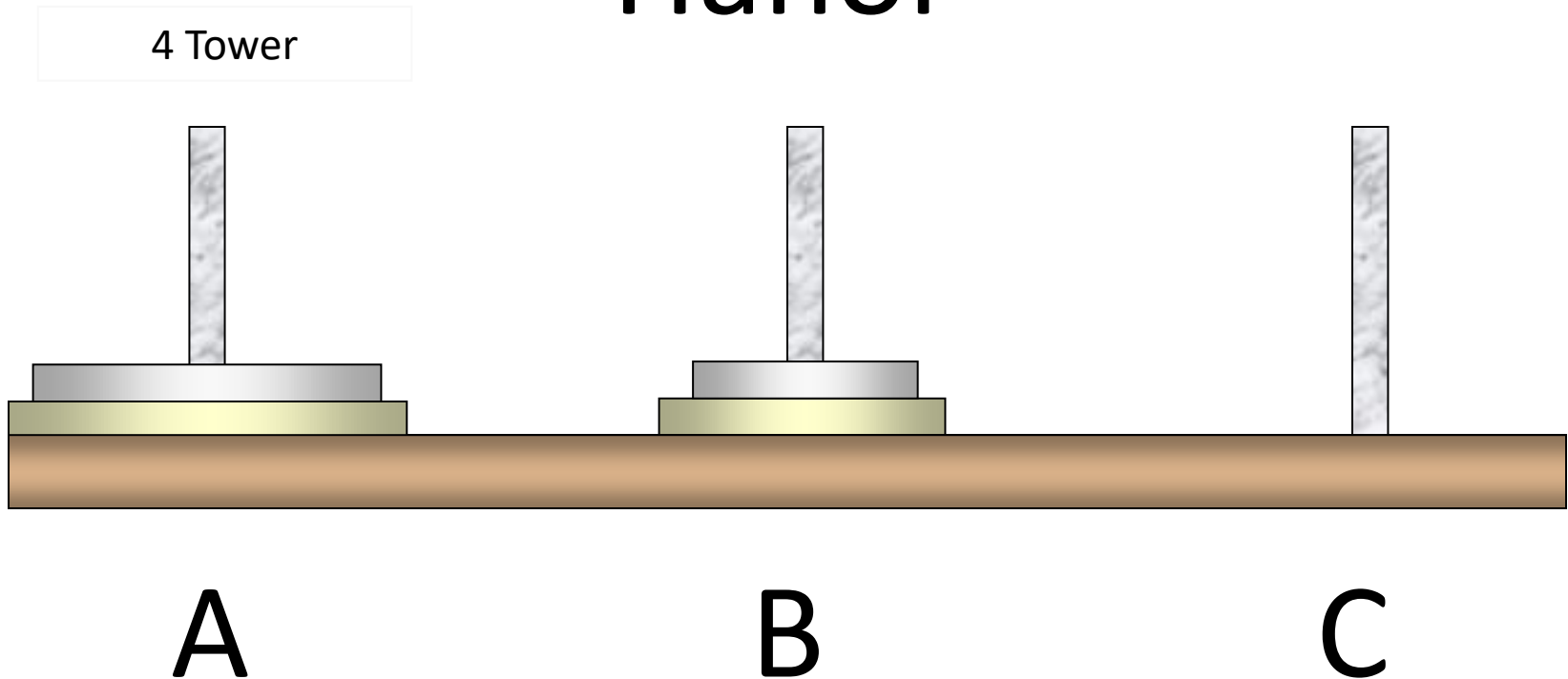


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4 Tower

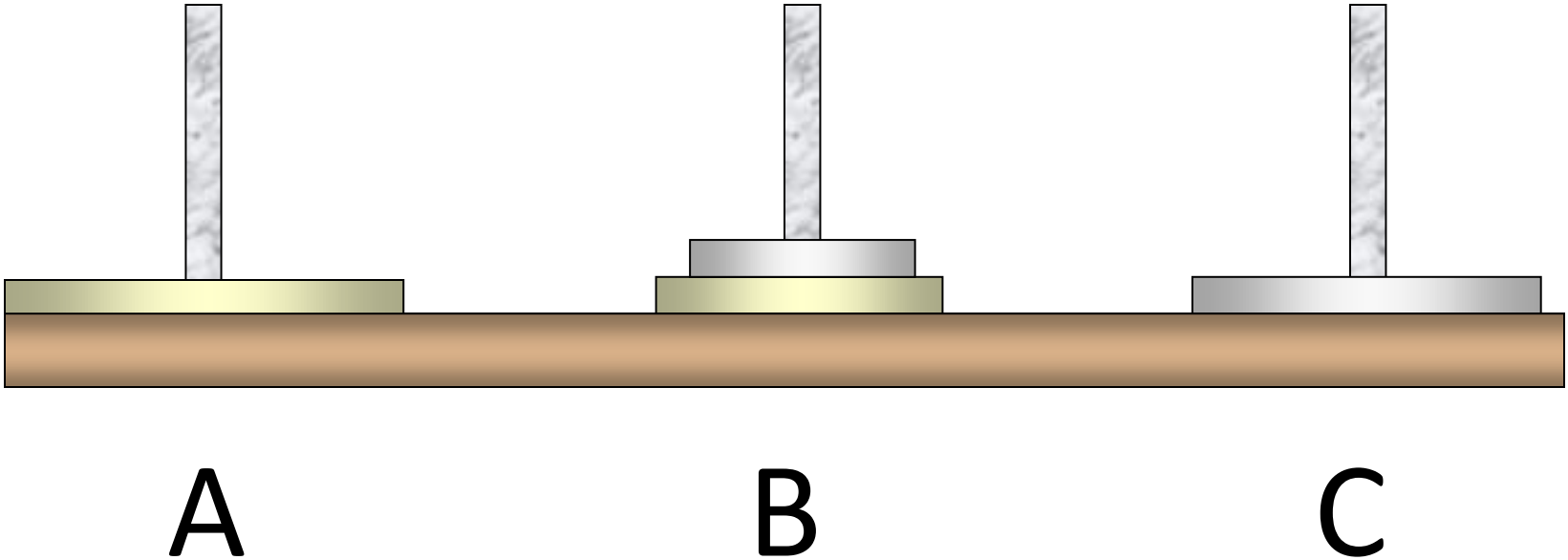


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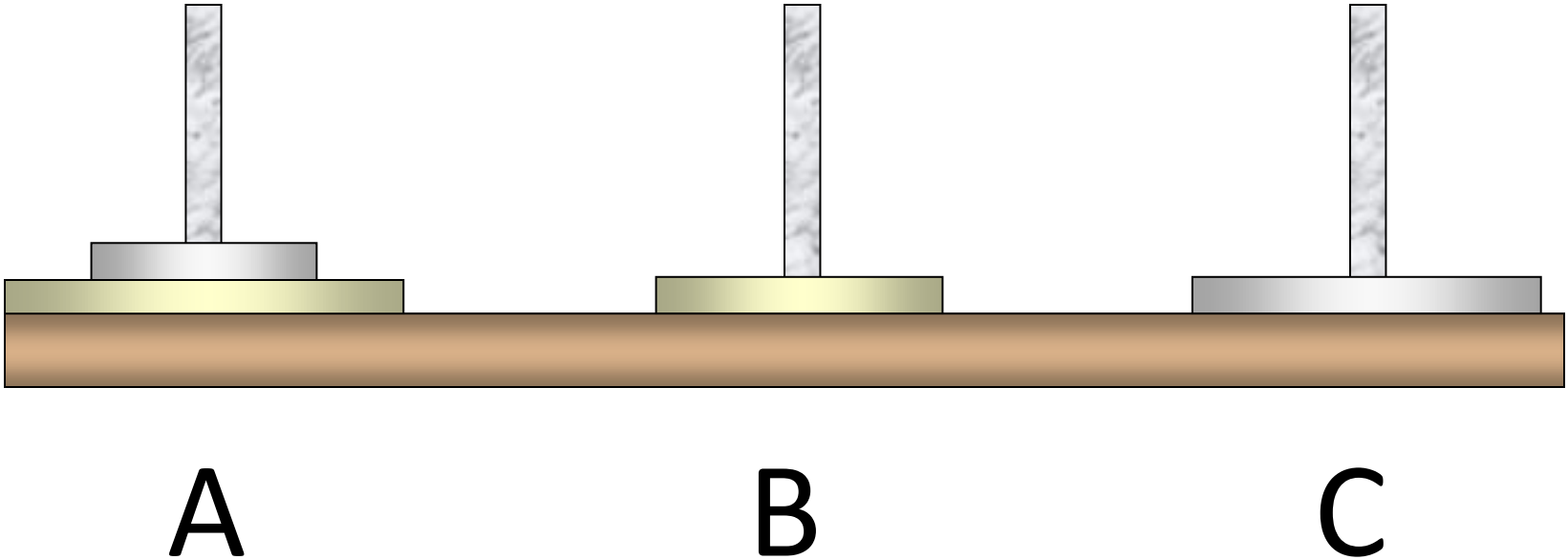
4 Tower





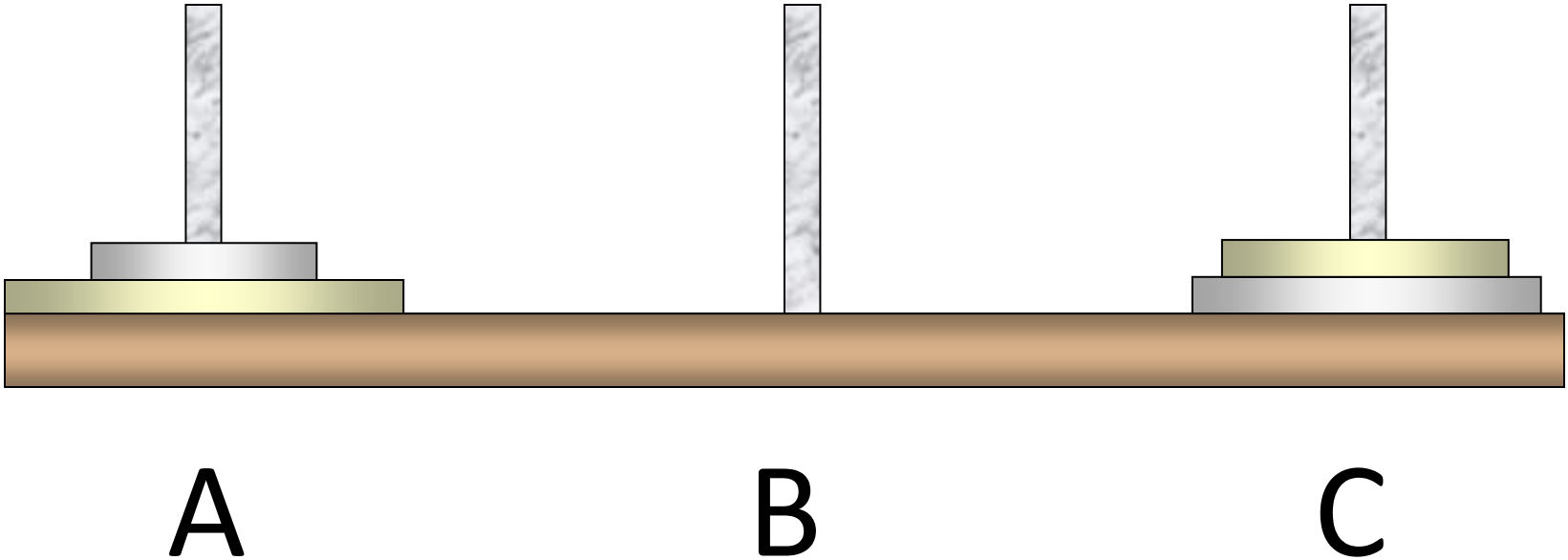
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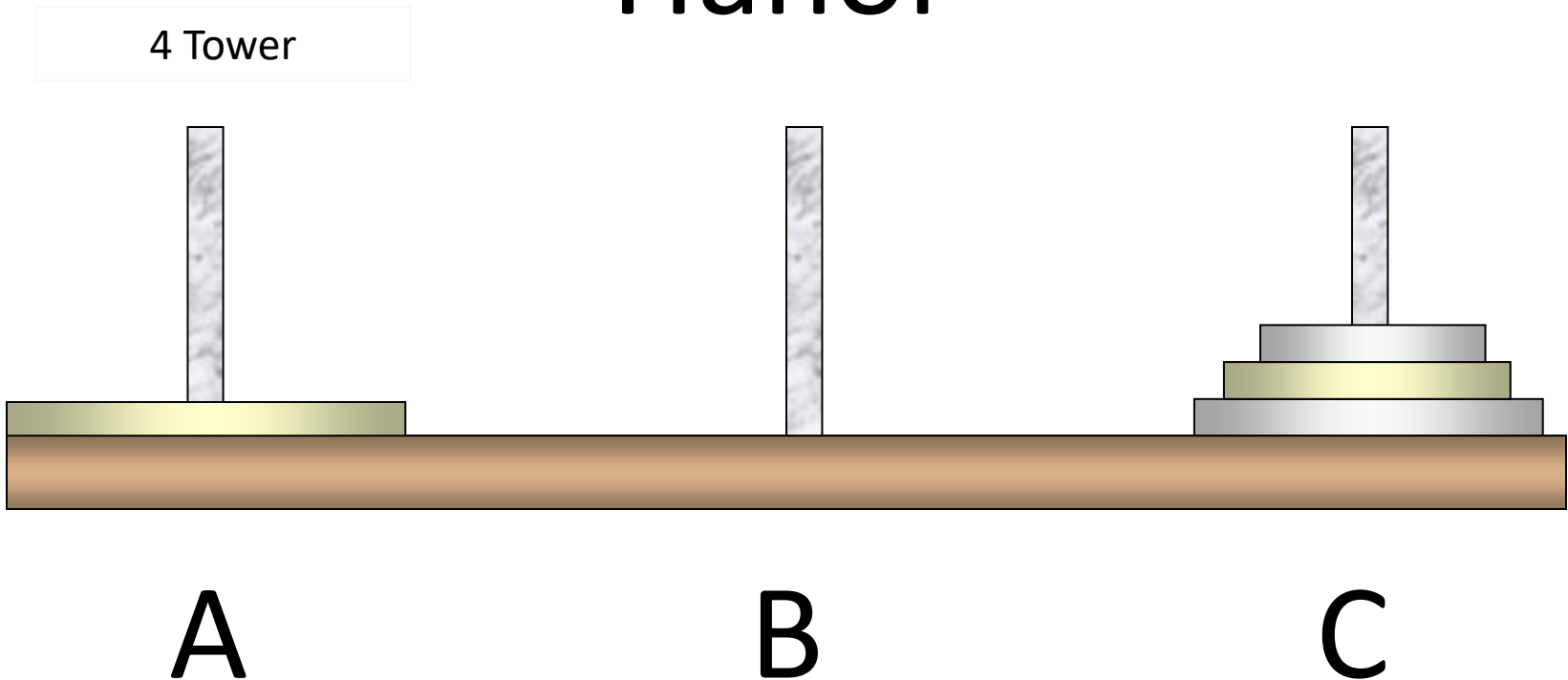


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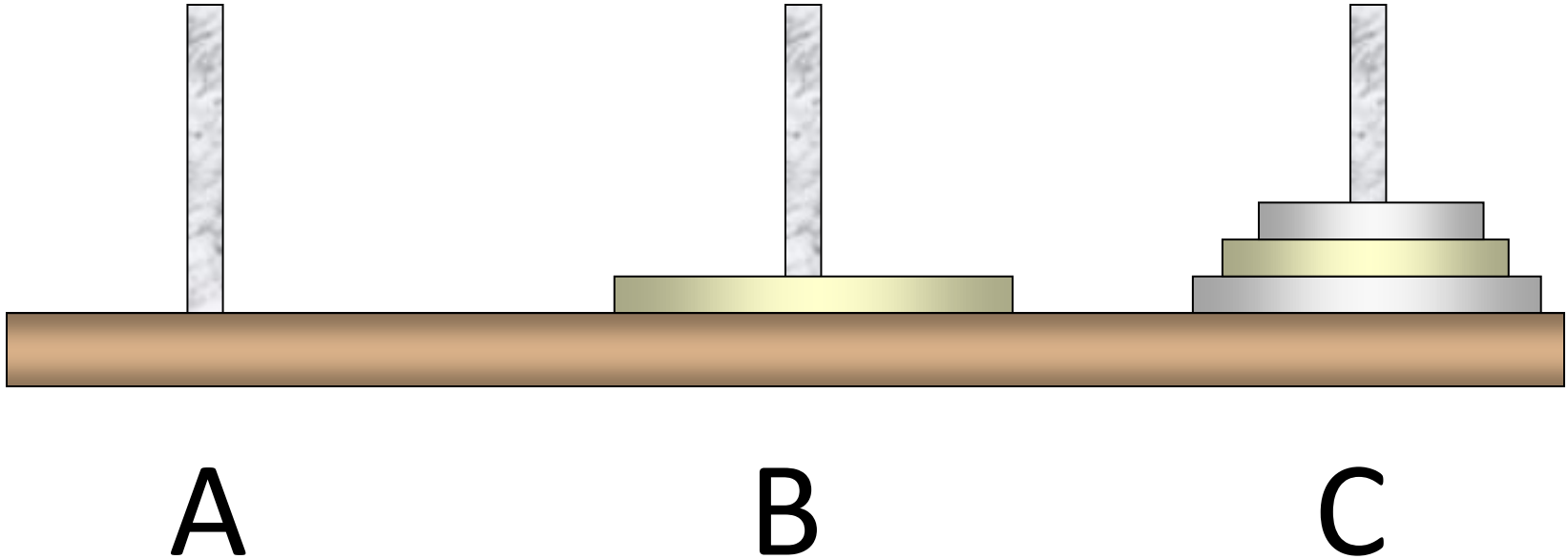


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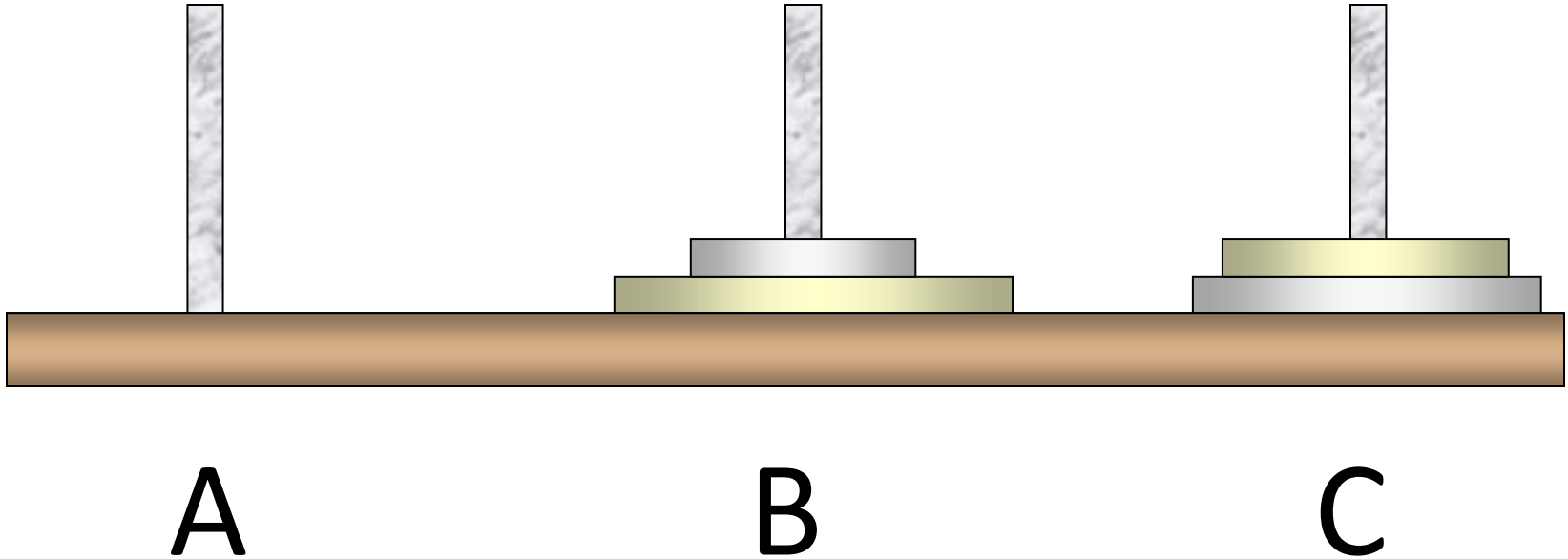
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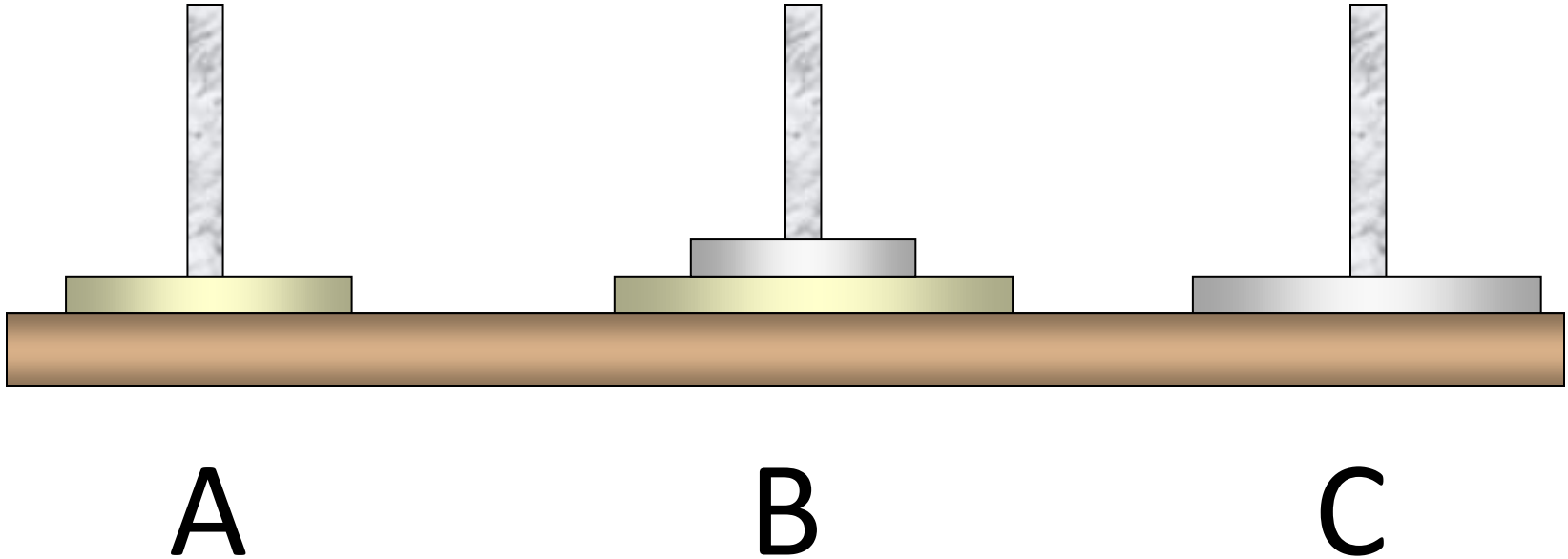
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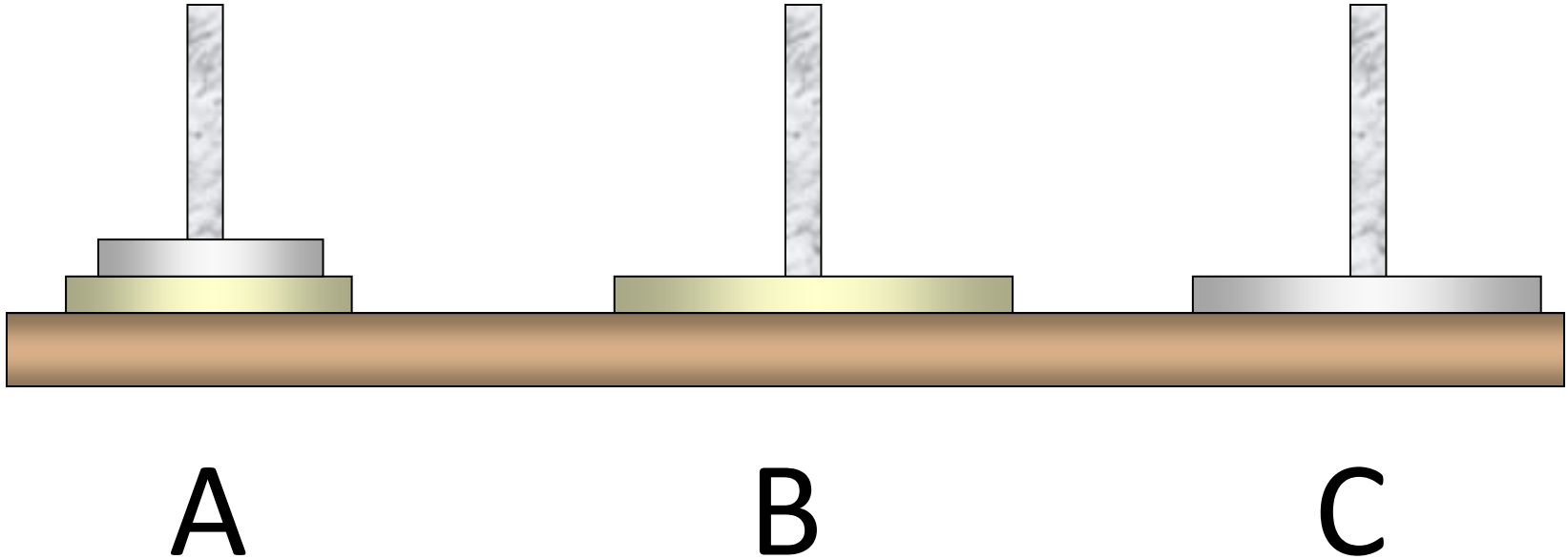
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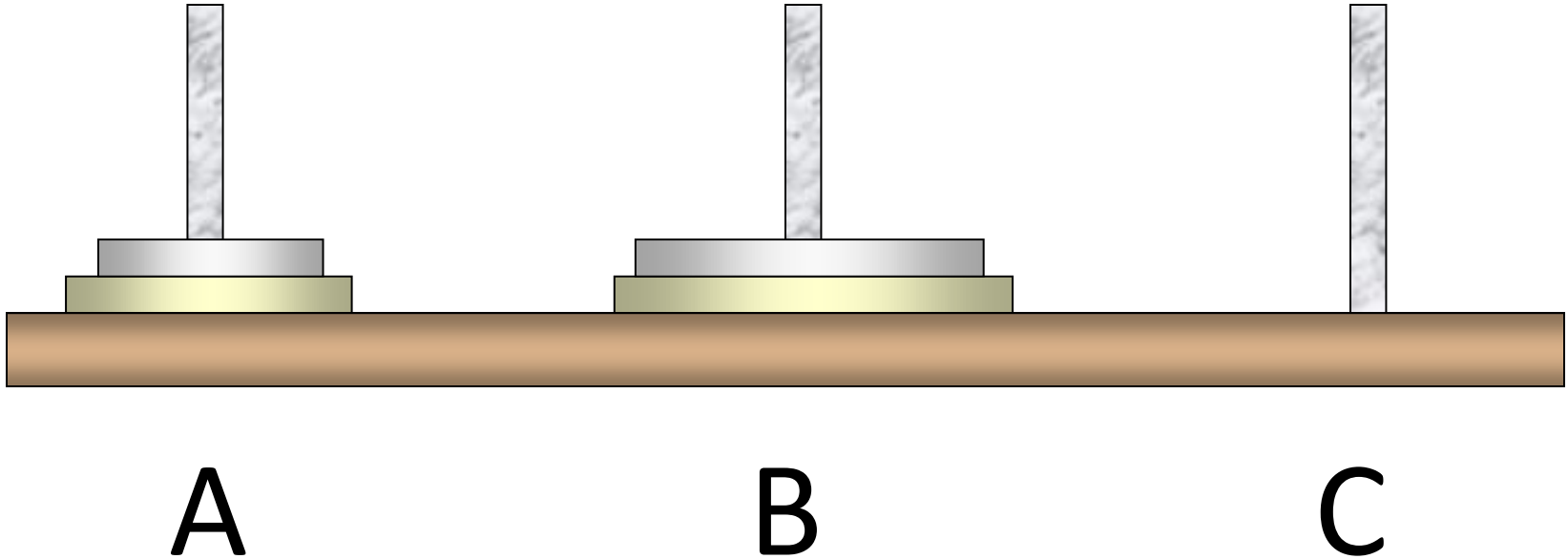
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4 Tower



# The Tower of Hanoi

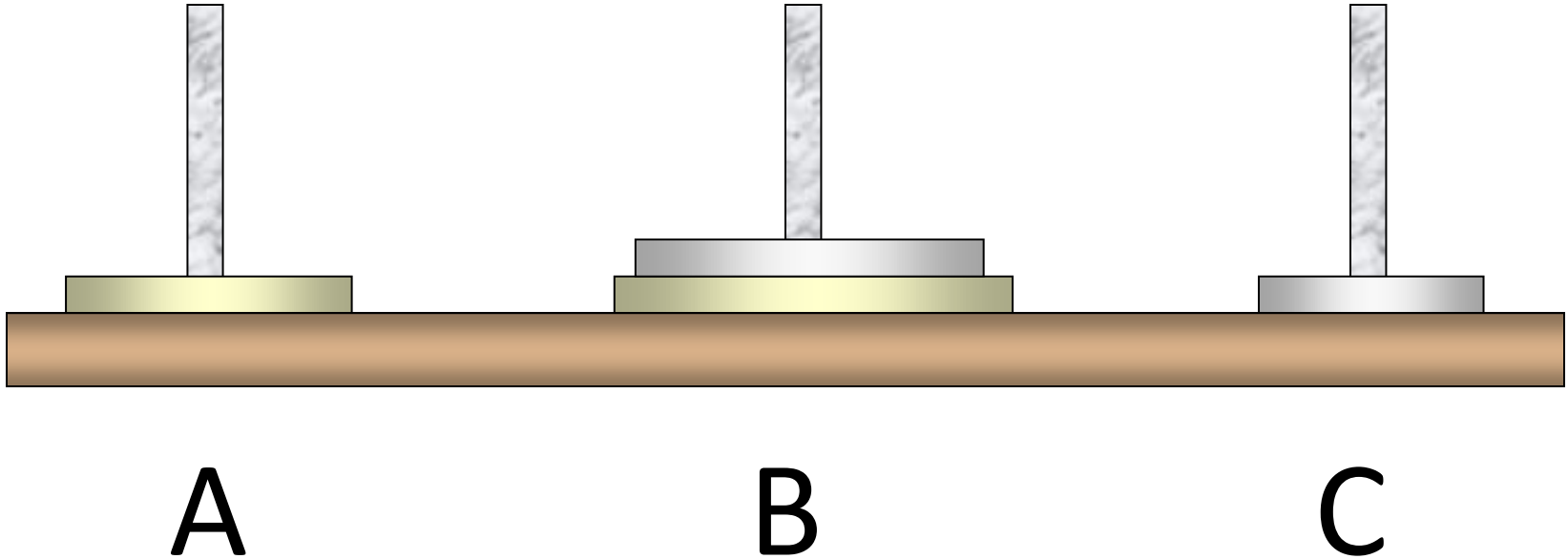
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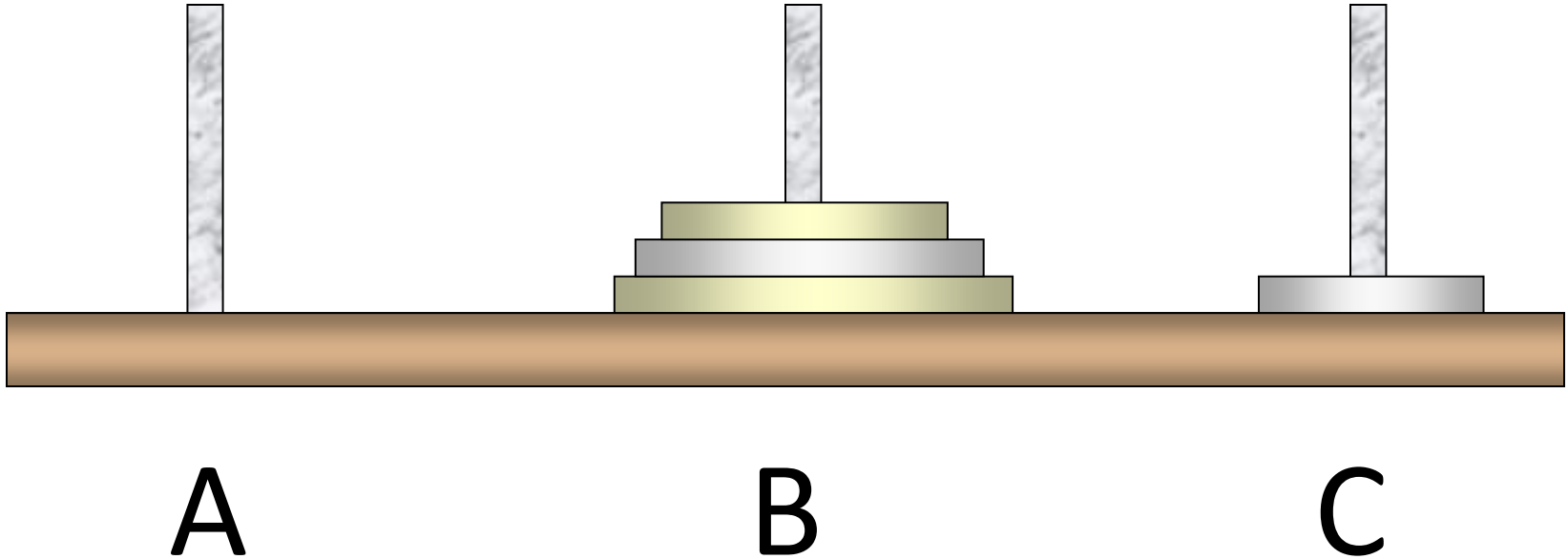
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4 Tower



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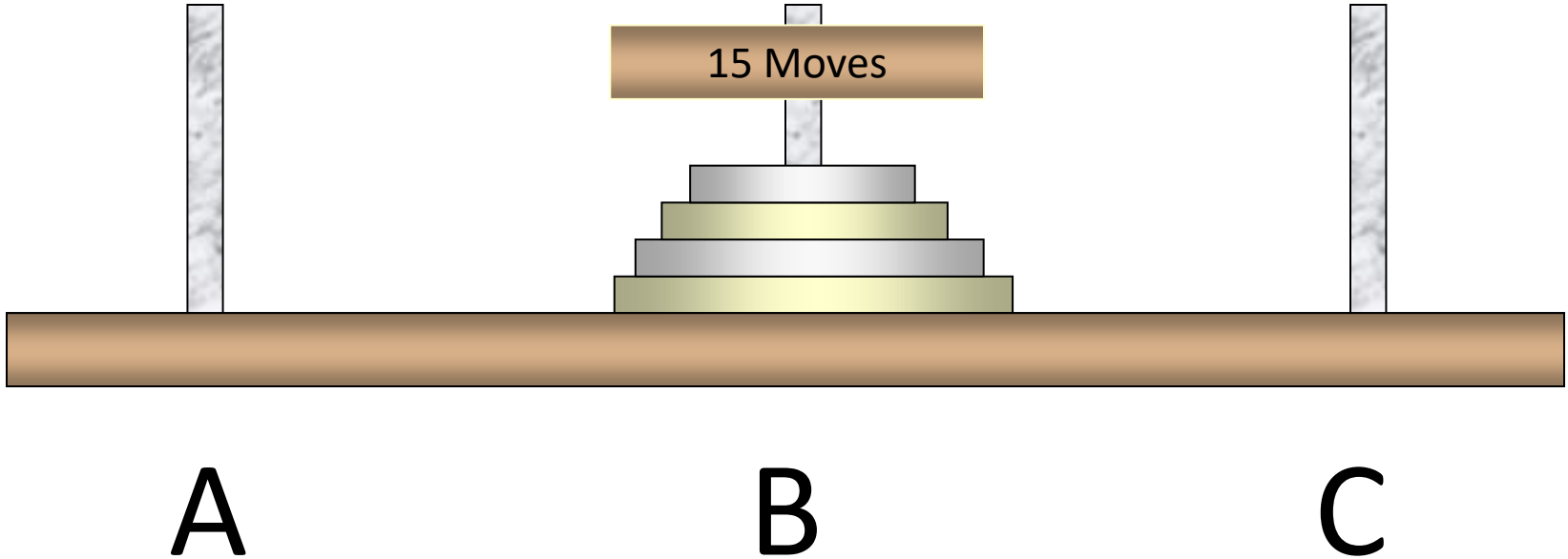
4 Tower



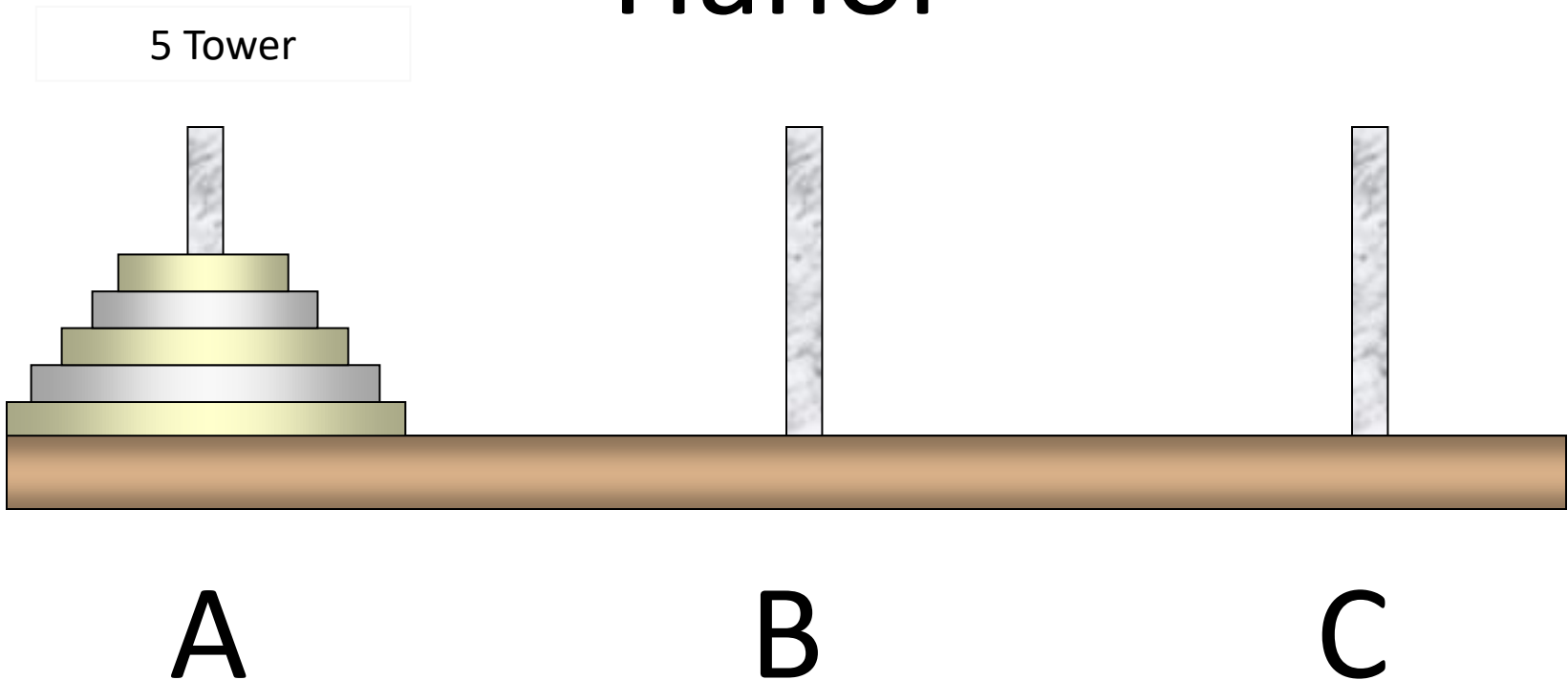
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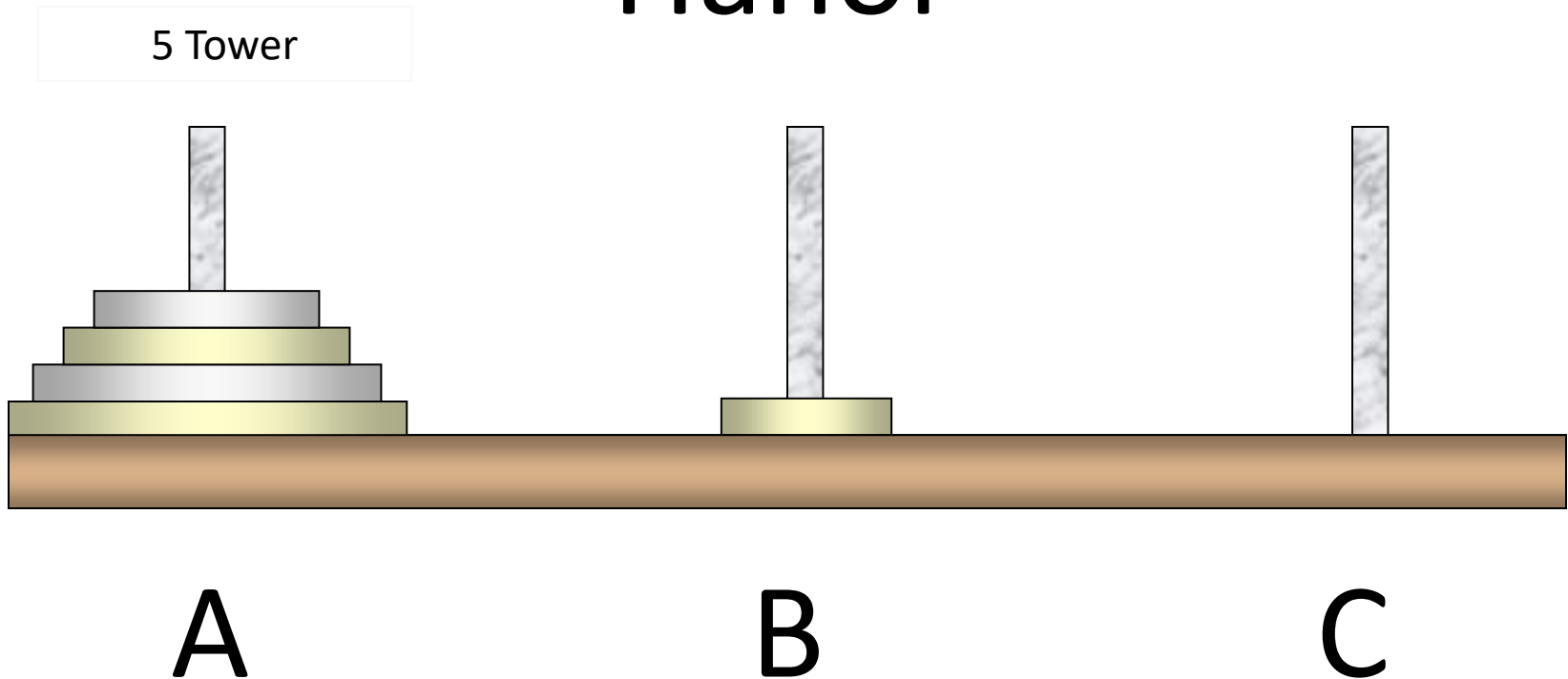
15 Moves



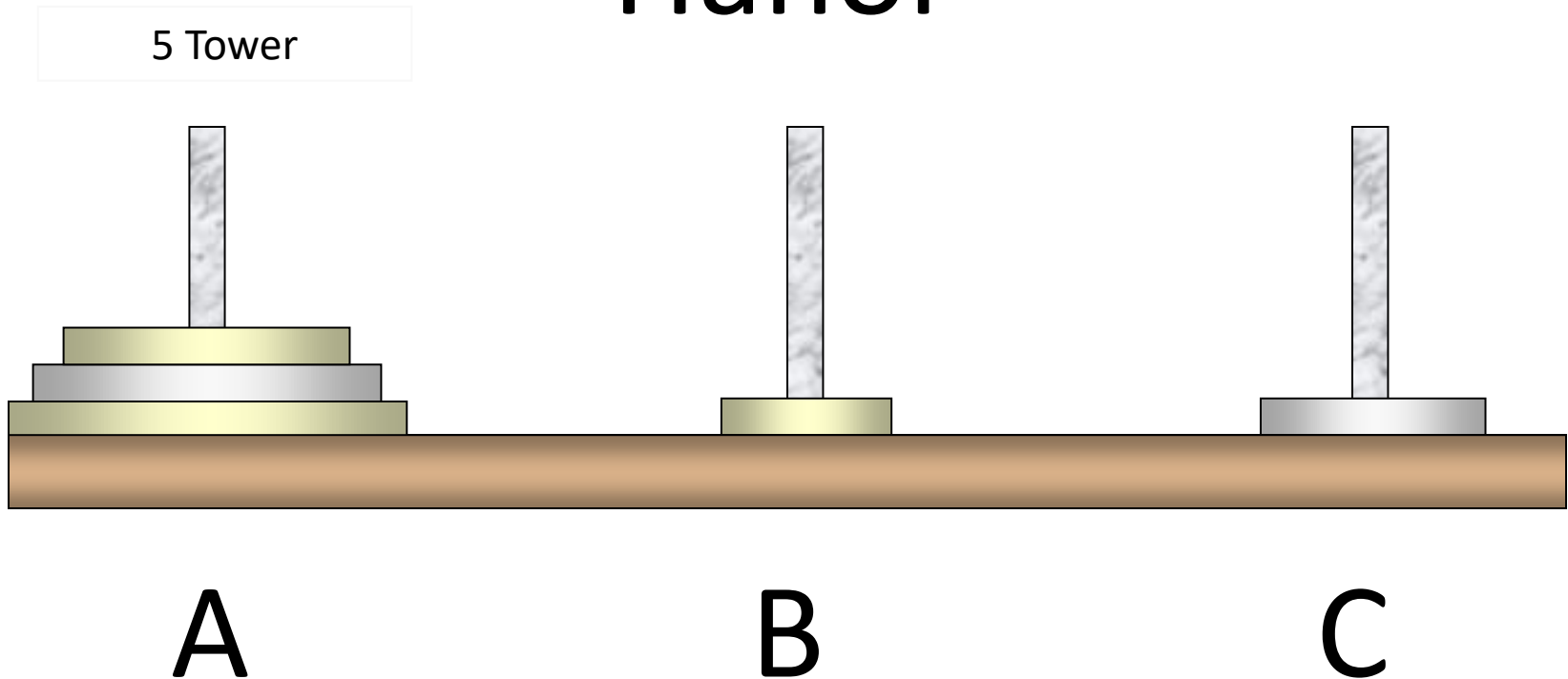
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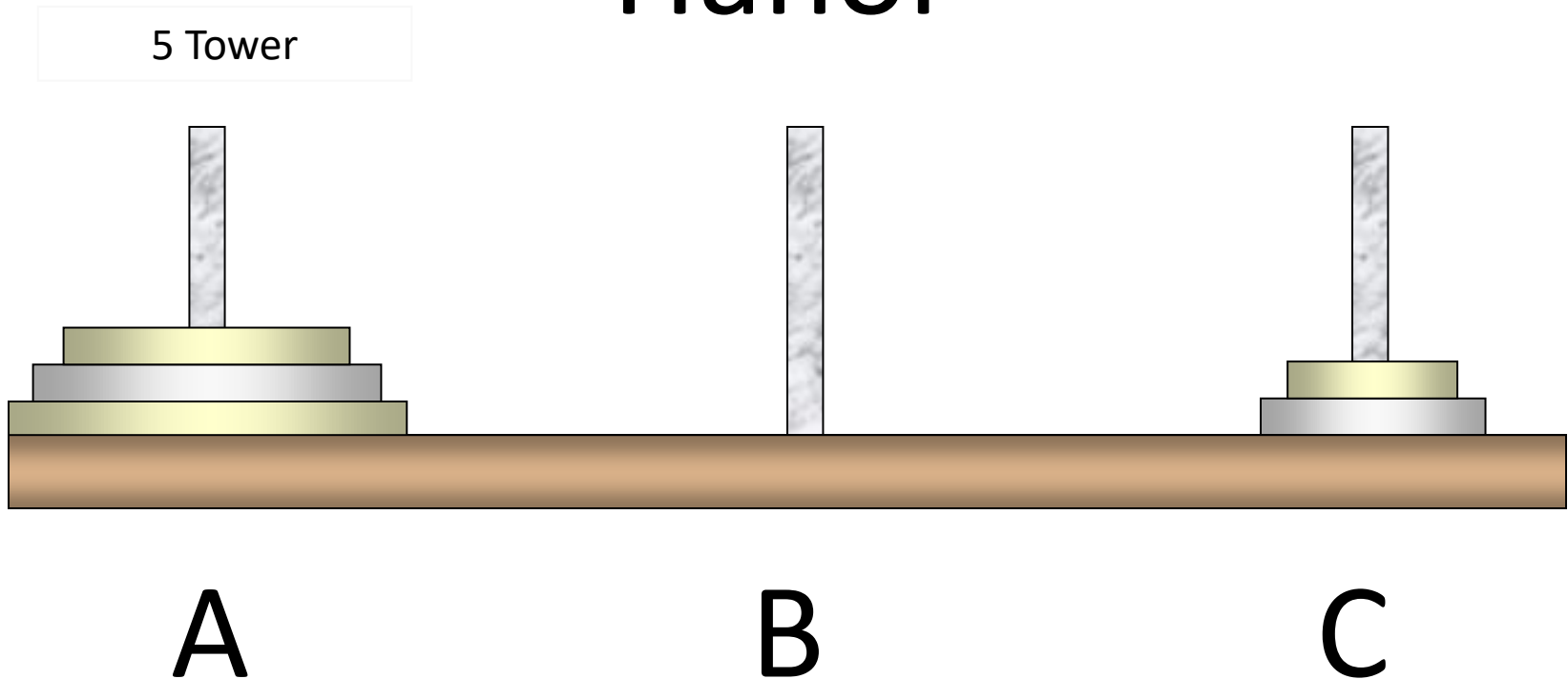
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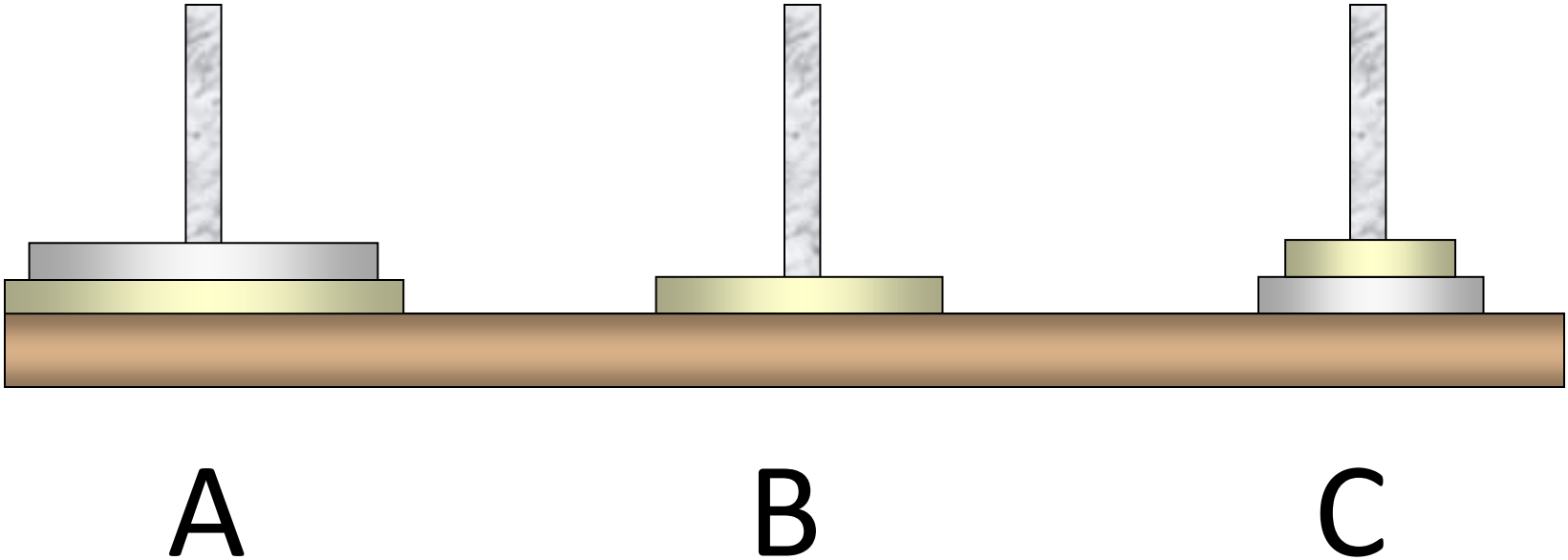


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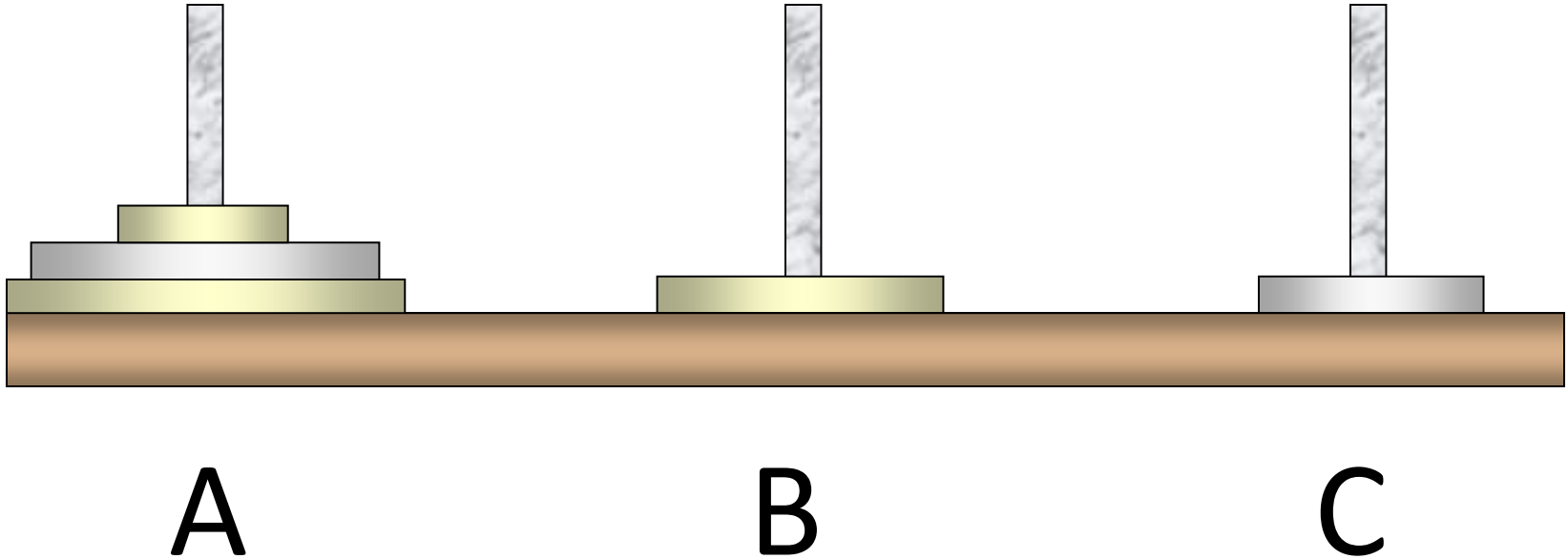
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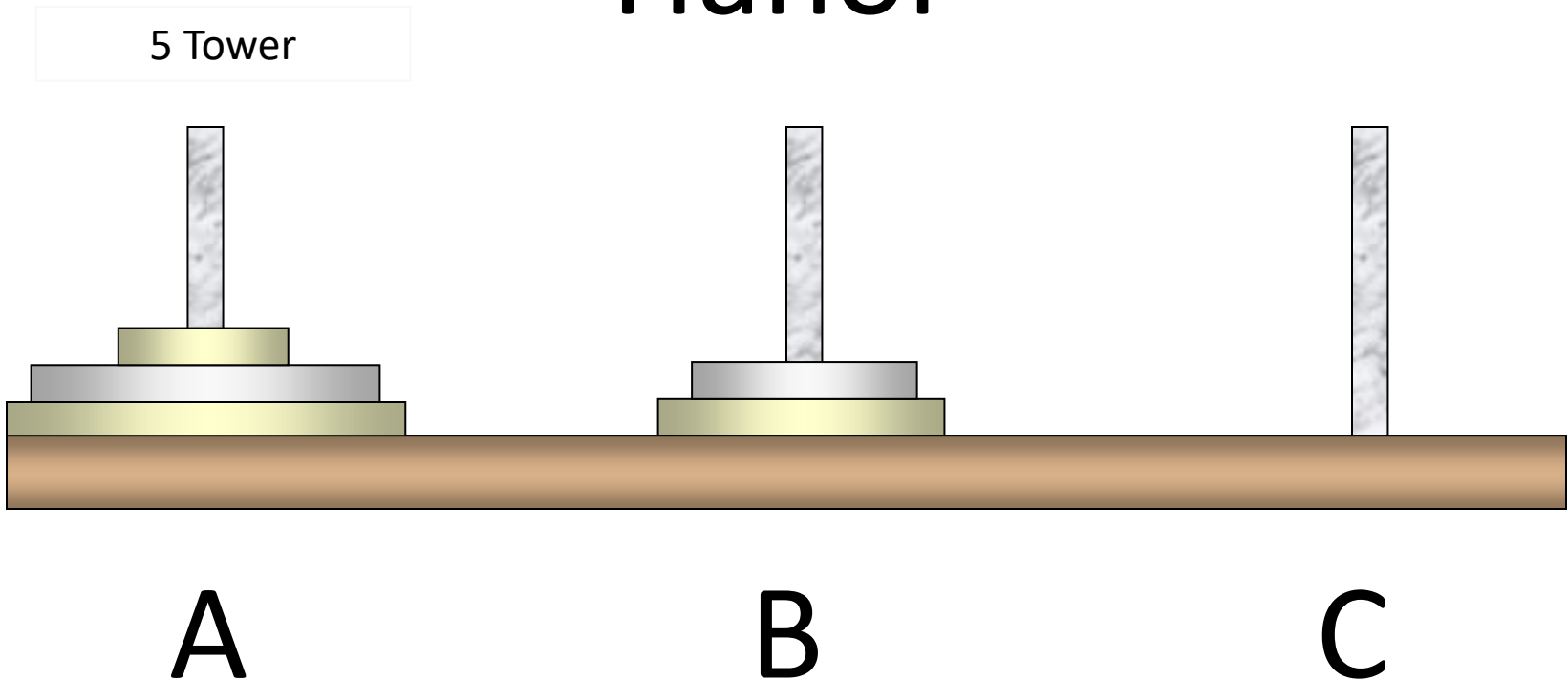


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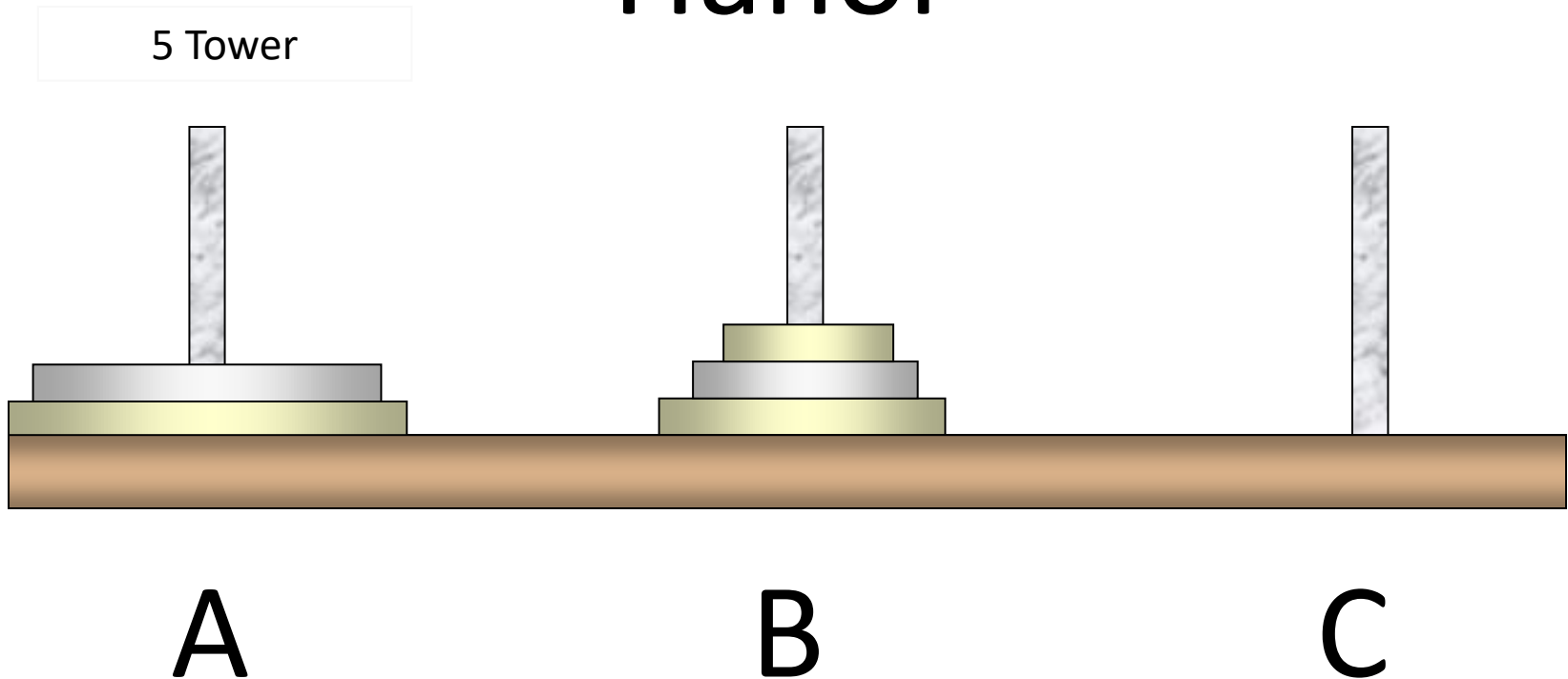
5 Tower



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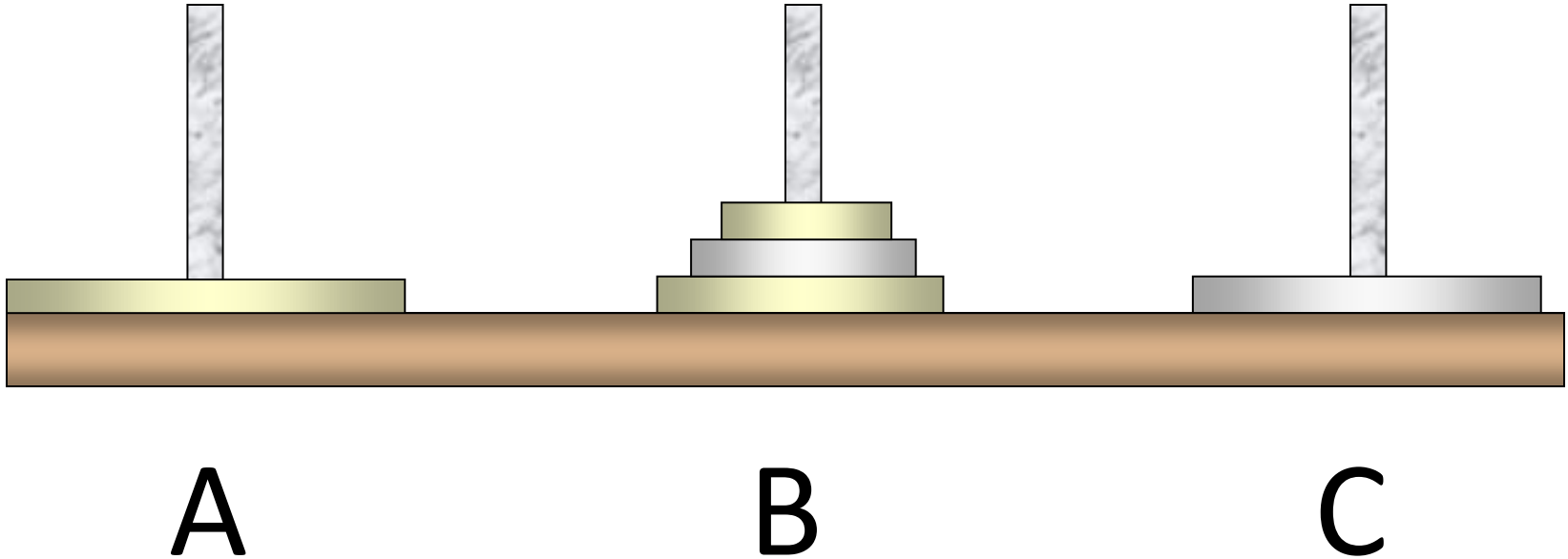


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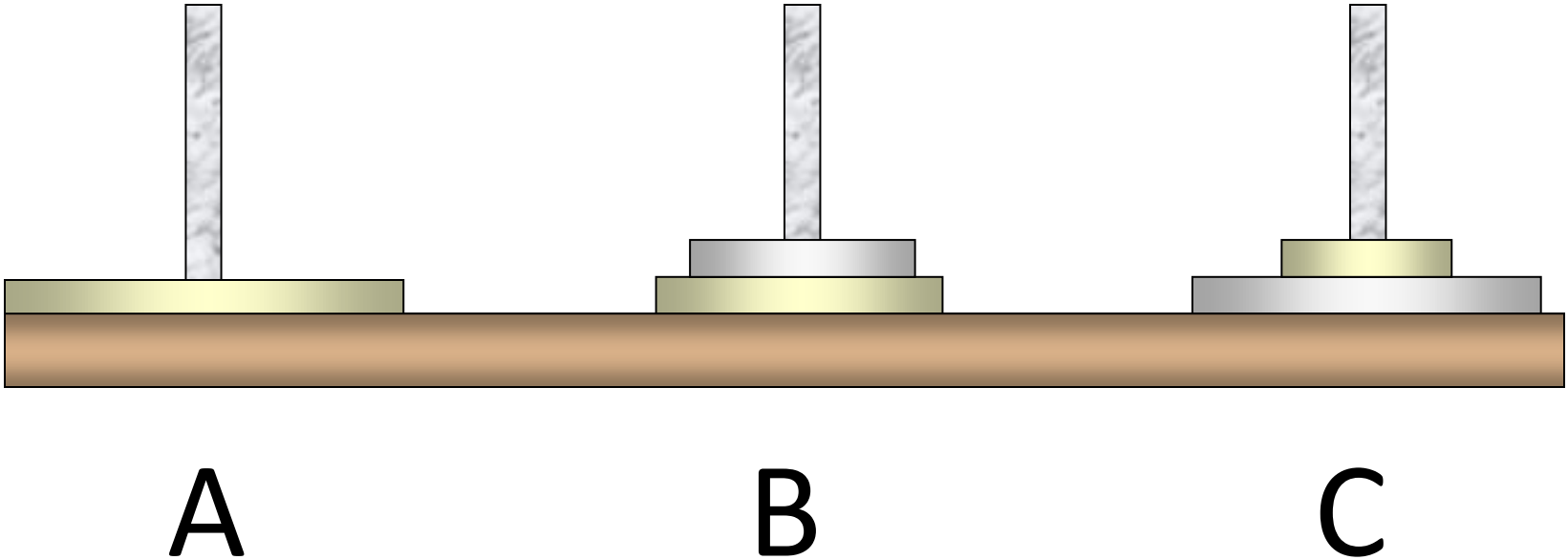
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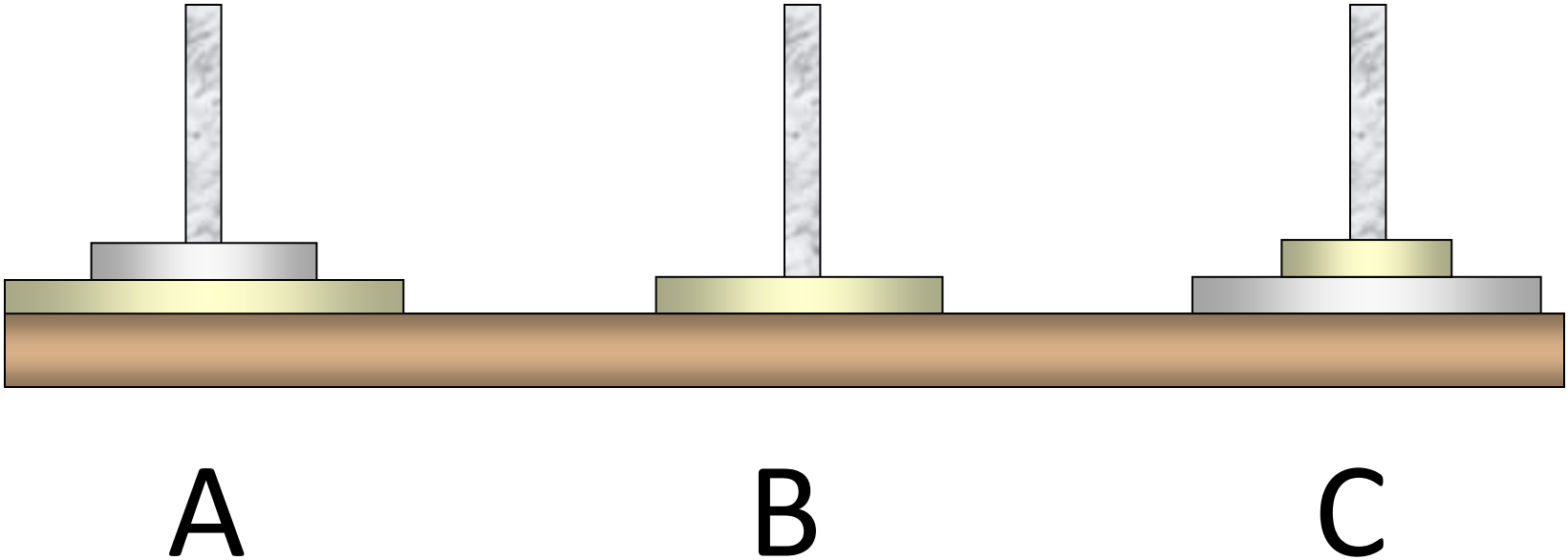
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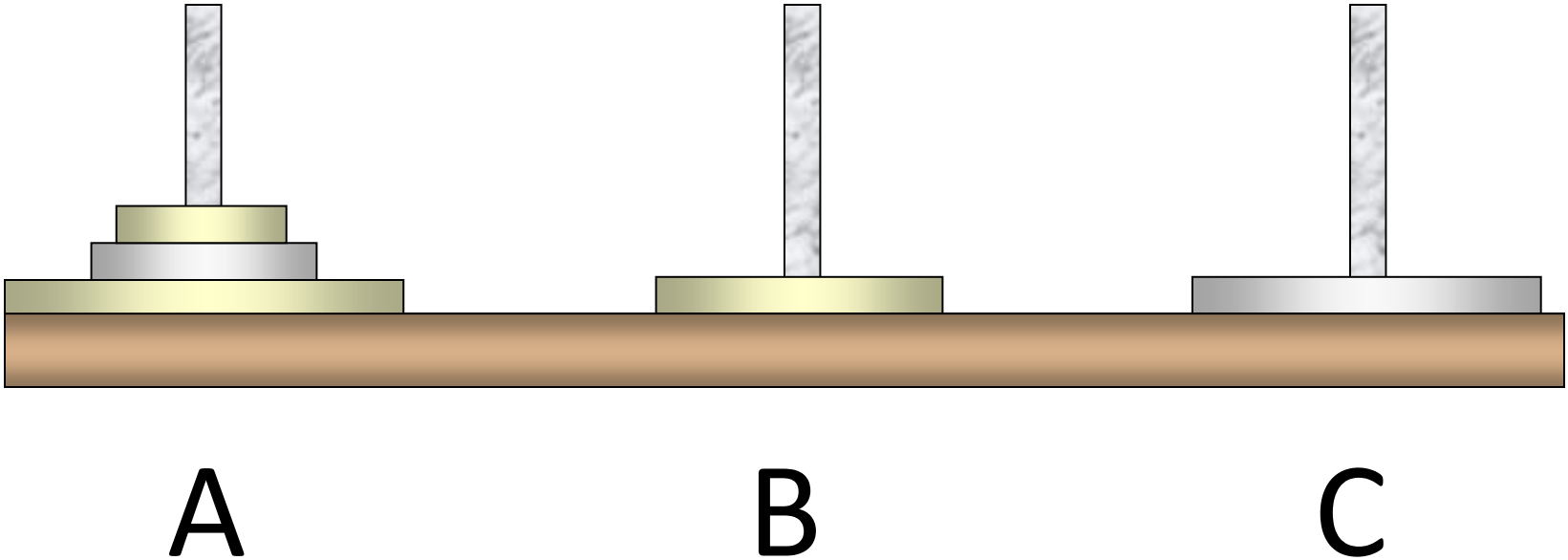
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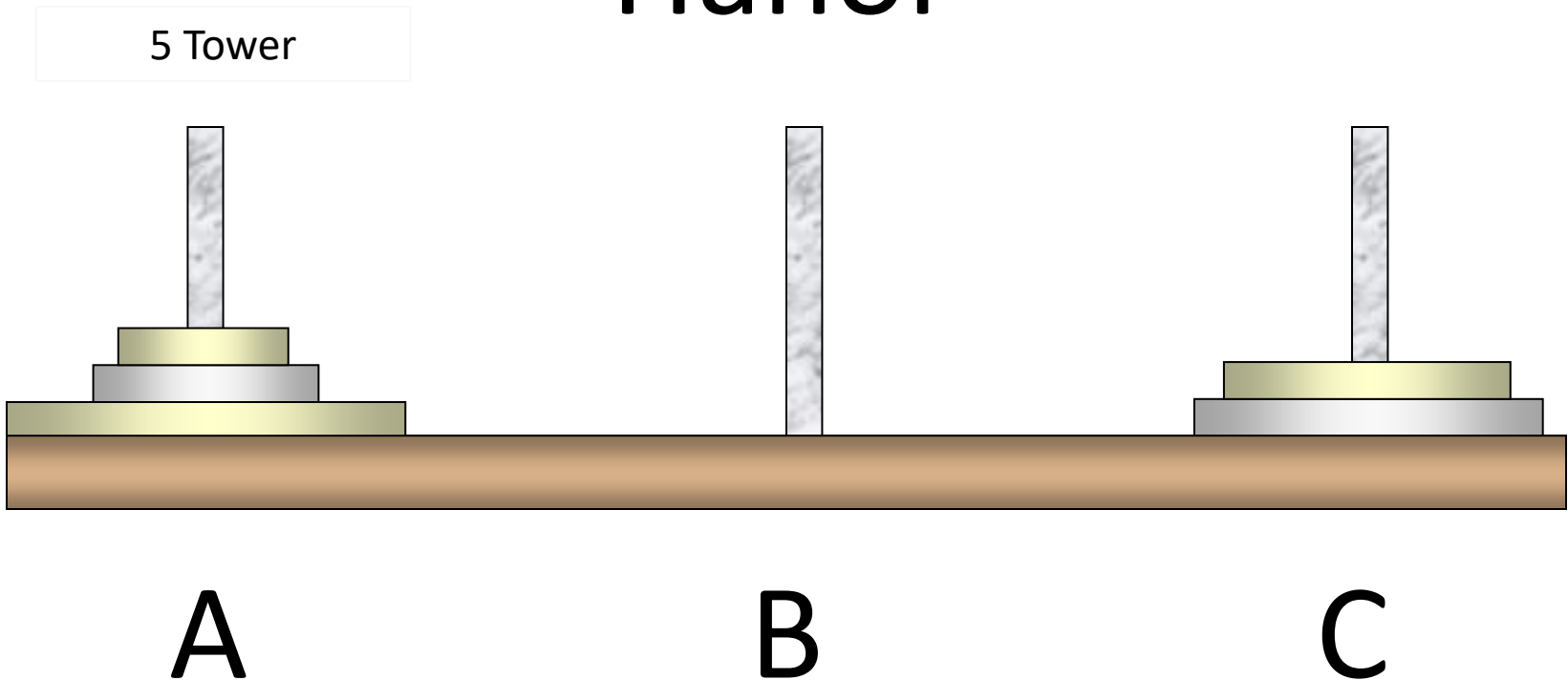


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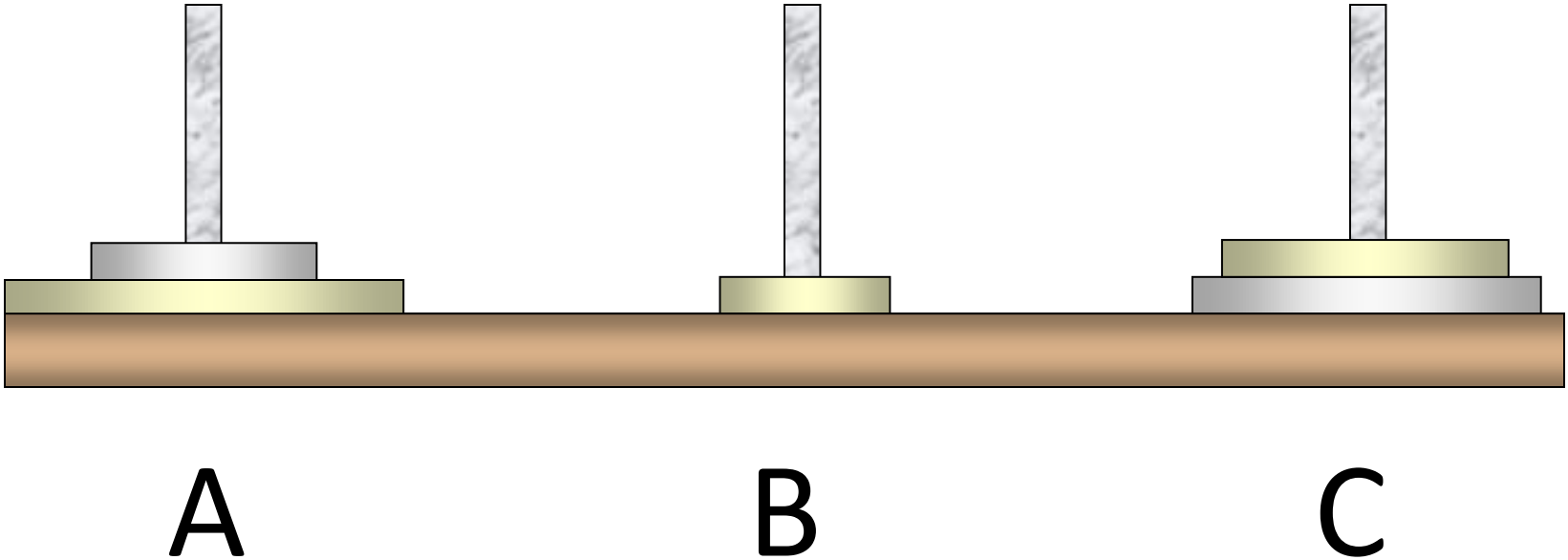
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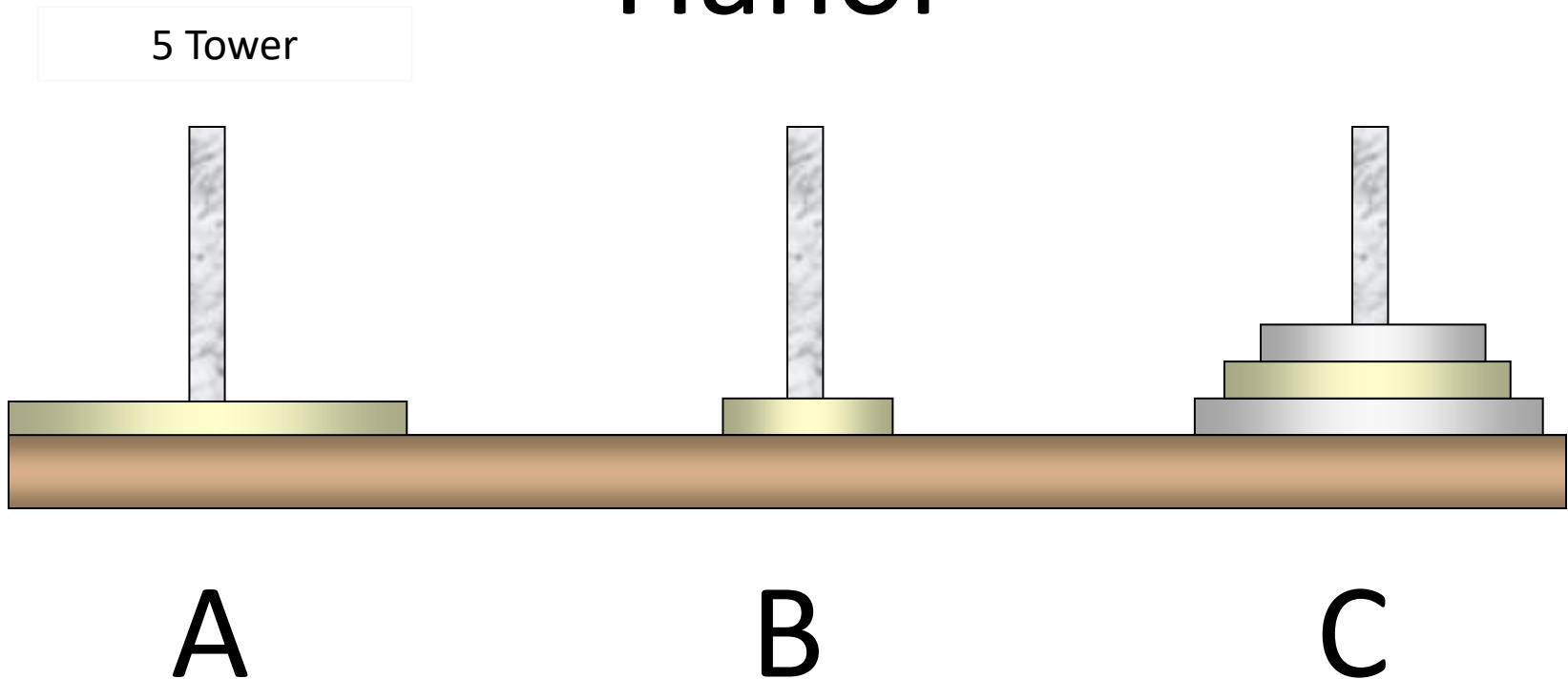


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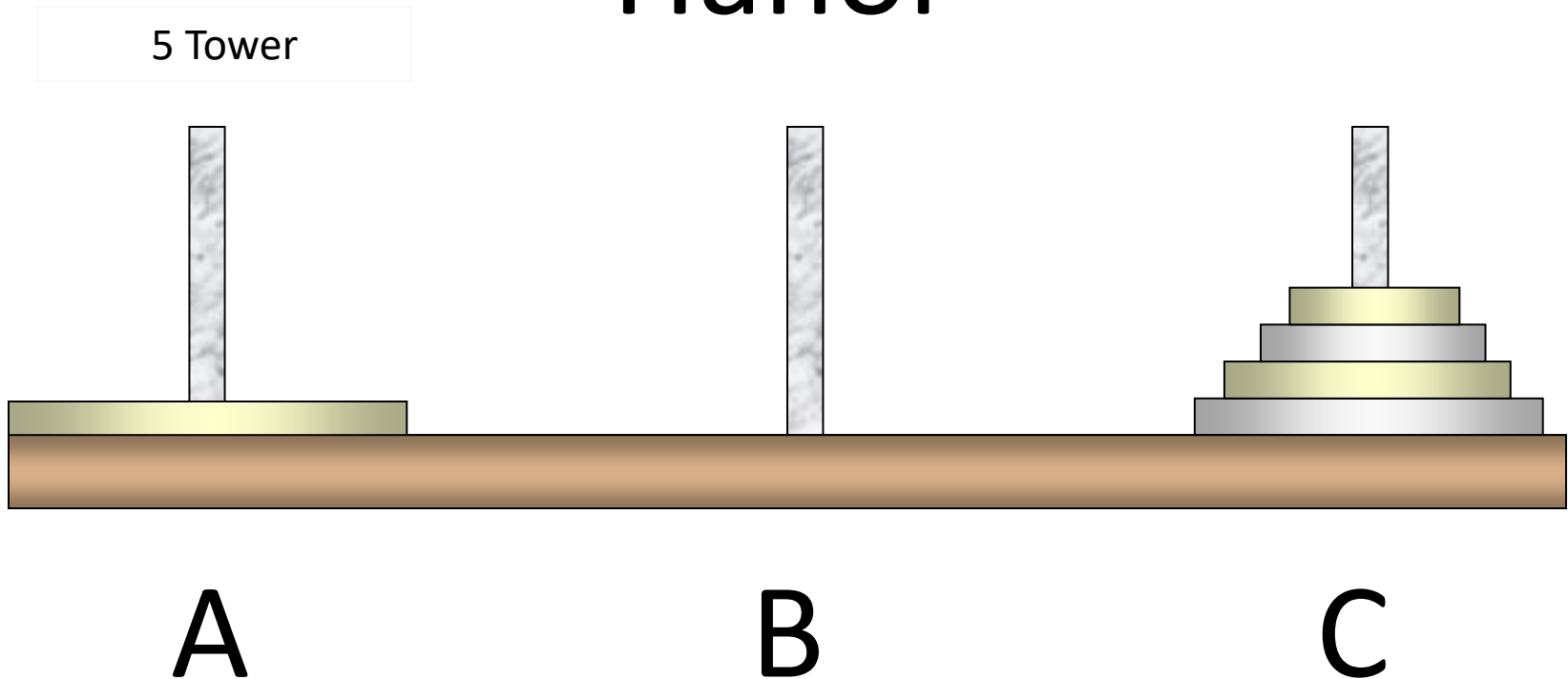
5 Tower



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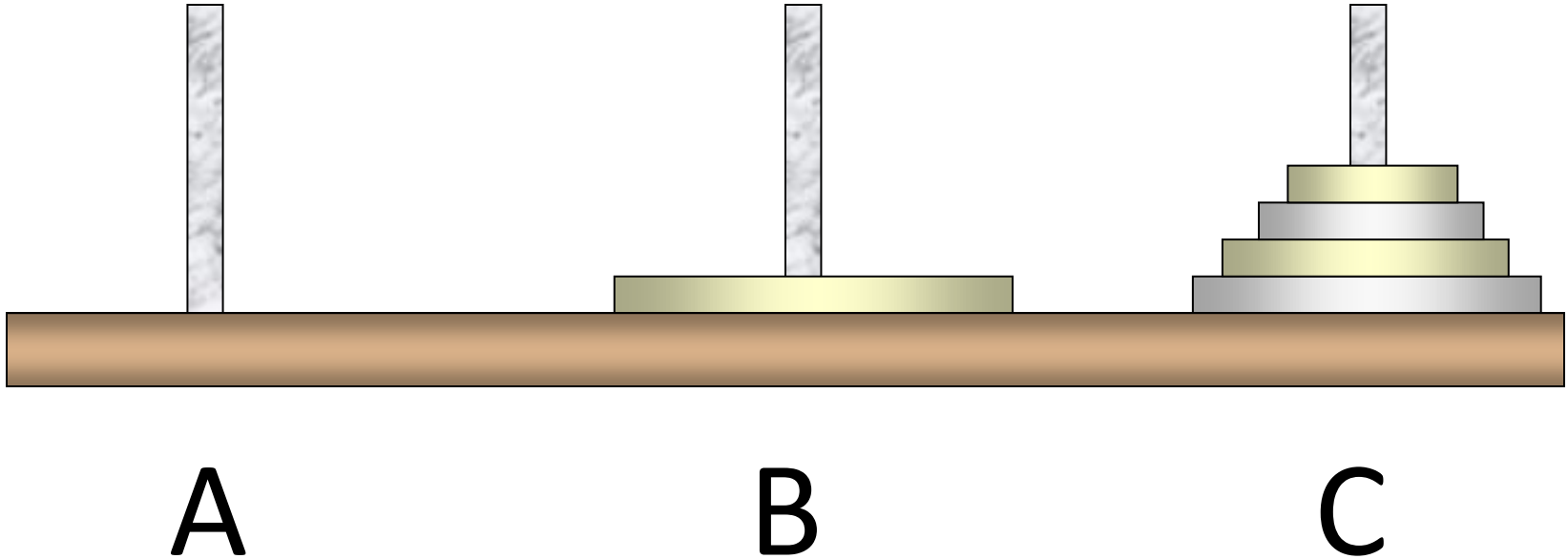


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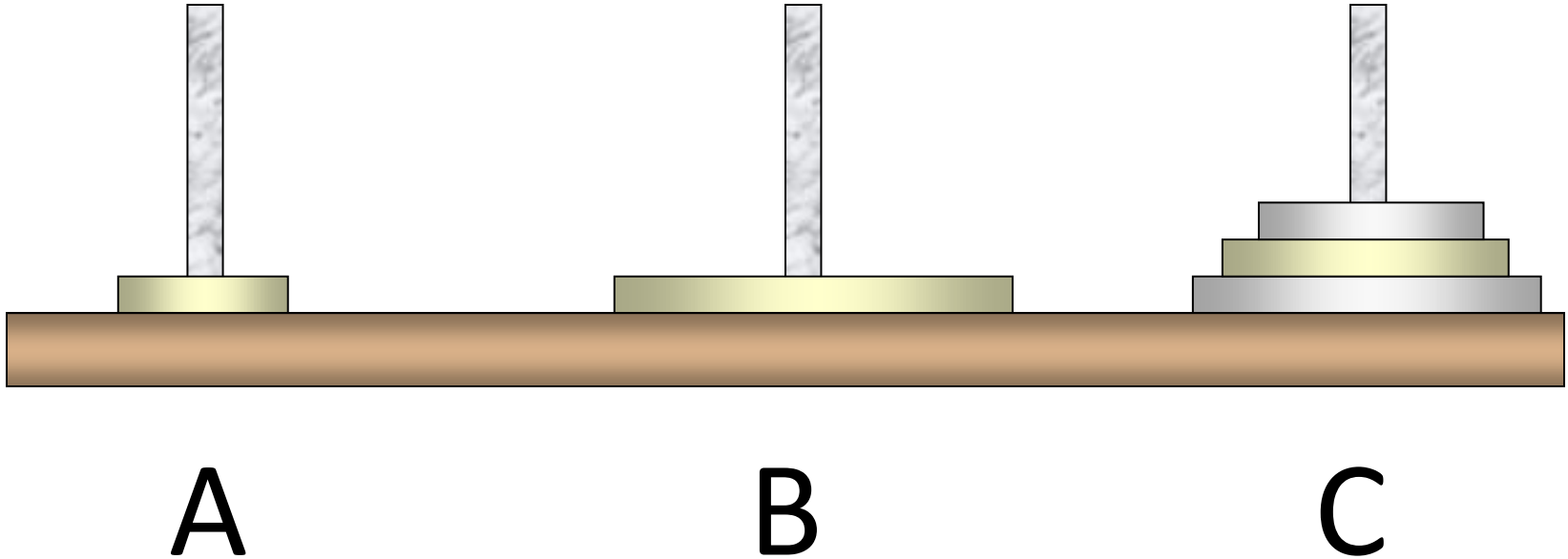
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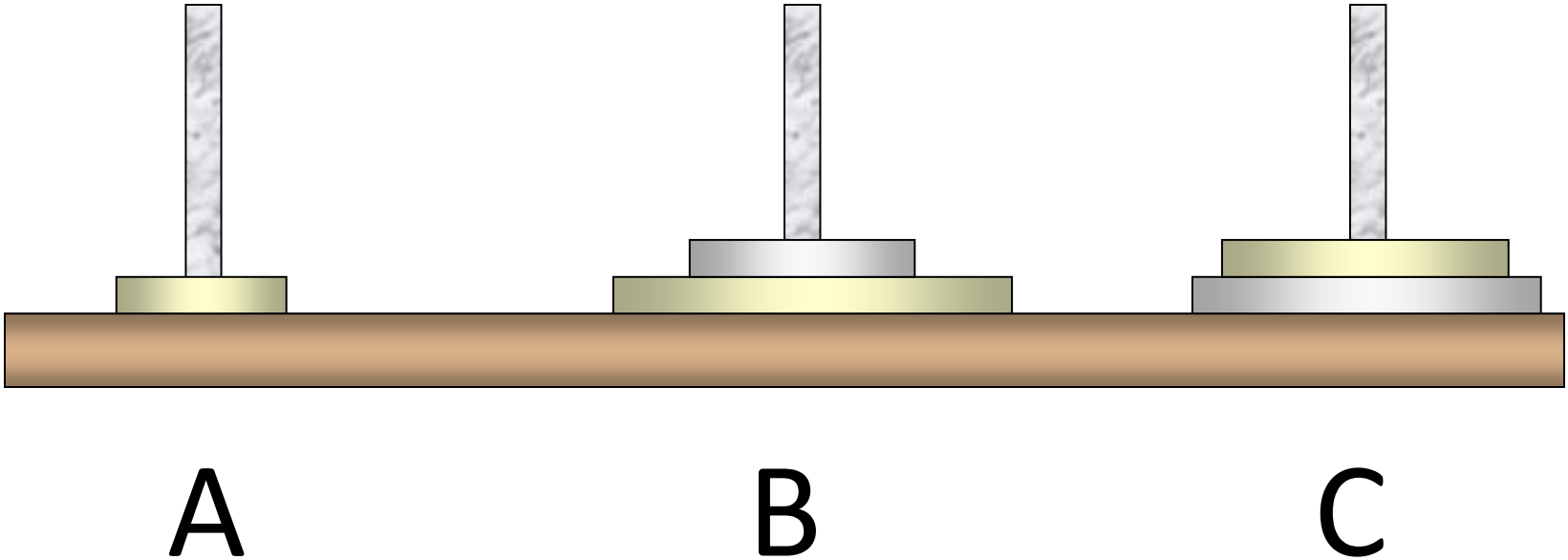
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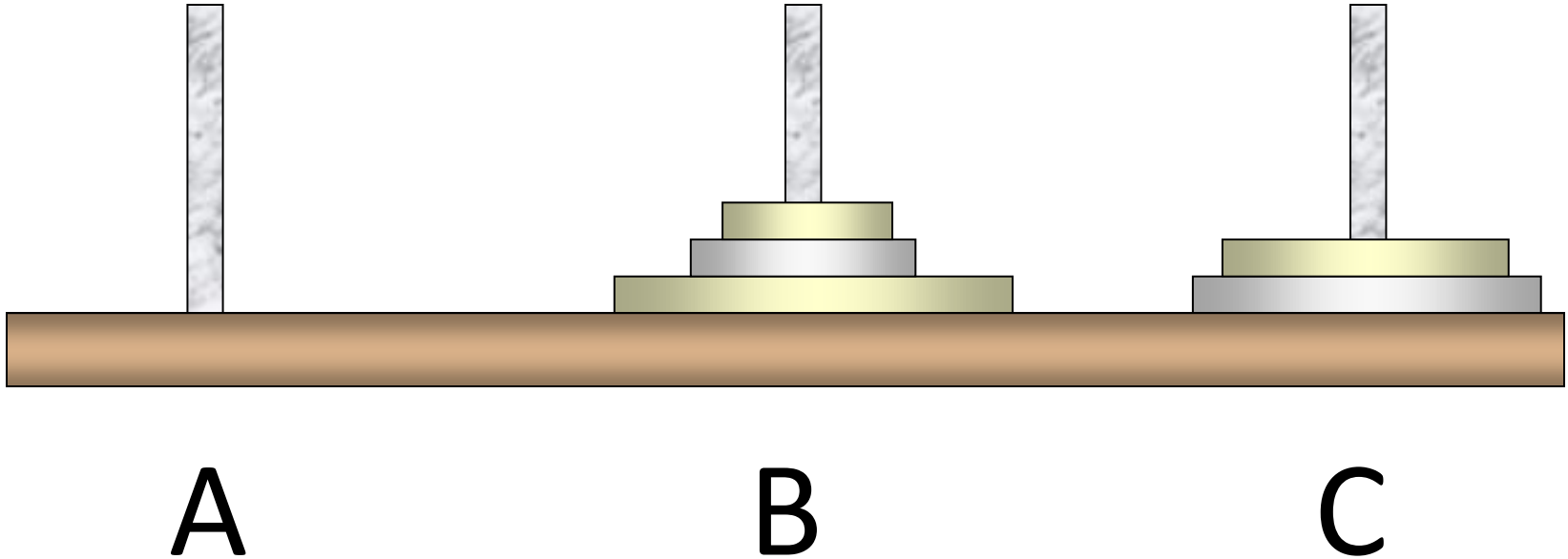
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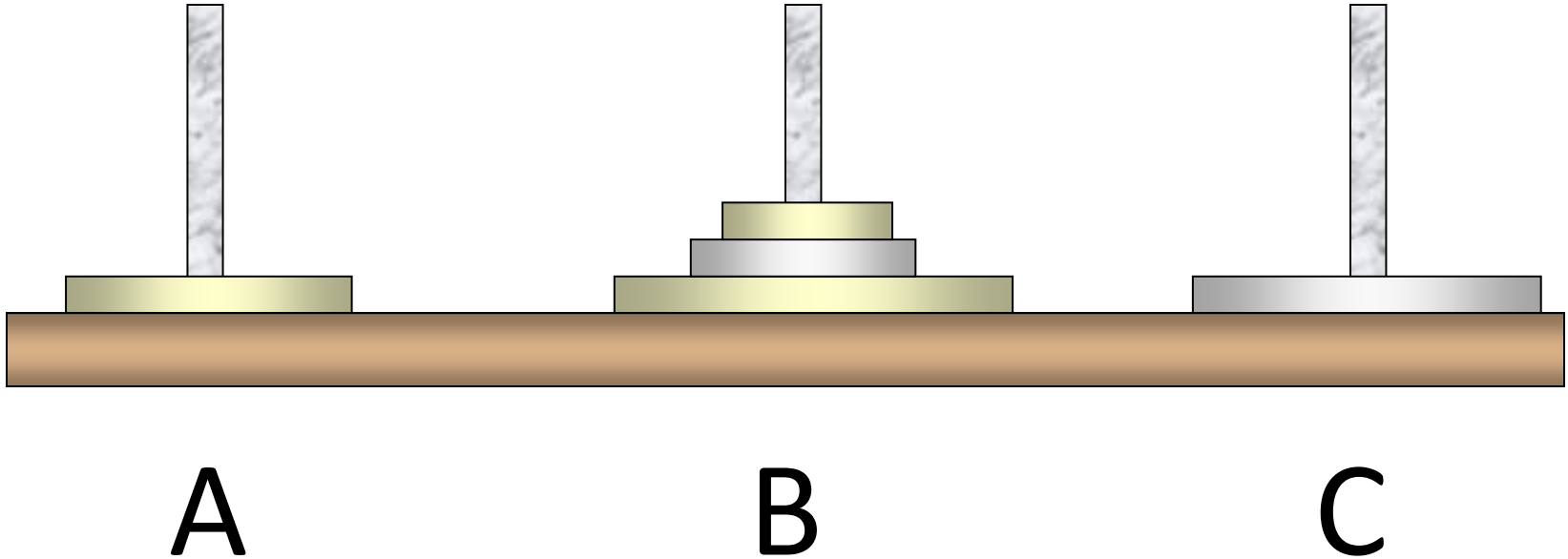
# The Tower of Hanoi

5 Tower



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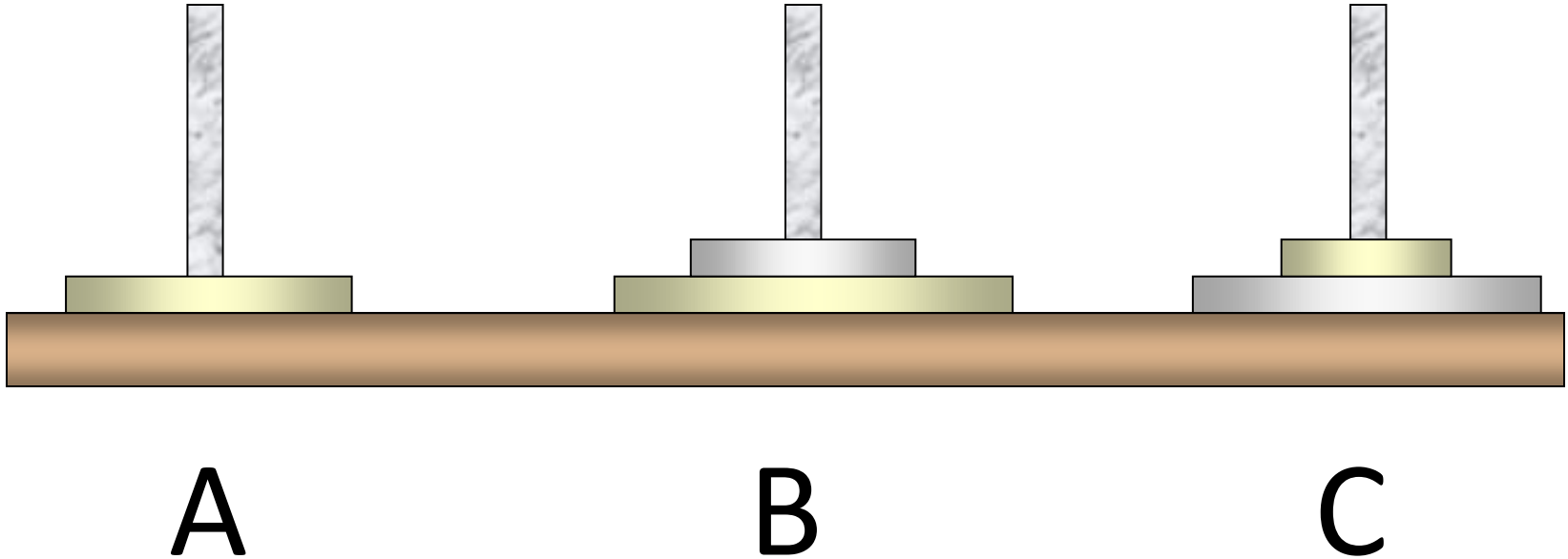
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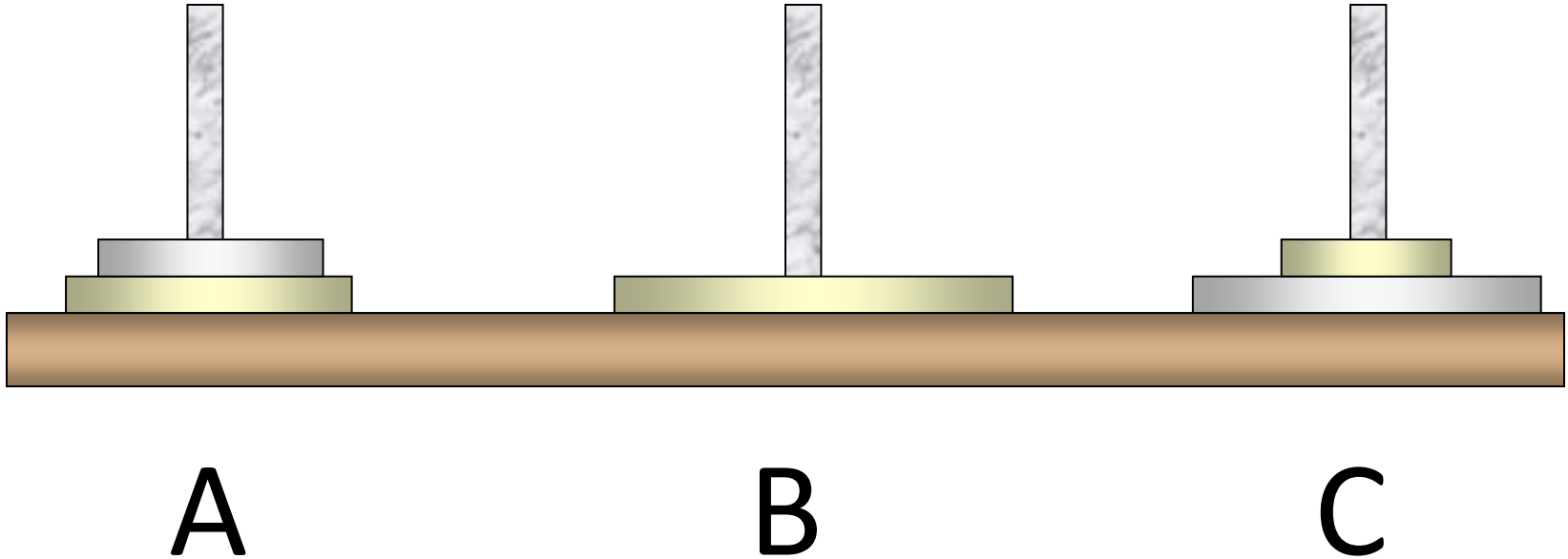
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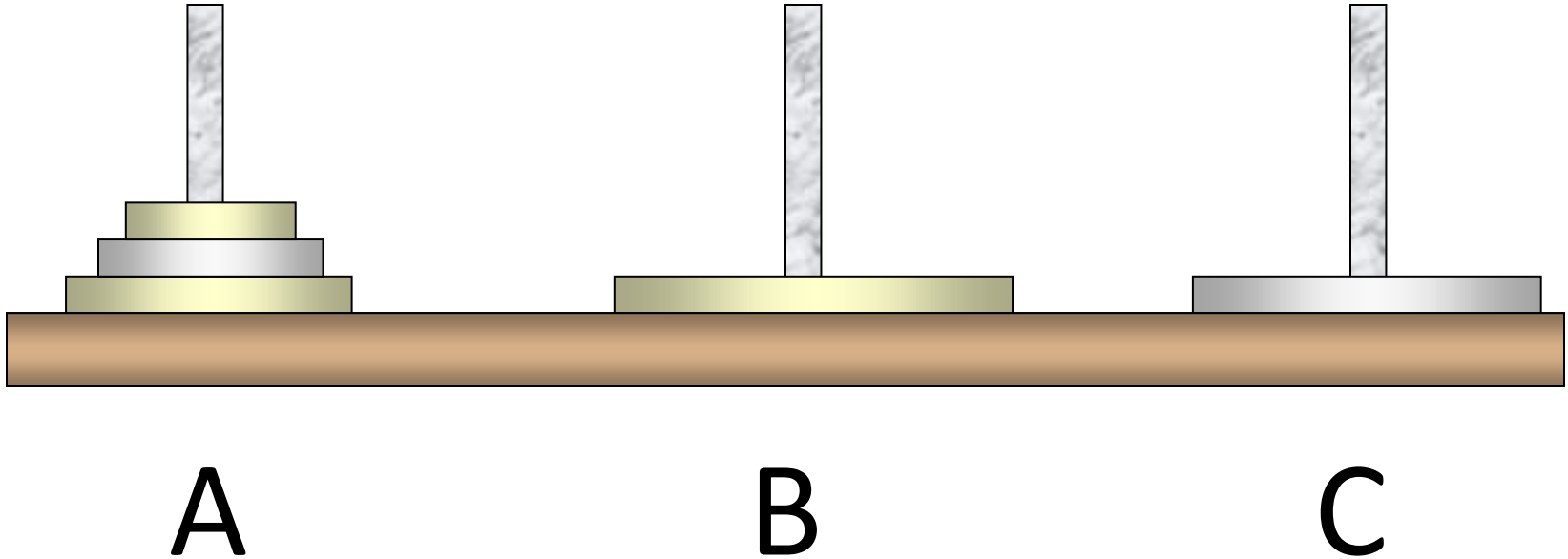
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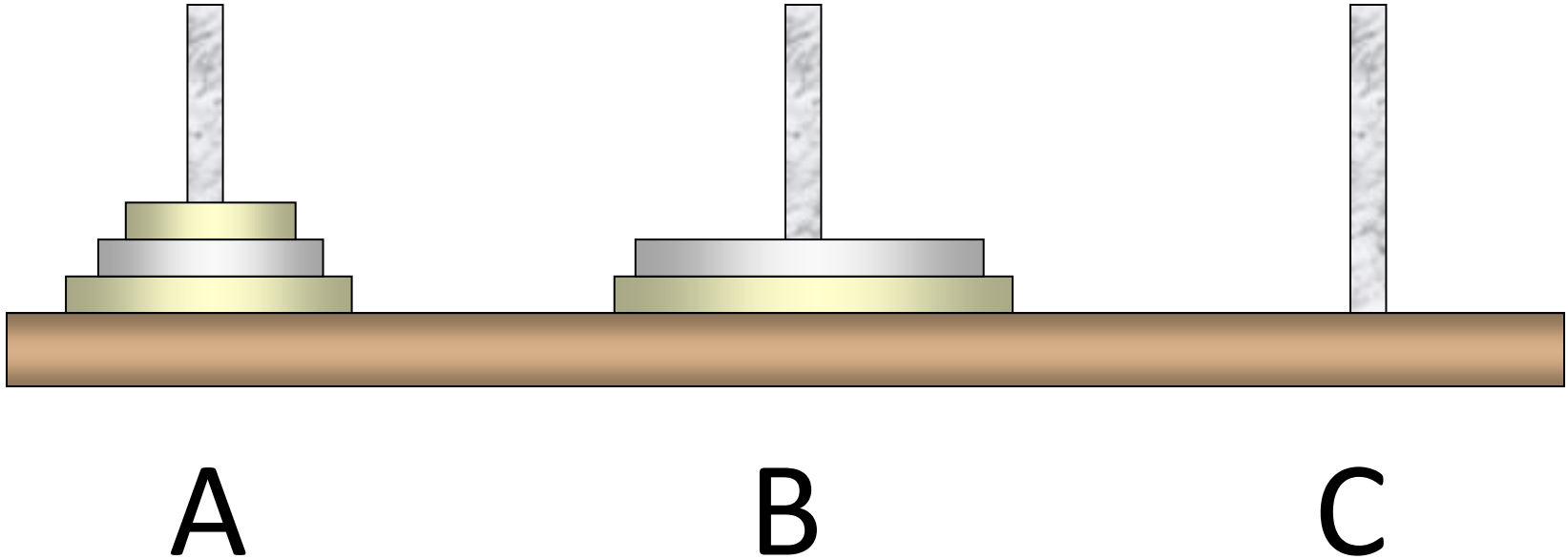
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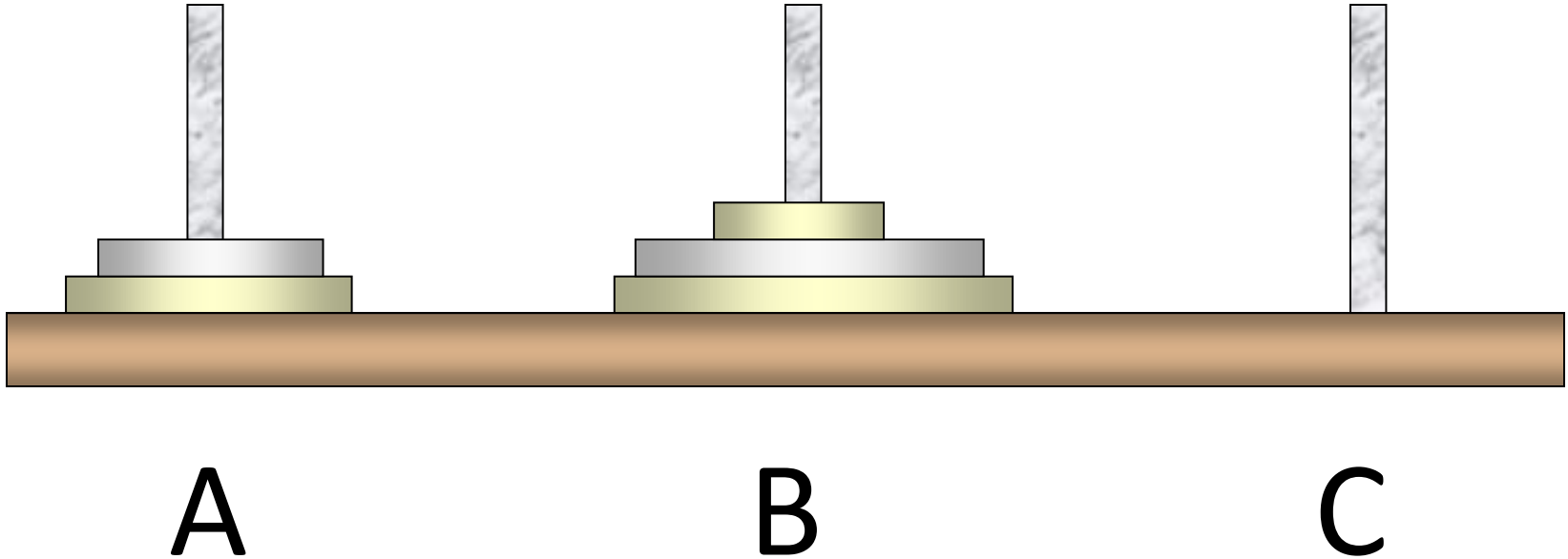
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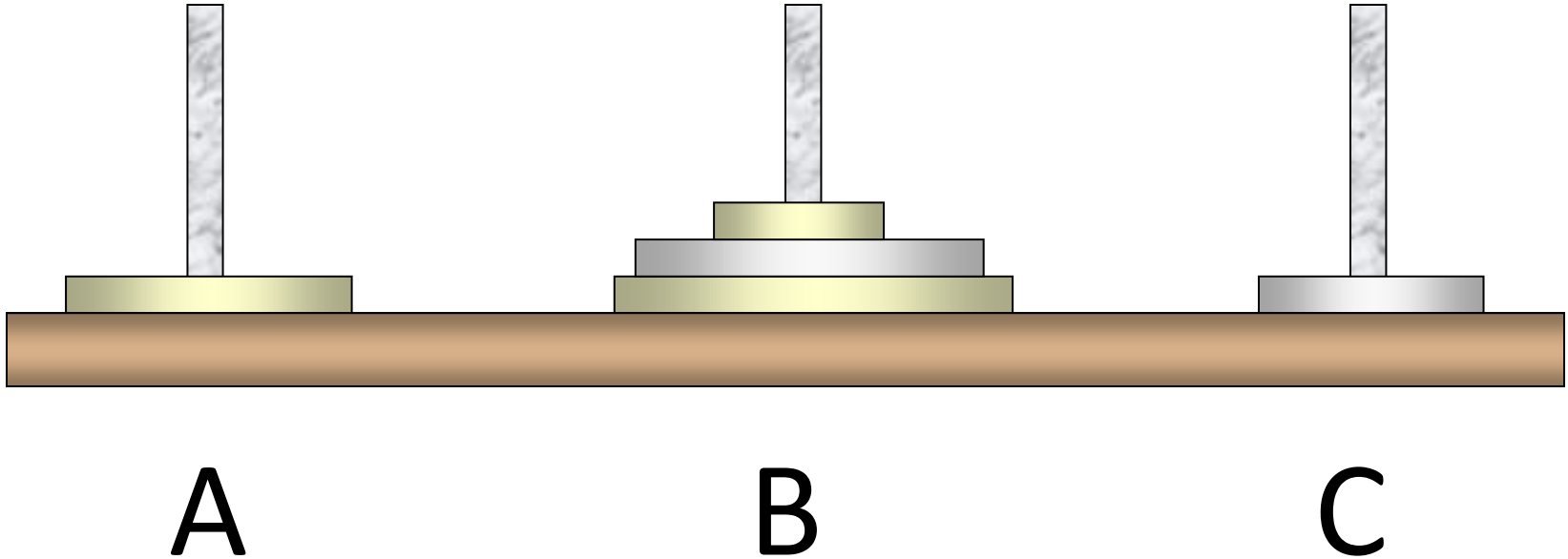
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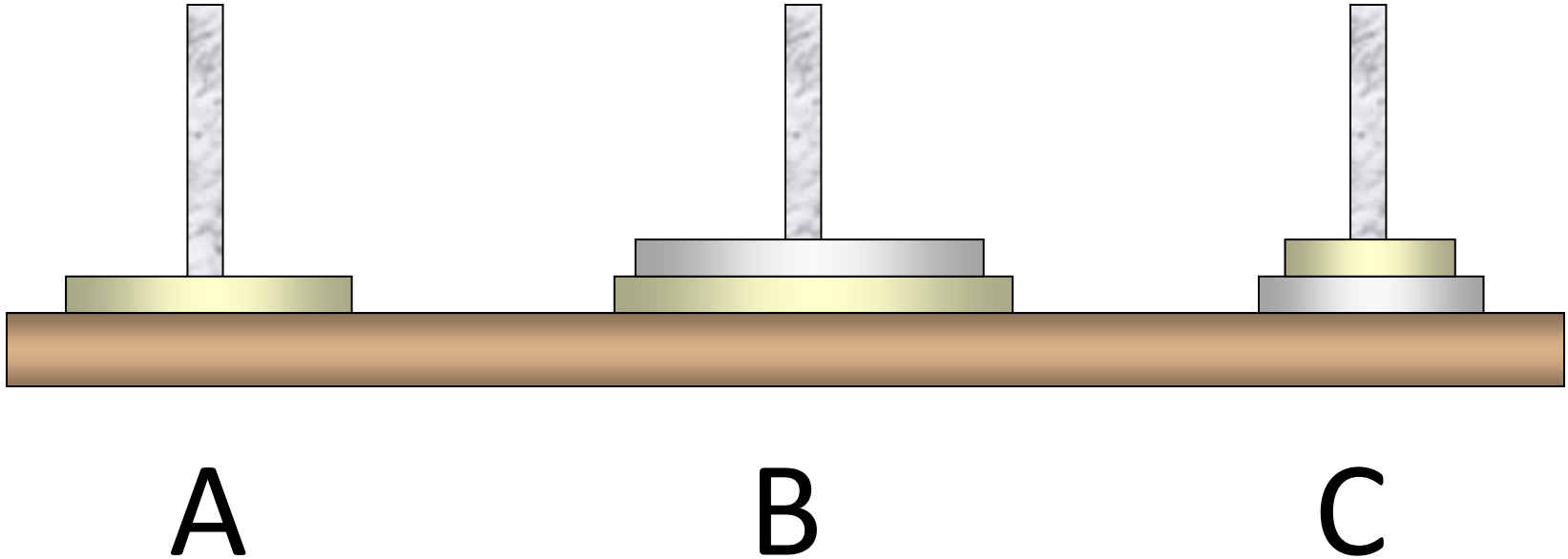
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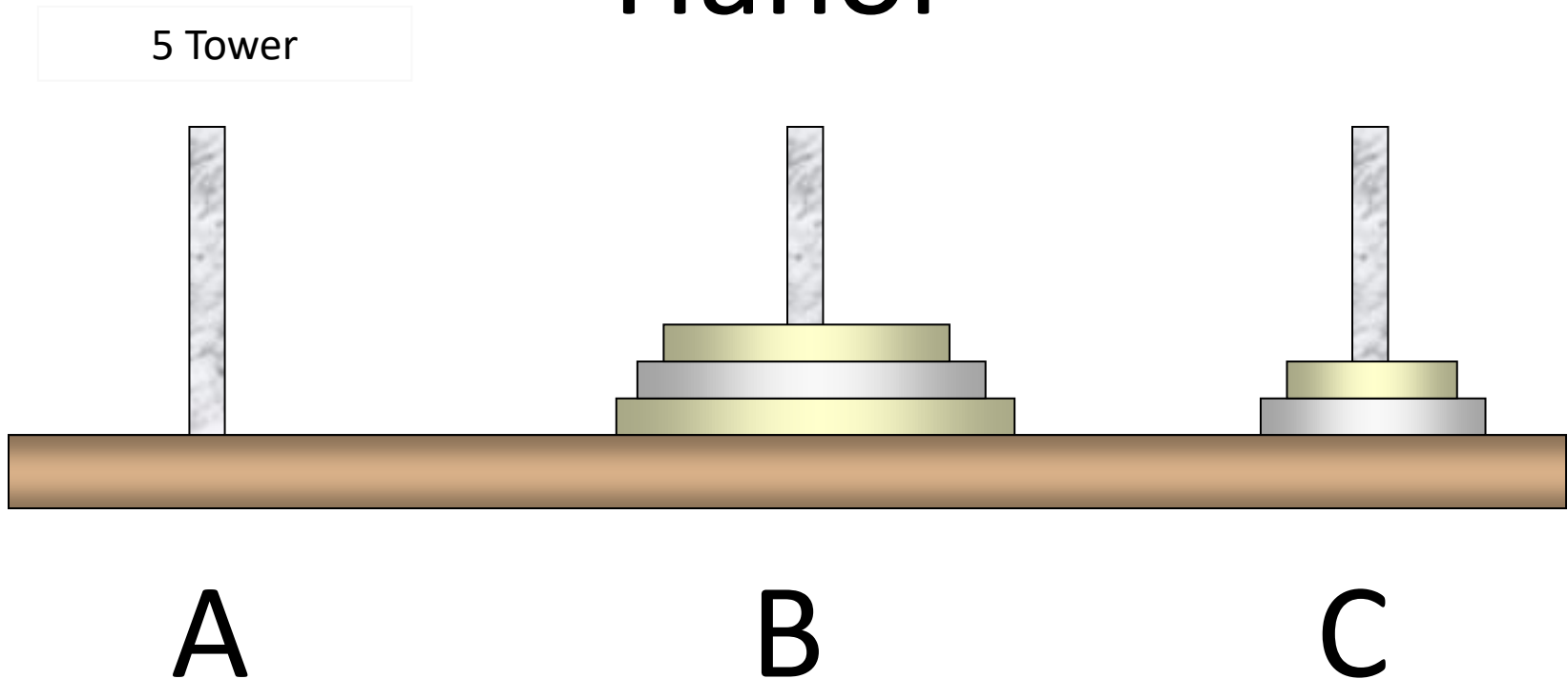


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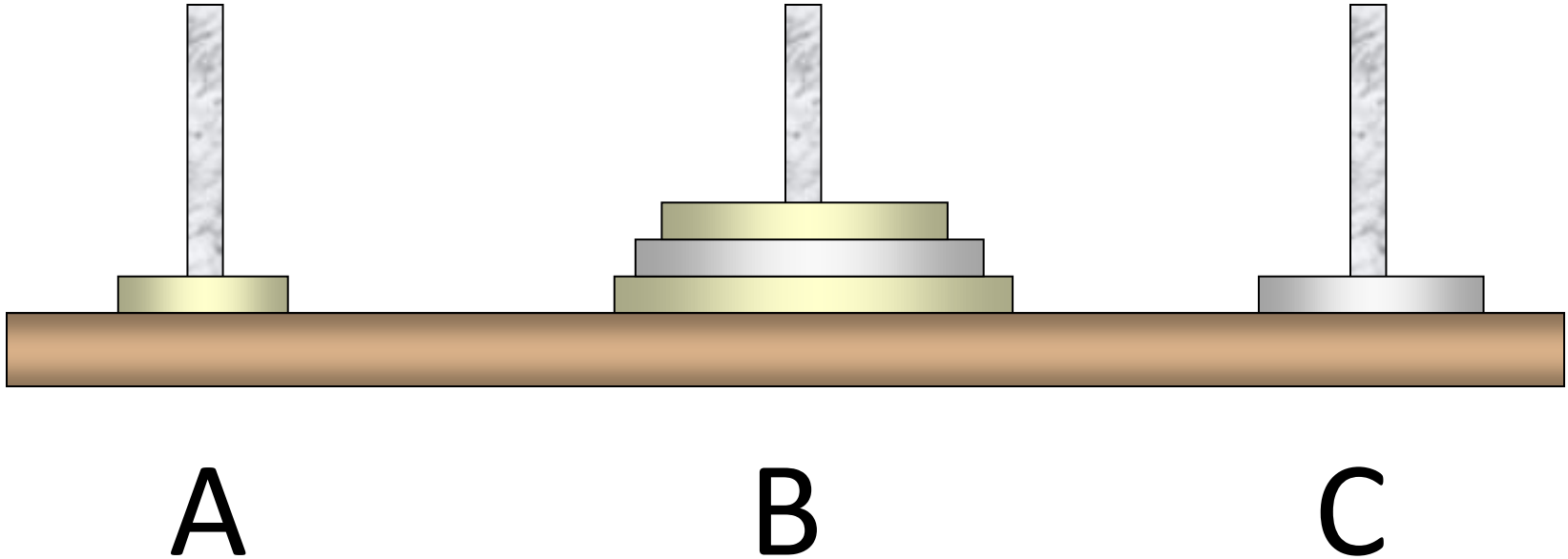
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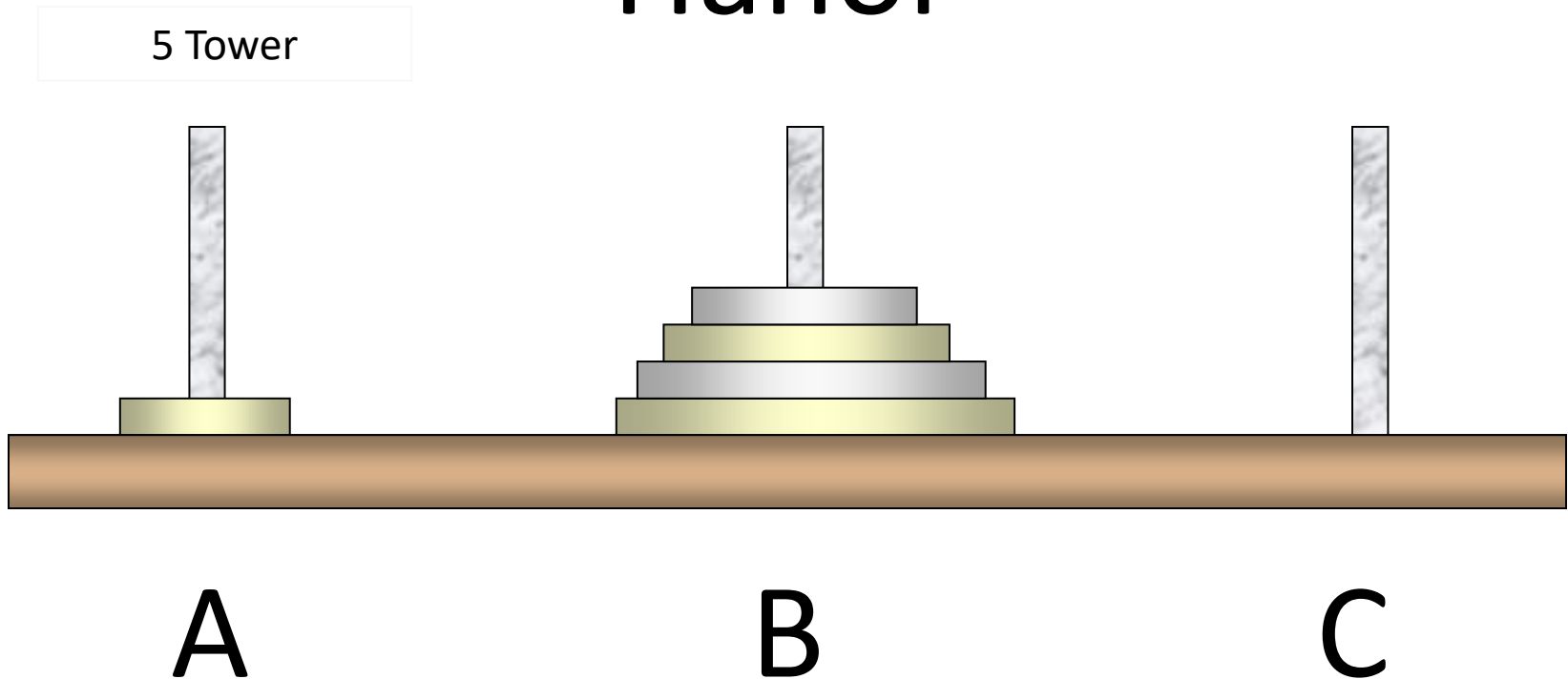


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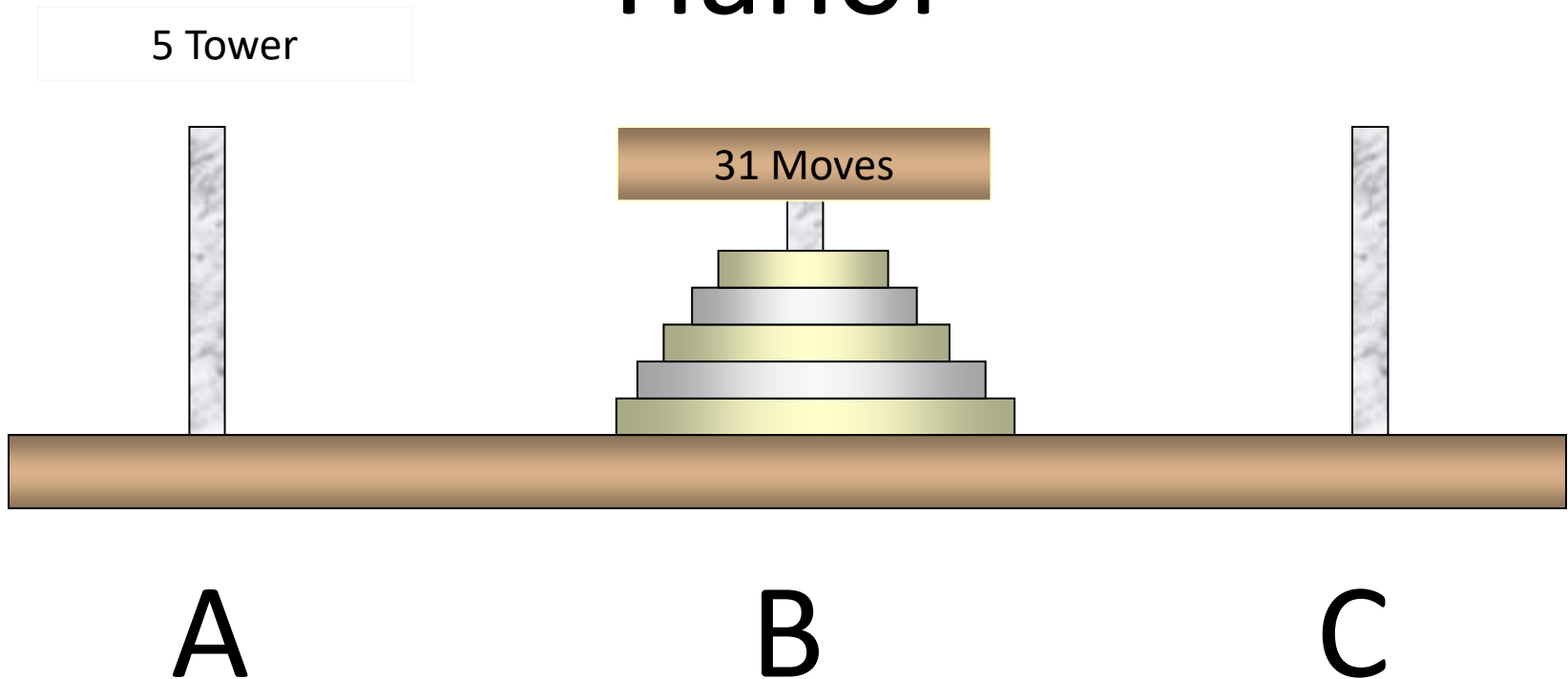
5 Tower



# The Tower of Hanoi

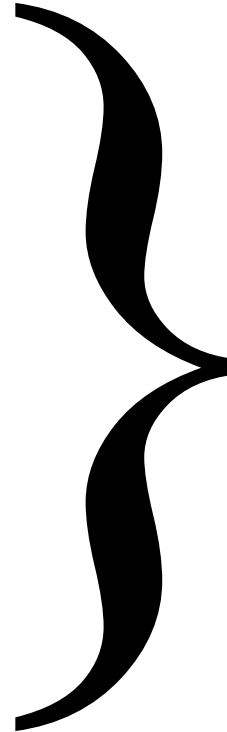


# The Tower of Hanoi



# The Tower of Hanoi

Discs	Moves
1	1
2	3
3	7
4	15
5	31
6	63
7	127
8	255



This is called a recursive function.

64	$2^{64} - 1$
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$n$	$2^n - 1$
-----	-----------

EXPRESSIONS

# Expressions

78

- An **algebraic expression** is a legal combination of operands and the operators.
- **Operand** is the *quantity* on which a mathematical operation is performed.
- **Operator** is a *symbol* which signifies a mathematical or logical operation.

# Expressions

79

- **INFIX:** expressions in which operands surround the operator.
- **POSTFIX:** operator comes after the operands, also known as *Reverse Polish Notation (RPN)*.
- **PREFIX:** operator comes before the operands, also known as *Polish Notation*.

## Example

- Infix:  $A+B$
- Postfix:  $AB+$
- Prefix:  $+AB$

# Infix Expression

80

- Appearance may be misleading, INFIX notations are not as simple as they seem
  
- To evaluate an infix expression, we need to consider
  - ▣ Operators' Priority
  - ▣ Associative property
  - ▣ Delimiters



# Infix Expression

81

- Infix Expression is hard to parse and difficult to evaluate.
- Postfix and prefix do not rely on operator priority and are easier to parse.

*An expression in infix form is thus converted into prefix or postfix form and then evaluated without considering the operators priority and delimiters.*

# Example

82

**Infix**

$A+B$

$(A+B) * (C + D)$

$A-B/(C*D^E)$

**PostFix**

$AB+$

$AB+CD+*$

$ABCDE^{*}/-$

**Prefix**

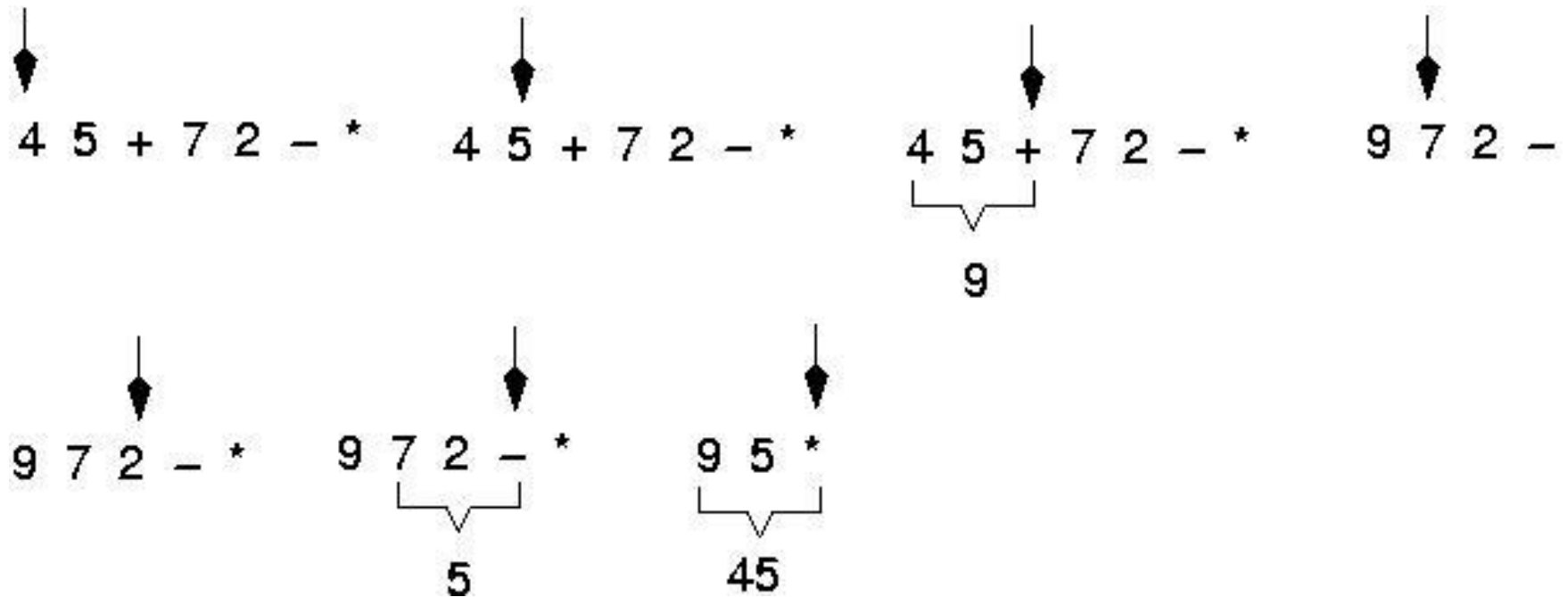
$+AB$

$*+AB+CD$

$-A/B*C^DE$

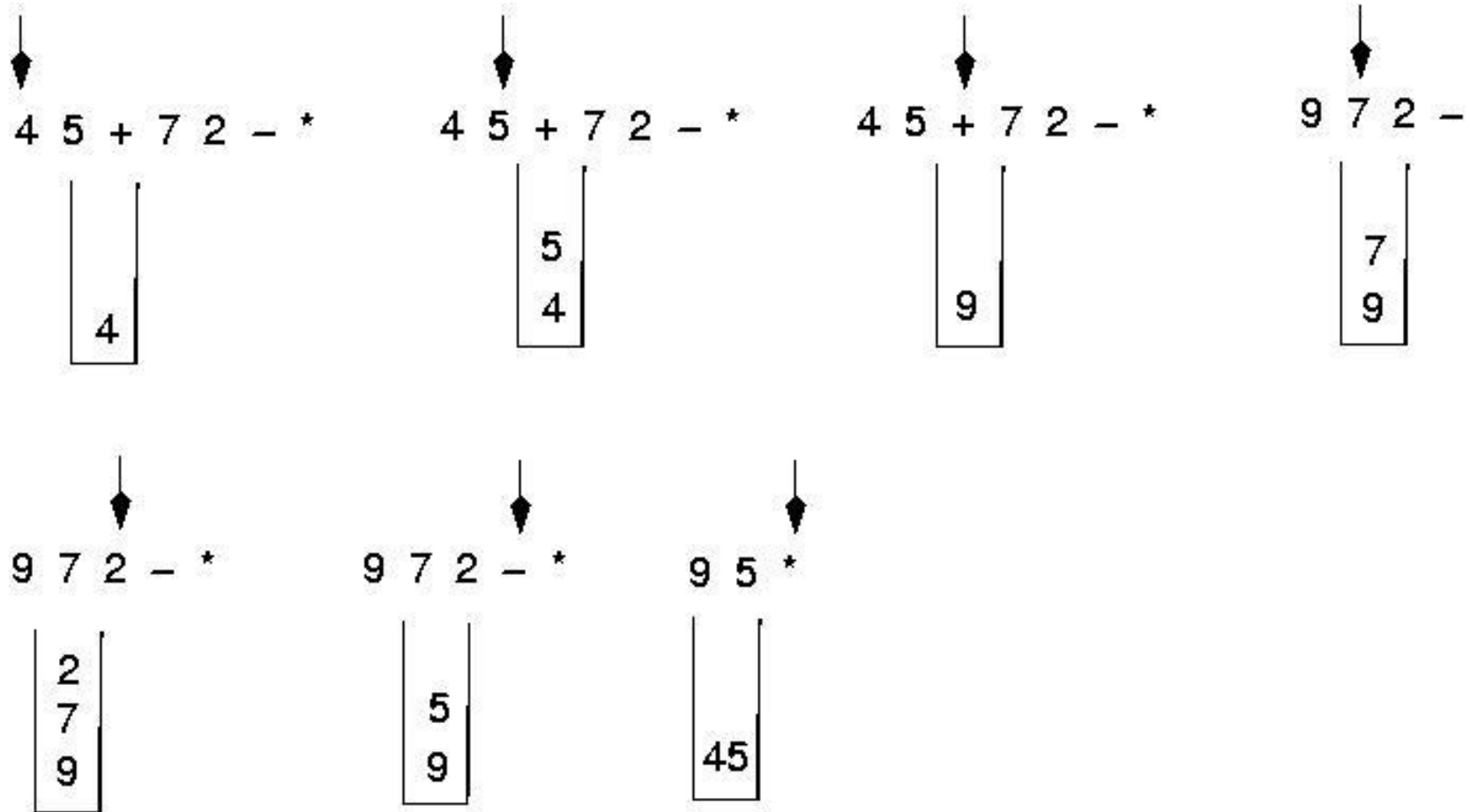
# Example

83



# Example

84



# Reading Materials

85

- Nell Dale – Chapter#4
- Schaum's Outlines – Chapter#6
- D. S. Malik – Chapter#7
- <http://www.cs.man.ac.uk/~pjj/cs2121/fix.html>