CS-218
DATA STRUCTURE

Dr. Hashim Yasin

National University of Computer and Emerging Sciences,

Faisalabad, Pakistan.

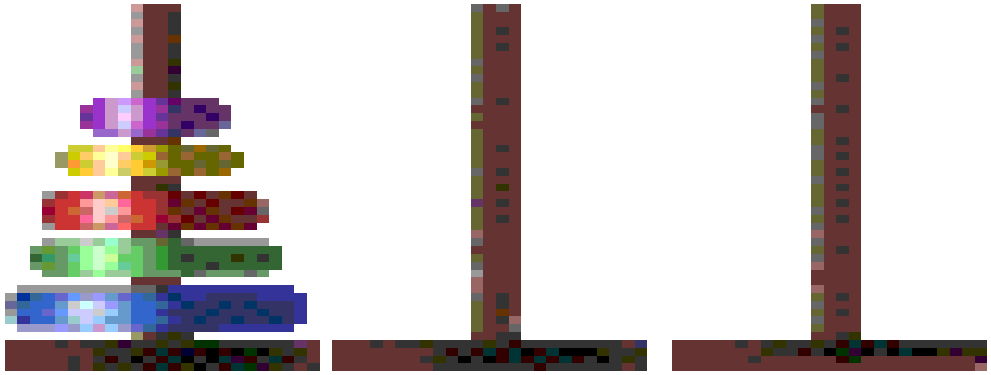
APPLICATION OF STACKS

Application of Stacks

- Tower of Hanoi
- Expressions
  - Infix: A+B-C
  - Postfix: AB+C-
  - Prefix: -+ABC
- Recursion

Tower of Hanoi

- GIVEN: Three poles
  - a set of discs on the first pole,
  - discs of different sizes,
  - the smallest discs at the top
- GOAL: move all the discs from the left pole to the right one.
- CONDITIONS: only one disc may be moved at a time.
  - A disc can be placed either on an empty pole or on top of a larger disc.



The Tower of Hanoi

Discs	Moves
1	1
2	3
3	7
4	15
5	31
6	63
7	127
8	255

This is called a recursive function.

64	$2^{64} - 1$

n	$2^n - 1$

EXPRESSIONS

Expressions

- An **algebraic expression** is a legal combination of operands and the operators.
  - **Operand** is the *quantity* on which a mathematical operation is performed.
  - **Operator** is a *symbol* which signifies a mathematical or logical operation.

Expressions

- **INFIX:** expressions in which operands surround the operator.
- **POSTFIX:** operator comes after the operands, also known as *Reverse Polish Notation (RPN)*.
- **PREFIX:** operator comes before the operands, also Known as Polish notation.

Example

- Infix: A+B
- Postfix: AB+
- Prefix: +AB

Example

Infix	PostFix	Prefix
A+B	AB+	+AB
(A+B) * (C + D)	AB+CD+*	*+AB+CD
A-B/(C*D^E)	ABCDE^*/-	-A/B*C^DE

FULLY PARENTHESIZED EXPRESSION

Fully Parenthesized Expression

- A Fully Parenthesized Expression (FPE) has exactly one set of Parentheses enclosing each operator and its operands.
- Which one is fully parenthesized?
  - (A + B) * C
  - ((A + B) * C)
  - ((A + B) * (C))

Algorithm: Infix to Postfix Conversion

Algorithm: Q is the given infix expression & we want P.

1. Scan Q from left to right and repeat steps 2 to 6 for each element of Q until the STACK is empty.
2. If an operand is encountered, add it to P
3. If a left parenthesis is encountered, push it onto STACK.
4. If an operator X is encountered, then:
   a. Repeatedly pop from STACK and add to P each operator which has same or higher precedence than X
   b. Push X on STACK

Algorithm: Infix to Postfix Conversion

5. If a right parenthesis is encountered, then:
   a. Repeatedly pop from STACK and add to P each operator until a left parenthesis is encountered
   b. Remove the left parenthesis. [Do not add it to P]
6. Exit

FPE Infix to Postfix

(((A + B) * (C - E)) / (F + G))

- stack: <empty>
- output: []

FPE Infix to Postfix

((A + B) * (C - E)) / (F + G))

- stack: (
- output: []

FPE Infix to Postfix

(A + B) * (C - E)) / (F + G))

- stack: ((
- output: []

FPE Infix to Postfix

A + B) * (C - E)) / (F + G))

- stack: (((
- output: []

FPE Infix to Postfix

+ B) * (C - E)) / (F + G))

- stack: (((
- output: [A]

FPE Infix to Postfix

B) * (C - E)) / (F + G))

- stack: (((+
- output: [A]

FPE Infix to Postfix

) * (C - E)) / (F + G))

- stack: (((+
- output: [A B]

FPE Infix to Postfix

* (C - E)) / (F + G))

- stack: ((
- output: [A B +]

FPE Infix to Postfix

(C - E)) / (F + G))

- stack: ((*
- output: [A B +]

FPE Infix to Postfix

C - E)) / (F + G))

- stack: ((* (
- output: [A B +]

FPE Infix to Postfix

- E)) / (F + G))

- stack: ((* (
- output: [A B + C]

FPE Infix to Postfix

E)) / (F + G))

- stack: ((* (-
- output: [A B + C]

FPE Infix to Postfix

)) / (F + G))

- stack: ((* (-
- output: [A B + C E]

FPE Infix to Postfix

) / (F + G))

- stack: ((*
- output: [A B + C E -]

FPE Infix to Postfix

/ (F + G))

- stack: (
- output: [A B + C E - *]

FPE Infix to Postfix

(F + G))

- stack: (/
- output: [A B + C E - *]

FPE Infix to Postfix

F + G))

- stack: (/ (
- output: [A B + C E - *]

FPE Infix to Postfix

+ G))

- stack: (/ (
- output: [A B + C E - * F]

FPE Infix to Postfix

G))

- stack: (/ (+
- output: [A B + C E - * F]

FPE Infix to Postfix

))

- stack: (/ (+
- output: [A B + C E - * F G]

FPE Infix to Postfix

)

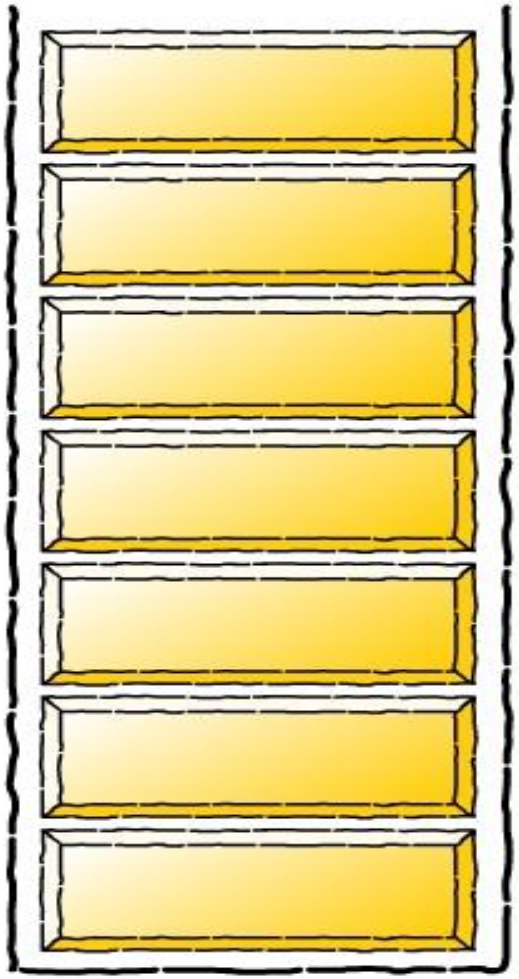
- stack: (/
- output: [A B + C E - * F G +]

FPE Infix to Postfix

- stack: <empty>
- output: [A B + C E - * F G + /]

EXAMPLE 2

Infix to Postfix Conversion

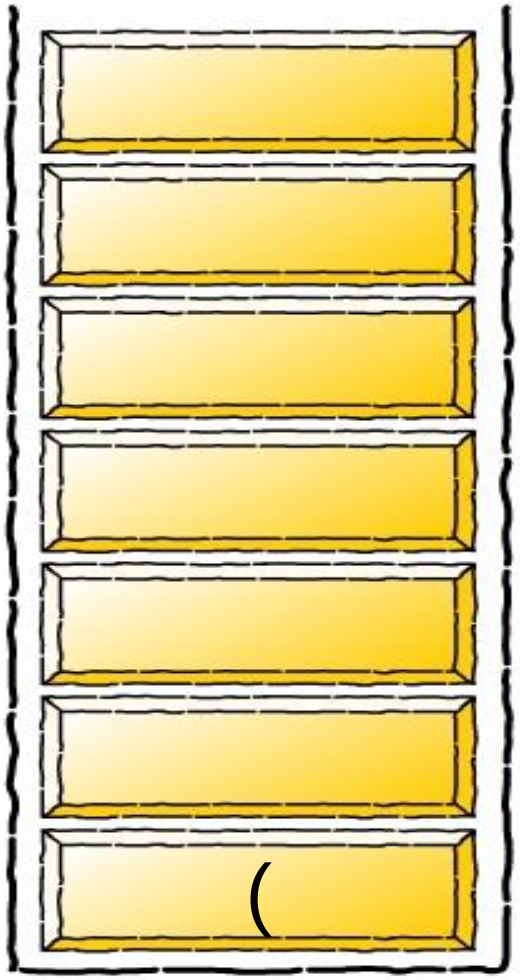


infixVect

(a + b – c) * d – (e + f)

postfixVect

Infix to Postfix Conversion

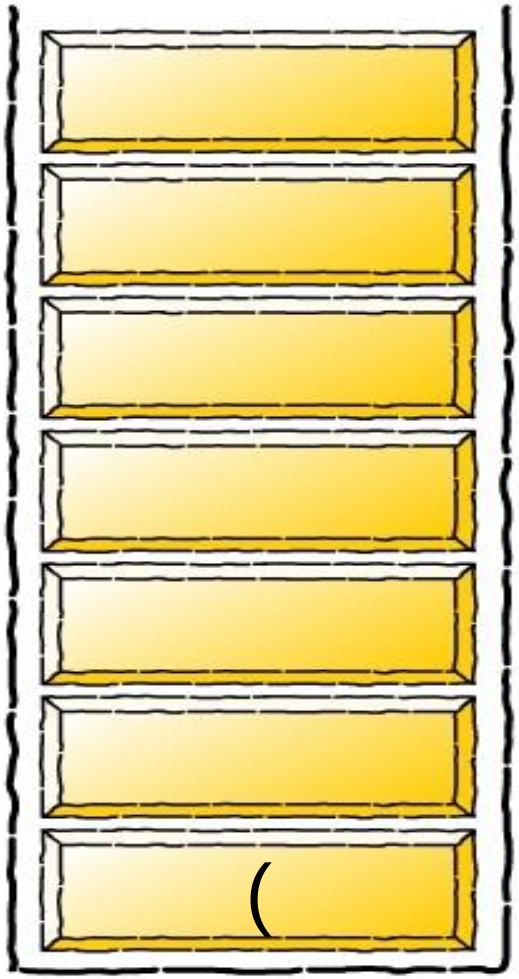


infixVect

a + b – c) * d – (e + f)

postfixVect

Infix to Postfix Conversion



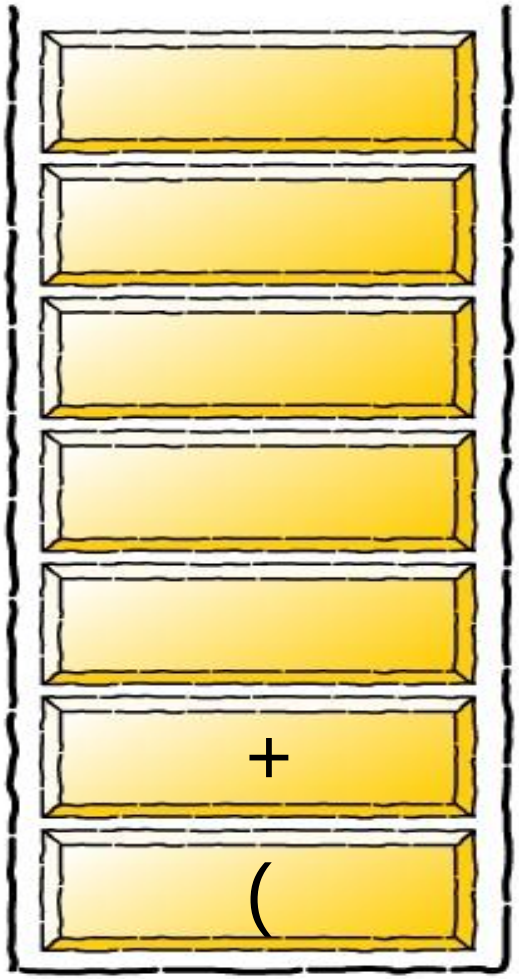
infixVect

+ b – c) * d – (e + f)

postfixVect

a

Infix to Postfix Conversion



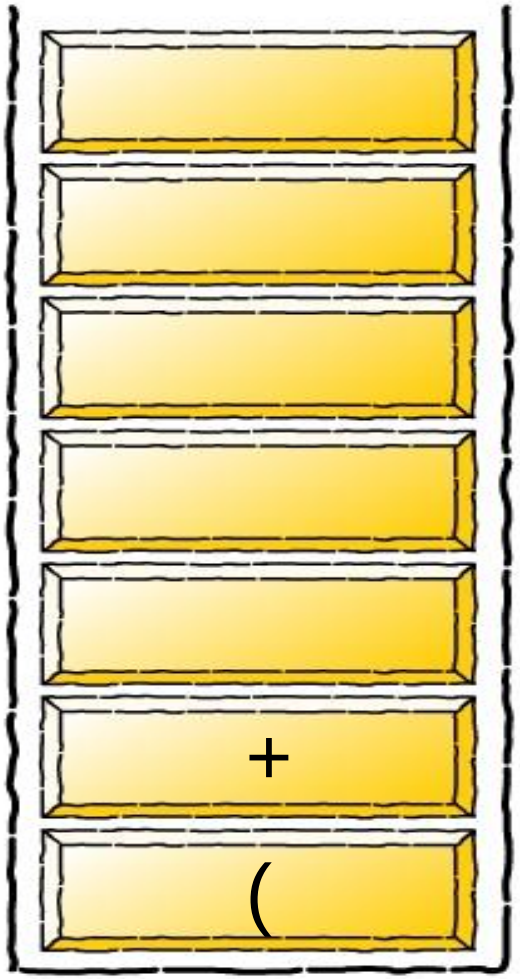
infixVect

b – c) * d – (e + f)

postfixVect

a

Infix to Postfix Conversion



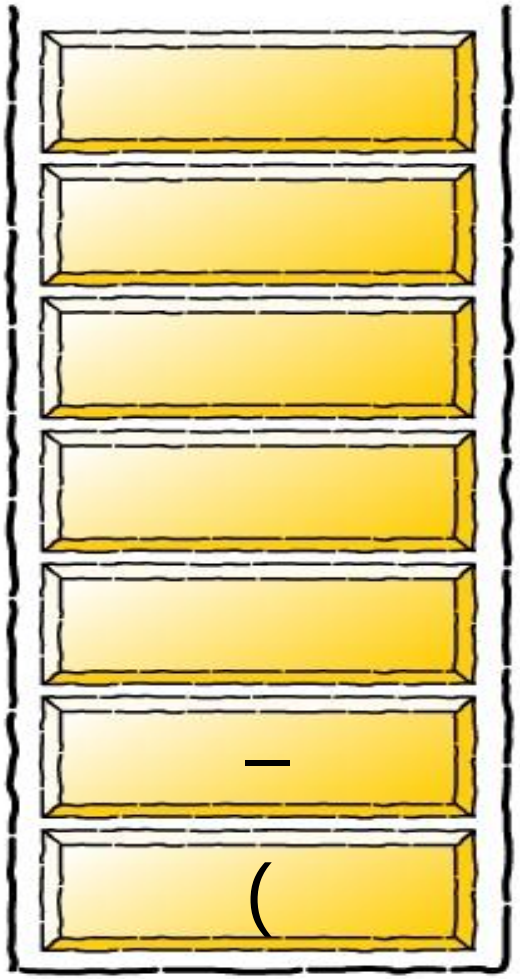
infixVect

– c) * d – (e + f)

postfixVect

a b

Infix to Postfix Conversion



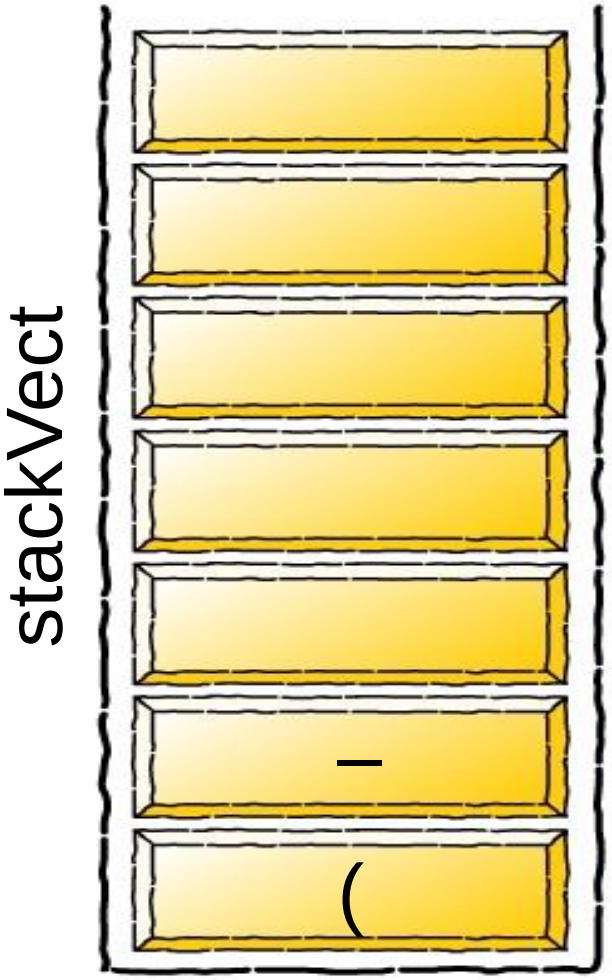
infixVect

c) * d – (e + f)

postfixVect

a b +

Infix to Postfix Conversion



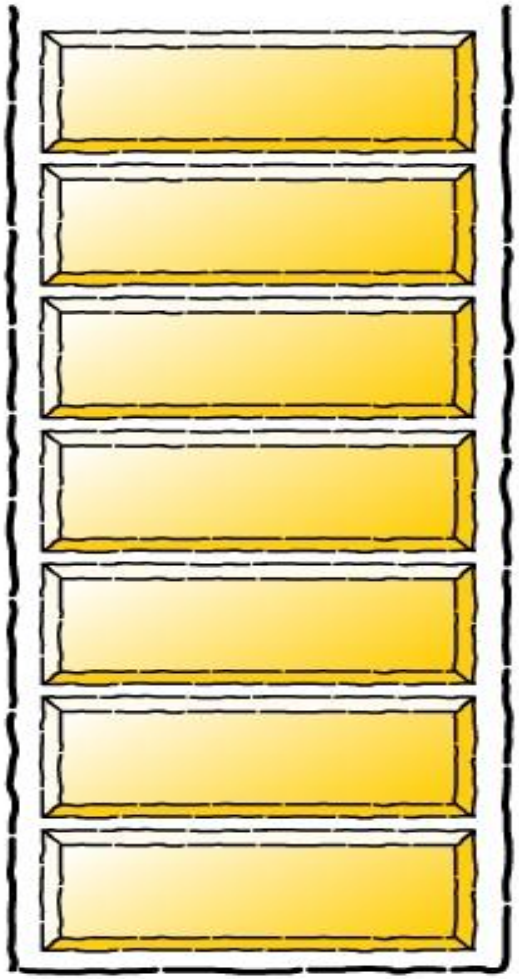
infixVect

) * d – (e + f)

postfixVect

a b + c

Infix to Postfix Conversion



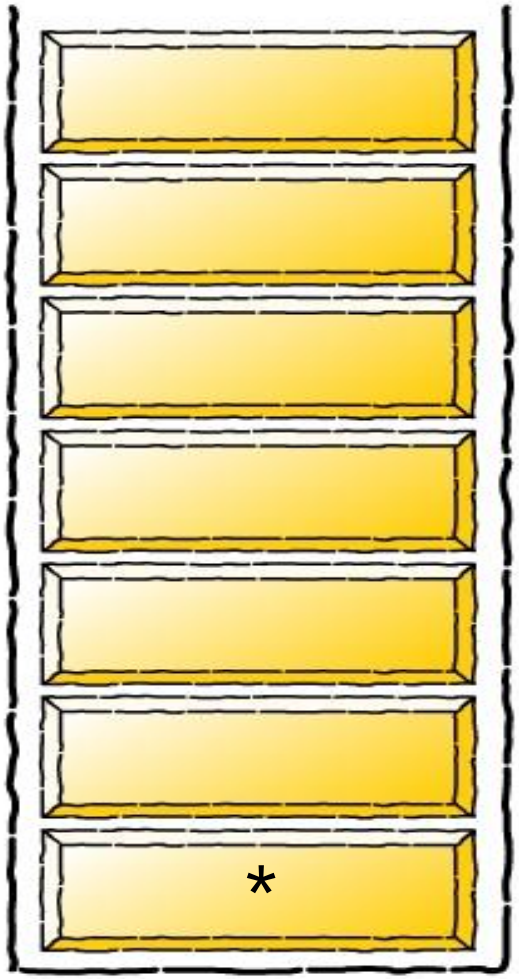
infixVect

* d – (e + f)

postfixVect

a b + c –

Infix to Postfix Conversion



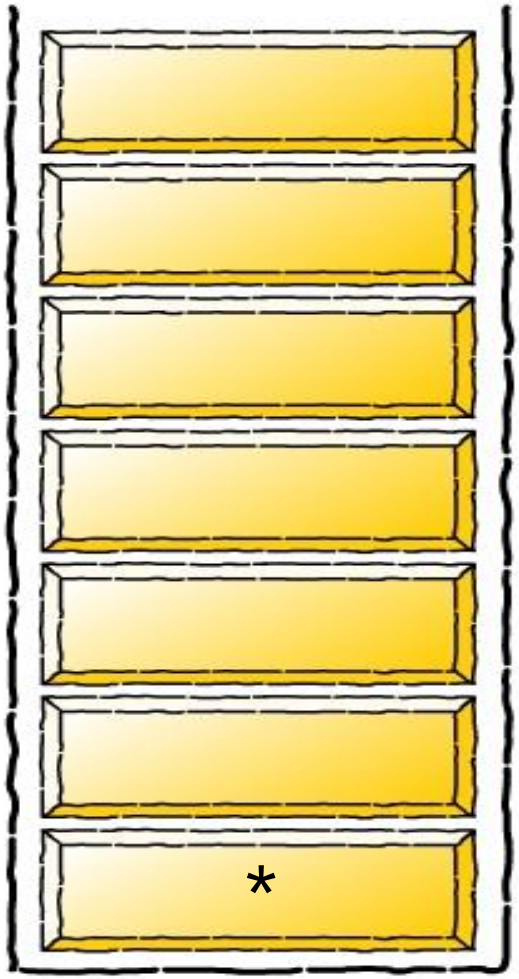
infixVect

d – (e + f)

postfixVect

a b + c –

Infix to Postfix Conversion



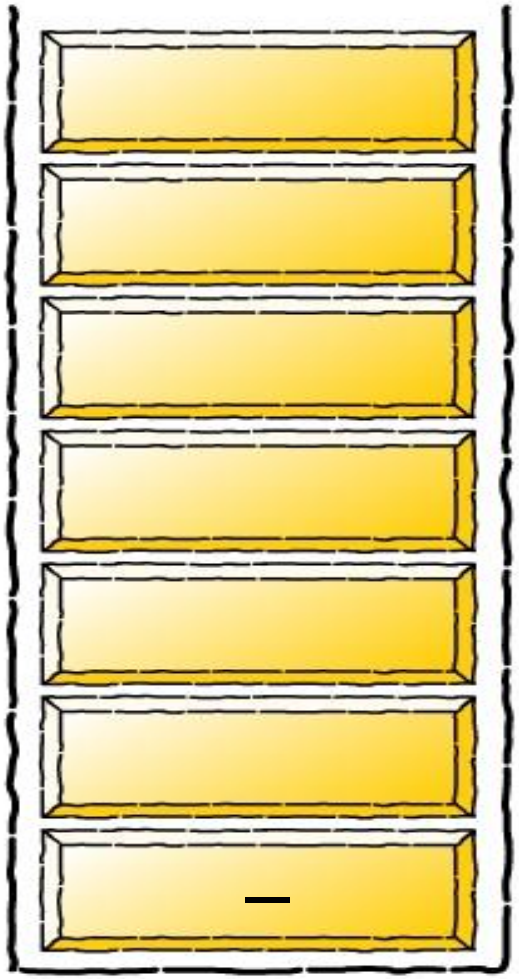
infixVect

– (e + f)

postfixVect

a b + c – d

Infix to Postfix Conversion



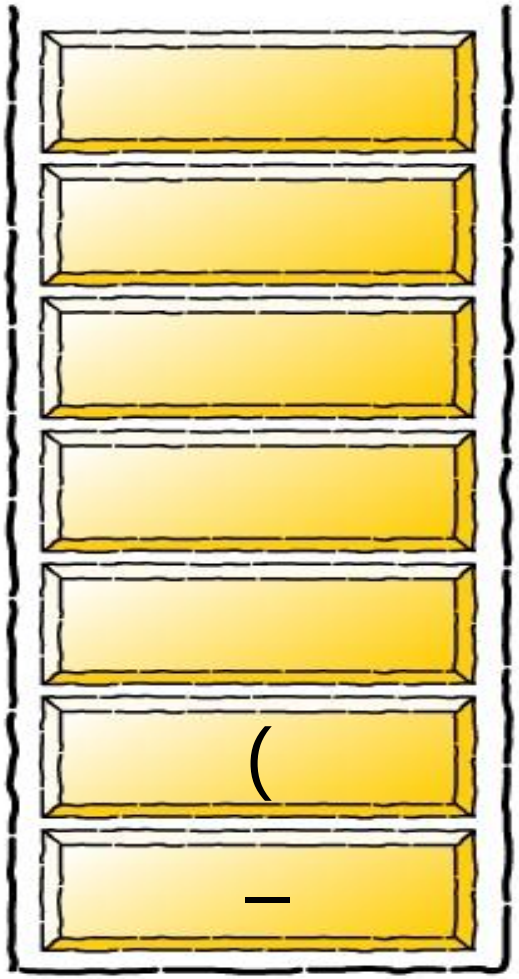
infixVect

(e + f)

postfixVect

a b + c – d *

Infix to Postfix Conversion



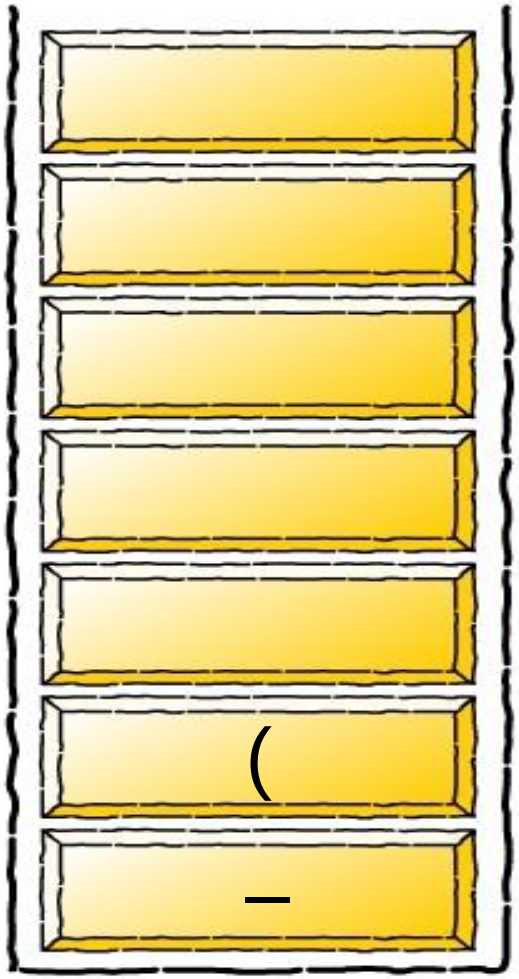
infixVect

e + f)

postfixVect

a b + c – d *

Infix to Postfix Conversion



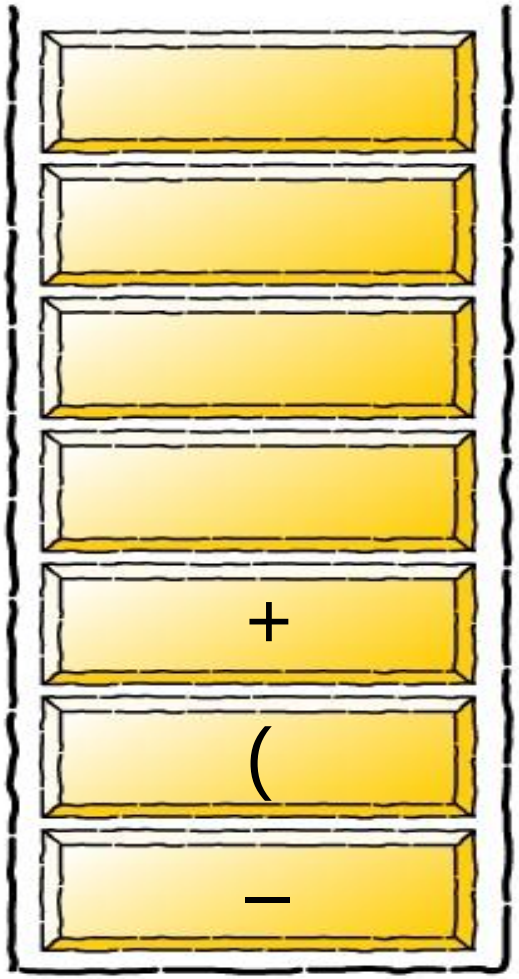
infixVect

+ f)

postfixVect

a b + c – d * e

Infix to Postfix Conversion



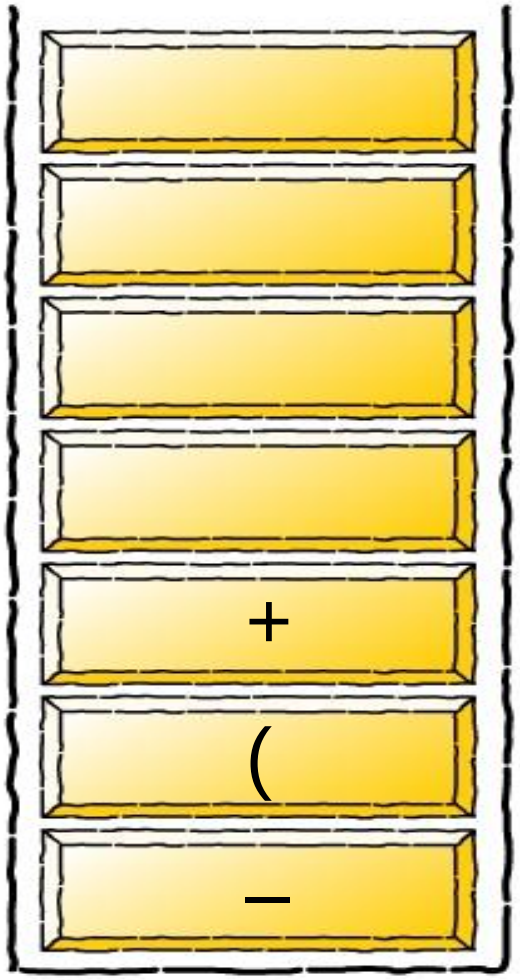
infixVect

f)

postfixVect

a b + c – d * e

Infix to Postfix Conversion



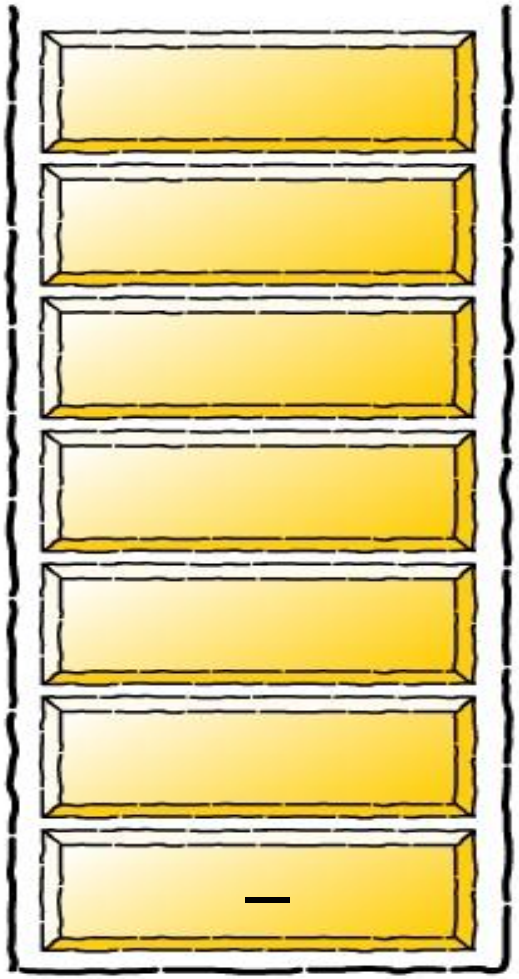
infixVect

)

postfixVect

a b + c – d * e f

Infix to Postfix Conversion

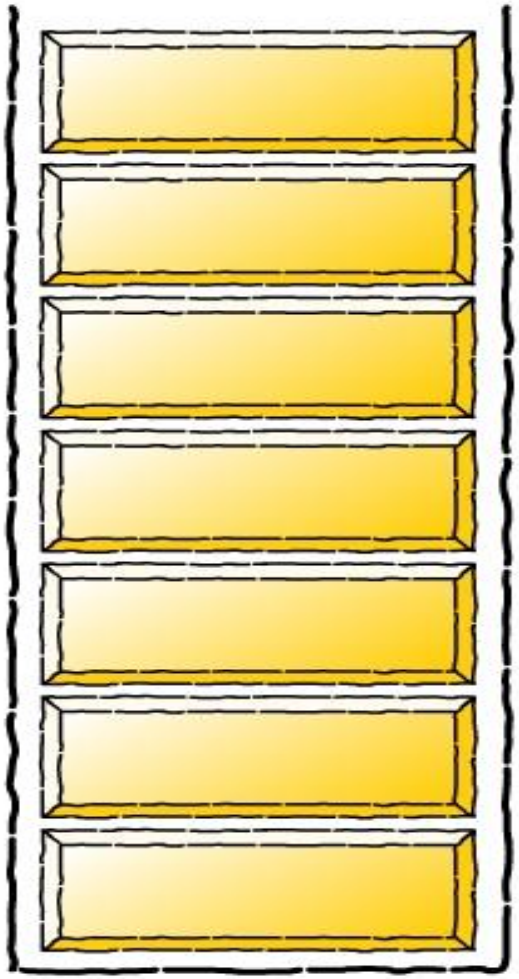


infixVect

postfixVect

a b + c – d * e f +

Infix to Postfix Conversion



infixVect

postfixVect

a b + c – d * e f + –

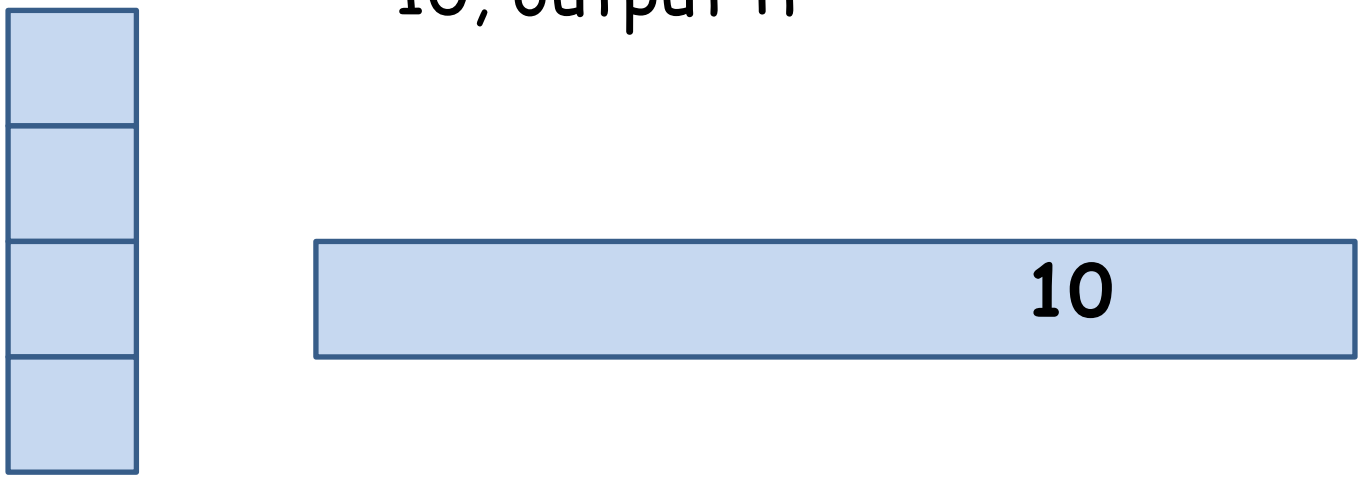
EXAMPLE 3

Infix to Postfix Conversion

Transform Infix to Postfix

Ex: 10 + 2 * 8 - 3

- We see the first number 10, output it

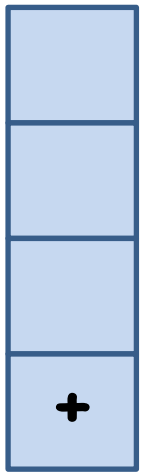


Infix to Postfix Conversion

Transform Infix to Postfix

Ex: 10 + 2 * 8 - 3

- We see the first operator +, push it into the stack

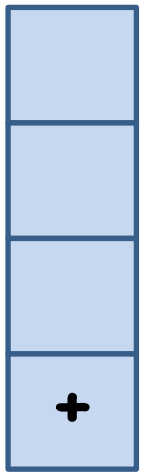


Infix to Postfix Conversion

Transform Infix to Postfix

Ex: 10 + 2 * 8 - 3

- We see the number 2, output it

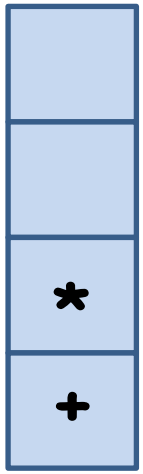


Infix to Postfix Conversion

Transform Infix to Postfix

Ex: 10 + 2 * 8 - 3

- We see the operator *, since the top operator in the stack, +, **has lower priority than** *, push(*)

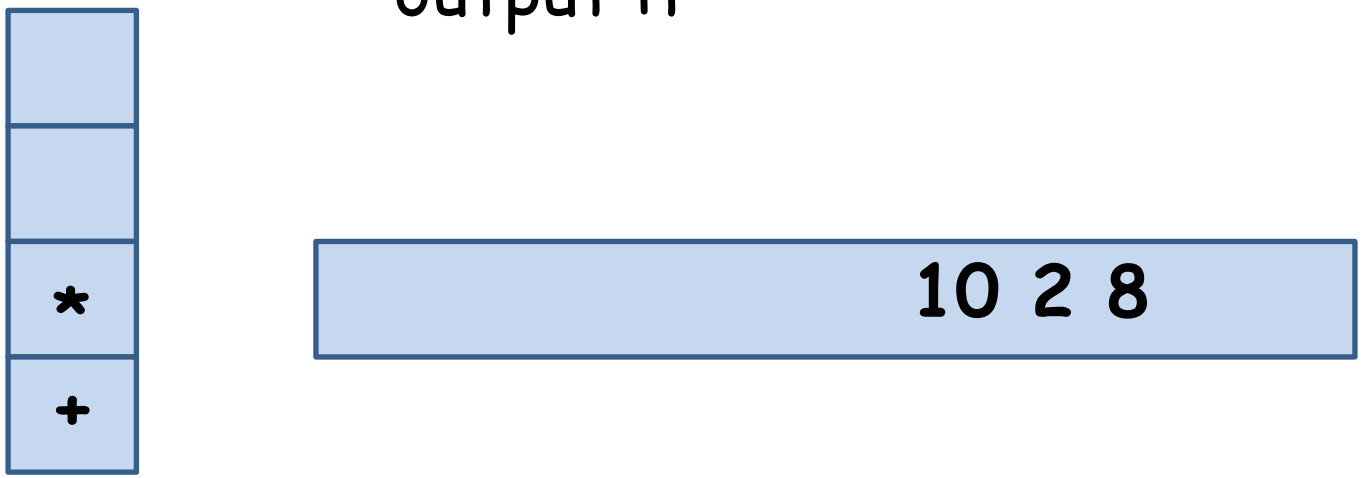


Infix to Postfix Conversion

Transform Infix to Postfix

Ex: 10 + 2 * 8 - 3

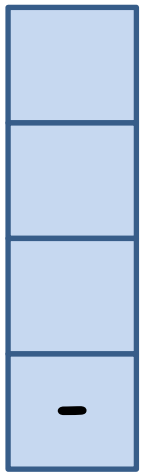
- We see the number 8, output it



Infix to Postfix Conversion

Transform Infix to Postfix

Ex: 10 + 2 * 8 - 3



- We see the operator -, because its **priority is lower** then *, we pop. Also, because + is on the left of it, we pop +, too. Then we push(-)

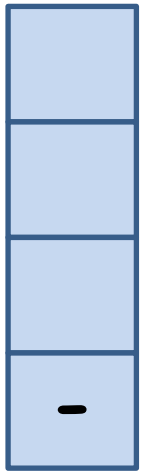
10 2 8 * +

Infix to Postfix Conversion

Transform Infix to Postfix

Ex: 10 + 2 * 8 - 3

- We see the number 3, output it



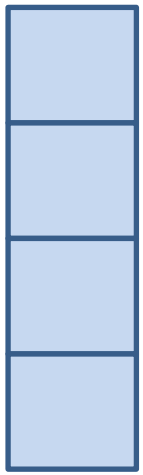
10 2 8 * + 3

Infix to Postfix Conversion

Transform Infix to Postfix

Ex: 10 + 2 * 8 - 3

- Because the expression is ended, we pop all the operators in the stack



10 2 8 * + 3 -

EVALUATING A POSTFIX EXPRESSION

Evaluating a Postfix Expression

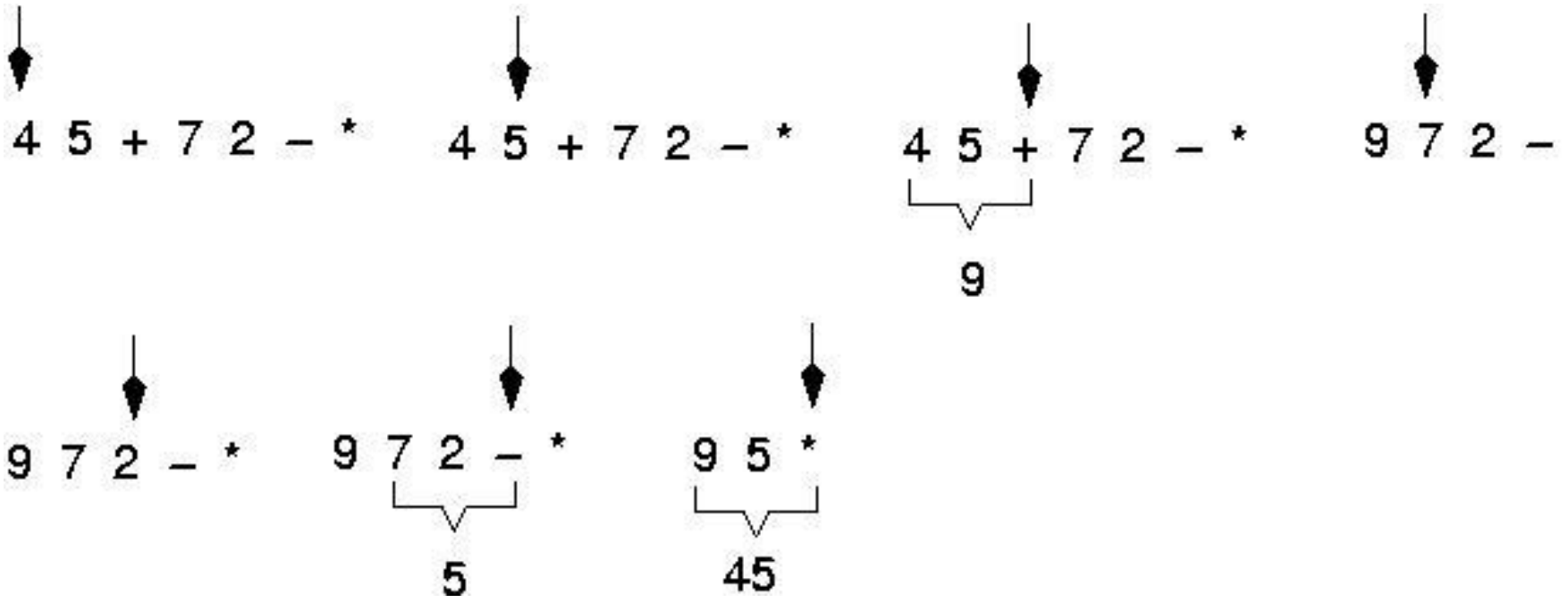
Algorithm: P is the given postfix expression.

1. Scan P from left to right and repeat steps 3 & 4 for each element of P until the sentinel ")" is encountered.
2. If an operand is encountered, push it on STACK
3. If an operator is encountered, then:
    a. Pop two operands from STACK: A & B
    b. Evaluate: A operator B
    c. Push result on STACK
4. Set value equal to the top element on STACK
5. Exit

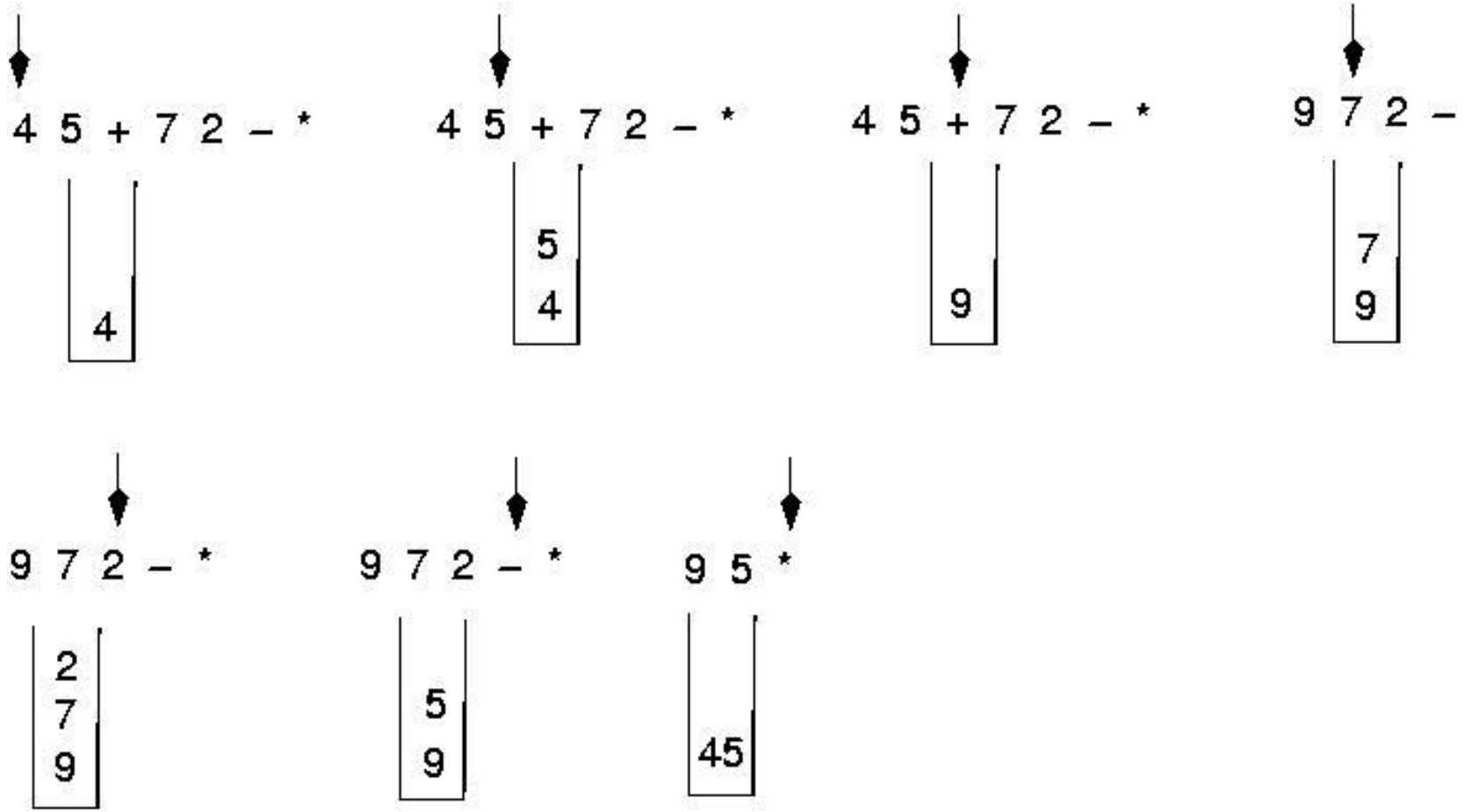
Evaluating a Postfix Expression


```
WHILE more input items exist
    Get an item
    IF item is an operand
      stack.Push(item)
    ELSE
      stack.Pop(operand2)
      stack.Pop(operand1)
      Compute result
      stack.Push(result)
stack.Pop(result)
```

Evaluating a Postfix Expression



Evaluating a Postfix Expression

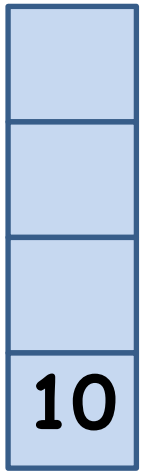


Evaluating a Postfix Expression

From Postfix to Answer

Ex: 10 2 8 * + 3 -

- First, push(10) into the stack

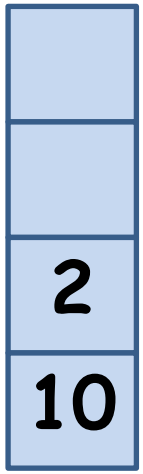


Evaluating a Postfix Expression

From Postfix to Answer

Ex: 10 2 8 * + 3 -

- Then, push(2) into the stack

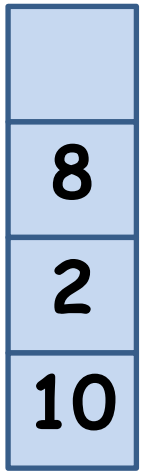


Evaluating a Postfix Expression

From Postfix to Answer

Ex: 10 2 8 * + 3 -

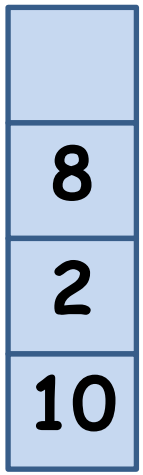
- Push(8) into the stack



Evaluating a Postfix Expression

From Postfix to Answer

Ex: 10 2 8 * + 3 -



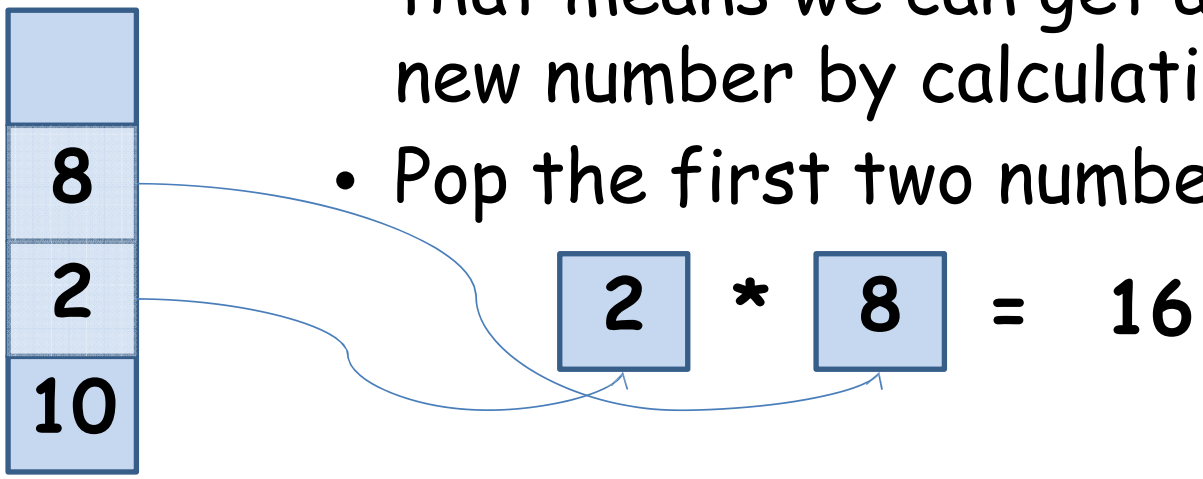
- Now we see an operator *, that means we can get an new number by calculation

Evaluating a Postfix Expression

From Postfix to Answer

Ex: 10 2 8 * + 3 -

- Now we see an operator *, that means we can get an new number by calculation
- Pop the first two numbers

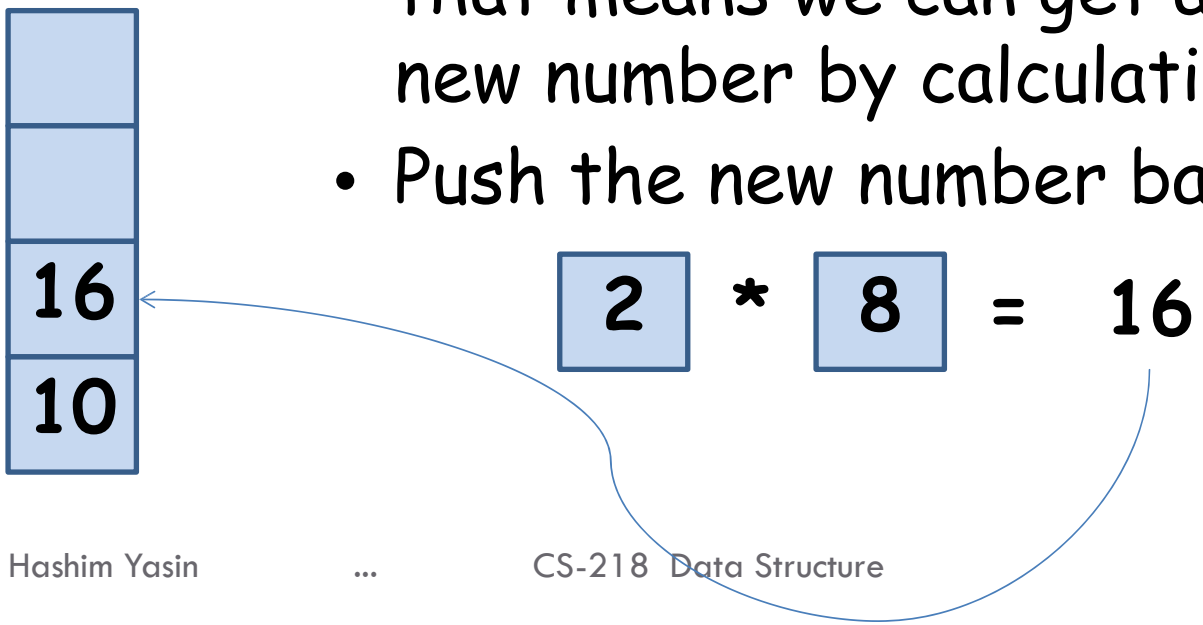


Evaluating a Postfix Expression

From Postfix to Answer

Ex: 10 2 8 * + 3 -

- Now we see an operator *, that means we can get an new number by calculation
- Push the new number back

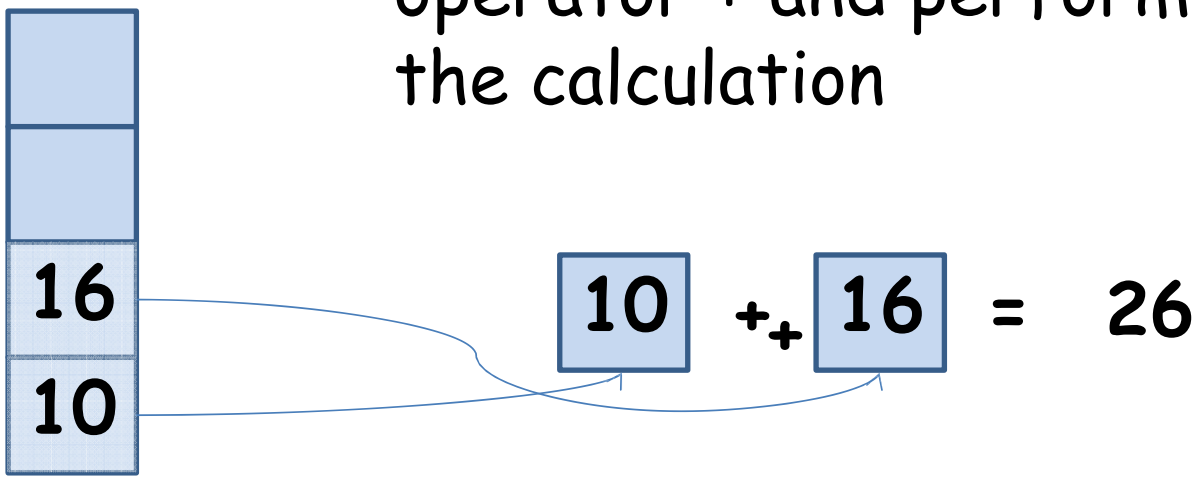


Evaluating a Postfix Expression

From Postfix to Answer

Ex: 10 2 8 * + 3 -

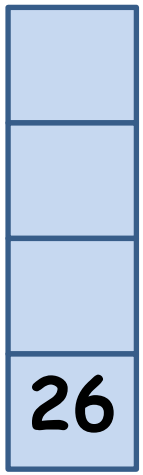
- Then we see the next operator + and perform the calculation



Evaluating a Postfix Expression

From Postfix to Answer

Ex: 10 2 8 * + 3 -



- Then we see the next operator + and perform the calculation
- Push the new number back

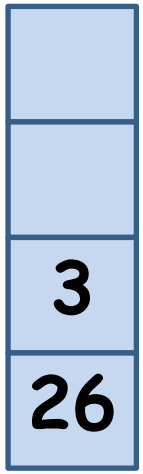
10 + 16 = 26

Evaluating a Postfix Expression

From Postfix to Answer

Ex: 10 2 8 * + 3 -

- We see the next number 3
- Push (3) into the stack

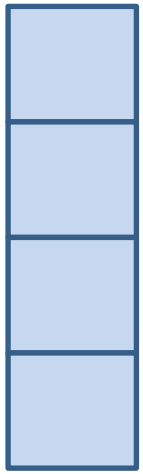


Evaluating a Postfix Expression

Compute the Answer

Ex: 10 2 8 * + 3 -

- The last operation

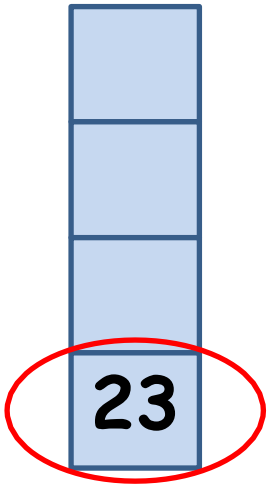


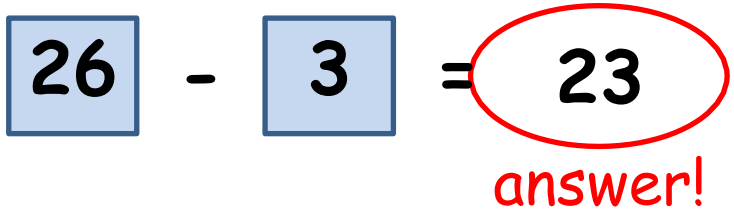
Evaluating a Postfix Expression

From Postfix to Answer

Ex: 10 2 8 * + 3 -

- The last operation





P= 623+-382/+*2$3+

S.N.	Symbol Scan	Operand 1	Operand 2	Value	STACK
1.	6				6
2.	2				6,2
3.	3				6,2,3
4.	+	2	3	5	6,5
5.	-	6	5	1	1

Next ?

P= 623+-382/+*2$3+

S.N.	Symbol Scan	Operand 1	Operand 2	Value	STACK
1.	6				6
2.	2				6,2
3.	3				6,2,3
4.	+	2	3	5	6,5
5.	-	6	5	1	1
6.	3				1,3
7.	8				1,3,8
8.	2				1,3,8,2
9.	/	8	2	4	1,3,4
10.	+	3	4	7	1,7
11.	*	1	7	7	7
12.	2				7,2
13.	$	7	2	49	49
14.	3				49,3
15.	+	49	3	52	52

Reading Materials

- Nell Dale – Chapter#4
- Schaum's Outlines – Chapter#6
- D. S. Malik – Chapter#7
- http://www.cs.man.ac.uk/~pjj/cs2121/fix.html