CS-2001
DATA STRUCTURE

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AVL TREE

AVL Trees

To maintain the height balanced property of the AVL tree after insertion or deletion, it is necessary to perform a *transformation* on the tree so that,

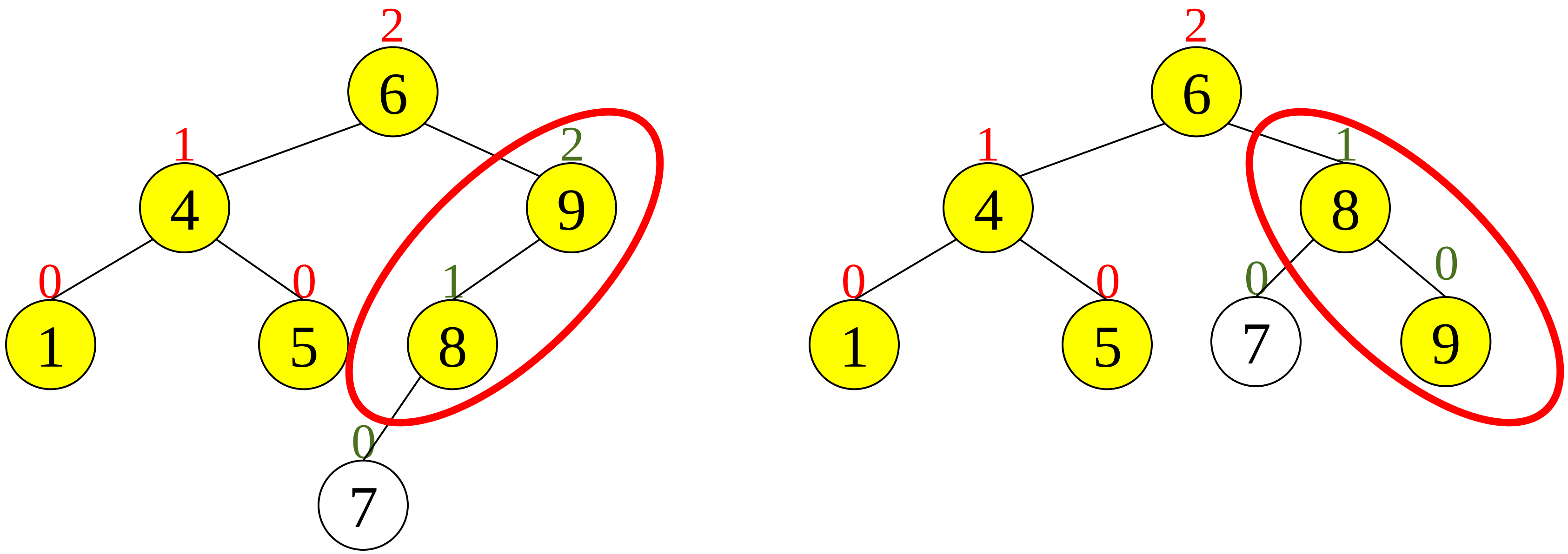
(1) the *in-order traversal of the transformed tree is the same as for the original tree* (i.e., the new tree remains a binary search tree).

(2) the tree after transformation is height-balanced.

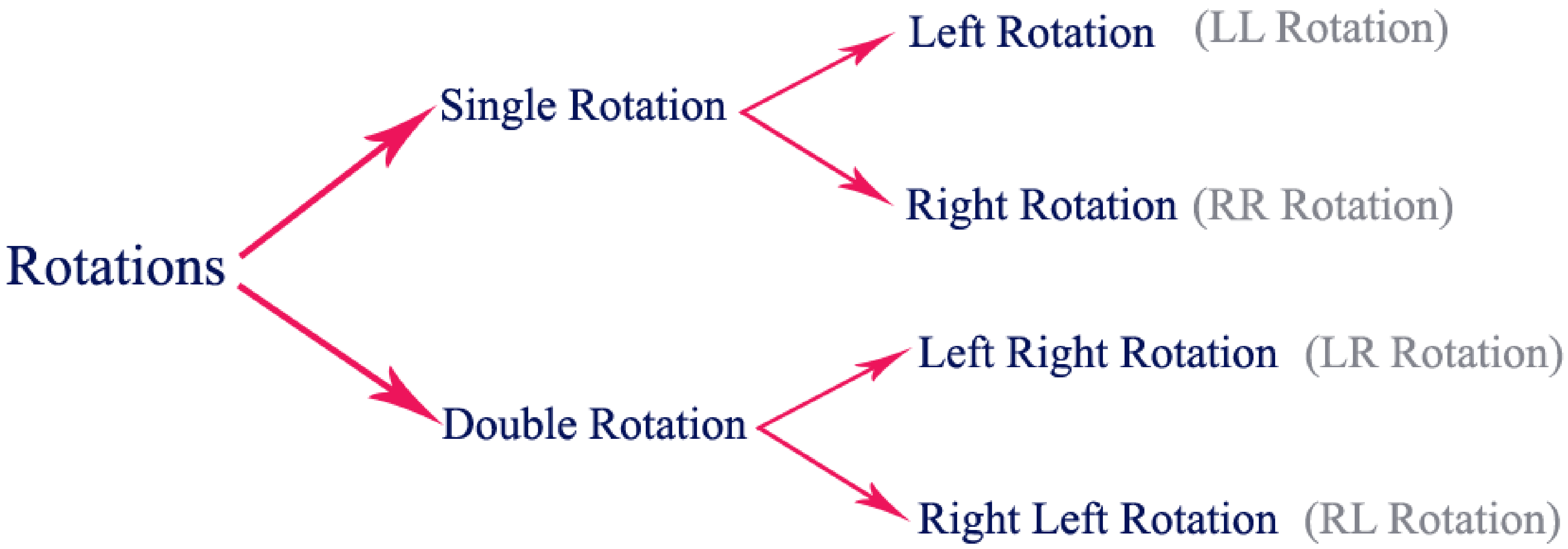
Insertion in AVL Trees

- Insert operation may cause balance factor to become 2 or –2 for some node
  - only nodes on the path from insertion point to root node have possibly changed in height
  - Follow the path up to the root, find the first node (i.e., deepest) whose new balance violates the AVL condition. Call this node *a*
  - If a new balance factor (the difference h_{left}-h_{right}) is 2 or –2, adjust tree by *rotation* around the node

AVL Tree



AVL Tree … Rotations

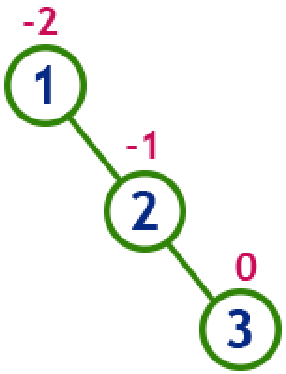


Rotations
Single Rotation
Double Rotation
Left Rotation (LL Rotation)
Right Rotation (RR Rotation)
Left Right Rotation (LR Rotation)
Right Left Rotation (RL Rotation)

LL Rotation

- In LL Rotation, every node moves *one position to **left** from the current position*.

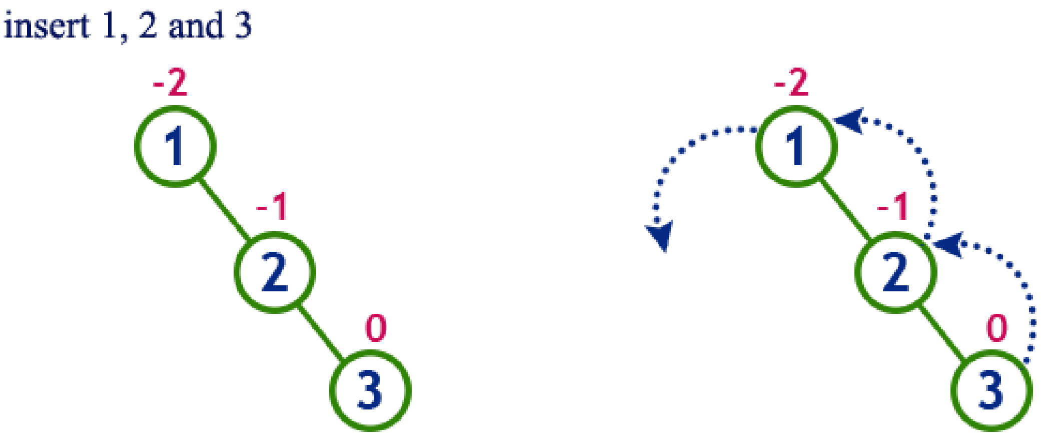
insert 1, 2 and 3



Tree is imbalanced

LL Rotation

- In LL Rotation, every node moves *one position to **left** from the current position*.

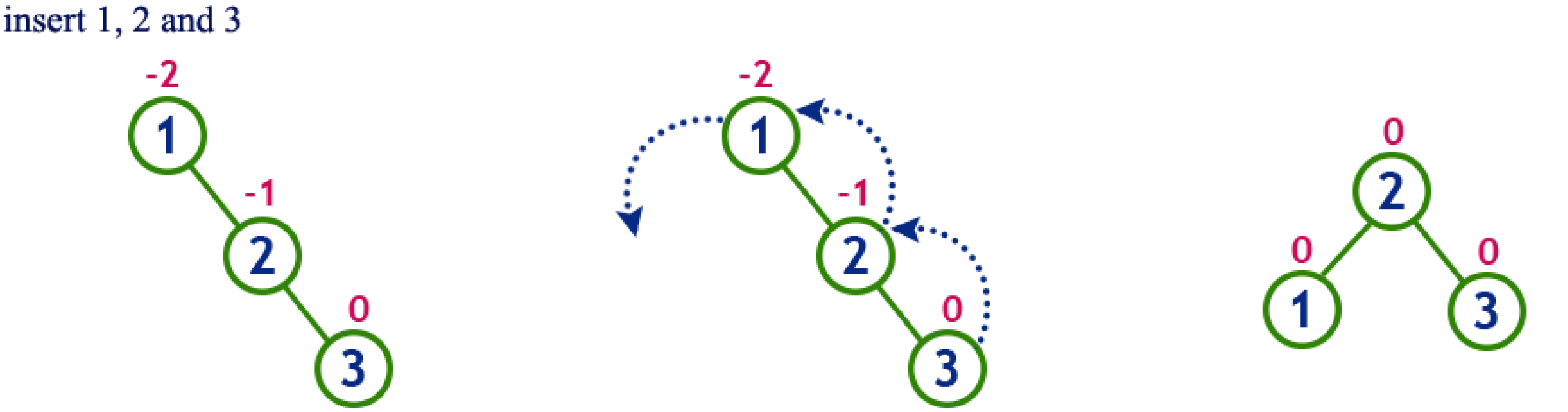


Tree is imbalanced

To make balanced we use LL Rotation which moves nodes one position to left

LL Rotation

- In LL Rotation, every node moves *one position to **left** from the current position*.



Tree is imbalanced

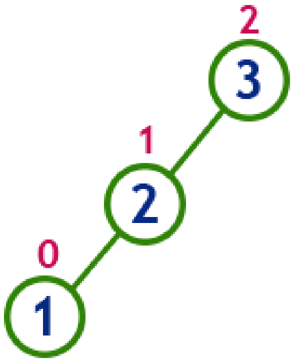
To make balanced we use LL Rotation which moves nodes one position to left

After LL Rotation Tree is Balanced

RR Rotation

- In RR Rotation, every node moves *one position to* ***right*** *from the current position*.

insert 3, 2 and 1



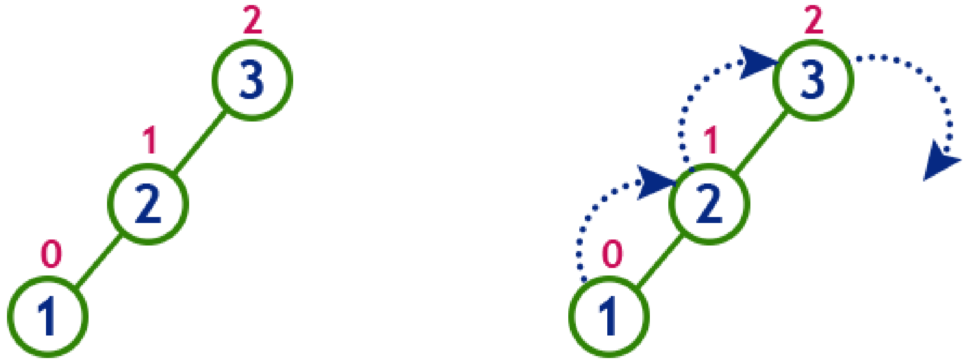
Tree is imbalanced

because node 3 has balance factor 2

RR Rotation

- In RR Rotation, every node moves *one position to* ***right*** *from the current position*.

insert 3, 2 and 1



Tree is imbalanced

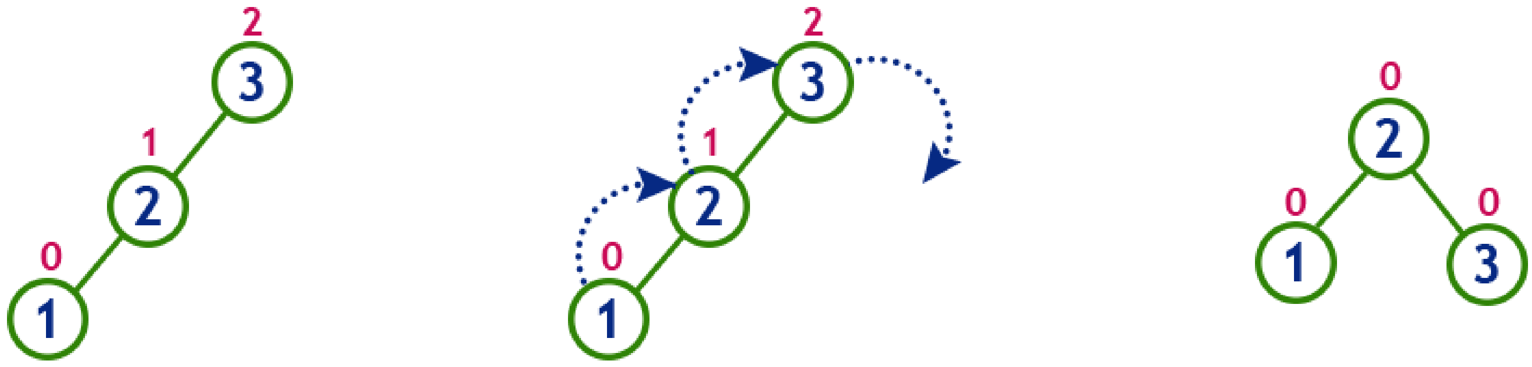
because node 3 has balance factor 2

To make balanced we use RR Rotation which moves nodes one position to right

RR Rotation

- In RR Rotation, every node moves *one position to* ***right*** *from the current position*.

insert 3, 2 and 1



Tree is imbalanced

because node 3 has balance factor 2

To make balanced we use RR Rotation which moves nodes one position to right

After RR Rotation Tree is Balanced

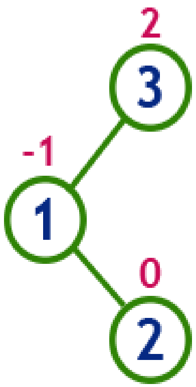
LR Rotation

- The LR Rotation is a *sequence of single left rotation followed by a single right rotation*.

- In LR Rotation, at first,
  - every node moves one position to the left and
  - one position to right from the current position.

LR Rotation

insert 3, 1 and 2

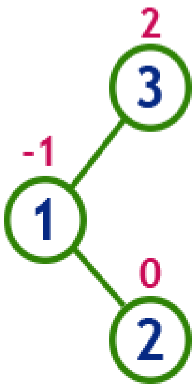


Tree is imbalanced

because node 3 has balance factor 2

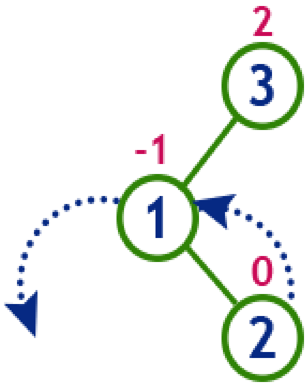
LR Rotation

insert 3, 1 and 2



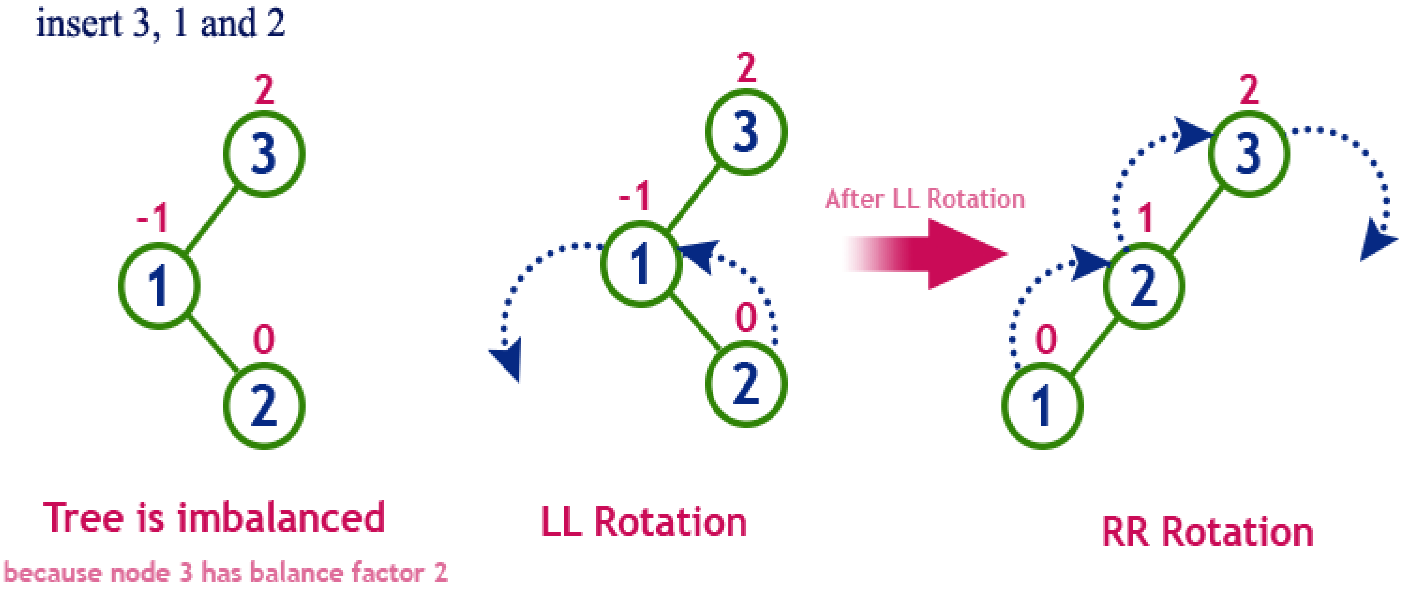
Tree is imbalanced

because node 3 has balance factor 2

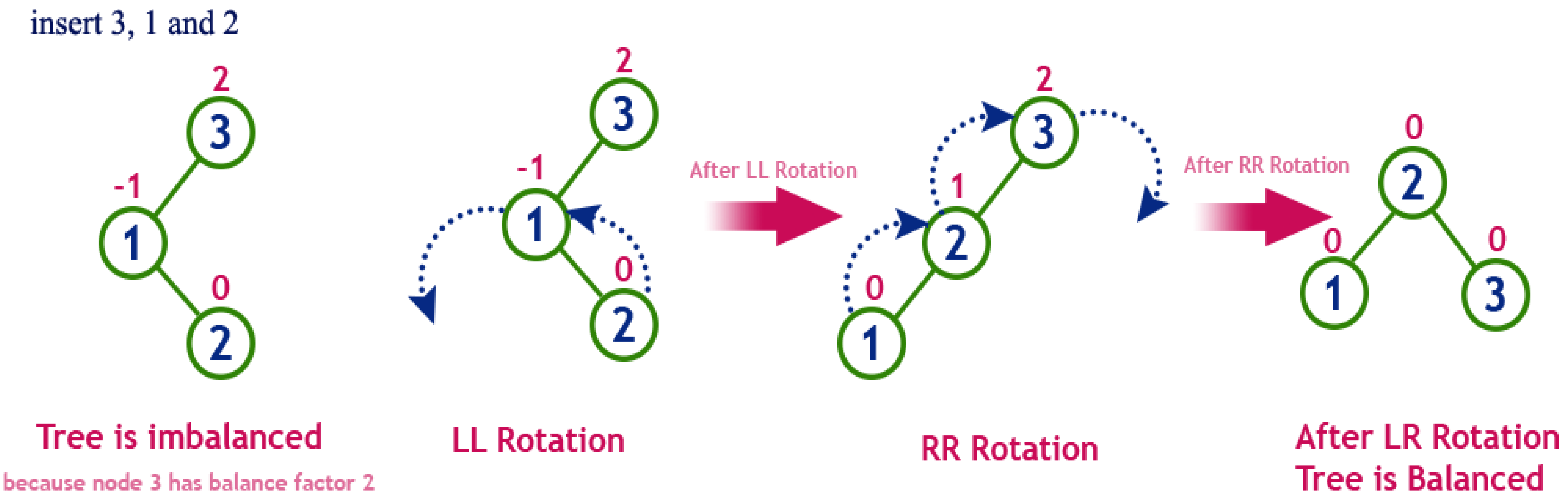


LL Rotation

LR Rotation


insert 3, 1 and 2
2
3
-1
1
0
2
Tree is imbalanced
because node 3 has balance factor 2
2
3
-1
1
0
2
LL Rotation
After LL Rotation
2
3
1
2
0
1
RR Rotation

LR Rotation


insert 3, 1 and 2
2
3
-1
1
0
2
2
3
-1
1
0
2
After LL Rotation
2
3
1
2
0
1
After RR Rotation
0
2
0
1
0
3
Tree is imbalanced
because node 3 has balance factor 2
LL Rotation
RR Rotation
After LR Rotation
Tree is Balanced

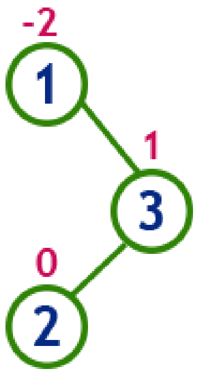
RL Rotation

- The RL Rotation is *sequence of single right rotation followed by single left rotation*.

- In RL Rotation, at first
  - every node moves one position to right and
  - one position to left from the current position.

RL Rotation

insert 1, 3 and 2

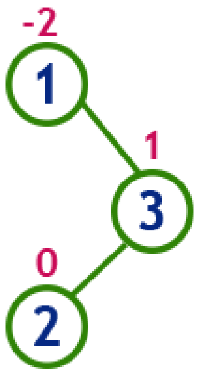


Tree is imbalanced

because node 1 has balance factor -2

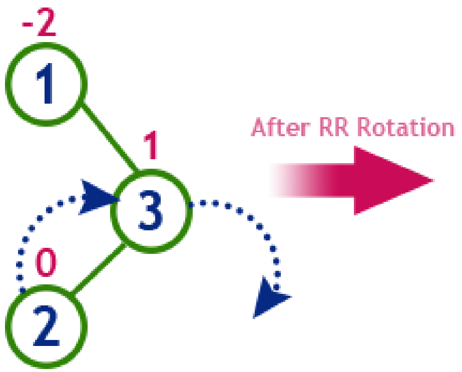
RL Rotation

insert 1, 3 and 2



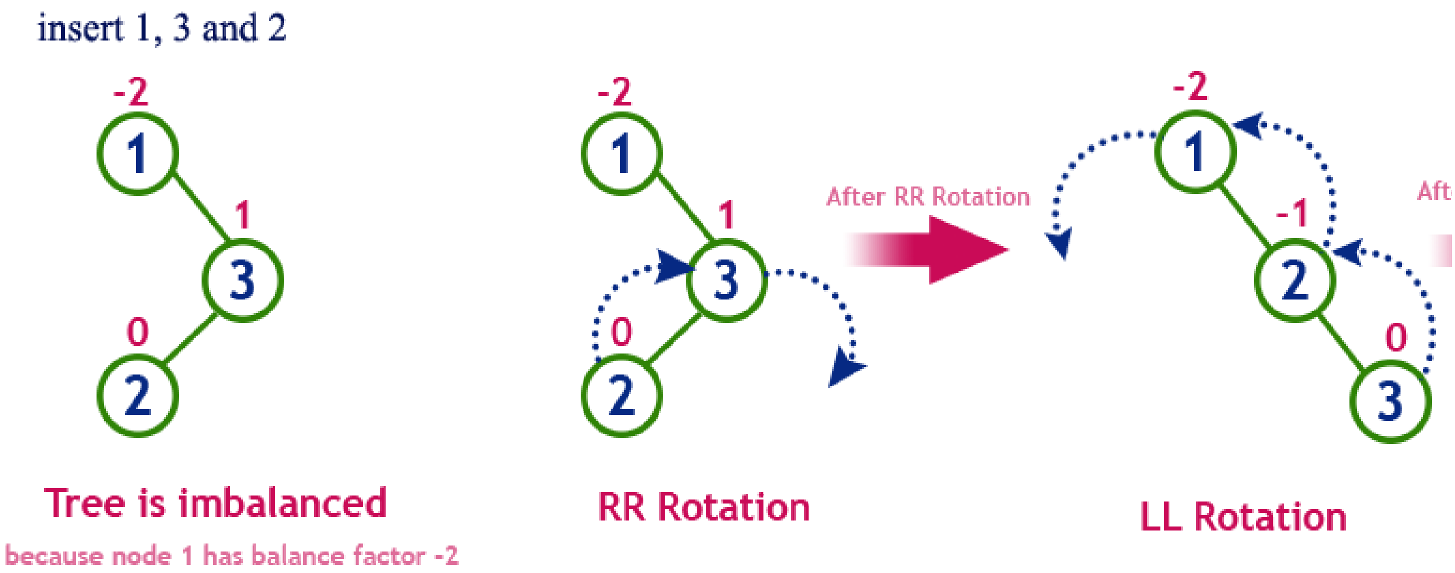
Tree is imbalanced

because node 1 has balance factor -2



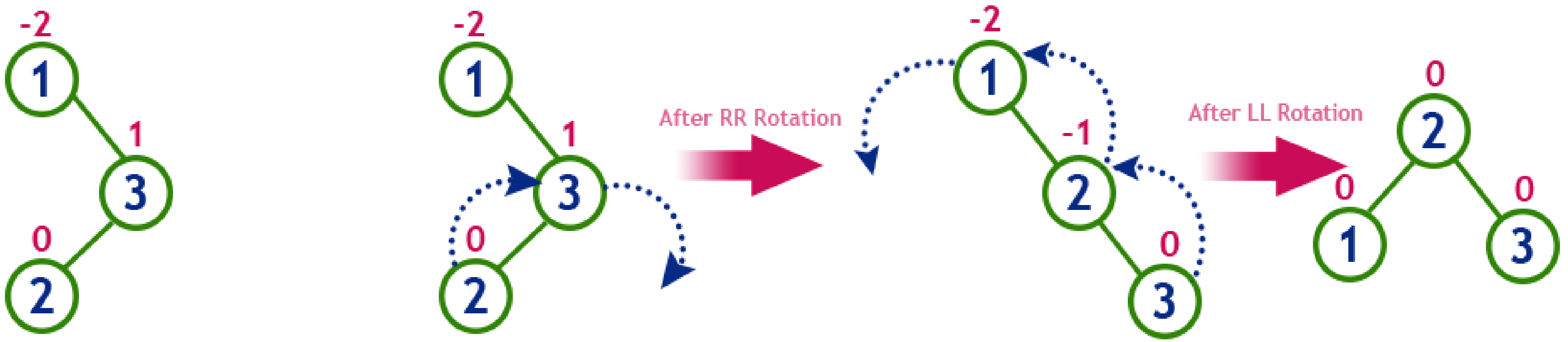
RR Rotation

RL Rotation


insert 1, 3 and 2
-2
1
1
3
0
2
Tree is imbalanced
because node 1 has balance factor -2
-2
1
1
3
0
2
RR Rotation
After RR Rotation
-2
1
-1
2
0
3
LL Rotation
Aft

RL Rotation

insert 1, 3 and 2



Tree is imbalanced

because node 1 has balance factor -2

RR Rotation

LL Rotation

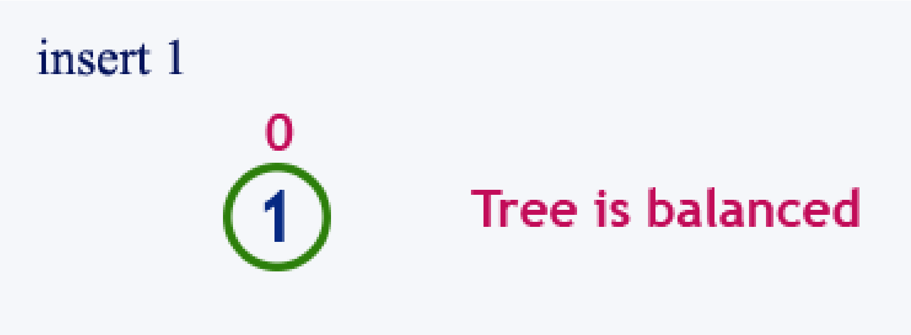
After RL Rotation
Tree is Balanced

EXAMPLE

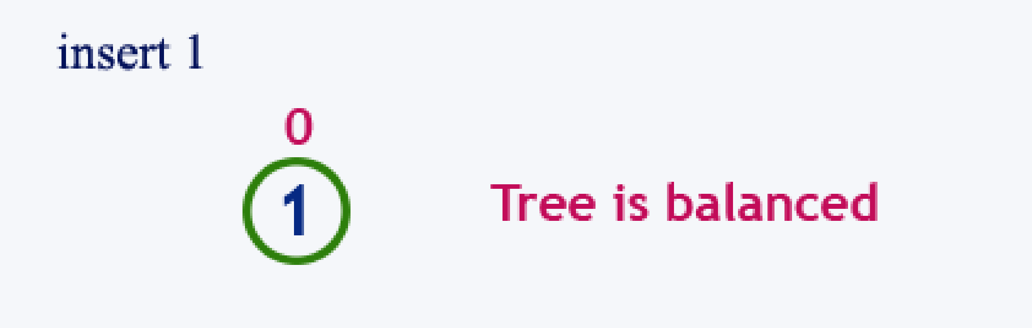
Example

Construct an AVL Tree by inserting numbers from 1 to 8.

Example

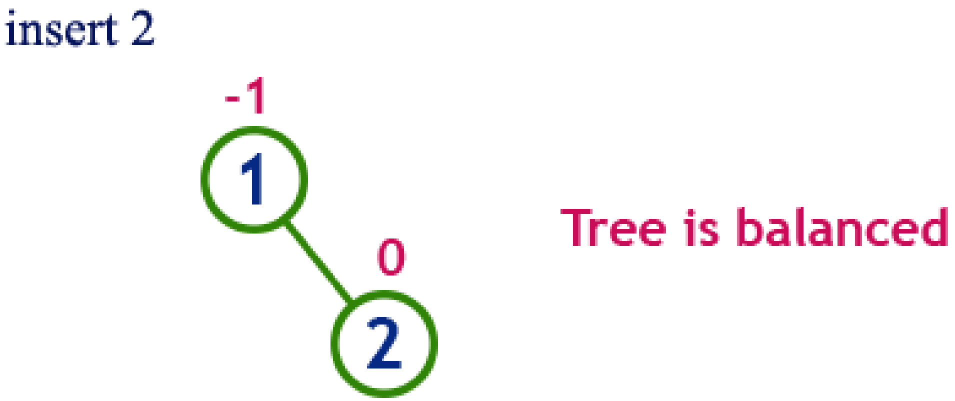


Example



insert 1

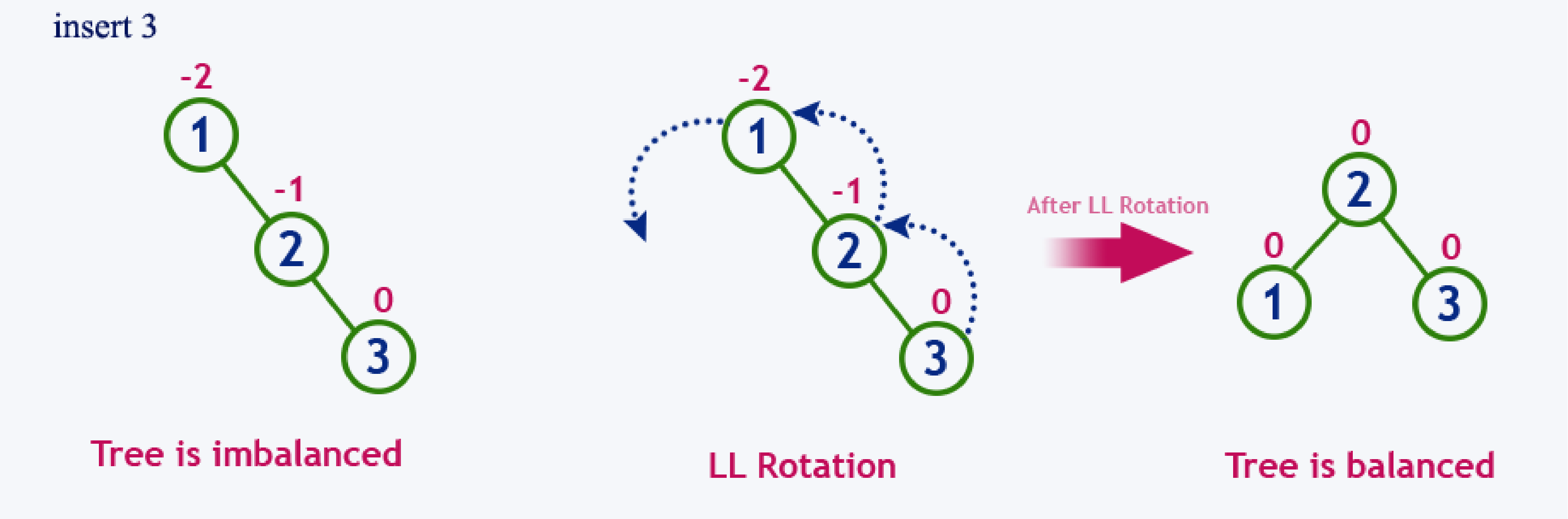
Tree is balanced



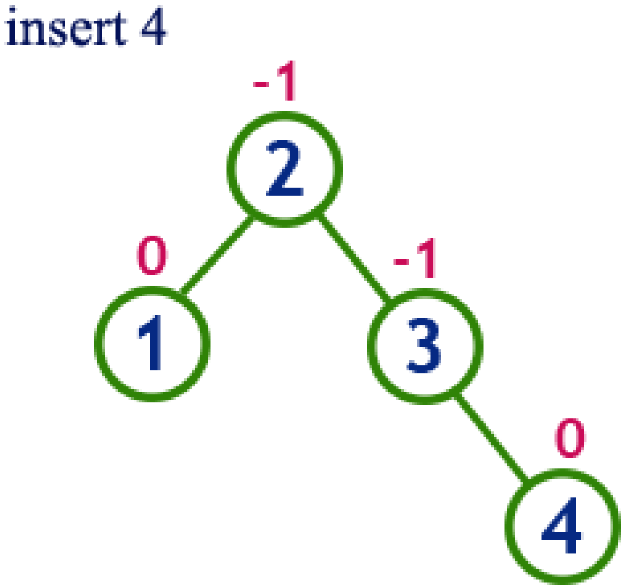
insert 2

Tree is balanced

Example

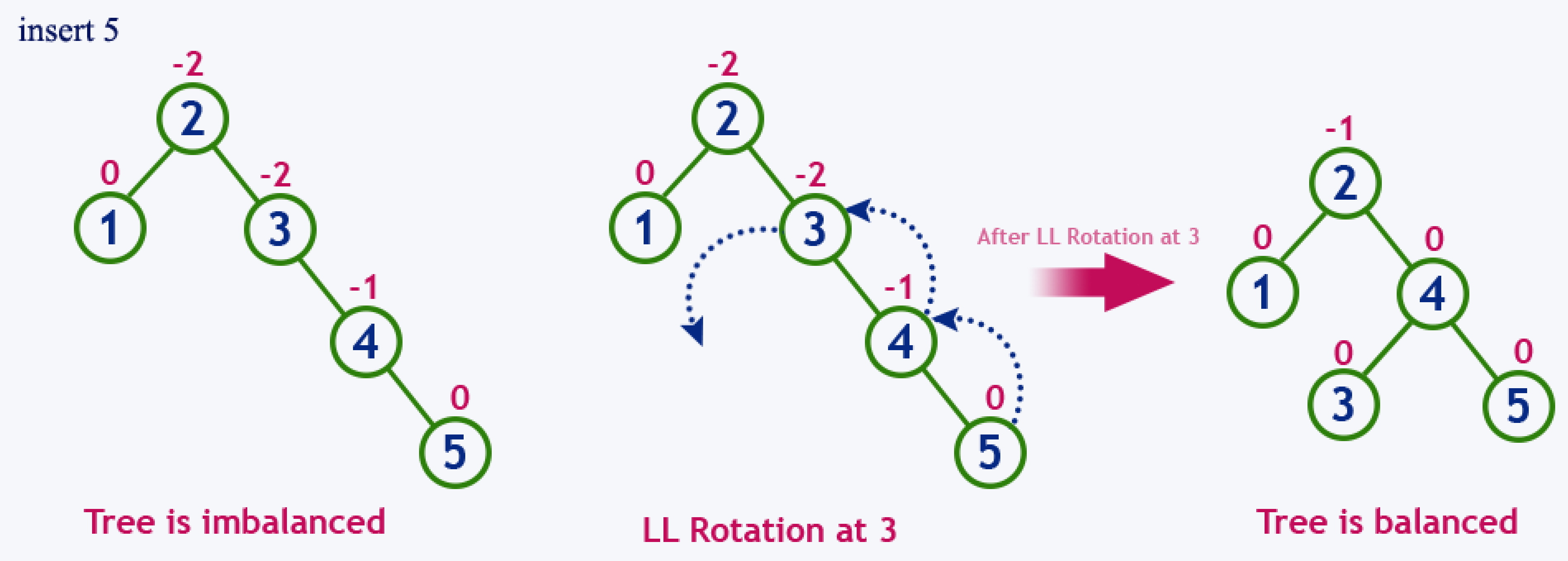

insert 3
-2
1
-1
2
0
3
Tree is imbalanced
-2
1
-1
2
0
3
LL Rotation
After LL Rotation
0
2
0
1
0
3
Tree is balanced

Example

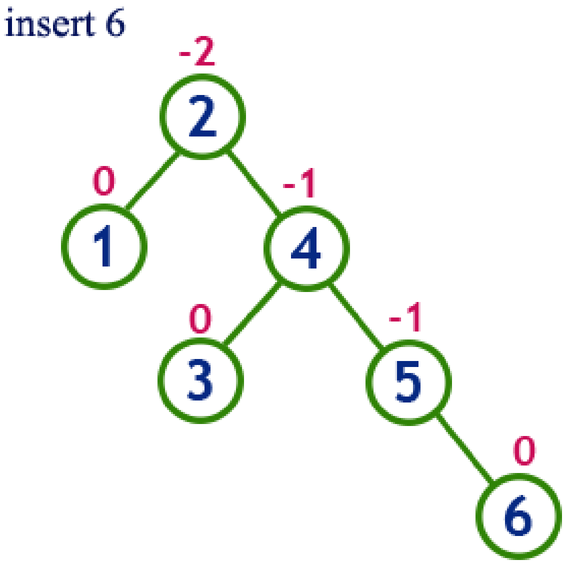


Tree is balanced

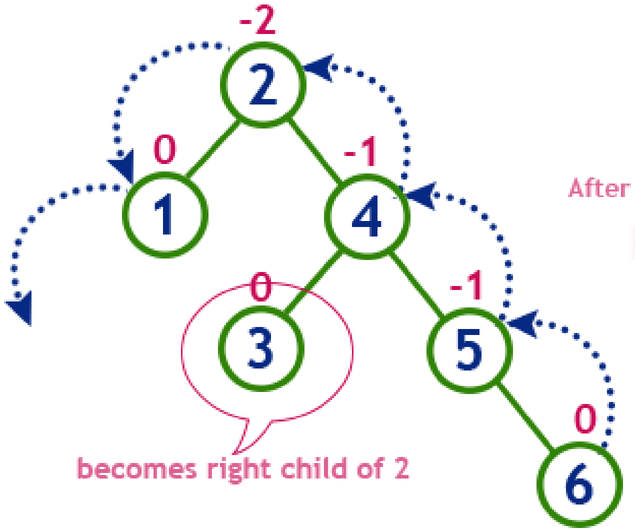
Example


insert 5
-2
2
0
1
-2
3
-1
4
0
5
Tree is imbalanced
-2
2
0
1
-2
3
-1
4
0
5
LL Rotation at 3
After LL Rotation at 3
-1
2
0
1
0
4
0
3
0
5
Tree is balanced

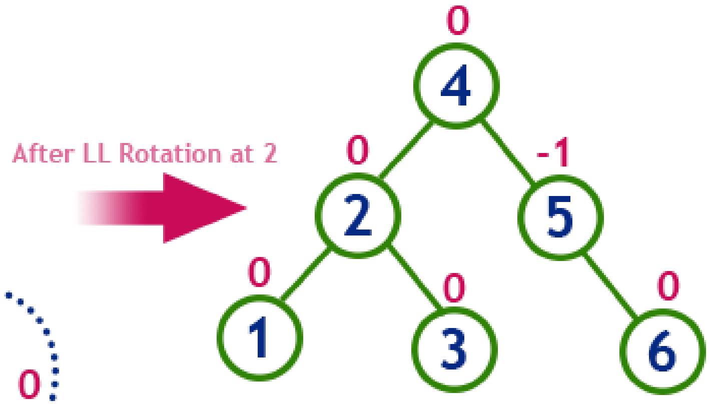
Example



Tree is imbalanced

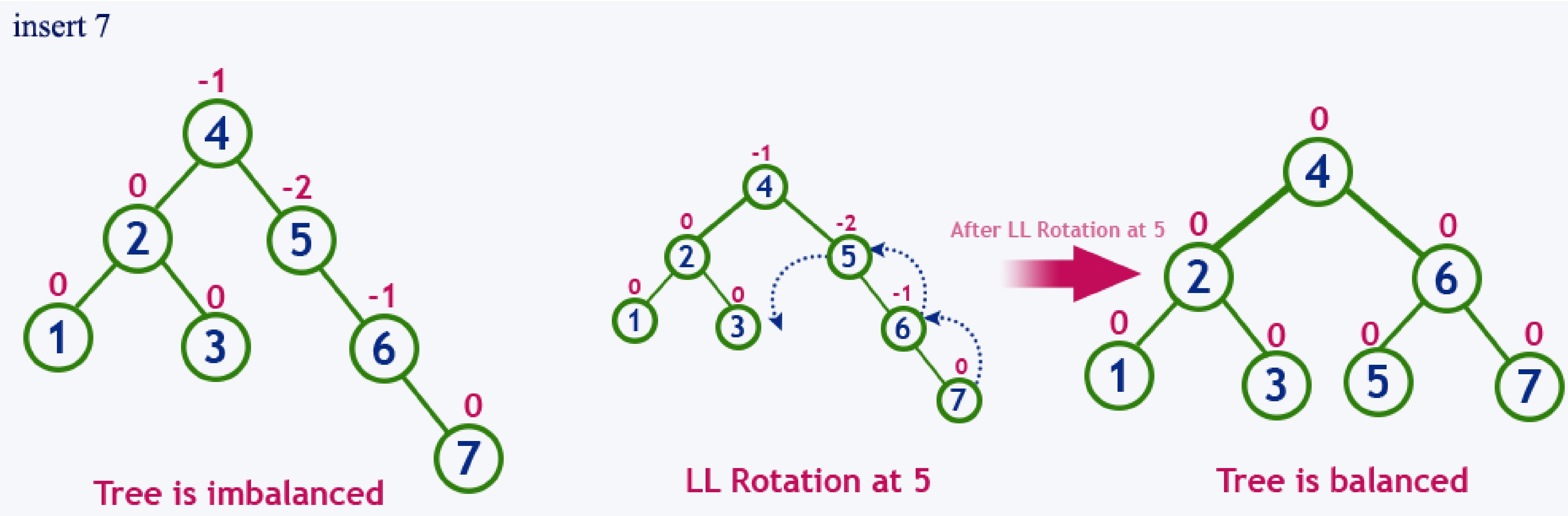


LL Rotation at 2

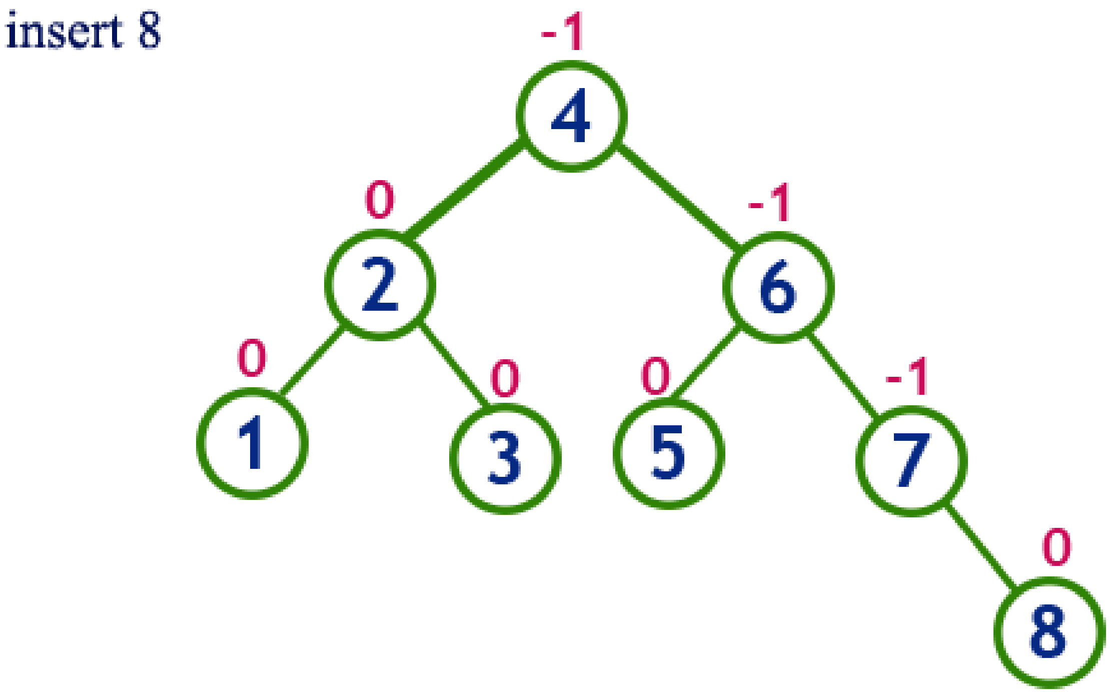


Tree is balanced

Example


insert 7
-1
4
0
2
-2
5
0
1
0
3
-1
6
0
7
Tree is imbalanced
-1
4
0
2
-2
5
0
1
0
3
-1
6
0
7
LL Rotation at 5
After LL Rotation at 5
0
4
0
2
0
6
0
1
0
3
0
5
0
7
Tree is balanced

Example



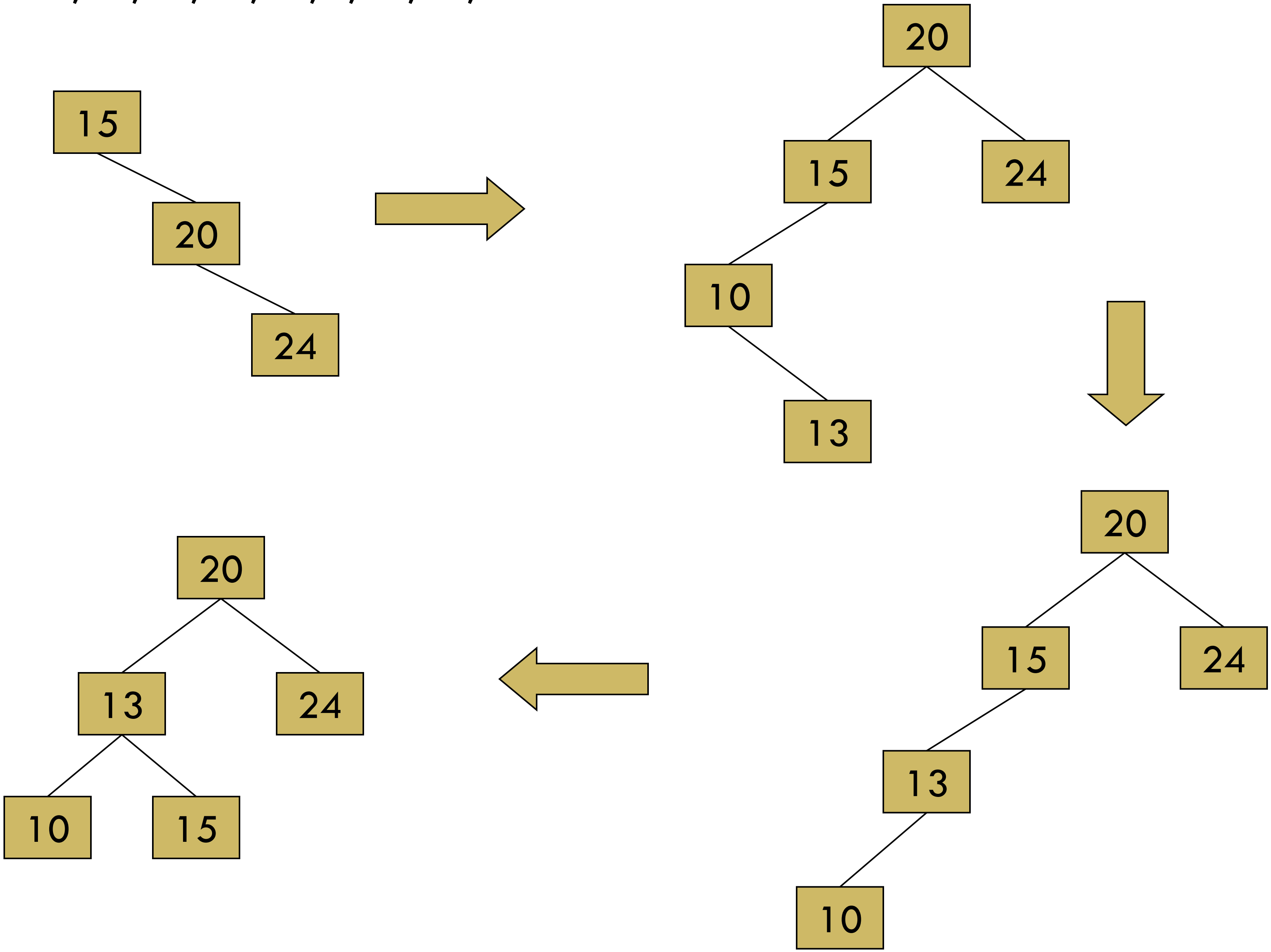
Tree is balanced

In Class Exercise

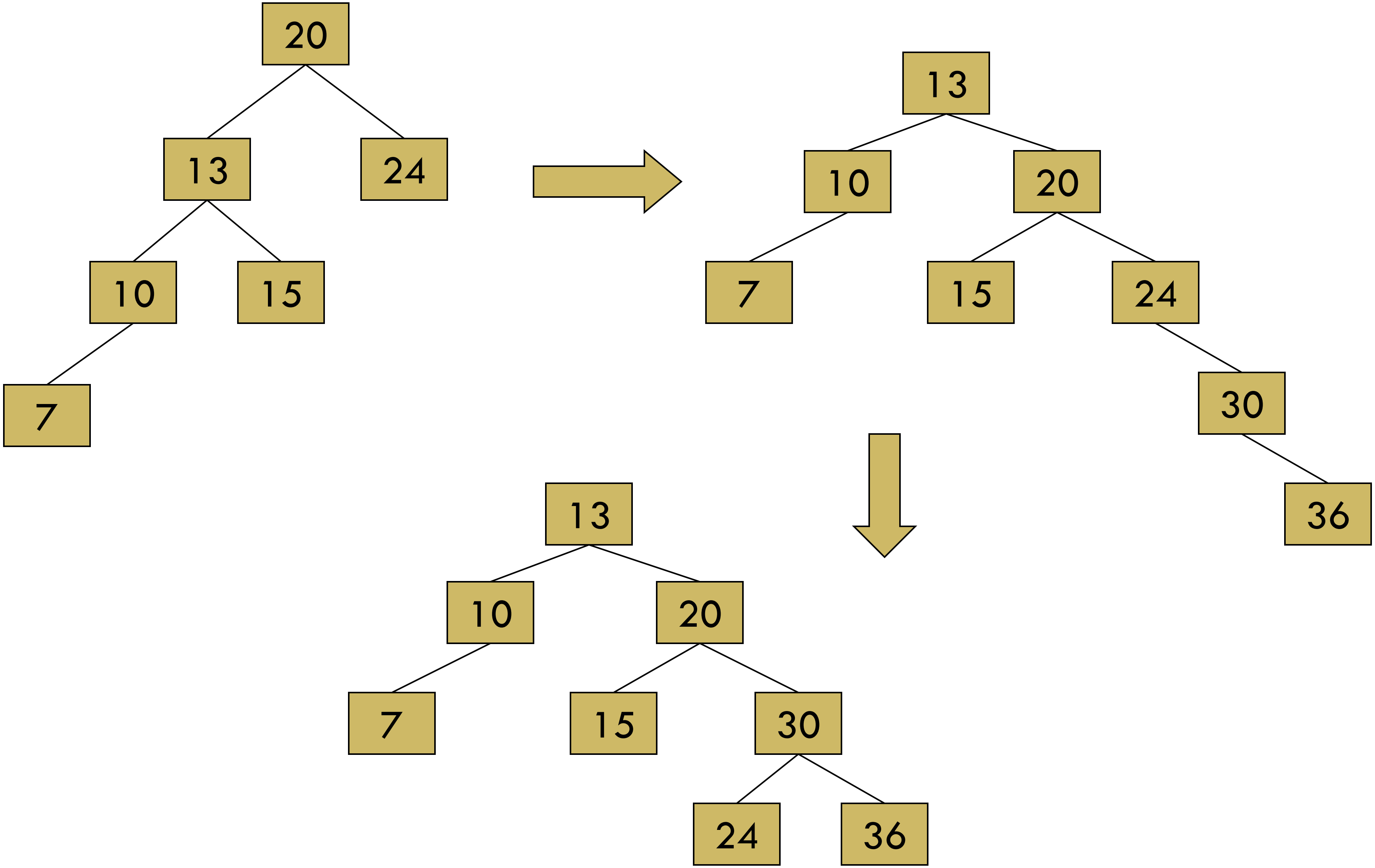
- Build an AVL tree with the following values:

  15, 20, 24, 10, 13, 7, 30, 36, 25

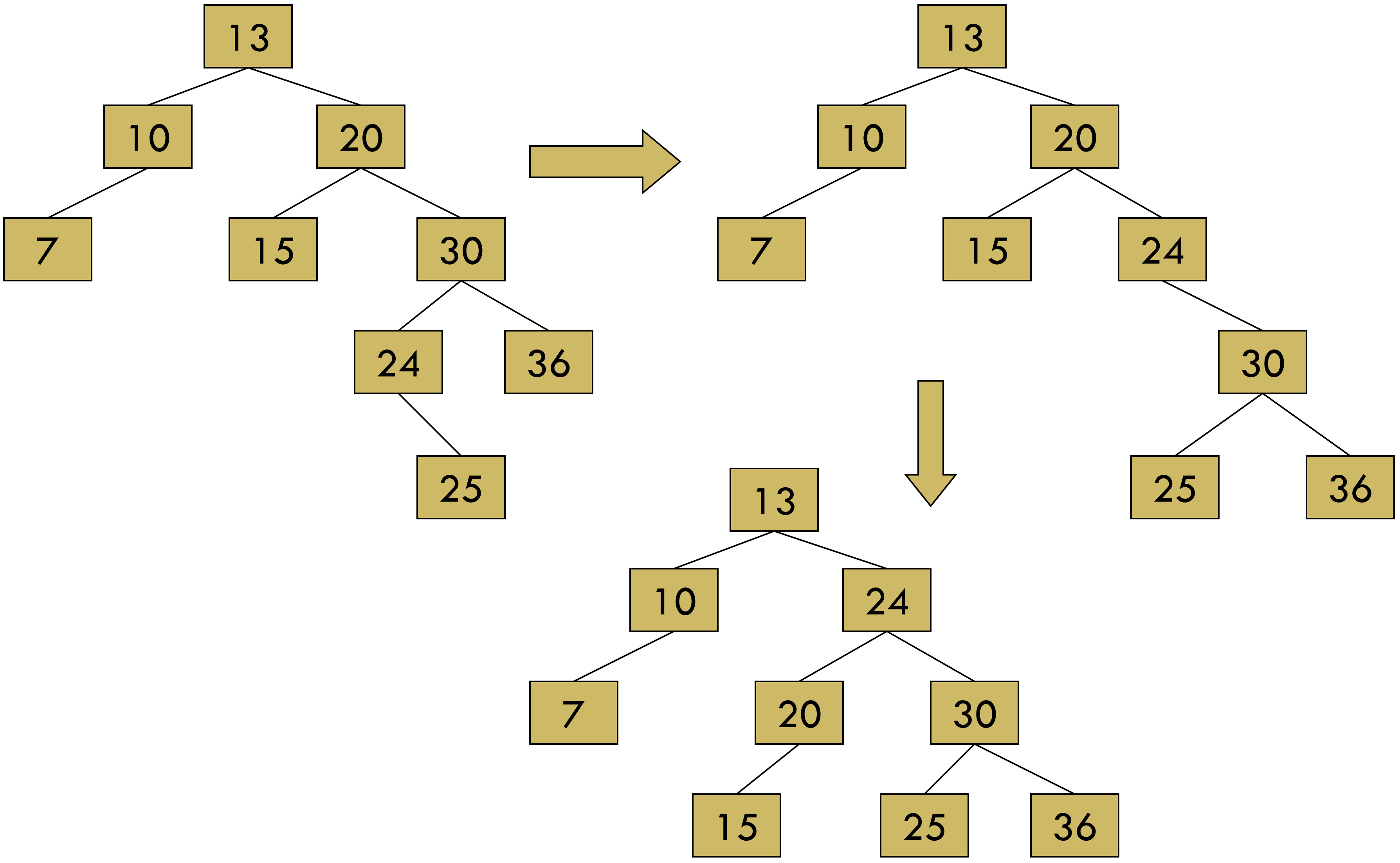
15, 20, 24, 10, 13, 7, 30, 36, 25



15, 20, 24, 10, 13, 7, 30, 36, 25



15, 20, 24, 10, 13, 7, 30, 36, 25



Reading Materials

- Schaum’s Outlines: Chapter # 7
- D. S. Malik: Chapter # 11
- Nell Dale: Chapter # 8
- Allen Weiss: Chapter # 4
- Tenebaum: Chapter # 5