

## **National University of Computer and Emerging Sciences**

October 27, 2023 MT1006 Differential Equations

## **Homework: First Order Ordinary Differential Equations**

Question 1: Identify the equation and solve them.

i. 
$$\frac{dy}{dx} + \frac{y}{x-2} = 5(x-2)y^{\frac{1}{2}}$$

**ii.** 
$$\frac{dy}{dx} = \frac{1}{y e^y - x}; y(0) = 1$$

iii. 
$$\frac{dy}{dx} = \frac{x \sec(\frac{y}{x}) + y}{x}$$

iv. 
$$(y^2 \cos x - 3x^2y - 2x)dx + (2y \sin x - x^3 + \ln y)dy = 0; y(0 = e)$$

v. 
$$(x^2 \sin x + 4y)dx + xdy = 0$$

$$vi. \quad (2y^2x - y)dx + xdy = 0$$

**vii.** 
$$\frac{dy}{dx} = \frac{y(y+\sin x)}{\left(\frac{1}{1+y^2} + \cos x - 2xy\right)}, y(0) = 1$$

**Question 2:** Suppose it is known that the population of the community in Problem 1 is 10,000 after 3 years. What was the initial population  $P_0$ ? What will be the population in 10 years? How fast is the population growing at t = 5 and t = 10?

**Question 3:** The radioactive isotope of lead, Pb - 209, decays at a rate proportional to the amount present at time t and has a half-life of 3.3 hours. If 1 gram of this isotope is present initially, how long will it take for 90% of the lead to decay?

**Question 4:** A small metal bar whose initial temperature was  $20^{\circ}$  *C* is dropped into a large container of boiling water. How long will it take the bar to reach  $90^{\circ}$  *C* if it is known that its temperature increases  $2^{\circ}$  in 1 second? How long will it take the bar to reach  $98^{\circ}$  *C*?

Good Luck