National University

Of Computer & Emerging Sciences-CFD Campus

Course Code: MT-1006

Course Title: Differential Equations

Fall 2023

Quiz#1

Maximum Marks: 10

Date: Sep 05, 2023 **Time:** 20 minutes

Name: Solution Manual **Roll No:**

Q.1 Determine the convergence or divergence of the sequences given.

a. $a_n = \dfrac{\ln n}{n^{1/n}}$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{\ln n}{n^{1/n}}$$

$$\because \lim_{n\to\infty} \ln n = \infty$$

&

$$\lim_{n\to\infty} n^{1/n} = 1$$

$\Rightarrow$

$$\lim_{n\to\infty} \frac{\ln n}{n^{1/n}} = \frac{\infty}{1} = \infty$$

$\Rightarrow$ Sequence $\{a_n\}$ diverges to ∞.

b. $b_n = \left(\dfrac{3}{2}\right)^{n-1} + \left(\dfrac{1}{6^{n-1}}\right)$

$$\lim_{n\to\infty} b_n = \lim_{n\to\infty} \left(\frac{3}{2}\right)^{n-1} + \lim_{n\to\infty} \left(\frac{1}{6^{n-1}}\right)$$

$$= \lim_{n\to\infty} \left(\frac{3}{2}\right)^{-1}\left(\frac{3}{2}\right)^{n} + \lim_{n\to\infty} \frac{6}{6^n}$$

$$= \frac{2}{3} \lim_{n\to\infty} \left(\frac{3}{2}\right)^{n} + 6 \lim_{n\to\infty} \frac{1}{6^n}$$

$$= \frac{2}{3}(\infty) + 6 \cdot \frac{1}{\infty}$$

$$\because \lim_{n\to\infty} x^n = \infty, \quad x > 1$$

&

$$= \infty + 6 \cdot 0$$

$$= \infty$$

Seq $\{b_n\}$ also diverges to ∞.

Q.2 Identify the type of infinite series $\sum_{n=1}^{\infty}\left(\left(\frac{3}{2}\right)^{n-1}+\frac{1}{6^{n-1}}\right)$ and find its sum, if possible.

$$\sum_{n=1}^{\infty}\left[\left(\frac{3}{2}\right)^{n-1}+\frac{1}{6^{n-1}}\right]$$

$$=\sum_{n=1}^{\infty}\left(\frac{3}{2}\right)^{n-1}+\sum_{n=1}^{\infty}\frac{1}{6^{n-1}} \quad \text{—— } A$$

where $\sum_{n=1}^{\infty}\left(\frac{3}{2}\right)^{n-1}$ is a ~~conve~~ divergent geometric series and its sum is not possible with $|r|=|3/2|=3/2>1$.

And $\sum_{n=1}^{\infty}\frac{1}{6^{n-1}}$ is a convergent G-series with $r=1/6$. and its sum is $\frac{a}{1-r}$.

Finally,

The given series also diverges and its sum is not possible.