10.6

Absolutly convergent

Find $\Sigma|x_n|$ and $\Sigma x_n$.

If $\Sigma|x_n|$ and $\Sigma x_n$

both convergent.

then we say $\Sigma x_n$ is

absolutly convergent.

Conditionaly convergent

$\Sigma x_n$ is convergent

but

$\Sigma|x_n|$ is divergent.

Diverging $\rightarrow$ Both $\Sigma x_n$ & $\Sigma|x_n|$ diverges.

i- If $\Sigma|a_n| \to c$

then $\Sigma a_n \to c$.

$\Rightarrow$ orginal series is absolutly convergent.

ii- If $\Sigma|a_n| \to \infty$, then

analyse orginal series,

if $\Sigma a_n \to c$

$\Rightarrow$ $\Sigma a_n$ is Conditionally cgnt.

iii- If $\Sigma|a_n| \to \infty$ and

$\Sigma a_n \to \infty$ Entire series

$\Rightarrow$ Divergent

Example

$$\sum_{n=1}^{\infty} \frac{\cos n\pi}{n^3} = -\frac{1}{1^3} + \frac{1}{2^3} - \frac{1}{3^3} + \cdots$$

$$\sum_{n=1}^{\infty}|a_n| = \sum_{n=1}^{\infty} \left| \frac{\cos n\pi}{n^3} \right| = \sum_{n=1}^{\infty} \frac{1}{n^3}$$ $|\cos n\pi| = 1$, $n \geq 1$

$$= \frac{1}{1^3} + \frac{1}{2^3} + \frac{1}{3^3} + \cdots$$

which is $p$-series

$\Rightarrow \sum_{n=1}^{\infty} |a_n|$ is convergent

so By absolute convergence test

$\Rightarrow \sum_{n=1}^{\infty} a_n$ is also convergent.

Th 18

$\Rightarrow \sum_{n=1}^{\infty} a_n$ is absolutely convergent.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt[3]{n}}$$

$$\sum_{n=1}^{\infty} |a_n| = \sum_{n=1}^{\infty} \frac{1}{(n)^{1/3}} = \cancel{\sum a_n}$$

p - series

$\sum_{n=1}^{\infty} |a_n|$ diverges.

Check original series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(n)^{1/3}} = \frac{1}{(1)^{1/3}} - \frac{1}{(2)^{1/3}} + \frac{1}{(3)^{1/3}} - \cdots$$

Use Alternating series Te

$$a_n = \frac{1}{n^{1/3}}$$

Check divergence Test

1 - $\lim_{n\to\infty} a_n = 0$

2 - $a_{n+1} \leq a_n$

$$\frac{1}{(n+1)^{1/3}} \leq \frac{1}{(n)^{1/3}}$$

$\Rightarrow$

$(n)^{1/3}$

$\Rightarrow$ original seri-
cgs. by (AST).

10/ $\sum a_n \to c$
while
$\sum |a_n| \to d$
$\Rightarrow \sum a_n$ is
Conditionally
convergent.

Example

$$\sum_{n=1}^{\infty} \frac{(-1)^n \cdot n^3}{n^3+5}$$

$\frac{1}{6}, \frac{8}{13}$

$$\sum |a_n| = \sum_{n=1}^{\infty} \frac{n^3}{n^3+5} = \cancel{\frac{1}{1+5/n^3}}$$

check divergence Test $\boxed{\lim_{n\to\infty} \frac{n^3}{n^3+5} = 1}$ $\neq 0$

$\Rightarrow$ series diverges.

> Consider orginal series

$$\sum_{n=1}^{\infty} \frac{(-1)^n n^3}{n^3+5} = -\frac{1}{6} + \frac{2^3}{2^3+5} - $$

AST

$\frac{n^3}{n^3+5} \not\to 0.$

$\Rightarrow \sum a_n$ dvg.

Ex # 10.6

(16) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{(0.1)^n}{n}$

$\frac{1}{(0.1)^\infty}$ $\infty$

Consider

$$\sum_{n=1}^{\infty} |a_n| = \sum_{n=1}^{\infty} \frac{(0.1)^n}{n}$$

Apply root test

$= \lim_{n\to\infty} \frac{0.1}{n^{1/n}} = 0.1 < 1$ $\Rightarrow$ cgs. abso-

Q.40

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{(n!)^2 3^n}{(2n+1)!}$$

Consider

$$\frac{(n!)^2 3^n}{(2n+1)!}.$$

Use Ratio Test:

$$= \frac{[(n+1)!]^2 3^{n+1}}{[2(n+1)+1]!} \times \frac{(2n+1)!}{\cancel{(n!)^2}\, \cancel{3^n}}$$

$$= \frac{(n+1)^2 \cancel{[(n)!]^2} \cdot 3 \cdot \cancel{(2n+1)!}}{(2n+3)!}$$

$$= \frac{}{(2n+3)(2n+2)\cancel{(2n+1)!}}$$

$2n+3$
$2n+2$
$4n^2+6n$
$+2n+6$

$$= \frac{3(n+1)^2}{(2n+3)(2n+2)} = \frac{3[n^2+1+2n]}{(2n+3)(2n+2)}$$

$$= \frac{1+\frac{1}{n^2}+\frac{2}{n}}{4+\frac{80}{n}+\frac{6}{n^2}}$$

$$= \frac{1}{4} < 1 \text{ Cgs}$$

Do Ex 10.5
Q 1 to 46 +
57 to 64

Example

(b) For $\sum_{n=1}^{\infty} \frac{\sin n}{n^2} = \frac{\sin 1}{1^2} + \frac{\sin 2}{2^2} + \dots + \frac{\sin n}{n^2} + \dots$

It contains +ive and -ive terms.

So take absolute values of above series,

$$\sum_{n=1}^{\infty} \left| \frac{\sin n}{n^2} \right| = \frac{|\sin 1|}{1^2} + \frac{|\sin 2|}{2^2} + \dots$$

$\because -1 \leq \sin n \leq 1$

but $|\sin n| \leq 1$

Use

Comparison test

Consider $a_n = \left| \frac{\sin n}{n^2} \right|$

$b_n = \frac{1}{n^2}$

$\sum b_n = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \dots$

$\because a_n \leq b_n$

$\because \sum b_n$ is convergent

p series. so by comparison

test $\sum a_n$ also cgs.

$\sin 1 = 0.017$

$\sin 1 < 1$

$\sin 2 = 0.0087$

$< 0.\frac{1}{2^2}$

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