Permutation

- A permutation is the choice of r things from a set of n things without replacement and where the order matters.
- Example

A permutation of a set S is a sequence that contains every element of S exactly once.

Let S = {a, b, c},

then here are all its permutations

$$(\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c})\ (\boldsymbol{a}, \boldsymbol{c}, \boldsymbol{b})\ (\boldsymbol{b}, \boldsymbol{a}, \boldsymbol{c})\ (\boldsymbol{b}, \boldsymbol{c}, \boldsymbol{a})\ (\boldsymbol{c}, \boldsymbol{a}, \boldsymbol{b})\ (\boldsymbol{c}, \boldsymbol{b}, \boldsymbol{a})$$

Combination

- In Combination order of arrangement is not important.
- Combinations are used when the same kind of things are to be sorted.

$$_{n}C_{r} = \frac{n!}{r!(n-r)!}$$

Example

- we select 5 cards at random from a deck of 52 cards. What is the probability that we will end up having a full house?

Solution

- A full house consists of three cards of one rank and two cards of another rank. There are 13 ranks in a deck of cards (Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King), and for each rank, there are 4 different suits (hearts, diamonds, clubs, spades).

Solution (cont)

- Number of ways to choose three cards of one rank:
- There are 13 ranks to choose from, and we need to select 3 of them. So, the number of ways to choose three cards of one rank is given by the combination formula:
- C(13, 1) * C(4, 3)
- = 13 * 4 = 52.

- After selecting three cards of one rank, we have 12 remaining ranks to choose from. We need to select 2 of them. So, the number of ways to choose two cards of another rank is given by the combination formula:
- C(12, 1) * C(4, 2)
- = 12 * 6
- = 72

Probability

- **Example:**

  Suppose we select 5 cards at random from a deck of 52 cards.

  What is the probability that we will end up having a full house?

  Doing this using the possibility tree will take some effort. But we can do this as follows:

$$|S| = \binom{n}{k}$$

$$|E| = 13.\binom{4}{3}.12.\binom{4}{2}$$

So, $\Pr[E] = \frac{13.12.\binom{4}{3}.\binom{4}{2}}{\binom{13}{5}} = \frac{13.12.4.6.5.4.3.2}{52.51.50.49.48} = \frac{18}{12495} \approx \frac{1}{694}$