Discrete Structures

Lecture # 06

Dr. Muhammad Ahmad

Department of Computer Science

FAST -- National University of Computer and Emerging Sciences. CFD Campus

"The Consequences of an Act Affect the Probability of its Occurring Again!"
- B. F. Skinner -

SET

A well defined collection of distinct objects is called a set.

The objects are called the elements or members of the set.

Sets are denoted by capital letters A,B,C … X,Y,Z.

SET

The elements of a set are represented by lower case letters a, b, c, … , x, y, z.

If an object x is a member of a set A we write $x \in A$, which reads "x belongs to A" or "x is in A" or "x is an element of A"

Otherwise we write $x \notin A$, which reads "x does not belong to A" or "x is not in A" or "x is not an element of A".

TABULAR FORM

Listing all the elements of a set, separated by commas and enclosed within braces or curly brackets {}.

EXAMPLES:

A = {1,2,3,4,5}
B = {2,4,6,8,…,50}
C = {1,3,5,7,9,…}

DISCRIPTIVE FORM

Stating in words the elements of the set.

EXAMPLES:

A = set of a first five Natural Numbers.
B = set of positive even integers less or equal to fifty.
C = set of positive odd integers.

SET BUILDER FORM

Writing in symbolic form the common characteristics shared by all the elements of the set.

EXAMPLES

$A = \{x \in N \mid x \leq 5\}$ N=Natural Number

$B = \{y \in E \mid 0 < y \leq 50\}$ E=Even Number

$C = \{x \in O \mid x > 0\}$ O=Odd Number

SET OF NUMBERS

1. Set of Natural Numbers

   $N = \{1, 2, 3, \ldots\}$

2. Set of Whole Numbers

   $W = \{0, 1, 2, 3, \ldots\}$

3. Set of Integers

   $Z = \{\ldots, -3, -2, -1, 0, +1, +2, +3, \ldots\}$

   $= \{0, \pm 1, \pm 2, \pm 3, \ldots\}$

SET OF NUMBERS

4. Set of Even Integers

   $E = \{0, \pm 2, \pm 4, \pm 6, \ldots\}$

5. Set of Odd Integers

   $O = \{\pm 1, \pm 3, \pm 5, \ldots\}$

6. Set of Prime Numbers

   $P = \{2, 3, 5, 7, 11, 13, 17, 19, \ldots\}$

7. Set of Rational Numbers

   $Q = \{x \mid x = p/q \ ; p, q \in Z, q \neq 0\}$

SUBSET

If A and B are two sets, A is called a subset of B, written $A \subseteq B$, if, and only if, every element of A is also an element of B.

Symbolically:

$$A \subseteq B \leftrightarrow \text{if } x \in A \text{ then } x \in B$$

1. So, if A has n elements, the maximum number of subsets of A is 2^n
2. On the other hand, the maximum number of non-empty sets is equal to $2^n - 1$.

SUBSET

REMARKS:

1. When $A \subseteq B$, then B is called a superset of A.
2. When $A \not\subseteq B$, then there exist at least one $x \in A$ such that $x \notin B$.
3. Every set is a subset of itself.

EXAMPLE

Let

$A = \{1, 3, 5\}$ $B = \{1, 2, 3, 4, 5\}$

$C = \{1, 2, 3, 4\}$ $D = \{3, 1, 5\}$

Then

$A \subseteq B$ $A = \{1, \ , 3, \ , 5\}$

$A \subseteq D$ $D = \{3, 1, 5\}$

$A \not\subseteq C$ $5 \in A$ but $5 \notin C$

PROPER SUBSET

Let A and B be sets. A is a proper subset of B, if, and only if, every element of A is in B but there is at least one element of B that is not in A.

Symbolically: $A \subset B$

EQUAL SETS

Two sets A and B are equal if, and only if, every element of A is in B and every element of B is in A and is denoted A = B.

Symbolically: $A = B \text{ iff } A \subseteq B \text{ and } B \subseteq A$

EQUAL SETS

EXAMPLE:

Let A = {1,2,3,6}

B = the set of positive divisors of 6

C = {3,1,6,2}

D = {1,2,2,3,6,6,6}

Then A,B,C, and D are all equal sets.

Point to ponder!!

1. Is $n(A)= n(D)$?
2. Are equal sets equivalent and vice versa?

NULL SET

A set which contains no element is called a null set, or an empty set or a void set.

Symbolically:

It is denoted by the Greek letter $\varnothing$(phi) or { }.

NULL SET

EXAMPLE

$A = \{x \mid x \text{ is a person taller than 10 feet}\}$

$A = \varnothing$

$B = \{x \mid x^2 = 4, x \text{ is odd}\}$

$B = \varnothing$

EXERCISE

(a)	x	$\in$	$\{x\}$	TRUE
(b)	$\{x\}$	$\subseteq$	$\{x\}$	TRUE
(c)	$\{x\}$	$\in$	$\{x\}$	FALSE
(d)	$\{x\}$	$\in$	$\{\{x\}\}$	TRUE
(e)	$\varnothing$	$\subseteq$	$\{x\}$	TRUE
(f)	$\varnothing$	$\in$	$\{x\}$	FALSE

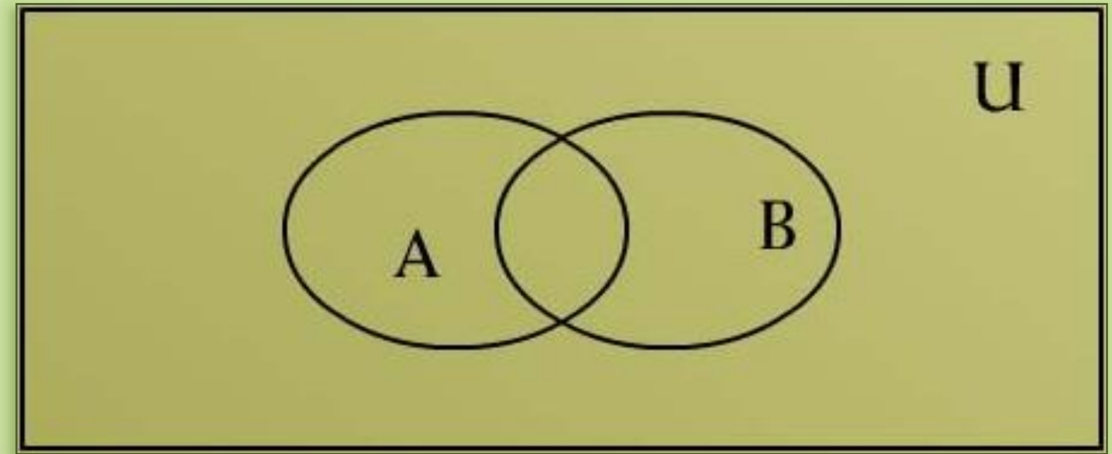
UNIVERSAL SET

The set of all elements under consideration is called the Universal Set.

The Universal Set is denoted by U.

VENN DIAGRAM

A Venn diagram is a graphical representation of sets by regions in the plane.



FINITE AND INFINITE SETS

A set S is said to be finite if it contains exactly m distinct elements where m denotes some non negative integer.

In such case we write

$$|S| = m \text{ or } n(S) = m$$

A set is said to be infinite if it is not finite.

FINITE AND INFINITE SETS

EXAMPLES

1. The set S of letters of English alphabets is finite and $|S| = 26$

2. The null set $\varnothing$ has no elements, is finite and $|\varnothing| = 0$

3. The set of positive integers $\{1, 2, 3, \ldots\}$ is infinite.

EXERCISE

1. A = {month in the year} FINITE

2. B = {even integers} INFINITE

3. C = {positive integers less than 1} FINITE

MEMBERSHIP TABLE

A table displaying the membership of elements in sets. To indicate that an element is in a set, a 1 is used; to indicate that an element is not in a set, a 0 is used.

A	A^c
1	0
0	1

UNION

Let A and B be subsets of a universal set U. The union of sets A and B is the set of all elements in U that belong to A or to B or to both, and is denoted $A \cup B$.

Symbolically:

$$A \cup B = \{x \in U \mid x \in A \text{ or } x \in B\}$$

UNION

EXAMPLE:

Let

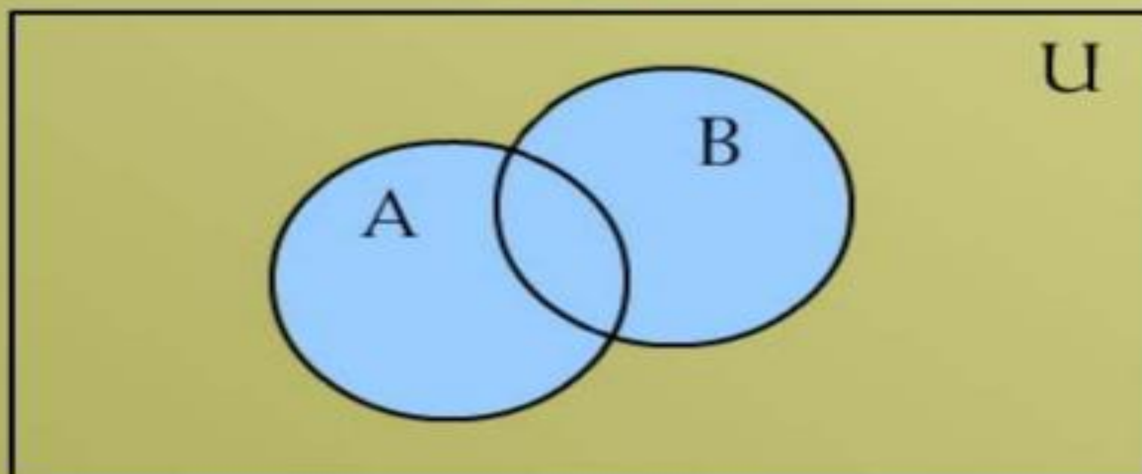
$$U = \{a, b, c, d, e, f, g\}$$
$$A = \{a, c, e, g\}$$
$$B = \{d, e, f, g\}$$

Then

$$A \cup B = \{a, c, e, g\} \cup \{d, e, f, g\}$$
$$= \{a, c, d, e, f, g\}$$

VENN DIAGRAM FOR

$A \cup B$



REMARK

1. $A \cup B = B \cup A$
2. $A \subseteq A \cup B$ and $B \subseteq A \cup B$

MEMBERSHIP TABLE FOR

$A \cup B$

A	B	$A \cup B$
1	1	1
1	0	1
0	1	1
0	0	0

INTERSECTION

Let A and B subsets of a universal set U. The intersection of sets A and B is the set of all elements in U that belong to both A and B and is denoted $A \cap B$.

Symbolically:

$$A \cap B = \{x \in U \mid x \in A \text{ and } x \in B\}$$

INTERSECTION

EXMAPLE

Let $U = \{a, b, c, d, e, f, g\}$

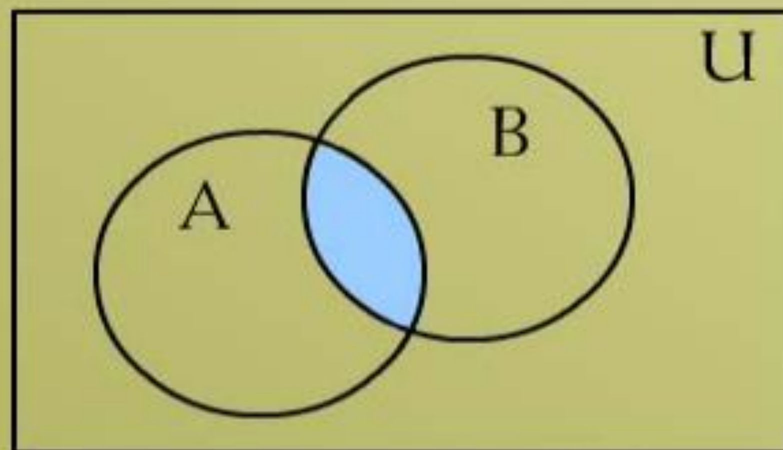
$A = \{a, c, e, g\}$

$B = \{d, e, f, g\}$

Then

$$A \cap B = \{a, c, e, g\} \cap \{d, e, f, g\}$$
$$= \{e, g\}$$

VENN DIAGRAM



$A \cap B$ is shaded

REMARK

1. $A \cap B = B \cap A$
2. $A \cap B \subseteq A$ and $A \cap B \subseteq B$
3. If $A \cap B = \varnothing$

   then A & B are called disjoint sets.

MEMBERSHIP TABLE FOR

$A \cap B$

A	B	$A \cap B$
1	1	1
1	0	0
0	1	0
0	0	0

SET DIFFERENCE

Let A and B be subsets of a universal set U. The difference of "A and B" (or relative complement of B in A) is the set of all element in U that belong to A but not to B, and is denoted by A-B or A/B.

Symbolically:

$$A - B = \{x \in U \mid x \in A \text{ and } x \notin B\}$$

SET DIFFERENCE

EXAMPLE:

Let $U = \{a, b, c, d, e, f, g\}$

$A = \{a, c, e, g\}$

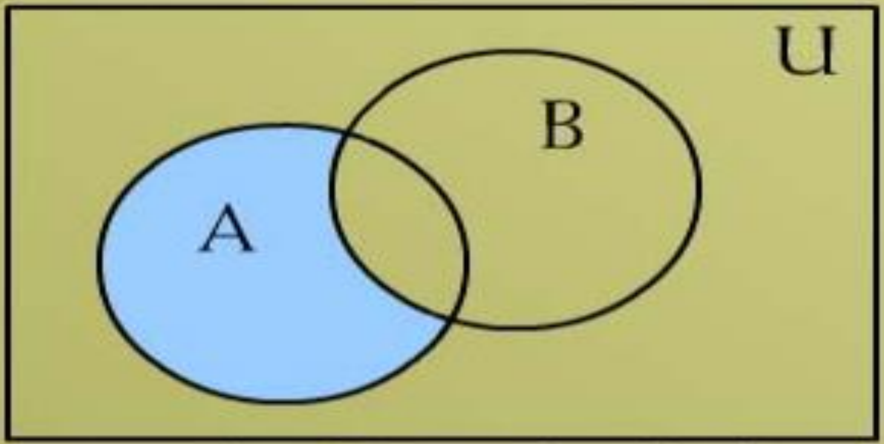
$B = \{d, e, f, g\}$

Then:

$$A - B = \{a, c, e, g\} - \{d, e, f, g\}$$

$$= \{a, c\}$$

VENN DIAGRAM



A - B is shaded

REMARKS:

1. $A - B \neq B - A$
2. $A - B \subseteq A$
3. $A - B$, $A \cap B$ and $B - A$ are mutually disjoint sets.

MEMBERSHIP TABLE FOR

A – B

A	B	A – B
1	1	0
1	0	1
0	1	0
0	0	0

COMPLEMENT

Let A be a subset of universal set U. The complement of A is the set of all element in U that do not belong to A, and is denoted A^c, $\overline{A}$ or A'

Symbolically:

$$A' = \{x \in U \mid x \notin A\}$$

COMPLEMENT

EXMAPLE

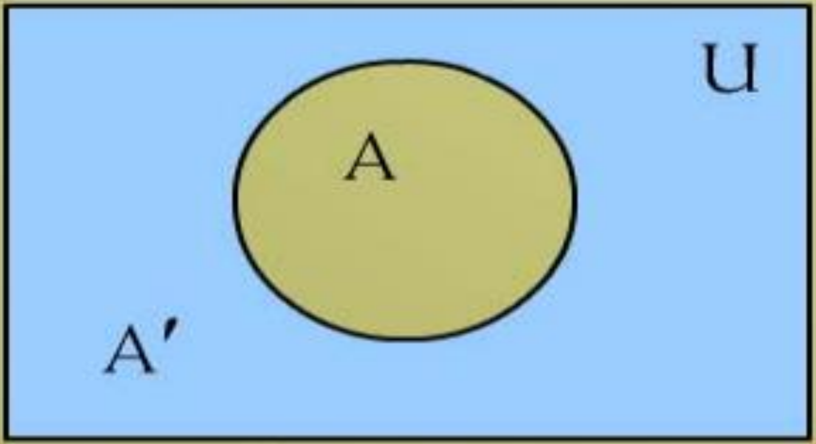
Let $U = \{a, b, c, d, e, f, g\}$

$A = \{a, c, e, g\}$

Then

$$A' = \{a, b, c, d, e, f, g\} - \{a, c, e, g\}$$
$$= \{b, d, f\}$$

VENN DIAGRAM



A′ is shaded

REMARKS:

1. $A' = U - A$
2. $A \cap A' = \varnothing$
3. $A \cup A' = U$

MEMBERSHIP TABLE FOR

A'

A	A'
1	0
0	1

EXERCISE

Let $U = \{1, 2, 3, \ldots, 10\}$

$X = \{1, 2, 3, 4, 5\}$

$Y = \{y \mid y = 2x, x \in X\}$

$Z = \{z \mid z^2 - 9z + 14 = 0\}$

Enumerate:

(i) $X \cap Y$

(ii) $Y \cup Z$

(iii) $X - Z$

(iv) Y'

(v) $X' - Z'$

(vi) $(X - Z)'$

Note that "y" and "z" both belongs to Universal set "U".

SOLUTION

Given

$$U = \{1, 2, 3, \ldots, 10\}$$
$$X = \{1, 2, 3, 4, 5\}$$

$$Y = \{y \in U \mid y = 2x, x \in X\}$$
$$= \{2, 4, 6, 8, 10\}$$

$$Z = \{z \in U \mid z^2 - 9z + 14 = 0\}$$
$$= \{2, 7\}$$

SOLUTION

$$(i)\ X \cap Y = \{1, 2, 3, 4, 5\} \cap \{2, 4, 6, 8, 10\}$$
$$= \{2, 4\}$$

$$(ii)\ Y \cup Z = \{2, 4, 6, 8, 10\} \cup \{2, 7\}$$
$$= \{2, 4, 6, 7, 8, 10\}$$

$$(iii)\ X - Z = \{1, 2, 3, 4, 5\} - \{2, 7\}$$
$$= \{1, 3, 4, 5\}$$

SOLUTION

(iv) $Y' = U - Y$

$= \{1, 2, 3, \ldots, 10\} - \{2, 4, 6, 8, 10\}$

$= \{1, 3, 5, 7, 9\}$

(v) $X' - Z'$

$= \{6, 7, 8, 9, 10\} - \{1, 3, 4, 5, 6, 8, 9, 10\}$

$= \{7\}$

(vi) $(X - Z)'$

$= U - (X - Z)$

$= \{1, 2, 3, \ldots, 10\} - \{1, 3, 4, 5\}$

$= \{2, 6, 7, 8, 9, 10\}$

EXERCISE

$$U = \{ x \in Z, 0 \leq x \leq 10\}$$

$$P = \{x \in U \mid x \text{ is a prime number}\}$$

$$Q = \{x \in U \mid x^2 < 70\}$$

(i) Draw a Venn diagram for the above

(ii) List the elements in $P^c \cap Q$

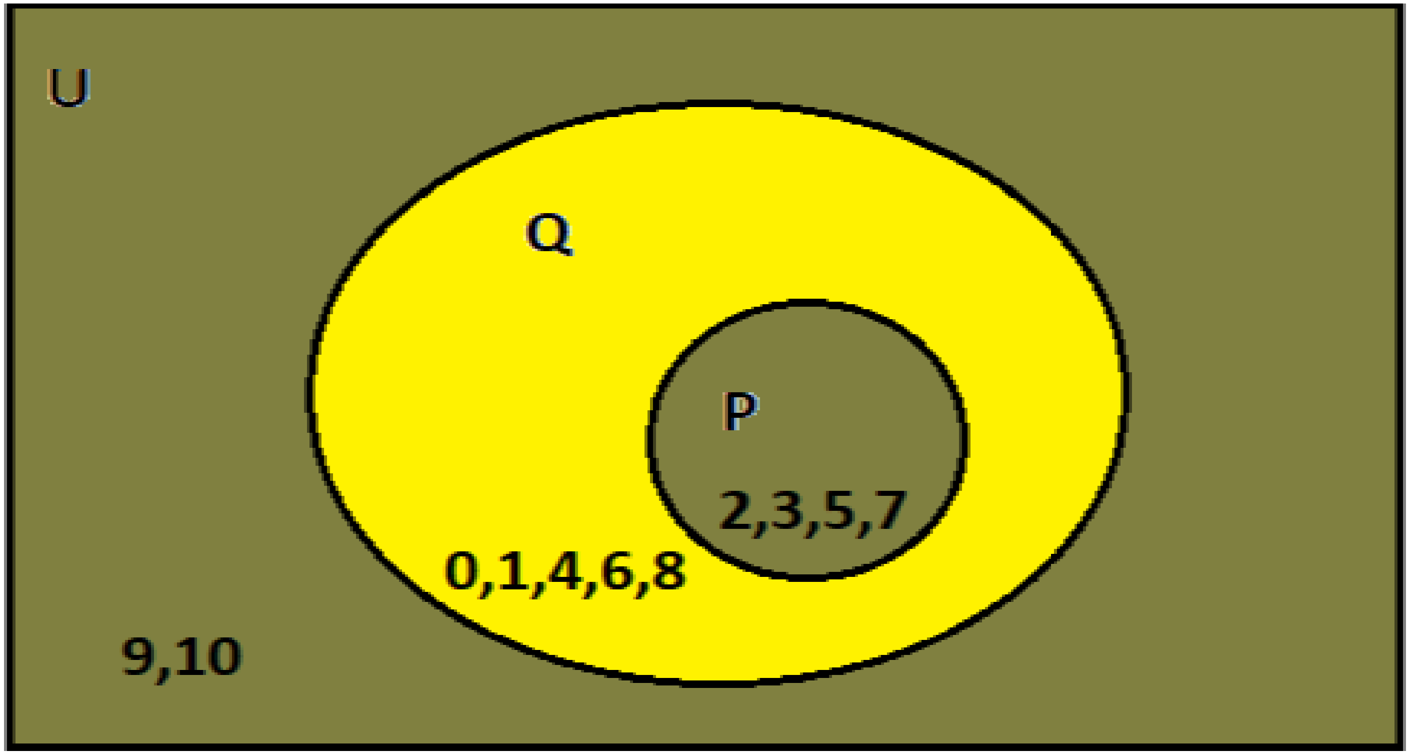
SOLUTION

$$U = \{ x \in \mathbb{Z}, 0 \le x \le 10 \}$$
$$= \{0, 1, 2, 3, \ldots, 10\}$$

$$P = \{x \in U \mid x \text{ is a prime number}\}$$
$$= \{2, 3, 5, 7\}$$

$$Q = \{x \in U \mid x^2 < 70\}$$
$$= \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$$

VENN DIAGRAM



The yellow shaded region is the desired result.

ELEMENTS OF

(ii) $P' \cap Q$

$P' = U - P$

$= \{0, 1, 2, 3, \ldots, 10\} - \{2, 3, 5, 7\}$

$= \{0, 1, 4, 6, 8, 9, 10\}$

and

$P' \cap Q$

$= \{0, 1, 4, 6, 8, 9, 10\} \cap \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$

$= \{0, 1, 4, 6, 8\}$

EXERCISE

Let $U = \{1, 2, 3, 4, 5\}$ $\quad C = \{1, 3\}$

Where A and B are non empty sets. Find A in each of the following:

(i) $A \cup B = U$ $\quad A \cap B = \varnothing$ and $B = \{1\}$

EXERCISE

(ii) $A \subset B$ and $A \cup B = \{4, 5\}$

(iii) $A \cap B = \{3\}$ $A \cup B = \{2, 3, 4\}$
and $B \cup C = \{1,2,3\}$

(iv) A and B are disjoint, B and C are disjoint, and the union of A and B is the set $\{1, 2\}$.

SOLUTION

(i) $A \cup B = U$ $\quad A \cap B = \varnothing$ and $\quad B = \{1\}$

SOLUTION:

$$\text{Since } A \cup B = U$$
$$= \{1, 2, 3, 4, 5\}$$
$$\text{and } \quad A \cap B = \varnothing$$

Therefore $\quad A = B'$

$$= \{1\}'$$
$$= \{2, 3, 4, 5\}$$

SOLUTION

(ii) $A \subset B$ and $A \cup B = \{4, 5\}$ also $C = \{1, 3\}$

SOLUTION:

When $A \subset B$

then $A \cup B = B$

$= \{4, 5\}$

Also A being a proper subset of B implies

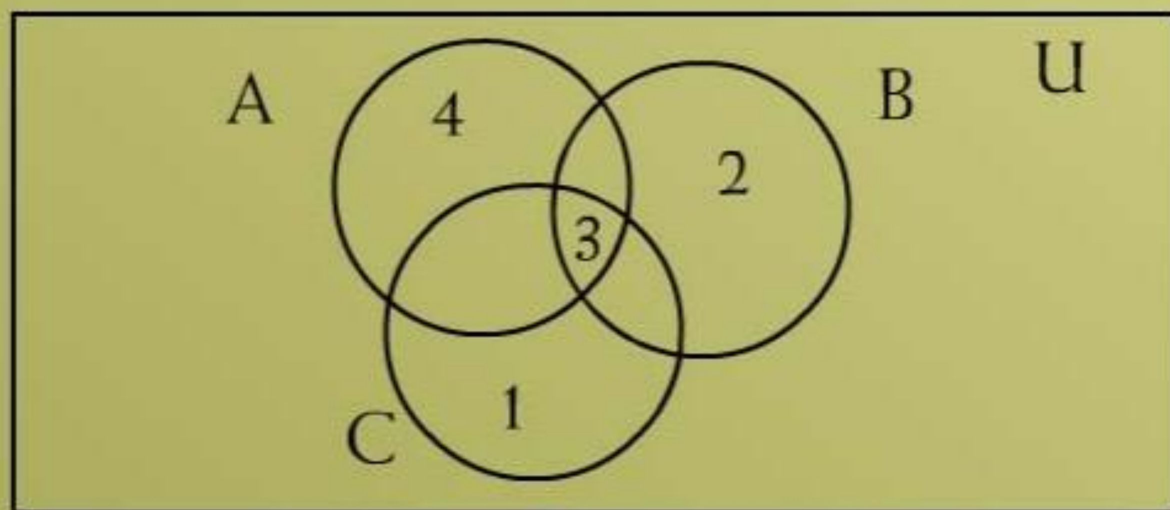
$A = \{4\}$ or $A = \{5\}$

Solution contd…

(iii) $A \cap B = \{3\}$ $A \cup B = \{2, 3, 4\}$

and

$B \cup C = \{1,2,3\}$ Also $C = \{1, 3\}$

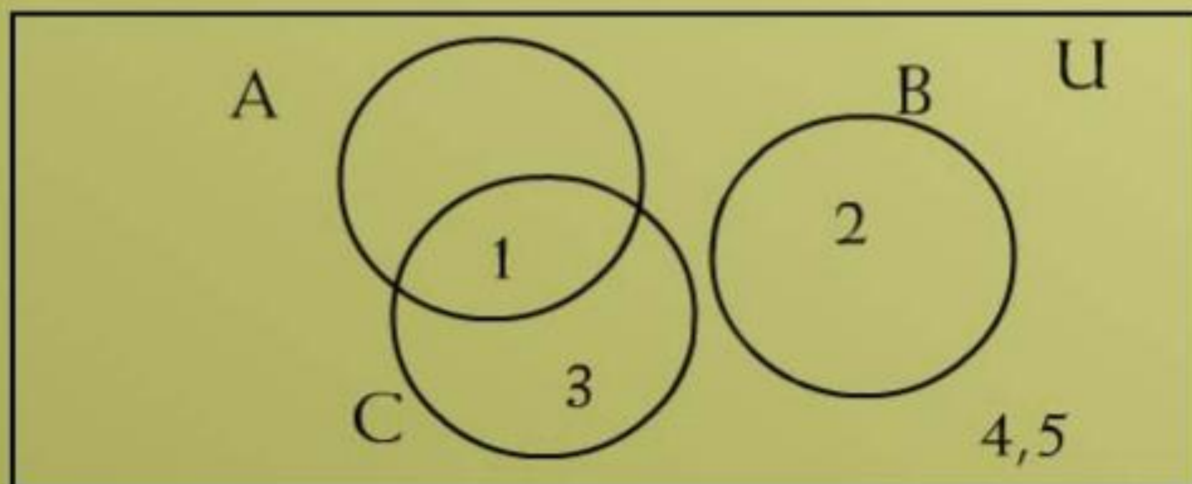


$A = \{3, 4\}$ $B = \{2, 3\}$

Solution contd…

(iv) $A \cap B = \varnothing$ $\quad B \cap C = \varnothing$

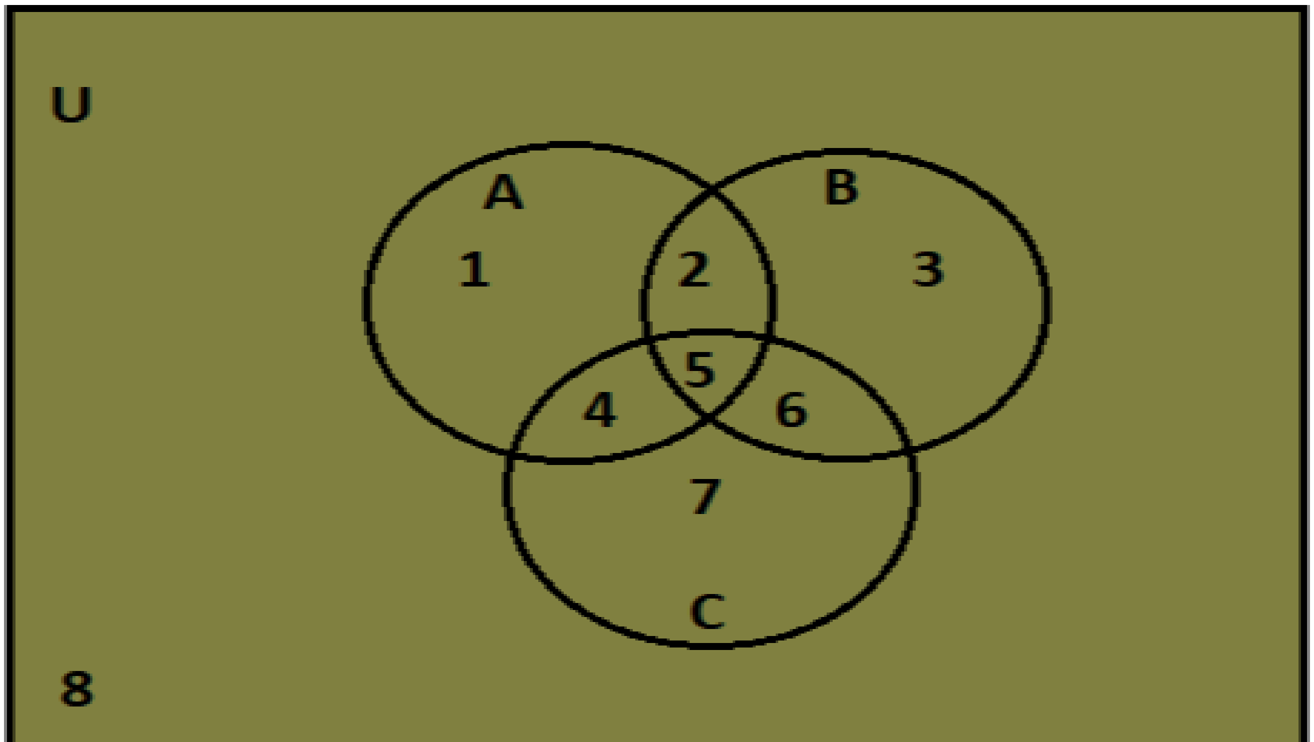
$A \cup B = \{1, 2\}$ Also $C = \{1, 3\}$



$A = \{1\}$

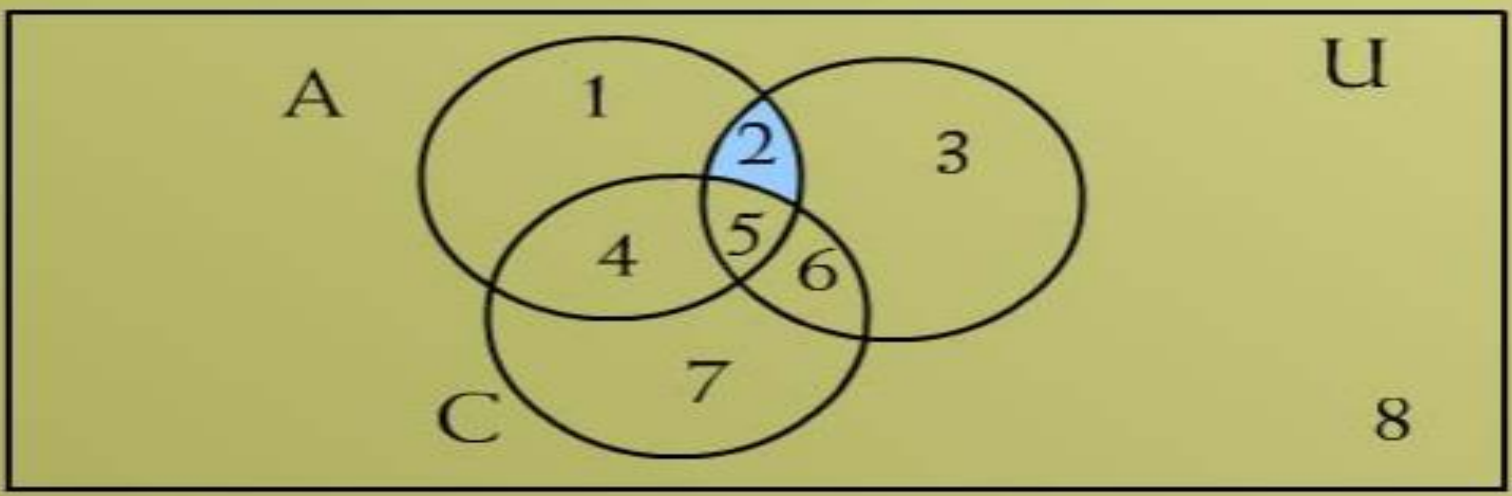
EXERCISE

(i)	$(A \cap B) \cap C'$	(ii)	$A' \cup (B \cup C)$
(iii)	$(A - B) \cap C$	(iv)	$(A \cap B') \cup C'$



VENN DIAGRAM FOR

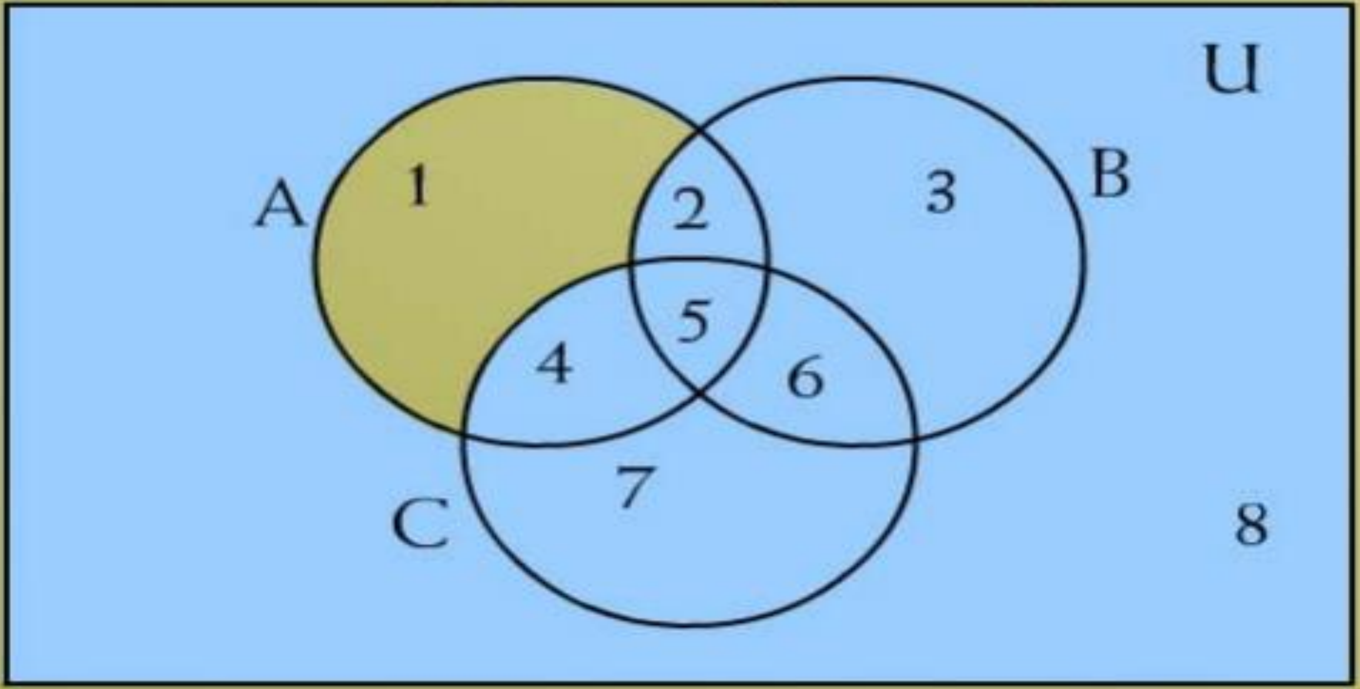
(i) $(A \cap B) \cap C'$



$(A \cap B) \cap C' = \{2\}$

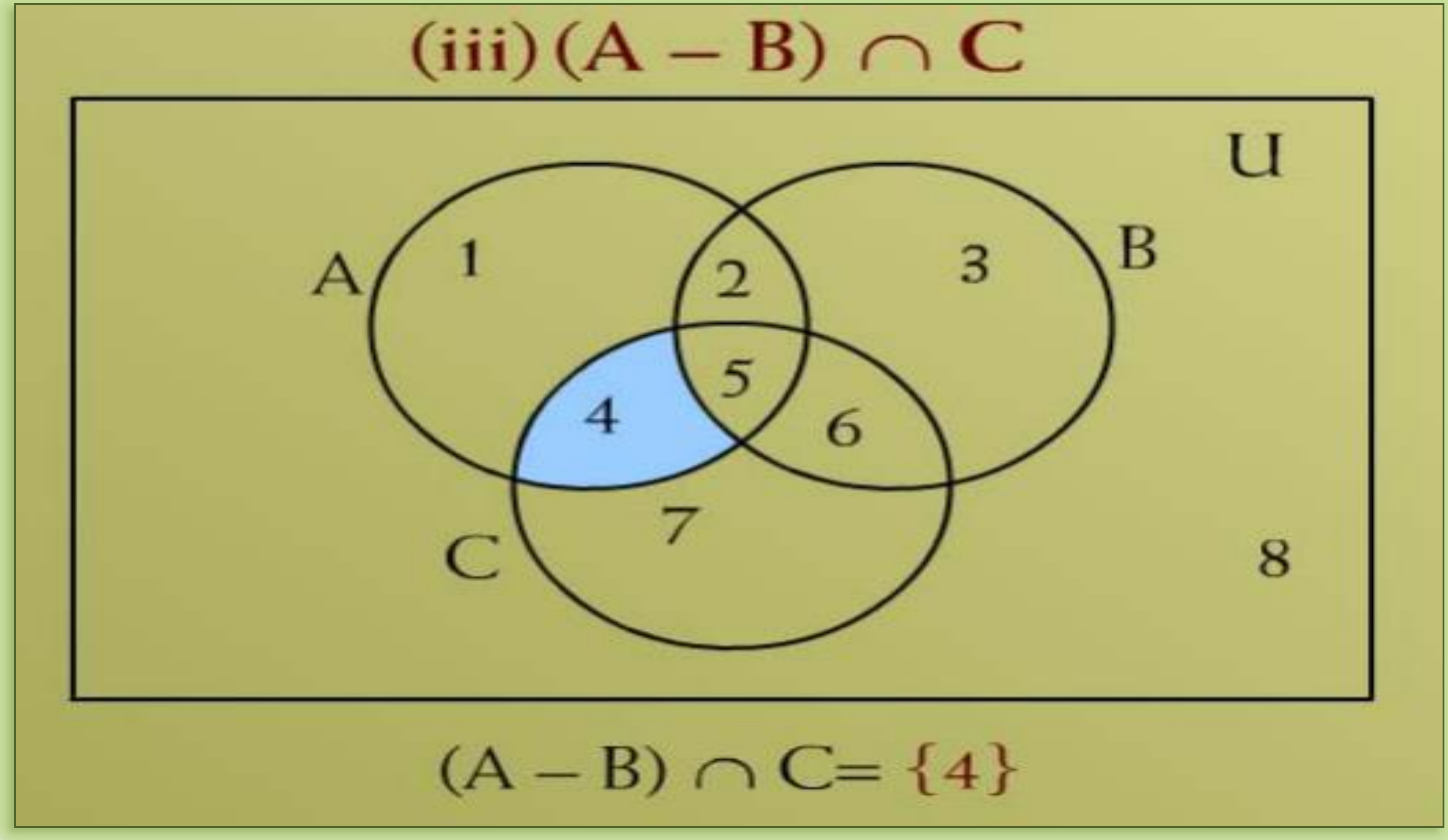
VENN DIAGRAM FOR

(ii) $A' \cup (B \cup C)$

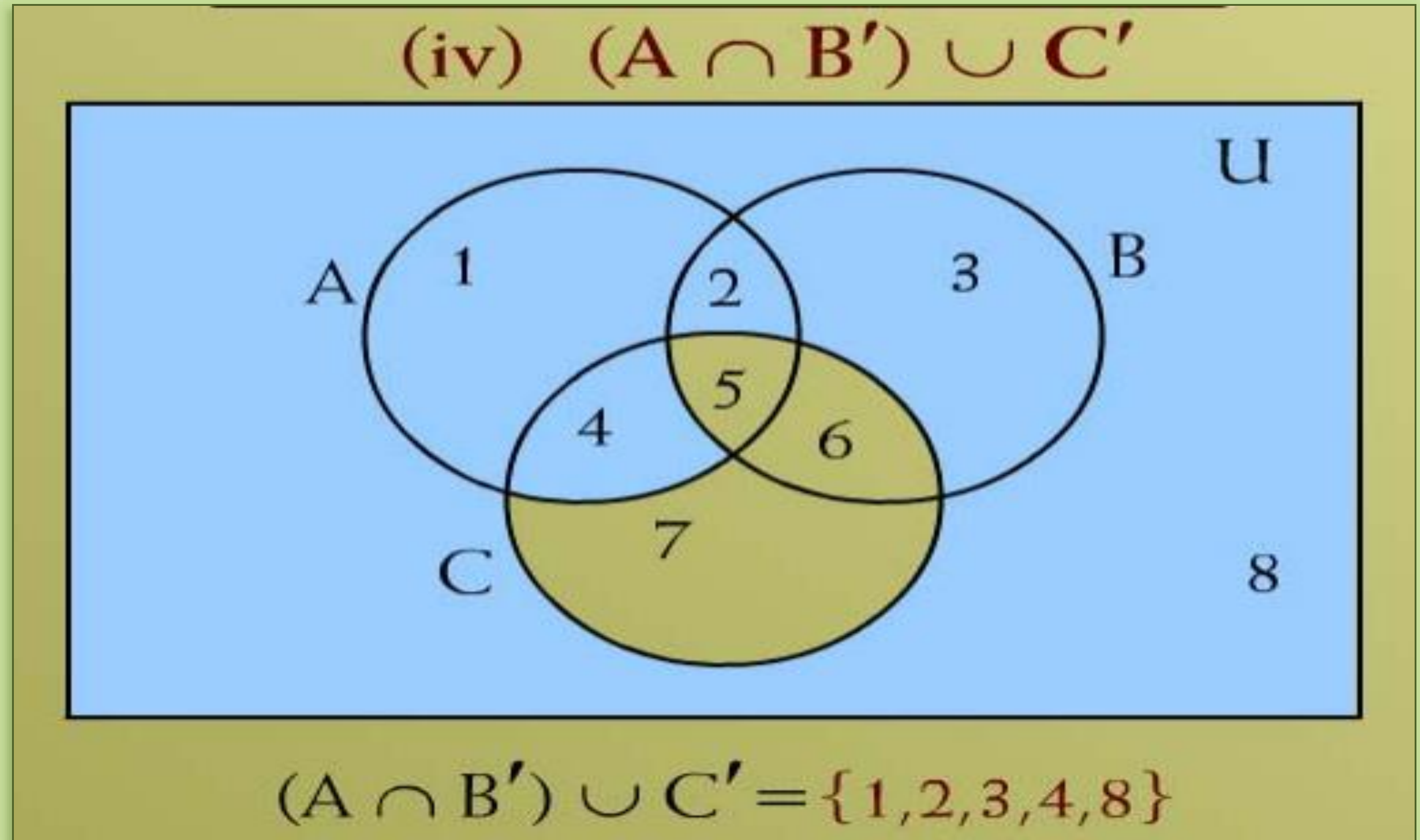


$A' \cup (B \cup C) = \{2, 3, 4, 5, 6, 7, 8\}$

VENN DIAGRAM FOR



VENN DIAGRAM FOR



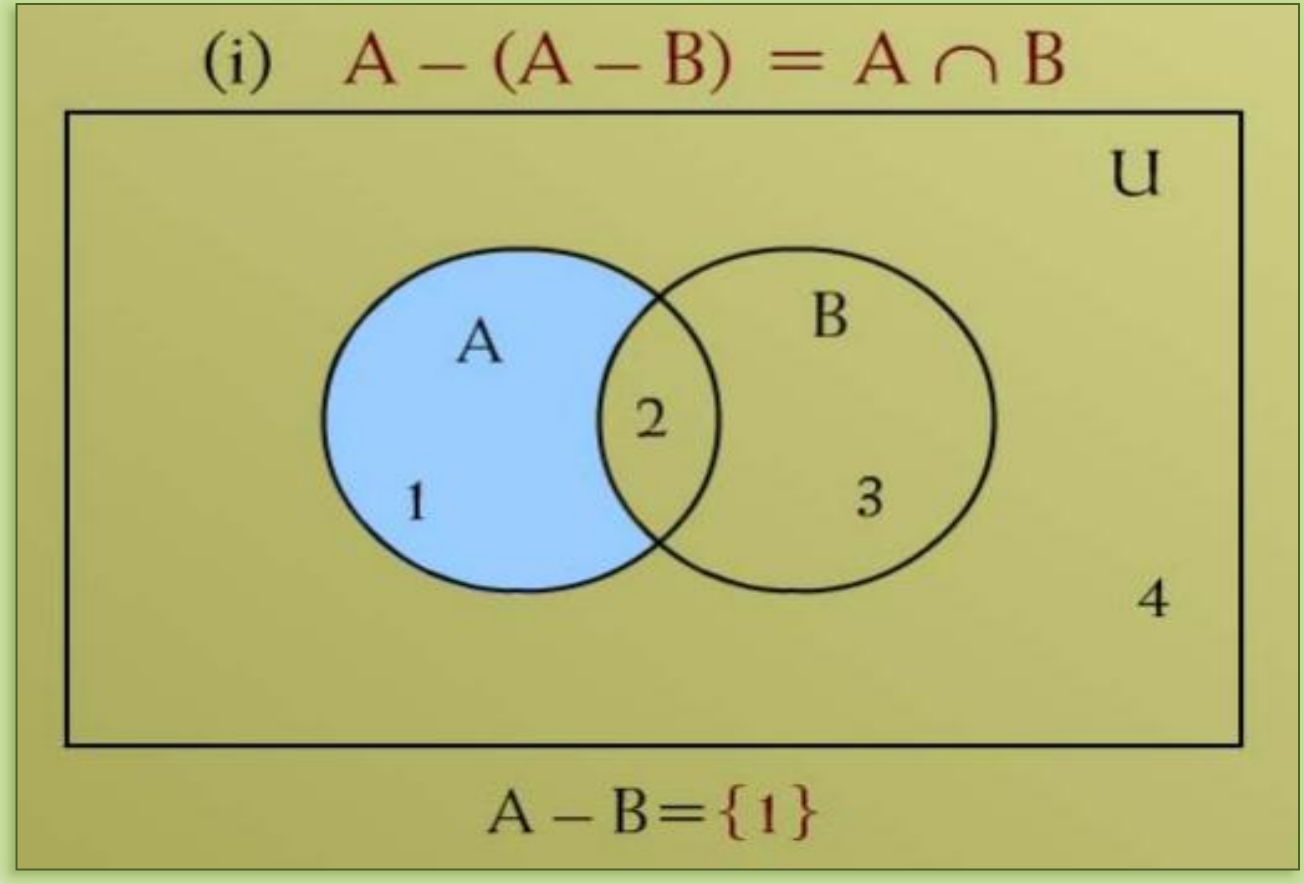
PROOFS USING VENN DIAGRAM

(i) $A - (A - B) = A \cap B$

(ii) $(A \cap B)' = A' \cup B'$

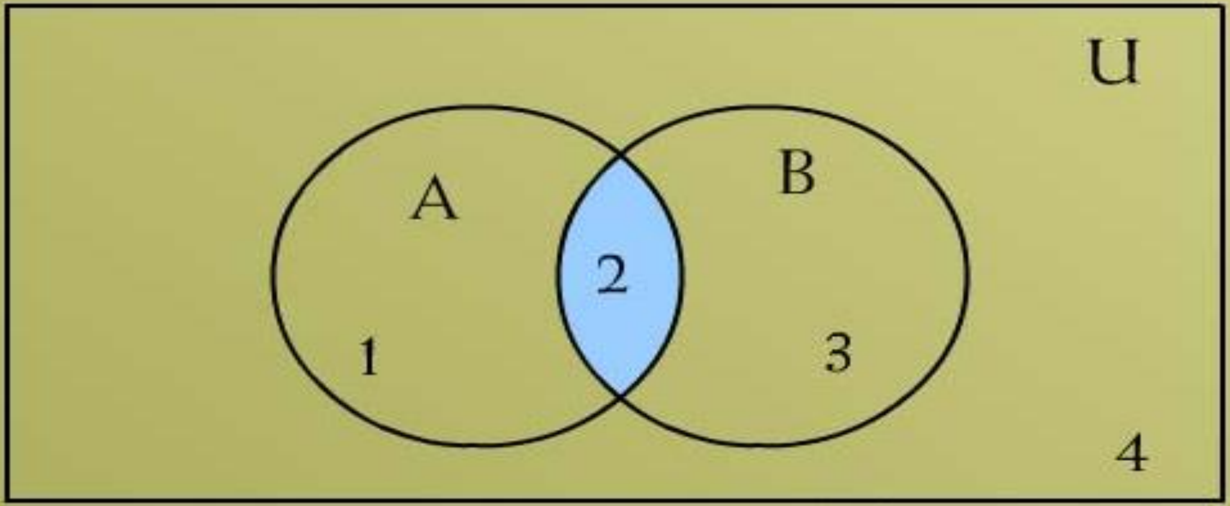
(iii) $A - B = A \cap B'$

SOLUTION



SOLUTION

(i) $A - (A - B) = A \cap B$

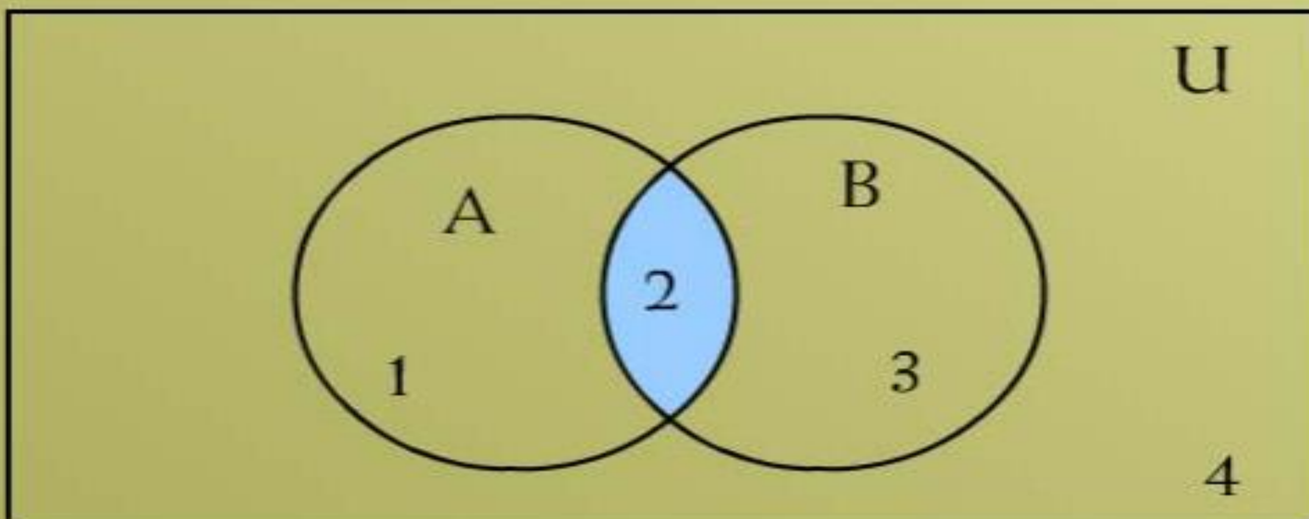


$A - (A - B) = \{2\}$

RESULT: $A - (A - B) = A \cap B$

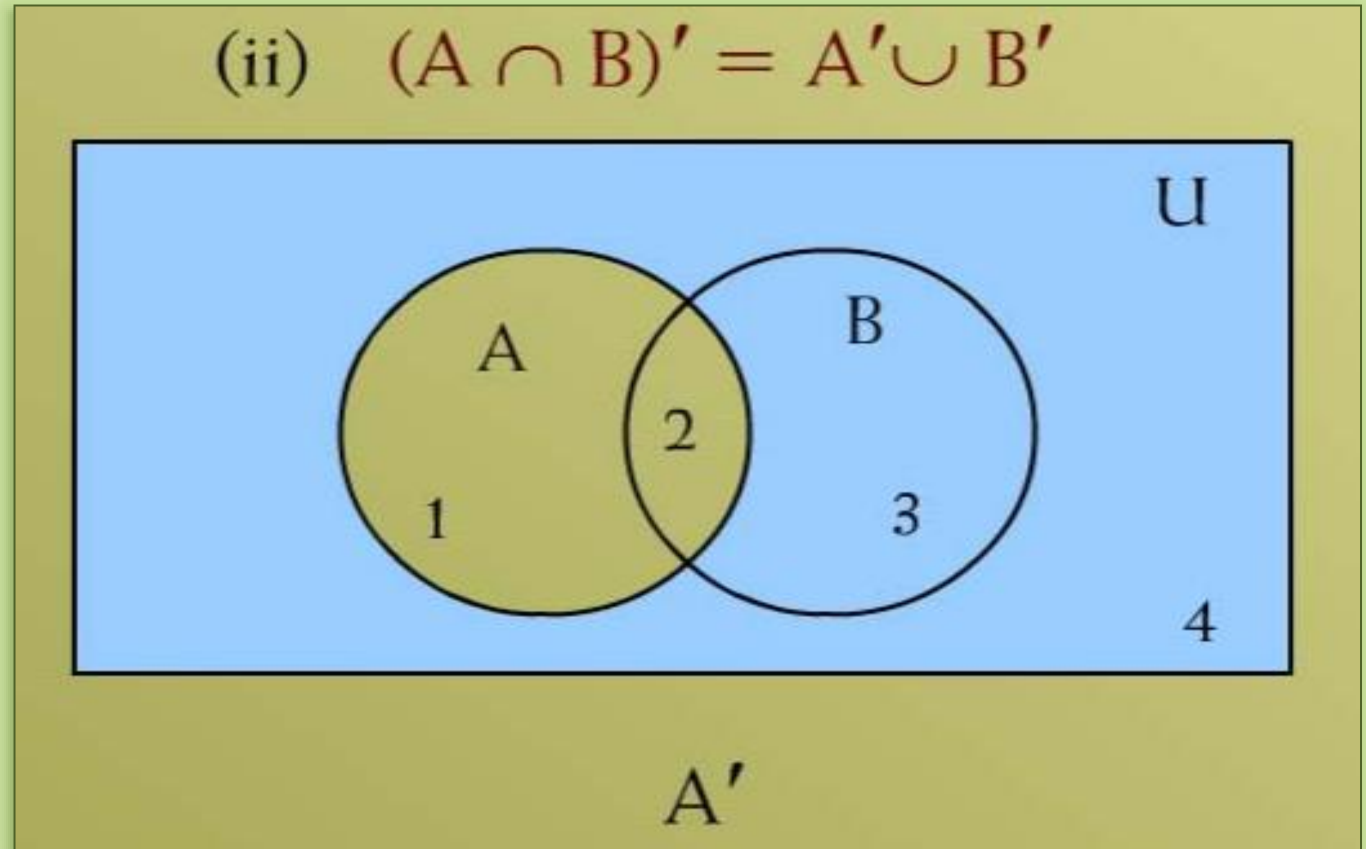
SOLUTION

(ii) $(A \cap B)' = A' \cup B'$

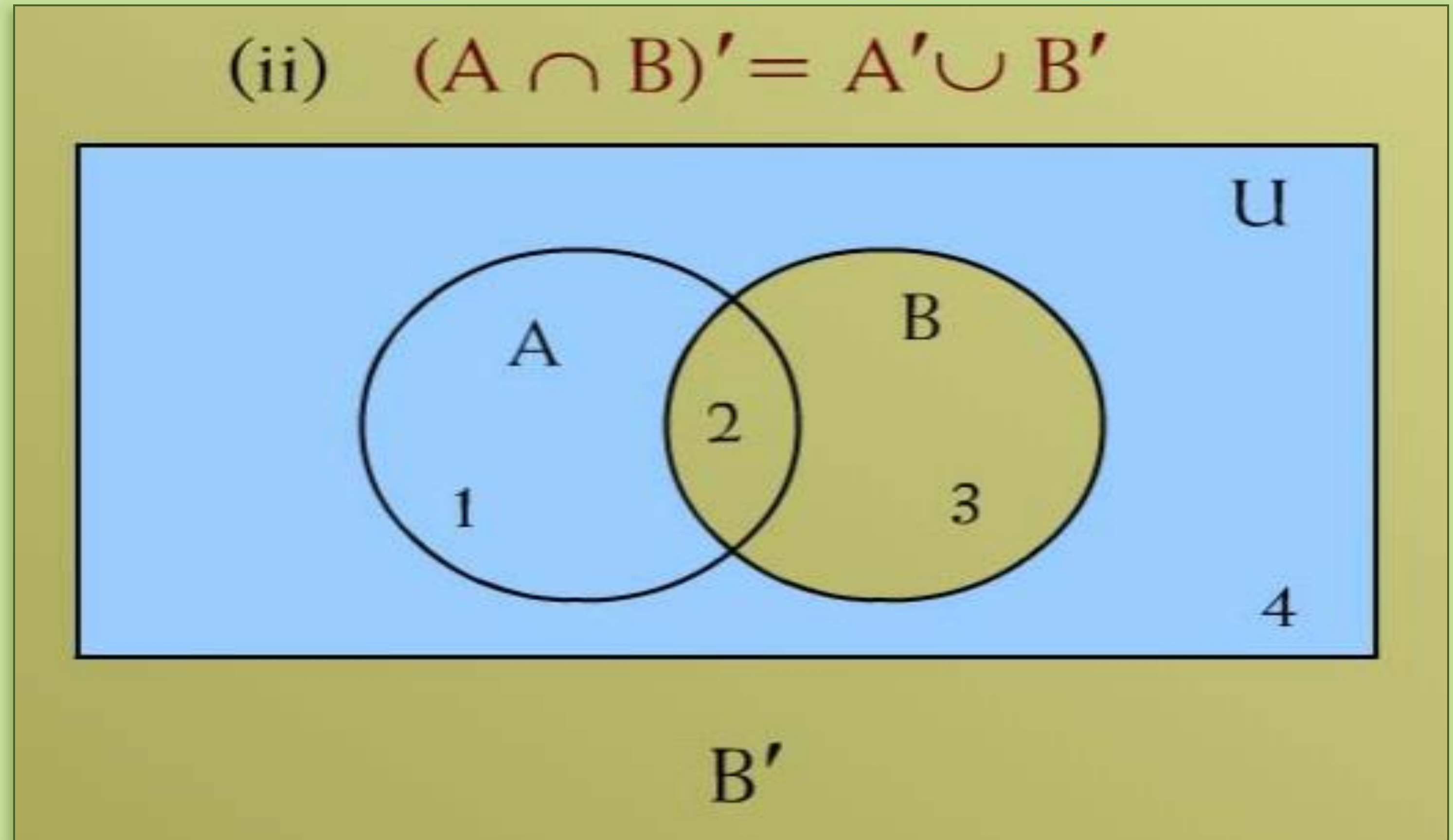


$A \cap B$

SOLUTION

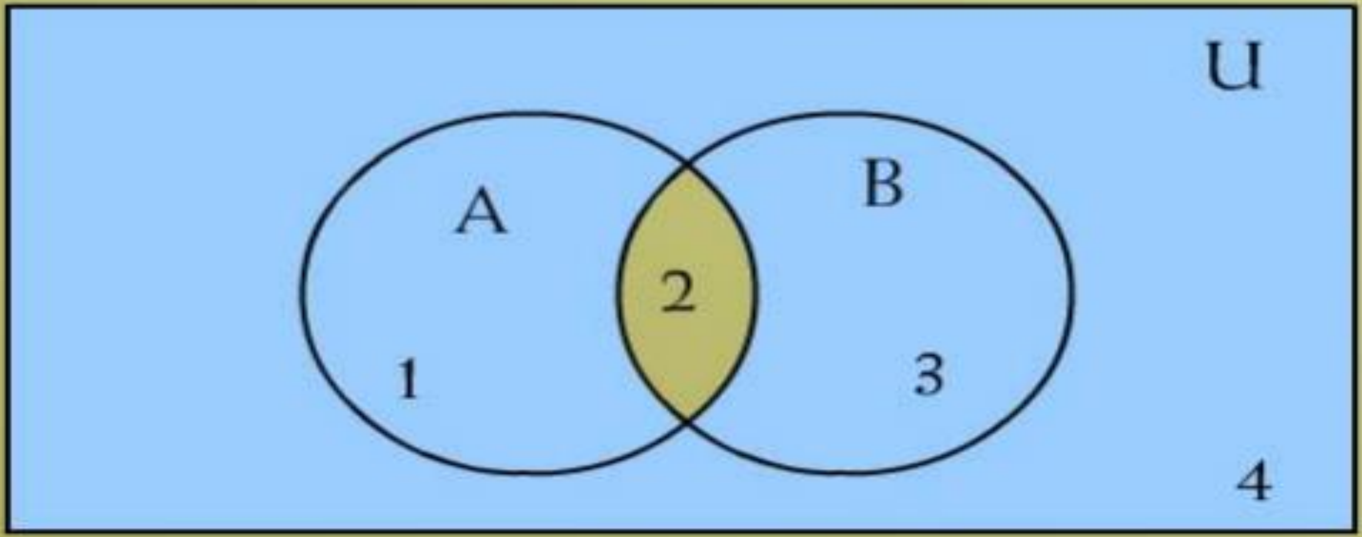


SOLUTION



SOLUTION

(ii) $(A \cap B)' = A' \cup B'$

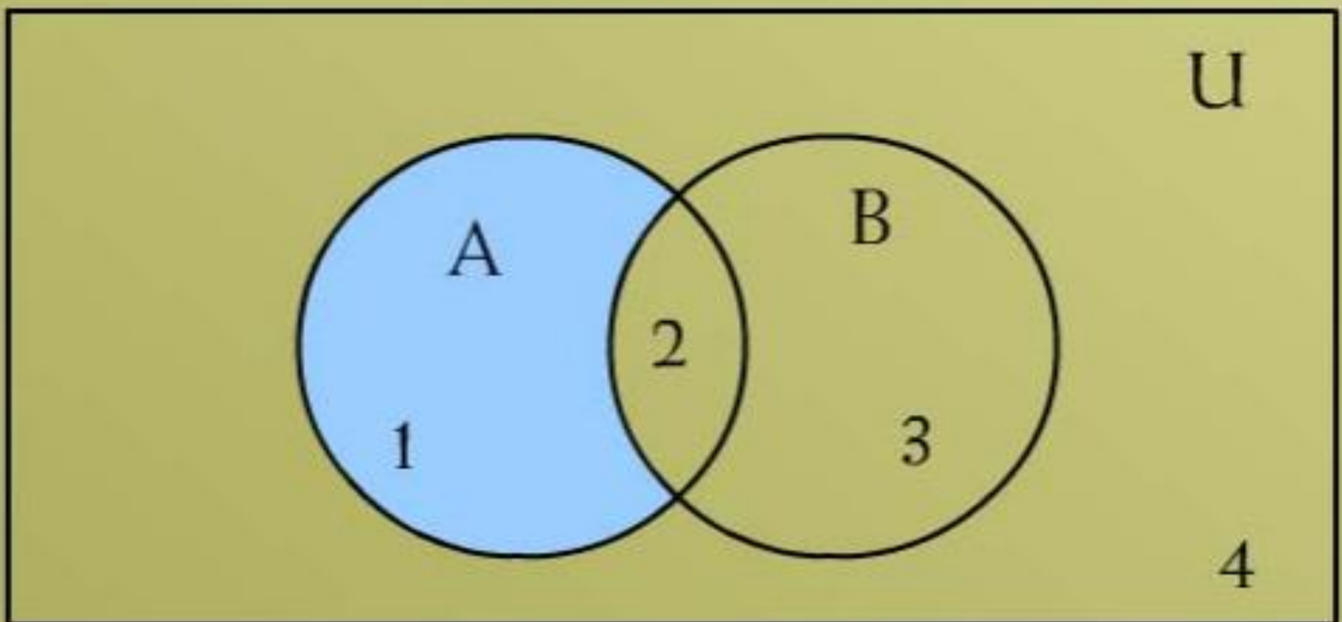


$A' \cup B'$

RESULT: $(A \cap B)' = A' \cup B'$

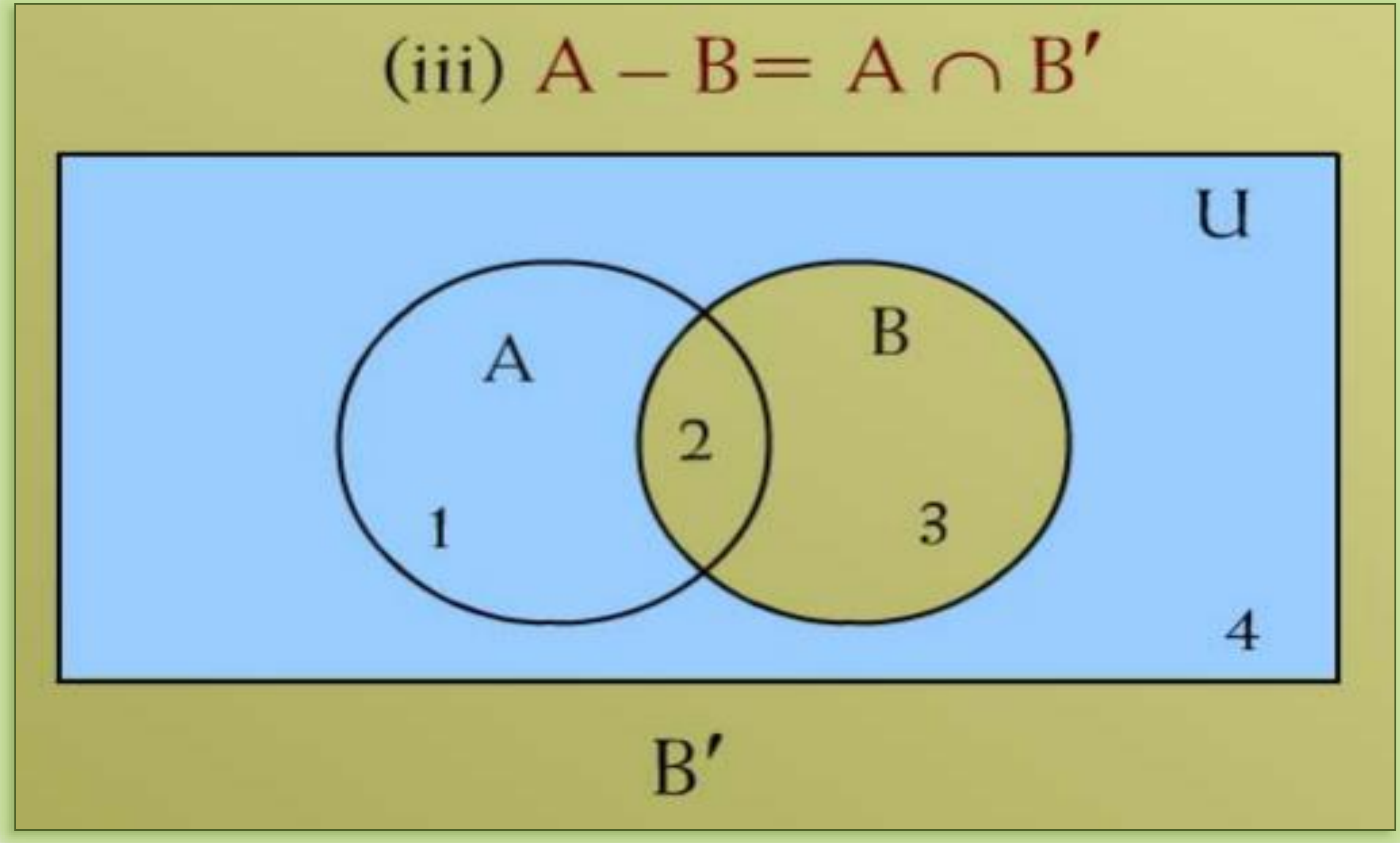
SOLUTION

(iii) $A - B = A \cap B'$



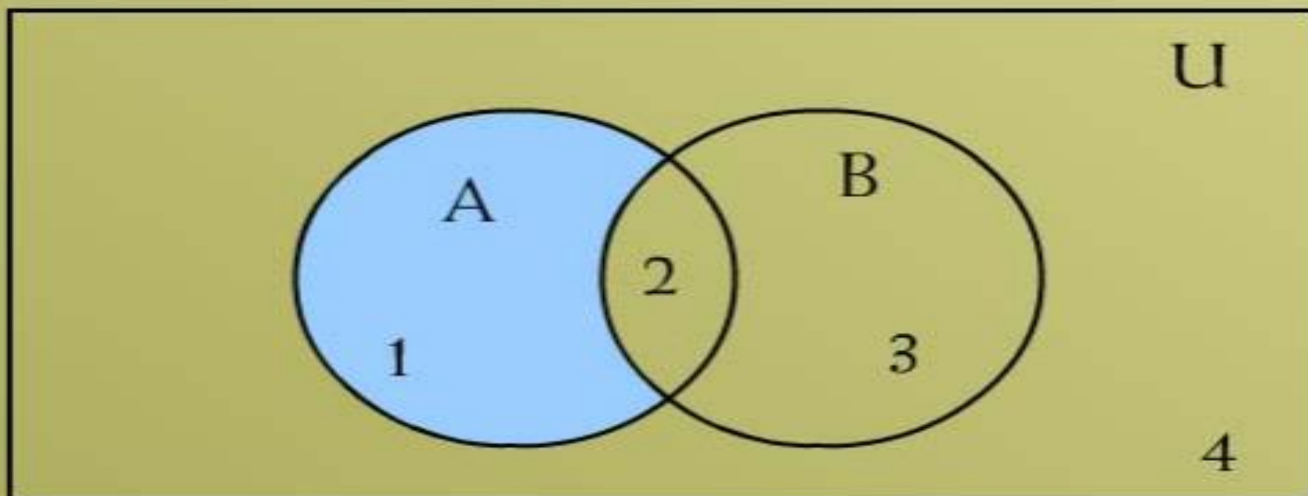
$A - B$

SOLUTION



SOLUTION

(iii) $A - B = A \cap B'$



$A \cap B'$

RESULT: $A - B = A \cap B'$

SET IDENTITIES

Let A, B, C be subsets of a universal set U.

1. **Idempotent Laws**

   a. $A \cup A = A$ b. $A \cap A = A$

2. **Commutative Laws**

   a. $A \cup B = B \cup A$ b. $A \cap B = B \cap A$

3. **Associative Laws**

   a. $A \cup (B \cup C) = (A \cup B) \cup C$

   b. $A \cap (B \cap C) = (A \cap B) \cap C$

SET IDENTITIES

4. **Distributive Laws**
   a. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
   b. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
5. **Identity Laws**
   a. $A \cup \varnothing = A$ b. $A \cap U = A$
6. **Complement Laws**
   a. $A \cup A' = U$ b. $A \cap A' = \varnothing$
   c. $U' = \varnothing$ d. $\varnothing' = U$

Domination Laws

a. $A \cup U = U$ b. $A \cap \varnothing = \varnothing$

SET IDENTITIES

7. **Double Complement Law**

   a. $(A')' = A$

8. **DeMorgan's Laws**

   a. $(A \cup B)' = A' \cap B'$

   b. $(A \cap B)' = A' \cup B'$

9. **Alternative Representation for Set Difference**

   a. $A - B = A \cap B'$

SET IDENTITIES

10. **Subset Laws**

a. $A \cup B \subseteq C$ iff $A \subseteq C$ and $B \subseteq C$

b. $C \subseteq A \cap B$ iff $C \subseteq A$ and $C \subseteq B$

11. **Absorption Laws**

a. $A \cup (A \cap B) = A$

b. $A \cap (A \cup B) = A$

EXERCISE

Let A, B and C be subsets of a universal set U.
Show that

1. $A \subseteq A \cup B$
2. $A - B \subseteq A$
3. If $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$

SOLUTION

1. $A \subseteq A \cup B$

SOLUTION:

Let x be an arbitrary element of A, that is $x \in A$.

$\Rightarrow x \in A$ or $x \in B$

$\Rightarrow x \in A \cup B$

But x is an arbitrary element of A.

$\therefore A \subseteq A \cup B$

Solution contd…

2. $A - B \subseteq A$

SOLUTION:

Let $x \in A - B$

$\Rightarrow x \in A$ and $x \notin B$ (by definition of $A - B$)

$\Rightarrow x \in A$ (in particular)

But x is an arbitrary element of $A - B$

$\therefore A - B \subseteq A$

Solution contd…

3. If $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$

SOLUTION:

Suppose that $A \subseteq B$ and $B \subseteq C$

Consider $x \in A$

$\Rightarrow x \in B$ (because $A \subseteq B$)

$\Rightarrow x \in C$ (because $B \subseteq C$)

But x is an arbitrary element of A

$\therefore \quad A \subseteq C$

EQUALITY OF SETS

Two sets are equal if first is subset of second and second is subset of first.

Now

$$A - B = A \cap B'$$

$$\Leftrightarrow A - B \subseteq A \cap B' \text{ and } A \cap B' \subseteq A - B$$

EXERCISE

Let A and B be subsets of a universal set U.
Prove that

$$A - B = A \cap B'$$

SOLUTION

Let

$$x \in A - B$$

$$\Rightarrow x \in A \text{ and } x \notin B$$

(definition of set difference)

Solution contd…

$\Rightarrow x \in A$ and $x \in B'$

(definition of complement)

$\Rightarrow x \in A \cap B'$

(definition of intersection)

But x is an arbitrary element of $A–B$

$\therefore \quad A–B \subseteq A \cap B'$..............(1)

Solution contd…

Conversely,

Let $y \in A \cap B'$

$\Rightarrow y \in A$ and $y \in B'$

(definition of intersection)

$\Rightarrow y \in A$ and $y \notin B$

(definition of complement)

Solution contd…

$\Rightarrow \; y \in A - B$

(definition of set difference)

But y is an arbitrary element of $A \cap B'$

$\therefore \qquad A \cap B' \subseteq A - B \ldots\ldots\ldots\ldots\ldots(2)$

From (1) and (2) it follows that

$A - B = A \cap B'$ (as required)

DEMORGAN'S LAW

$$(A \cup B)' = A' \cap B'$$

PROOF

First we show that $(A \cup B)' \subseteq A' \cap B'$

Let $x \in (A \cup B)'$

$\Rightarrow x \notin A \cup B$

(definition of complement)

$\Rightarrow x \notin A$ and $x \notin B$

(DeMorgan's Law of Logic)

Proof contd…

$\Rightarrow x \in A'$ and $x \in B'$

(definition of complement)

$\Rightarrow x \in A' \cap B'$

(definition of intersection)

But x is an arbitrary element of $(A \cup B)'$

$\therefore \quad (A \cup B)' \subseteq A' \cap B' \ldots\ldots\ldots(1)$

Proof contd…

Conversely,

Let $y \in A' \cap B'$

$\Rightarrow y \in A'$ and $y \in B'$

(definition of intersection)

$\Rightarrow y \notin A$ and $y \notin B$

(definition of complement)

$\Rightarrow y \notin A \cup B$

(DeMorgan's Law of Logic)

Proof contd…

$\Rightarrow \; y \in (A \cup B)'$

(definition of complement)

But y is an arbitrary element of $A' \cap B'$

$\therefore \quad A' \cap B' \subseteq (A \cup B)'$ ....(2)

From (1) and (2) we have

$(A \cup B)' = A' \cap B'$ (proved)

ASSOCIATIVE LAW

$$A \cap (B \cap C) = (A \cap B) \cap C$$

PROOF

First we show that $A \cap (B \cap C) \subseteq (A \cap B) \cap C$

Consider $x \in A \cap (B \cap C)$

$\Rightarrow x \in A$ and $x \in B \cap C$

(definition of intersection)

$\Rightarrow x \in A$ and $x \in B$ and $x \in C$

(definition of intersection)

Proof contd…

$\Rightarrow x \in A \cap B$ and $x \in C$

(definition of intersection)

$\Rightarrow x \in (A \cap B) \cap C$

(definition of intersection)

But x is an arbitrary element of $A \cap (B \cap C)$

$\therefore \; A \cap (B \cap C) \subseteq (A \cap B) \cap C \ldots\ldots\ldots(1)$

Proof contd…

Conversely,

Let $y \in A \cap (B \cap C)$

$\Rightarrow y \in A \cap B$ and $y \in C$

(definition of intersection)

$\Rightarrow y \in A$ and $y \in B$ and $y \in C$

(definition of intersection)

$\Rightarrow y \in A$ and $y \in B \cap C$

(definition of intersection)

Proof contd…

$\Rightarrow y \in A \cap (B \cap C)$

(definition of intersection)

But y is an arbitrary element of $(A \cap B) \cap C$

$\therefore \quad (A \cap B) \cap C \subseteq A \cap (B \cap C)$ …….(2)

From (1) and (2) we conclude that

$A \cap (B \cap C) = (A \cap B) \cap C$ (proved)

EXERCISE

For all subset A and B of a universal set U, prove that

$$(A - B) \cup (A \cap B) = A$$

PROOF:

$$L.H.S = (A - B) \cup (A \cap B)$$

$$= (A \cap B') \cup (A \cap B)$$

(Alternative representation for set difference)

Proof contd…

$= A \cap (B' \cup B)$

Distributive Law

$= A \cap U$

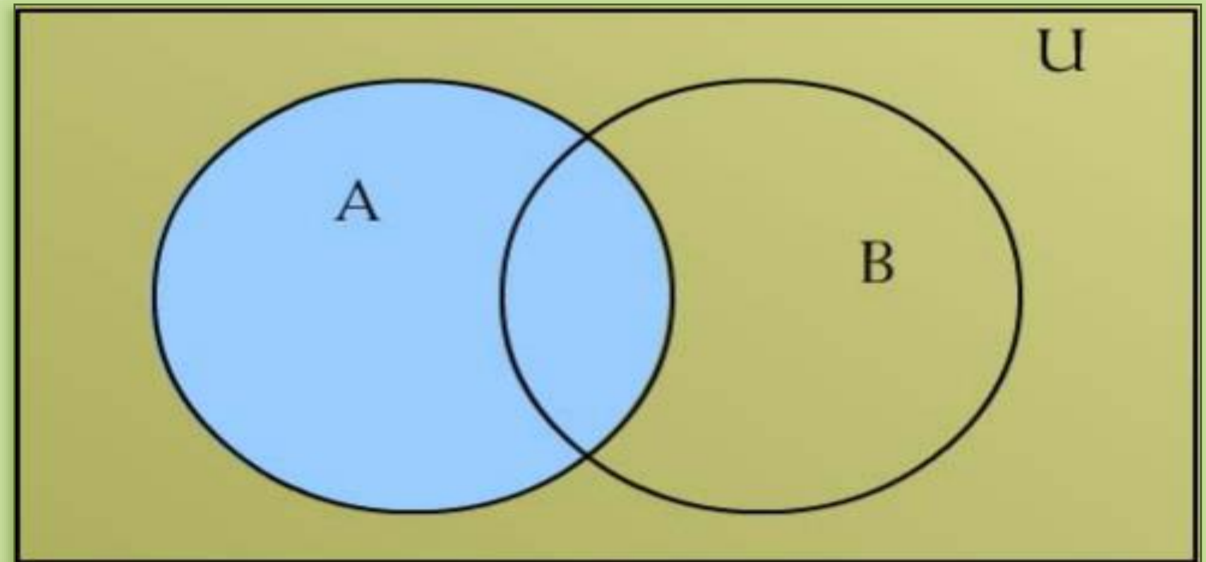
Complement Law

$= A$

Identity Law

$= R.H.S$

VENN DIAGRAM



$$(A - B) \cup (A \cap B) = A$$

EXERCISE

$$A - (A - B) = A \cap B$$

SOLUTION

$$L.H.S = A - (A - B)$$

$$= A - (A \cap B')$$

Alternative representation for set difference

$$= A \cap (A \cap B')'$$

Alternative representation for set difference

Solution contd…

$$\text{L.H.S} = A \cap (A' \cup (B')')$$

DeMorgan's Law

$$= A \cap (A' \cup B)$$

Double Complement Law

$$= (A \cap A') \cup (A \cap B)$$

Distributive Law

$$= \varnothing \cup (A \cap B)$$

Complement Law

$$= A \cap B$$ Identity Law

$$= \text{R.H.S}$$

EXERCISE

$$(A - B) - C = (A - C) - B$$

SOLUTION

$$L.H.S = (A - B) - C$$

$$= (A \cap B') - C$$

Alternative representation of set difference

$$= (A \cap B') \cap C'$$

Alternative representation of set difference

$$= A \cap (B' \cap C') \qquad \text{Associative Law}$$

Solution contd…

$$LHS = A \cap (C' \cap B')$$ Commutative Law

$$= (A \cap C') \cap B'$$ Associative Law

$$= (A - C) \cap B'$$

Alternative representation of set difference

$$= (A - C) - B$$

Alternative representation of set difference

$$= RHS$$ (proved)

PROVING SET IDENTITIES BY MEMBERSHIP TABLE

1. $A - (A - B) = A \cap B$

2. $(A \cap B)' = A' \cup B'$

3. $A - B = A \cap B'$

SOLUTION

$$A - (A - B) = A \cap B$$

A	B	$A - B$	$A - (A - B)$	$A \cap B$
1	1	0	1	1
1	0	1	0	0
0	1	0	0	0
0	0	0	0	0

SOLUTION

$$A - (A - B) = A \cap B$$

A	B	A – B	A – (A – B)	A∩B
1	1	0		
1	0	1		
0	1	0		
0	0	0		

SOLUTION

$$A - (A - B) = A \cap B$$

A	B	$A - B$	$A - (A - B)$	$A \cap B$
1	1	0	1	
1	0	1	0	
0	1	0	0	
0	0	0	0	

SOLUTION

$$A - (A - B) = A \cap B$$

A	B	$A - B$	$A - (A - B)$	$A \cap B$
1	1			1
1	0			0
0	1			0
0	0			0

SOLUTION

$$(A \cap B)' = A' \cup B'$$

A	B	$A \cap B$	$(A \cap B)'$	A'	B'	$A' \cup B'$
1	1	1	0	0	0	0
1	0	0	1	0	1	1
0	1	0	1	1	0	1
0	0	0	1	1	1	1

SOLUTION

$$A - B = A \cap B'$$

A	B	$A - B$	B'	$A \cap B'$
1	1	0	0	0
1	0	1	1	1
0	1	0	0	0
0	0	0	1	0