CS-2001

Programing Fundamentals

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Recursion

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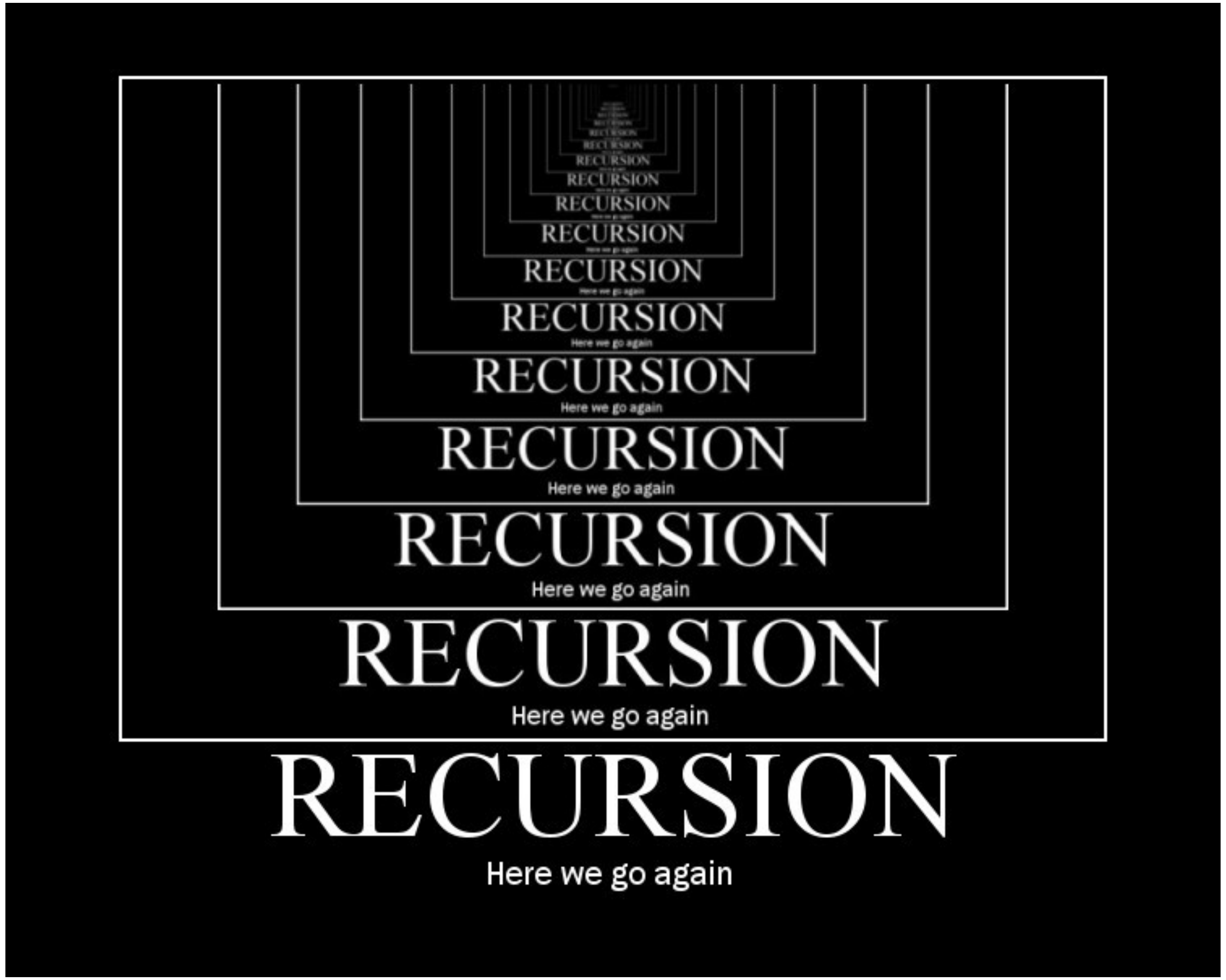

What is Recursion?

what is recursion

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Introduction to Recursion - Data Structure and Algorithm ...

19-Sept-2022 — A **recursive function solves a particular problem by calling a copy of itself and solving smaller subproblems of the original problems**. Many more ...

1): Terminates when the base case becomes t... 3): Every recursive call needs extra space i...
2): Used with functions 4): Smaller code size

What is Recursion?

Recursion:

A problem solving technique in which problems are solved by reducing them to smaller problems *of the same form*.

Why Recursion?

1. Great style
2. Powerful tool
3. Master of control flow

Pedagogy

Many simple examples

Recursion in Programming

In programming, recursion simply means that a function will call itself:

```
int main() {
     main();
     return 0;
}
```

SEG FAULT!

(this is a terrible example, and will crash!)

main() isn't supposed to call itself, but if we do write this program, what happens?

We'll get back to programming in a minute...

Recursion in Real Life?

Recursion

- How to solve a jigsaw puzzle recursively ("solve the puzzle")
  - Is the puzzle finished? If so, stop.
  - Find a correct puzzle piece and place it.
  - Solve the puzzle

ridiculously hard puzzle

In C++:

```
int numStudentsBehind(Student curr) {
if (noOneBehind(curr)) {
        return 0;
    } else {
        Student personBehind = curr.getBehind();
        return numStudentsBehind(personBehind) + 1
   }
}
```

Recursive call!

Recursion ?

- A recursive function is a function that calls itself either directly or indirectly through another function.
  - A piece the function knows how to do and
  - A piece the function doesn’t knows how to do.

- Because this new problem looks like the original problem the function launches (calls) a fresh (new) copy of itself to go to work on smaller problems. This is referred to as a recursive call and is also called recursion step.

Recursion ?

- Given a positive integer n, n factorial is defined as the product of all integers between n and 1. for example…

5! = 5 * 4 * 3 * 2 * 1 = 120

0! = 1

so

n! = 1 if (n== 0)

n! = n * (n-1) * (n-2) * (n-3) …… * 1 if n> 0

Iteration ?

So we can present an algorithm that accepts integer n and returns the value of n! as follows.

```
int fact = 1, n;

cin>>n;

for (int i = 1; i <= n; i++)

{

   fact = fact * i;

 }
```

Such an algorithm is called iterative algorithm because it calls itself for the explicit (precise) repetition of some process until a certain condition is met.

Iterative Solution ?

In above program **n!** is calculated like as follows.

0! = 1

1! = 1

2! = 2 * 1

3! = 3 * 2 * 1

4! = 4 * 3 * 2 * 1

Recursive Solution ?

Let us see how the recursive definition of the factorial can be used to evaluate 5!.

1. 5! = 5 * 4!
2. 4! = 4 * 3!
3. 3! = 3 * 2!
4. 2! = 2 * 1!
5. 1! = 1 * 0!
6. 0! = 1

In above each case is reduced to a simpler case until we reach the case 0! Which is defined directly as 1. we may therefore backtrack from line # 6 to line # 1 by returning the value computed in one line to evaluate the result of previous line.

Recursive Solution ?

```
int factorial( int numb )
{
   if( numb <= 0 )
        return 1;
   else
        return numb * factorial( numb - 1 );
}//------------------------------------------
void main()
{
   int n;
   cout<<" \n Enter no for finding its Factorial.\n";
   cin>>n;
   cout<<"\n Factorial of "<<n<<" is : "<<factorial( n );
  return 0;
}
```

In C++:

The structure of recursive functions is typically like the following:

```
recursiveFunction() {
    if (test for simple case) {
        Compute the solution without recursion
    } else {
        Break the problem into subproblems of the same form
        Call recursiveFunction() on each subproblem
        Reassamble the results of the subproblems
    }
}
```

In C++:

Every recursive algorithm involves at least **two** cases:

- base case: The simple case; an occurrence that can be answered directly; the case that recursive calls reduce to.

- recursive case: a more complex occurrence of the problem that cannot be directly answered, but can be described in terms of smaller occurrences of the same problem.

1. Your code must have a case for all valid inputs

2. You must have a base case that makes no recursive calls

3. When you make a recursive call it should be to a simpler instance and make forward progress towards the base case.

More Examples!

The power() function:

Write a recursive function that takes in a number (x) and an exponent (n) and returns the result of x^n

Powers

$$x^0 = 1$$

$$x^n = x \cdot x^{n-1}$$

Powers

Each previous call waits for the next call to finish (just like any function).

```
cout << power(5, 3) << endl;
```

```
// first call: power (5, 3)
// second call: power (5, 2)
// third call: power (5, 1)
// fourth call: power (5, 0)
int power(int x, int exp) {
    if (exp == 0) {
        return 1;
    } else {
        return x * power(x, exp - 1);
    }
}
```

Powers

- Each previous call waits for the next call to finish (just like any function).

```
cout << power(5, 3) << endl;
```

```
// first call: power (5, 3)
// second call: power (5, 2)
// third call: power (5, 1)
// fourth call: power (5, 0)
int power(int x, int exp) {
    if (exp == 0) {
        return 1;
    } else {
        return x * power(x, exp - 1);
    }
}
```

This call returns 1

Powers

- Each previous call waits for the next call to finish (just like any function).

```
cout << power(5, 3) << endl;
```

```
// first call: power (5, 3)
in
in
}
```

```
// second call: power (5, 2)
i
}
```

```
// third call: power (5, 1)
int power(int x, int exp) {  if (exp
      == 0) {
     } else { return 1;
          return x * power(x, exp - 1);
     }
}
```

equals 1 from call

this entire statement returns 5 * 1

• Each previous call waits for the next call to finish (just like any function).

```
cout << power(5, 3) << endl;
```

```
// first call: power (5, 3)
i
n
}
```

```
// second call: power (5, 2)  int
power(int x, int exp) {
    if (exp == 0) {
        return 1;
    } else {
        return x * power(x, exp - 1);
    }
}
```

equals 5 from call

this entire statement returns 5 * 5

Powers

- Each previous call waits for the next call to finish (just like any function).

```
cout << power(5, 3) << endl;
```

```
// first call: power (5, 3)  int power(int
x, int exp) {
      if (exp == 0) {  return 1;
      } else {
            return x * power(x, exp - 1);
      }
}
```

equals 25 from call

this entire statement returns 5 * 25

the original function call was to this one, so it returns 125, which is 5^3

```
int power(int x, int exp) {  if(exp == 0)
      {
            // base case  return
            1;
      } else {
            if (exp % 2 == 1) {
            // if exp is odd
                  return x * power(x, exp - 1);
            } else {
                  // else, if exp is even  int y =
                  power(x, exp / 2);  return y * y;
            }
      }
}
```

Exponentiation by squaring

Big O???

O(log n) -- yay!

Mystery Recursion: Trace this function

```
int mystery  (int n) {
     if (n <10) {
          return n;
     }
     else {
          int a = n/10;
          int b = n % 10;
          return mystery(a + b);
     }
}
```

What is the result of mystery(648)?

A. 8
B. 9
C. 54
D. 72
E. 648

Mystery Recursion: Trace this function

```
int mystery(int n) { // n = 648
        if (n < 10) {
            return n;
        } else {
            int a = n/10; // a = 64
            int b = n % 10; // b = 8
            return mystery(a + b); // mystery(72);
        }
}
```

Mystery Recursion: Trace this function

```
int mystery(int n) { // n = 648
    int mystery(int n) { // n = 72
            if (n < 10) {
             return n;
        } else {
             int a = n/10; // a = 7
             int b = n % 10; // b = 2
             return mystery(a + b); // mystery(9);
}       }
    }
```

Mystery Recursion: Trace this function

```
int mystery(int n) { // n = 648
int mystery(int n) { // n = 72
int mystery(int n) { // n = 9
        if (n < 10) {
          return n; // return 9;
        } else {
          int a = n/10;
          int b = n % 10;
          return mystery(a + b);
        }
}
}
}
```

Mystery Recursion: Trace this function

```
int mystery(int n) { // n = 648
    int mystery(int n) { // n = 72
            if (n < 10) {
              return n;
        } else {
              int a = n/10; // a = 7
              int b = n % 10; // b = 2
}             return mystery(a + b); // mystery(9);
        }
                                returns 9
    }
```

Mystery Recursion: Trace this function

```
int mystery(int n) { // n = 648
        if (n < 10) {
            return n;
    } else {
        int  a = n/10; // a  = 64
        int  b = n % 10; // b =  8
          return mystery(a + b); //
    }
}
```

returns 9

What is the result of mystery(648)?

A. 8

B. 9

C. 54

D. 72

E. 648

More Examples! isPalindrome(string s)

Write a recursive function isPalindrome accepts a string and returns true if it reads the same forwards as backwards.

isPalindrome("**madam**") → true
isPalindrome("**racecar**") → true
isPalindrome("**step on no pets**") → true
isPalindrome("**Java**") → false
isPalindrome("**byebye**") →false

Three Musts of Recursions

1. Your code must have a case for all valid inputs

2. You must have a base case that makes no recursive calls

3. When you make a recursive call it should be to a simpler instance and make forward progress towards the base case.

isPalindrome(string s)

```
// Returns true if the given string reads the same
// forwards as backwards.
// Trivially true for empty or 1-letter strings.
bool isPalindrome(const string& s) {
    if (s.length() < 2) { // base case
        return true;
    } else { // recursive case
        if (s[0] != s[s.length() - 1]) {
        return false;
        }
        string middle = s.substr(1, s.length() - 2);
        return isPalindrome(middle);
    }
}
```

Flashback to 106A: Hailstone

```
// Couts the sequence of numbers from n to one
// produced by the Hailstone (aka Collatz) procedure
void hailstone(int n) {
	cout << n << endl;
	if(n == 1) {
		return;
	} else {
		if(n % 2 == 0) {
			// n is even so we repeat with n/2
			hailstone(n / 2);
		} else {
			// n is odd so we repeat with 3 * n + 1
			hailstone(3 * n + 1);
		}
	}
}
```

Flashback to 106A: Hailstone

```
// Couts the sequence of numbers from n to one
// produced by the Hailstone (aka Collatz) procedure
void hailstone(int n) {
    cout << n << endl;
    if(n == 1) {
        return;
```

3. When you make a recursive call it should be to a simpler instance and make forward progress towards the base case.

```
            // n is odd so we repeat with 3 * n + 1
            hailstone(3 * n + 1);
        }
    }
}
```

Is this simpler???

Flashback to 106A: Hailstone

`hailstone(int n)`

Hailstone has been checked for values up to 5×10^{18} but

no one has proved that it always reaches 1!

There is a cash prize for proving it!

The prize is $1400.

Flashback to 106A: Hailstone

Print the sequences of numbers that you take to get from N until 1, using the Hailstone (Collatz) production rules:

If n == 1, you are done.
If n is even your next number is n / 2.
If n is odd your next number is 3*n + 1.

n=3; 10, 5, 16, 8, 4, 2, 1, 4, 2, 1, …
n=4; 2, 1, 4, 2, 1, …
n=7; 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1, 4, 2, 1,

Fibonacci Sequence: Recursion

Print the nth terms of a fibonacci sequence using recursion:

- Fibonacci Series:
  - 0,1,1,2,3,5,8,13,21,.....etc
- first two terms:
  - fo = 0, f1 = 1

$$F_0 = 0$$

$$F_1 = 1$$

$$F_n = F_{n-1} + F_{n-2}$$

Fibonacci Sequence: Recursion

```
int fibonacci(int n)
{
    if (n <=1)
        return n; // base cases
    else
        return fibonacci(n-1) + fibonacci(n-2); // recursive step
}
int main()
{
    cout << fibonacci(6) <<endl;
    return 0;
}
```

Towers of Hanoi: C++ Code

```
void TOH(int n,char Sour, char Aux,char Des)
{
    if(n==1)
        { cout<<"Move Disk "<<n<<" from "<<Sour<<" to "<<Des<<endl;      return; }

    TOH(n-1,Sour,Des,Aux);
    cout<<"Move Disk "<<n<<" from "<<Sour<<" to "<<Des<<endl;
    TOH(n-1,Aux,Sour,Des);
}

int main()
{
    int n;
    cout<<"Enter no. of disks:";
    cin>>n;
    TOH(n,'A','B','C');

    return 0;
}
```

Efficiency of Recursion

- In general, a non recursive version of a program will execute more efficiently in terms of time and space than a recursive version. This is because the overhead involved in entering and exiting a new block (system stack) is avoided in the non recursive version.

- However, sometimes a recursive solution is the most natural and logical way of solving a problem as we will soon observe while studying different Tree data structures.

Recap

- **Recursion**
  - Break a problem into smaller subproblems of the same form, and call the same function again on that smaller form.
  - Super powerful programming tool
  - Not always the perfect choice, but often a good one
  - Some beautiful problems are solved recursively

- **Three Musts for Recursion:**

1. Your code must have a case for all valid inputs
2. You must have a base case that makes no recursive calls
3. When you make a recursive call it should be to a simpler instance and make forward progress towards the base case.

Practice Tasks

1. Given a string as input, find all the subsequences of word, separated by commas,

 * where a subsequence is a string of letters found in word

 * in the same order that they appear in word.

subsequences("**abc**") might return "**abc,ab,bc,ac,a,b,c,”**