

### Quiz 3 SOLUTION

1. If  $L$  and  $N$  are languages and  $L$  can be reduced to  $N$ . It is known that  $L$  is not decidable but recognizable then what can you say about the complement of  $N$ ? Is it recognizable or decidable or cannot be known and why? Answer in 2-3 lines. No marks without justification.

If  $L$  can be reduced to  $N$  and  $L$  is known to be a recognizable language, then  $N$  is also a recognizable language.

From theorem discussed in class, we know if a language is recognizable, then its complement will be unrecognizable.

Therefore  $N$ -complement will be unrecognizable.

2. Find out if the following problem is Turing Decidable or Turing Recognizable or Turing Unrecognizable?

**Problem: Given two Turing machines  $M_1$  and  $M_2$ . Find out if  $L(M_1) \cap L(M_2) = \Phi$ .**

Give valid justification of your answer by giving an algorithm.

Using the following algorithm

- a. For every  $w \in \Sigma^*$ 
  - i. Run  $M_1$  for  $w$
  - ii. Run  $M_2$  for  $w$
  - iii. If  $M_1$  accepts and  $M_2$  also accepts, reject
- b. Accept

We can see then the loop in statement a will run forever and we will never goto statement b, therefore if

1.  $L(M_1) \cap L(M_2) \neq \Phi$ , the algorithm will reject
2.  $L(M_1) \cap L(M_2) = \Phi$ , the algorithm will loop forever

Hence the given problem is not Recognizable