Design and Analysis of Algorithms

Spring 2024

Practice Problems

Problem 1

Rank the following functions by order of growth; that is, find an arrangement $g_1, g_2, \ldots g_{20}$ of the functions satisfying $g_1 = O(g_2), g_2 = O(g_3), \ldots, g_{19} = O(g_{20})$. Partition your list into equivalence classes such that functions f (n) and g(n) are in the same class if and only if f (n) = θ(g(n)).

$\sqrt{2}^{lgn}$	n^2	$n!$	$(lgn)!$	$(3/2)^n$
n^3	$(lgn)^2$	$\lg(n!)$	2^{2^n}	$lnln(n)$
$n.2^n$	2^{lgn}	e^n	4^{lgn}	$(n+1)!$
n^n	2^2	$nlgn$	1	n

Problem 2

Indicate, for each pair of expressions .(A,B) in the table below, whether A is O,Ω or θ of B. Assume that k >= 1, ϵ > 0, and c > 1 are constants. Your answer should be in the form of the table with "yes" or "no" written in each box

A	B	O	Θ	Ω
$\log^k(n)$	$n^{1-\epsilon}$			
n^k	$5c^n$			
2^n	2^{n+1}			
$2^{n/2}$	2^n			
n^{lgc}	c^{lgn}			
$n^{1/2}$	$\lg(n^2)$			
lg(n!)	$\lg(n^n)$			

Problem 3

Solve the following recurrences and compute the asymptotic upper bounds. Assume that T(n) is a constant for sufficiently small n. Make your bounds as tight as possible.

a. $T(n) = 2T\left(\frac{n}{2}\right) + n^4$

b. $T(n) = T\left(\frac{7n}{10}\right) + n$

c. $T(n) = 2T(n-1) + c$