Design and Analysis of Algorithms

Spring 2024

Practice Problems

Problem 1

Rank the following functions by order of growth; that is, find an arrangement $g_1, g_2, \ldots g_{20}$ of the functions satisfying $g_1=O(g_2), g_2 = O(g_3), \ldots, g_{19} = O(g_{20})$. Partition your list into equivalence classes such that functions f (n) and g(n) are in the same class if and only if $f(n) = \theta(g(n))$.

$\sqrt{2}^{lgn}$	n^2	$n!$	$(lgn)!$	$(3/2)^n$
n^3	$(lgn)^2$	$\lg(n!)$	2^{2^n}	$lnln(n)$
$n.2^n$	2^{lgn}	e^n	4^{lgn}	$(n+1)!$
n^n	2^2	$nlgn$	1	n

Solution

2^{2^n}	
n^n	
$(n+1)!$	
$n!$	
e^n	
$n.2^n$	
$(3/2)^n$	
$(lgn)!$	
n^3	
n^2	4^{lgn}
$\lg(n!)$	$nlgn$
2^{lgn}	n
$\sqrt{2}^{lgn}$	
$(lgn)^2$	
$lnln(n)$	
2^2	1

Problem 2

Indicate, for each pair of expressions .(A,B) in the table below, whether A is O,Ω or θ of B. Assume that $k >= 1$, $\epsilon > 0$, and $c > 1$ are constants. Your answer should be in the form of the table with "yes" or "no" written in each box

A	B	O	Θ	Ω
$\log^k(n)$	$n^{1-\epsilon}$	X		
n^k	$5c^n$	X		

2^n	2^{n+1}	X	X	X
$2^{n/2}$	2^n	X		
$n^{\lg c}$	$c^{\lg n}$	X	X	X
$n^{1/2}$	$\lg(n^2)$			X
$\lg(n!)$	$\lg(n^n)$	X	X	X

Problem 3

Solve the following recurrences and compute the asymptotic upper bounds. Assume that T(n) is a constant for sufficiently small n. Make your bounds as tight as possible.

a. $T(n) = 2T\left(\frac{n}{2}\right) + n^4 = 2^k T\left(\frac{n}{2^k}\right) + n^4 \sum_{i=0}^{k-1} (\frac{1}{8})^i = O(n^4)$

b. $T(n) = T\left(\frac{7n}{10}\right) + n = T\left(\frac{n}{\left(\frac{10}{7}\right)^i}\right) + n \sum_{i=0}^{k-1} (\frac{7}{10})^i = O(n)$

c. $T(n) = 2T(n-1) + c = 2^k T(n-k) + c \sum_{i=0}^{k-1} (2)^i = O(2^n)$