National University of Computer and Emerging Sciences, Lahore Campus

	Course:	**Design and Analysis of Algorithms**	**Course Code:**	**CS302**
	Program:	**BS(Computer Science)**	**Semester:**	**Spring 2020**
	Duration:	**4 Hours + 30 minutes for uploading exam**	**Total Marks:**	**100**
	Paper Date:	**09-July-2020**	**Weight**	**50**
	Section:	**ALL**	**Page(s):**	**5**
	Exam:	**Final Exam**		

Instruction/Notes: Attempt the examination on white paper and submit handwritten neat and concise answers. Submit paper as one PDF file by combining images as one PDF file. Name of file should be your roll number and name.
Your explanation is very important and will be used to assess your own contribution in this exam.
You have 4 hours to solve this exam and extra 30 min to scan and upload your solution. The paper should be uploaded no later than 1:30 pm.
You have choice in solving any one of the Problems 3a or 3b.
Similarly you have choice to solve any one of the Problems 5a or 5b

Problem 1: [20 Marks]

Matt Damon: Another Rescue: NASA sent you and Matt Damon on a space mission and the inevitable happened. Matt went missing! AGAIN!

Now, you are on a star in a far off galaxy. There are n planets to your left and n planets to your right. Matt is on one of these planets. You are low on fuel so you can't go after him. So you decide to use your knowledge of Computer Science to find his current location, using the available data. His space suit has a built in thermal sensor that is transmitting the current temperature of his location. You have lost all other communication with him.

NASA has sent you two reports from previous expeditions of this galaxy. One of the reports contains temperatures of all the planets on your left, in decreasing order. The second report has temperatures of all the planets on your right, in increasing order. Each planet has a distinct value of temperature. You also know that the presence of a human body raises the temperature of that planet by 5 degrees. Using this information, devise an algorithm to detect Matt's location that takes no more than O(log n) time.

First give a 3-4 line explanation of your idea and then give detailed pseudo code of your algorithm. Also show that how your designed algorithm is O(log n)

Problem 2: [20 Marks]

When XYZ Sugar Mill opens each morning at 8:00 AM, there is queue of $\boldsymbol{n}$ vehicles waiting to unload that have travelled from different farms. The XYZ sugar mill owns these vehicles they just pay the drivers by the hours of unloading, and thus would like to find an efficient order in which to unload the vehicles. Let $\boldsymbol{u_i}$ be the time required to unload vehicle i. Keep in mind that the drivers are indifferent to the order in which vehicles are unloaded since they are paid by the hour so if they have to wait more than will get paid more as well.

So, the XYZ Sugar Mill wants to minimize

$$T_n = \sum_{i=1}^{n} t_i$$

$Where\ T_n\ is\ the\ total\ time ¿ unload\ n\ vehicles.$

$t_i = T_{i-1} + u_i$

The time required to unload a vehicle depends on its capacity and the type and size of unloading hatches. For example, some vehicles have large, automatically operated hatches; others have small, manually operated hatches. The waiting time of a truck while other trucks are being unloaded (T_{i-1}) is also counted in its time (t_i).

Your task is to design and algorithm to let the XYZ Sugar Mill optimize its chosen objective function given $\boldsymbol{u_i}$, where $\boldsymbol{1 \leq i \leq n}$.

i. Write pseudo code (or steps) that defines how the algorithm works
ii. What kind of algorithm is it: Divide and Conquer, Greedy, dynamic programming or Brute Force etc?
iii. What is the running time of the algorithm? Explain in few sentences.

Solve any of the following two Problems (3(a) OR 3(b)) [20 Marks]

Problem 3(a):

Suppose your Design and Analysis of Algorithms professor has given you a really tough take home exam having total n challenging problems. You have to solve the problems in the given sequence. Each problem carries different marks. You have limited time and you cannot attempt all problems.

Your professor has offered to give hints for each problem. The marks for correct solution of each problem are reduced if you ask for hint for any problem. Each problem's hint is different according to the problem so the marks are also reduced differently for each problem.

If you decide to solve the problem without any hint then due to stress of solving the problem you will have to skip the next problem.

Your goal is to find which problems should be attempted and for which problem hint should be used in order to maximize total marks.

Suppose the exam has 5 problems. Following table gives total marks of original problem O_i and marks if hint is used H_i for each problem.

	Problem 1	Problem 2	Problem 3	Problem 4	Problem 5
$\mathbf{H_i}$	7	2	5	15	10
$\mathbf{O_i}$	10	10	15	50	25

The optimal solution for this exam is to attempt first problem using hint, attempt second problem without hint and attempt fourth problem without hint.

Optimal = H1+O2+O4 = 7+10+50 = 67

Note that since we have decided to solve second problem without hint so we have to skip the third problem. Similarly we also skip fifth problem.

i. Show that following algorithm does not give optimal answer by giving a counter example. You should show the optimal solution and the solution found by this algorithm.

```
   i =0, totalMarks = 0
   while ( i <= n ) {
      if  ( O[i] > H[i] + O[i+1]) {
                  Print “Attempt Problem i without hint”
                  totalMarks += O[i]
            }
       else {
                  totalMarks += H[i] + O[i+1]
            }
      i += 2
     }
```

ii. Give an efficient algorithm using dynamic programming to solve the given problem of selecting questions in order to maximize total marks. Also write its time complexity.

OR

Problem 3(b):

Suppose your Design and Analysis of Algorithms professor has given you a really tough take home exam having total n challenging questions. Each question has different marks and different amount of time to solve the question. The time required to solve the question is also given with each question. It is not necessary that the question requiring more time has more marks. There is no relation between the time required to solve the question and its total marks. The professor has announced that she will not give partial credit to incorrect or incomplete solutions and there will be binary marking. The paper is so lengthy that you cannot attempt all questions in given time so you have to choose the questions wisely.

Devise an efficient algorithm to select which questions should be solved in order to maximize total marks. You are given n questions, with marks and time (minutes) for each question. Total time (T minutes) to solve the paper is also given.

For example, suppose the following table gives the 4 questions of the exam. The total time to solve the paper is 180 minutes.

	Problem 1	**Problem 2**	**Problem 3**	**Problem 4**
$time_i$	50	70	60	100
$marks_i$	100	200	50	400

Optimal solution = 200 + 400 = 600 marks

i. Show that following algorithm does not give optimal answer by giving a counter example. You should show the optimal solution and the solution found by this algorithm.

```
 Sort all questions by marks/time ratio in descending order
 and number them from 1 to n such that the problem numbered
```

```
1 has highest marks/time ratio.
remianingTime = totalTime
totalMarks = 0
i = 1
While (i <= n AND reminingTime >= 0) {
  If (time[i] <= remianingTime) {
       totalMarks += marks[i]
       remianingTime = remianingTime  - time[i]
    }

i = i+1
}
Return totalMarks
```

ii. Give an efficient algorithm using dynamic programming to solve the given problem of selecting questions in order to maximize total marks. Also write its time complexity.

Problem 4: [20 Marks]

[Town Planning] The government of Pakistan is planning to build a new town near Lahore to provide houses to homeless people. Initially, it was planned that all the roads will be two way. However, the newly elected government changed the plan and mark all the roads as one way roads. The directions of the roads are marked in a way that that all places are reachable from the Town Hall. Due to this change, not all the places in the town are reachable from each other. In other words there are some pair of places that has no directed path between them. This means some new roads must be constructed to provide path between all those pair of places that are not connected in this one way road network. The government wants to spend minimum amount of money on this project. You are asked to analyze the current road network and add minimum possible new roads such that all pair of places has a path between them. Formulate this problem as a graph problem and give a linear time algorithm that can compute the new edges (roads) given the existing road network.

First give a 3-4 line explanation of your idea and then give detailed pseudo code of your algorithm. Also show that how your designed algorithm is linear.

Solve any of the following two Problems (5(a) OR 5(b)) [20 Marks]

Problem 5(a):

Agents Alice and Bob have been stationed in the enemy territory at houses A and B respectively. At one point during their mission, Alice needs to hand over a bag containing high-valued secret items to Bob. The contents of this bag are top secret and a third party cannot be trusted to deliver it. It is decided that Alice and Bob will meet each other at a third location, a hotel, where the bag will be handed over. The agency has already prepared a map of the enemy territory. It contains the homes A, B, and the n hotels in the area:, h_1, h_2, …, h_n. Like most road maps, this map is a directed graph, but it also contains edge weights. The weight of an edge represents the risk – a positive number with the higher values indicating higher risk – of being intercepted by enemy agents on that particular road. The total risk of a path is simply the sum of the edge-weights on that path.

i. The agency wishes to tell Alice and Bob which hotel, h, to meet at so that the total risk, for both of them combined, is minimized. Such a hotel would be the *safest hotel*. Note that both Alice and Bob would need to travel to h to make the delivery possible. The problem is that the map is too large for manual processing. Therefore, you have been tasked to write an algorithm that can find the safest hotel. Your goal is to write an algorithm that takes no more than $O((|V|+|E|)\lg|V|)$ to accomplish this task.

ii. Unfortunately, before the delivery could take place the enemy spies have grown suspicious of Bob. His house at B is constantly monitored. He simply cannot step out. The job for Alice has therefore changed. She must now drop the bag at hotel, h, and come back to A, so that Bob could collect it at a later time when the situation has improved. The definition of the safest hotel has changed too. It is now the hotel, h, that minimizes the total risk for Alice to make the drop. Note that Alice must travel to h and come back to her house at A: the total risk will be the risk of the round-trip. Your goal now is to write an algorithm that takes no more than $O((|V|+|E|)\lg|V|)$ to accomplish this task.

OR

Problem 5(b):

The city of Mountana is planning a chair-lift network between N destination points in the city. Since this is a mountainous city, chair-lift promises to be a good alternative to traditional modes of transportation. The city's governor has obtained a map of the N destinations along with the direct aerial distance between every pair of them. This distance is the length of the cable that would be needed to connect two destinations and hence represents the cabling cost of the connection. This large map has been handed over to you to suggest a most cost-effective way of creating the chair-lift network that makes it possible for the citizens to travel from any one destination to any other. All chair-lift cables are bidirectional.

i. Model this problem as a graph problem. Define a graph $G=(V, E)$. What type of a graph is this? What do the vertices, V, and the edges E and their weights represent in this graph?

ii. Which algorithm will you use to find the most cost-effective chair-lift network? What will be the running time, in terms of $|V|$ and $|E|$, of this algorithm?

iii. The ideal network may not be possible due to various issues. For example, the wind speed between two destinations or the elevation difference between them might make it too difficult to connect them directly even if the cost is low. After providing the program that produces the best chair-lift network, you are now asked to add the functionality that will help the governor's office to modify the network by choosing to disallow a certain connection in the existing network. When a connection is disallowed, the chair-lift network cannot use that connection anymore. Your algorithm should update the best network accordingly. It should take linear time to do so, i.e. $O(|V|+|E|)$.

iv. According to a new survey conducted by the governor's office, it has been estimated that a direct connection between any two destinations in the city can in fact be cabled in \$10, 000, regardless of the distance between them. This means that the cost of each connection becomes a fixed amount of \$10, 000. Additionally, k of these connections have been sponsored by a beverage company and can each be cabled in only \$5, 000. Without even computing the ideal network, can you tell what could be the minimum possible cost of the network under the new scenario?