$T(n) = 7T\left(\frac{n}{2}\right) + n^2$

$T\left(\frac{n}{2}\right) = 7\ T\left(\frac{n}{4}\right) + \left(\frac{n}{2}\right)^2$

$T\left(\frac{n}{4}\right) = 7T\left(\frac{n}{8}\right) + \left(\frac{n}{4}\right)^2$

$T(n) = 7\left[7T\left(\frac{n}{4}\right) + \frac{n^2}{4}\right] + n^2$

$= 7^2 T\left(\frac{n}{4}\right) + \frac{n^2}{4} + n^2.$

$= 7^2\left[7T\left(\frac{n}{8}\right) + \frac{n^2}{16}\right] + \frac{n^2}{4} + n^2$

$= 7^3\ T\left(\frac{n}{8}\right) + \frac{n^2}{16} + \frac{n^2}{4} + n^2$

$= 7^K\ T\left(\frac{n}{2^K}\right) + \frac{n^2}{4^K}$

$\because \frac{n}{2^K} = 1 \Rightarrow n = 2^K \Rightarrow \boxed{K = \log n}$

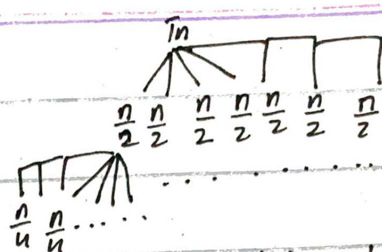
$= 7^{\log n}\ T(1) + \frac{n^2}{4^{\log n}} \Rightarrow$

$= 7^{\log n} + n^2/4^{\log n}$

$= n^{\log 7} + \frac{n^2}{(2^{\log n})^2} \Rightarrow n^{\log 7} + \frac{\cancel{n^2}}{\cancel{(n)^2}} \Rightarrow \boxed{n^{\log 7}} = n^{2.8}$

$= n^3$

$(\because 2^{\log n} \Rightarrow n)$



levels	problem size.	nodes	cost	Total
0	n	1	$(n)^2$	
1	$n/2$	7	$(n/2)^2$	
2	$n/4$	7^2	⋮	
⋮	⋮		⋮	
n	$n/2^K$	7^K		
i	$n/2^i$	7^i	$\left(\frac{n}{2^i}\right)^2$	$\left(\frac{n}{2^i}\right)^2 \times 7^i$

$\left(\frac{n}{2^i}\right)^2 = 1$

$\frac{n^2}{(2^i)^2} = 1$

$n^2 = (2^i)^2.$

$n^2 = 2^{2i}$

$\log_2 n^2 = 2i$

$\log_7 n = i.$

$\boxed{7^{\log n}}$

$$T(n) = T(n-2) + \sqrt{n}$$

(Tn)
$T(n-2$
$T(n-4)$
$T(n-8)$.

$$T(n-2) = T(n-4) + \sqrt{n-2}.$$

$$T(n-4) = T(n-6) + \sqrt{n-4}$$

$$T(n-6) = T(n-8) + \sqrt{n-6}.$$

$$T(n) = (T(n-4) + \sqrt{n-2}) + \sqrt{n}.$$

$$= (T(n-6) + \sqrt{n-4}) + \sqrt{n-2} + \sqrt{n}.$$

$$= (T(n-8) + \sqrt{n-6}) + \sqrt{n-4} + \sqrt{n-2} + \sqrt{n}.$$

$$= T(n-8) + \sqrt{n-6} + \sqrt{n-4} + \sqrt{n-2} + \sqrt{n}.$$

$$= T(n-2K) + \sqrt{n} + \sqrt{n-2} + \sqrt{n-4} + \sqrt{n-6} \ldots\ldots$$

$$n - 2K = 1.$$

$$n - 1 = 2K. \Rightarrow \boxed{\frac{n-1}{2} = K}$$

$$= T\left(n - 2\left(\frac{n-1}{2}\right)\right) \Rightarrow T\left(\frac{2n-2n-2}{2}\right) \Rightarrow T(1).$$

$$= T(1) + \sqrt{n} + \sqrt{n-2} + \sqrt{n-4} \ldots\ldots$$

Now, For the series as.

$$\int_0^n \sqrt{n}\, dn \Rightarrow \frac{2}{3} n^{3/2}. \quad \text{so,}$$

$$T(n) = T(1) + \frac{2}{3} n^{3/2}$$

$$\boxed{T(n) = \Theta(n^{3/2})}$$

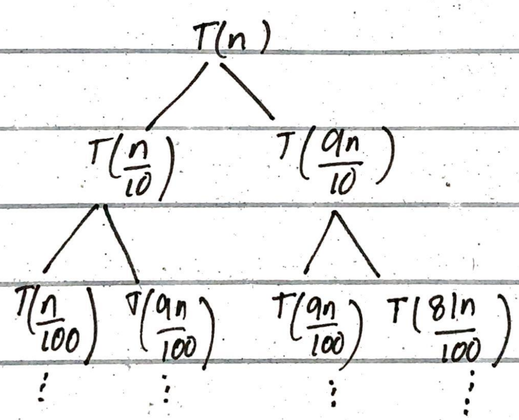
c. $T\left(\frac{n}{10}\right) + T\left(\frac{9n}{10}\right) + O(n).$

$$T\left(\frac{n}{10}\right) = T\left(\frac{n}{100}\right) + T\left(\frac{9n}{100}\right) + O\left(\frac{n}{10}\right)$$

$$T\left(\frac{9n}{10}\right) = T\left(\frac{9n}{100}\right) + T\left(\frac{81n}{100}\right) + O\left(\frac{9n}{10}\right).$$

$$T(n) = T\left(\frac{n}{100}\right) + T\left(\frac{9n}{100}\right) + O\left(\frac{n}{10}\right) + T\left(\frac{9n}{100}\right) + T\left(\frac{8n}{100}\right) + O\left(\frac{9n}{10}\right) + O(n).$$

$$= T\left(\frac{n}{100}\right) + 2T\left(\frac{9n}{100}\right) + T\left(\frac{8n}{100}\right) + O\left(\frac{n}{10}\right) + O\left(\frac{9n}{10}\right) + O(n).$$



$$\sum_{i=0}^{\log n-1} \left(\frac{9n}{100^i}\right)^{2i}$$

$$\Rightarrow \frac{9^i n}{100^i}$$

$$\Rightarrow i = \log_{\frac{9}{100}} n$$

$$\boxed{= O(n \log n)}$$

steps.	problem size.	Nodes	cost of node	Total cost
0	$T(0)$.	1	$T(n)$	$T(n)$
1	$\frac{9n}{10}$	2	$\frac{9n}{10}$	$\left(\frac{9n}{10}\right)(2)$
2	$81n/100$	4	$\frac{81n}{100}$	$\left(\frac{81n}{100}\right)^4$
⋮	⋮	⋮		
n				
⋮				
i	$\frac{9n}{100^i}$	2^i	$\frac{9n}{100^i}$	$\left(\frac{9n}{100^i}\right)^{2i}$

$$\sum_{i=0}^{\lg n-1} \left(\frac{9n}{100^i}\right)^{2i} + O(n).$$

Total work - done from all these is

$$\boxed{O(n \log_{10/9} n)}$$