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C.W

Date: 3-9-2024

Numerical Computing (5).

- Gauss Forward $0 < p < 1$
- Gauss Backward $-1 < p < 0$
- Stirling's Formula $-\frac{1}{2} \le p \le \frac{1}{2}$
- Bessel's Formula $\frac{1}{4} < p \le \frac{3}{4}$
- Laplace - Everett Formula $0 < p < 1$

Lagrange's Interpolation Formula:

$$y = f(x) = \left[\frac{(x-x_1)(x-x_2)(x-x_3)\ldots\ldots(x-x_n)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)\ldots(x_0-x_n)}\right](y_0) +$$

$$\left[\frac{(x-x_1)(x-x_2)(x-x_3)\ldots\ldots(x-x_n)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)\ldots(x_1-x_n)}\right](y_1) +$$

$$\left[\frac{(x-x_1)(x-x_2)(x-x_3)\ldots\ldots\ldots(x-x_n)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)\ldots\ldots\ldots(x_2-x_n)}\right](y_2) + \ldots$$

$$\ldots\ldots\left[\frac{(x-x_0)(x-x_1)(x-x_2)\ldots\ldots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)(x_n-x_2)\ldots(x_n-x_{n-1})}\right](y_n)$$

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e.g.

x_0	y_0
x_1	y_1

$$f(x) = \left[\frac{x - x_1}{x_0 - x_1}\right](y_0) + \left[\frac{x - x_0}{x_1 - x_0}\right](y_1)$$

x_0	y_0
x_1	y_1
x_2	y_2

$$f(x) = \left[\frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}\right](y_0) + \left[\frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}\right](y_1) + \left[\frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}\right](y_2)$$

Example 4.31:

Find the parabola thru the points $(\underset{x_0}{0},\overset{y_0}{1}), (\underset{x_1}{1},\overset{y_1}{3}), (\underset{x_2}{3},\overset{y_2}{55})$ using Lagrange's Formula.

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$$f(x) = \left[\frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}\right](y_0) + \left[\frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}\right](y_1) + \left[\frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}\right](y_2)$$

	x	y	
x_0	0	1	y_0
x_1	1	3	y_1
x_2	3	55	y_2

$$f(x) = \left[\frac{(x-1)(x-3)}{(0-1)(0-3)}\right](1) + \left[\frac{(x-0)(x-3)}{(1-0)(1-3)}\right](3) + \left[\frac{(x)(x-1)}{(3)(2)}\right](55)$$

$$= \frac{x^2-x-3x+3}{(3)} + \left[\frac{x^2-3x}{-2}\right](3) + \left[\frac{x^2-x}{6}\right](55)$$

$$\Rightarrow \frac{x^2-4x+3}{3} + \frac{3x^2-9x}{-2} + \frac{55x^2-55x}{6}$$

$$\Rightarrow \frac{x^2-4x+3}{3} - \frac{3}{2}(x^2-3x) + \frac{55}{6}(x^2-x)$$

$$\Rightarrow \frac{2x^2-8x+6-9x^2+27x+55x^2-55x}{6}$$

$$\Rightarrow \frac{48x^2-36x+6}{6} \Rightarrow 8x^2-6x+1$$

$$f'(x) = 16x-6$$

$$x = \frac{\cancel{6}\,3}{\cancel{16}\,8}, \quad x = 0$$

$$f''(x) = 16 > 0 \quad (\text{minima})$$

* Inverse Lagrange Polynomial, Swap places of x & y.

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⇒ Numerical Differentiation :

Newton Fwd formula :

$$y = f(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \dots$$

$$p = \frac{x - x_0}{h}$$

$$x = x_0 + ph. \rightsquigarrow \frac{dx}{dp} = h.$$

$$f(x_0+ph) = y_0 + p\Delta y_0 + \left(\frac{p^2-p}{2!}\right)\Delta^2 y_0 + \left(\frac{p^3-3p^2+2p}{6}\right)\Delta^3 y_0 + \dots$$

Differentiate w.r.t 'p' on both sides.

$$\frac{d}{dp}\, y(x_0+ph) \longrightarrow \frac{d}{dx}\frac{dx}{dp}\ h$$

$$\frac{dy}{dp} = \frac{dy}{dx}\cdot\frac{dx}{dp}\ h$$

$$\frac{dy}{dp} = \frac{d}{dx}$$

$$\frac{dy}{dx}\cdot\frac{dx}{dp} \Rightarrow h\cdot\frac{dy}{dx}$$

$$h\frac{dy}{dx} = \Delta y_0 + \left(\frac{2p-1}{2}\right)\Delta^2 y_0 + \left(\frac{3p^2-6p+2}{3!}\right)\Delta^3 y_0 + \dots$$

$$\frac{dy}{dx} = \frac{1}{h}\left[\Delta y_0 + \left(\frac{2p-1}{2}\right)\Delta^2 y_0 + \left(\frac{3p^2-6p+2}{3!}\right)\Delta^3 y_0 + \dots\right]$$

Differentiate Both sides w.r.t 'p'

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$\therefore \frac{d}{dp}\left\{\frac{dy}{dx}\right\}$ $h \cdot \frac{d^2y}{dx^2} = \frac{1}{h}\left\{\Delta^2 y_0 + \frac{6p-6}{6}\Delta^3 y_0 + \cdots\right\}$

$\frac{d}{dx} \cdot \frac{dx}{dp}$ h $\frac{d^2y}{dx^2} = \frac{1}{h^2}\left\{\Delta^2 y_0 + (p-1)\Delta^3 y_0 + \cdots\right\}$

$h \cdot \frac{d}{dx}\left\{\frac{dy}{dx}\right\}$

$h \cdot \frac{d^2y}{dx^2}$