# STIRLING INTERPOLATION FORMULA:

$$y_p = y_0 + \sum_{i=1}^{n}\left[\binom{p+i-1}{2i-1}\delta^{2i-1}\mu y_0 + \left(\frac{p}{2i}\right)\binom{p+i-1}{2i-1}\delta^{2i} y_0\right]$$

For collocation at $p=-n,\ldots,n$

Assume that collocation is at $p=-3,\ldots,3$

| | | | | | | |
|---|---|---|---|---|---|---|
| $-3$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ |
| $x_{-3}$ | $x_{-2}$ | $x_{-1}$ | $x_0$ | $x_1$ | $x_2$ | $x_3$ |
| $y_{-3}$ | $y_{-2}$ | $y_{-1}$ | $y_0$ | $y_1$ | $y_2$ | $y_3$ |

$\underline{\underline{n=3}}$

$$y_p = y_0 + \sum_{i=1}^{3}\left[\binom{p+i-1}{2i-1}\delta^{2i-1}\mu y_0 + \left(\frac{p}{2i}\right)\binom{p+i-1}{2i-1}\delta^{2i} y_0\right]$$

$$= y_0 + \binom{p}{1}\delta\mu y_0 + \left(\frac{p}{2}\right)\binom{p}{1}\delta^2 y_0$$
$$+ \binom{p+1}{3}\delta^3\mu y_0 + \left(\frac{p}{4}\right)\binom{p+1}{3}\delta^4 y_0$$
$$+ \binom{p+2}{5}\delta^5\mu y_0 + \left(\frac{p}{6}\right)\binom{p+2}{5}\delta^6 y_0$$

Here $\binom{p}{1} = \frac{p!}{1!(p-1)!} = \frac{p(p-1)!}{1!(p-1)!} = \frac{p}{1!}$

$$\binom{p+1}{3} = \frac{(p+1)!}{3!(p-2)!} = \frac{(p+1)p(p-1)(p-2)!}{3!(p-2)!} = \frac{p(p^2-1)}{3!}$$

(5):

$$\binom{P+2}{5} = \frac{(P+2)!}{5!\,(P-3)!} = \frac{(P+2)(P+1)P(P-1)(P-2)(P-3)!}{5!\,(P-3)!}$$

$$= \frac{P(P^2-1)(P^2-4)}{5!}$$

$$y_p = y_0 + \frac{P}{1!}\delta\mu y_0$$

$$+ \frac{P}{2}\frac{P}{1!}\delta^2 y_0$$

$$+ \frac{P(P^2-1)}{3!}\delta^3\mu y_0$$

$$+ \frac{P}{4}\frac{P(P^2-1)}{3!}\delta^4 y_0$$

$$+ \frac{P(P^2-1)(P^2-4)}{5!}\delta^5\mu y_0$$

$$+ \frac{P}{6}\frac{P(P^2-1)(P^2-4)}{5!}\delta^6 y_0$$

(c)

| | | | |
|---|---|---|---|
| $x_0-h$ | $f(x_0-h)$ | $x_{-1}$ | $y_{-1}$ |
| $x_0-h/2$ | $f(x_0-h/2)$ | $x_{-1/2}$ | $y_{-1/2}$ |
| $x_0$ | $f(x_0)$ | $x_0$ | $y_0$ |
| $x_0+h/2$ | $f(x_0+h/2)$ | $x_{1/2}$ | $y_{1/2}$ |
| $x_0+h$ | $f(x_0+h)$ | $x_1$ | $y_1$ |

$$\delta f(x_0) = f\left(x_0+\frac{h}{2}\right) - f\left(x_0-\frac{h}{2}\right)$$

$$\mu f(x_0) = \frac{f(x_0+h/2) + f(x_0-h/2)}{2}$$

$$\delta y_0 = y_{1/2} - y_{-1/2}$$

$$\mu y_0 = \frac{y_{-1/2} + y_{1/2}}{2}$$

$$\delta \cdot \mu y_0 = \frac{y_{-1/2} + y_{1/2}}{2}$$

Applying $\delta$ on both sides

$$\delta \mu y_0 = \delta\left(\frac{y_{-1/2} + y_{1/2}}{2}\right)$$

$$= \frac{\delta y_{-1/2} + \delta y_{1/2}}{2}$$

(4)

Again Applying $\delta^2$ on b.s

$$\delta^3 \mu y_0 = \frac{\delta^3 y_{-1/2} + \delta^3 y_{1/2}}{2}$$

Again applying $\delta^2$ on b.s

$$\delta^5 \mu y_0 = \frac{\delta^5 y_{-1/2} + \delta^5 y_{1/2}}{2}$$

$$y_p = y_0 + \frac{p}{1!}\left(\frac{\delta y_{-1/2} + \delta y_{1/2}}{2}\right)$$

$$+ \frac{p^2}{2!}\delta^2 y_0$$

$$+ \frac{p(p^2-1)}{3!}\left(\frac{\delta^3 y_{-1/2} + \delta^3 y_{1/2}}{2}\right)$$

$$+ \frac{p^2(p^2-1)}{4!}\delta^4 y_0$$

$$+ \frac{p(p^2-1)(p^2-4)}{5!}\left(\frac{\delta^5 y_{-1/2} + \delta^5 y_{1/2}}{2}\right)$$

$$+ \frac{p^2(p^2-1)(p^2-4)}{6!\ \ 6!}\delta^6 y_0$$

$+ \cdots$

e

$$+ \vdots$$

$$+ \frac{p(p^2-1^2)(p^2-2)\cdots[p^2-(n-1)^2]}{(2n-1)!}\left(\frac{\delta^{2n-1}y_{-\frac{1}{2}}+\delta^{2n-1}y_{\frac{1}{2}}}{2}\right)$$

$$+ \frac{p^2(p^2-1^2)(p^2-2^2)\cdots[p^2-(n-1)^2]}{(2n)!}\delta^{2n}y_0$$

$$|P| \le 0.25$$

$$\rightarrow \quad -0.25 \le P \le 0.25$$

(7) Approximate $f(0.43)$ using the following data and the stirling formula:

| | | | | |
|---|---|---|---|---|
| 0.0 | 0.2 | 0.4 | 0.6 | 0.8 |
| 1 | 1.2214 | 1.49182 | 1.82212 | 2.22554 |

| | | | | | | |
|---|---|---|---|---|---|---|
| -2 | 0 | 1 | | | | |
| | | | 0.2214 | | | |
| -1 | 0.2 | 1.2214 | | 0.04902 | | |
| | | | $\delta y_{-1/2}$ 0.27042 | | $\delta^3 y_{-1/2}$ 0.01086 | |
| 0 | 0.4 | 1.49182 $y_0$ | | 0.05988 $\delta^2 y_0$ | | 0.00238 $\delta^4 y_0$ |
| | | | 0.3303 $\delta y_{1/2}$ | | 0.01324 $\delta^3 y_{1/2}$ | |
| 1 | 0.6 | 1.82212 | | 0.07312 | | |
| | | | 0.40342 | | | |
| 2 | 0.8 | 2.22554 | | | | |

$X = 0.43 \quad x_0 = 0.4 \quad y_0 = 1.49182, \; h = 0.2$

$$p = \frac{X - x_0}{h} = \frac{0.43 - 0.4}{0.2} = 0.15$$

$$y_p = 1.49182 + 0.15\left(\frac{0.27042+0.3303}{2}\right)$$

$$+\frac{(0.15)^2}{2} * 0.05988$$

$$+\frac{0.15(0.15^2-1)}{6}\left(\frac{0.01086+0.01324}{2}\right)$$

$$+\frac{(0.15)^2(0.15^2-1)}{24} * 0.00238$$

$$y_p = 1.49182 + 0.045054 + 0.00067365$$
$$-0.00029447 1875 - 0.0000021810 46875$$

$$= 1.5372 50997$$