Mon Tue Wed Thu Fri Sat Sun

C.W

Date: 10-9-2024

Numerical Computing (7).

=> Numerical Integration:

-> Romberg Integration is a combination for both Trapezoidal and Simpson's Rule

Error Estimation for Trapezoidal Rule:

$$E = -\frac{(b-a)}{12} \cdot h^2 \cdot f''(\eta) \qquad a < \eta < b$$

$O(h^2)$ -> Order

$$|E| = \frac{h^2}{12} \cdot f''(\eta)$$

$$\frac{h^2}{12} \cdot f''(0) < |E| < \frac{h^2}{12} \cdot f''(1)$$

=> Composite Simpson's Rule: (2nd Degree polynomial)

=> Flip Over.

Date:

$$\Rightarrow \int_{x_0}^{x_2} P_2(x)\cdot dx + \int_{x_2}^{x_4} P_2(x)\cdot dx + \int_{x_4}^{x_6} P_2(x)\cdot dx$$

$$+ \ldots\ldots\ldots \int_{x_{2n-2}}^{x_{2n}} P_2(x)\cdot dx$$

$$\Rightarrow \int_{x_0}^{x_2} P_2(x)\cdot dx = \int_{x_0}^{x_2} \left[f_0 + t\Delta f_0 + \frac{t(t-1)}{2!}\Delta^2 f_0 \right]\cdot dx$$

$\therefore$ We know

$$t = \frac{x - x_0}{h}$$

$$dt = \frac{1}{h}\, dx$$

① at $x = x_0$

$t = 0$

② at $x = x_2$

$$t = \frac{x_2 - x_0}{h}$$

$$= \frac{2\cancel{h}}{\cancel{h}} = 2$$

$$\Rightarrow \int_0^2 \left[f_0 + t\Delta f_0 + \left(\frac{t^2}{2} - \frac{t}{2}\right)\Delta^2 f_0 \right] h\cdot dt$$

$$\Rightarrow h\left[tf_0 + \frac{t^2}{2}\Delta f_0 + \left(\frac{t^3}{6} - \frac{t^2}{4}\right)\Delta^2 f_0 \right]_0^2$$

$$\Rightarrow h\left[2f_0 + 2\Delta f_0 + \left(\frac{8}{6} - 1\right)\Delta^2 f_0\right]$$

$$\Rightarrow h\left[2f_0 + 2[f_1 - f_0] + \frac{1}{3}\left(f_2 - 2f_1 + f_0\right)\right]$$

$$\Rightarrow h\left[\frac{1}{3} \cdot f_0 + \frac{4}{3} f_1 + \frac{1}{3} f_2\right]$$

$$\Rightarrow \frac{h}{3}\left[f_0 + 4f_1 + f_2\right]$$

$\vdots$

$$\Delta(\Delta f_0) = \Delta(f_1 - f_0)$$

$$\Delta^2 f_0 = \Delta f_1 - \Delta f_0$$

$$= (f_2 - f_1) - (f_1 - f_0)$$

$$= f_2 - 2f_1 + f_0$$

$$h = \frac{b-a}{2N}$$

$$= \frac{h}{3}\left[f_0 + 4f_1 + f_2\right] + \frac{h}{3}\left[f_2 + 4f_3 + f_4\right] + \frac{h}{3}\left[f_4 + 4f_5 + f_6\right] + \frac{h}{3}\left[f_{2n-2} + 4f_{2n-1} + f_{2n}\right]$$

$$h = \frac{b-a}{2N}$$

$$\int_a^b f(x) \cdot dx = \frac{h}{3}\left[f_0 + f_{2N} + 4\left[f_1 + f_3 + f_5 + \cdots + f_{2N-1}\right] + 2\left[f_2 + f_4 + f_6 + \cdots + f_{2N-2}\right]\right]$$

Mon Tue Wed Thu Fri Sat Sun

Date:

Example 05 :

$$\int_0^1 e^{-x} \cdot dx$$

$h = 0.5$ $\qquad$ $2N = \frac{b-a}{h} \Rightarrow 2N = 2 \Rightarrow N = 1$

$0 \qquad 0.5 \qquad 1 = 2N$

$a = \qquad\qquad\qquad\qquad = b$

$x_0 \qquad x_1 \qquad x_2$

$y_0 = e^0 \qquad y_1 = e^{-0.5} \qquad y_2 = e^{-1}$

$$\int_0^1 e^{-x} \cdot dx = h/3 \left[y_0 + y_2 + 4y_1 \right]$$

$= 0.63233$

$h = 0.25 \Rightarrow 2N = \frac{1-0}{0.25} \Rightarrow 2N = 4$

$N = 2$

$0 \qquad 0.25 \qquad 0.5 \qquad 0.75 \qquad 1$

$x_0 \Rightarrow \qquad x_1 \qquad x_2 \qquad x_3 \qquad x_4 \rightarrow y_4 = e^{-1}$

$y_0 = e^0 \qquad y_1 = e^{-0.25} \qquad y_2 = e^{-0.5} \qquad y_3 = e^{-0.75}$

$= \cancel{h/3 \left(y_0 + y_8 + 4(y_1 + y_3 + y_5 + y_7) + 2(y_2 + y_4 + y_6) \right)}$

$= h/3 \left[y_0 + y_4 + 4(y_1 + y_3) + 2(y_2) \right]$

$= 0.63213$

CamScanner

Mon Tue Wed Thu Fri Sat Sun Date:

Example 04 :

$$\int_{1}^{2.5} e^{-x} \cdot dx$$

tolerance = 0.5×10^{-5}

$h = ?$

$N = ?$

Trapezoidal

$$|E| = \frac{h^2}{12}(b-a) \cdot f''(\eta)$$

$$= \frac{1.5h^2}{12} \cdot f''(\eta) \qquad 1.0 < \eta < 2.5$$

$$\frac{1.5h^2}{12} \cdot f''(2.5) \leq 0.5 \times 10^{-5}$$

$$\sqrt{h^2} \leq \sqrt{\frac{(5 \times 10^{-5})(12)}{1.5 \times e^{-2.5}}}$$

$$h = 0.00573$$

$$N = \frac{b-a}{h} = \frac{1.5}{Ans} = 261.77577$$

Simpson's

$$|E| = \left(\frac{b-a}{180}\right) h^4 \cdot f^{iv}(\eta) \Rightarrow N = \frac{b-a}{2h} = \frac{1.5}{2(\quad)}$$

$$\Rightarrow h \leq \sqrt[4]{\frac{180(0.5 \times 10^{-5})}{1.5 \times e^{2.5}}}$$

≈ 8.95 intervals