

## Numerical (3)

### Computing

$\theta$	$y = \sin \theta$	$\Delta y$	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
10 $\theta_0$	0.1736 $y_0$	$\Delta y_0$	$\Delta^2 y_0$	$\Delta^3 y_0$	$\Delta^4 y_0$
20 $\theta_1$	0.3420 $y_1$	0.1684	-0.0104	-0.0048	
30 $\theta_2$	0.5000 $y_2$	0.1580	-0.0152	$\Delta^3 y_1$	0.0004
40 $\theta_3$	0.6428 $y_3$	0.1428	$\Delta^2 y_2$	-0.0044	$\Delta^4 y_1$
50 $\theta_4$	0.7660 $y_4$	0.1232 $\Delta y_3$	-0.0196		

$y(25) = ?$

Using Forward Difference Formula:

Given:

$x = 25$

$x_0 = \theta_0 = 10^\circ$

$y_0 = 0.1736$

$\Delta^1 y_0 = 0.1684$

$\theta = \theta_0 + ph$

$25 = 10 + ph$   
 $1.5 = p$

$$y_p = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!} \Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!} \Delta^3 y_0$$

$$+ \frac{p(p-1)(p-2)(p-3)}{4!} \Delta^4 y_0$$

$$y = 0.1736 + 1.5(0.1684) + \frac{(1.5)(0.5)(-0.0104)}{2!}$$

$$+ \frac{(1.5)(0.5)(-0.5)(-0.0048)}{3!}$$

$$+ \frac{(1.5)(0.5)(-0.5)(-1.5)(0.0004)}{4!}$$

$$\sin(45^\circ) = y_p = 0.422609375$$

$$\text{Exact} = 0.4226$$

$$\text{Abs. Error} = |0.4226 - 0.4226| = 0$$

$$\text{Approx} = 0.4226$$

Relative

$$\text{Error} = \text{[scribble]} 0$$

$$y(45^\circ) = Q = 45^\circ$$

$$x = x_n + ph$$

$$p = \frac{x - x_n}{h} = \frac{45 - 50}{10} = -0.5$$

$$y_p = y_4 + p \nabla y_4 + \frac{p(p+1)}{2!} \nabla^2 y_4 + \frac{p(p+1)(p+2)}{3!} \nabla^3 y_4$$

$$+ \frac{p(p+1)(p+2)(p+3)}{4!} \nabla^4 y_4$$

$$y_{45} = 0.7660 + (-0.5)(0.1232) + \frac{(-0.5)(0.5)(-0.0196)}{2!} + \frac{(-0.5)(0.5)(1.5)(-0.0004)}{3!}$$

$$+ \frac{(-0.5)(0.5)(1.5)(2.5)(0.0004)}{4!} = 0.7071$$

$$\sin 45^\circ = 0.7071$$

No errors.

$x_{-4}$	$y_{-4}$	
$x_{-3}$	$y_{-3}$	
$x_{-2}$	$y_{-2}$	
$x_{-1}$	$y_{-1}$	
$x_0$	$y_0$	
$x_1$	$y_1$	
$x_2$	$y_2$	
$x_3$	$y_3$	
$x_4$	$y_4$	

Diagram illustrating the Gauss Forward and Gauss Backward interpolation methods. The Gauss Forward method is shown for  $x_0$  and  $x_1$ , with  $\Delta y_0$  and  $\Delta^2 y_0$  indicated. The Gauss Backward method is shown for  $x_{-1}$  and  $x_{-2}$ , with  $\Delta y_{-1}$  and  $\Delta^2 y_{-1}$  indicated. The central difference formula is also shown for  $x_0$  and  $x_1$ , with  $\Delta y_0$  and  $\Delta^2 y_0$  indicated.

## Interpolation

Equally  
Spread  
Data

un-equally  
spaced data

For Equally Spread Data:

- Newton Forward Difference Formula:  $x = x_0 + ph$
- Newton Backward Difference Formula:  $x = x_n + ph$
- Central Difference Formula:  $x = x_0 + ph$

For un-equally spaced data:

- Newton Divided Difference Formula
- Lagrange Formula

For Equally Spread Data (continued):

- Gauss Forward
- Gauss Backward
- Stirling Formula
- Laplace Everett's Formula
- Bessel Formula