Q Use Taylor Method of order 4 to approximate y and z at $t=0.1, 0.2$ for the following system of first order odes:

$$\frac{dy}{dt} = t+z \qquad y(0)=2$$

$$\frac{dz}{dt} = t-y^2 \qquad z(0)=1$$

Here $f(t,y,z) = t+z$

$g(t,y,z) = t-y^2$

and $h=0.1$

$t_0=0 \quad y_0=2 \quad z_0=1$

$t_1=0.1 \quad y_1=? \quad z_1=?$

$t_2=0.2 \quad y_2=? \quad z_2=?$

For $i=0$ to 1

$$y_{i+1} = y_i + hy_i' + \frac{h^2}{2!}y_i'' + \frac{h^3}{3!}y_i''' + \frac{h^4}{4!}y_i^{(iv)}$$

$$z_{i+1} = z_i + hz_i' + \frac{h^2}{2!}z_i'' + \frac{h^3}{3!}z_i''' + \frac{h^4}{4!}z_i^{(iv)}$$

$y' = t + z$

$y_0' = t_0 + z_0$

$= 0 + 1$

$= 1$

$z' = t - y^2$

$z_0' = t_0 - y_0^2$

$= 0 - 2^2$

$= -4$

$y'' = 1 + z'$

$y_0'' = 1 + z_0'$

$= 1 - 4$

$= -3$

$z'' = 1 - 2yy'$

$z_0'' = 1 - 2y_0y_0'$

$= 1 - 2 * 2 * 1$

$= -3$

$y''' = z''$

$y_0''' = z_0''$

$= -3$

$z''' = -2yy'' - 2y'y'$

$z_0''' = -2y_0y_0'' - 2(y_0')^2$

$= -2 * 2 * (-3) - 2(1)^2$

$= 10$

$y^{(iv)} = z'''$

$= 10$

$z^{(iv)} = -2yy''' - 2y'y'' - 4y'y''$

$= -2yy''' - 6y'y''$

$z_0^{(iv)} = -2y_0y_0''' - 6y_0'y_0''$

$= -2 * 2 * (-3) - 6 * 1 * (-3)$

$= 30$

$i=0$

$$y_1 = y_0 + h y_0' + \frac{h^2}{2!} y_0'' + \frac{h^3}{3!} y_0''' + \frac{h^4}{4!} y_0^{(iv)}$$

$$y_1 = 2 + 0.1*1 + \frac{0.1^2}{2}*(-3) + \frac{0.1^3}{6}*(-3) + \frac{0.1^4}{24}*10$$

$$= 2.08454$$

$$z_1 = z_0 + h z_0' + \frac{h^2}{2!} z_0'' + \frac{h^3}{3!} z_0''' + \frac{h^4}{4!} z_0^{(iv)}$$

$$= 1 + 0.1*(-4) + \frac{0.1^2}{2}*(-3) + \frac{0.1^3}{6}*10 + \frac{0.1^4}{24}*30$$

$$= 0.58679$$

$y_1' = t_1 + z_1$

$= 0.1 + 0.58679$

$= 0.68679$

$z_1' = t_1 - y_1^2$

$= 0.1 - 2.08454^2$

$= -4.24531$

$y_1'' = 1 + z_1'$

$= 1 - 4.24531$

$= -3.24531$

$z_1'' = 1 - 2y_1 y_1'$

$= 1 - 2 * 2.08454 * 0.68679$

$= -1.86328$

$y_1''' = z_1''$

$= -1.86328$

$z_1''' = -2y_1 y_1'' - 2(y_1')^2$

$= -2 * 2.08454 * (-3.24531) - 2(0.68679)$

$= 12.15638$

$y_1^{(iv)} = z_1'''$

$= 12.15638$

$z_1^{(iv)} = -2y_1 y_1''' - 6y_1' y_1''$

$= -2 * 2.08454 * (-1.86328) - 6 * 0.68679 * (-3.24531)$

$= 21.14124$

i=1

$$y_2 = y_1 + h y_1' + \frac{h^2}{2!} y_1'' + \frac{h^3}{3!} y_1''' + \frac{h^4}{4!} y_1^{(iv)}$$

$$= 2.08454 + 0.1 * 0.68679 + \frac{0.1^2}{2} * (-3.24531) + \frac{0.1^3}{6} * (-1.86328) + \frac{0.1^4}{24} * 12.15638$$

$$= 2.13673$$

$$z_2 = z_1 + h z_1' + \frac{h^2}{2!} z_1'' + \frac{h^3}{3!} z_1''' + \frac{h^4}{4!} z_1^{(iv)}$$

$$= 0.58679 + 0.1 * (-4.24531) + \frac{0.1^2}{2} * (-1.86328) + \frac{0.1^3}{6} * 12.15638 + \frac{0.1^4}{24} * 21.14124$$

$$= 0.15506$$