Centered Differences

Gauss Forward Formula:

If the polynomial is of even degree $2n$ and collocation is at $p = -n, \ldots, n$, then Gauss forward formula is:

$$y_p = y_0 + \sum_{i=1}^{n} \left[\binom{p+i-1}{2i-1} \delta^{2i-1} y_{1/2} + \binom{p+i-1}{2i} \delta^{2i} y_0 \right]$$

And if the polynomial is of odd degree $2n + 1$ and collocation is at $p = -n, \ldots, n + 1$, then it becomes:

$$y_p = \sum_{i=0}^{n} \left[\binom{p+i-1}{2i} \delta^{2i} y_0 + \binom{p+1}{2i+1} \delta^{2i+1} y_{1/2} \right]$$

Gauss Backward Formula:

If the polynomial is of even degree $2n$ and collocation is at $p = -n, \ldots, n$, then Gauss backward formula is:

$$y_p = y_0 + \sum_{i=1}^{n} \left[\binom{p+i-1}{2i-1} \delta^{2i-1} y_{-1/2} + \binom{p+i}{2i} \delta^{2i} y_0 \right]$$

The Gaussian formulas are used for interpolation in the middle of the difference matrix (close to x_0). The first formula (forward interpolation) is applied for $X > x_0$ and second (backward interpolation) for $X < x_0$.

One principal use of the two formulas of Gauss is in deriving Stirling's formula.

Stirling’s Formula:

It is one of the most heavily applied forms of the collocation polynomial. Arithmetic mean of Gauss Forward & Gauss backward formulas yields:

$$y_p = y_0 + \sum_{i=1}^{n} \left[\binom{p+i-1}{2i-1} \delta^{2i-1} \mu y_0 + \frac{p}{2i} \binom{p+i-1}{2i-1} \delta^{2i} y_0 \right]$$

and is a very popular formula for collocation at $p = -n, \ldots, n$. The formula is used for interpolation in the middle of the difference matrix for the values of p close to zero. In practical applications it is used for $|p| \leq 0.25$.

Everett’s Formula:

Formula for Everett can be obtained by rearranging the ingredients of the Gauss forward formula of odd degree and takes the form:

$$y_p = \sum_{i=0}^{n} \left[\binom{p+i}{2i+1} \delta^{2i} y_1 - \binom{p+i-1}{2i+1} \delta^{2i} y_0 \right]$$

Collocation is at $p = -n, \ldots, n+1$.

Bessel’s Formula:

Bessel’s formula is a rearrangement of Everett's and can be written as:

$$y_p = \sum_{i=0}^{n} \left[\binom{p+i-1}{2i} \delta^{2i} \mu y_{1/2} + \frac{(p-0.5)}{2i+1} \binom{p+i-1}{2i} \delta^{2i+1} y_{1/2} \right]$$

Collocation is at $p = -n, \ldots, n+1$. The formula is used for interpolation in the middle of the difference matrix for the values of p close to 0.5. In practical applications it is used for $0.25 \leq |p| \leq 0.75$.

Note: For all centered differences, the value of p is calculated by the formula: $p = \frac{X - x_0}{h}$.

Reference Books:

[1] Introduction to Numerical Analysis by *Francis B. Hildebrand*, 2nd edition, Dover Publications, INC.
[2] Numerical Analysis by *Francis Scheid*, 2nd edition, McGraw Hill.
[3] Computational Mathematics -Worked Examples and Problems With Elements of Theory by ***N.V. Kopchenova & I. A. Maron***, Mir Publishers, Moscow.
[4] Numerical Analysis by *Richard L. Burden* & *J. Douglas Faires*, 10th edition, Brooks/Cole, Cengage Learning.