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C.W

Date: 5-9-2024

Numerical Computing (6)

=> Numerical Differentiation (Continued) .....

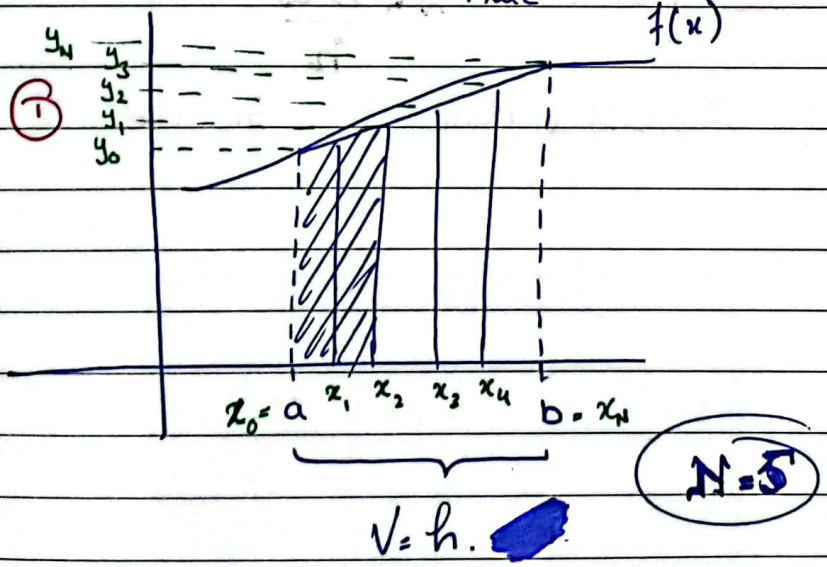
=> Differentiation of Gregory-Newton Backward Formula.

=> Numerical Integration:

Numerical Integration

① Composite Trapezoidal Rule

② Composite Simpson's Rule.



① $N = 5$

$V = h$.

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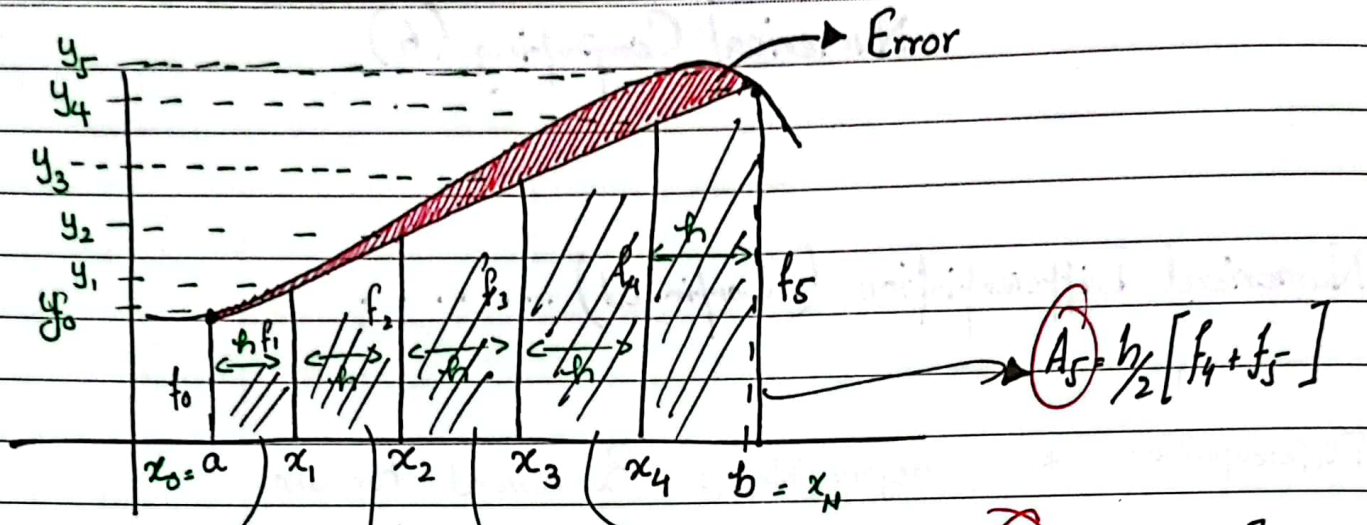

$A_5 = \frac{h}{2}[f_4 + f_5]$

$A_1 = \frac{h}{2}[f_0 + f_1]$

$A_4 = \frac{h}{2}[f_3 + f_4]$

$A_2 = \frac{h}{2}[f_1 + f_2]$

$A_3 = \frac{h}{2}[f_2 + f_3]$

$A_N = \frac{h}{2}[f_{N-1} + f_N]$

$$\int_a^b f(x)\,dx = \frac{h}{2}\left[f_0 + f_N + 2\left(f_1 + f_2 + f_3 + \dots + f_{N-1}\right)\right]$$

→ Composite Trapezoidal Rule. => $N = \frac{b-a}{h}$

* Error is estimated in Composite Trapezoidal Rule thru the use of Taylor Series.

Example 0.2:

$$I = \int_0^1 e^{-x}\,dx$$

$x = 0 = a$, $x_N = 1 = b$ $\quad f_0 = e^{-x_0} = e^0 = 1$ $\quad f_1 = e^{-1} = 0.36787944$

* $h = 1$, $N = 1$

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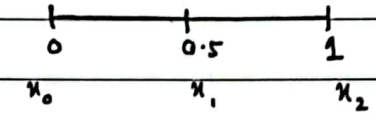
$$\int_0^1 e^{-u}\,du = \frac{h}{2}\left[f_0 + f_1\right]$$

$$= \frac{1}{2}\left[1 + e^{-1}\right] = 0.68393972$$

* $h = 0.5$, $N = 2$



$f_0 = 1$ $\qquad$ $f_2 = e^{-1}$

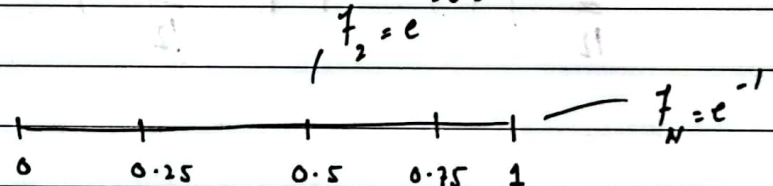
$f_1 = e^{-0.5}$

$$\int_0^1 e^{-u}\,du = \frac{h}{2}\left[1 + e^{-1} + 2\left[e^{-0.5}\right]\right] = 0.64523519$$

Exact

$= 0.63212055588$

* $h = 0.25$, $N = 4$



$f_0 = 1$ $\qquad$ $f_1 = e^{-0.25}$ $\qquad$ $f_3 = e^{-0.75}$

$$\Rightarrow \frac{0.25}{2}\left[1 + e^{-1} + 2\left(e^{-0.25} + e^{-0.5} + e^{-0.75}\right)\right]$$

$$\Rightarrow 0.635409429$$

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* $h = 0.125$, $N = 8$

$$\Rightarrow \int_0^1 e^{-x} \cdot dx = \frac{0.125}{2}\left[1 + e^{-1} + 2\left(e^{-0.125} + e^{-0.25} + e^{-0.375} + e^{-0.5} + e^{-0.625} + e^{-0.75} + e^{-0.875}\right)\right]$$

$$\Rightarrow 0.6329434182$$

$$E = \frac{(b-a)}{12} \cdot h^2 \cdot f''(x)$$

$$|E| = \frac{h^2}{12} \cdot f''(x)$$

for $h = 1$:

$$\frac{1}{12} \cdot f''(0) < |E| < \frac{1}{12} \cdot f''(1).$$