Mon Tue Wed Thu Fri Sat Sun

C.W

Date: 29/8/2024

Numerical Computing (4)

Example 4.10:

	x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
x_{-1}	2	y_{-1} 3.818			
			-1.395		
x_0	3	y_0 2.423 *□		-2.055 *□	
			-3.45 *		3.738 *
x_1	4	y_1 -1.027 *□		1.683 *□	
			-1.767		
x_2	5	y_2 -2.794			

Gauss Backward

Gauss Forward

centre

Stirling y_0

mean

→ Bessel

□ laplace everett $(q = 1-p)$

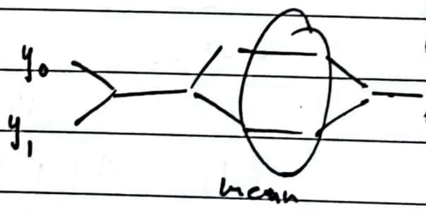
find y at $x = 3.5$.

$x = x_0 + ph$

$\frac{x - x_0}{h} = p$

for $x_0 = 3$: $\frac{3.5 - 3}{1} = p \Rightarrow$ 0.5

for $x_0 = 4$: $\frac{3.5 - 4}{1} \Rightarrow p$ -0.5



FORMULAE ON NEXT PG. ONWARDS.

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① Gauss Forward:

$$y_p = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_{-1} + \frac{p(p+1)(p-1)}{3!}\Delta^3 y_{-1} +$$

$$y_p = 2.423 + 0.5(-3.45) + \frac{(0.5)(0.5-1)(-2.055)}{2} + \frac{(0.5)(0.5+1)(0.5-1)(3.738)}{6}$$

② Gauss Backward:

$$y_p = y_0 + p(\Delta y_{-1}) + \left[\text{[redacted]}\ \frac{(p-1)}{2!}\right]\Delta^2 y_{-1} + \left[\frac{p(p-1)}{2!} + \frac{p(p-1)(p-2)}{3!}\right]$$

$$y_p = 2.423 + 0.5(-1.395) + \frac{(0.5)(0.5-1)(-2.055)}{2!} \cdot (\Delta^3 y_{-1}) + \ldots\ldots$$

⑤ Stirling Formula:

$$y_p = y_0 + p\left[\frac{\Delta y_0 + \Delta y_{-1}}{2}\right] + \frac{p^2}{2!}\Delta^2 y_{-1} + \frac{p(p^2-1^2)}{3!}\left[\frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2}\right] + \ldots$$

$$y_p = 2.423 + 0.5\left(\frac{-1.395 - 3.45}{2}\right) + \frac{(0.5)^2}{2}(-2.055)$$

④ Bessel's Formula:

$$y_p = \frac{y_0 + y_1}{2} + \left(p - \tfrac{1}{2}\right)\Delta y_0 + \frac{p(p-1)}{2!}\left(\frac{\Delta^2 y_{-1} + \Delta^2 y_0}{2}\right) + \frac{\left(p-\tfrac{1}{2}\right)p(p-1)}{3!} \cdot \Delta^3 y_{-1} + \ldots\ldots$$

$$y_p = \frac{2.423 - 1.027}{2} + \left(0.5 - \tfrac{1}{2}\right)(-3.45) + \frac{(0.5)(0.5-1)}{2}\left(\frac{-2.055 + 1.683}{2}\right) + \frac{\left(0.5 - \tfrac{1}{2}\right)(0.5)(0.5-1)(3.738)}{6}$$

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5 Laplace - Everett:

$$y_p = q\cdot y_0 + \frac{q(q^2-1^2)}{3!}\cdot(\Delta^2\cdot y_{-1}) + q\frac{(q^2-1^2)(q^2-2^2)}{5!}\Delta^4 y_{-2} + \dots$$

$$\dots + p y_1 + \frac{p(p^2-q^2)}{3!}\Delta^2 y_0 + \frac{p(p^2-1^2)(p^2-2^2)}{5!}\cdot\Delta^4 y_{-1} + \dots$$

$$y_p = 0.5(2.423) + \frac{0.5(0.5^2-1^2)}{6}(-2.055) + 0.5(-1.027) + \frac{0.5(0.5^2-1^2)}{6}\cdot(1.683) =$$

Unequally Spaced Data:
- → Newton Divided Difference Formula:
- → Lagrange's Formula:

1 Newton Divided Difference:

$$\left.\begin{aligned} \frac{y_1-y_0}{x_1-x_0} &= f(x_0, x_1) \\ \frac{y_2-y_1}{x_2-x_1} &= f(x_1, x_2) \end{aligned}\right\} \text{First Difference}$$

$$\left.\begin{aligned} &\frac{f(x_1, x_2) - f(x_0, x_1)}{x_2 - x_0} = f(x_0, x_1, x_2) \\ &\frac{f(x_2, x_3) - f(x_1, x_2)}{x_3 - x_1} = f(x_1, x_2, x_3) \end{aligned}\right\} \text{Second Difference}$$

$$y = f(x) = y_0 + (x - x_0) \cdot f(x_0, x_1) + (x - x_0)(x - x_1) \cdot f(x_0, x_1, x_2) +$$

$$(x - x_0)(x - x_1)(x - x_2) \cdot f(x_0, x_1, x_2, x_3) + \dots\dots$$

$$+ (x - x_0)(x - x_1) \dots\dots (x - x_{n-1}) \cdot f(x_0, x_1, x_2, \dots\dots, x_n).$$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
5	150				
		121			
7	392		24		
		265		$\frac{32-24}{13-5} = 1$	
11	1452		32		$\frac{1-1}{17-5}$
		457		$\frac{10}{10} = 1$	
13	2366		42		$\Rightarrow 0$
		709			
17	5202				

Ex 4.24

$f(9) = ??$ $\quad x = 9, \; x_0 = 5$

$$y = 150 + (9-5)(121) + (9-5)(9-7)(24) + (9-5)(9-7)(9-11)(1)$$

$$= 810$$