QUESTION:

Use appropriate Lagrange interpolating polynomials of degrees one, two, and three to approximate f (0.9) if f (0.6) = −0.17694460, f (0.7) = 0.01375227, f (0.8) = 0.22363362, f (1.0) = 0.65809197.

SOLUTION:

Here x=0.9

Since $0.8 < 0.9 < 1$, rearrange the data as follows:

x	f (x)
$x_0 = 0.8$	$y_0 = 0.22363362$
$x_1 = 1.0$	$y_1 = 0.65809197$
$x_2 = 0.7$	$y_2 = 0.01375227$
$x_3 = 0.6$	$y_3 = -0.17694460$

Lagrange interpolating polynomial of degree n is:

$$P_n(x) = \sum_{i=0}^{n} y_i * \prod_{\substack{j=0 \\ j\neq i}}^{n} \left(\frac{x - x_j}{x_i - x_j}\right)$$

For degree 1, put n=1

$$P_1(x) = \sum_{i=0}^{1} y_i * \prod_{\substack{j=0 \\ j\neq i}}^{1} \left(\frac{x - x_j}{x_i - x_j}\right)$$

$$= y_0 * \prod_{\substack{j=0 \\ j\neq 0}}^{1} \left(\frac{x - x_j}{x_0 - x_j}\right) + y_1 * \prod_{\substack{j=0 \\ j\neq 1}}^{1} \left(\frac{x - x_j}{x_1 - x_j}\right)$$

$$= y_0 * \left(\frac{x - x_1}{x_0 - x_1}\right) + y_1 * \left(\frac{x - x_0}{x_1 - x_0}\right)$$

$$P_1(x) = 0.22363362 * \left(\frac{0.9 - 1}{0.8 - 1}\right) + 0.65809197 * \left(\frac{0.9 - 0.8}{1 - 0.8}\right)$$

$$= 0.22363362 * 0.5 + 0.65809197 * 0.5$$

$$= 0.440862795$$

For degree 2, put n=2

$$P_2(x) = \sum_{i=0}^{2} y_i * \prod_{\substack{j=0 \\ j\neq i}}^{2} \left(\frac{x - x_j}{x_i - x_j}\right)$$

$$= y_0 * \prod_{\substack{j=0 \\ j\neq 0}}^{2} \left(\frac{x - x_j}{x_0 - x_j}\right) + y_1 * \prod_{\substack{j=0 \\ j\neq 1}}^{2} \left(\frac{x - x_j}{x_1 - x_j}\right) + y_2 * \prod_{\substack{j=0 \\ j\neq 2}}^{2} \left(\frac{x - x_j}{x_2 - x_j}\right)$$

$$= y_0 * \left(\frac{x - x_1}{x_0 - x_1}\right)\left(\frac{x - x_2}{x_0 - x_2}\right) + y_1 * \left(\frac{x - x_0}{x_1 - x_0}\right)\left(\frac{x - x_2}{x_1 - x_2}\right) + y_2 * \left(\frac{x - x_0}{x_2 - x_0}\right)\left(\frac{x - x_1}{x_2 - x_1}\right)$$

$$P_2(0.9) = 0.22363362 * \left(\frac{0.9 - 1}{0.8 - 1}\right)\left(\frac{0.9 - 0.7}{0.8 - 0.7}\right) + 0.65809197 * \left(\frac{0.9 - 0.8}{1 - 0.8}\right)\left(\frac{0.9 - 0.7}{1 - 0.7}\right) + 0.01375227 * \left(\frac{0.9 - 0.8}{0.7 - 0.8}\right)\left(\frac{0.9 - 1}{0.7 - 1}\right)$$

$$= 0.22363362 * 1 + 0.65809197 * 0.3333333333 + 0.01375227 * (-0.3333333333)$$

$$= 0.22363362 + 0.21936399 - 0.00458409$$
$$= 0.43841352$$

For degree 3, put n=3

$$P_3(x) = \sum_{i=0}^{3} y_i * \prod_{\substack{j=0 \\ j\neq i}}^{3} \left(\frac{x - x_j}{x_i - x_j}\right)$$

$$= y_0 * \prod_{\substack{j=0 \\ j\neq 0}}^{3} \left(\frac{x - x_j}{x_0 - x_j}\right) + y_1 * \prod_{\substack{j=0 \\ j\neq 1}}^{3} \left(\frac{x - x_j}{x_1 - x_j}\right) + y_2 * \prod_{\substack{j=0 \\ j\neq 2}}^{3} \left(\frac{x - x_j}{x_2 - x_j}\right) + y_3 * \prod_{\substack{j=0 \\ j\neq 2}}^{3} \left(\frac{x - x_j}{x_3 - x_j}\right)$$

$$= y_0 * \left(\frac{x - x_1}{x_0 - x_1}\right)\left(\frac{x - x_2}{x_0 - x_2}\right)\left(\frac{x - x_3}{x_0 - x_3}\right)$$
$$+ y_1 * \left(\frac{x - x_0}{x_1 - x_0}\right)\left(\frac{x - x_2}{x_1 - x_2}\right)\left(\frac{x - x_3}{x_1 - x_3}\right)$$
$$+ y_2 * \left(\frac{x - x_0}{x_2 - x_0}\right)\left(\frac{x - x_1}{x_2 - x_1}\right)\left(\frac{x - x_3}{x_2 - x_3}\right)$$
$$+ y_3 * \left(\frac{x - x_0}{x_3 - x_0}\right)\left(\frac{x - x_1}{x_3 - x_1}\right)\left(\frac{x - x_2}{x_3 - x_2}\right)$$

$$P_3(0.9) = 0.22363362 * \left(\frac{0.9 - 1}{0.8 - 1}\right)\left(\frac{0.9 - 0.7}{0.8 - 0.7}\right)\left(\frac{0.9 - 0.6}{0.8 - 0.6}\right)$$
$$+ 0.65809197 * \left(\frac{0.9 - 0.8}{1 - 0.8}\right)\left(\frac{0.9 - 0.7}{1 - 0.7}\right)\left(\frac{0.9 - 0.6}{1 - 0.6}\right)$$
$$+ 0.01375227 * \left(\frac{0.9 - 0.8}{0.7 - 0.8}\right)\left(\frac{0.9 - 1}{0.7 - 1}\right)\left(\frac{0.9 - 0.6}{0.7 - 0.6}\right)$$

$$-0.17694460 * \left(\frac{0.9-0.8}{0.6-0.8}\right)\left(\frac{0.9-1}{0.6-1}\right)\left(\frac{0.9-0.7}{0.6-0.7}\right)$$

$$= 0.22363362 * 1.5 + 0.65809197 * 0.25 + 0.01375227 * (-1) - 0.17694460 * 0.25$$
$$= 0.33545043 + 0.164522993 - 0.01375227 - 0.04423615$$
$$= 0.441985003$$