National University of Computer and Emerging Sciences, Lahore Campus

Course:	Theory of Automata	**Course Code:**	CS-3005	
Program:	BS (Computer Science)	**Semester:**	Fall 2022	
Duration:	180 Minutes	**Total Marks:**	80	
Paper Date:	17-December-2022	**Weight**	40 %	
Section:	ALL	**Page(s):**	16	
Exam:	Final Term	**Roll No.**		

Instruction/Notes:
1. Answer in the space provided, showing all the work.
2. Rough Sheets are not allowed.
3. In case of confusion or ambiguity make a reasonable assumption.
4. Attempt all Questions

Section 1: (Short Question Answers) [25 Marks]

Q1: What is the cardinality of L? [3 Marks]

$L = \{ w$ over $\Sigma \mid |w| > 5$ and $|w| \leq 10 \}$.

$\Sigma = \{0,1,2\}$

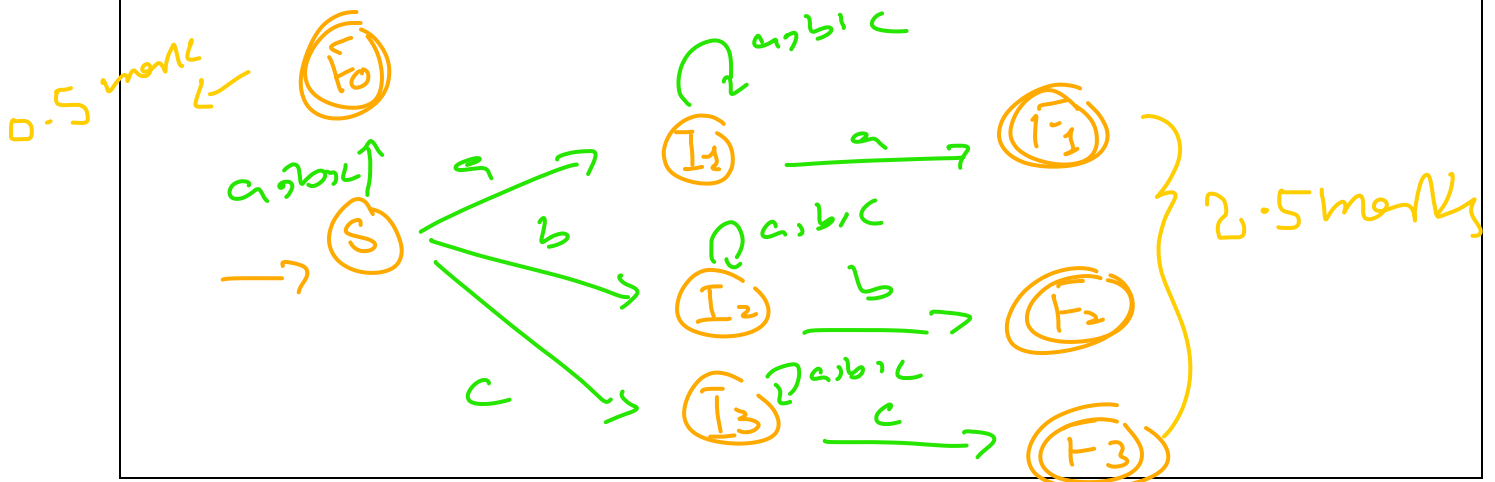> **Note:** Cardinality means the total number of elements in the given set.
>
> 88209
>
> 3 marks
>
> $\Sigma^6 + \Sigma^7 + \Sigma^8 + \Sigma^9 + \Sigma^{10} = 3$

Q2: Design a Finite Automaton (DFA or NFA) for the following language. [3 Marks]

$L = \{ x \mid x$ over $\{a, b, c\}$ x starts and ends with same alphabet $\}$

> **Note:** Pick a suitable sub-category from FA and design the machine accordingly.
>
> 0.5 mark
>
> 2.5 marks

Q3: What will be the language of the following grammar? [7 Marks]

$$L \rightarrow ALB \mid AABB$$

$$A \rightarrow aAb \mid aaabbb$$

$$B \rightarrow ccBd \mid \wedge$$

Note: You are required to write answer in a proper format. For Example, see Q1 statement. You are expected to write a proper answer based on CFG given above. Lengthy Statements are not required here.

$$L = \{ (a^i b^i)^k (c^{2j} d^j)^k ; i \geq 3 ; j \geq 0 ; k \geq 2 \}$$
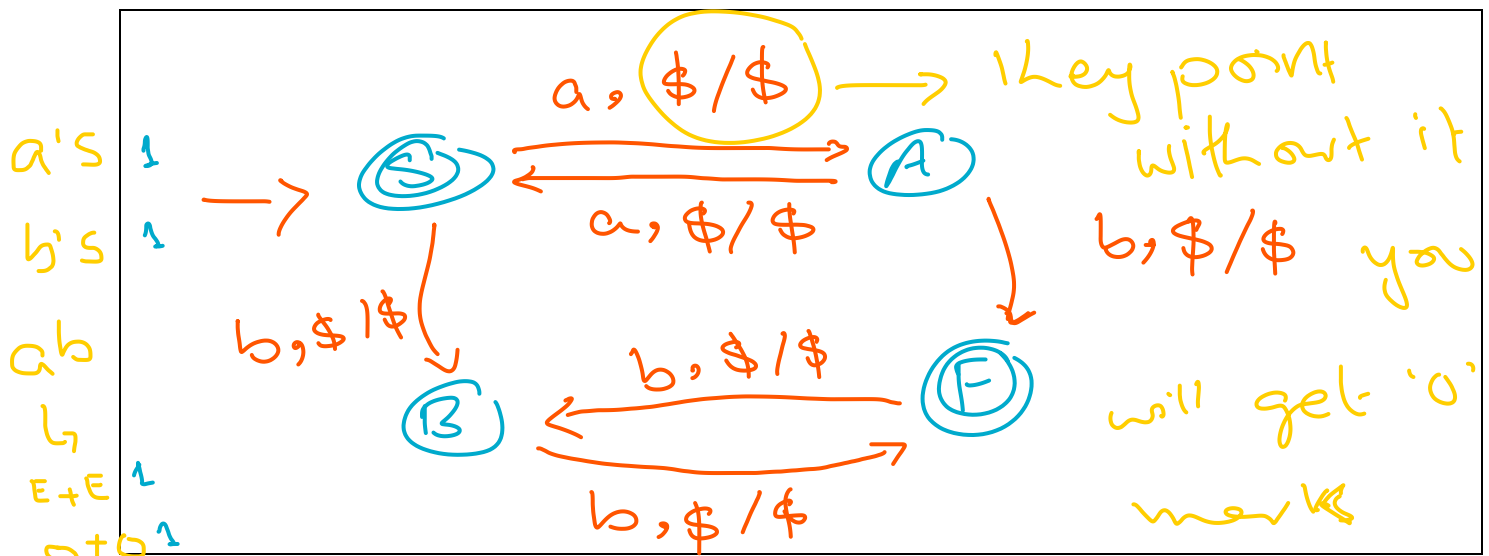
Q4: Write a Regular Expression for the following Language. [4 Marks]

$$L = \{ x \mid x \text{ over } \{a, b\} \ x \text{ contains } 'aba' \text{ and } 'bab' \text{ as a substring} \}$$

$$R.E = \{ \ abab + baba + (a+b)*aba(a+b)*bab(a+b)* + (a+b)*bab(a+b)*aba(a+b)* \ \}$$

Q5: Design the transition diagram of a PDA for the following Language? [4 Marks]

$$L = \{ a^n b^m ; n + m = \text{even} \}$$

a's   1
b's   1
ab
↳
E+E   1
oto   1

a, $/$

a, $/$

a, $/$

b, $/$

b, $/$

b, $/$

they point
without it
b, $/$ you
will get '0'
marks

Q6: What will be the Regular Expression for the following Finite Automaton? [4 Marks]

Start State = A & Final State = {A,C}

States(q)	δ(q,a)	δ(q,b)
A	B	A
B	B	C
C	D	C
D	D	D

Note: Use State Elimination Method for extraction of Regular Expression. Write Final Regular Expression in the space provided below. Delete the given states in the following order, first State A then B then C & then D.

Regular Expression:

$$b^* + b^* a^+ b^+ \Big/ b^* (\wedge + a^+ b^+)$$

Exact marks ④

No partial marks

Section 2: (Long / Detailed Solving Question Answers) [55 Marks]

Q1: Develop 3 multi-tape TM having 2 inputs X and Y (X and Y ε {0,1}*) [15 Marks]

X is on tape 1 and Y is on tape 2. Y slides over the X with the step of 1. Each time it computes the exclusive nor (XNOR) of the corresponding overlapping bits and note down the number of 1's (only) in tape 3 as shown below:

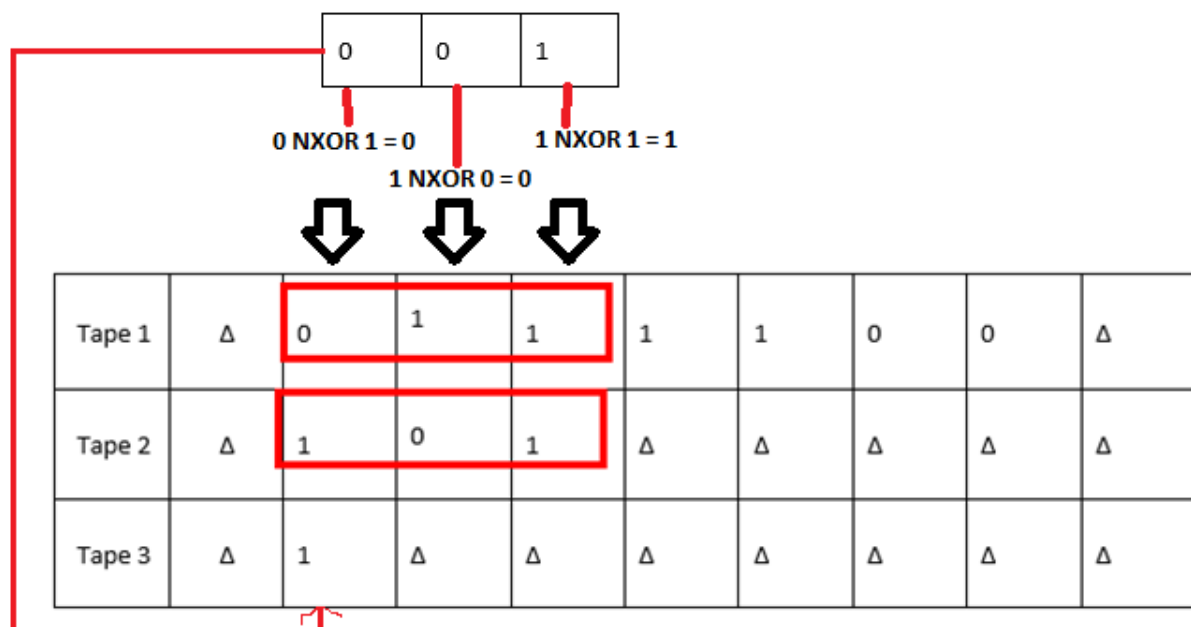| A	B	A XNOR B
0	0	1
0	1	0
1	0	0
1	1	1

Truth Table for XNOR for inputs A and B
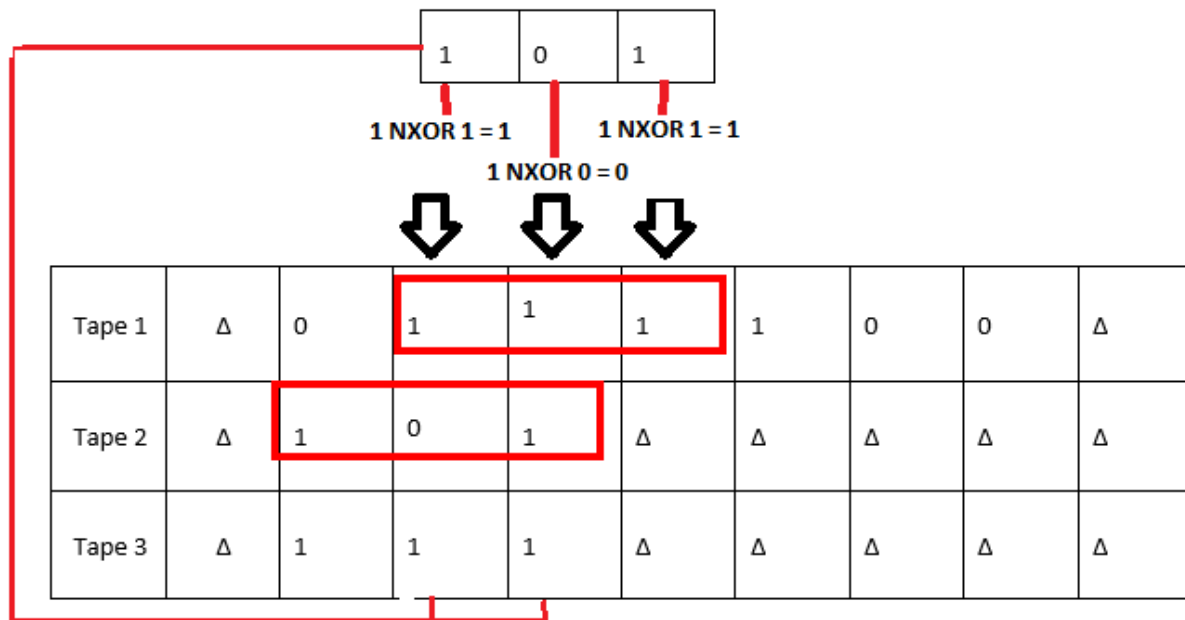
Initial configuration of 3 multi-tape TM

Tape 1: X	Δ	0	1	1	1	1	0	0	Δ
Tape 2: Y	Δ	1	0	1	Δ	Δ	Δ	Δ	Δ
Tape 3: Output	Δ	Δ	Δ	Δ	Δ	Δ	Δ	Δ	Δ

Y will slide 5 times on X (in this example)

First time (first slide)

Second time (second slide)

1 NXOR 1 = 1   1 NXOR 1 = 1
1 NXOR 0 = 0

Tape 1	Δ	0	1	1	1	1	0	0	Δ
Tape 2	Δ	1	0	1	Δ	Δ	Δ	Δ	Δ
Tape 3	Δ	1	1	1	Δ	Δ	Δ	Δ	Δ

.

.

.

(last and 5th slide) Eventually Output will be

Tape 1	Δ	0	1	1	1	1	0	0	Δ	Δ
Tape 2	Δ	1	0	1	Δ	Δ	Δ	Δ	Δ	Δ
Tape 3	Δ	1	1	1	1	1	1	1	1	Δ

Provide the algorithm first that will explain your logic in simple statement and then draw TM:

Note: *Be clear and to the point*. Clearly mention where your pointers are. **No marks if algorithm is incorrect.**

Algorithm:

△△△/△△△, RRR

11△/x11△, RRR

X1△/x1△, RSS
X0△/x0△, RSS

00△/x01, RRR

01△/x1△, RRS
100/x0△, RSS

11△/11△, LLS
10△/10△, LLS
01△/01△, LLS
00△/00△, LLS

1△△/1△△, LLS
0△△/0△△, LLS

11△/11△, RRR
00△/001, RRR

01△/01△, RRS
10△/10△, RRS

△△△/△△△, SSS

Q2: Dry run the single-tape Turing machine on page 10 and give the content of the tape after running it (When TM halts). [15 Marks = 10 + 5] ?

The initial configuration of the TM is given below

Δ	1	0	1	1	Δ	0	0	1	0	Δ	Δ	.....	Δ

⬆ head/pointer

① pointer
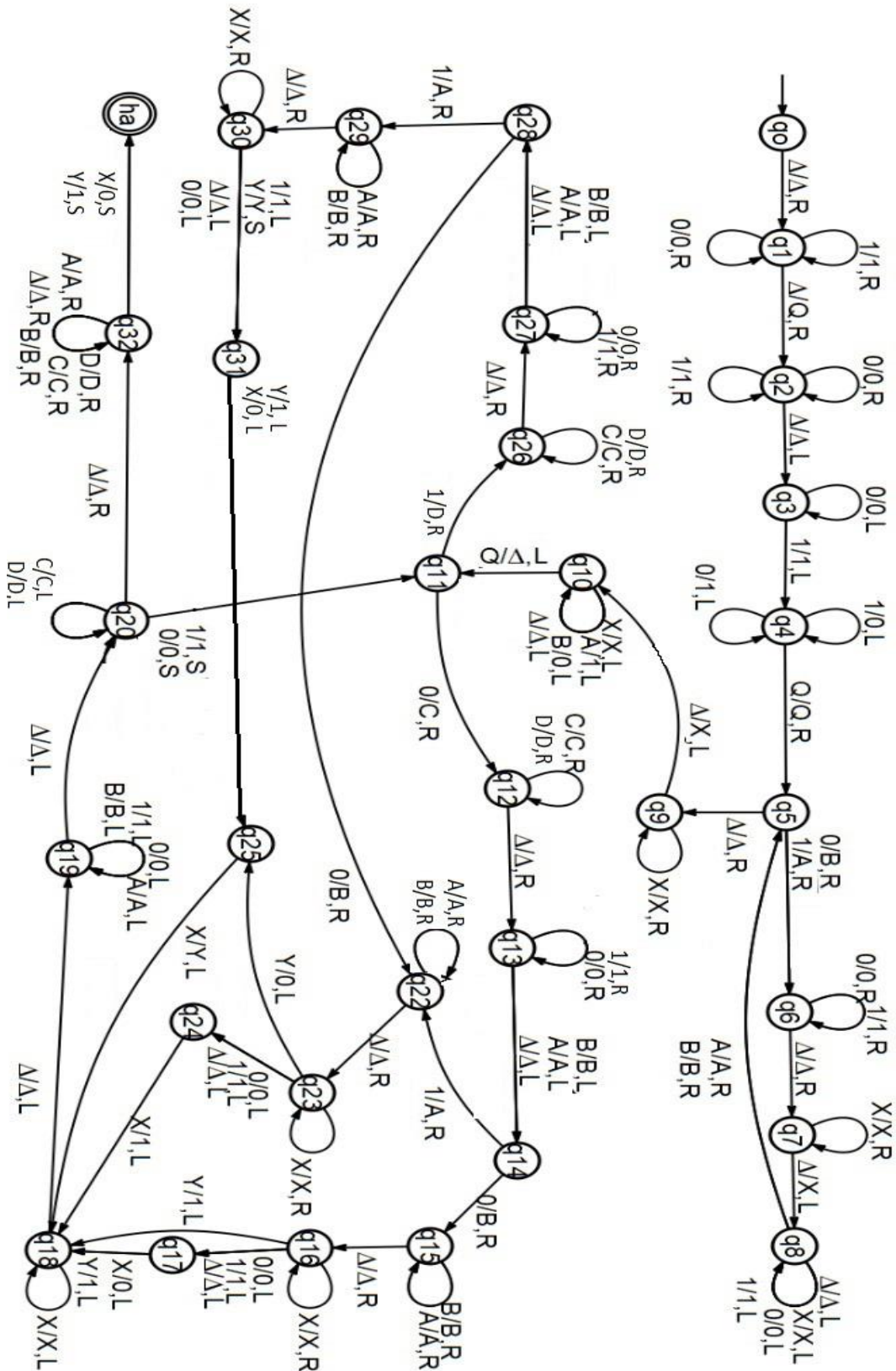
2

Answer:

Δ	D	C	D	D	Δ	A	A	A	B	Δ	1	1	0	0	1	Δ			

Clearly show where will be the head/pointer when TM halts

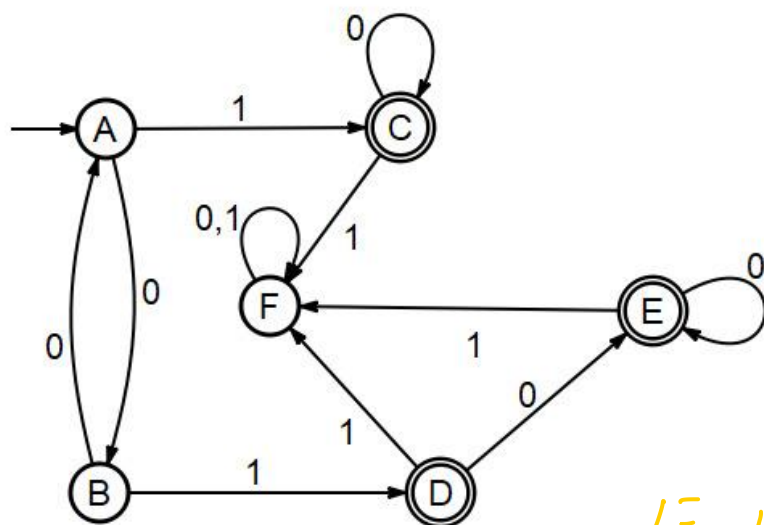2   5

What is TM doing? (Explain in not more than 2 lines. Be brief and to the point. No mark for stories)
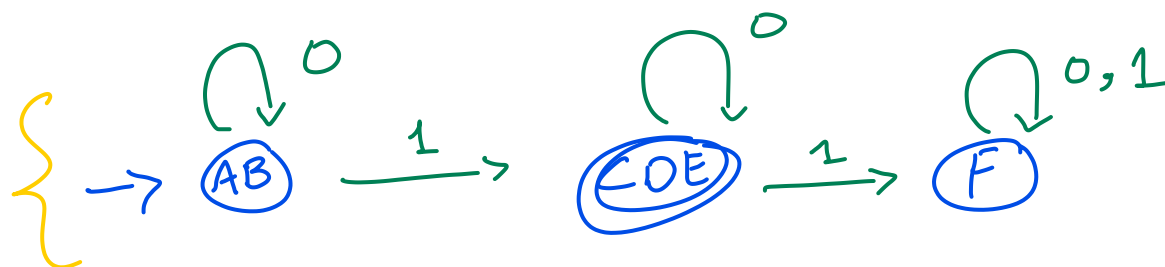
Q3. For the DFA pictured in the figure below, use the minimization algorithm discussed in the class to find a minimum-state DFA recognizing the same language. [10 Marks]

DFA:

Each row = 1 mark

Minimized DFA:

4 marks

$$R.E = \{0^* 1 0^*\}$$

	A	B	C	D	E	F
A	-					
B	[ ]	-				
C	✓	✓	-			
D	✓	✓	[ ]	-		
E	✓	✓	[ ]	[ ]	-	
F	✓	✓	✓	✓	✓	-

A,B

C,D

C,E
D,E

→ Possible Grouping

1 x 6 = 6

Note: Use only the cell required

Q4. Let G be the following CFG:

3 marks

$$S \rightarrow AaB \mid aB$$
$$A \rightarrow a \mid Aa$$
$$B \rightarrow b \mid C$$
$$C \rightarrow bC \mid a$$

X -> a
Y -> b
Z -> AX
S -> ZB | XB
A -> AX | a
B -> YC | a | b
C -> YC | a

CNF-
Grammar

Determine whether the string "abba" is a member of L(G) using CYK Algorithm.   [10 Marks]

j=4	S	-	-	-
j=3	-	B,C	-	-
j=2	S	-	B,C	-
j=1	X,A,B,C	Y,B	Y,B	X,A,B,C
	a	b	b	a

Note: Use only the cell required

'abba' belongs to Language.

Each = 1.5 marks = 1.5×4 = 6
row

=> 1 marks

Q5. Tell whether the following Language is context free (CFL) or non- context free (non- CFL). If it is CFL provide PDA else prove it using Pumping Lemma

$$L = \{a^{m^2} b^m \mid m \geq 0\}$$

[5 Marks]

$$x = a^{p^2} b^p$$

$u =$
$v^i =$
$w_\circ =$
$y^i =$
$z =$

$\wedge$	a^{p^2}	$a^{p^2 - v - s - t}$
$a^{p^2 - r - s}$	$b^{p - r - s}$	a^v
a^v	b^v	a^s
a^s	b^s	a^t
b^p	$\wedge$	b^p

....

Selecting string = 1 marks

division s = 2 marks

proof = 2 marks