QUESTION: Use Newton's method to find real zero of the following function accurate to within 10^{-7}.

$$f(x) = 2x\,cos2x - (x-2)^2$$

SOLUTION:

$f(2) = 4\,cos4 = -2.6145744,$

$f(3) = 6\,cos6 - 1 = 4.7610217$

Real zero of $f(x)$ lies between 2 and 3. Let $p_0 = 2.5$.

$$f(x) = 2x\,cos2x - (x-2)^2$$

$$f'(x) = -4x\,sin2x + 2\,cos2x - 2(x-2)$$

Newton's formula is:

$$p_n = p_{n-1} - \frac{f(p_{n-1})}{f'(p_{n-1})} \qquad \text{for } n \geq 1$$

[Reference: Algorithm 2.3 on page 68 of Numerical Analysis by Burden Faires 9th edition]

Python Code:

```
import numpy as np

import sys


def f(x): return 2 * x * np.cos(2 * x) - (x-2) ** 2


def fprime(x): return -4 * x * np.sin(2 * x) + 2 * np.cos(2 * x) - 2 * (x - 2)


p0 = float(input("Enter p0: "))
n = 1
N = 10
tol = 1e-7

print('n         pn                      f(pn)')
while n <= N:
    a = f(p0)
    b = fprime(p0)
    p = p0 - a / b
    print(f'{n} \
        {p:.10f} \
         {f(p):.10f}')
    if abs(f(p)) <= tol:
        print('Newton method has converged')
        print('final p =', p)
        print('final f(p) =', f(p))
        sys.exit(0)
    else:
        p0 = p
    n += 1

print('zero not found to the desired tolerance')
```

Output:

```
Enter p0: 2.5
```

n=1:

$$a = f(p0)$$
$$= f(2.5)$$
$$= 2 * 2.5 * \cos(2 * 2.5) - (2.5 - 2)^2$$
$$= 1.1683109273$$

$$b = fprime(p0)$$
$$= fprime(2.5)$$
$$= -4 * 2.5 * \sin(2 * 2.5) + 2\cos(2 * 2.5) - 2(2.5 - 2)$$
$$= 9.1565671176$$

$$p = p0 - a / b$$
$$= 2.5 - 1.1683109273 / 9.1565671175$$
$$= 2.3724073212$$

n	pn	f(pn)
1	2.3724073212	0.0151395834

$$abs(f(p)) = 2 * 2.3724073212 * \cos(2 * 2.3724073212) - (2.3724073212 - 2)^2$$
$$= 0.0151395834 > tol$$

p0 = 2.3724073212

n=2:

$$a = f(p0)$$
$$= f(2.3724073212)$$
$$= 2 * 2.3724073212 * \cos(2 * 2.3724073212) - (2.3724073212 - 2)^2$$
$$= 0.0151395834$$

$$b = fprime(p0)$$
$$= fprime(2.3724073212)$$
$$= -4 * 2.3724073212 * \sin(2 * 2.3724073212) + 2\cos(2 * 2.3724073212) - 2(2.3724073212 - 2)$$
$$= 8.8046662298$$

$$p = p0 - a / b$$
$$= 2.372407321 - 0.015139583 / 8.8046662298$$
$$= 2.3706878257$$

n	pn	f(pn)
1	2.3724073212	0.0151395834
2	2.3706878257	0.0000079869

$$abs(f(p)) = 2 * 2.3706878257 * \cos(2 * 2.3706878257) - (2.3706878257 - 2)^2$$
$$= 0.0000079869 > tol$$

p0 = 2.3706878257

n=3:

$$a = f(p0)$$
$$= f(2.3706878257)$$
$$= 2 * 2.3706878257 * \cos(2 * 2.3706878257) - (2.3706878257 - 2)^2$$
$$= 0.0000079869$$

$$b = fprime(p0)$$
$$= fprime(2.3706878257)$$
$$= -4 * 2.3706878257 * \sin(2 * 2.3706878257) + 2\cos(2 * 2.3706878257) - 2(2.3706878257 - 2)$$
$$= 8.7953573222$$

$$p = p0 - a / b$$
$$= 2.3706878257 - 0.0000079869 / 8.7953573222$$
$$= 2.3706869177$$

n	pn	f(pn)
1	2.3724073212	0.0151395834
2	2.3706878257	0.0000079869
3	2.3706869177	0.0000000000

$$abs(f(p)) = 2 * 2.3706869177 * \cos(2 * 2.3706869177) - (2.3706869177 - 2)^2$$
$$= 2.24761e - 12 < tol$$

Newton method has converged
final p = 2.370686917662517
final f(p) = 2.2476187577780138e-12

Process finished with exit code 0