QUESTION: Use Secant method to find real zero of the following function accurate to within 10^{-7}.

$$f(x) = 2x\, cos2x - (x-2)^2$$

SOLUTION:

$f(2) = 4\, cos4 = -2.6145744,$

$f(3) = 6\, cos6 - 1 = 4.7610217$

The formula for secant method is:

$$p_n = p_{n-1} - \frac{f(p_{n-1})(p_{n-1} - p_{n-2})}{f(p_{n-1}) - f(p_{n-2})} \qquad \text{for } n \geq 2$$

[Reference: Algorithm 2.4 on page 72 of Numerical Analysis by Burden Faires 9th edition]

Python Code:

```
import numpy as np

import sys


def f(x): return 2 * x * np.cos(2 * x) - (x-2) ** 2


p0 = float(input("Enter p0: "))
p1 = float(input("Enter p1: "))

n = 1
N = 10
tol = 1e-7

print('n          p                      f(p)')
while n <= N:
    q0 = f(p0)
    q1 = f(p1)
    p2 = p1 - q1 * (p1 - p0) / (q1 - q0)
    print(f'{n} \
        {p2:.10f} \
         {f(p2):.10f}')
    if abs(f(p2)) <= tol or abs(p2 - p1) <= tol:
        print('Secant method has converged')
        print('The real zero of f(x) =', p2)
        print('Value of function at real zero =', f(p2))
        sys.exit(0)
    else:
        p0 = p1
        p1 = p2
    n += 1

print('zero not found to the desired tolerance')
```

Output:

```
Enter p0 : 2
Enter p1 : 3
```

n=1:

$$q0 = f(p0)$$
$$= f(2)$$
$$= 4\,cos4$$
$$= -2.6145744834$$

$$q1 = f(p1)$$
$$= f(3)$$
$$= 6\,cos6 - 1$$
$$= 4.7610217199$$

$$p2 = 3 - 4.7610217199 * (3 - 2)/(4.7610217199 + 2.6145744834)$$
$$= 2.3544899167$$

n	p	f(p)
1	2.3544899167	-0.1417166747

$$|f(p2)| = |f(2.3544899167)|$$
$$= |2 * 2.3544899167 * cos2 * 2.3544899167 - (2.3544899167 - 2)^2|$$
$$= 0.1417166747 > tol$$

or

$$|p2 - p1| = |2.3544899167 - 3|$$
$$= 0.64551008334 > tol$$

```
p0 = 3
p1 = 2.3544899167
```

n=2:

$$q0 = f(p0)$$
$$= f(3)$$
$$= 4.7610217199$$

$$q1 = f(p1)$$
$$= f(2.3544899167)$$
$$= -\,0.1417166747$$

$$p2 = 2.3544899167 + 0.1417166747 * (2.3544899167 - 3)/(-0.1417166747 - 4.7610217199)$$
$$= 2.3731487834$$

n	p	f(p)
1	2.3544899167	-0.1417166747
2	2.3731487834	0.0216693873

$$|f(p2)| = |f(2.3731487834)|$$
$$= |2 * 2.3731487834 * \cos(2 * 2.3731487834) - (2.3731487834 - 2)^2|$$
$$= 0.0216693873 > tol$$

or

$$|p2 - p1| = |2.3731487834 - 2.3544899167|$$
$$= 0.0186588668 > tol$$

```
p0 = 2.3544899167
p1 = 2.3731487834
```

n=3:

$$q0 = f(p0)$$
$$= f(2.3544899167)$$
$$= -0.1417166747$$

$$q1 = f(p1)$$
$$= f(2.3731487834)$$
$$= 0.0216693873$$

$$p2 = 2.3731487834 - 0.0216693873 * (2.3731487834 - 2.3544899167)/(0.0216693873 + 0.1417166747)$$
$$= 2.3706741157$$

n	p	f(p)
1	2.3544899167	-0.1417166747
2	2.3731487834	0.0216693873
3	2.3706741157	-0.0001125971

$$|f(p2)| = |f(2.3706741157)|$$
$$= |2 * 2.3706741157 * \cos(2 * 2.3706741157) - (2.3706741157 - 2)^2|$$
$$= 0.0001125971 > tol$$

or

$$|p2 - p1| = |2.3706741157 - 2.3731487834|$$
$$= 0.0024746677 > tol$$

```
p0 = 2.3731487834
p1 = 2.3706741157
```

n=4:

$$q0 = f(p0)$$
$$= f(2.3731487834)$$
$$= 0.0216693873$$

$$q1 = f(p1)$$
$$= f(2.3706741157)$$
$$= -0.0001125971$$

$$p2 = 2.3706741157 + 0.0001125971 * (2.3706741157 - 2.3731487834)/(-0.0001125971 - 0.0216693873)$$
$$= 2.3706869080$$

n	p	f(p)
1	2.3544899167	-0.1417166747
2	2.3731487834	0.0216693873
3	2.3706741157	-0.0001125971
4	2.3706869080	-0.0000000853

$$|f(p2)| = |f(2.3706869080)|$$
$$= |2 * 2.3706869080 * \cos(2 * 2.3706869080) - (2.3706869080 - 2)^2|$$
$$= 0.0000000853 < tol$$

Secant method has converged
The real zero of f(x) = 2.370686907966889
Value of function at real zero = -8.52742195467382e-08

Process finished with exit code 0