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Mon ☐ Tue ☒ Wed ☐ Thu ☐ Fri ☐ Sat ☐ Sun ☐

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Numerical Computing (7)

→ Numerical Integration;

→ Romberg Integration is a combination for both Trapezoidal and Simpson's Rule

Error Estimation for Trapezoidal Rule:

$$E = - \left(\frac{b-a}{12} \right) \cdot h^2 \cdot f''(\eta) \quad a < \eta < b$$

$O(h^2)$ \rightarrow Order

$$|E| = \frac{h^2}{12} \cdot f''(\eta)$$

$$\frac{h^2}{12} \cdot f''(0) < |E| < \frac{h^2}{12} \cdot f''(1)$$

→ Composite Simpson's Rule: (2nd Degree polynomial)

⇒ Flip Over.

$$\rightarrow \int_{x_0}^{x_2} P_2(x) \cdot dx + \int_{x_2}^{x_4} P_2(x) \cdot dx + \int_{x_4}^{x_6} P_2(x) \cdot dx$$

$$+ \dots + \int_{x_{2n-2}}^{x_{2n}} P_2(x) \cdot dx$$

$$\rightarrow \int_{x_0}^{x_2} P_2(x) \cdot dx = \int_{x_0}^{x_2} \left[f_0 + t \Delta f_0 + \frac{t(t-1)}{2!} \Delta^2 f_0 \right] \cdot dx$$

\therefore We know

$$t = \frac{x - x_0}{h}$$

① at $x = x_0$
 $t = 0$

$$dt = \frac{1}{h} dx$$

② at $x = x_2$

$$t = \frac{x_2 - x_0}{h}$$

$$= \frac{2h}{h} = 2$$

$$\rightarrow \int_0^2 \left[f_0 + t \Delta f_0 + \left(\frac{t^2}{2} - \frac{t}{2} \right) \Delta^2 f_0 \right] h \cdot dt$$

$$\rightarrow h \left[t f_0 + \frac{t^2}{2} \Delta f_0 + \left(\frac{t^3}{6} - \frac{t^2}{4} \right) \Delta^2 f_0 \right]_0^2$$

$$\Rightarrow h \left[2f_0 + 2\Delta f_0 + \left(\frac{8}{6} - 1 \right) \Delta^2 f_0 \right]$$

$$\Rightarrow h \left[2f_0 + 2[f_1 - f_0] + \frac{1}{3} (f_2 - 2f_1 + f_0) \right]$$

$$\Rightarrow h \left[\frac{1}{3} f_0 + \frac{4}{3} f_1 + \frac{1}{3} f_2 \right]$$

$$\Rightarrow \frac{h}{3} [f_0 + 4f_1 + f_2]$$

$$\Delta(\Delta f_0) = \Delta(f_1 - f_0)$$

$$\Delta^2 f_0 = \Delta f_1 - \Delta f_0$$

$$= (f_2 - f_1) - (f_1 - f_0)$$

$$= f_2 - 2f_1 + f_0$$

$$h = \frac{b-a}{2N}$$

$$= \frac{h}{3} [f_0 + 4f_1 + f_2] +$$

$$\frac{h}{3} [f_2 + 4f_3 + f_4] + \frac{h}{3} [f_4 + 4f_5 + f_6] + \frac{h}{3} [f_{2N-2} + 4f_{2N-1} + f_{2N}]$$

$$\int_a^b f(x) dx = \frac{h}{3} \left[f_0 + f_{2N} + 4 \left[f_1 + f_3 + f_5 + \dots + f_{2N-1} \right] + 2 \left[f_2 + f_4 + f_6 + \dots + f_{2N-2} \right] \right]$$

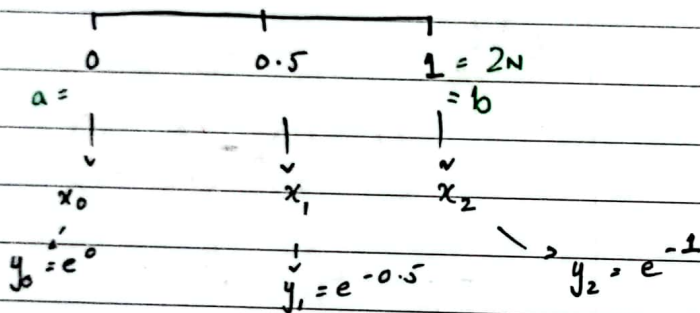
Example 05:

$$\int_0^1 e^{-x} \cdot dx$$

$h = 0.5$

$$2N = \frac{b-a}{h} = \Rightarrow 2N = 2 \Rightarrow$$

$N = 1$



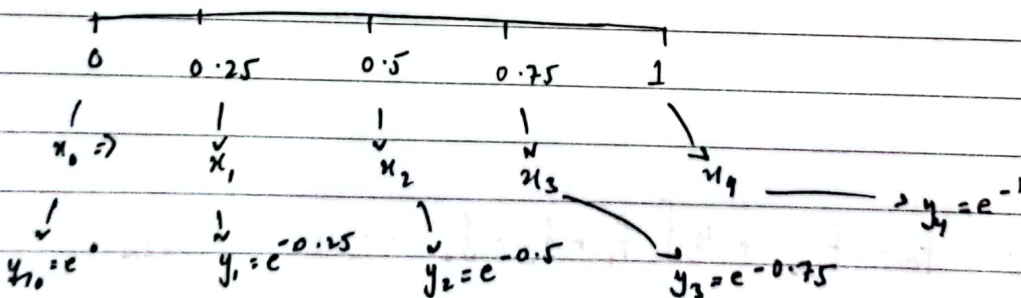
$$\int_0^1 e^{-x} \cdot dx = \frac{h}{3} [y_0 + y_2 + 4y_1] = 0.63233$$

$h = 0.25$

\Rightarrow

$$2N = \frac{1-0}{0.25} = \Rightarrow$$

$2N = 4$
 $N = 2$



~~$$= \frac{h}{3} [y_0 + y_4 + 4(y_1 + y_3 + y_5 + y_7) + 2(y_2 + y_6 + y_8)]$$~~

$$= \frac{h}{3} [y_0 + y_4 + 4(y_1 + y_3) + 2(y_2)]$$

$= 0.63213$

Example 04:

$$\int_1^{2.5} e^{2x} \cdot dx$$

tolerance = 0.5×10^{-5}

$h = ?$

$N = ?$

Trapezoidal

$$|E| = \frac{h^2}{12} (b-a) \cdot f''(\eta)$$

$$= \frac{1.5h^2}{12} \cdot f''(\eta)$$

$$1.0 < \eta < 2.5$$

$$\frac{1.5h^2}{12} \cdot f''(2.5) \leq 0.5 \times 10^{-5}$$

$$\sqrt{h} \leq \sqrt{\frac{(5 \times 10^{-5})(12)}{1.5 \times e^{-2.5}}}$$

$$h = 0.00573$$

$$N = \frac{b-a}{h} = \frac{1.5}{0.00573} = 261.7757$$

Simpson's

$$|E| = \left(\frac{b-a}{180} \right) h^4 \cdot f^{(4)}(\eta)$$

$$\Rightarrow N = \frac{b-a}{2h} = \frac{1.5}{2h}$$

$$\Rightarrow h \leq \sqrt[4]{\frac{180(0.5 \times 10^{-5})}{1.5 \times e^{2.5}}}$$

≈ 8.95 intervals