QUESTION:

Solve the following system using "CROUT'S ALGORITHM for tridiagonal system with SINGLE SUBSCRIPT":

$$\begin{aligned} 2x_1 - x_2 &= 1, \\ x_1 + 2x_2 - x_3 &= 2, \\ 2x_2 + 4x_3 - x_4 &= -1, \\ 2x_4 - x_5 &= -2, \\ x_4 + 2x_5 &= -1. \end{aligned}$$

$$\begin{bmatrix} 2 & -1 & 0 & 0 & 0 \\ 1 & 2 & -1 & 0 & 0 \\ 0 & 2 & 4 & -1 & 0 \\ 0 & 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \\ -2 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} a_0 = 2 & d_0 = -1 & 0 & 0 & 0 \\ c_0 = 1 & a_1 = 2 & d_1 = -1 & 0 & 0 \\ 0 & c_1 = 2 & a_2 = 4 & d_2 = -1 & 0 \\ 0 & 0 & c_2 = 0 & a_3 = 2 & d_3 = -1 \\ 0 & 0 & 0 & c_3 = 1 & a_4 = 2 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} b_0 = 1 \\ b_1 = 2 \\ b_2 = -1 \\ b_3 = -2 \\ b_4 = -1 \end{bmatrix}$$

Python code:

```
import numpy as np

a = [2, 2, 4, 2, 2]
n = len(a)
c = [1, 2, 0, 1, ]
d = [-1, -1, -1, -1]
b = [1, 2, -1, -2, -1]
l = np.zeros((n, 1))
y = np.zeros((n, 1))
x = np.zeros((n, 1))

#STEP 1: Calculating diagonal of lower triangular matrix:

l[0] = a[0]
for i in range(1, n):
    l[i] = a[i] - c[i - 1] * d[i - 1] / l[i - 1]

print('l=')
print(l)

#STEP 2: Calculating Y from LY=b

y[0] = b[0] / l[0]
for i in range(1, n):
    y[i] = (b[i] - c[i - 1] * y[i - 1]) / l[i]

print('y=')
print(y)

#STEP 3: Calculating X from UX=Y

x[n - 1] = y[n - 1]
for i in range(n - 2, -1, -1):
    x[i] = y[i] - d[i] * x[i + 1] / l[i]

print('x=')
print(x)
```

Output:

a= [2, 2, 4, 2, 2]
a[0] a[1] a[2] a[3] a[4]

n=4

c= [1, 2, 0, 1]
c[0] c[1] c[2] c[3]

d= [-1, -1, -1, -1]
d[0] d[1] d[2] d[3]

b= [1, 2, -1, -2, -1]
b[0] b[1] b[2] b[3] b[4]

#Step 1: Calculating Diagonal of Lower triangular matrix

L[0] = a[0]
= 2

i=1 l[1] = a[1]-c[0]*d[0]/l[0]
= 2-1*(-1)/2
= 2.5

i=2 l[2] = a[2]-c[1]*d[1]/l[1]
= 4-2*(-1)/2.5
= 4.8

i=3 l[3] = a[3]-c[2]*d[2]/l[2]
= 2-0*(-1)/4.8
= 2

i=4 l[4] = a[4]-c[3]*d[3]/l[3]
= 2-1*(-1)/2
= 2.5

$$l = \begin{bmatrix} 2 \\ 2.5 \\ 4.8 \\ 2 \\ 2.5 \end{bmatrix}$$

#Step 2: Calculating Y from LY=b

y[0] = b[0]/l[0]
= 1/2
= 0.5

i=1 y[1] = (b[1] - c[0] * y[0])/l[1]
= (2 - 1 * 0.5)/2.5
= 0.6

i=2 y[2] = (b[2] - c[1] * y[1])/l[2]
= (-1 - 2 * 0.6)/4.8
= -0.4583333333

i=3 y[3] = (b[3] - c[2] * y[2])/l[3]
= (-2 - 0 * (-0.4583333333))/2
= -1

i=4 y[4] = (b[4] - c[3] * y[3])/l[4]
= (-1 - 1 * (-1))/2.5
= 0

$$y = \begin{bmatrix} 0.5 \\ 0.6 \\ -0.4583333333 \\ -1 \\ 0 \end{bmatrix}$$

#Step 3: Calculating X from UX=Y

X[4] = y[4]
X[4] = 0

i=3 x[3] = y[3] – d[3] * x[4]/l[3]
= -1 – (-1) * 0/2
= -1

i=2 x[2] = y[2] – d[2] * x[3]/l[2]
= -0.4583333333 – (-1) * (-1)/4.8
= -0.6666666667

i=1 x[1] = y[1] – d[1] * x[2]/l[1]
= 0.6 – (-1) * (-0.6666666667)/2.5
= 0.3333333333

i=0 x[0] = y[0] – d[0] * x[1]/l[0]
= 0.5 – (-1) * 0.3333333333/2
= 0.6666666667

$$x = \begin{bmatrix} 0.6666666667 \\ 0.3333333333 \\ -0.6666666667 \\ -1 \\ 0 \end{bmatrix}$$