QUESTION: Use method of False Position to find real zero of the following function accurate to within 10^{-7}.

$$f(x) = 2x\,cos2x - (x-2)^2$$

SOLUTION:

[Reference: Algorithm 2.5 on page 74 of Numerical Analysis by Burden Faires 9th edition]

Python Code:

```
import numpy as np
import sys


def f(x): return 2 * x * np.cos(2 * x) - (x-2) ** 2


p0 = float(input("Enter p0: "))
p1 = float(input("Enter p1: "))

if f(p0) * f(p1) > 0:
    sys.exit('Function has the same sign at end points')
else:
    print('Real zero of f(x) lies between p0=', p0, ' and p1=', p1)

i = 1
N = 10
TOL = 1e-7

while i <= N:
    q0 = f(p0)
    q1 = f(p1)
    p2 = p1 - q1 * (p1 - p0) / (q1 - q0)
    q2 = f(p2)

    if abs(q2) <= TOL:
        print('False-position method has converged')
        print('The real zero of f(x) =', p2)
        print('Value of function at real zero =', f(p2))
        sys.exit(0)

    if q2 * q0 < 0:
        p1 = p2
    else:
        p0 = p2

    print('Real zero of f(x) lies between p0=', f'{p0:.10f}', ' and p1= ', f'{p1:.10f}')

    i += 1

print('zero not found to the desired tolerance')
```

Output:

$Enter\ p0:\ 2$
$Enter\ p1:\ 3$

$$
\begin{aligned}
f(p0)*f(p1) &= f(2)*f(3)\\
&= -2.6145744834*4.7610217199\\
&= -12.4480459037 < 0
\end{aligned}
$$

Real zero of $f(x)$ lies between $p0 = 2.0$ and $p1 = 3.0$

i=1:

$$
\begin{aligned}
q0 &= f(p0)\\
&= f(2)\\
&= 4\,cos4\\
&= -2.6145744834
\end{aligned}
$$

$$
\begin{aligned}
q1 &= f(p1)\\
&= f(3)\\
&= 6\,cos6 - 1\\
&= 4.7610217199
\end{aligned}
$$

$$
\begin{aligned}
p2 &= 3 - 4.7610217199*(3-2)/(4.7610217199 + 2.6145744834)\\
&= 2.3544899167
\end{aligned}
$$

$$
\begin{aligned}
q2 &= f(2.3544899167)\\
&= 2*2.3544899167*cos2*2.3544899167 - (2.3544899167 - 2)^2\\
&= -0.1417166747
\end{aligned}
$$

$$abs(q2) = 0.1417166747 > TOL$$

$$
\begin{aligned}
q2*q0 &= -0.1417166747*(-2.6145744834)\\
&= 0.3705288015 > 0
\end{aligned}
$$

$p0\ = 2.3544899167$

Real zero of $f(x)$ lies between $p0 =\ 2.3544899167\ and\ p1 =\ 3.0000000000$

i=2:

$$
\begin{aligned}
q0 &= f(p0)\\
&= f(2.3544899167)\\
&= -0.1417166747
\end{aligned}
$$

$$
\begin{aligned}
q1 &= f(p1)\\
&= f(3)\\
&= 4.7610217199
\end{aligned}
$$

$p2 = 3 - 4.7610217199 * (3 - 2.3544899167)/(4.7610217199 + 0.1417166747)$
$= 2.3731487834$

$q2 = f(2.3731487834)$
$= 2 * 2.3731487834 * \cos(2 * 2.3731487834) - (2.3731487834 - 2)^2$
$= 0.0216693873$

$abs(q2) = 0.0216693873 > TOL$

$q2 * q0 = 0.0216693873 * (-0.1417166747)$
$= -0.00307091351 < 0$

$p1 = 2.3731487834$

Real zero of $f(x)$ lies between $p0 = 2.3544899167$ and $p1 = 2.3731487834$

i=3:

$q0 = f(p0)$
$= f(2.3544899167)$
$= -0.1417166747$

$q1 = f(p1)$
$= f(2.3731487834)$
$= 0.0216693873$

$p2 = 2.3731487834 - 0.0216693873 * (2.3731487834 - 2.3544899167)/(0.0216693873 + 0.1417166747)$
$= 2.3706741157$

$q2 = f(2.3706741157)$
$= 2 * 2.3706741157 * \cos(2 * 2.3706741157) - (2.3706741157 - 2)^2$
$= -0.0001125971$

$abs(q2) = 0.0001125971 > TOL$

$q2 * q0 = -0.0001125971 * (-0.1417166747)$
$= 1.5957e - 05 > 0$

$p0 = 2.3706741157$

Real zero of $f(x)$ lies between $p0 = 2.3706741157$ and $p1 = 2.3731487834$

i=4:

$q0 = f(p0)$
$= f(2.3706741157)$
$= -0.0001125971$

$q1 = f(p1)$

$= f(2.3731487834)$
$= 0.0216693873$

$p2 = 2.3731487834 - 0.0216693873 * (2.3731487834 - 2.3706741157)/(0.0216693873 + 0.0001125971)$
$= 2.3706869080$

$q2 = f(2.3706869080)$
$= 2 * 2.3706869080 * \cos(2 * 2.3706869080) - (2.3706869080 - 2)^2$
$= -0.0000000853$

$abs(q2) = 0.0000000853 > TOL$

False-position method has converged
The real zero of f(x) = 2.370686907966889
Value of function at real zero = -8.52742195467382e-08

Process finished with exit code 0