Mon Tue Wed Thu Fri Sat Sun

C.w

Date: 12-9-2024

Numerical Computing (8)

-> Numerical Integration:

• Romberg Integration:

Richardson Extrapolation Process:

• Trapezoidal Extrapolation:

$h = h/2, h/4, h/8$

$$I_T^m(h) = \frac{4^m I_T^{m-1}(h/2) - I_T^{m-1}(h)}{4^m - 1}$$

$m = 0, 1, 2, 3, \ldots$

where

$$I_T^0(h) = I_h$$

$$I_T^0(h/2) = I(h/2)$$

$$\int_0^1 e^{-x} \cdot dx$$

$a = 0$

$b = 1$

$f(x) = e^{-x}$

$h = 1.0, 0.5, 0.25, 0.125$

$h = 1.0 \Rightarrow I_h$

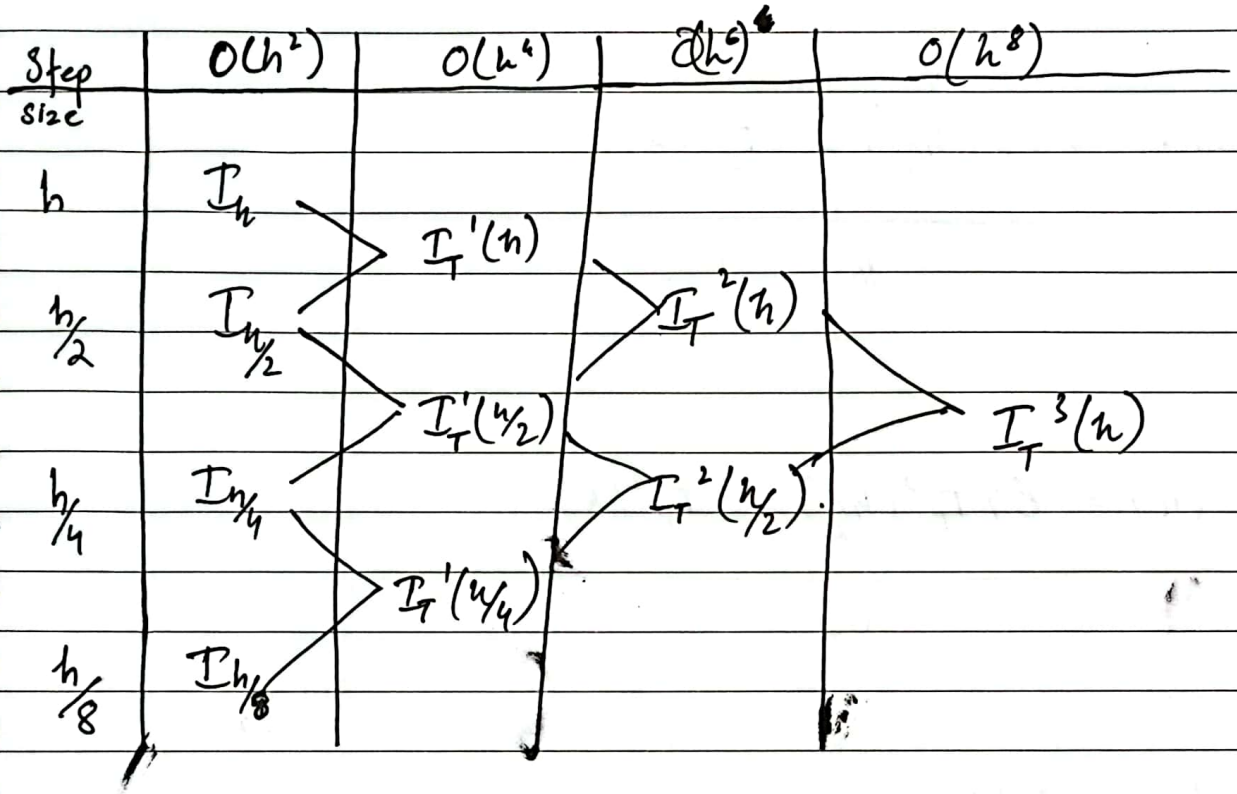
$h \Rightarrow 0.5 \Rightarrow I_{h/2}$

$h \Rightarrow 0.25 \Rightarrow I_{h/4}$

$h \Rightarrow 0.125 \Rightarrow I_{h/8}$

Mon Tue Wed Thu Fri Sat Sun

Date:



Starting trapezoidal extrapolation for m = 1;

$$I_T'(h) = \frac{4 I_T^0(h/2) - I_T^0(h)}{3} \Rightarrow$$

$$\frac{4(0.64523519) - 0.68393972}{3} \Rightarrow \boxed{0.632333680} \quad I'_{h, h/2}$$

$$I_T'(h/2) = \frac{4 I_T^0(h/4) - I_T^0(h/2)}{3} \Rightarrow \boxed{0.632134753} \quad I'_{h/2, h/4}$$

$$I_T'(h/4) = \frac{4 I_T^0(h/8) - I_T^0(h/4)}{3} \Rightarrow \boxed{0.632121414 6\ldots} \quad I'_{h/4, h/8}$$

Mon Tue Wed Thu Fri Sat Sun

Date:

for $m = 2$;

$$I_T^2(h) = \frac{16\, I_T^1(h/2) - I_T^1(h)}{15}$$

for $m = 3$;

$$I_T^3(h) = \frac{64\, I_T^2(h/2) - I_T^2(h)}{63}$$

$I^2_{h, h/2, h/4}$

$$I_T^2(h) = \frac{16(0.6321341753) - 0.6323233680}{15} \Rightarrow \boxed{0.6321208875}$$

$$I_T^2(h/2) = \frac{16(0.6321214144) - 0.6321341753}{15} \Rightarrow$$

$I^2_{h/2, h/4, h/8}$

$$\Rightarrow 0.6321205632 \Rightarrow \boxed{0.6321205632}$$

$$I_T^3(h) = \frac{64(0.6321205632) - 0.6321208875}{63} \Rightarrow \boxed{0.6321205583}$$

$I^3_{h, h/2, h/4, h/8}$

Mon Tue Wed Thu Fri Sat Sun

Date:

Simpson's Extrapolation:

$$I_s^m(h) = \frac{4^{m+1} I_s^{m-1}(h/2) - I_s^{m-1}(h)}{4^{m+1} - 1}$$

; $m = 1, 2, 3, \ldots$

where $I_T^0(h) = I_n$

$I_T^0(h/2) = I_{h/2}$

Step size	$O(h)$	$O(h^4)$	$O(h^6$	$O(h^{10})$
h	I_h	$I_s'(h)$	$I_s^2(h)$	$I_{(s)}^3(h)$
$h/2$	$I_{h/2}$	$I'_{(s)}(h/2)$	$I_0^2(h/2)$	
$h/4$	$I_{h/4}$	$I_s'(h/4)$		
$h/8$	$I_{h/8}$			