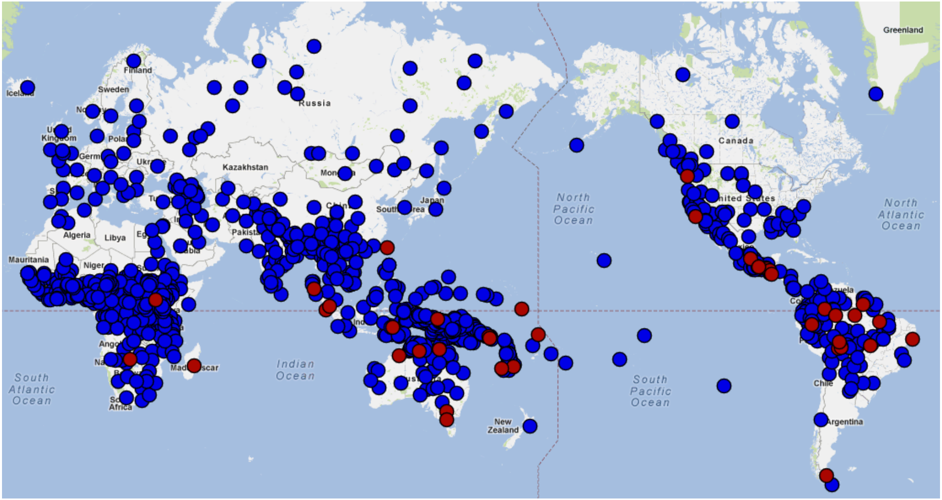
Greenbergian Universals and Bayesian inference

Jenny Culbertson

Simulating Language, 7 March, 2019

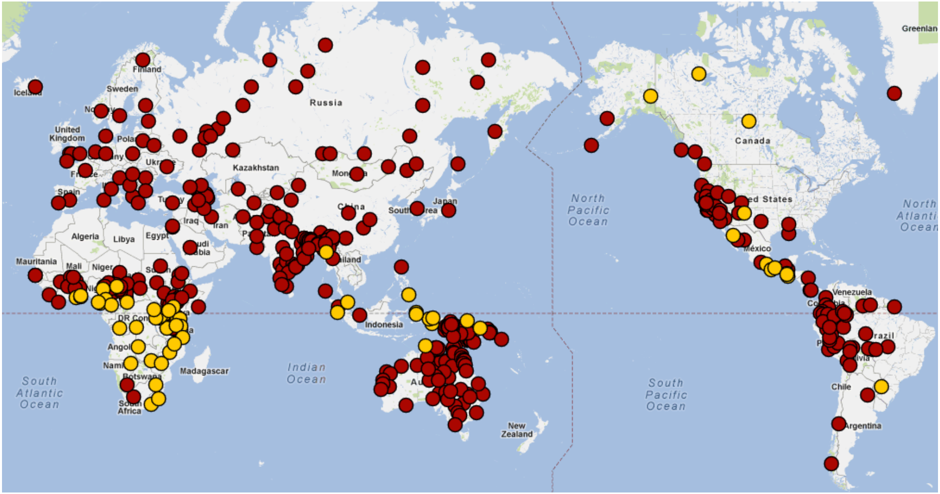
Greenberg's Universal 1

1. SOV, SVO, VSO (not VOS, OSV, OVS)



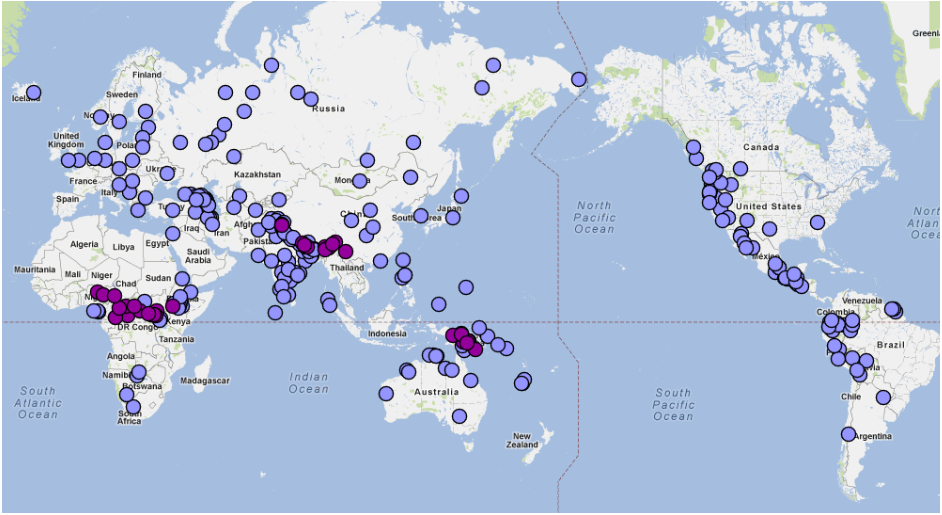
Greenberg's Universal 26

26. Suffixes (not prefixes)



Greenberg's Universal 18

18. If Adjective-Noun → Numeral-Noun



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- Lots of confounding factors (e.g.,...?)

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    - Non-uniform preference among patterns
    - (Could be innate or learned)
    - (Could be general or specialized for language)

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- Lots of confounding factors (e.g.,...?)
- But could indicate cognitive biases
  - Cognitive bias = prior bias
  - Non-uniform preference among patterns
  - (Could be innate or learned)
  - (Could be general or specialized for language)
- How to investigate? Preferences in a single generation??

Universal 18

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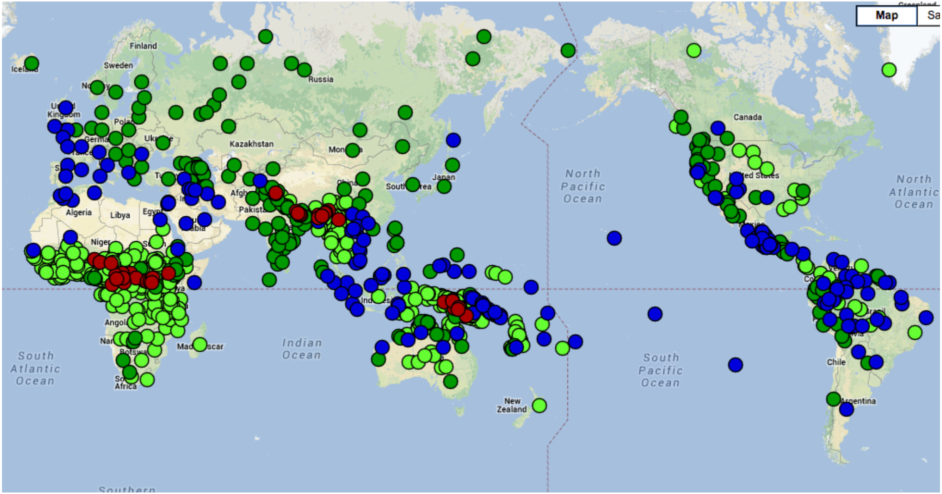

Universal 18

- Actually, there is more than one asymmetry here...

	N-Adj	Adj-N
Num-N	17%	27%
N-Num	52%	4%

Universal 18

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- Related to another bias you've read about??

Setting up the experiment

- The conditions

	N-Adj	Adj-N
Num-N	3	1
N-Num	2	4

- Easy or hard to learn...?

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Setting up the experiment

- The conditions

	N-Adj	Adj-N
Num-N	3	1
N-Num	2	4

- Easy or hard to learn...?
- Adding in regularization...
  - 70% dominant pattern, 30% minority pattern
  - What would regularization look like in this case?

Formulating hypotheses

- Training = listening to Adj-N, N-Adj, Num-N, N-Num phrases
- Testing = producing phrases

Formulating hypotheses

- Training = listening to Adj-N, N-Adj, Num-N, N-Num phrases
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- Three reasonable hypotheses...

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H1. Learning involves tracking input statistics

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H1. Learning involves tracking input statistics

H2. Learners regularization variation

H3. Learners regularize but only orders that are easy to learn

Formulating hypotheses

- In terms of Bayesian inference...

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H2. Input likelihood $\times$ regularization prior

H3. Input likelihood $\times$ regularization prior $\times$ order prior

Making predictions

- Three predicted outcomes...

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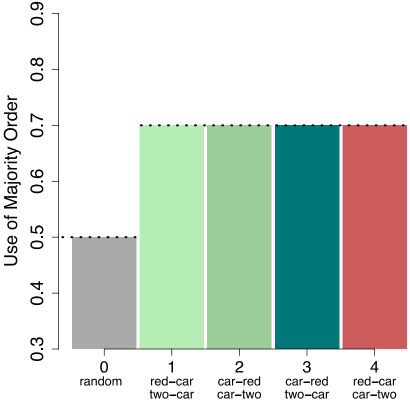
1. Probability matching



Making predictions

- Three reasonable outcomes...

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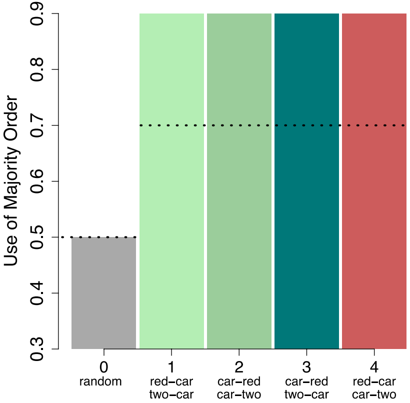
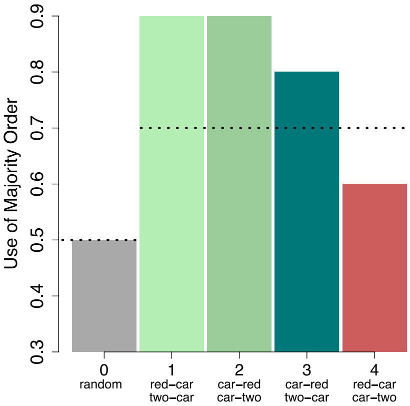
2. Across the board regularization



Making predictions

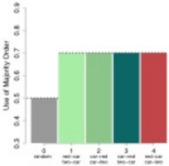
- Three reasonable outcomes...

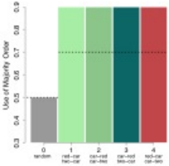
1. Probability matching
2. Across the board regularization
3. Regularization modulated by order

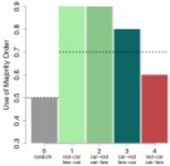


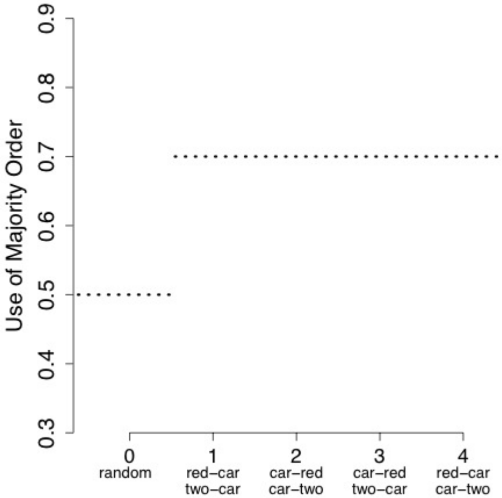
Results

- Participants: 65 native-English-speaking undergrads









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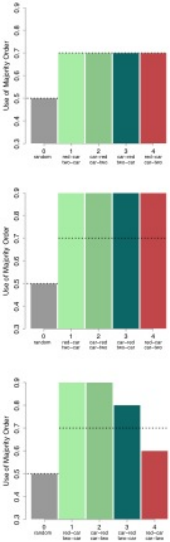

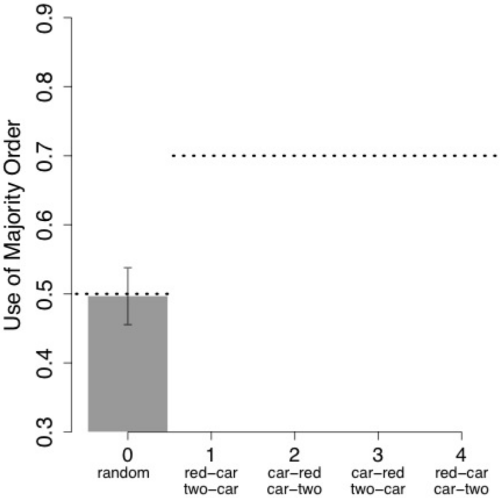

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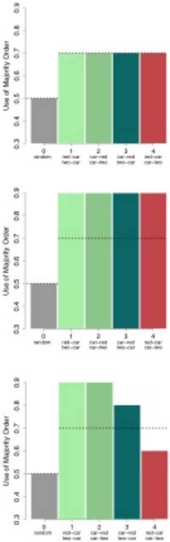

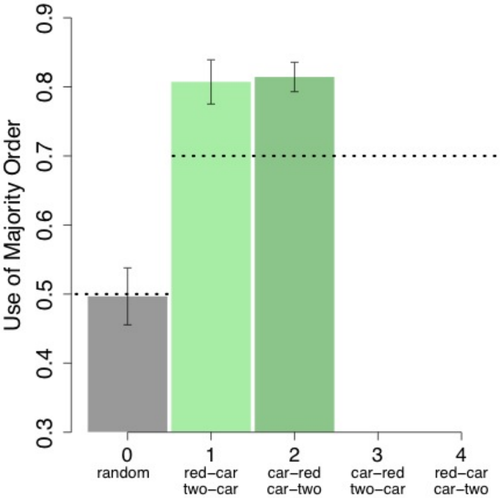

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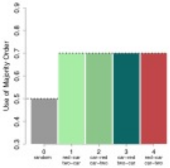

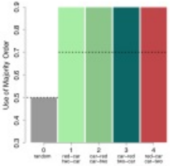

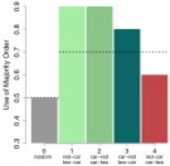

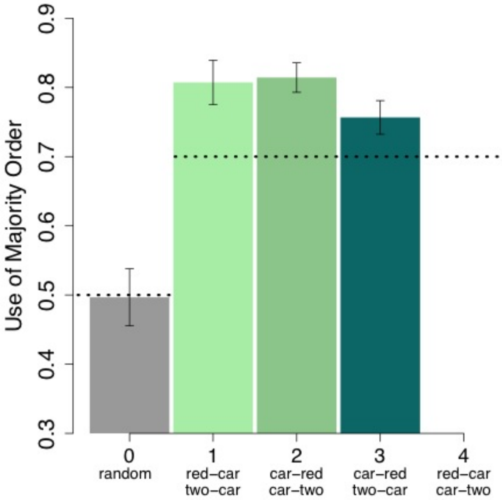

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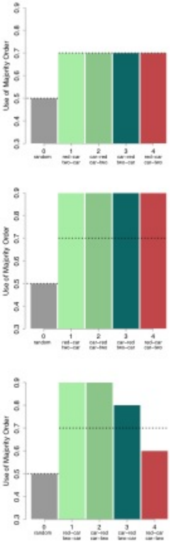
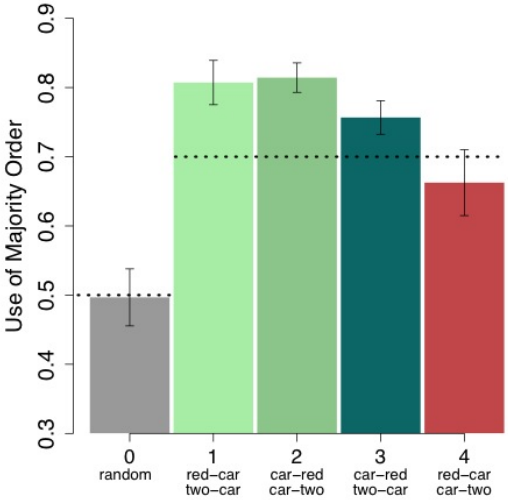
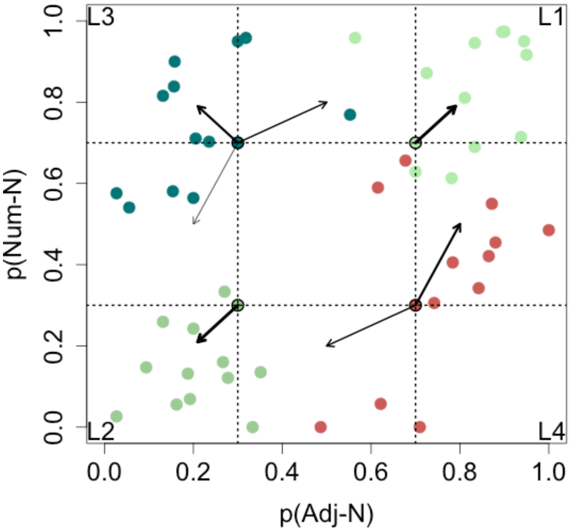




Individual learner outcomes



Formulating hypotheses

- In terms of Bayesian inference...

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- Regularization prior
- Ordering prior

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  binomial(5 heads $|p=$0.5, 10 tosses) $= 0.2$

  binomial(5 heads $|p=$0.9, 10 tosses) $= 0.001$

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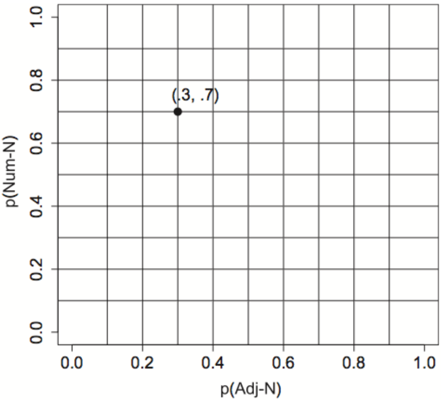
binomial(28 Adj-N $|p=$0.7, 40 Adj) $=$ 0.14

binomial(28 Adj-N $|p=$0.3, 40 Adj) $=$ 0.0000018

Likelihood

- Adj *and* Num ordering
  - Grid of possible probability combos
  - Each assigns likelihood to a set of counts
  - (Total likelihood just multiplies Adj and Num likelihoods)



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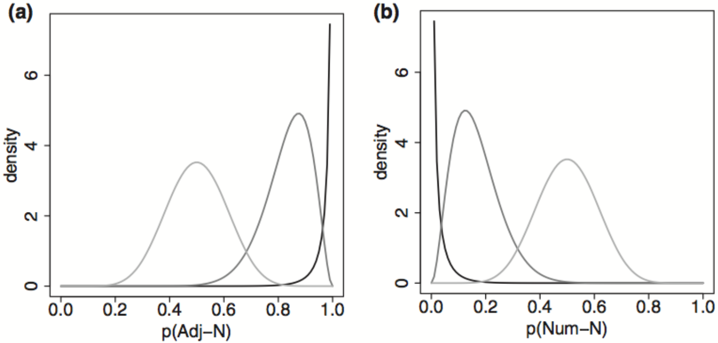

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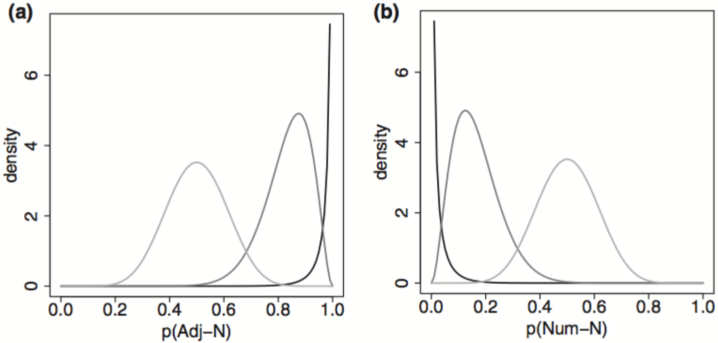
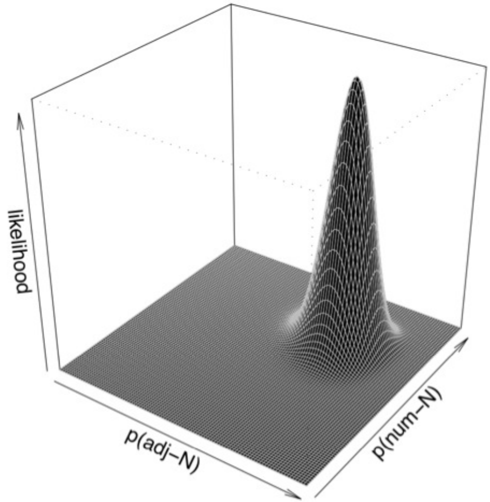


- *Asymmetrical* beta distributions: skewed parameters → one-way regularization

Effect of prior on posterior

- Likelihood alone vs. likelihood $\times$ regularization prior



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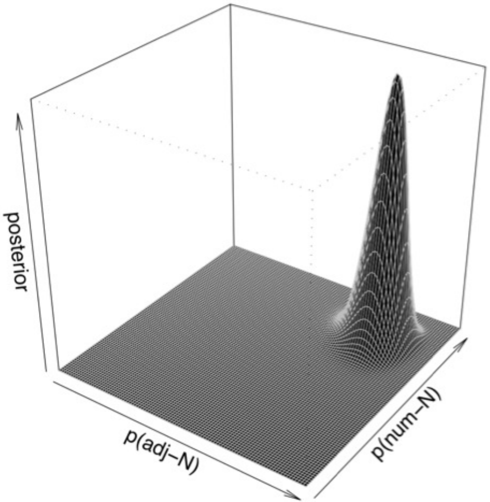

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- Conceptually: prior *counts*, e.g. of Adj-N utterances

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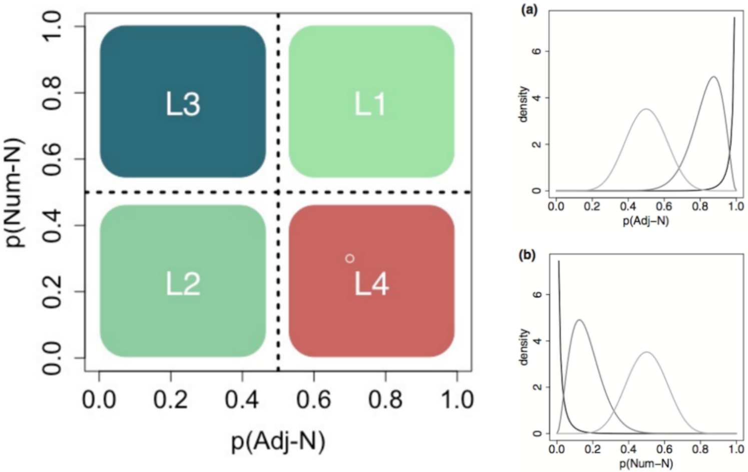

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  [what would a biased one look like??]

Complete prior

- Complete prior probability of a grammar p(Adj-N), p(Num-N) is a sum over four beta combinations of:
  - prior probability of p(Adj-N) given regularization bias $\times$
  - prior probability of p(Num-N) given regularization bias $\times$
  - prior probability of particular combination of betas
- e.g., prior for p(Adj-N)=0.8, p(Num-N)=0.2

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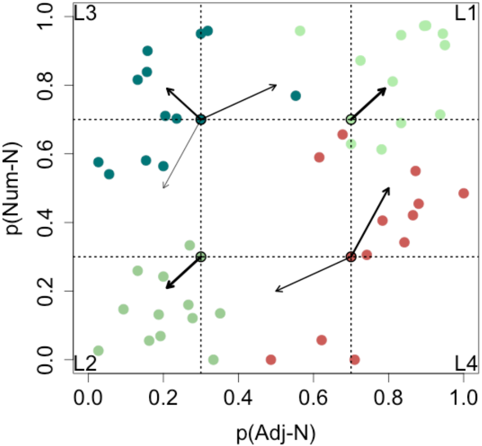
\end{aligned}$$

- ...Low component prior $\rightarrow$ posterior will move away from that area of grammar space

Looking for prior biases

- What parameters make the testing data most likely?



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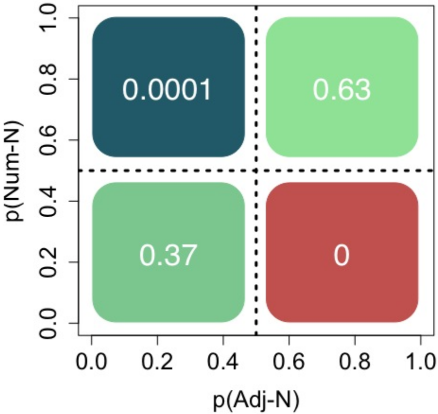
- Prior probability of pattern types:



Posterior (finally!)

- What kinds of p(Adj-N), p(Num-N) pairs are learners likely to acquire given set of prior parameters?

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- What kinds of p(Adj-N), p(Num-N) pairs are learners likely to acquire given set of prior parameters?

- Prior probability of p(Adj-N)=high, p(Num-N)=high is high
- Prior probability of p(Adj-N)=low, p(Num-N)=low is high
- Prior probability of p(Adj-N)=low, p(Num-N)=high is pretty low
- Prior probability of p(Adj-N)=high, p(Num-N)=low is zero!

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Num-N	17%	27%
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For the lab...

- Calculate posterior distributions
- Recreate model predictions
- Investigate the effect of the prior parameters on predicted grammars
- Extra-credit: iterate it

Readings

- [Culbertson et al., 2012] <link to paper>
- [Culbertson and Smolensky, 2012] <link to paper>

Culbertson, J. and Smolensky, P. (2012). A Bayesian model of biases in artificial language learning: The case of a word-order universal. *Cognitive Science*, 36(8):1468–1498.

Culbertson, J., Smolensky, P., and Legendre, G. (2012). Learning biases predict a word order universal. *Cognition*, 122:306–329.