Mandelbrot Set 的生成和探索

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Abstract

In this paper, we gave a brief introduction of Mandelbrot set, explained its mathematical thoery. Then we implemented the algorithem using C++, listed pictures as emaxple, and draw a conclusion as the end.

1 Introduction

Mandelbrot Set is a set that for each complex C, it can generate a iteration according to a certain iteration rule. And depending on the sequence of the iteration, we give it a corresponding color. A picture full of beauty of math then can be presented.

2 Background of the Problem

Amarican Mathematician Benoit B. Mandelbrot first came up with this facinating set in 1975, and he called it "Devil's polymer". [2]

3 Mathematical Theory

According to the equation:

$$Z_{n+1} = Z_n^2 + C \tag{1}$$

For each $C \in \mathbb{C}$, start iteration from $0.0 + 0.0j$, and if the series is convergent, then C is in Mandelbrot Set. [4]

4 Algorithem

Algorithm 1 Iteration in Mandelbrot Set

Input: A maximal iteration number N, points of a picture

Output: Colors of each point in the picture

```
for each point of the picture do
    z = 0.0 + 0.0j
    c = the complex number represented by the point [1]
    for int i=1 to N do
        if |z|>2 then
            This point is not convergent, give it color according to i
            Break to go to the next point
        else
            z = z^2 + c
        end if
        if the loop reaches its natural end then
            Give this point a corresponding color like black
        end if
    end for
end for
```

5 Example

Mandelbrot Set with different maximal iteration times:

(a) $N = 10$ (b) $N = 100$ (c) $N = 1000$

图 1: Mandelbrot Set with different maximal iteration times

This indicates that the larger the maximal iteration time is, the smoother the boundary seems to be in the picture, and if we zoom in, we can see more

details as the maximal iteration time is larger.
Choose part of the picture and zoom in:

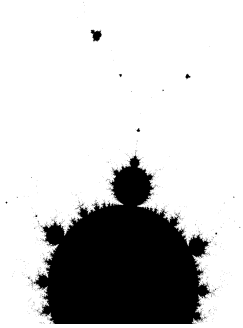

图 2: Zoom in

We can see that Mandelbrot Set has similar structure when zoomed in at infinity.
Paint the points according to their iteration times:

(a) Whole (b) Zoom in

图 3: Colored Mandelbrot Set

Paint Mandelbrot Set according to specific iteration-time-funcion can create beautiful pictures.

6 Conclusion

Mandelbrot Set can be presented as a beautiful picture, and it can always have the similar structure and clear details no matter how many the magnificantion times is, which is called ”self-similarity” [3]. Morever, com-

bined with well-designed color function according to the maximal iteration time, Mandelbrot Set can demonstrate unimaginable pattern.

Reference

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