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SWE 485 - Selected Topics in Software Engineering

MAP COLORING PROBLEM

Phase 1

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Introduction

Map coloring problem

The Map Coloring Problem is a classic combinatorial problem in graph theory and computer science. It involves assigning colors to regions on a map in such a way that no two adjacent regions share the same color. The primary objective is to find a valid coloring for the map while adhering to this constraint.. Map coloring problem states that given a graph G {V, E} where V and E are the set of vertices and edges of the graph, all vertices in V need to be colored in such a way that no two adjacent vertices must have the same color. [1]

Real-World Applications of the coloring problem:

- Cartography: In creating maps for geographical regions, ensuring neighboring areas have distinct colors aids in visual clarity.
- Scheduling: Regions can represent tasks or events, and coloring helps optimize scheduling to prevent conflicts.
- Frequency Allocation: In wireless communication, the assignment of frequencies to adjacent regions without interference aligns with the map coloring concept.

Objective

The objective of this report is to employ Constraint Satisfaction Problem (CSP) formalization to solve the Map Coloring Problem. By defining the variables and domains, constraints, and objective function as a first step. this report aims to define the needed components to create a mathematical representation of the problem that is amenable to the application of search algorithms.

Problem definition

key elements of a Constraint Satisfaction Problem (CSP):

1. **Variables**: A set of regions that will be colored (X).

In the map coloring problem, each region on the map is treated as a variable. Let's assume we have a map with Rn regions, each of which needs to be colored. We can denote these regions (variables) as $R1, R2, R3, \ldots, Rn$

2. **Domains**:
   - Definition: Domains define the possible values that each variable can take. It represents the feasible options or potential assignments for the variables.

The domain for each variable is the set of colors that can be assigned to each region. Assuming we have a set of Cm colors available, we can represent the domain of colors as $C=\{c1, c2, c3, \ldots, cm\}$.

3. **Constraints**:
   - Definition: Constraints define the relationships, limitations, or conditions that must be satisfied by the variable assignments to be considered valid solutions.
   - Constraints in the Map Coloring Problem refer to the rules that must be followed when assigning colors to regions on a map. In this context, constraints ensure that no two adjacent regions share the same color. So If Ri and Rj share a common border then the constraints would be $Ci \neq Cj$
4. **Objective function:**
   - Definition: In the context of the search algorithms, the objective function is the function we aim to maximize or minimize, it could represent the cost of the solution, the measure of fitness, or the overall quality of a candidate solution within the search space.[2]
   - Map coloring problem objective function: The objective of the Map Coloring Problem is to find a valid assignment of colors to regions that satisfies all constraints. There might not be a numerical objective function in the traditional sense, but the goal is to find a solution that adheres to the constraints.

Example

Since the map coloring problem is a general problem and it doesn't have a single scenario, we considered providing an example to better illustrate each element in the CSP :

Let's consider a simple example with four regions: R1,R2,R3,R4, and three colors: C1,C2,C3.

Variables and Domains: In this example, each of the four regions is a variable that we need to assign a color to. Therefore, the variables are**:** R1,R2,R3,R4**.**

The domain for each variable is the set of colors that can be assigned to it. Since we have three colors available (*C*1, *C*2,*C*3), the domain for each of the regions is the same, consisting of these three colors.

Hence, the domains for the variables are as follows:

Domain for *R*1: D (R1)={ *C*1,*C*2,*C*3**}**

Domain for *R*2: D (R2) ={ *C*1,*C*2,*C*3**}**

Domain for *R*3: D (R3) ={ *C*1,*C*2,*C*3**}**

Domain for *R*4: D (R4) ={ *C*1,*C*2,*C*3**}**

Constraints:

In the context of the Map Coloring Problem, the main constraint is that for every pair of adjacent regions, the colors assigned to them must be different. This constraint can be expressed as follows:

If there is an edge between two regions Ri and Rj (represented as (Ri, Rj) in the graph), then the colors assigned to Ri and Rj, denoted as Ci and Cj, must be different. This constraint can be formally stated as Ci $\neq$ Cj.

For the example with four regions (R1, R2, R3, R4) and three colors (C1, C2, C3), the constraints can be specified as:

- R1 and R2 are adjacent: C(R1) $\neq$ C(R2)

- R1 and R3 are adjacent: C(R1) $\neq$ C(R3)

- R1 and R4 are adjacent: C(R1) $\neq$ C(R4)

- R2 and R3 are adjacent: C(R2) $\neq$ C(R3)

- R2 and R4 are adjacent: C(R2) $\neq$ C(R4)

- R3 and R4 are adjacent: C(R3) $\neq$ C(R4)

Objective Function:

- Find a valid assignment of colors to R1,R2,R3,R4 that satisfies all the constraints.

References

[1] Map colouring algorithm (no date) Tutorialspoint. Available at: https://www.tutorialspoint.com/data_structures_algorithms/map_colouring_algorithm.htm (Accessed: 03 March 2024).

[2] Abbas AliAbbas Ali 56633 gold badges1010 silver badges1717 bronze badges and nbronbro 40.2k1212 gold badges103103 silver badges185185 bronze badges (1964) What is an objective function?, Artificial Intelligence Stack Exchange. Available at: https://ai.stackexchange.com/questions/9005/what-is-an-objective-function (Accessed: 03 March 2024).