ASSIGNMENT - I

NAME: FAZIL. S
ROLL NO: 221501031
CLASS: AIML - 'A'
SUBJECT: COMPUTATIONAL STATISTICS

If $f(x,y) = k(x+y), 0<x<1, 0<y<1$ is a joint density function find k.

soln:-

To find value of k

take double integral,

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy = \int_0^1\int_0^1 k(x+y)\,dx\,dy$$

$$= k\int_0^1 \left[\frac{x^2}{2}+yx\right]_0^1 dy \Rightarrow k\int_0^1\left(\frac{1}{2}+y\right)dy$$

$$= k\left[\frac{1}{2}y+\frac{y^2}{2}\right]_0^1 \Rightarrow \frac{k}{2}[1+1] \Rightarrow \frac{k}{2}\times 2 = 1$$

$$\boxed{k=1}$$

2) If covariance matrix is given by $\Sigma = \begin{bmatrix} 4 & 6 & -2 \\ 6 & 9 & 0 \\ -2 & 0 & 25 \end{bmatrix}$ obtain standard deviation matrix

Covariance matrix

$$\Sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{pmatrix} = \begin{bmatrix} 4 & 6 & -2 \\ 6 & 9 & 0 \\ -2 & 0 & 25 \end{bmatrix}$$

std matrix

$$V^{12} = \begin{pmatrix} \sqrt{\sigma_{11}} & 0 & 0 \\ 0 & \sqrt{\sigma_{22}} & 0 \\ 0 & 0 & \sqrt{\sigma_{33}} \end{pmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

3) Explain type I & Type 2 misclassification

Type I $\longrightarrow$ when it belongs to i_I but assigned to P_{II}.

Type II → Belongs to P_{II}

4) Find apparent error rate given $n_{1c} = 0$, $n_{1m} =$

Apparent Error rate $= \dfrac{n_1 m + n_2 m}{n_1 c + n m + n_2 c} = \dfrac{2+2}{10+2+10+12} = \dfrac{4}{24}$

$= \dfrac{1}{6} = 0.1667$

5) The scores for individuals 1 & 2 on $P=6$ binary variable are

	X_1	X_2	X_3	X_4	X_5	X_6
Individual I :	0	0	0	1	1	1
Individual II :	1	1	1	0	1	0

Soln:-

Identify the matches & mismatches lets now X_1 individual has 0 individual 2 has 1 (mismatch)

count the matches & mismatches

No. of matches : 1 (only X_5 matches)

No. of mismatches : 5 (son X_1, X_2, X_3, X_4, X_5)

	Individual 1 (0)	Individual 2 (1)
Individual (0)	0	3
Individual (1)	2	1

Part-B

1) The Joint pmf of (X, Y) is given by $f(x,y) = \dfrac{x+y}{2}$

$x = 1, 2$ and 3; $y = 1$ & 3. Find all marginal & conditional probability

$y = 1, 2$ let's calculate $P(X = x)$ for each x

$X = 1$ $P(X=2) = f(1,1) + f(1,2) = \frac{5}{21}$

$X = 2$ $P(X=2) = f(2,1) + f(2,2) = \frac{7}{21}$

for $y = 1$

$P(y=1) = f(1,1) + f(2,1) + f(3,1) = \frac{9}{21}$

for $y = 1$: $P(X = 1/y = 1) = \frac{2}{9}$, $P(X = 2/Y = 1) = \frac{1}{3}$

$P(X = 3/Y = 1) = \frac{4}{9}$; $P(X = 1) / Y = 2) = 1/4$

b) Let x be a $N_3(u, \Sigma)$ with $\omega^T = [3, 1, 4]$ and

$\Sigma = \begin{bmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ which of following are independent

Explain a) x_2 & x_3

b) $\frac{x_1 + x_2}{2}$ and x_3 c) x_2 & $x_2 - \frac{5}{2} x_1 - x_3$

Soln:-

(A) x_2 & x_3 are independent ($\because d_{23} = 0$)

(B) This is a linear equation with co-efficient $(\frac{1}{2}, \frac{1}{2}, 0)$ & $(0, 0, 1)$ by property of linear equation

i.e $(\frac{1}{2}, \frac{1}{2}, 0)(0, 0, 1) \sim N[A^T\mu, A^T \Sigma A]$

now, $A' = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \therefore A = \begin{bmatrix} 1/2 & 0 \\ 1/2 & 0 \\ 0 & 1 \end{bmatrix}$

$$A^T \mu = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -3 \\ 1 \\ 4 \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

$$A^T \Sigma A = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 1/2 & 0 \\ 0 & 2 \end{pmatrix}$$

$\therefore \cancel{\phi} \dfrac{x_1 + x_2}{2}$ and x_5 are independent

(c) This is a linear equation with coefficient $(0,1,0)(-5/2,1,-1)$ by prop. I, follows no with mean $A^T \mu$ & covariance $A^T \Sigma A$

$$A^T \mu = \begin{pmatrix} 0 & 1 & 0 \\ -5/2 & 1 & -1 \end{pmatrix} \begin{pmatrix} -3 \\ 1 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \\ 9/2 \end{pmatrix}$$

$$A^T \Sigma A = \begin{pmatrix} 5 & 10 \\ 15/2 & 29/2 \end{pmatrix}$$

$\therefore X_2$ and $x_2 - \cancel{x_2}\, x_1 - x_3$ are not independent

$\therefore \Sigma_{12} = \Sigma_{21} \neq 0$.

iii) Let x be a $N_3(\mu, \Sigma)$ with $\mu = [2, -3, 1]$ and $\Sigma = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 2 \\ 1 & 2 & 2 \end{bmatrix}$ find the conditional distribution X_3, given that $X_1 = x_1$ & $X_2 = x_2$

$$\Sigma = \left(\begin{array}{c|cc} \sigma_{33} & \sigma_{31} & \sigma_{32} \\ \hline \sigma_{12} & \sigma_{11} & \sigma_{13} \\ \sigma_{23} & \sigma_{21} & \sigma_{22} \end{array}\right) = \left(\begin{array}{c|cc} 2 & 1 & 2 \\ \hline 1 & 1 & 1 \\ 2 & 1 & 3 \end{array}\right)$$

Consider the hypothetical distance b/w pairs of five object as follow

$$D = \begin{array}{c} \\ a \\ b \\ c \\ d \\ \\ \end{array} \begin{bmatrix} a & b & c & d \\ 0 & 9 & 3 & 6 \\ 9 & 0 & 7 & 5 \\ 6 & 5 & 9 & 0 \\ 11 & 10 & 2 & 3 \end{bmatrix}$$

Single linkage method:-

minimum distance : $\boxed{d_{ec} = 2}$

$d(ec)a = \min\{d_{ea}, d_{ca}\} = \min\{11, 3\} = 3$

$d(ec)d = \min\{d_{eb}, d_{cb}\} = \{10, 7\} = 7$

$\boxed{d(ec)a = 3}$

New matrix:-

$$\begin{array}{c} \\ d(ec)a \\ b \\ d \end{array} \begin{bmatrix} d(ec)a & b & d \\ 0 & 7 & 6 \\ 7 & 0 & 5 \\ 6 & 5 & 0 \end{bmatrix} \quad d(bd) = 5$$

$\therefore d(ce)a = \min\{d(b, (e)d), d((e), a\}$

$= \{7, 6\} = 6$

complete linkage method:-

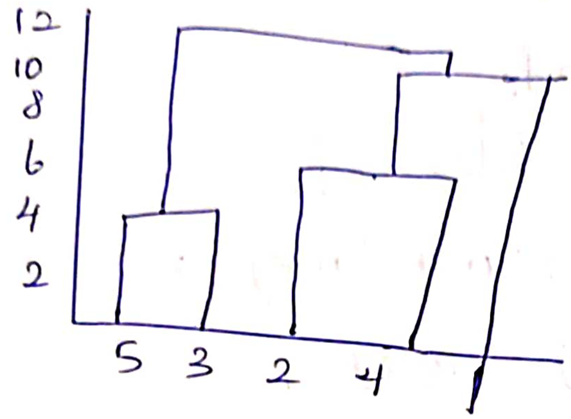
$d(53)1 = \max\{d_{51}, d_{32}\} = 11$

$d(53)2 = \max\{d_{12}, d_{32}\} = 11$

$d(53)4 = \max\{d_{54}, d_{34}\} = 9$

new matrix

$$\begin{array}{c|cc} & 1(24) & 53 \\ \hline 1(24) & 0 & 11 \\ 53 & 11 & 0 \end{array}$$



② soln:-

centroid (mean)

	$\overline{x_1}$	$\overline{x_2}$
AB	$\frac{5+1}{2}=3$	$\frac{4-2}{2}=1$
CD	$\frac{7+3}{2}=1$	$\frac{1+1}{2}=1$

step 2:-

	AB	CD	minimum distance
A(5,4)	13	25	13 (A ∈ AB)
B(1,-2)	13	9	9 (B ∈ CD)
C(-1,1)	16	4	4 (C ∈ CD)
D(3,1)	0	4	0 (D ∈ AB)

Again step 1:-

	$\overline{X_1}$	$\overline{X_2}$
AD	4	2.5
BC	0	-0.5

	AD (4,2.5)	BC (1,-0.5)	min-distance
A(5,4)	3.25	45.25	3.25 (A ∈ AD)
B(1,-2)	29.25	3.25	3.25 (B ∈ BC)
C(-1,1)	27.25	3.25	3. (C ∈ BC)
D(2,1)	3.25	11.25	3.25 (D ∈ AB)

conclusion

AD	BC
A	B
D	C