NCERT Solutions for Class 10

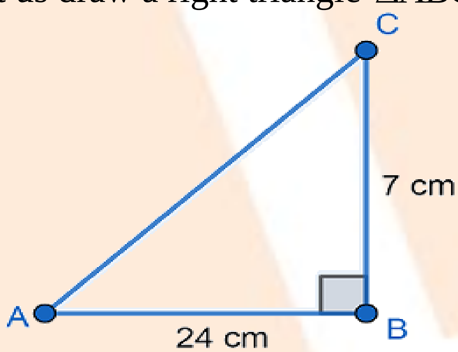
Maths

Chapter 8 – Introduction to Trigonometry

Exercise 8.1

1. In ΔABC right angled at B, $AB = 24$ cm, $BC = 7$ cm. Determine
(i). $\sin A, \cos A$

Ans: Given that in right angle triangle ΔABC, $AB = 24$ cm, $BC = 7$ cm.
Let us draw a right triangle ΔABC, also $AB = 24$ cm, $BC = 7$ cm. We get



We have to find $\sin A, \cos A$.
We know that for right triangle

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}} \text{ and}$$

$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

Now, we need to apply the Pythagoras theorem to find the measure of hypotenuse.
In ΔABC, by Pythagoras theorem ,

$$(\text{hypotenuse})^2 = (\text{base})^2 + (\text{perpendicular})^2$$

Here, $AB = 24$ cm, $BC = 7$ cm
We get

$$\Rightarrow (AC)^2 = (AB)^2 + (BC)^2$$

$$\Rightarrow (AC)^2 = (24)^2 + (7)^2$$

$$\Rightarrow (AC)^2 = 576 + 49$$

$\Rightarrow (AC)^2 = 625 \text{ cm}^2$

$\Rightarrow AC = 25 \text{ cm}$

Now,

$\sin\theta = \dfrac{\text{opposite side}}{\text{hypotenuse}}$

$\Rightarrow \sin A = \dfrac{BC}{AC}$

$\therefore \sin A = \dfrac{7}{25}$

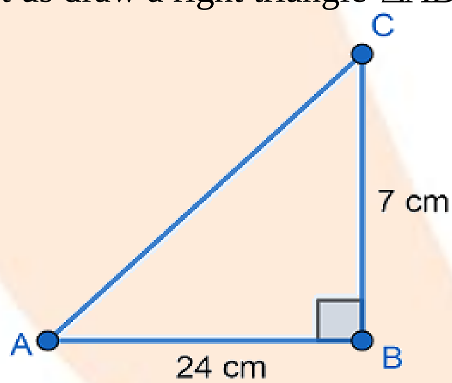
$\cos\theta = \dfrac{\text{adjacent side}}{\text{hypotenuse}}$

$\Rightarrow \cos A = \dfrac{AB}{AC}$

$\therefore \cos A = \dfrac{24}{25}$

(ii). $\sin C, \cos C$

Ans: Given that in right angle triangle ΔABC, $AB = 24$ cm, $BC = 7$ cm.
Let us draw a right triangle ΔABC, also $AB = 24$ cm, $BC = 7$ cm. We get



We have to find $\sin C, \cos C$.
We know that for right triangle

$\sin\theta = \dfrac{\text{opposite side}}{\text{hypotenuse}}$ and

$\cos\theta = \dfrac{\text{adjacent side}}{\text{hypotenuse}}$

Now, we need to apply the Pythagoras theorem to find the measure of hypotenuse.

In ΔABC, by Pythagoras theorem ,

$$(\text{hypotenuse})^2=(\text{base})^2+(\text{perpendicular})^2$$

Here, $AB=24\text{ cm},\ BC=7\text{ cm}$

We get

$$\Rightarrow (AC)^2=(AB)^2+(BC)^2$$

$$\Rightarrow (AC)^2=(24)^2+(7)^2$$

$$\Rightarrow (AC)^2=576+49$$

$$\Rightarrow (AC)^2=625\text{ cm}^2$$

$$\Rightarrow AC=25\text{ cm}$$

Now,

$$\sin\theta=\frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\Rightarrow \sin C=\frac{AB}{AC}$$

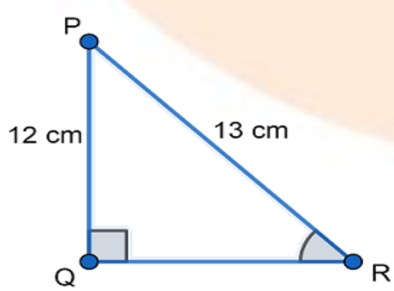
$$\therefore \sin C=\frac{24}{25}$$

$$\cos\theta=\frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\Rightarrow \cos C=\frac{BC}{AC}$$

$$\therefore \cos A=\frac{7}{25}$$

2. In the given figure find $\tan P-\cot R$.



Ans: Given in the figure,

$\text{PQ} = 12 \text{ cm}$

$\text{PQ} = 13 \text{ cm}$

We know that for right triangle

$\tan\theta = \dfrac{\text{opposite side}}{\text{adjacent side}}$ and

$\cot\theta = \dfrac{\text{adjacent side}}{\text{opposite side}}$

Now, we need to apply the Pythagoras theorem to find the measure of adjacent side/base.

In ΔPQR, by Pythagoras theorem ,

$(\text{hypotenuse})^2 = (\text{base})^2 + (\text{perpendicular})^2$

We get

$\Rightarrow (\text{PR})^2 = (\text{PQ})^2 + (\text{QR})^2$

$\Rightarrow (13)^2 = (12)^2 + (\text{QR})^2$

$\Rightarrow 169 = 144 + (\text{QR})^2$

$\Rightarrow (\text{QR})^2 = 169 - 144$

$\Rightarrow (\text{QR})^2 = 25 \text{ cm}^2$

$\Rightarrow \text{QR} = 5 \text{ cm}$

Now,

$\tan \text{P} = \dfrac{\text{opposite side}}{\text{adjacent side}}$

$\Rightarrow \tan \text{P} = \dfrac{\text{QR}}{\text{PQ}}$

$\therefore \tan \text{P} = \dfrac{5}{12}$

$\cot \text{R} = \dfrac{\text{adjacent side}}{\text{opposite side}}$

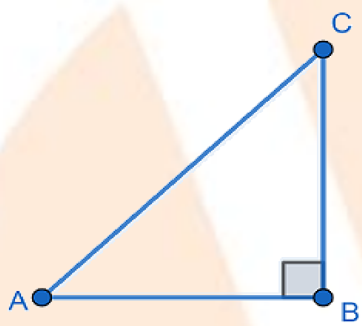
$\Rightarrow \cot \text{R} = \dfrac{\text{QR}}{\text{PQ}}$

$\therefore \cot \text{R} = \dfrac{5}{12}$

$$\Rightarrow \tan P - \cot R = \frac{5}{12} - \frac{5}{12}$$
$$\therefore \tan P - \cot R = 0$$

3. If $\sin A = \frac{3}{4}$, calculate $\cos A$ and $\tan A$.

Ans: Let us consider a right angled triangle ΔABC. We get



Given that $\sin A = \frac{3}{4}$.

We know that $\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$.

From the above figure, we get

$$\sin A = \frac{BC}{AC}$$

Therefore, we get

$\Rightarrow BC = 3$ and

$\Rightarrow AC = 4$

Now, we have to find the values of $\cos A$ and $\tan A$.

We know that $\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$ and $\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$.

Now, we need to apply the Pythagoras theorem to find the measure of adjacent side/base.

In ΔABC, by Pythagoras theorem ,

$$(\text{hypotenuse})^2 = (\text{base})^2 + (\text{perpendicular})^2$$

Here, $AC = 4$ cm, $BC = 3$ cm

We get

$$\Rightarrow (AC)^2 = (AB)^2 + (BC)^2$$
$$\Rightarrow 4^2 = AB^2 + 3^2$$

$\Rightarrow 16 = AB^2 + 9$
$\Rightarrow AB^2 = 16 - 9$
$\Rightarrow AB^2 = 7$
$\Rightarrow AB = \sqrt{7}$ cm

Now, we get

$\cos A = \dfrac{AB}{AC}$

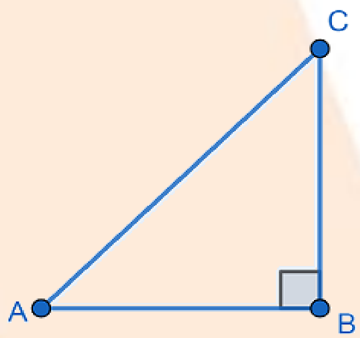
$\therefore \cos A = \dfrac{\sqrt{7}}{4}$

And $\tan A = \dfrac{BC}{AB}$

$\therefore \tan A = \dfrac{3}{\sqrt{7}}$

4. Given $15\cot A = 8$. Find $\sin A$ and $\sec A$.

Ans: Let us consider a right angled triangle ΔABC. We get



Given that $15\cot A = 8$.

We get $\cot A = \dfrac{8}{15}$.

We know that $\cot\theta = \dfrac{\text{adjacent side}}{\text{opposite side}}$.

From the above figure, we get

$\cot A = \dfrac{AB}{BC}$

Therefore, we get

$\Rightarrow BC = 15$ and
$\Rightarrow AB = 8$

Now, we have to find the values of $\sin A$ and $\sec A$.

We know that $\sin\theta=\frac{\text{opposite side}}{\text{hypotenuse}}$ and $\sec\theta=\frac{\text{hypotenuse}}{\text{adjacent side}}$.

Now, we need to apply the Pythagoras theorem to find the measure of hypotenuse.

In ΔABC, by Pythagoras theorem ,

$(\text{hypotenuse})^2=(\text{base})^2+(\text{perpendicular})^2$

We get

$\Rightarrow (AC)^2=(AB)^2+(BC)^2$

$\Rightarrow AC^2=8^2+15^2$

$\Rightarrow AC^2=64+225$

$\Rightarrow AC^2=289$

$\Rightarrow AC=17\text{ cm}$

Now, we get

$\sin A=\frac{BC}{AC}$

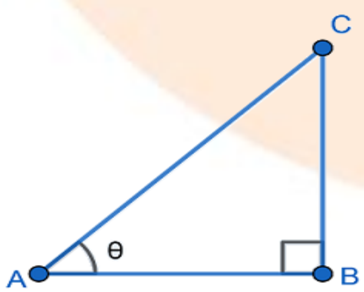
$\therefore \sin A=\frac{15}{17}$

And $\sec A=\frac{AC}{AB}$

$\therefore \sec A=\frac{17}{8}$

5. Given $\sec\theta=\frac{13}{12}$, calculate all other trigonometric ratios.

Ans: Let us consider a right angled triangle ΔABC. We get



Given that $\sec\theta=\frac{13}{12}$.

We know that $\sec\theta = \dfrac{\text{hypotenuse}}{\text{adjacent side}}$.

From the above figure, we get

$$\sec\theta = \frac{AC}{AB}$$

Therefore, we get

$\Rightarrow AC = 13$ and

$\Rightarrow AB = 12$

Now, we need to apply the Pythagoras theorem to find the measure of the perpendicular/opposite side.

In ΔABC, by Pythagoras theorem ,

$$(\text{hypotenuse})^2 = (\text{base})^2 + (\text{perpendicular})^2$$

We get

$$\Rightarrow (AC)^2 = (AB)^2 + (BC)^2$$

$$\Rightarrow 13^2 = 12^2 + BC^2$$

$$\Rightarrow 169 = 144 + BC^2$$

$$\Rightarrow BC^2 = 25$$

$$\Rightarrow BC = 5 \text{ cm}$$

Now, we know that

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

Here, $\sin\theta = \dfrac{BC}{AC}$

$$\therefore \sin\theta = \frac{5}{13}$$

We know that $\cos\theta = \dfrac{\text{adjacent side}}{\text{hypotenuse}}$

Here, $\cos\theta = \dfrac{AB}{AC}$

$$\therefore \cos\theta = \frac{12}{13}$$

We know that $\tan\theta = \dfrac{\text{opposite side}}{\text{adjacent side}}$

Here, $\tan\theta = \dfrac{BC}{AB}$

$\therefore \tan\theta = \frac{5}{12}$

We know that $\operatorname{cosec}\theta = \frac{\text{hypotenuse}}{\text{opposite side}}$

Here, $\operatorname{cosec}\theta = \frac{AC}{BC}$

$\therefore \operatorname{cosec}\theta = \frac{13}{5}$

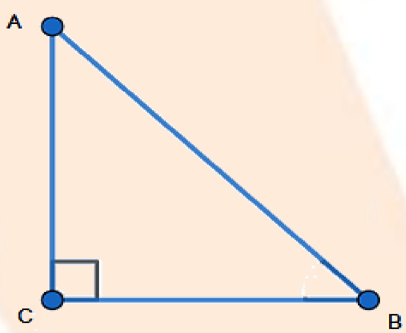
We know that $\cot\theta = \frac{\text{adjacent side}}{\text{opposite side}}$

Here, $\cot\theta = \frac{AB}{BC}$

$\therefore \cot\theta = \frac{12}{5}$.

6. If $\angle A$ and $\angle B$ are acute angles such that $\cos A = \cos B$, then show that $\angle A = \angle B$.

Ans: Let us consider a right angled triangle ΔABC. We get



Given that $\cos A = \cos B$.

In a right triangle ΔABC, we know that

$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$

Here,

$\cos A = \frac{AC}{AB}$

And $\cos B = \frac{BC}{AB}$

As given $\cos A = \cos B$, we get

$$\Rightarrow \frac{AC}{AB} = \frac{BC}{AB}$$

$$\Rightarrow AC = AB$$

Now, we know that angles opposite to the equal sides are also equal in measure.

Then, we get

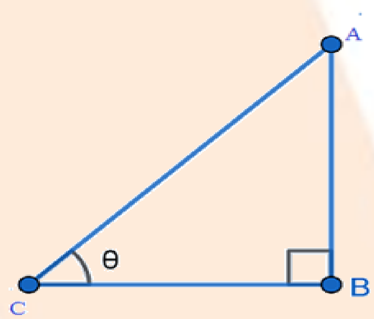
$$\angle A = \angle B$$

Hence proved.

7. Evaluate the following if $\cot\theta = \frac{7}{8}$

(i). $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

Ans: Let us consider a right angled triangle ΔABC. We get



Now, in a right triangle we know that $\cot\theta = \frac{\text{adjacent side}}{\text{opposite side}}$.

Here, from the figure $\cot\theta = \frac{BC}{AB}$.

We get

$AB = 8$ and

$BC = 7$

Now, we need to apply the Pythagoras theorem to find the measure of hypotenuse.

In ΔABC, by Pythagoras theorem ,

$$\left(\text{hypotenuse}\right)^2=\left(\text{base}\right)^2+\left(\text{perpendicular}\right)^2$$

We get

$$\Rightarrow \left(AC\right)^2=\left(AB\right)^2+\left(BC\right)^2$$

$$\Rightarrow \left(AC\right)^2=8^2+7^2$$

$$\Rightarrow \left(AC\right)^2=64+49$$

$$\Rightarrow \left(AC\right)^2=113$$

$$\Rightarrow AC=\sqrt{113}$$

Now, we know that

$$\sin\theta=\frac{\text{opposite side}}{\text{hypotenuse}}$$

Here, we get

$$\sin\theta=\frac{AB}{AC}=\frac{8}{\sqrt{113}}$$ and

$$\cos\theta=\frac{\text{adjacent side}}{\text{hypotenuse}}$$

Here, we get

$$\cos\theta=\frac{BC}{AC}=\frac{7}{\sqrt{113}}$$

Now, we have to evaluate

$$\frac{\left(1+\sin\theta\right)\left(1-\sin\theta\right)}{\left(1+\cos\theta\right)\left(1-\cos\theta\right)}$$

Applying the identity $\left(a+b\right)\left(a-b\right)=a^2-b^2$, we get

$$\Rightarrow \frac{\left(1+\sin\theta\right)\left(1-\sin\theta\right)}{\left(1+\cos\theta\right)\left(1-\cos\theta\right)}=\frac{1-\sin^2\theta}{1-\cos^2\theta}$$

Substituting the values, we get

$$\Rightarrow \frac{\left(1+\sin\theta\right)\left(1-\sin\theta\right)}{\left(1+\cos\theta\right)\left(1-\cos\theta\right)}=\frac{1-\left(\frac{8}{\sqrt{113}}\right)^2}{1-\left(\frac{7}{\sqrt{113}}\right)^2}$$

$$\Rightarrow \frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{1-\frac{64}{113}}{1-\frac{49}{113}}$$

$$\Rightarrow \frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{\frac{113-64}{113}}{\frac{113-49}{113}}$$

$$\Rightarrow \frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{\frac{49}{113}}{\frac{64}{113}}$$

$$\therefore \frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{49}{64}$$

(ii). $\cot^2\theta$

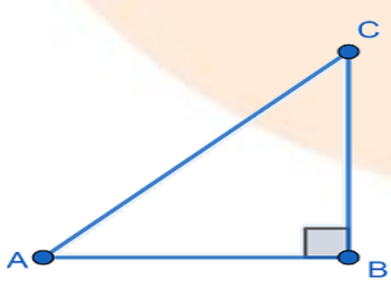
Ans: Given that $\cot\theta = \frac{7}{8}$.

Now, $\cot^2\theta = \left(\frac{7}{8}\right)^2$

$\therefore \cot^2\theta = \frac{49}{64}$

8. If $3\cot A = 4$, check whether $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ or not.

Ans: Let us consider a right angled triangle ΔABC. We get



Given that $3\cot A = 4$.

We get $\cot A = \frac{4}{3}$.

We know that $\cot\theta = \frac{\text{adjacent side}}{\text{opposite side}}$.

From the above figure, we get

$$\cot A = \frac{AB}{BC}$$

Therefore, we get

$\Rightarrow BC = 3$ and

$\Rightarrow AB = 4$

Now, we need to apply the Pythagoras theorem to find the measure of hypotenuse.

In ΔABC, by Pythagoras theorem ,

$$(\text{hypotenuse})^2 = (\text{base})^2 + (\text{perpendicular})^2$$

We get

$$\Rightarrow (AC)^2 = (AB)^2 + (BC)^2$$

$$\Rightarrow AC^2 = 4^2 + 3^2$$

$$\Rightarrow AC^2 = 16 + 9$$

$$\Rightarrow AC^2 = 25$$

$$\Rightarrow AC = 5$$

Now, let us consider LHS of the expression $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$, we get

$$LHS = \frac{1-\tan^2 A}{1+\tan^2 A}$$

Now, we know that $\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$

Here, we get

$$\tan A = \frac{BC}{AB} = \frac{3}{4}$$

Substitute the value, we get

$$\Rightarrow \frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2}$$

$$\Rightarrow \frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-\frac{9}{16}}{1+\frac{9}{16}}$$

$$\Rightarrow \frac{1-\tan^2 A}{1+\tan^2 A} = \frac{\frac{16-9}{16}}{\frac{16+9}{16}}$$

$$\Rightarrow \frac{1-\tan^2 A}{1+\tan^2 A} = \frac{\frac{7}{16}}{\frac{25}{16}}$$

$$\Rightarrow \frac{1-\tan^2 A}{1+\tan^2 A} = \frac{7}{25}$$

Now, let us consider RHS of the expression $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$, we get

$\text{RHS} = \cos^2 A - \sin^2 A$

We know that $\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$ and $\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$.

Here, we get

$\sin A = \frac{BC}{AC} = \frac{3}{5}$

And $\cos A = \frac{AB}{AC} = \frac{4}{5}$

Substitute the values, we get

$$\Rightarrow \cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$$

$$\Rightarrow \cos^2 A - \sin^2 A = \frac{16}{25} - \frac{9}{25}$$

$$\Rightarrow \cos^2 A - \sin^2 A = \frac{7}{25}$$

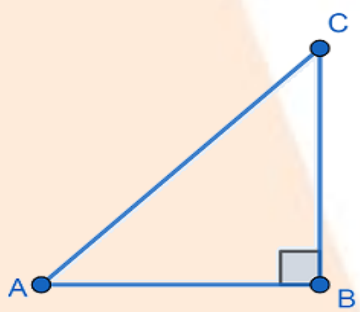
Hence, we get LHS=RHS

$$\therefore \frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A .$$

9. In ABC, right angled at B. If $\tan A = \frac{1}{\sqrt{3}}$, find the value of

(i). $\sin A \cos C + \cos A \sin C$

Ans: Let us consider a right angled triangle ΔABC. We get



Given that $\tan A = \frac{1}{\sqrt{3}}$.

In a right triangle, we know that $\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$

Here, from the figure we get

$$\tan A = \frac{BC}{AB} = \frac{1}{\sqrt{3}}$$

We get $BC = 1$ and $AB = \sqrt{3}$.

Now, we need to apply the Pythagoras theorem to find the measure of hypotenuse.

In ΔABC, by Pythagoras theorem ,

$$(\text{hypotenuse})^2 = (\text{base})^2 + (\text{perpendicular})^2$$

We get

$$\Rightarrow (AC)^2 = (AB)^2 + (BC)^2$$
$$\Rightarrow AC^2 = \left(\sqrt{3}\right)^2 + 1^2$$
$$\Rightarrow AC^2 = 3 + 1$$
$$\Rightarrow AC^2 = 4$$
$$\Rightarrow AC = 2$$

We know that $\sin\theta = \dfrac{\text{opposite side}}{\text{hypotenuse}}$ and $\cos\theta = \dfrac{\text{adjacent side}}{\text{hypotenuse}}$.

Here, we get

$\sin A = \dfrac{BC}{AC} = \dfrac{1}{2}$ and $\sin C = \dfrac{AB}{AC} = \dfrac{\sqrt{3}}{2}$

And $\cos A = \dfrac{AB}{AC} = \dfrac{\sqrt{3}}{2}$ and $\cos C = \dfrac{BC}{AC} = \dfrac{1}{2}$

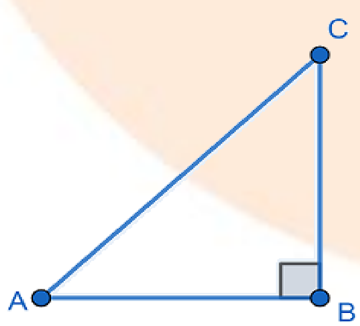
Now, we have to find the value of the expression $\sin A\cos C + \cos A\sin C$.

Substituting the values we get

$$\Rightarrow \sin A\cos C + \cos A\sin C = \frac{1}{2}\times\frac{1}{2} + \frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2}$$
$$\Rightarrow \sin A\cos C + \cos A\sin C = \frac{1}{4} + \frac{3}{4}$$
$$\Rightarrow \sin A\cos C + \cos A\sin C = \frac{4}{4}$$
$$\therefore \sin A\cos C + \cos A\sin C = 1$$

(ii). $\mathbf{\cos A\cos C - \sin A\sin C}$

Ans: Let us consider a right angled triangle ΔABC. We get



Given that $\tan A = \dfrac{1}{\sqrt{3}}$.

In a right triangle, we know that $\tan\theta = \dfrac{\text{opposite side}}{\text{adjacent side}}$

Here, from the figure we get

$$\tan A = \frac{BC}{AB} = \frac{1}{\sqrt{3}}$$

We get $BC = 1$ and $AB = \sqrt{3}$.

Now, we need to apply the Pythagoras theorem to find the measure of hypotenuse.

In ΔABC, by Pythagoras theorem ,

$$(\text{hypotenuse})^2 = (\text{base})^2 + (\text{perpendicular})^2$$

We get

$$\Rightarrow (AC)^2 = (AB)^2 + (BC)^2$$

$$\Rightarrow AC^2 = \left(\sqrt{3}\right)^2 + 1^2$$

$$\Rightarrow AC^2 = 3 + 1$$

$$\Rightarrow AC^2 = 4$$

$$\Rightarrow AC = 2$$

We know that $\sin\theta = \dfrac{\text{opposite side}}{\text{hypotenuse}}$ and $\cos\theta = \dfrac{\text{adjacent side}}{\text{hypotenuse}}$.

Here, we get

$$\sin A = \frac{BC}{AC} = \frac{1}{2} \text{ and } \sin C = \frac{AB}{AC} = \frac{\sqrt{3}}{2}$$

And $\cos A = \dfrac{AB}{AC} = \dfrac{\sqrt{3}}{2}$ and $\cos C = \dfrac{BC}{AC} = \dfrac{1}{2}$

Now, we have to find the value of the expression $\cos A \cos C - \sin A \sin C$.

Substituting the values we get

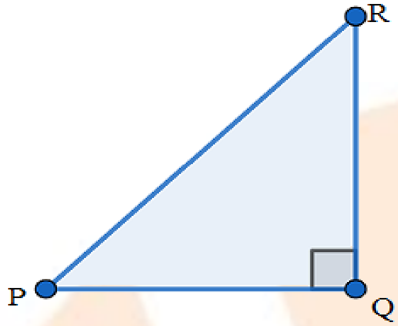
$$\Rightarrow \cos A \cos C - \sin A \sin C = \frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{\sqrt{3}}{2}$$

$$\Rightarrow \cos A \cos C - \sin A \sin C = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4}$$

$$\therefore \Rightarrow \cos A \cos C - \sin A \sin C = 0$$

10. In ΔPQR, right angled at Q, $PR + QR = 25$ cm and $PQ = 5$ cm. Determine the values of $\sin P, \cos P$ and $\tan P$.

Ans: Let us consider a right angled triangle ΔPQR, we get



Given that $PR + QR = 25\text{ cm}$ and $PQ = 5\text{ cm}$.

Let $QR = 25 - PR$

Now, applying the Pythagoras theorem in ΔPQR, we get

$(\text{hypotenuse})^2 = (\text{base})^2 + (\text{perpendicular})^2$

We get

$\Rightarrow (PR)^2 = (PQ)^2 + (QR)^2$

$\Rightarrow PR^2 = 5^2 + (25 - PR)^2$

$\Rightarrow PR^2 = 25 + 25^2 + PR^2 - 50PR$

$\Rightarrow PR^2 = PR^2 + 25 + 625 - 50PR$

$\Rightarrow 50PR = 650$

$\Rightarrow PR = 13\text{ cm}$

Therefore,

$QR = 25 - 13$

$\Rightarrow QR = 12\text{ cm}$

Now, we know that in right triangle,

$\sin\theta = \dfrac{\text{opposite side}}{\text{hypotenuse}}$, $\cos\theta = \dfrac{\text{adjacent side}}{\text{hypotenuse}}$ and $\tan\theta = \dfrac{\text{opposite side}}{\text{adjacent side}}$.

Here, we get

$\sin P = \dfrac{QR}{PR}$

$\therefore \sin P = \dfrac{12}{13}$

$\cos P = \dfrac{PQ}{PR}$

$\therefore \cos P = \frac{5}{13}$

$\tan P = \frac{QR}{PQ}$

$\therefore \tan P = \frac{12}{5}$

11. State whether the following are true or false. Justify your answer.

(i) The value of $\tan A$ is always less than 1.

Ans: The given statement is false. The value of $\tan A$ depends on the length of sides of a right triangle and sides of a triangle may have any measure.

(ii) For some value of angle A, $\sec A = \frac{12}{5}$.

Ans: We know that in right triangle $\sec A = \frac{\text{hypotenuse}}{\text{adjacent side of } \angle A}$.

We know that in right triangle hypotenuse is the largest side.
Therefore, the value of $\sec A$ must be greater than 1.

In the given statement $\sec A = \frac{12}{5}$, which is greater than 1.

Therefore, the given statement is true.

(iii) $\cos A$ is the abbreviation used for the cosecant of angle A.

Ans: The given statement is false because $\cos A$ is the abbreviation used for the cosine of angle A. Abbreviation used for the cosecant of angle A is $\operatorname{cosec} A$.

(iv) $\cot A$ is the product of $\cot$ and A.

Ans: $\cot A$ is the abbreviation used for the cotangent of angle A. Hence the given statement is false.

(v) For some angle θ, $\sin\theta = \frac{4}{3}$.

Ans: We know that in right triangle $\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$.

We know that in right triangle hypotenuse is the largest side.
Therefore, the value of $\sin\theta$ must be less than 1.

In the given statement $\sin\theta = \frac{4}{3}$, which is greater than 1.

Therefore, the given statement is false.

Exercise 8.2

1. Evaluate the following:

(i) $\sin 60° \cos 30° + \sin 30° \cos 60°$

Ans: With the help of trigonometric ratio table we can find the values of standard trigonometric angles. The trigonometric ratio table is as follows:

Exact Values of Trigonometric Functions				
Angle θ		$\sin\theta$	$\cos\theta$	$\tan\theta$
Degrees	Radians			
$0°$	0	0	1	0
$30°$	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
$45°$	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
$60°$	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$90°$	$\frac{\pi}{2}$	1	0	Not defined

We have to evaluate $\sin 60° \cos 30° + \sin 30° \cos 60°$.

Substitute the values from the above table, we get

$$\Rightarrow \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2}$$

$\Rightarrow \frac{3}{4}+\frac{1}{4}$

$\Rightarrow \frac{4}{4}$

$\therefore \sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ = 1$.

(ii) $2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

Ans: With the help of trigonometric ratio table we can find the values of standard trigonometric angles. The trigonometric ratio table is as follows:

Exact Values of Trigonometric Functions				
Angle θ		$\sin\theta$	$\cos\theta$	$\tan\theta$
Degrees	Radians			
0°	0	0	1	0
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	$\frac{\pi}{2}$	1	0	Not defined

We have to evaluate $2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$.
Substitute the values from the above table, we get

$\Rightarrow 2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$

$\Rightarrow 2+\frac{3}{4}-\frac{3}{4}$

$\Rightarrow 2$

$\therefore 2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ = 2$.

(iii) $\dfrac{\cos 45°}{\sec 30° + \operatorname{cosec} 30°}$

Ans: With the help of trigonometric ratio table we can find the values of standard trigonometric angles. The trigonometric ratio table is as follows:

Exact Values of Trigonometric Functions				
Angle θ		$\sin\theta$	$\cos\theta$	$\tan\theta$
Degrees	Radians			
$0°$	0	0	1	0
$30°$	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
$45°$	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
$60°$	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$90°$	$\frac{\pi}{2}$	1	0	Not defined

We have to evaluate $\dfrac{\cos 45°}{\sec 30° + \operatorname{cosec} 30°}$.

Substitute the values from the above table, we get

$$\Rightarrow \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}}+2}$$

$$\Rightarrow \frac{\frac{1}{\sqrt{2}}}{\frac{2+2\sqrt{3}}{\sqrt{3}}}$$

$$\Rightarrow \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2+2\sqrt{3}}$$

Multiplying and dividing by $\sqrt{3}-1$, we get

$$\Rightarrow \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2+2\sqrt{3}} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$\Rightarrow \frac{\sqrt{3}\left(\sqrt{3}-1\right)}{\sqrt{2}\left(2+2\sqrt{3}\right)\left(\sqrt{3}-1\right)}$$

$$\Rightarrow \frac{\sqrt{3}\left(\sqrt{3}-1\right)}{2\sqrt{2}\left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right)}$$

$$\Rightarrow \frac{3-\sqrt{3}}{2\sqrt{2}\left(\left(\sqrt{3}\right)^{2}-1^{2}\right)}$$

$$\Rightarrow \frac{3-\sqrt{3}}{2\sqrt{2}\left(3-1\right)}$$

$$\Rightarrow \frac{3-\sqrt{3}}{4\sqrt{2}}$$

$$\therefore \frac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ} = \frac{3-\sqrt{3}}{4\sqrt{2}}$$

(iii) $\mathbf{\dfrac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ - \cot 45^\circ}}$

Ans: With the help of trigonometric ratio table we can find the values of standard trigonometric angles. The trigonometric ratio table is as follows:

Exact Values of Trigonometric Functions				
Angle θ		$\sin\theta$	$\cos\theta$	$\tan\theta$
Degrees	Radians			
0°	0	0	1	0
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1

60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	$\frac{\pi}{2}$	1	0	Not defi ned

We have to evaluate $\frac{\sin 30° + \tan 45° - \operatorname{cosec} 60°}{\sec 30° + \cos 60° - \cot 45°}$.

Substitute the values from the above table, we get

$$\Rightarrow \frac{\frac{1}{2}+1-\frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}}+\frac{1}{2}+1}$$

$$\Rightarrow \frac{\frac{3}{2}-\frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}}+\frac{3}{2}}$$

$$\Rightarrow \frac{\frac{3\sqrt{3}-4}{2\sqrt{3}}}{\frac{3\sqrt{3}+4}{2\sqrt{3}}}$$

$$\Rightarrow \frac{3\sqrt{3}-4}{3\sqrt{3}+4}$$

Multiplying and dividing by $3\sqrt{3}-4$, we get

$$\Rightarrow \frac{3\sqrt{3}-4}{3\sqrt{3}+4} \times \frac{3\sqrt{3}-4}{3\sqrt{3}-4}$$

Now, applying the identity $(a+b)(a-b) = a^2 - b^2$, we get

$$\Rightarrow \frac{\left(3\sqrt{3}-4\right)^2}{\left(3\sqrt{3}\right)^2 - 4^2}$$

$$\Rightarrow \frac{\left(3\sqrt{3}-4\right)^2}{\left(3\sqrt{3}\right)^2-4^2}$$

$$\Rightarrow \frac{27+16-24\sqrt{3}}{27-16}$$

$$\Rightarrow \frac{43-24\sqrt{3}}{11}$$

$$\therefore \frac{\sin 30^\circ+\tan 45^\circ-\operatorname{cosec} 60^\circ}{\sec 30^\circ+\cos 60^\circ-\cot 45^\circ}=\frac{43-24\sqrt{3}}{11}$$

(iv) $\mathbf{\dfrac{5\cos^2 60^\circ+4\sec^2 30^\circ-\tan^2 45^\circ}{\sec^2 30^\circ+\cos^2 30^\circ}}$

Ans: With the help of trigonometric ratio table we can find the values of standard trigonometric angles. The trigonometric ratio table is as follows:

Exact Values of Trigonometric Functions				
Angle θ		$\sin\theta$	$\cos\theta$	$\tan\theta$
Degrees	Radians			
0°	0	0	1	0
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	$\frac{\pi}{2}$	1	0	Not defined

We have to evaluate $\dfrac{5\cos^2 60^\circ+4\sec^2 30^\circ-\tan^2 45^\circ}{\sec^2 30^\circ+\cos^2 30^\circ}$.

Substitute the values from the above table, we get

$$\Rightarrow \frac{5\left(\frac{1}{2}\right)^2+4\left(\frac{2}{\sqrt{3}}\right)^2-1^2}{\left(\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$\Rightarrow \frac{5\left(\frac{1}{4}\right)+4\left(\frac{4}{3}\right)-1}{\left(\frac{1}{4}\right)+\left(\frac{3}{4}\right)}$$

$$\Rightarrow \frac{\frac{15+64-12}{12}}{\frac{1+3}{4}}$$

$$\Rightarrow \frac{\frac{15+64-12}{12}}{\frac{1+3}{4}}$$

$$\Rightarrow \frac{\frac{15+64-12}{12}}{\frac{4}{4}}$$

$$\Rightarrow \frac{\frac{67}{12}}{1}$$

$$\therefore \frac{5\cos^2 60°+4\sec^2 30°-\tan^2 45°}{\sec^2 30°+\cos^2 30°}=\frac{67}{12}.$$

2. Choose the correct option and justify your choice.

(i) $\frac{2\tan 30°}{1+\tan^2 30°}=$ ....

(A) $\sin 60°$

(B) $\cos 60°$

(C) $\tan 60°$

(D) $\sin 30°$

Ans: The given expression is $\frac{2\tan 30°}{1+\tan^2 30°}$.

We know that from the trigonometric ratio table we have $\tan 30^\circ = \frac{1}{\sqrt{3}}$.

Substitute the value in the given expression we get

$$\frac{2\tan 30^\circ}{1+\tan^2 30^\circ} = \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1+\left(\frac{1}{\sqrt{3}}\right)^2}$$

$$\Rightarrow \frac{2\tan 30^\circ}{1+\tan^2 30^\circ} = \frac{\frac{2}{\sqrt{3}}}{1+\frac{1}{3}}$$

$$\Rightarrow \frac{2\tan 30^\circ}{1+\tan^2 30^\circ} = \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}}$$

$$\Rightarrow \frac{2\tan 30^\circ}{1+\tan^2 30^\circ} = \frac{\sqrt{3}}{2}$$

From the trigonometric table we know that

$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\cos 60^\circ = \frac{1}{2}$$

$$\tan 60^\circ = \sqrt{3}$$

$$\sin 30^\circ = \frac{1}{2}$$

Hence, $\frac{2\tan 30^\circ}{1+\tan^2 30^\circ} = \sin 60^\circ$.

Therefore, option (A) is the correct answer.

(ii) $\frac{1-\tan^2 45°}{1+\tan^2 45°}=$ ....

(A) $\tan 90°$

(B) 1

(C) $\sin 45°$

(D) 0

Ans: The given expression is $\frac{1-\tan^2 45°}{1+\tan^2 45°}$.

We know that from the trigonometric ratio table we have $\tan 45° = 1$.

Substitute the value in the given expression we get

$$\frac{1-\tan^2 45°}{1+\tan^2 45°}=\frac{1-1^2}{1+1^2}$$

$$\Rightarrow \frac{1-\tan^2 45°}{1+\tan^2 45°}=\frac{1-1}{1+1}$$

$$\Rightarrow\Rightarrow \frac{1-\tan^2 45°}{1+\tan^2 45°}=\frac{0}{2}$$

$$\therefore \frac{1-\tan^2 45°}{1+\tan^2 45°}=0$$

Therefore, option (D) is the correct answer.

(ii) $\sin 2A = 2\sin A$ is true when $A =$ ....

(A) $0°$

(B) $30°$

(C) $45°$

(D) $60°$

Ans: The given expression is $\sin 2A = 2\sin A$.

We know that from the trigonometric ratio table we have

$$\sin 0° = 0$$

$$\sin 30° = \frac{1}{2}$$

$$\sin 45° = \frac{1}{\sqrt{2}}$$

$$\sin 60° = \frac{\sqrt{3}}{2}$$

$$\sin 90° = 1$$

The given statement is true when $A = 0°$.

Substitute the value in the given expression we get

$\Rightarrow \sin 2A = 2\sin A$

$\Rightarrow \sin 2\times 0° = 2\sin 0°$

$0 = 0$

Therefore, option (A) is the correct answer.

(iii) $\mathbf{\dfrac{2\tan 30°}{1-\tan^2 30°} = ...}$

(A) $\mathbf{\sin 60°}$

(B) $\mathbf{\cos 60°}$

(C) $\mathbf{\tan 60°}$

(D) $\mathbf{\sin 30°}$

Ans: The given expression is $\dfrac{2\tan 30°}{1-\tan^2 30°}$.

We know that from the trigonometric ratio table we have $\tan 30° = \dfrac{1}{\sqrt{3}}$.

Substitute the value in the given expression we get

$$\frac{2\tan 30°}{1-\tan^2 30°} = \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1-\left(\frac{1}{\sqrt{3}}\right)^2}$$

$$\Rightarrow \frac{2\tan 30°}{1-\tan^2 30°} = \frac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}}$$

$$\Rightarrow \frac{2\tan 30°}{1-\tan^2 30°} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}}$$

$$\Rightarrow \frac{2\tan 30°}{1-\tan^2 30°} = \sqrt{3}$$

From the trigonometric table we know that

$\sin 60° = \dfrac{\sqrt{3}}{2}$

$\cos 60^\circ = \frac{1}{2}$

$\tan 60^\circ = \sqrt{3}$

$\sin 30^\circ = \frac{1}{2}$

Hence, $\frac{2\tan 30^\circ}{1-\tan^2 30^\circ} = \tan 60^\circ$.

Therefore, option (C) is the correct answer.

3. If $\tan(A+B)=\sqrt{3}$ and $\tan(A-B)=\frac{1}{\sqrt{3}}$, $0^\circ < A+B \le 90^\circ$. Find A and B.

Ans: Given that $\tan(A+B)=\sqrt{3}$ and $\tan(A-B)=\frac{1}{\sqrt{3}}$.

From the trigonometric ratio table we know that $\tan 60^\circ = \sqrt{3}$ and $\tan 30^\circ = \frac{1}{\sqrt{3}}$

Then we get

$\tan(A+B)=\sqrt{3}$

$\Rightarrow \tan(A+B)=\tan 60^\circ$

$\Rightarrow A+B=60^\circ$ ……….(1)

Also, $\tan(A-B)=\frac{1}{\sqrt{3}}$

$\Rightarrow \tan(A-B)=\tan 30^\circ$

$\Rightarrow A-B=30^\circ$ ……….(2)

Adding eq. (1) and (2), we get

$2A=90^\circ$

$\therefore A=45^\circ$

Substitute the obtained value in eq. (1), we get

$45^\circ + B = 60^\circ$

$\Rightarrow B = 60^\circ - 45^\circ$

$\therefore B = 15^\circ$

Therefore, the values of A and B is 45° and 15° respectively.

4. State whether the following are true or false. Justify your answer.
(i) $\sin(A+B) = \sin A + \sin B$.
Ans: Let us assume $A = 30^\circ$ and $B = 60^\circ$.
Now, let us consider LHS of the given expression, we get
$\sin(A+B)$
Substitute the assumed values in the LHS, we get
$\sin(A+B) = \sin(30^\circ + 60^\circ)$
$\Rightarrow \sin(A+B) = \sin(90^\circ)$
From the trigonometric ratio table we know that $\sin 90^\circ = 1$, we get
$\Rightarrow \sin(A+B) = 1$
Now, let us consider the RHS of the given expression and substitute the values, we get
$\sin A + \sin B = \sin 30^\circ + \sin 60^\circ$
From the trigonometric ratio table we know that $\sin 30^\circ = \frac{1}{2}$ and $\sin 60^\circ = \frac{\sqrt{3}}{2}$, we get
$\Rightarrow \sin A + \sin B = \frac{1}{2} + \frac{\sqrt{3}}{2}$
$\Rightarrow \sin A + \sin B = \frac{1+\sqrt{3}}{2}$
Thus, $\text{LHS} \neq \text{RHS}$.
Therefore, the given statement is false.

(ii) The value of $\sin\theta$ increases as θ increases.
Ans: The value of sine from the trigonometric ratio table is as follows:
$\sin 0^\circ = 0$
$\sin 30^\circ = \frac{1}{2} = 0.5$

$\sin 45^\circ = \dfrac{1}{\sqrt{2}} = 0.707$

$\sin 60^\circ = \dfrac{\sqrt{3}}{2} = 0.866$

$\sin 90^\circ = 1$

Therefore, we can conclude that the value of $\sin\theta$ increases as θ increases.
Therefore, the given statement is true.

(iii) The value of $\cos\theta$ increases as θ increases.

Ans: The value of cosine from the trigonometric ratio table is as follows:

$\cos 0^\circ = 1$

$\cos 30^\circ = \dfrac{\sqrt{3}}{2} = 0.866$

$\cos 45^\circ = \dfrac{1}{\sqrt{2}} = 0.707$

$\cos 60^\circ = \dfrac{1}{2} = 0.5$

$\cos 90^\circ = 0$

Therefore, we can conclude that the value of $\cos\theta$ decreases as θ increases.
Therefore, the given statement is false.

(iv) $\sin\theta = \cos\theta$ for all values of θ.

Ans: The trigonometric ratio table is given as follows:

Exact Values of Trigonometric Functions				
Angle θ		$\sin\theta$	$\cos\theta$	$\tan\theta$
Degrees	Radians			
0°	0	0	1	0
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

$90°$	$\frac{\pi}{2}$	1	0	Not defi ned

From the above table we can conclude that $\sin\theta = \cos\theta$ is true only for $\theta = 45°$

$\sin\theta = \cos\theta$ is not true for all values of θ.

Therefore, the given statement is false.

(iv) $\cot A$ is not defined for $A = 0°$.

Ans: We know that $\cot A = \dfrac{\cos A}{\sin A}$.

If $A = 0°$, then $\cot 0° = \dfrac{\cos 0°}{\sin 0°}$

From trigonometric ratio table we get

$\sin 0° = 0$ and $\cos 0° = 1$

We get

$\cot 0° = \dfrac{1}{0}$, which is undefined.

Therefore, the given statement is true.

Exercise 8.3

1. Evaluate the following:

(i) $\dfrac{\sin 18°}{\cos 72°}$

Ans: The given expression is $\dfrac{\sin 18°}{\cos 72°}$.

The given expression can be written as $\dfrac{\sin(90° - 72°)}{\cos 72°}$.

Now, we can apply the identity $\sin(90° - \theta) = \cos\theta$, we get

$$\frac{\sin 18°}{\cos 72°} = \frac{\sin(90° - 72°)}{\cos 72°}$$

$$\Rightarrow \frac{\sin 18°}{\cos 72°} = \frac{\cos 72°}{\cos 72°}$$

$$\therefore \frac{\sin 18°}{\cos 72°} = 1$$

(ii) $\dfrac{\tan 26°}{\cot 64°}$

Ans: The given expression is $\dfrac{\tan 26°}{\cot 64°}$.

The given expression can be written as $\dfrac{\tan(90° - 64°)}{\cot 64°}$.

Now, we can apply the identity $\tan(90° - \theta) = \cot\theta$, we get

$$\frac{\tan 26°}{\cot 64°} = \frac{\tan(90° - 64°)}{\cot 64°}$$

$$\Rightarrow \frac{\tan 26°}{\cot 64°} = \frac{\cot 64°}{\cot 64°}$$

$$\therefore \frac{\tan 26°}{\cot 64°} = 1$$

(iii) $\cos 48° - \sin 42°$

Ans: The given expression is $\cos 48° - \sin 42°$.

The given expression can be written as $\cos(90° - 42°) - \sin 42°$.

Now, we can apply the identity $\cos(90° - \theta) = \sin\theta$, we get

$$\cos 48° - \sin 42° = \cos(90° - 42°) - \sin 42°$$

$$\Rightarrow \cos 48° - \sin 42° = \sin 42° - \sin 42°$$

$$\therefore \cos 48° - \sin 42° = 0$$

(iv) $\operatorname{cosec} 31° - \sec 59°$

Ans: The given expression is $\operatorname{cosec} 31° - \sec 59°$.

The given expression can be written as $\operatorname{cosec}(90° - 59°) - \sec 59°$.

Now, we can apply the identity $\operatorname{cosec}(90° - \theta) = \sec\theta$, we get

$$\operatorname{cosec} 31° - \sec 59° = \operatorname{cosec}(90° - 59°) - \sec 59°$$

$$\Rightarrow \operatorname{cosec} 31° - \sec 59° = \sec 59° - \sec 59°$$

$$\therefore \operatorname{cosec} 31° - \sec 59° = 0$$

2. Show that

(i) $\tan 48° \tan 23° \tan 42° \tan 67° = 1$

Ans: The given expression is $\tan 48° \tan 23° \tan 42° \tan 67° = 1$.

Let us consider LHS of the given expression, we get

$\tan 48° \tan 23° \tan 42° \tan 67°$

The above expression can be written as

$\Rightarrow \tan(90° - 42°)\tan(90° - 67°)\tan 42° \tan 67°$

Now, we can apply the identity $\tan(90° - \theta) = \cot\theta$, we get

$\Rightarrow \cot 42° \cot 67° \tan 42° \tan 67°$

Now, we know that $\cot A = \dfrac{1}{\tan A}$, we get

$\Rightarrow \dfrac{1}{\tan 42° \tan 67°} \times \tan 42° \tan 67°$

$\Rightarrow 1$

$\Rightarrow \text{RHS}$

$\therefore \tan 48° \tan 23° \tan 42° \tan 67° = 1$

(ii) $\cos 38° \cos 52° - \sin 38° \sin 52° = 0$

Ans: The given expression is $\cos 38° \cos 52° - \sin 38° \sin 52° = 0$.

Let us consider LHS of the given expression, we get

$\cos 38° \cos 52° - \sin 38° \sin 52°$

The above expression can be written as

$\Rightarrow \cos(90° - 52°)\cos(90° - 38°) - \sin 38° \sin 52°$

Now, we can apply the identity $\cos(90° - \theta) = \sin\theta$, we get

$\Rightarrow \sin 52° \sin 38° - \sin 38° \sin 52°$

$\Rightarrow 0$

$\Rightarrow \text{RHS}$

$\therefore \cos 38° \cos 52° - \sin 38° \sin 52° = 0$

3. Find the value of A, if $\tan 2A = \cot(A - 18°)$, where $2A$ is an acute angle.

Ans: Given $\tan 2A = \cot(A - 18°)$ ....(1)

Now, we know that $\cot(90° - \theta) = \tan\theta$.

Here, we can write $\tan 2A = \cot(90° - 2A)$

Substitute the value in eq. (1), we get

$\Rightarrow \cot(90° - 2A) = \cot(A - 18°)$

Equating both angles, we get

$\Rightarrow (90° - 2A) = (A - 18°)$

$\Rightarrow 90° + 18° = A + 2A$

$\Rightarrow 108° = 3A$

$\Rightarrow 3A = 108°$

$\therefore A = 36°$

4. Prove that $\mathbf{A + B = 90°}$, if $\mathbf{\tan A = \cot B}$.

Ans: Given that $\mathbf{\tan A = \cot B}$.

Now, substitute $\tan A = \cot(90° - A)$ in the given expression, we get

$\Rightarrow \cot(90° - A) = \cot B$

Equating both angles, we get

$\Rightarrow (90° - A) = B$

$\Rightarrow 90° = B + A$

$\therefore A + B = 90°$

Hence proved

5. Find the value of $\mathbf{A}$, if $\mathbf{\sec 4A = \operatorname{cosec}(A - 20°)}$, where $\mathbf{4A}$ is an acute angle.

Ans: Given $\sec 4A = \operatorname{cosec}(A - 20°)$ ....(1)

Now, we know that $\operatorname{cosec}(90° - \theta) = \sec\theta$.

Here, we can write $\sec 4A = \operatorname{cosec}(90° - 4A)$

Substitute the value in eq. (1), we get

$\Rightarrow \operatorname{cosec}(90° - 4A) = \operatorname{cosec}(A - 20°)$

Equating both angles, we get

$\Rightarrow (90° - 4A) = (A - 20°)$

$\Rightarrow 90° + 20° = A + 4A$

$\Rightarrow 110° = 5A$

$\Rightarrow 5A = 110°$

$\therefore A = 22°$

6. If $\mathbf{A, B}$ and $\mathbf{C}$ are interior angles of a triangle $\mathbf{ABC}$, then show that $\mathbf{\sin\left(\frac{B + C}{2}\right) = \cos\frac{A}{2}}$.

Ans: Given that A, B and C are interior angles of a triangle ABC.
We know that sum of interior angles of a triangle is always $180°$.
Then, we get
$\Rightarrow \angle A + \angle B + \angle C = 180°$
$\Rightarrow \angle B + \angle C = 180° - \angle A$
Now, divide both sides of the equation by 2, we get
$$\Rightarrow \frac{\angle B + \angle C}{2} = \frac{180° - \angle A}{2}$$
$$\Rightarrow \frac{\angle B + \angle C}{2} = 90° - \frac{\angle A}{2}$$
Applying the sine function to the both sides of the equation, we get
$$\Rightarrow \sin\left(\frac{\angle B + \angle C}{2}\right) = \sin\left(90° - \frac{\angle A}{2}\right)$$
Now, we know that $\sin(90° - \theta) = \cos\theta$.
$$\therefore \sin\left(\frac{B + C}{2}\right) = \cos\frac{A}{2}.$$
Hence proved

7. Express $\sin 67° + \cos 75°$ in terms of trigonometric ratios of angles between $0°$ and $45°$.
Ans: Given expression $\sin 67° + \cos 75°$.
Now, we know that $\cos(90° - \theta) = \sin\theta$.
The given expression can be written as
$\sin 67° + \cos 75° = \cos(90° - 23°) + \cos(90° - 15°)$
$\therefore \sin 67° + \cos 75° = \cos 23° + \cos 15°$
Therefore, we get the expression in terms of trigonometric ratios of angles between $0°$ and $45°$.

Exercise 8.4
1. Express the trigonometric ratios $\sin A, \sec A$ and $\tan A$ in terms of $\cot A$
Ans: For a right triangle we have an identity $\operatorname{cosec}^2 A = 1 + \cot^2 A$.
Let us consider the above identity, we get
$\operatorname{cosec}^2 A = 1 + \cot^2 A$
Now, reciprocating both sides we get
$$\Rightarrow \frac{1}{\operatorname{cosec}^2 A} = \frac{1}{1 + \cot^2 A}$$

Now, we know that $\frac{1}{\operatorname{cosec}^2 A} = \sin^2 A$, we get

$$\Rightarrow \sin^2 A = \frac{1}{1+\cot^2 A}$$

$$\Rightarrow \sin A = \pm\frac{1}{\sqrt{1+\cot^2 A}}$$

Now, we know that sine value will be negative for angles greater than $180°$, for a triangle sine value is always positive with respect to an angle. Then we will consider only positive value.

$$\therefore \sin A = \frac{1}{\sqrt{1+\cot^2 A}}$$

We know that $\tan A = \frac{1}{\cot A}$

Also, we will use the identity $\sec^2 A = 1+\tan^2 A$, we get

$$\sec^2 A = 1+\tan^2 A$$

$$\Rightarrow \sec^2 A = 1+\frac{1}{\cot^2 A}$$

$$\Rightarrow \sec^2 A = \frac{\cot^2 A+1}{\cot^2 A}$$

$$\Rightarrow \sec A = \frac{\sqrt{\cot^2 A+1}}{\sqrt{\cot^2 A}}$$

$$\therefore \sec A = \frac{\sqrt{\cot^2 A+1}}{\cot A}$$

2. Write all the other trigonometric ratios of $\angle A$ in terms of $\sec A$.

Ans: We know that $\cos A = \frac{1}{\sec A}$.

$$\therefore \cos A = \frac{1}{\sec A}$$

For a right triangle we have an identity $\sin^2 A + \cos^2 A = 1$.

Let us consider the above identity, we get

$$\sin^2 A + \cos^2 A = 1$$

Now, we know that $\cos A = \frac{1}{\sec A}$, we get

$$\Rightarrow \sin^2 A = 1 - \cos^2 A$$

$$\Rightarrow \sin^2 A = 1 - \frac{1}{\sec^2 A}$$

$$\Rightarrow \sin A = \sqrt{1 - \left(\frac{1}{\sec A}\right)^2}$$

$$\therefore \sin A = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$$

Also, we will use the identity $\sec^2 A = 1 + \tan^2 A$, we get

$$\tan^2 A = \sec^2 A - 1$$

$$\therefore \tan A = \sqrt{\sec^2 A - 1}$$

Now, we know that $\cot A = \frac{\cos A}{\sin A}$, we get

$$\Rightarrow \cot A = \frac{\frac{1}{\sec A}}{\frac{\sqrt{\sec^2 A - 1}}{\sec A}}$$

$$\therefore \cot A = \frac{1}{\sqrt{\sec^2 A - 1}}$$

We know that $\operatorname{cosec} A = \frac{1}{\sin A}$, we get

$$\therefore \operatorname{cosec} A = \frac{\sec A}{\sqrt{\sec^2 A - 1}}$$

3. Evaluate the following:

(i) $\mathbf{\dfrac{\sin^2 63° + \sin^2 27°}{\cos^2 17° + \cos^2 73°}}$

Ans: The given expression is $\dfrac{\sin^2 63° + \sin^2 27°}{\cos^2 17° + \cos^2 73°}$.

The above expression can be written as

$$\frac{\sin^2 63° + \sin^2 27°}{\cos^2 17° + \cos^2 73°} = \frac{\left[\sin(90° - 27°)\right]^2 + \sin^2 27°}{\left[\cos(90° - 73°)\right]^2 + \cos^2 73°}$$

Now, we can apply the identity $\cos(90° - \theta) = \sin\theta$ and $\sin(90° - \theta) = \cos\theta$, we get

$$\Rightarrow \frac{\sin^2 63° + \sin^2 27°}{\cos^2 17° + \cos^2 73°} = \frac{\cos^2 27° + \sin^2 27°}{\sin^2 73° + \cos^2 73°}$$

Now, by applying the trigonometric identity $\sin^2\theta + \cos^2\theta = 1$, we get

$$\Rightarrow \frac{\sin^2 63° + \sin^2 27°}{\cos^2 17° + \cos^2 73°} = \frac{1}{1}$$

$$\therefore \frac{\sin^2 63° + \sin^2 27°}{\cos^2 17° + \cos^2 73°} = 1$$

(ii) $\mathbf{\sin 25° \cos 65° + \cos 25° \sin 65°}$

Ans: The given expression is $\sin 25° \cos 65° + \cos 25° \sin 65°$.

The above expression can be written as

$$\sin 25° \cos 65° + \cos 25° \sin 65° = \sin 25° \cos(90° - 25°) + \cos 25° \sin(90° - 25°)$$

Now, we can apply the identity $\cos(90° - \theta) = \sin\theta$ and $\sin(90° - \theta) = \cos\theta$, we get

$$\Rightarrow \sin 25° \cos 65° + \cos 25° \sin 65° = \sin 25° \sin 25° + \cos 25° \cos 25°$$

$$\Rightarrow \sin 25° \cos 65° + \cos 25° \sin 65° = \sin^2 25° + \cos^2 25°$$

Now, by applying the trigonometric identity $\sin^2\theta + \cos^2\theta = 1$, we get

$$\Rightarrow \sin 25° \cos 65° + \cos 25° \sin 65° = 1$$

$$\therefore \sin 25° \cos 65° + \cos 25° \sin 65° = 1$$

4. Choose the correct option and justify your choice:

(i) $\mathbf{9\sec^2 A - 9\tan^2 A =}$ **…….**

(A) 1

(B) 9

(C) 8
(D) 0
Ans: The given expression is $9\sec^2 A - 9\tan^2 A$.
The given expression can be written as
$\Rightarrow 9\sec^2 A - 9\tan^2 A = 9\left(\sec^2 A - \tan^2 A\right)$
Now, we will use the identity $\sec^2 A = 1 + \tan^2 A$, we get
$\sec^2 A - \tan^2 A = 1$
$\Rightarrow 9\sec^2 A - 9\tan^2 A = 9(1)$
$\therefore 9\sec^2 A - 9\tan^2 A = 9$
Therefore, option (B) is the correct answer.

(ii) $\mathbf{(1+\tan\theta+\sec\theta)(1+\cot\theta-\mathrm{cosec}\,\theta)}$
(A) 0
(B) 1
(C) 2
(D) –1
Ans: The given expression is $(1+\tan\theta+\sec\theta)(1+\cot\theta-\mathrm{cosec}\,\theta)$.
We know that the trigonometric functions have values as:
$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$, $\cot\theta = \dfrac{\cos\theta}{\sin\theta} = \dfrac{1}{\tan\theta}$, $\mathrm{cosec}\,\theta = \dfrac{1}{\sin\theta}$ and $\sec\theta = \dfrac{1}{\cos\theta}$
Substituting these values in the given expression, we get

$$\Rightarrow (1+\tan\theta+\sec\theta)(1+\cot\theta-\mathrm{cosec}\,\theta) = \left(1+\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}\right)\left(1+\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}\right)$$

$$\Rightarrow (1+\tan\theta+\sec\theta)(1+\cot\theta-\mathrm{cosec}\,\theta) = \left(\frac{\cos\theta+\sin\theta+1}{\cos\theta}\right)\left(\frac{\sin\theta+\cos\theta-1}{\sin\theta}\right)$$

Now, by applying the identity $(a+b)(a-b) = a^2 - b^2$, we get

$$\Rightarrow (1+\tan\theta+\sec\theta)(1+\cot\theta-\mathrm{cosec}\,\theta) = \frac{(\sin\theta+\cos\theta)^2 - 1^2}{\sin\theta\cos\theta}$$

$$\Rightarrow (1+\tan\theta+\sec\theta)(1+\cot\theta-\mathrm{cosec}\,\theta) = \frac{\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta-1}{\sin\theta\cos\theta}$$

Now, by applying the trigonometric identity $\sin^2\theta+\cos^2\theta = 1$, we get

$$\Rightarrow (1+\tan\theta+\sec\theta)(1+\cot\theta-\mathrm{cosec}\,\theta) = \frac{1+2\sin\theta\cos\theta-1}{\sin\theta\cos\theta}$$

$$\Rightarrow (1+\tan\theta+\sec\theta)(1+\cot\theta-\operatorname{cosec}\theta)=\frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta}$$

$$\therefore (1+\tan\theta+\sec\theta)(1+\cot\theta-\operatorname{cosec}\theta)=2$$

Therefore, option (C) is the correct answer.

(iii) $(\sec A+\tan A)(1-\sin A)=$ ....

(A) $\sec A$

(B) $\sin A$

(C) $\operatorname{cosec} A$

(D) $\cos A$

Ans: Given expression is $(\sec A+\tan A)(1-\sin A)$.

We know that $\tan\theta=\frac{\sin\theta}{\cos\theta}$ and $\sec\theta=\frac{1}{\cos\theta}$

Substituting these values in the given expression, we get

$$(\sec A+\tan A)(1-\sin A)=\left(\frac{1}{\cos A}+\frac{\sin A}{\cos A}\right)(1-\sin A)$$

$$\Rightarrow (\sec A+\tan A)(1-\sin A)=\left(\frac{1+\sin A}{\cos A}\right)(1-\sin A)$$

$$\Rightarrow (\sec A+\tan A)(1-\sin A)=\left(\frac{(1+\sin A)(1-\sin A)}{\cos A}\right)$$

Now, by applying the identity $(a+b)(a-b)=a^2-b^2$, we get

$$\Rightarrow (\sec A+\tan A)(1-\sin A)=\left(\frac{1^2-\sin^2 A}{\cos A}\right)$$

Now, we know that $\sin^2\theta+\cos^2\theta=1$, we get

$$\Rightarrow (\sec A+\tan A)(1-\sin A)=\left(\frac{\cos^2 A}{\cos A}\right)$$

$$\therefore (\sec A+\tan A)(1-\sin A)=\cos A$$

Therefore, option (D) is the correct answer.

(iv) $\frac{1+\tan^2 A}{1+\cot^2 A}$

(A) $\sec^2 A$

(B) -1

(C) $\cot^2 A$
(D) $\tan^2 A$

Ans: Given expression is $\frac{1+\tan^2 A}{1+\cot^2 A}$.

We know that the trigonometric functions have values as:

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{1}{\tan\theta}$.

Substituting these values in the given expression, we get

$$\frac{1+\tan^2 A}{1+\cot^2 A} = \frac{1+\frac{\sin^2 A}{\cos^2 A}}{1+\frac{\cos^2 A}{\sin^2 A}}$$

$$\Rightarrow \frac{1+\tan^2 A}{1+\cot^2 A} = \frac{\frac{\cos^2 A+\sin^2 A}{\cos^2 A}}{\frac{\sin^2 A+\cos^2 A}{\sin^2 A}}$$

Now, we know that $\sin^2\theta+\cos^2\theta=1$, we get

$$\Rightarrow \frac{1+\tan^2 A}{1+\cot^2 A} = \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}}$$

$$\Rightarrow \frac{1+\tan^2 A}{1+\cot^2 A} = \frac{\sin^2 A}{\cos^2 A}$$

$$\Rightarrow \frac{1+\tan^2 A}{1+\cot^2 A} = \tan^2 A$$

Therefore, option (D) is the correct answer.

5. Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

(i) $\left(\text{cosec}\,\theta - \cot\theta\right)^2 = \frac{1-\cos\theta}{1+\cos\theta}$

Ans: Given expression is $\left(\text{cosec}\,\theta - \cot\theta\right)^2 = \frac{1-\cos\theta}{1+\cos\theta}$.

Let us consider the LHS of the given expression, we get

$\text{LHS} = (\operatorname{cosec}\theta - \cot\theta)^2$

Now, we know that $\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{1}{\tan\theta}$ and $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$.

By substituting the values, we get

$$\Rightarrow (\operatorname{cosec}\theta - \cot\theta)^2 = \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2$$

$$\Rightarrow (\operatorname{cosec}\theta - \cot\theta)^2 = \left(\frac{1-\cos\theta}{\sin\theta}\right)^2$$

$$\Rightarrow (\operatorname{cosec}\theta - \cot\theta)^2 = \frac{(1-\cos\theta)^2}{\sin^2\theta}$$

Now, we know that $\sin^2\theta + \cos^2\theta = 1$, we get

$$\Rightarrow (\operatorname{cosec}\theta - \cot\theta)^2 = \frac{(1-\cos\theta)^2}{1-\cos^2\theta}$$

Now, by applying the identity $(a+b)(a-b) = a^2 - b^2$, we get

$$\Rightarrow (\operatorname{cosec}\theta - \cot\theta)^2 = \frac{(1-\cos\theta)^2}{(1-\cos\theta)(1+\cos\theta)}$$

$$\Rightarrow (\operatorname{cosec}\theta - \cot\theta)^2 = \frac{(1-\cos\theta)}{(1+\cos\theta)}$$

$$\Rightarrow (\operatorname{cosec}\theta - \cot\theta)^2 = \text{RHS}$$

$$\therefore (\operatorname{cosec}\theta - \cot\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$$

Hence proved

(ii) $\boldsymbol{\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = 2\sec A}$

Ans: Given expression is $\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = 2\sec A$.

Let us consider the LHS of the given expression, we get

$$\text{LHS} = \frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A}$$

Now, taking LCM, we get

$$\Rightarrow \frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}=\frac{\cos^2 A+(1+\sin A)(1+\sin A)}{(1+\sin A)\cos A}$$

$$\Rightarrow \frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}=\frac{\cos^2 A+\sin^2 A+2\sin A+1}{(1+\sin A)\cos A}$$

Now, we know that $\sin^2\theta+\cos^2\theta=1$, we get

$$\Rightarrow \frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}=\frac{1+2\sin A+1}{(1+\sin A)\cos A}$$

$$\Rightarrow \frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}=\frac{2+2\sin A}{(1+\sin A)\cos A}$$

$$\Rightarrow \frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}=\frac{2(1+\sin A)}{(1+\sin A)\cos A}$$

$$\Rightarrow \frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}=\frac{2}{\cos A}$$

We know that $\sec\theta=\frac{1}{\cos\theta}$, we get

$$\Rightarrow \frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}=2\sec A$$

$$\Rightarrow \frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}=\text{RHS}$$

$$\therefore \frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}=2\sec A$$

Hence proved

(iii) $\boldsymbol{\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\sec\theta\,\mathrm{cosec}\,\theta}$

Ans: Given expression is $\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\sec\theta\,\mathrm{cosec}\,\theta$.

Let us consider the LHS of the given expression, we get

$$\text{LHS}=\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}$$

Now, we know that $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{1}{\tan\theta}$.

By substituting the values, we get

$$\Rightarrow \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = \left(\frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}} \right)$$

$$\Rightarrow \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = \left(\frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}} \right)$$

$$\Rightarrow \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = \left(\frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} + \frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)} \right)$$

$$\Rightarrow \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = \frac{1}{(\sin\theta-\cos\theta)} \left(\frac{\sin^2\theta}{\cos\theta} + \frac{\cos^2\theta}{\sin\theta} \right)$$

$$\Rightarrow \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = \frac{1}{(\sin\theta-\cos\theta)} \left(\frac{\sin^3\theta-\cos^3\theta}{\sin\theta\cos\theta} \right)$$

Now, by applying the identity $a^3-b^3=(a-b)(a^2+ab+b^2)$, we get

$$\Rightarrow \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = \frac{1}{(\sin\theta-\cos\theta)} \left[\frac{(\sin\theta-\cos\theta)(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta)}{\sin\theta\cos\theta} \right]$$

Now, we know that $\sin^2\theta+\cos^2\theta=1$, we get

$$\Rightarrow \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = \frac{1}{(\sin\theta-\cos\theta)} \left[\frac{(\sin\theta-\cos\theta)(1+\sin\theta\cos\theta)}{\sin\theta\cos\theta} \right]$$

$$\Rightarrow \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = \frac{(1+\sin\theta\cos\theta)}{\sin\theta\cos\theta}$$

$$\Rightarrow \frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=\frac{1}{\sin\theta\cos\theta}+\frac{\sin\theta\cos\theta}{\sin\theta\cos\theta}$$

$$\Rightarrow \frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=\frac{1}{\sin\theta\cos\theta}+1$$

We know that $\operatorname{cosec}\theta=\frac{1}{\sin\theta}$ and $\sec\theta=\frac{1}{\cos\theta}$, we get

$$\Rightarrow \frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=\sec\theta\operatorname{cosec}\theta+1$$

$$\Rightarrow \frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\sec\theta\operatorname{cosec}\theta$$

$$\Rightarrow \frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=\text{RHS}$$

$$\therefore \frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\sec\theta\operatorname{cosec}\theta$$

Hence proved

(iv) $\mathbf{\frac{1+\sec A}{\sec A}=\frac{\sin^2 A}{1-\cos A}}$

Ans: Given expression is $\frac{1+\sec A}{\sec A}=\frac{\sin^2 A}{1-\cos A}$.

Let us consider the LHS of the given expression, we get

$$\text{LHS}=\frac{1+\sec A}{\sec A}$$

Now, we know that $\sec\theta=\frac{1}{\cos\theta}$.

By substituting the value, we get

$$\Rightarrow \frac{1+\sec A}{\sec A}=\frac{1+\frac{1}{\cos A}}{\frac{1}{\cos A}}$$

$$\Rightarrow \frac{1+\sec A}{\sec A}=\frac{\frac{\cos A+1}{\cos A}}{\frac{1}{\cos A}}$$

$$\Rightarrow \frac{1+\sec A}{\sec A}=\cos A+1$$

Multiply and divide by $\left(1-\cos A\right)$, we get

$$\Rightarrow \frac{1+\sec A}{\sec A}=\frac{\left(1+\cos A\right)\left(1-\cos A\right)}{\left(1-\cos A\right)}$$

Now, by applying the identity $\left(a+b\right)\left(a-b\right)=a^2-b^2$, we get

$$\Rightarrow \frac{1+\sec A}{\sec A}=\frac{1-\cos^2 A}{\left(1-\cos A\right)}$$

Now, we know that $\sin^2\theta+\cos^2\theta=1$, we get

$$\Rightarrow \frac{1+\sec A}{\sec A}=\frac{\sin^2 A}{\left(1-\cos A\right)}$$

$$\Rightarrow \frac{1+\sec A}{\sec A}=\text{RHS}$$

$$\therefore \frac{1+\sec A}{\sec A}=\frac{\sin^2 A}{1-\cos A}$$

Hence proved

(v) $\mathbf{\dfrac{\cos A-\sin A+1}{\cos A+\sin A-1}=\operatorname{cosec} A+\cot A}$

Ans: Given expression is $\dfrac{\cos A-\sin A+1}{\cos A+\sin A-1}=\operatorname{cosec} A+\cot A$.

Now, let us consider the LHS of the given expression, we get

$$\text{LHS}=\frac{\cos A-\sin A+1}{\cos A+\sin A-1}$$

Dividing numerator and denominator by $\sin A$, we get

$$\Rightarrow \frac{\cos A-\sin A+1}{\cos A+\sin A-1}=\frac{\frac{\cos A}{\sin A}-\frac{\sin A}{\sin A}+\frac{1}{\sin A}}{\frac{\cos A}{\sin A}+\frac{\sin A}{\sin A}-\frac{1}{\sin A}}$$

Now, we know that $\cot\theta=\frac{\cos\theta}{\sin\theta}=\frac{1}{\tan\theta}$ and $\operatorname{cosec}\theta=\frac{1}{\sin\theta}$, we get

$$\Rightarrow \frac{\cos A-\sin A+1}{\cos A+\sin A-1}=\frac{\cot A-1+\operatorname{cosec} A}{\cot A+1-\operatorname{cosec} A}$$

Now, by applying the identity $\operatorname{cosec}^2 A=1+\cot^2 A$, substitute $1=\cot^2 A-\operatorname{cosec}^2 A$, we get

$$\Rightarrow \frac{\cos A-\sin A+1}{\cos A+\sin A-1}=\frac{\cot A-\left(\cot^2 A-\operatorname{cosec}^2 A\right)+\operatorname{cosec} A}{\cot A+\cot^2 A-\operatorname{cosec}^2 A-\operatorname{cosec} A}$$

$$\Rightarrow \frac{\cos A-\sin A+1}{\cos A+\sin A-1}=\frac{\cot A-\cot^2 A+\operatorname{cosec}^2 A+\operatorname{cosec} A}{\cot A+\cot^2 A-\operatorname{cosec}^2 A-\operatorname{cosec} A}$$

$$\Rightarrow \frac{\cos A-\sin A+1}{\cos A+\sin A-1}=\frac{\left(\cot A-1+\operatorname{cosec} A\right)^2}{\cot^2 A-1+\operatorname{cosec}^2 A+2\operatorname{cosec} A}$$

$$\Rightarrow \frac{\cos A-\sin A+1}{\cos A+\sin A-1}=\frac{\left(\cot A-1+\operatorname{cosec} A\right)^2}{\cot^2 A-1+\operatorname{cosec}^2 A+2\operatorname{cosec} A}$$

$$\Rightarrow \frac{\cos A-\sin A+1}{\cos A+\sin A-1}=\frac{2\operatorname{cosec}^2 A+2\cot A\operatorname{cosec} A-2\cot A-2\operatorname{cosec} A}{\cot^2 A-1+\operatorname{cosec}^2 A+2\operatorname{cosec} A}$$

$$\Rightarrow \frac{\cos A-\sin A+1}{\cos A+\sin A-1}=\frac{2\operatorname{cosec} A\left(\cot A-\operatorname{cosec} A\right)-2\left(\cot A-\operatorname{cosec} A\right)}{\cot^2 A-1+\operatorname{cosec}^2 A+2\operatorname{cosec} A}$$

$$\Rightarrow \frac{\cos A-\sin A+1}{\cos A+\sin A-1}=\frac{\left(2\operatorname{cosec} A-2\right)\left(\cot A-\operatorname{cosec} A\right)}{1-1+2\operatorname{cosec} A}$$

$$\Rightarrow \frac{\cos A-\sin A+1}{\cos A+\sin A-1}=\frac{\left(2\operatorname{cosec} A-2\right)\left(\cot A-\operatorname{cosec} A\right)}{2\operatorname{cosec} A}$$

$$\Rightarrow \frac{\cos A-\sin A+1}{\cos A+\sin A-1}=\operatorname{cosec} A+\cot A$$

$$\Rightarrow \frac{\cos A-\sin A+1}{\cos A+\sin A-1}=\text{RHS}$$

$$\therefore \frac{\cos A-\sin A+1}{\cos A+\sin A-1}=\operatorname{cosec} A+\cot A$$

Hence proved

(vi) $\sqrt{\dfrac{\mathbf{1+\sin A}}{\mathbf{1-\sin A}}}=\mathbf{\sec A+\tan A}$

Ans: Given expression is $\sqrt{\dfrac{1+\sin A}{1-\sin A}}=\sec A+\tan A$.

Let us consider the LHS of the given expression, we get

$$\text{LHS}=\sqrt{\frac{1+\sin A}{1-\sin A}}$$

Now, multiply and divide the expression by $\sqrt{1+\sin A}$, we get

$$\Rightarrow \sqrt{\frac{1+\sin A}{1-\sin A}}=\sqrt{\frac{(1+\sin A)(1+\sin A)}{(1-\sin A)(1+\sin A)}}$$

Now, by applying the identity $(a+b)(a-b)=a^2-b^2$, we get

$$\Rightarrow \sqrt{\frac{1+\sin A}{1-\sin A}}=\sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}}$$

Now, we know that $\sin^2\theta+\cos^2\theta=1$, we get

$$\Rightarrow \sqrt{\frac{1+\sin A}{1-\sin A}}=\frac{1+\sin A}{\sqrt{\cos^2 A}}$$

$$\Rightarrow \sqrt{\frac{1+\sin A}{1-\sin A}}=\frac{1+\sin A}{\cos A}$$

$$\Rightarrow \sqrt{\frac{1+\sin A}{1-\sin A}}=\frac{1}{\cos A}+\frac{\sin A}{\cos A}$$

$$\Rightarrow \sqrt{\frac{1+\sin A}{1-\sin A}}=\sec A+\tan A$$

$$\Rightarrow \sqrt{\frac{1+\sin A}{1-\sin A}} = \text{RHS}$$

$$\therefore \sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$$

Hence proved

(vii) $\mathbf{\dfrac{\sin\theta - 2\sin^3\theta}{2\cos\theta - \cos\theta} = \tan\theta}$

Ans: Given expression is $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos\theta - \cos\theta} = \tan\theta$.

Let us consider the LHS of the given expression, we get

$$\text{LHS} = \frac{\sin\theta - 2\sin^3\theta}{2\cos\theta - \cos\theta}$$

Taking common terms out, we get

$$\Rightarrow \frac{\sin\theta - 2\sin^3\theta}{2\cos\theta - \cos\theta} = \frac{\sin\theta\left(1-2\sin^2\theta\right)}{\cos\theta\left(2\cos^2\theta - 1\right)}$$

Now, we know that $\sin^2\theta + \cos^2\theta = 1$, we get

$$\Rightarrow \frac{\sin\theta - 2\sin^3\theta}{2\cos\theta - \cos\theta} = \frac{\sin\theta\left(1-2\sin^2\theta\right)}{\cos\theta\left(2\left(1-2\sin^2\theta\right)-1\right)}$$

$$\Rightarrow \frac{\sin\theta - 2\sin^3\theta}{2\cos\theta - \cos\theta} = \frac{\sin\theta\left(1-2\sin^2\theta\right)}{\cos\theta\left(2-2\sin^2\theta-1\right)}$$

$$\Rightarrow \frac{\sin\theta - 2\sin^3\theta}{2\cos\theta - \cos\theta} = \frac{\sin\theta\left(1-2\sin^2\theta\right)}{\cos\theta\left(1-2\sin^2\theta\right)}$$

$$\Rightarrow \frac{\sin\theta - 2\sin^3\theta}{2\cos\theta - \cos\theta} = \frac{\sin\theta}{\cos\theta}$$

$$\Rightarrow \frac{\sin\theta - 2\sin^3\theta}{2\cos\theta - \cos\theta} = \tan\theta$$

$$\Rightarrow \frac{\sin\theta - 2\sin^3\theta}{2\cos\theta - \cos\theta} = \text{RHS}$$

$$\therefore \frac{\sin\theta - 2\sin^3\theta}{2\cos\theta - \cos\theta} = \tan\theta$$

Hence proved

(ix) $\left(\sin A+\operatorname{cosec} A\right)^2+\left(\cos A+\sec A\right)^2=7+\tan^2 A+\cot^2 A$

Ans: Given expression is

$\left(\sin A+\operatorname{cosec} A\right)^2+\left(\cos A+\sec A\right)^2=7+\tan^2 A+\cot^2 A$.

Let us consider the LHS of the given expression, we get

$\text{LHS}=\left(\sin A+\operatorname{cosec} A\right)^2+\left(\cos A+\sec A\right)^2$

Now, by applying the identity $(a+b)^2=a^2+2ab+b^2$, we get

$\Rightarrow\left(\sin A+\operatorname{cosec} A\right)^2+\left(\cos A+\sec A\right)^2=\sin^2 A+\operatorname{cosec} A^2+2\sin A\operatorname{cosec} A+\cos^2 A+\sec^2 A+$

$\Rightarrow\left(\sin A+\operatorname{cosec} A\right)^2+\left(\cos A+\sec A\right)^2=\sin^2 A+\cos^2 A+\operatorname{cosec} A^2+\sec^2 A+2\sin A\operatorname{cosec} A+$

We know that $\sin^2\theta+\cos^2\theta=1,\ \operatorname{cosec}\theta=\frac{1}{\sin\theta}$ and $\sec\theta=\frac{1}{\cos\theta}$, we get

$\Rightarrow\left(\sin A+\operatorname{cosec} A\right)^2+\left(\cos A+\sec A\right)^2=1+\operatorname{cosec}^2\theta+\sec^2\theta+2\sin A\frac{1}{\sin A}+2\cos A\frac{1}{\cos A}$

$\Rightarrow\left(\sin A+\operatorname{cosec} A\right)^2+\left(\cos A+\sec A\right)^2=1+\left(1+\cot^2 A+1+\tan^2 A\right)+2+2$

$\Rightarrow\left(\sin A+\operatorname{cosec} A\right)^2+\left(\cos A+\sec A\right)^2=7+\tan^2 A+\cot^2 A$

$\Rightarrow\left(\sin A+\operatorname{cosec} A\right)^2+\left(\cos A+\sec A\right)^2=\text{RHS}$

$\therefore\left(\sin A+\operatorname{cosec} A\right)^2+\left(\cos A+\sec A\right)^2=7+\tan^2 A+\cot^2 A$

Hence proved

(x) $\left(\operatorname{cosec} A-\sin A\right)\left(\sec A-\cos A\right)=\frac{1}{\tan A+\cot A}$

Ans: Given expression is $\left(\operatorname{cosec} A-\sin A\right)\left(\sec A-\cos A\right)=\frac{1}{\tan A+\cot A}$.

Let us consider the LHS of the given expression, we get

$\text{LHS}=\left(\operatorname{cosec} A-\sin A\right)\left(\sec A-\cos A\right)$

We know that $\operatorname{cosec}\theta=\frac{1}{\sin\theta}$ and $\sec\theta=\frac{1}{\cos\theta}$, we get

$\Rightarrow\left(\operatorname{cosec} A-\sin A\right)\left(\sec A-\cos A\right)=\left(\frac{1}{\sin A}-\sin A\right)\left(\frac{1}{\cos A}-\cos A\right)$

$$\Rightarrow (\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \left(\frac{1-\sin^2 A}{\sin A}\right)\left(\frac{1-\cos^2 A}{\cos A}\right)$$

Now, we know that $\sin^2\theta + \cos^2\theta = 1$, we get

$$\Rightarrow (\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \left(\frac{\cos^2 A}{\sin A}\right)\left(\frac{\sin^2 A}{\cos A}\right)$$

$$\Rightarrow (\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \sin A \cos A$$

Now, consider the RHS of the given expression, we get

$$\text{RHS} = \frac{1}{\tan A + \cot A}$$

Now, we know that $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{1}{\tan\theta}$.

$$\Rightarrow \frac{1}{\tan A + \cot A} = \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}}$$

$$\Rightarrow \frac{1}{\tan A + \cot A} = \frac{1}{\frac{\sin^2 A + \cos^2 A}{\sin A \cos A}}$$

$$\Rightarrow \frac{1}{\tan A + \cot A} = \frac{\sin A \cos A}{\sin^2 A + \cos^2 A}$$

Now, we know that $\sin^2\theta + \cos^2\theta = 1$, we get

$$\Rightarrow \frac{1}{\tan A + \cot A} = \sin A \cos A$$

Here, we get LHS=RHS

$$\therefore (\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$$

Hence proved

(xi) $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right)=\left(\frac{1-\tan A}{1-\cot A}\right)^2$

Ans: Given expression is $\frac{1+\tan^2 A}{1+\cot^2 A}=\left(\frac{1-\tan A}{1-\cot A}\right)^2$.

Let us consider the LHS of the given expression, we get

$$\text{LHS}=\frac{1+\tan^2 A}{1+\cot^2 A}$$

By applying the identities $\sec^2 A=1+\tan^2 A$ and $\operatorname{cosec}^2 A=1+\cot^2 A$, we get

$$\Rightarrow \frac{1+\tan^2 A}{1+\cot^2 A}=\frac{\sec^2 A}{\operatorname{cosec}^2 A}$$

We know that $\operatorname{cosec}\theta=\frac{1}{\sin\theta}$ and $\sec\theta=\frac{1}{\cos\theta}$, we get

$$\Rightarrow \frac{1+\tan^2 A}{1+\cot^2 A}=\frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}}$$

$$\Rightarrow \frac{1+\tan^2 A}{1+\cot^2 A}=\frac{\sin^2 A}{\cos^2 A}$$

$$\Rightarrow \frac{1+\tan^2 A}{1+\cot^2 A}=\tan^2 A$$

Now, consider the RHS of the given expression, we get

$$\text{RHS}=\left(\frac{1-\tan A}{1-\cot A}\right)^2$$

Now, we know that $\cot\theta=\frac{1}{\tan\theta}$, we get

$$\Rightarrow \left(\frac{1-\tan A}{1-\cot A}\right)^2=\left(\frac{1-\tan A}{1-\frac{1}{\tan A}}\right)^2$$

$$\Rightarrow\left(\frac{1-\tan A}{1-\cot A}\right)^2=\left(\frac{1-\tan A}{\frac{\tan A-1}{\tan A}}\right)^2$$

$$\Rightarrow\left(\frac{1-\tan A}{1-\cot A}\right)^2=(-\tan A)^2$$

$$\Rightarrow\left(\frac{1-\tan A}{1-\cot A}\right)^2=\tan^2 A$$

Here, we get LHS=RHS

$$\therefore\frac{1+\tan^2 A}{1+\cot^2 A}=\left(\frac{1-\tan A}{1-\cot A}\right)^2$$

Hence proved