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1 k-space construction

The authors of the paper are considering a truncated k-space of radius $\Lambda \sim 20 \times 2\pi/L$. So my k-space is a 2×2 array with each dimension running from $-\Lambda$ to Λ to begin with. Later, I only compute and diagonalise the hamiltonian if the complete condition is satisfied, $k_x^2 + k_y^2 \leq \Lambda^2$. This should restrict my space in the way they did.

2 Equation (4): Conductivity

The authors of the paper have given the Kubo formula

$$g = -\frac{i2\pi\hbar^2}{L^2} \sum_{n,n'} \frac{f(E_n) - f(E_{n'})}{E_n - E_{n'}} \frac{\langle n|v_x|n'\rangle\langle n'|v_x|n\rangle}{E_n - E_{n'} + i\eta} \tag{1}$$

where $v_x = v_F \sigma_x$

This, to me, looks like each pair (k_x, k_y) will have two eigenstates $|n\rangle$ and $|n'\rangle$, and we're summing over all their permutations. When we're considering the terms that use the same eigenstate, i.e., have the same eigenvalues, the denominator happens to be zero. To resolve this, I considered that term as the limit to which the eigenvalues are equal. In this limit, the term $(f(E_n) - f(E_{n'}))/(E_n - E_{n'})$ is the slope of the zero-temperature Fermi-Dirac function, which is zero everywhere except at the Fermi surface. So I chose to rewrite eq.(1) as

$$g = -\frac{i2\pi\hbar^2 v_F^2}{L^2} \sum_{k_x^2 + k_y^2 \leq \Lambda^2} \frac{|\langle n'|\sigma_x|n\rangle|^2}{E_n - E_{n'}} \left(\frac{1}{E_n - E_{n'} + i\eta} + \frac{1}{E_{n'} - E_n + i\eta} \right)$$

which simplifies to

$$g = -\frac{4\pi\hbar^2 v_F^2 \eta}{L^2} \sum_{k_x^2 + k_y^2 \leq \Lambda^2} \frac{|\langle n'|\sigma_x|n\rangle|^2}{(E_n - E_{n'})\left((E_n - E_{n'})^2 + \eta^2\right)} \tag{2}$$