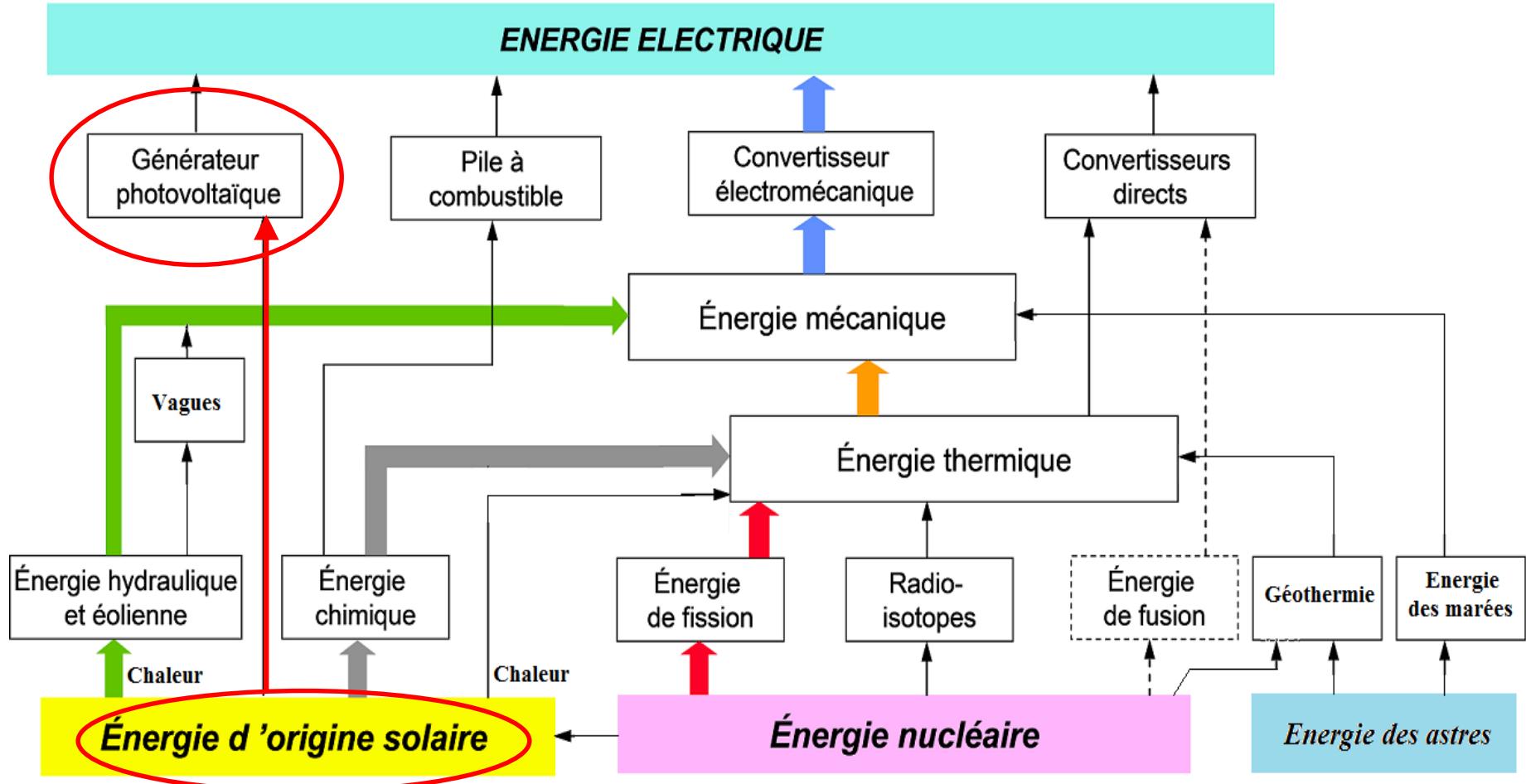


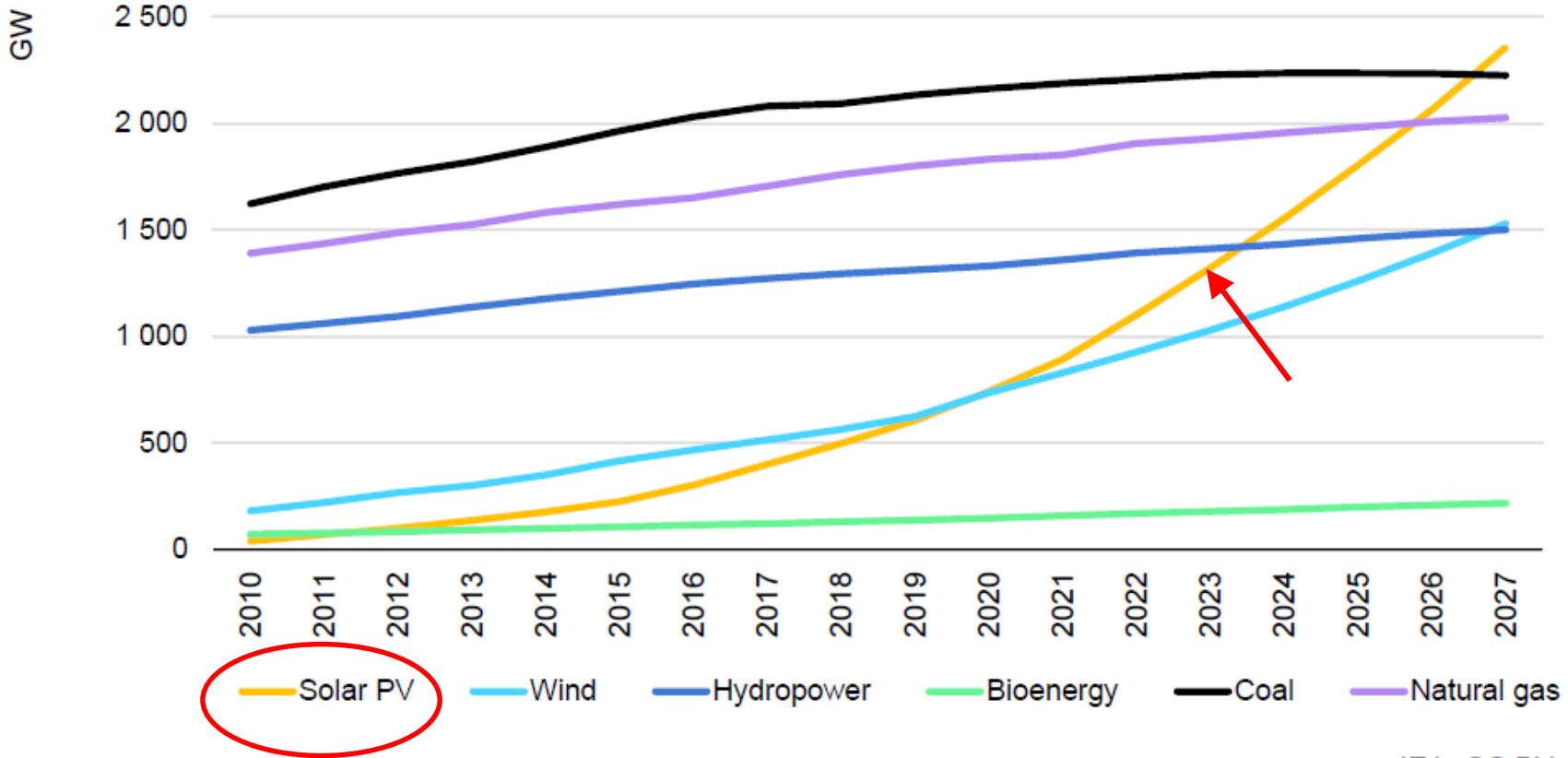
Photovoltaic Systems (Solar PV)



Electric power generation chains

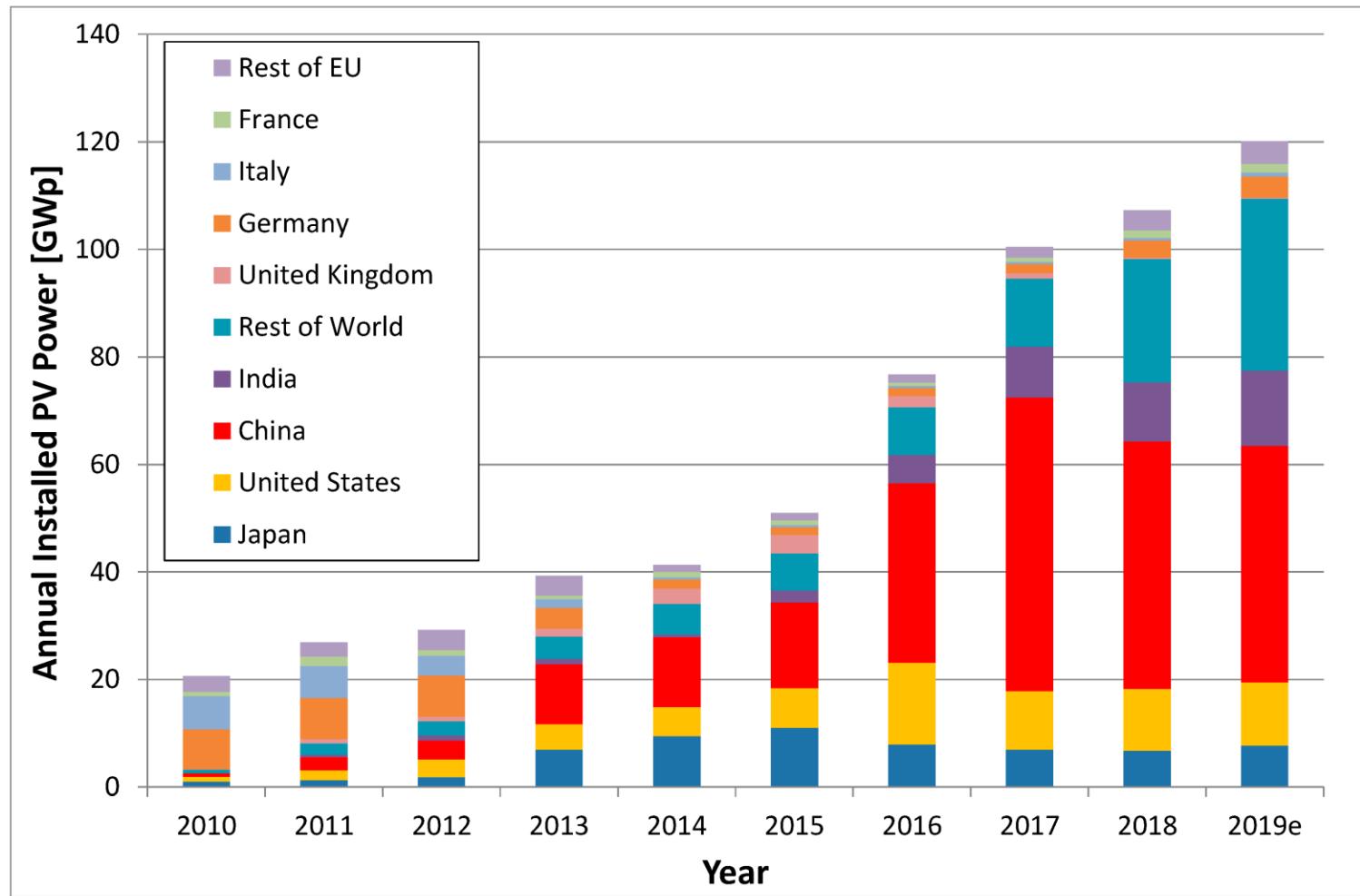


Electricity generation: installed power in the world



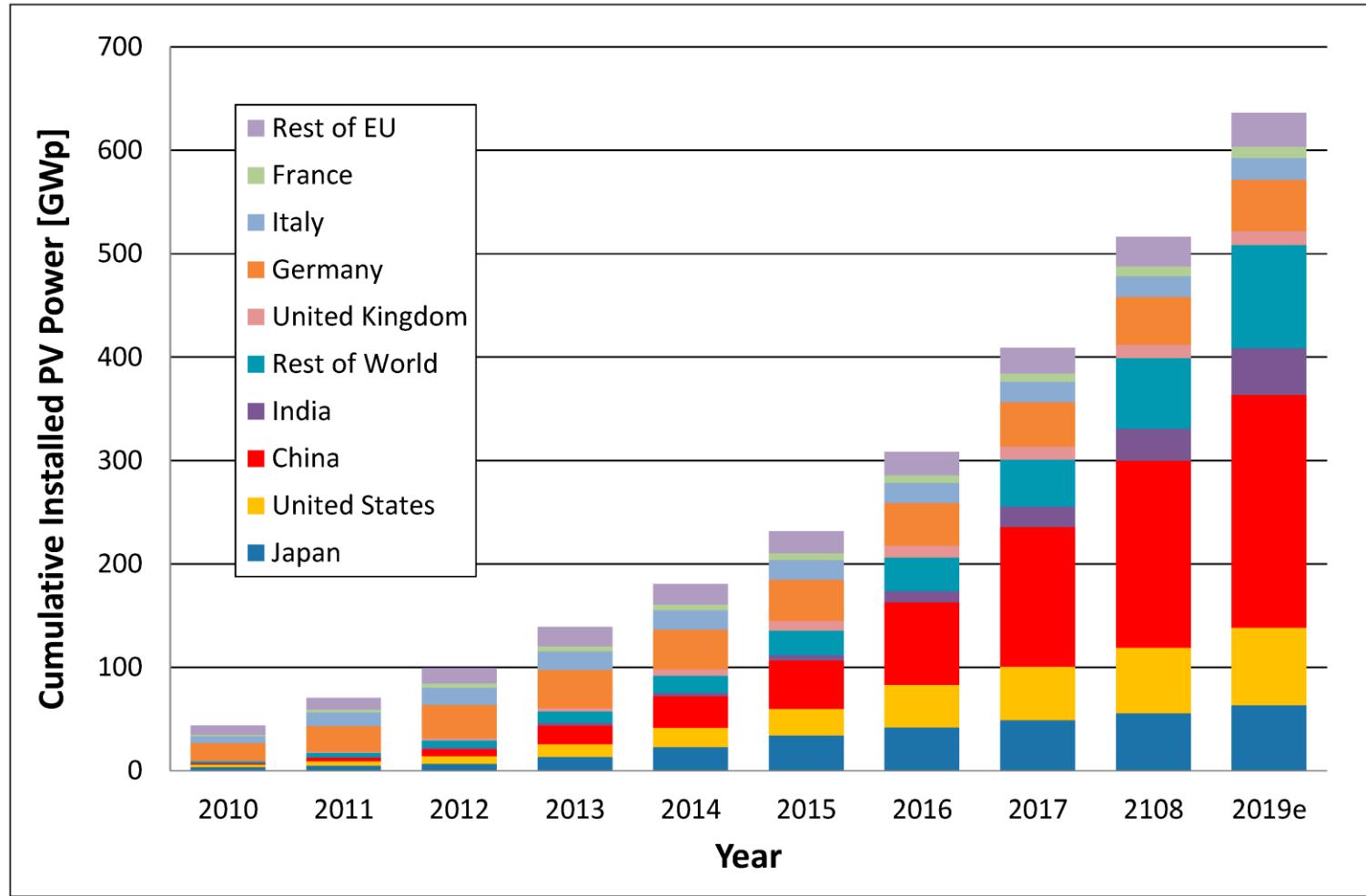
IEA. CC BY 4.0.

Photovoltaics: installed power in the world



In 2018, over 100 GW of new PV power capacity was added. The annual PV capacity addition in 2018 was more than the total cumulative installed PV capacity installed until the mid of 2012.

Photovoltaics: installed power in the world



Total installed power (estimated) : **630 GWp** (2019)

Expected installed power in 2025: 970 GWp (source: IEA)

<https://globalenergymonitor.org/projects/global-solar-power-tracker/>

LENVI 2007

Among the largest PV plants of the world



Bhadla Solar Park – 2245MW – India

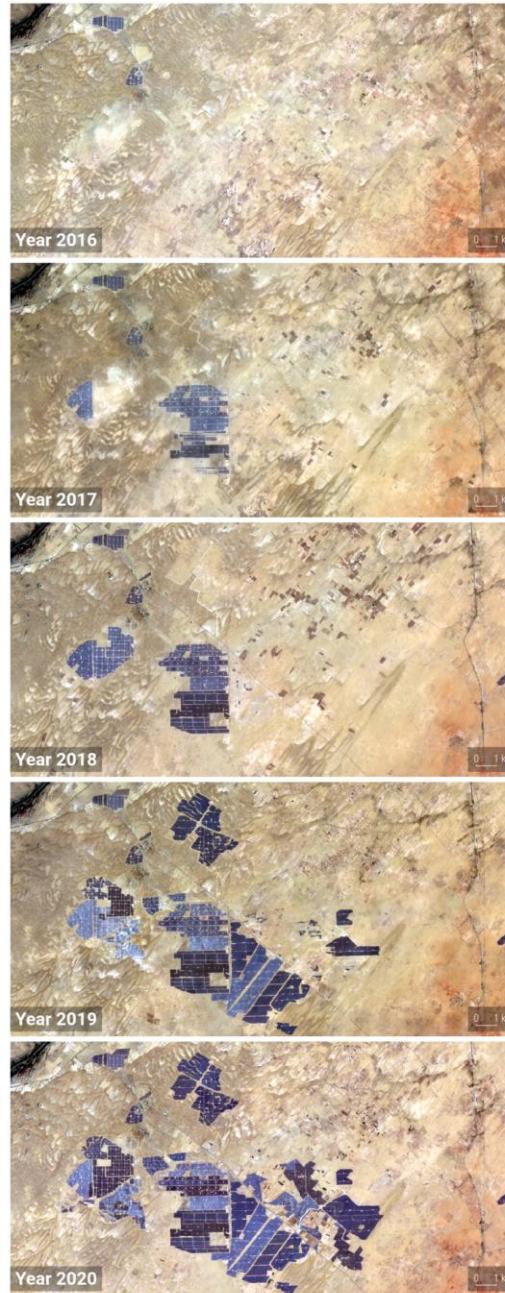


Tengger Desert Solar Park – 1500MW – China



Pavagada Solar Park – 1400MW – India

Pavagada Solar Park – 1400MW – India



Photovoltaic power plants cluster development near Bhadla, Rajasthan, India.
Satellite image source: Copernicus Sentinel Hub, Sentinel-2, True color
a) 2016-05-21, L1C product; b) 2017-05-26, L2A product; c) 2018-07-10, L2A product;
d) 2019-05-06, L2A product; e) 2020-05-10, L2A product
Composed by Solaris, Rendered by QGIS.



(China) 100 MW

Photovoltaïcs: installed power in Belgium



PHOTOVOLTAIQUE

Sélectionner tout

Bruxelles

Flandre

Wallonie

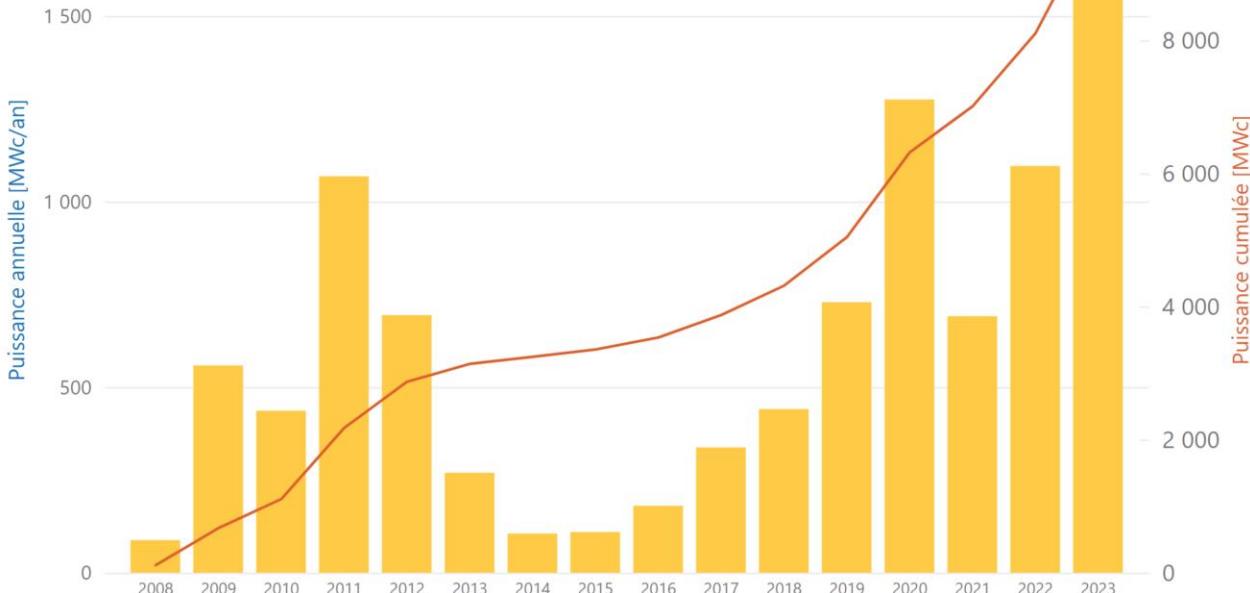
Années

2008

2023

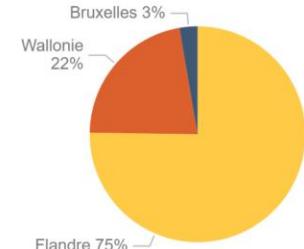


Evolution des puissances installées

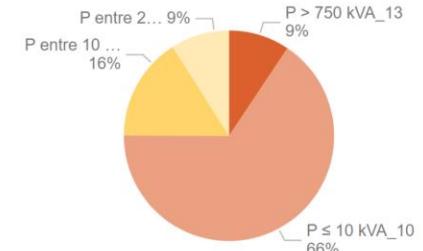


En 2023

Répartition du parc par Région



Répartition selon la puissance [kWc]



(i.e. a 75 km² area, which means more than 10500 football fields)

<https://www.renouvelle.be/fr/faits-chiffres/observatoire-photovoltaïque/>

LENVI 2007

Load factor (utilization factor) of a power generation installation

Load factor (also known as **utilization factor**)

$$F_L = \frac{E}{E_N} = \frac{\int_0^T P(t)dt}{P_N T} = \frac{T_u}{T}$$

$$P_N T_u = \int_0^T P(t)dt$$

P_N = Nominal (rated) installed power

In the case of PV installations P_N usually refers to the power produced in standard conditions (to be detailed further in this course). It is often – but improperly - called “peak power” and is expressed/given in Wp (Watt-peak)

T = Observation period (generally $T = 1$ year = 8760 h)

T_u = Equivalent utilization time = generation time under nominal power, leading to the same amount of energy as produced over period T ($T_u \leq T$)

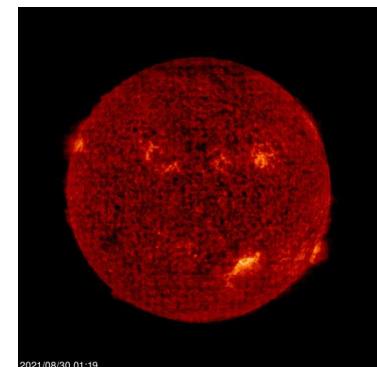
Photovoltaïcs: energy produced in Belgium



Total PV energy produced in 2023: 9140 GWh (= 9,14 TWh); $F_L \approx 10,5\%$

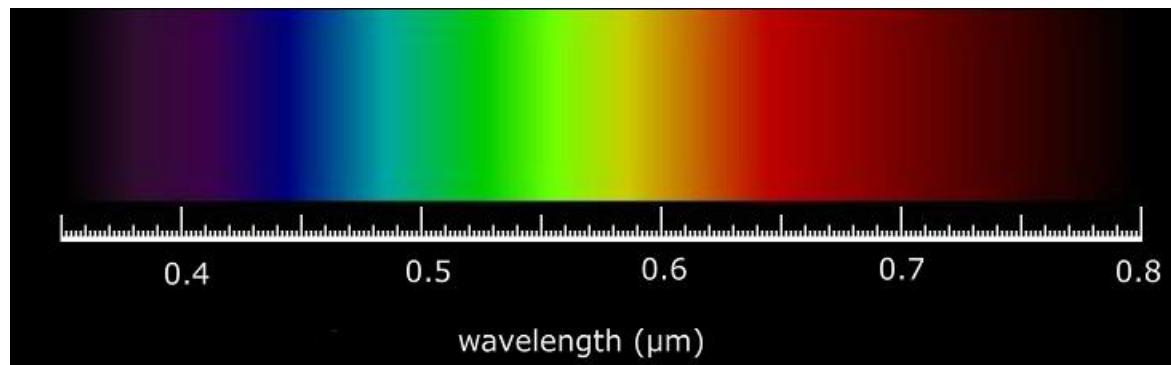
N.B. Total electrical energy consumption in Belgium: ≈ 82 TWh

<https://www.renouvelle.be/fr/faits-chiffres/observatoire-photovoltaïque/>



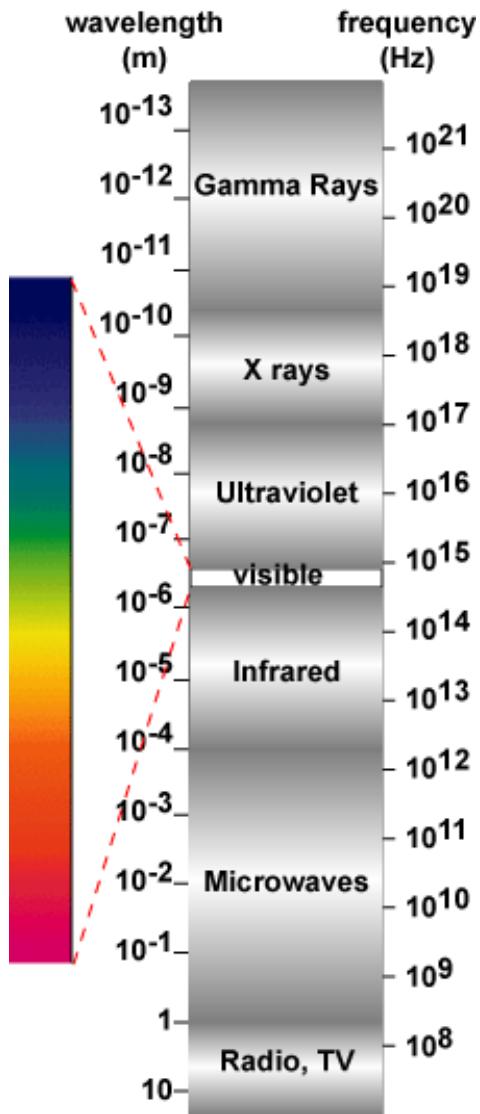
Characterization of the resource (1)

Physical characteristics of sunlight



The dual nature of solar radiation

Electromagnetic radiation



Particle (photon) flow

Solar radiation consists of *photons carrying energy* given by :

$$E_{ph} = h\nu = \frac{hc}{\lambda}$$

$$h = 6,626 \times 10^{-34} [\text{J}\cdot\text{s}] \text{ (Planck's constant)}$$

$$c = 2,998 \times 10^8 [\text{m/s}] \text{ (Speed of light)}$$

$$hc = 1,99 \times 10^{-25} [\text{J}\cdot\text{m}]$$

Expression using the eV unit: $1 \text{ eV} = 1,602 \times 10^{-19} \text{ J}$

$$hc = (1,99 \times 10^{-25} \text{ J.m}) \times (1 \text{ eV} / 1,602 \times 10^{-19} \text{ J}) = 1,24 \times 10^{-6} \text{ eV.m}$$

$$= (1,24 \times 10^{-6} \text{ eV.m}) \times (10^6 \mu\text{m/m}) = \underline{1,24 \text{ eV} \cdot \mu\text{m}}$$

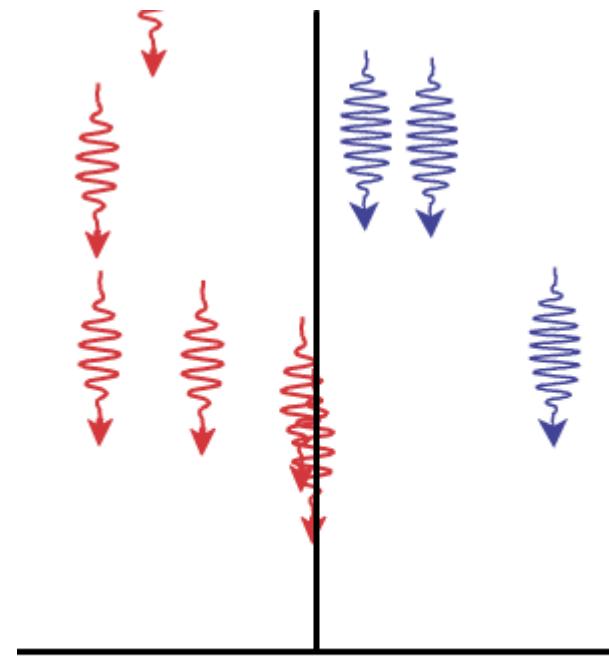
$$E_{ph}(\text{eV}) = \frac{1,24}{\lambda(\mu\text{m})}$$

Power density of solar radiation: Irradiance

Photon flux:

$$\Phi = \frac{\# \text{ photons}}{m^2 s}$$

Power density = Irradiance: $H = \Phi \frac{hc}{\lambda} \quad [\frac{W}{m^2}]$



For the same light intensity, blue light requires fewer photons since the energy content of each photon is greater.

The *photon flux of high energy (or short wavelength) photons needed to give a certain radiant power density will be lower than the photon flux of low energy (or long wavelength) photons required to give the same radiant power density.*

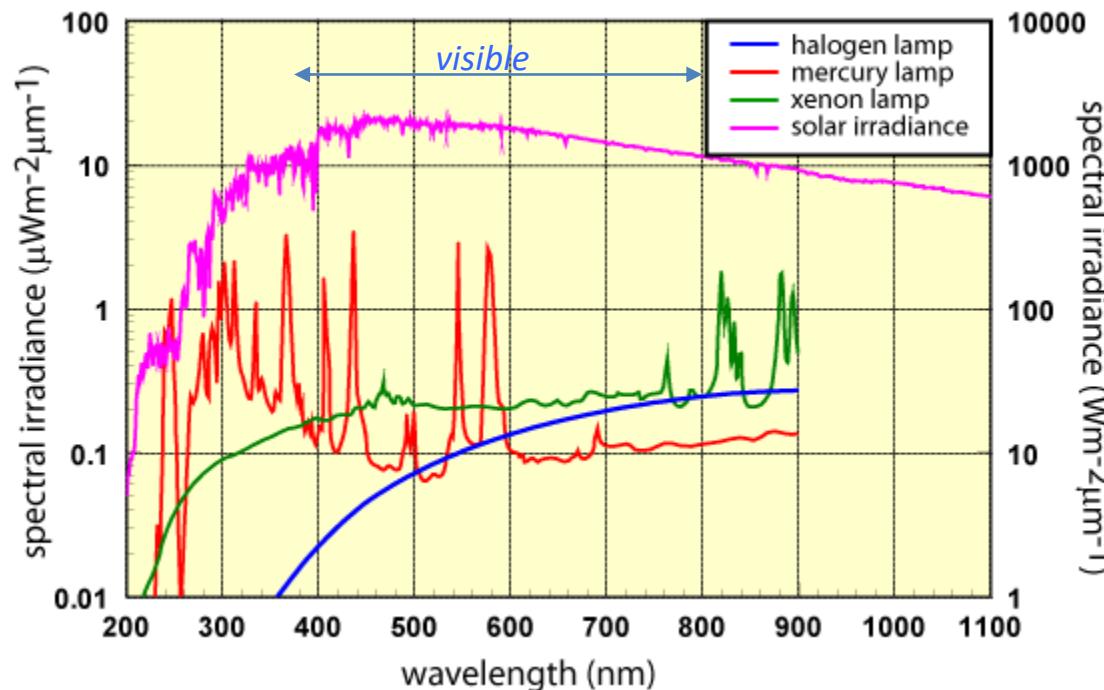
Spectral irradiance

The spectral irradiance as a **function of photon wavelength** (or energy), denoted by F , is the most common way of characterizing a light source.

It gives the power density at a particular wavelength. The units of spectral irradiance are in $\text{Wm}^{-2}\mu\text{m}^{-1}$.

The Wm^{-2} term is the power density at the wavelength λ (μm).

Therefore, the m^{-2} refers to the surface area of the light emitter and the μm^{-1} refers to the wavelength of interest.



The spectral irradiance of artificial light sources (left axis) compared to the spectral irradiance from the sun (right axis).

Light sources as blackbody emitters

Many commonly encountered light sources, including the sun and light bulbs, are closely modelled as "blackbody" emitters.

A blackbody absorbs all radiation incident on its surface and emits radiation based on its temperature.

(Blackbodies derive their name from the fact that, *if they do not emit radiation in the visible range*, they appear black due to the complete absorption of all wavelengths.)

The spectral irradiance from a blackbody is given by Planck's radiation law :

$$F(\lambda) = \frac{2\pi hc^2}{\lambda^5 \left(\exp\left(\frac{hc}{k\lambda T}\right) - 1 \right)}$$

[W m⁻² m⁻¹]

$h = 6,626 \cdot 10^{-34}$ [J s] (Planck)

$k = 1,380649 \cdot 10^{-23}$ [J K⁻¹] (Boltzmann)



M. Planck
1858-1947

The total power density from a blackbody is determined by integrating the spectral irradiance over all wavelengths:

$$H = \sigma T^4$$

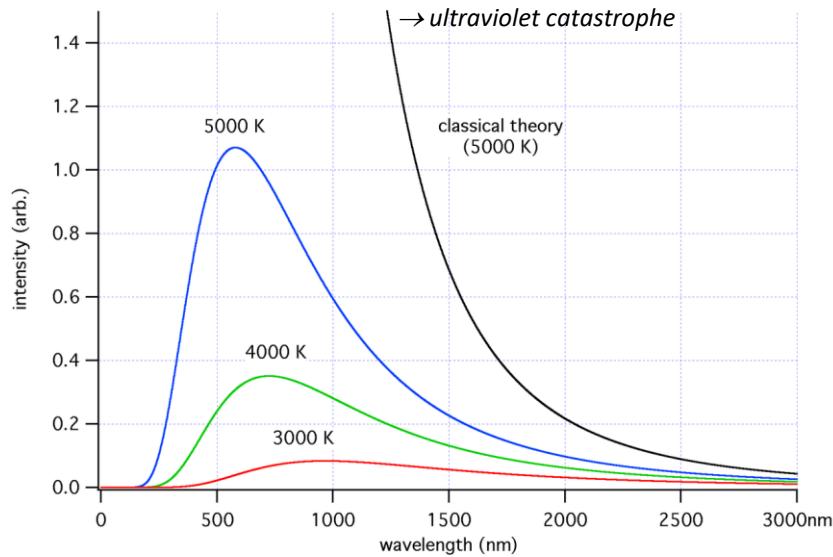
[W m⁻²]

where σ is the Stefan-Boltzmann constant and T is the temperature of the blackbody in kelvin.

$$F(\lambda) = \frac{2c}{\lambda^4} kT$$

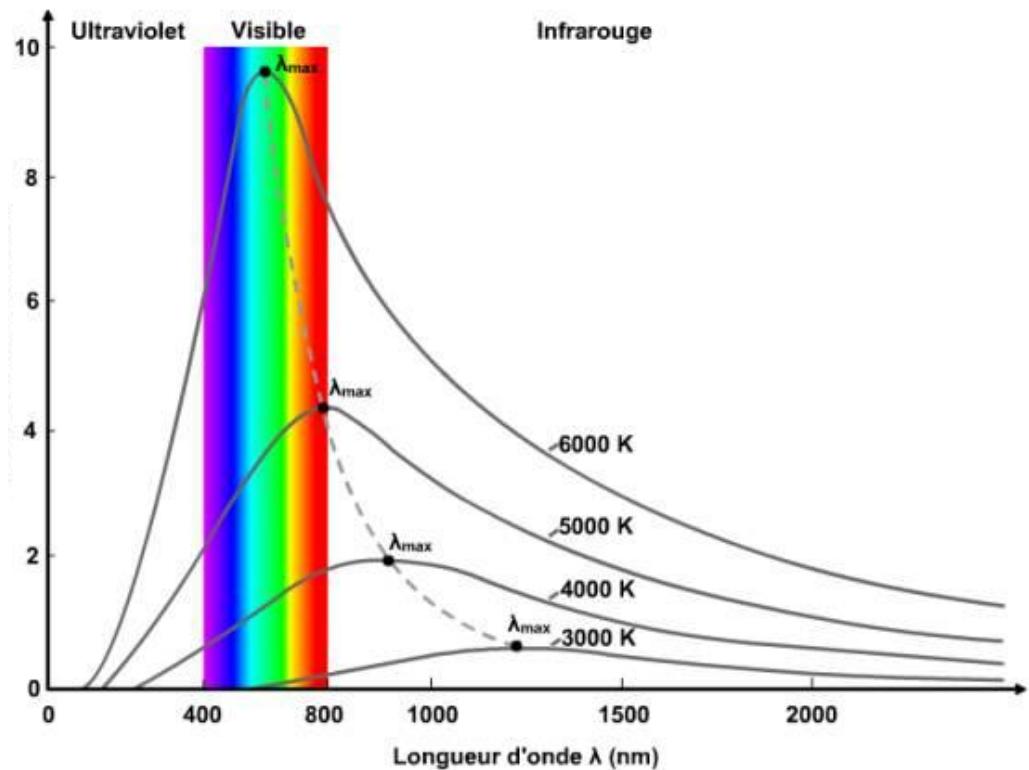
$$F(\lambda) = \frac{2\pi hc^2}{\lambda^5 \left(\exp\left(\frac{hc}{k\lambda T}\right) - 1 \right)}$$

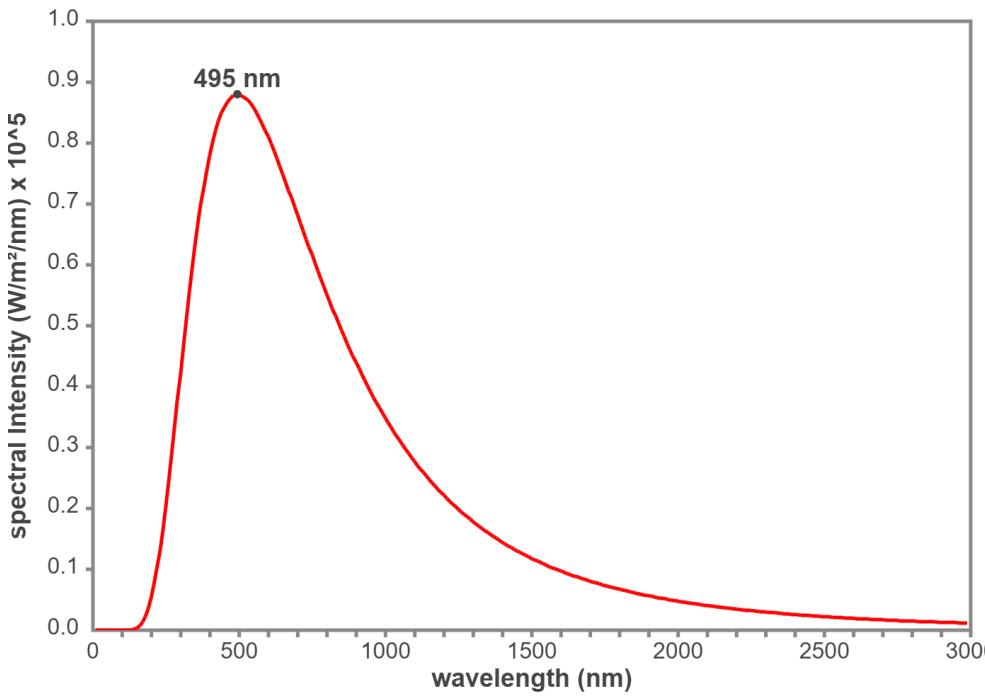
Classical (Rayleigh-Jeans)



(Spectral power density)

Quantum (Planck)





The spectral irradiance of a blackbody @ 5800 K (calculated according to Planck's law)

An additional important parameter of a blackbody source is *the wavelength where the spectral irradiance is the highest*, or, in other words *the wavelength where most of the power is emitted*.

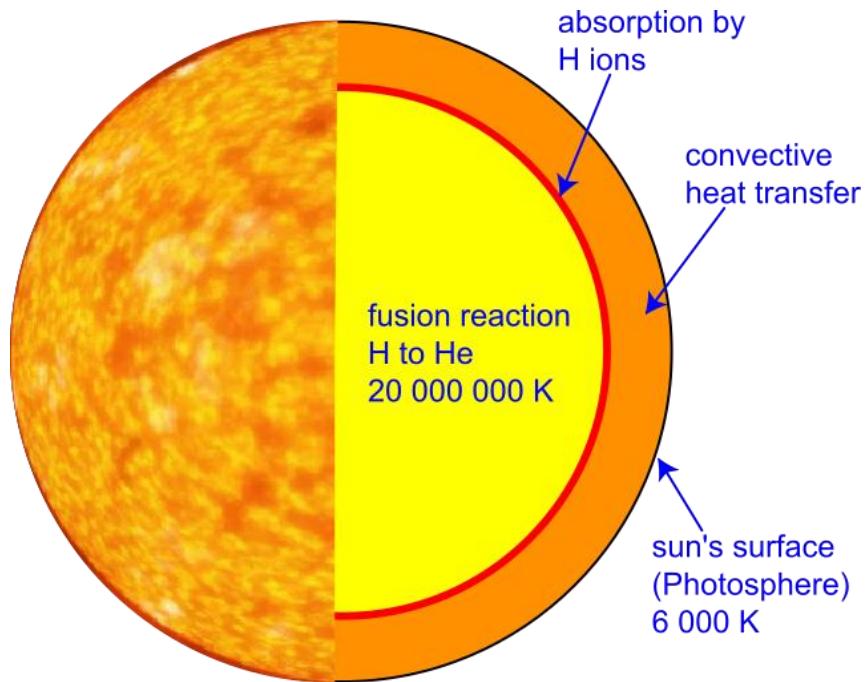
The peak wavelength of the spectral irradiance is determined by differentiating the spectral irradiance and solving the derivative when it equals 0.

The result is known as Wien's law:

$$\lambda_p (\mu m) \cong \frac{2900}{T}$$

The resource: the sun

The sun is a hot sphere of gas whose internal temperatures reach over 20 million kelvin due to nuclear fusion reactions at the sun's core which convert hydrogen to helium. The radiation from the inner core is not visible since it is strongly absorbed by a layer of hydrogen atoms closer to the sun's surface. Heat is transferred through this layer by convection



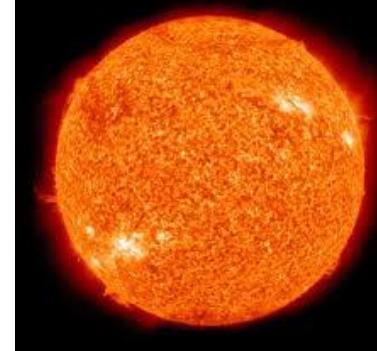
The surface of the sun, called the photosphere, is at a temperature of about 6000K and closely approximates a blackbody.

Astronomers use 5778 K when classifying the sun as a star.

It is common to use the approximation of 5800 K.

N.B. About nuclear fusion:

<https://www.iaea.org/bulletin/what-is-fusion-and-why-is-it-so-difficult-to-achieve>



The resource: the sun

The sun can be considered as a **blackbody in thermal equilibrium** (that is, at a constant temperature ≈ 5800 K) **which emits electromagnetic radiation**.

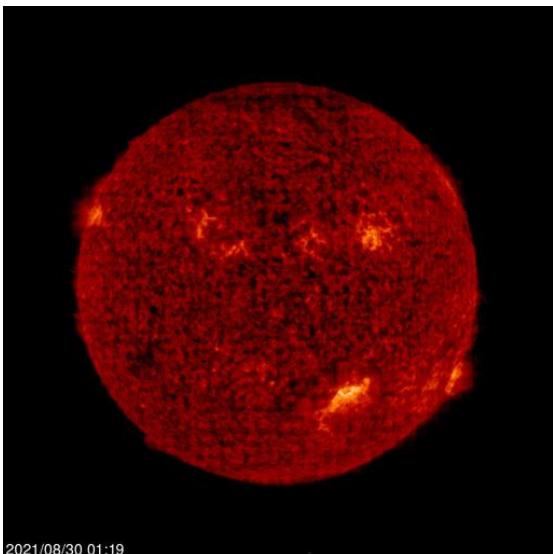
This solar radiation consists of **photons carrying energy** given by : $E_{ph} = h\nu = h\frac{c}{\lambda}$

The radiation is emitted according to Planck's law, meaning that it has a spectrum that is determined by the temperature alone, not by the body's shape or composition.

A blackbody in thermal equilibrium has two notable properties:

- It is an *ideal* emitter: at every frequency, it emits as much energy as – or more energy than – any other body at the same temperature.
- It is a *diffuse* emitter: the energy is radiated isotropically, independent of direction.

The resource: the sun



2021/08/30 01:19

Using the previous equations and a temperature of 5800 K, gives a **surface power density** of

$$H_{\text{sun}} = 64 \times 10^6 \text{ W/m}^2.$$

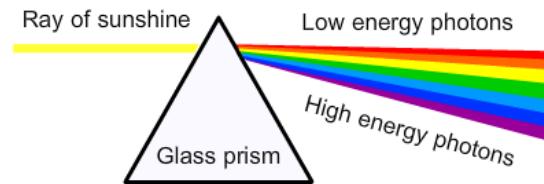
The total power emitted by the sun is calculated by multiplying the emitted power density by the surface area of the sun.

The sun has a radius of $695 \times 10^6 \text{ m}$, giving a surface area of $6.07 \times 10^{18} \text{ m}^2$.

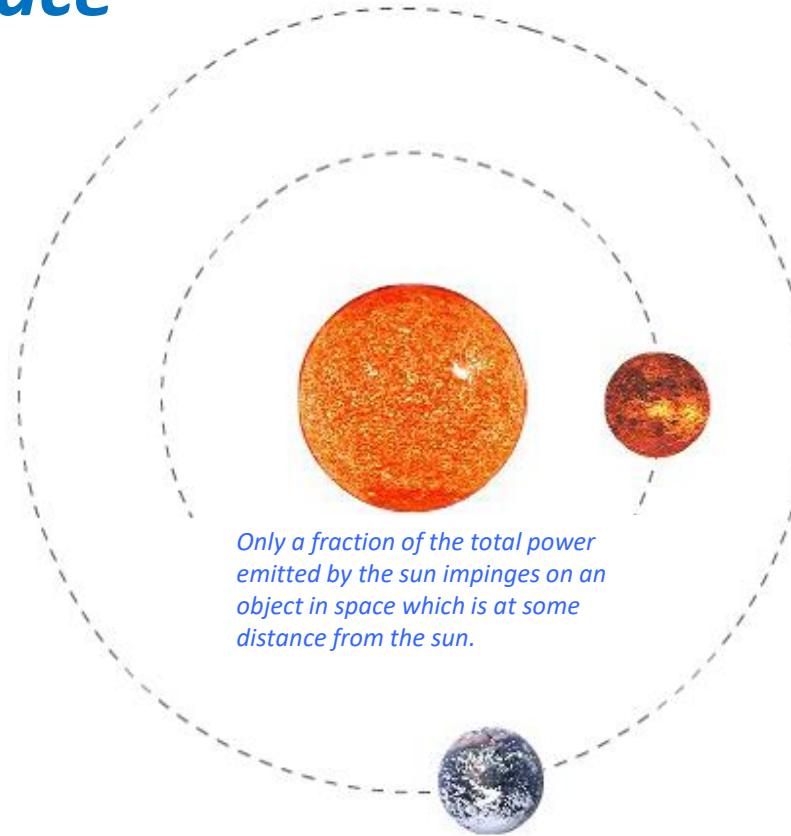
Thus the total power output of the sun is 64×10^6 times $6.07 \times 10^{18} \text{ m}^2$, which is equal to $3.9 \times 10^{26} \text{ W}$.

(Clearly an enormous amount of power compared to power uses on Earth)

The total power emitted from the sun is composed not of a single wavelength, but is composed of many wavelengths and therefore appears white or yellow to the human eye. These different wavelengths can be seen by passing light through a dispersion prism, or water droplets in the case of a rainbow. Different wavelengths show up as different colors, but not all the wavelengths can be seen since some are "invisible" to the human eye.



Solar radiation in space



Only a fraction of the total power emitted by the sun impinges on an object in space which is at some distance from the sun.

At the sun's surface, the power density is that of a blackbody at about 5800 K and the total power from the sun is this value multiplied by the sun's surface area (see previous slide).

However, *at some distance from the sun, the total power from the sun is now spread out over a much larger surface area and therefore the solar irradiance on an object in space decreases as the object moves further away from the sun.*

The solar irradiance on an object at some distance D from the sun is found by dividing the total power emitted from the sun by the surface area over which the sunlight falls.

The total solar radiation emitted by the sun is given by σT^4 multiplied by the surface area of the sun ($4\pi R_{\text{sun}}^2$) where R_{sun} is the radius of the sun.

The surface area over which the power from the sun falls will be $4\pi D^2$ (D is the distance of the object from the (center of the) sun).

Therefore, the solar radiation intensity, H_0 in (W/m^2), incident on an object is:

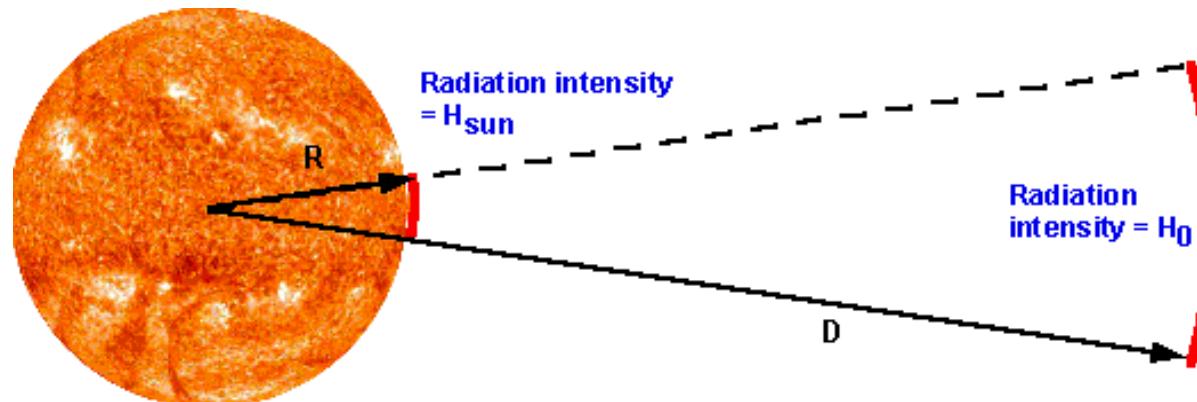
$$H_0 = \frac{R_{\text{sun}}^2}{D^2} H_{\text{sun}} \quad (\text{W/m}^2)$$

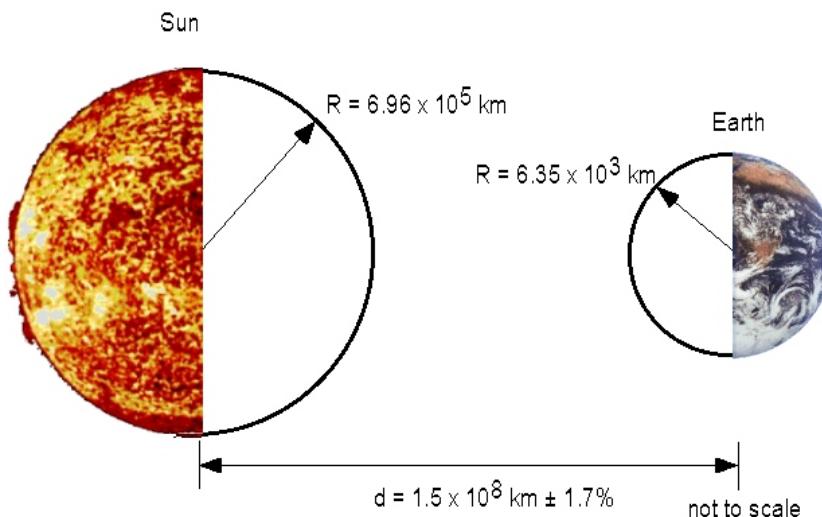
where:

H_{sun} is the power density at the sun's surface (in W/m^2) as determined by Stefan-Boltzmann's blackbody equation;

R_{sun} is the radius of the sun in meters;

D is the distance from the sun in meters





The solar radiation outside the earth's atmosphere is calculated using the radiant power density (H_{sun}) at the sun's surface (6.4×10^7 W/m²), the radius of the sun (R_{sun}), and the distance between the earth and the sun. The calculated **solar irradiance at the Earth's atmosphere is about 1.366 kW/m²**.

The actual power density varies slightly since the Earth-Sun distance changes as the Earth moves in its elliptical orbit around the sun, and because the sun's emitted power is not constant. The power variation due to the elliptical orbit is about 3.4%, with the largest solar irradiance in January and the smallest solar irradiance in July.

$$\frac{H}{H_{\text{constant}}} = 1 + 0.033 \cos\left(\frac{360(n - 2)}{365}\right)$$

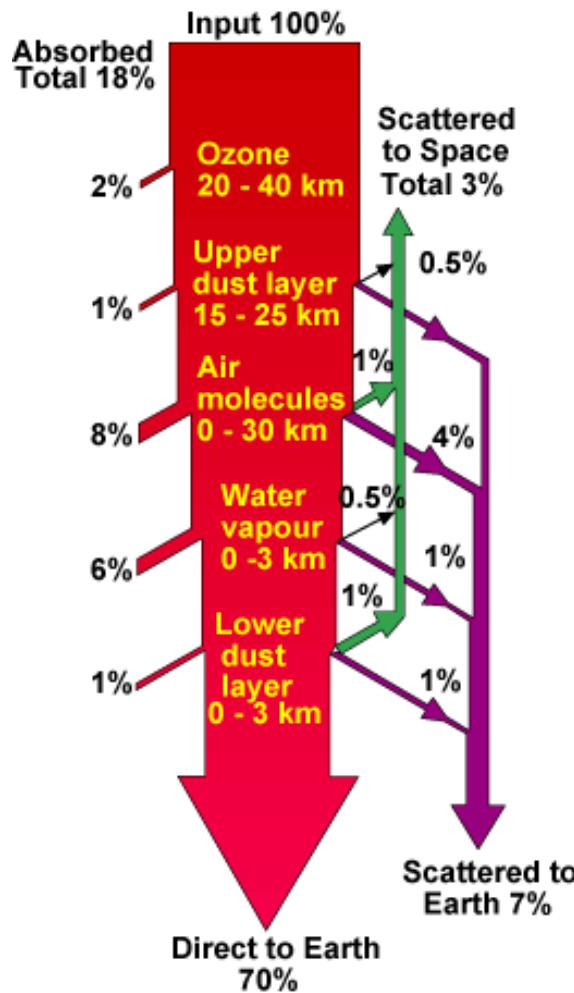
where:

H is the radiant power density outside the Earth's atmosphere (in W/m²);

H_{constant} is the value of the solar constant, 1.353 kW/m²; and n is the day of the year.

These variations are typically small and for photovoltaic applications the solar irradiance can be considered constant.

Terrestrial solar radiation: atmospheric effects

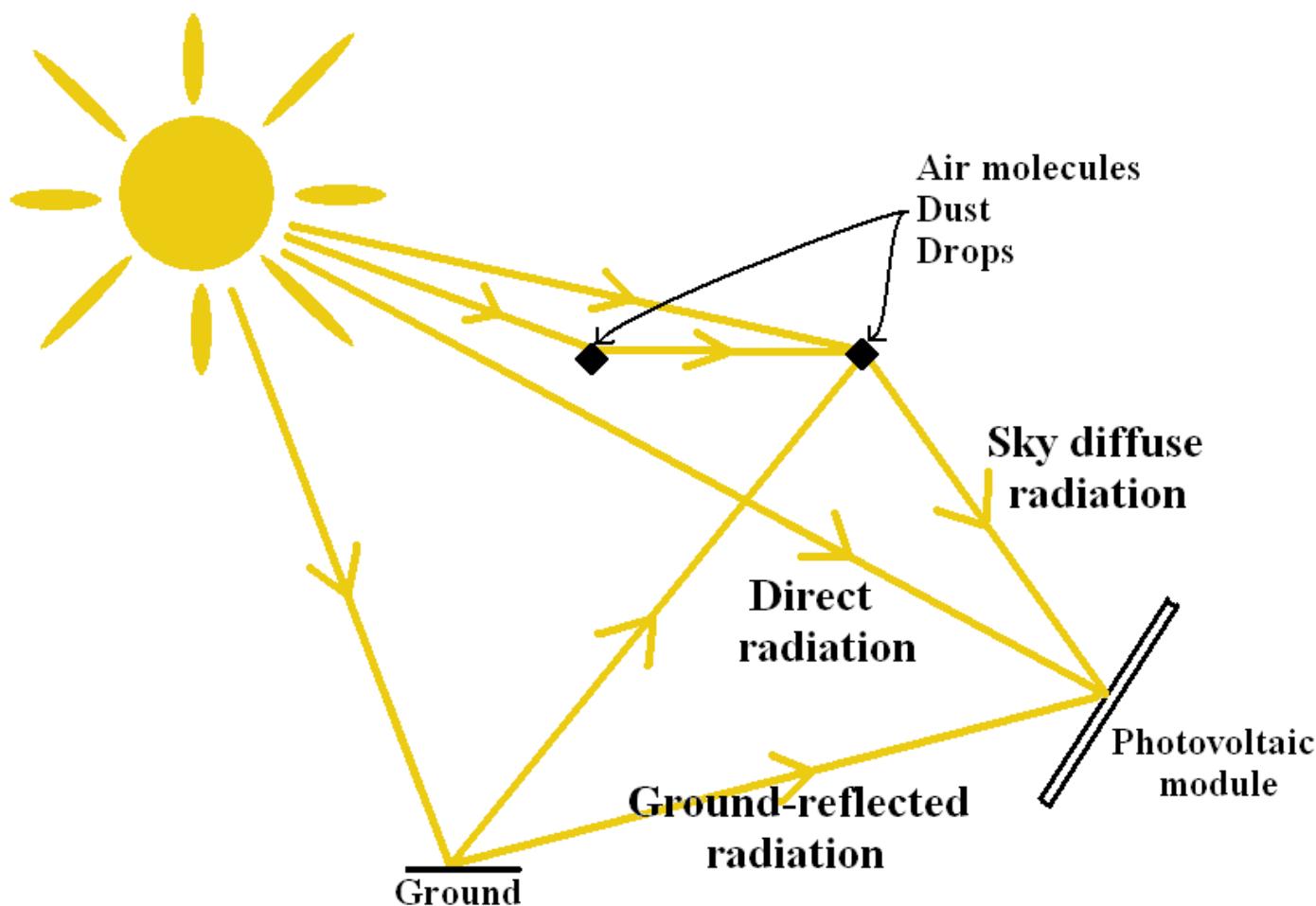


Atmospheric effects have several impacts on the solar radiation at the Earth's surface.

The **major effects for photovoltaic applications** are:

- a reduction in the power of the solar radiation due to absorption, scattering and reflection in the atmosphere;
- a change in the spectral content of the solar radiation due to greater absorption or scattering of some wavelengths;
- the introduction of a diffuse or indirect component into the solar radiation;
- local variations in the atmosphere (such as water vapor, clouds and pollution) which have additional effects on the incident power, spectrum and directionality.

Terrestrial solar radiation: direct radiation and atmospheric effects



Terrestrial solar radiation



While the solar radiation incident on the Earth's atmosphere is relatively constant, the radiation at the Earth's surface varies widely due to:

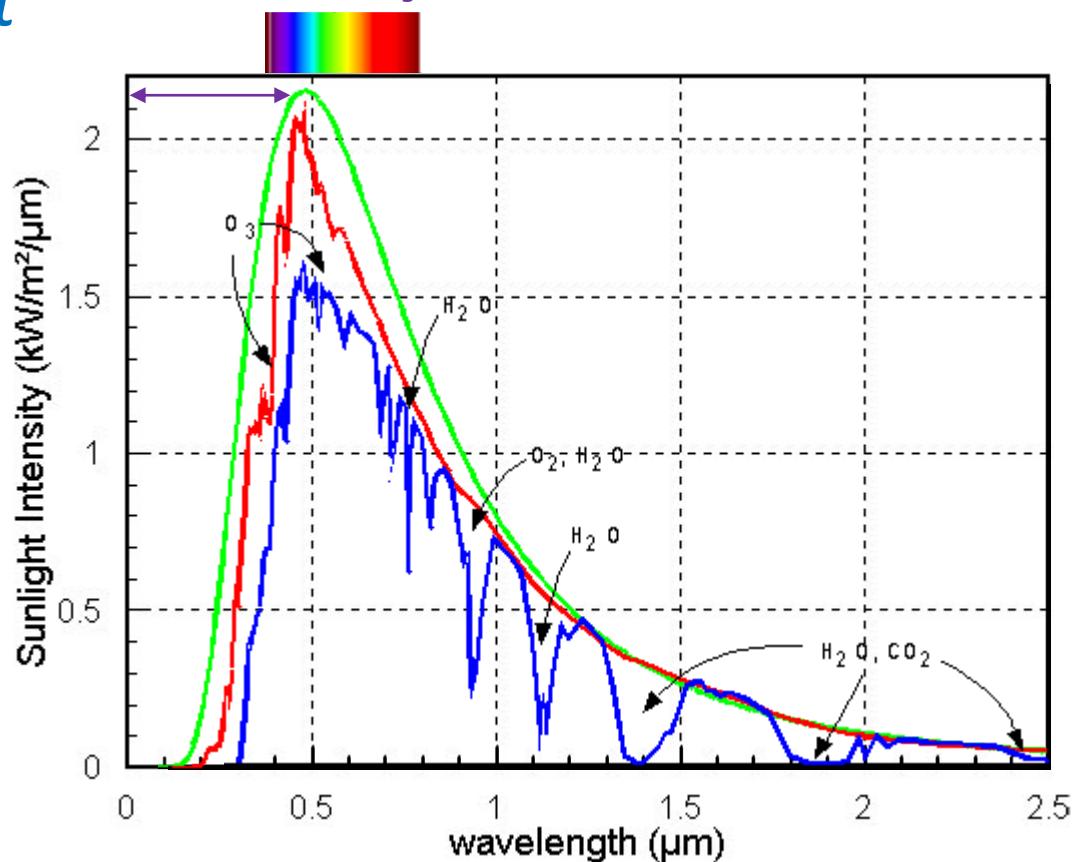
- *atmospheric effects*, including absorption and scattering;
- *local variations in the atmosphere*, such as water vapor, clouds, and pollution;
- *latitude* of the location;
- the *season* of the year and the *time* of day.

The above effects have several impacts on the solar radiation received at the Earth's surface. These changes include *variations in the overall power received, the spectral content of the light and the angle from which light is incident on a surface*.

In addition, a key change is that the *variability of the solar radiation* at a particular location increases dramatically. The variability is due to both local effects such as clouds and seasonal variations, as well as other effects such as the length of the day at a particular latitude. Desert regions tend to have lower variations due to local atmospheric phenomena such as clouds. Equatorial regions have low variability between seasons.

Spectral content

Visible wavelengths



Comparison of solar radiation outside the Earth's atmosphere (red curve) with the amount of solar radiation reaching the Earth itself (blue curve). Green curve indicates the blackbody spectral irradiance curve.
(Note: the human eye has naturally evolved to the point where sensitivity is greatest at the most intense wavelengths)

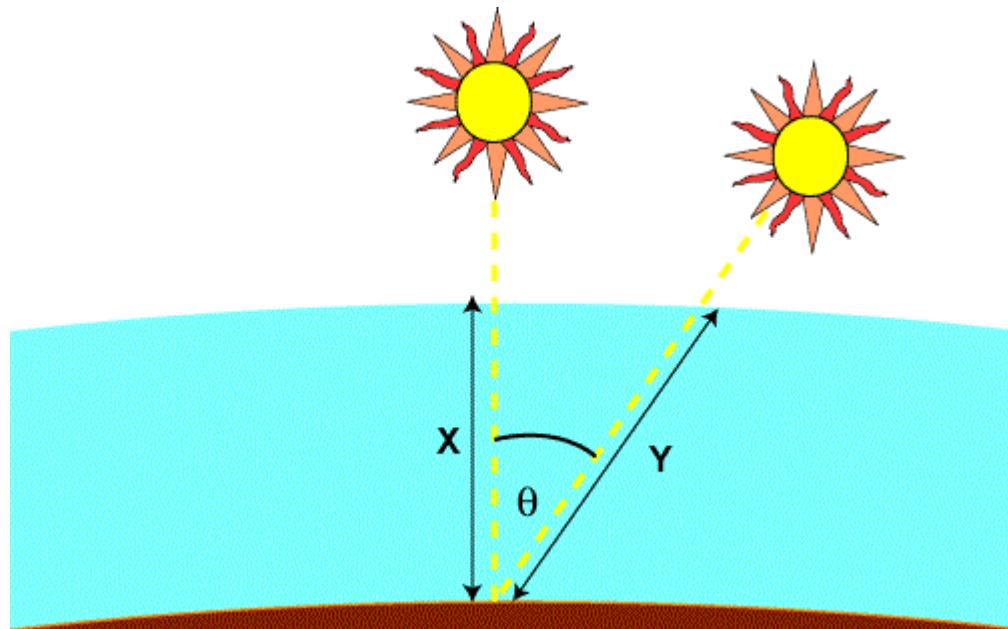
As solar radiation passes through the atmosphere, gasses, dust and aerosols absorb the incident photons. Specific gasses, notably ozone (O₃), carbon dioxide (CO₂), and water vapor (H₂O), have very high absorption of photons that have energies close to the bond energies of these atmospheric gases. This absorption yields deep troughs in the spectral radiation curve.

The Air Mass is the path length which light takes through the atmosphere normalized to the shortest possible path length (that is, when the sun is directly overhead).

The Air Mass quantifies the reduction in the power of light as it passes through the atmosphere and is absorbed by air and dust.

$$AM = \frac{1}{\cos(\theta)}$$

where θ is the *angle from the vertical*, called the zenith angle. (see further on slide n° 43)



The air mass represents the proportion of atmosphere that the light must pass through before striking the Earth relative to its overhead path length, and is equal to Y/X.

→ *Definition of Standard Spectra*

The efficiency of a solar cell is sensitive to variations in both the power and the spectrum of the incident light.

To facilitate an accurate comparison between solar cells measured at different times and locations, a standard spectrum and power density has been defined for both radiation outside the Earth's atmosphere and at the Earth's surface.

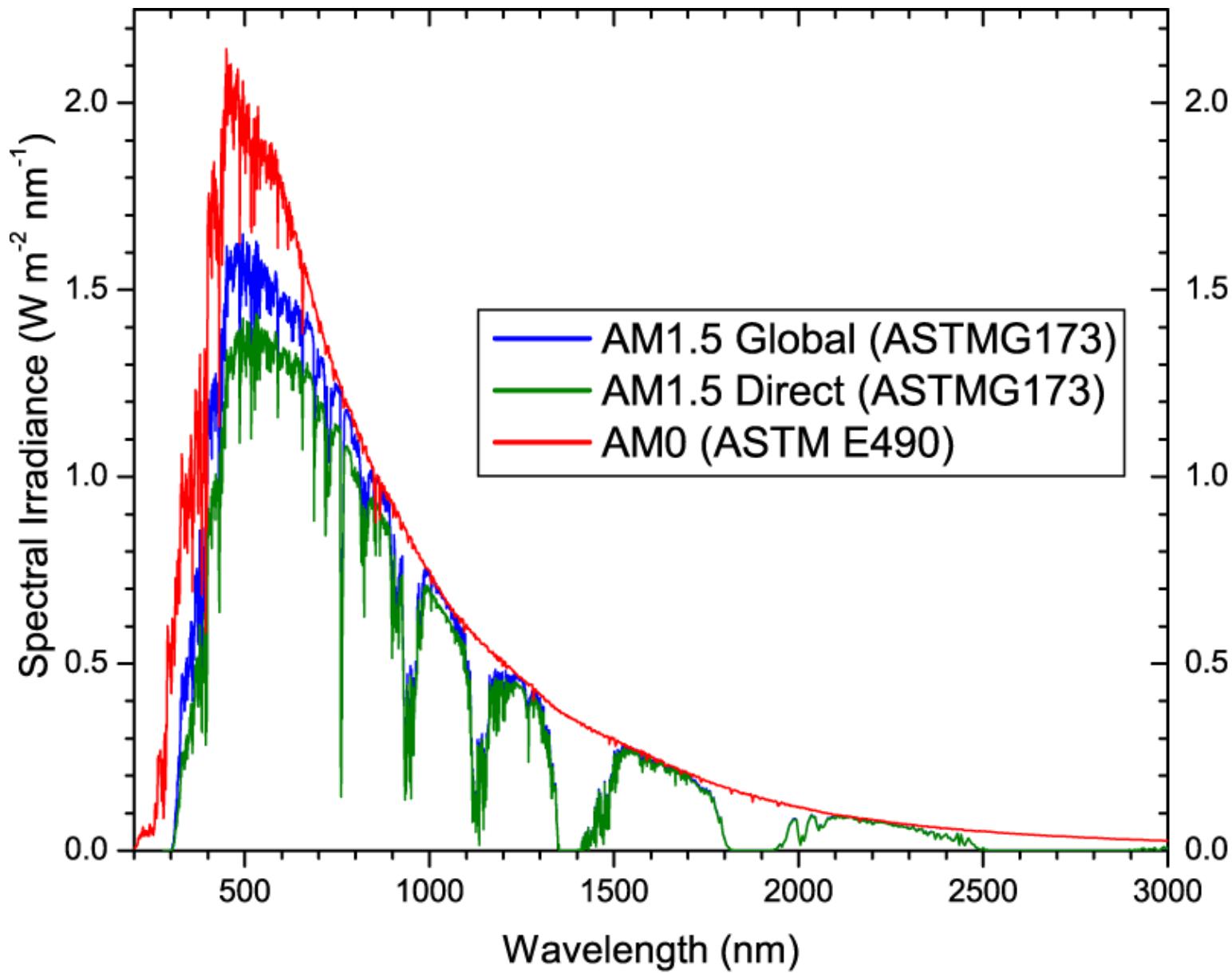
- The **standard spectrum outside the Earth's atmosphere** is called **AM0**, because at no stage does the light pass through the atmosphere. This spectrum is typically used to predict the expected performance of cells in space.
- The **standard spectrum at the Earth's surface** is called **AM1.5G**, (the G stands for “global” and includes both direct and diffuse radiation) or **AM1.5D** (which includes direct radiation only).

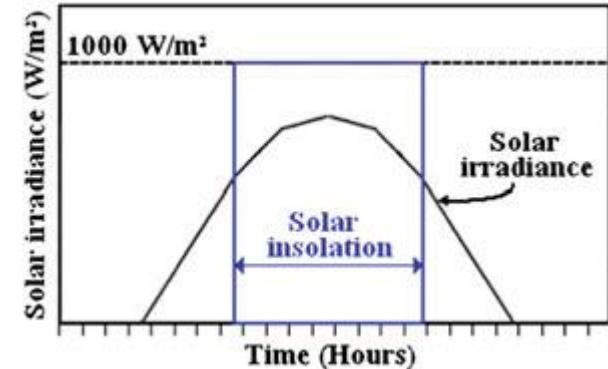
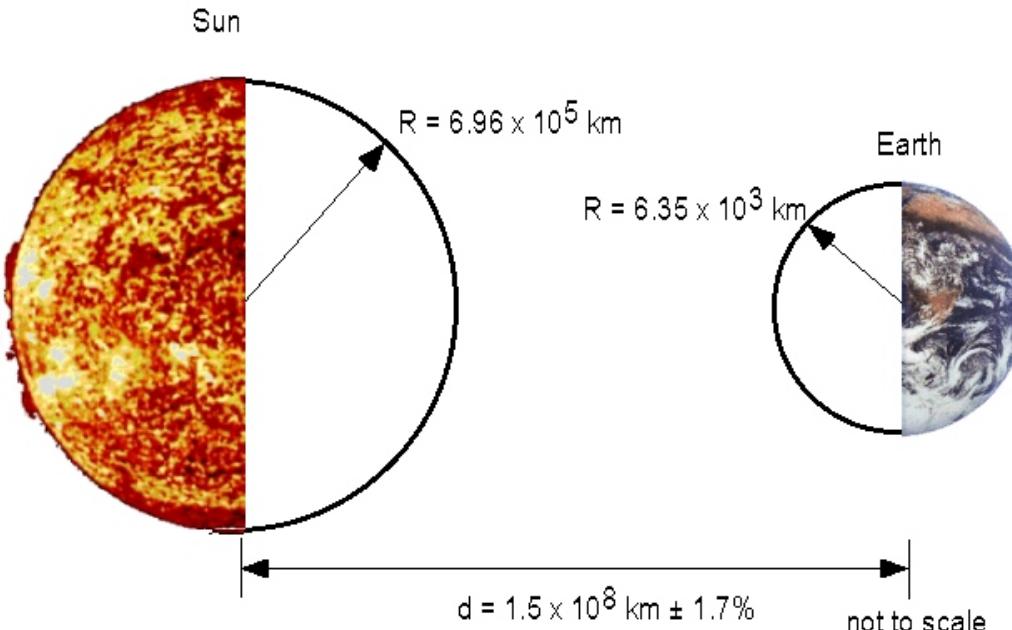
The intensity of AM1.5D radiation can be approximated by reducing the AM0 spectrum by 28% (18% due to absorption and 10% to scattering). The global spectrum is 10% higher than the direct spectrum. These calculations give approximately 970 W/m² for AM1.5G.

However, *the standard AM1.5G spectrum has been normalized to give 1kW/m² due to the convenience of the round number and the fact that there are inherently variations in incident solar radiation.*

N.B. « 1.5 » means that the standard spectra have been arbitrarily defined as to correspond to an air mass $AM = 1.5$ (i.e. $\theta = 48,2^\circ$)

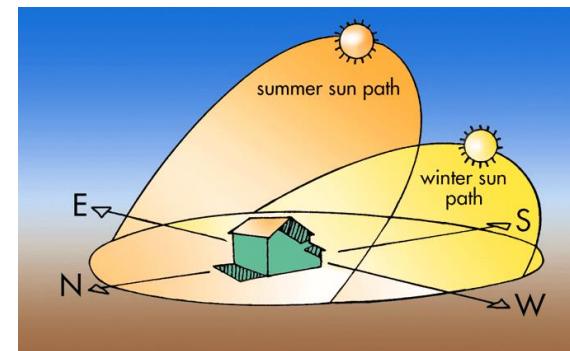
→ Standard spectra





N.B. Referring to a base « standard conditions » irradiance of 1 kW/m^2 , irradiation is sometimes expressed in hours (h) and called “solar insolation”

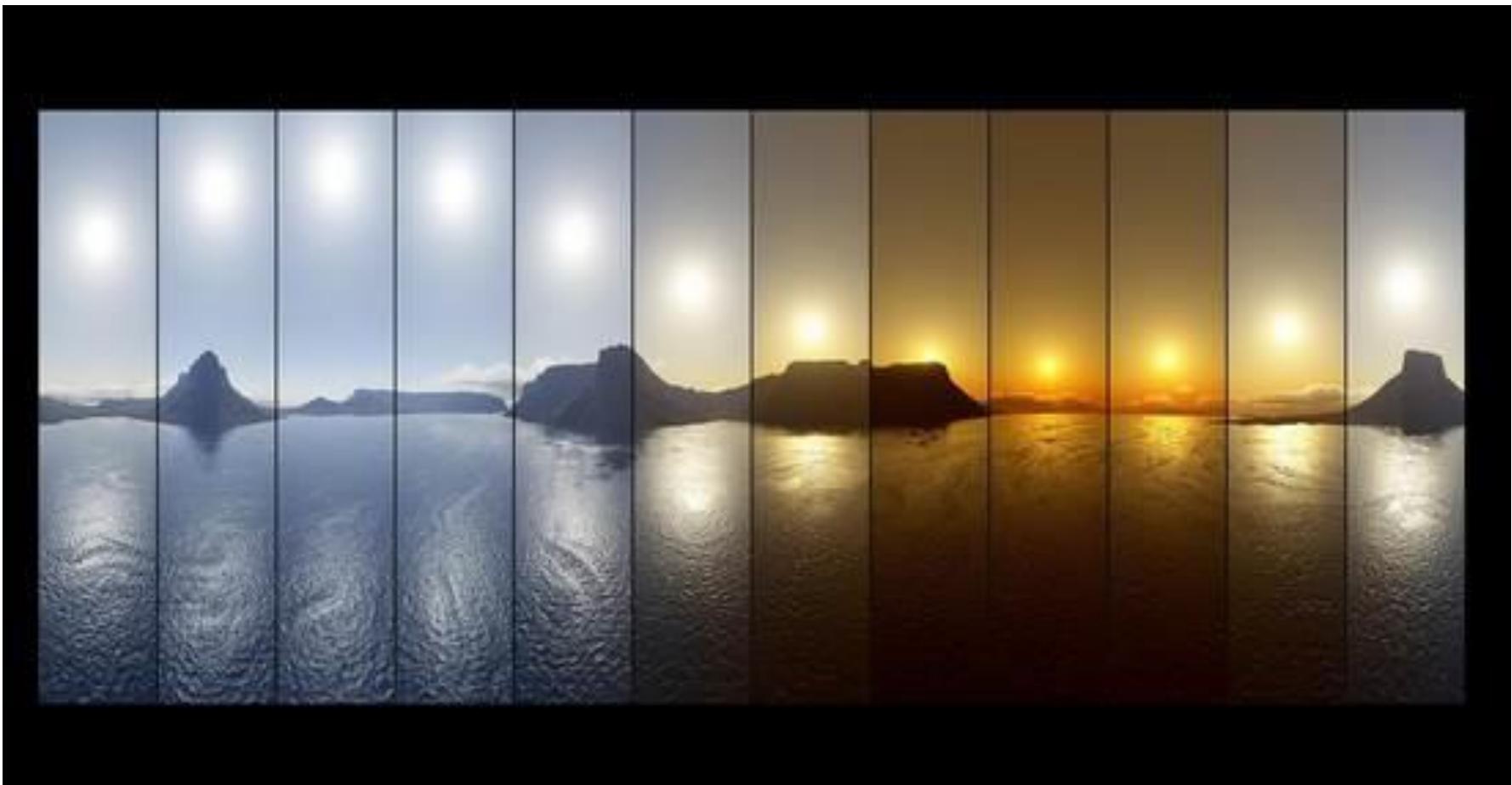
- **Irradiance** (French: *Éclairage*) = *flux of solar radiation (power) on a given surface (W/m²)*
 - $\approx 1,366 \text{ kW/m}^2$ at the outer atmosphere
 - $\approx 1 \text{ kW/m}^2$ (peak) on the earth's surface (horizontal plane), in Belgium
- **Irradiation** (French: *Insolation*) = time integral of irradiance (Wh/m^2) = *solar irradiance energy received on a given surface in a given time*



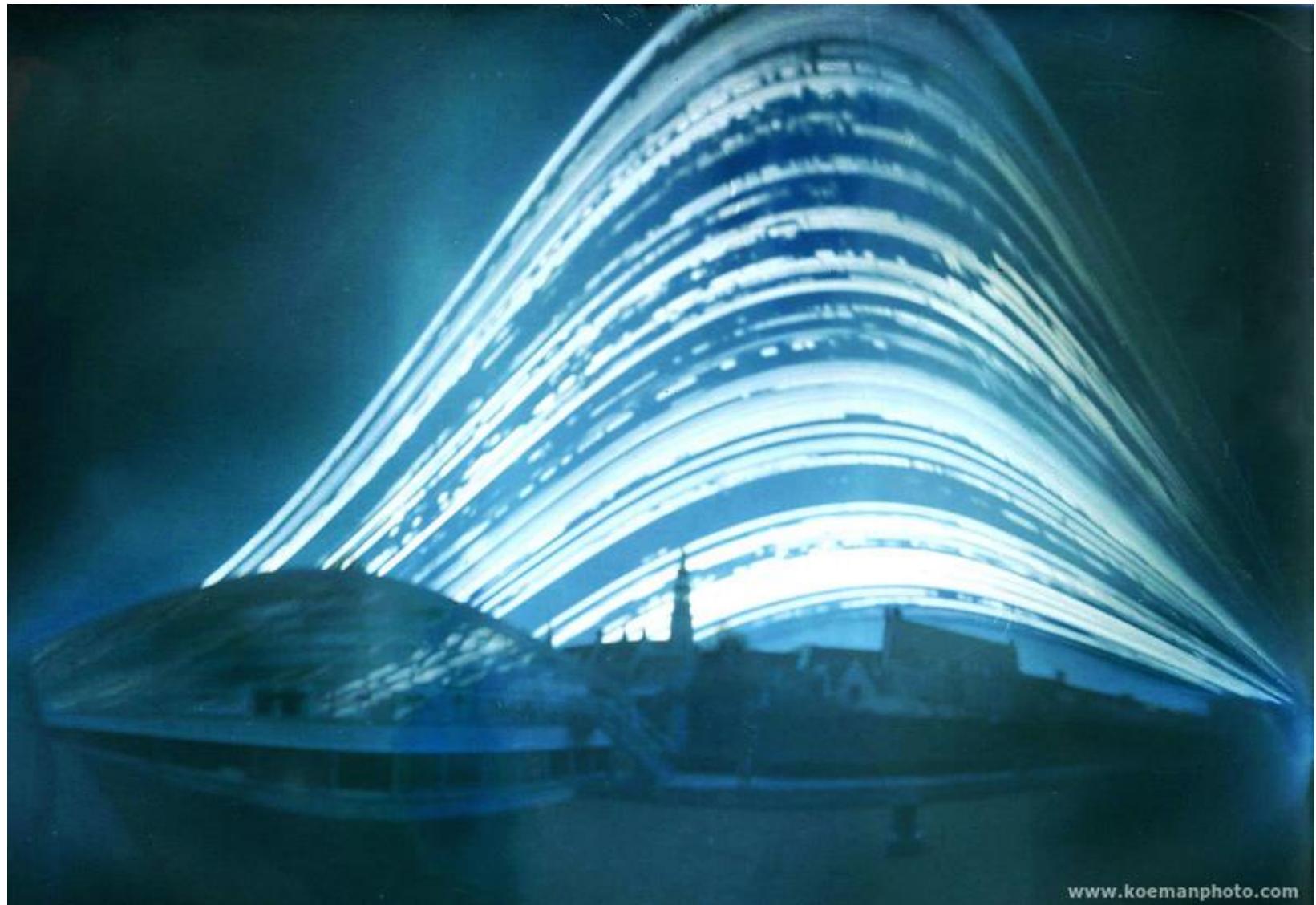
Characterization of the resource (2)

*Apparent motion of the sun in the sky
and its consequences on PV generation*

Apparent motion of the sun in the sky: the sun path

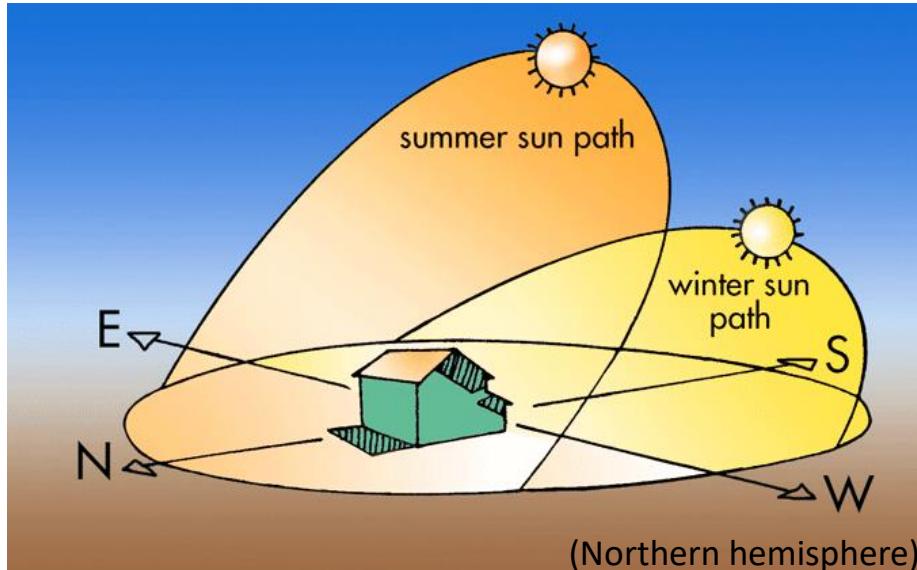


Apparent motion of the sun in the sky: the sun path



www.koemanphoto.com

Apparent motion of the sun in the sky: the sun path



Characterization by two angles:
Elevation (altitude) & Azimuth

The apparent motion of the sun, caused by the rotation of the Earth about its axis, changes the angle at which the direct component of light will strike the Earth.

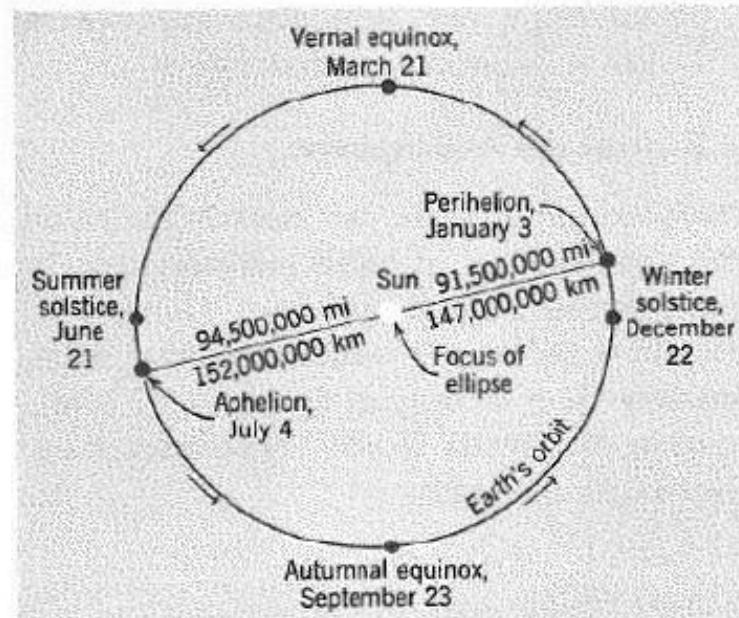
From a fixed location on Earth, the sun appears to move throughout the sky. The position of the sun depends on the location of a point on Earth, the time of day and the time of year.

This apparent motion of the sun has a major impact on the amount of power received by a solar collector.

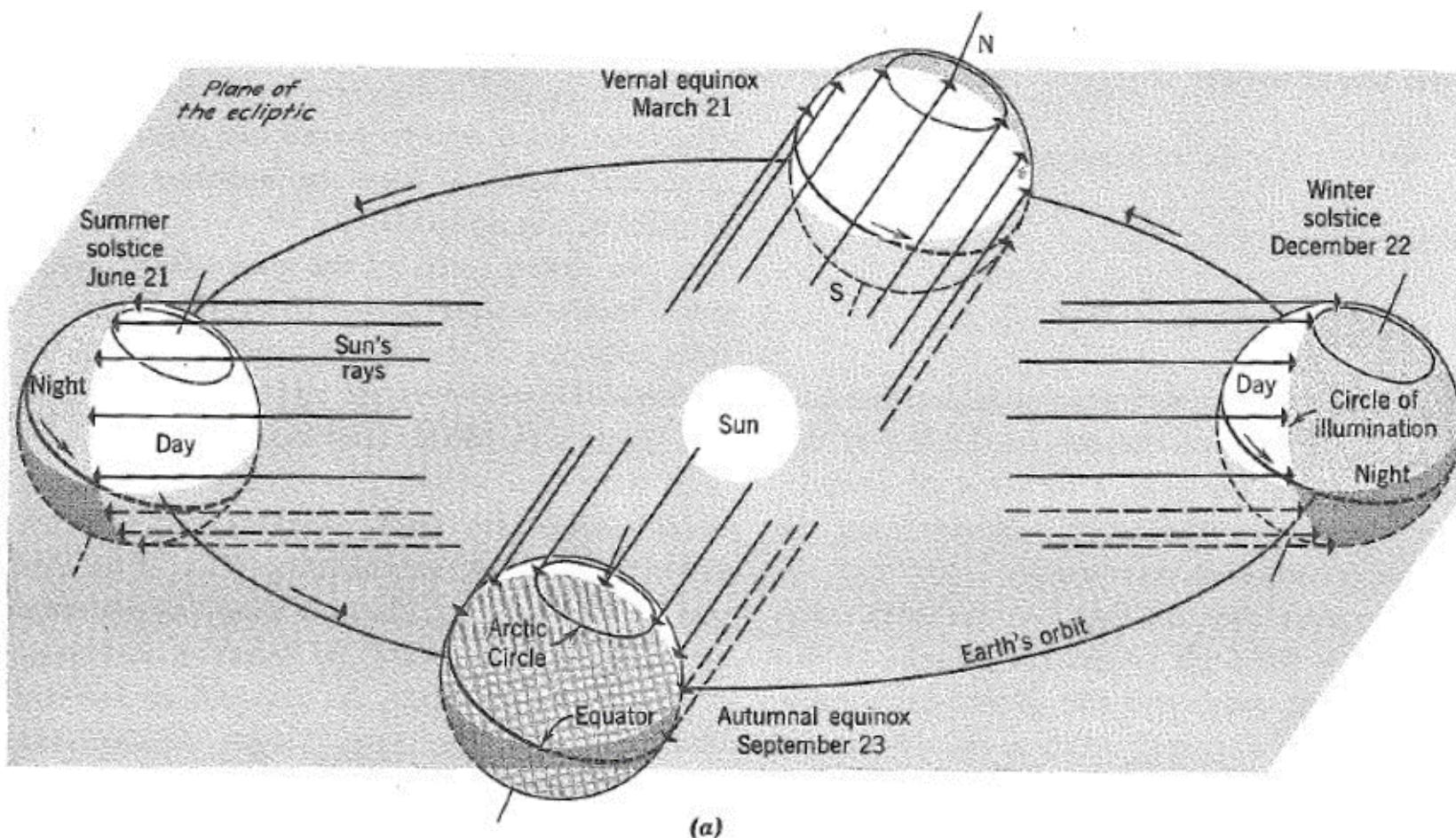
When the sun's rays are perpendicular to the absorbing surface, the power density on the surface is equal to the incident power density. However, as the angle between the sun and the absorbing surface changes, the intensity on the surface is reduced. When the module is parallel to the sun's rays (and the angle to the module normal = 90°) the intensity of light essentially falls to zero. For intermediate angles, the relative power density is $\cos(\theta)$ where θ is the angle between the sun's rays and the module normal.

Earth – Sun movement

Figure 1–5. Earth-sun relationships. (a) Earth's orbit around the sun and illumination characteristics at the equinoxes and solstices. (b) The angle of the sun in the sky at different latitudes at the equinox (upper) and the northern hemisphere winter solstice (lower). (From Strahler, A.N., *Physical Geography*, Copyright © 1969 by John Wiley and Sons. Reproduced by permission.)



Earth – Sun movement

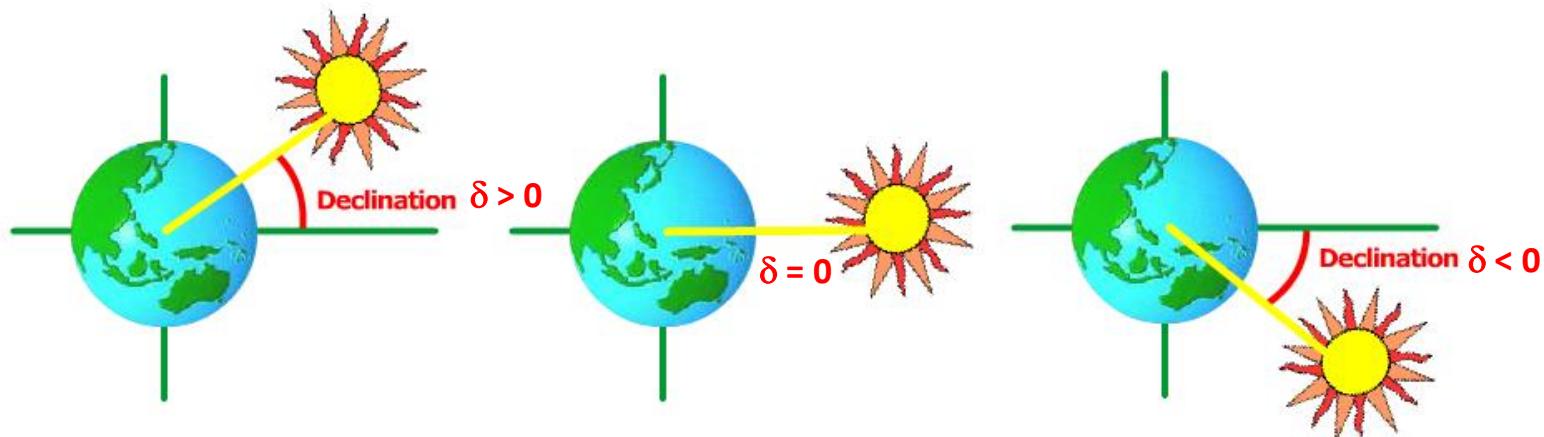


https://www.youtube.com/watch?v=taHTA7S_JGk

Mathematical description of the Earth – Sun movement: the declination angle δ

The declination of the sun is the angle between the equator and a line drawn from the center of the Earth to the center of the sun.

Summer solstice in the northern hemisphere. The declination angle (δ) is at its maximum and is 23.45° .



Spring equinox in the northern hemisphere and autumn equinox in the southern hemisphere. The declination angle (δ) is 0° .

Winter solstice in the northern hemisphere and summer solstice in the southern hemisphere. The declination angle (δ) is -23.45° .

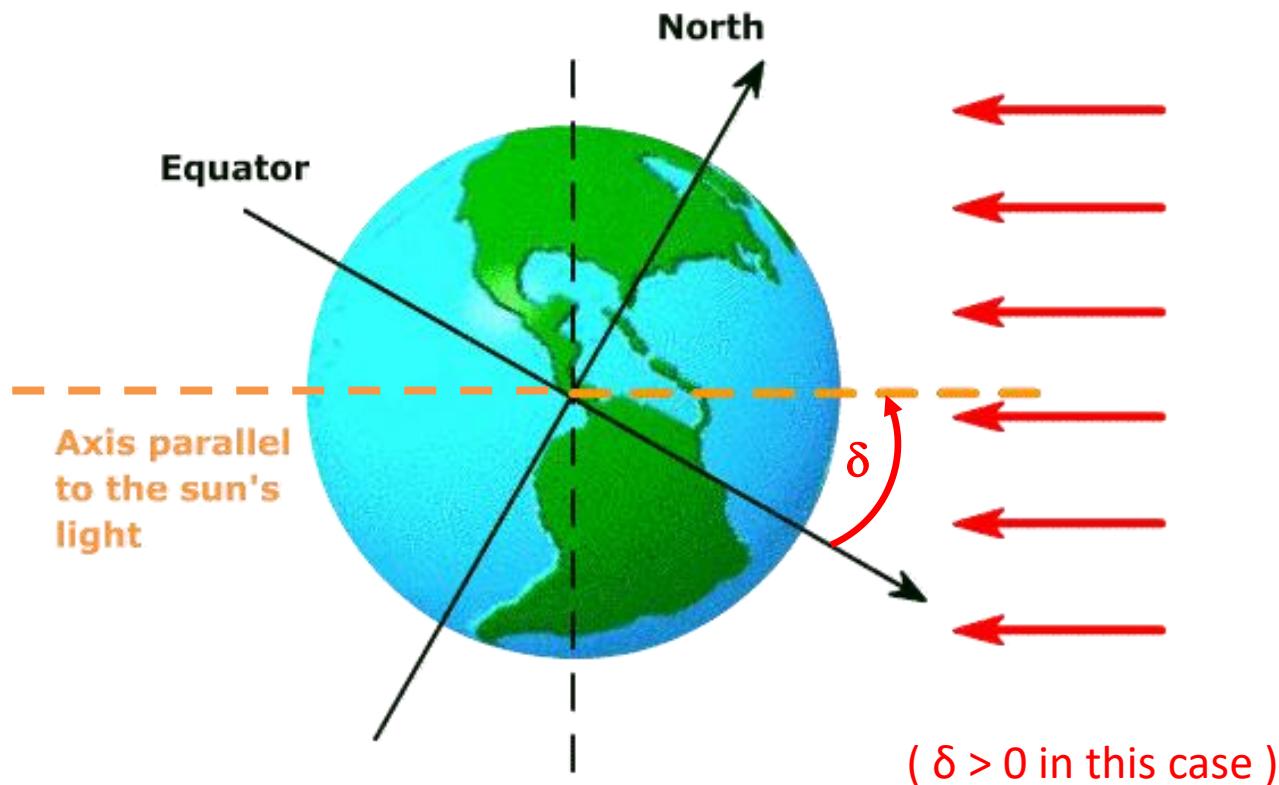
The declination angle, denoted by δ , varies seasonally due to the tilt of the Earth on its axis of rotation and the rotation of the Earth around the sun.

If the Earth were not tilted on its axis of rotation, the declination would always be 0° . However, the Earth is tilted by 23.45° and the declination angle varies plus or minus this amount.
(Only at the spring and fall equinoxes is the declination angle equal to 0°)

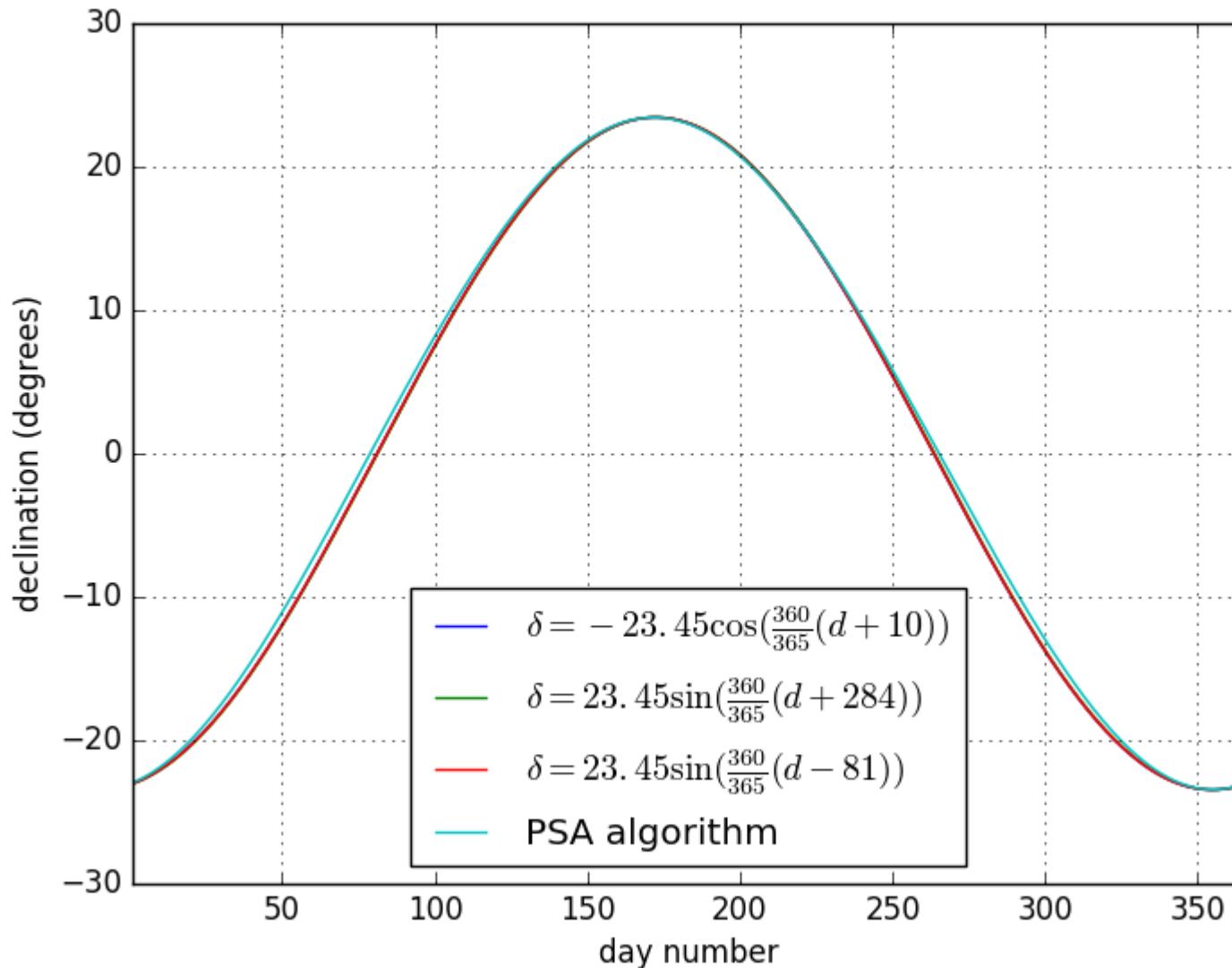
$$\delta = -23.45^\circ \times \cos\left(\frac{360}{365} \times (d + 10)\right)$$

(d = day number)

The tilt of the earth compared to the sun, given by the declination angle δ , depends on the season. Shown here is the maximum declination angle, occurring at summer solstice in the northern hemisphere and winter solstice in the southern hemisphere



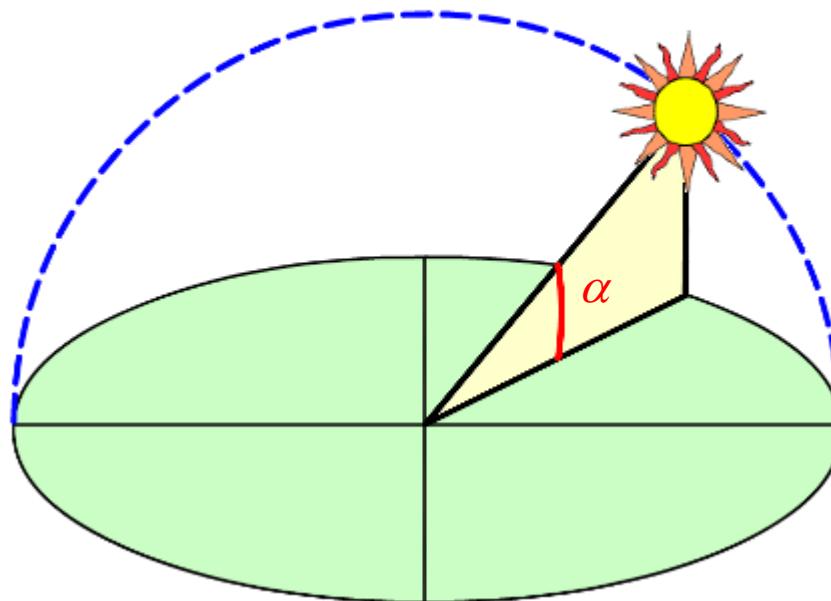
Earth – Sun movement: declination angle



Declination angle: various models found in the literature

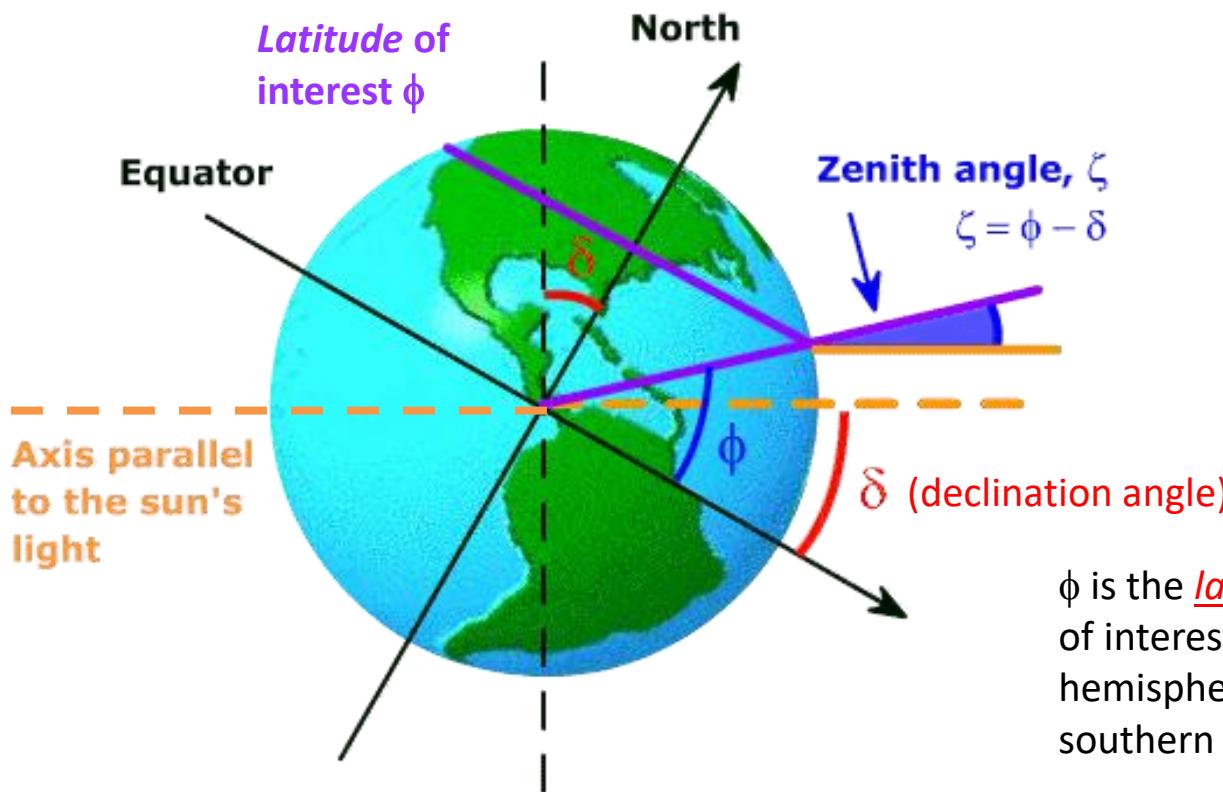
Mathematical description of the Earth – Sun movement: the elevation angle (altitude angle) α

- The elevation angle (also known as altitude angle) is the angular height of the sun in the sky, measured from the horizontal.
- The elevation is 0° at sunrise and can go up to 90° when the sun is directly overhead (which occurs for example at the equator on the spring and fall equinoxes).
- The elevation angle varies throughout the day.
- It also depends on the latitude of a particular location and the day of the year.



Zenith angle ζ at solar noon

The zenith angle, ζ , at solar noon is defined as the angle between the incident sunlight and the particular location and is given by $\phi - \delta$.

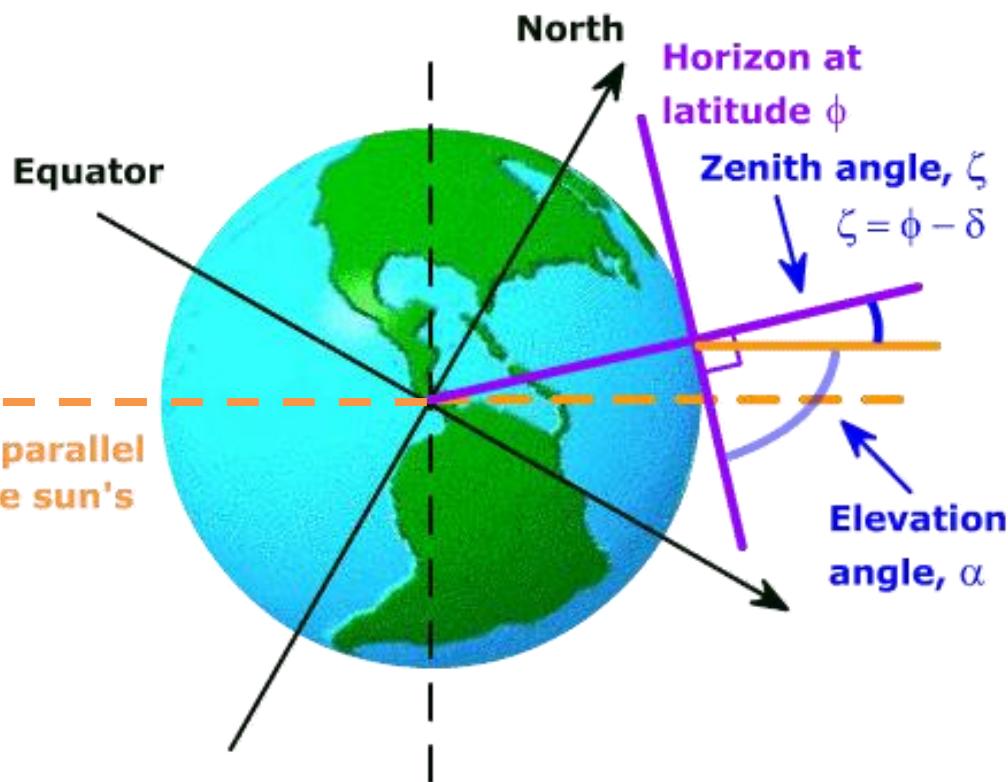


ϕ is the *latitude* of the location of interest (+ for the northern hemisphere and - for the southern hemisphere),

δ is the *declination angle*, which depends on the day of the year.

Maximum elevation angle α (elevation angle at solar noon)

The elevation or altitude angle, α , is defined from the horizontal plane and is given by 90° – the zenith angle, or 90° – $(\phi - \delta)$

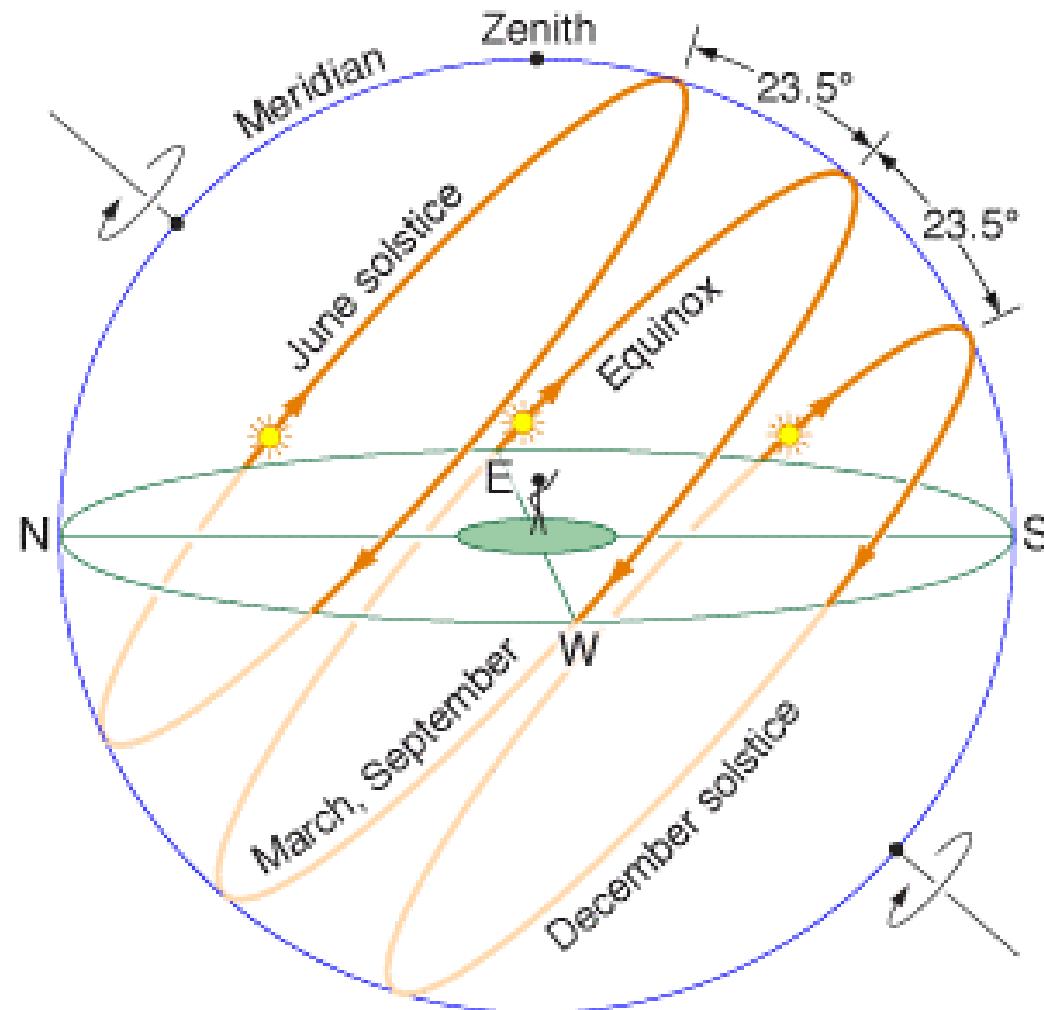


$$\begin{aligned}\alpha_{\max} &= 90^\circ - \zeta \\ &= 90^\circ - (\phi - \delta)\end{aligned}$$

ϕ is the *latitude* of the location of interest (+ for the northern hemisphere and - for the southern hemisphere),

δ is the *declination angle*, which depends on the day of the year.

Variation of the maximum elevation angle α (elevation angle at solar noon)



Elevation angle α at any time ?

(Mathematical description of its variation throughout the day)

$$\alpha_{max} = 90^\circ - \varphi + \delta$$

φ = latitude

δ = declination angle

- At the Tropic of Cancer on the (northern hemisphere) summer solstice, the sun is directly overhead and the elevation angle is 90° .
- While the maximum elevation angle is used in very simple PV system design, more accurate PV system simulation requires the knowledge of how the elevation angle varies throughout the day :

$$\alpha = \sin^{-1} [\sin\delta \sin\varphi + \cos\delta \cos\varphi \cos(HRA)]$$

declination angle

latitude

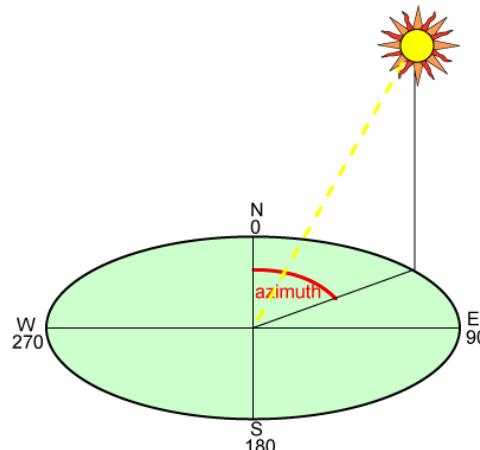
$HRA = \text{« Hour Angle »}$
 $= \underline{\text{image of the Local Solar Time}}$
 (LST)

Mathematical description of the Earth – Sun movement

Variation of the compass direction of the sunlight: the azimuth angle Θ

- The azimuth angle Θ is the compass direction from which the sunlight is coming.
- At solar noon, the sun is always directly south in the northern hemisphere and directly north in the southern hemisphere.
- The azimuth angle varies throughout the day.
- At the equinoxes, the sun rises directly east and sets directly west regardless of the latitude, thus making the azimuth angles 90° at sunrise and 270° at sunset.
- In general however, the azimuth angle varies with the latitude and time of year

$$\Theta = \cos^{-1} \left[\frac{\sin \delta \cos \varphi - \cos \delta \sin \varphi \cos(HRA)}{\cos \alpha} \right]$$



*HRA = « Hour Angle »
= image of the Local Solar Time (LST)*

N.B. Azimuth angle is measured clockwise from North

Detailed mathematical description of the Earth – Sun movement: Local Solar Time and Hour Angle

- ***Local Solar Time (LST) vs Local Time (LT)***

Twelve noon local solar time (LST) is defined as when the sun is highest in the sky.

Local time (LT) usually varies from LST because of

- human adjustments such as *time zones* and *daylight saving*,
- the eccentricity of the Earth's orbit.

- ***Hour Angle (HRA)***

The Hour Angle converts the local solar time (LST) into an angle being the number of degrees which the sun moves across the sky. This angle is referred to solar noon.

By definition, the Hour Angle is 0° at solar noon.

Since the Earth rotates 15° per hour ($15^\circ = 360^\circ/24$ hours), each hour away from solar noon corresponds to an angular motion of the sun in the sky of 15° .

In the morning the hour angle is negative, in the afternoon the hour angle is positive.

$$HRA = 15^\circ(LST - 12)$$

Calculation of Local Solar Time LST (as related to the Local Time) (1)

How to calculate the local solar time(LST) from the local time (LT) ?

Time correction

$$LST = LT + \frac{TC}{60}$$

- Time Correction component n°1: Local Standard Time Meridian (LSTM)

The Local Standard Time Meridian (LSTM) is a reference meridian used for a particular time zone:

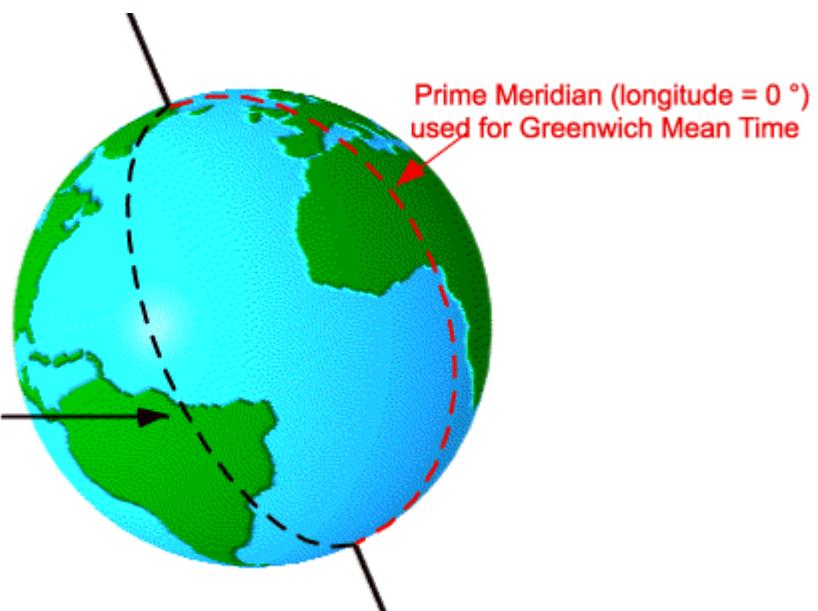
$$LSTM = 15^{\circ} \Delta T_{UTC}$$

where ΔT_{UTC} is the difference of the *Local Time (LT)* from Universal Coordinated Time (UTC) in hours.

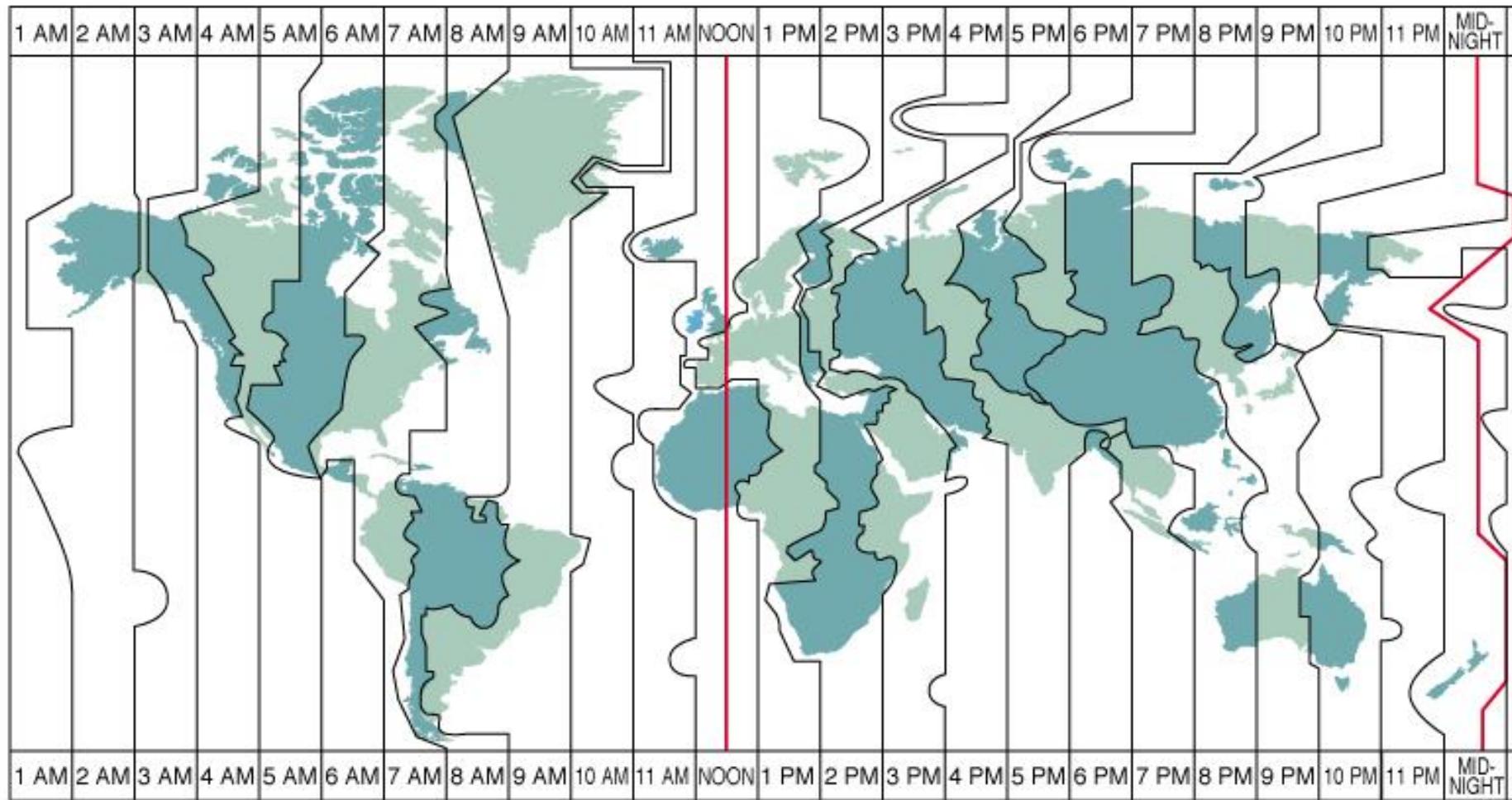
ΔT_{UTC} is also equal to the time zone.

Note: $15^{\circ} = 360^{\circ}/24$ hours.

LSTM (Local Standard Time Meridian) used in a local time zone. Shown here is the LSTM for the time zone incorporating parts of Brazil and Greenland.



Calculation of Local Solar Time (as related to the Local Time) (2)



© Addison-Wesley Longman

$$LSTM = 15^0 \Delta T_{UTC}$$

where ΔT_{UTC} is the difference of the *Local Time (LT)* from Universal Coordinated Time (UTC) in hours.
 ΔT_{UTC} is also equal to the time zone.

Calculation of Local Solar Time (as related to the Local Time) (3)

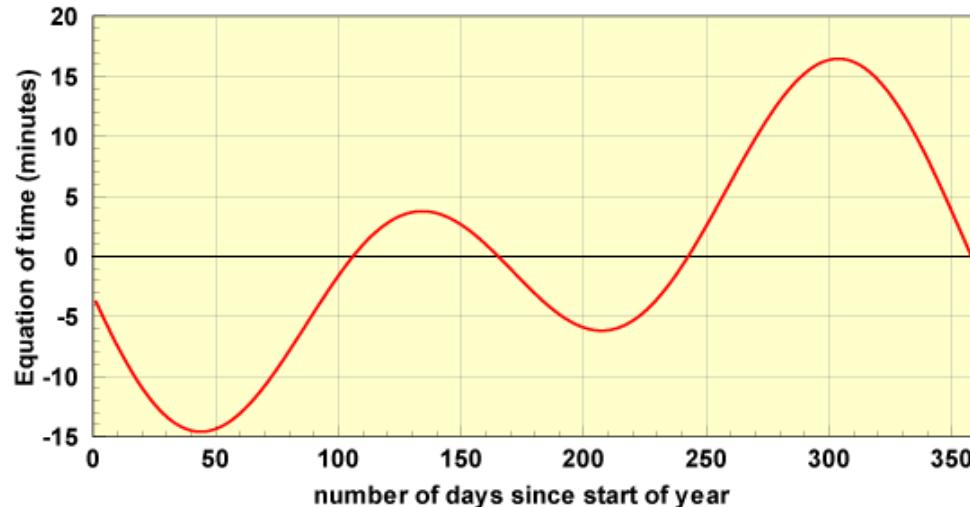
- Time Correction component n°2: Equation of time (EoT)

The equation of time (EoT) (given in minutes) is an empirical equation that corrects for the *eccentricity of the Earth's orbit* and the *Earth's axial tilt*. An approximation accurate to within $\frac{1}{2}$ minute is :

$$EoT = 9.87 \sin(2B) - 7.53 \cos(B) - 1.5 \sin(B)$$

$$B = \frac{360}{365}(d - 81)$$

B is expressed in degrees and d is the number of days since the start of the year



Calculation of Local Solar Time (as related to the Local Time) (4)

- *Net Time Correction Factor (TC)*

The net Time Correction Factor (in minutes) accounts for the variation of the Local Solar Time (LST) within a given time zone due to the longitude variations within the time zone and also incorporates the *EoT* i.e. it is a combination of corrections n°1 & 2 introduced in the previous slides:

$$TC = 4(\text{Longitude} - LSTM) + EoT$$

The factor of 4 (minutes) comes from the fact that the Earth rotates 1° every 4 minutes.

- *Local Solar time (LST):*

$$LST = LT + \frac{TC}{60}$$

- *Hour Angle:*

$$HRA = 15^\circ(LST - 12)$$

The Hour Angle converts the local solar time (LST) into the number of degrees which the sun moves across the sky. This angle is referred to solar noon.

By definition, the Hour Angle is 0° at solar noon.

Mathematical description of the Earth-Sun movement: summary of the calculation procedure

Input data:

- day number (d = number of days since the beginning of the year)
- local time (LT)
- geographical position: Latitude (φ) and Longitude

Sun position calculation procedure:

Intermediate quantities for the calculation:

- ΔT_{UTC} = difference of the Local Time (LT) from Universal Coordinated Time (UTC), in hours
 - Local Standard Time Meridian (LSTM): $LSTM = 15^0 \Delta T_{UTC}$
 - Equation of time: $B = \frac{360}{365} (d - 81)$
$$EoT = 9.87 \sin(2B) - 7.53 \cos(B) - 1.5 \sin(B)$$
- Time correction factor: $TC = 4(\text{Longitude} - LSTM) + EoT$
- Local Solar Time & **Hour Angle**: $LST = LT + \frac{TC}{60}$ $HRA = 15^\circ(LST - 12)$
- **Declination angle**:
$$\delta = -23.45^\circ \times \cos\left(\frac{360}{365} \times (d + 10)\right)$$

Mathematical description of the Earth: summary of the model

Final characterizing quantities:

Elevation angle: $\alpha = \sin^{-1} [\sin\delta \sin\varphi + \cos\delta \cos\varphi \cos(HRA)]$

Azimuth angle: $\theta = \cos^{-1} \left[\frac{\sin\delta \cos\varphi - \cos\delta \sin\varphi \cos(HRA)}{\cos\alpha} \right]$

Apparent motion of the sun in the sky: the sun path

Characterized by two angles: azimuth & elevation (or altitude)

Sun Positions

40 degrees North Latitude

measured by

Azimuth &
Altitude

Jun 21

8 AM

7 AM

6 AM

5 AM

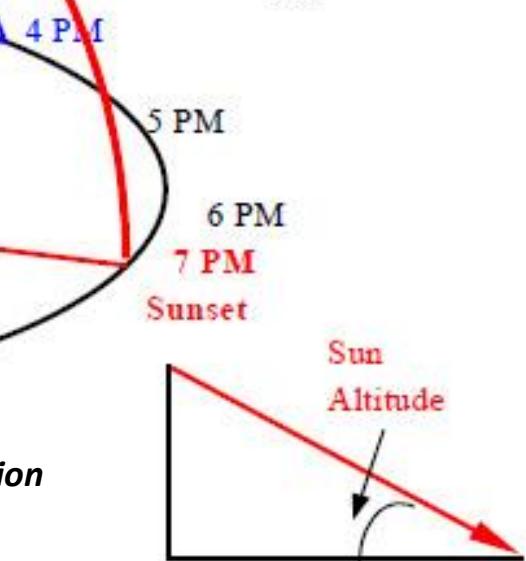
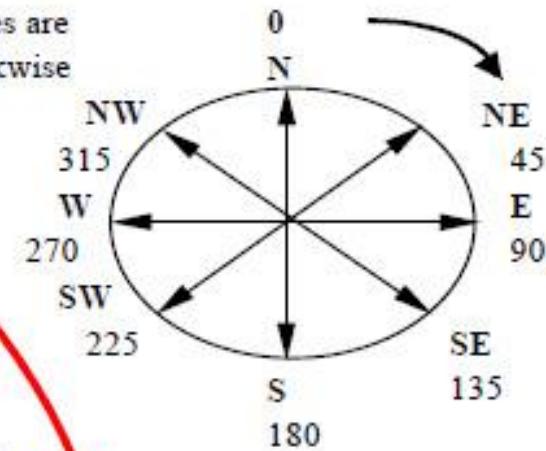
Jun 21

Dec 22

12 Noon

Midnight
North AZ = 0

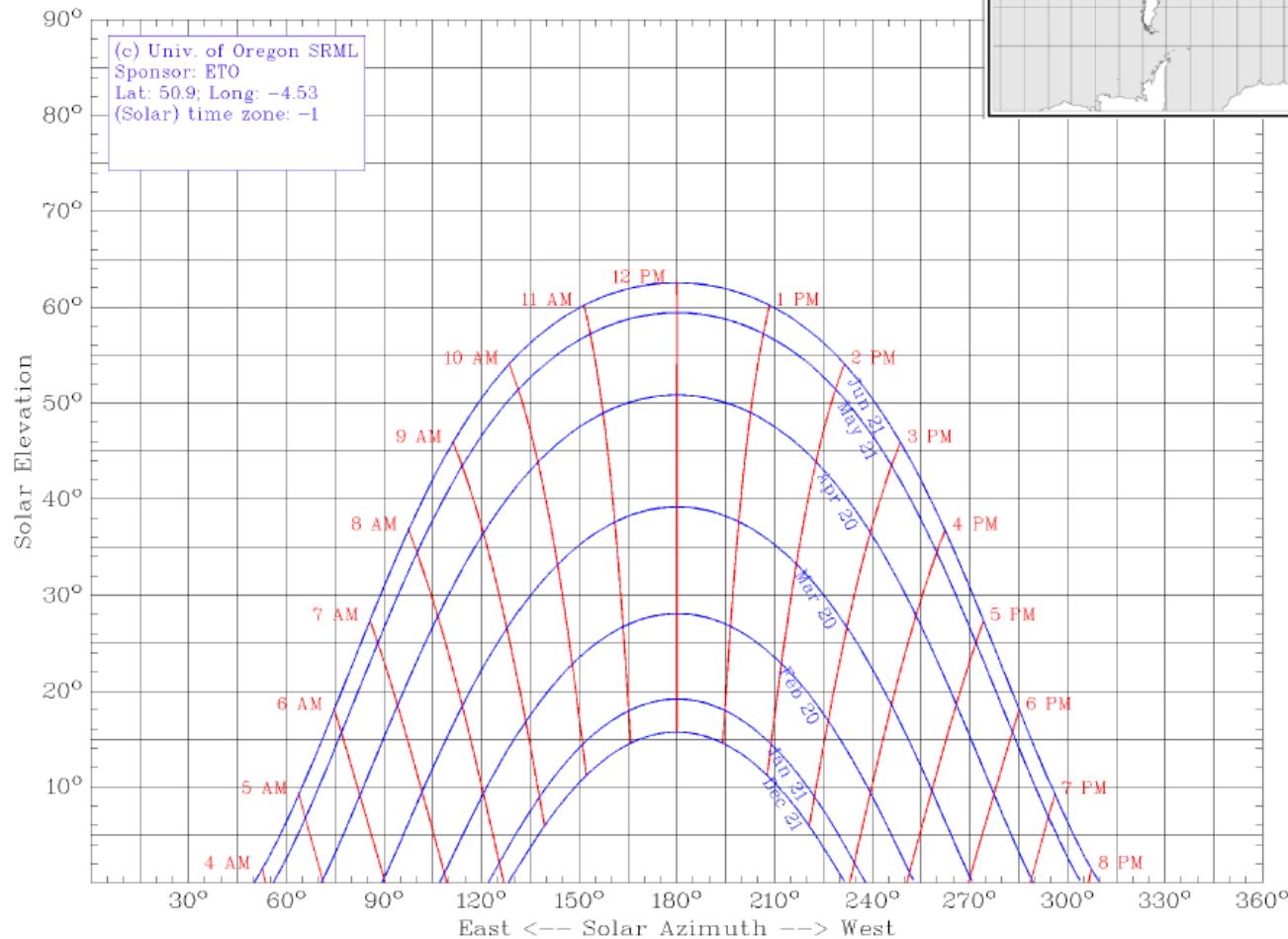
Azimuth Angles are
measured clockwise
from North

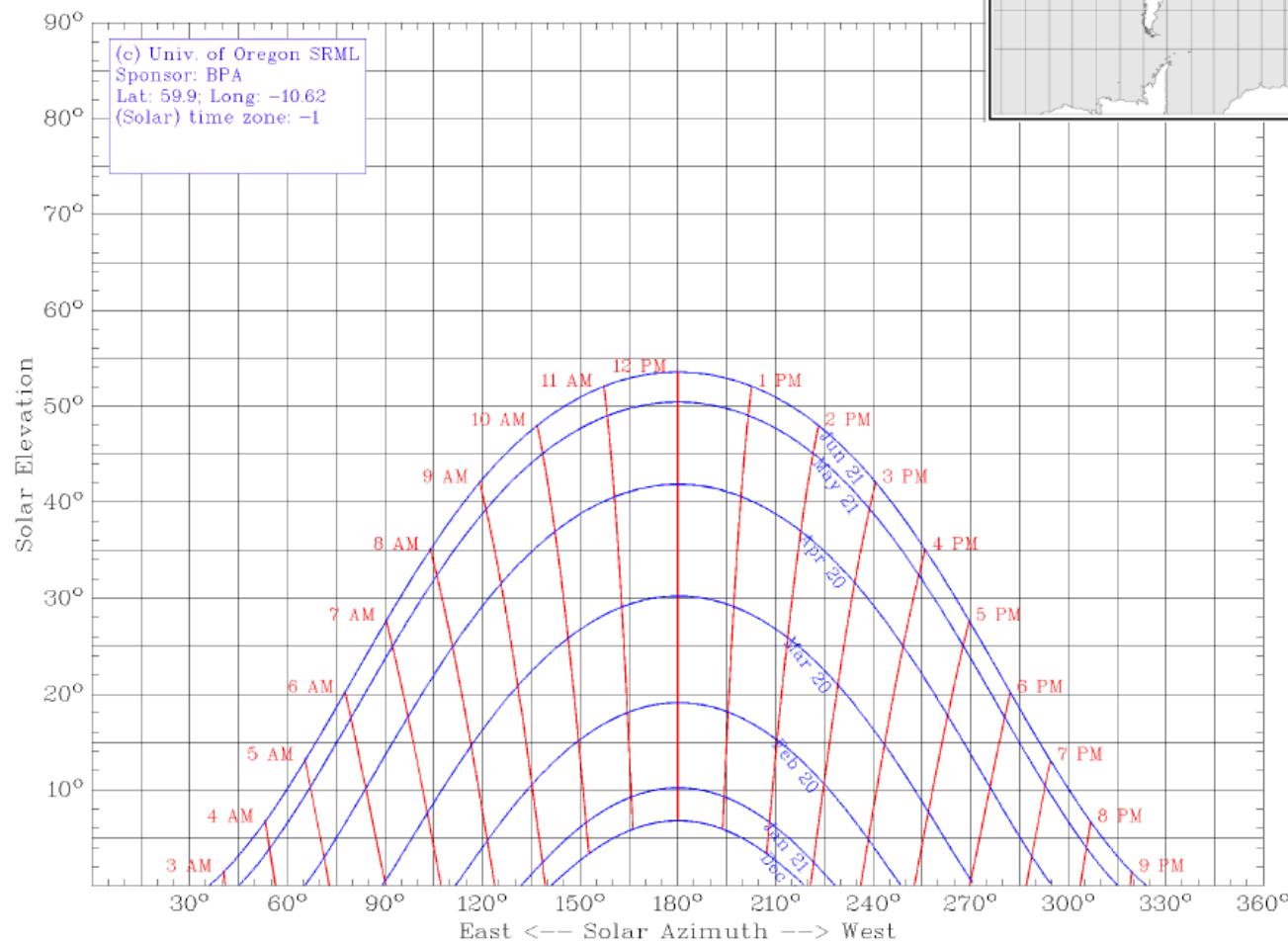


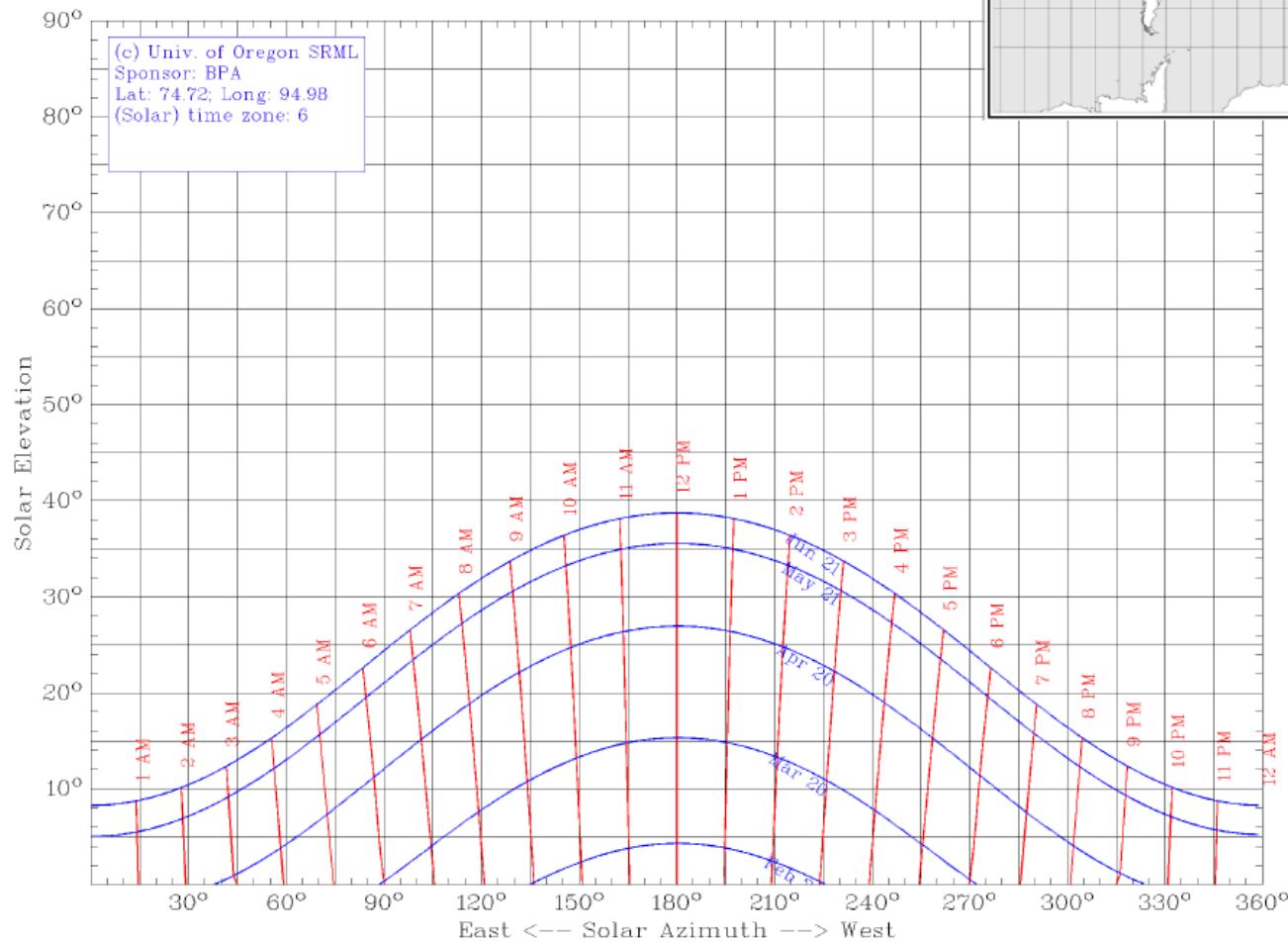
Altitude = Elevation

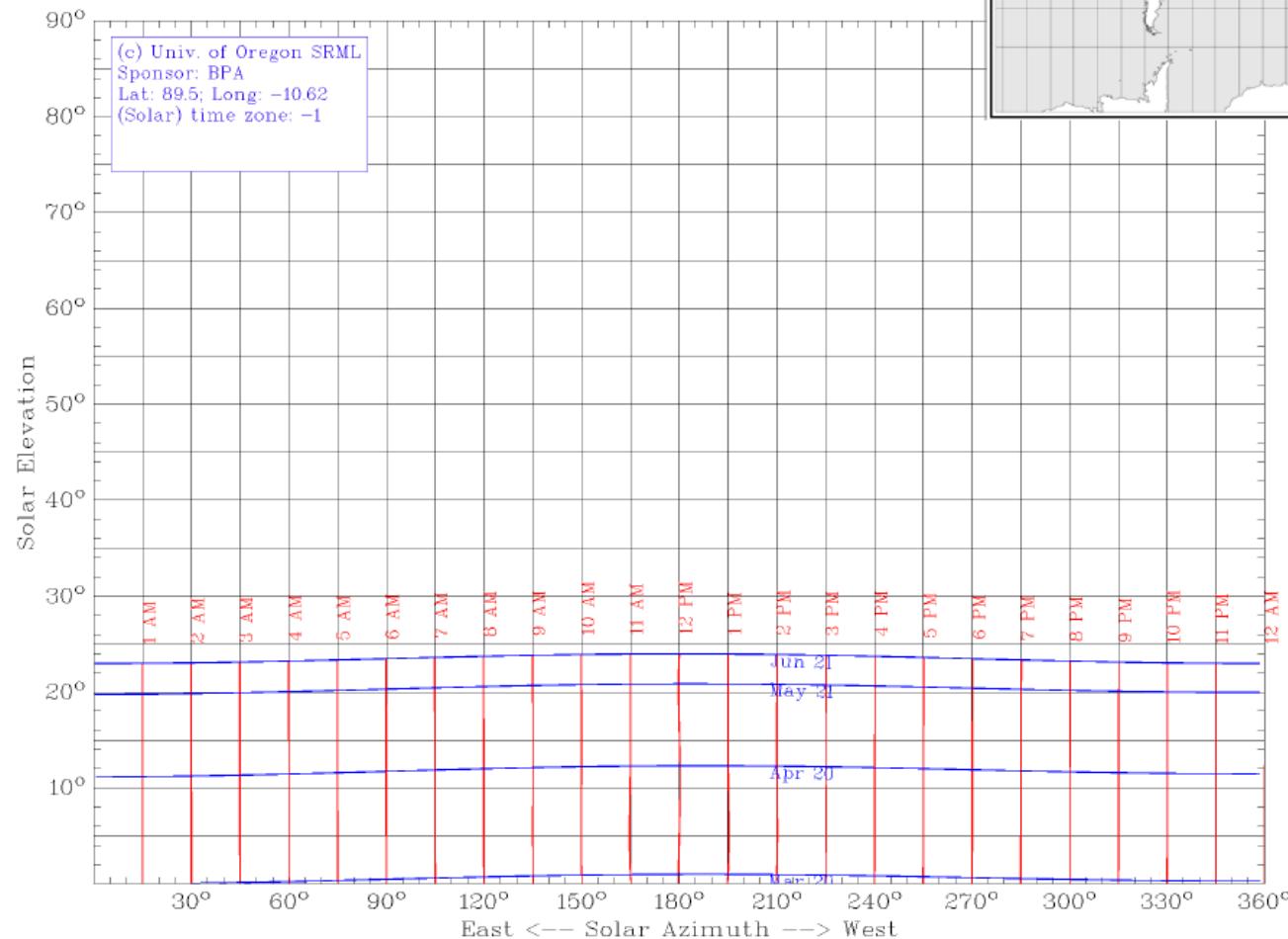
Sun position calculator:

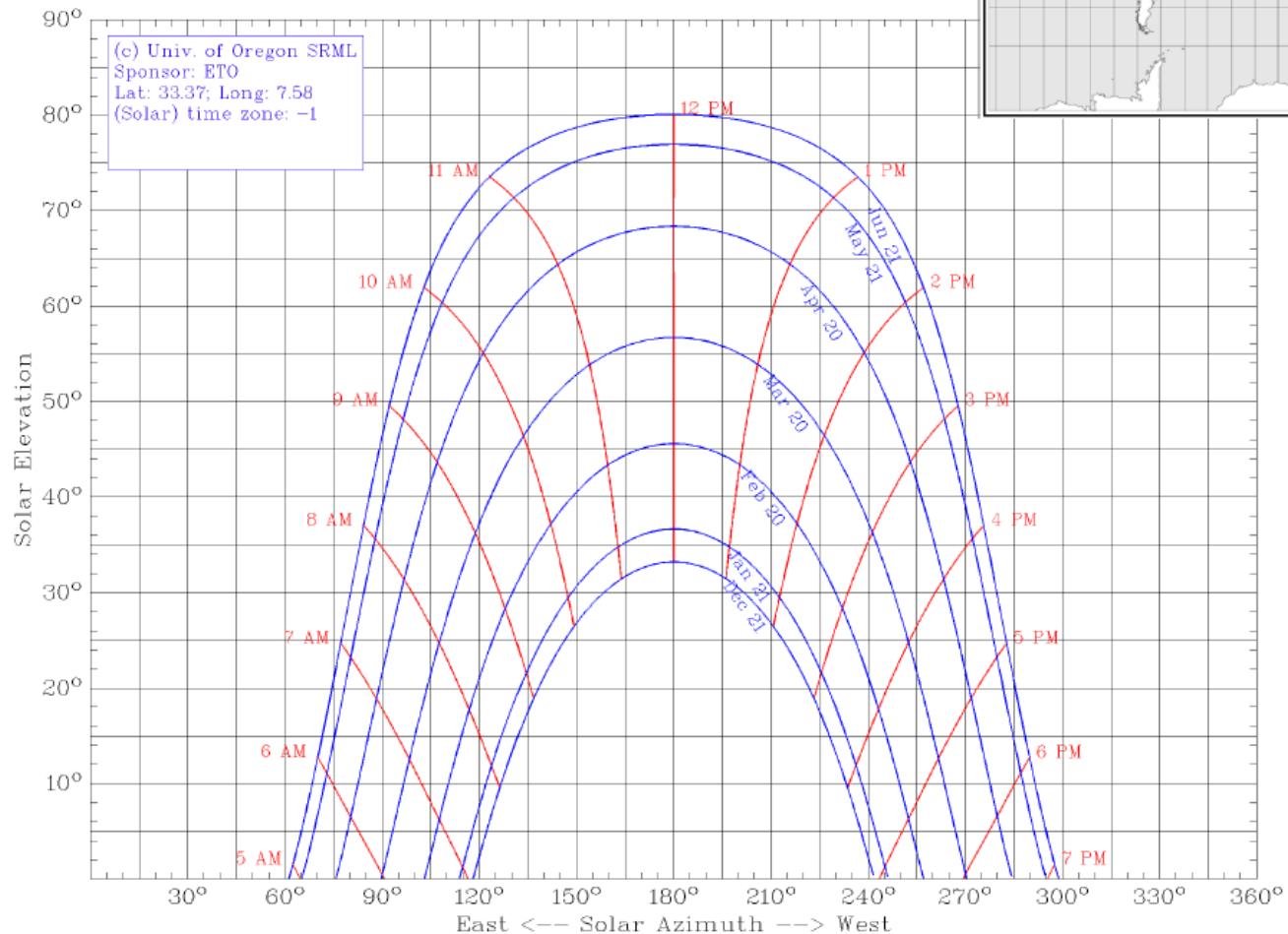
<https://www.pveducation.org/pvcdrom/properties-of-sunlight/sun-position-calculator>

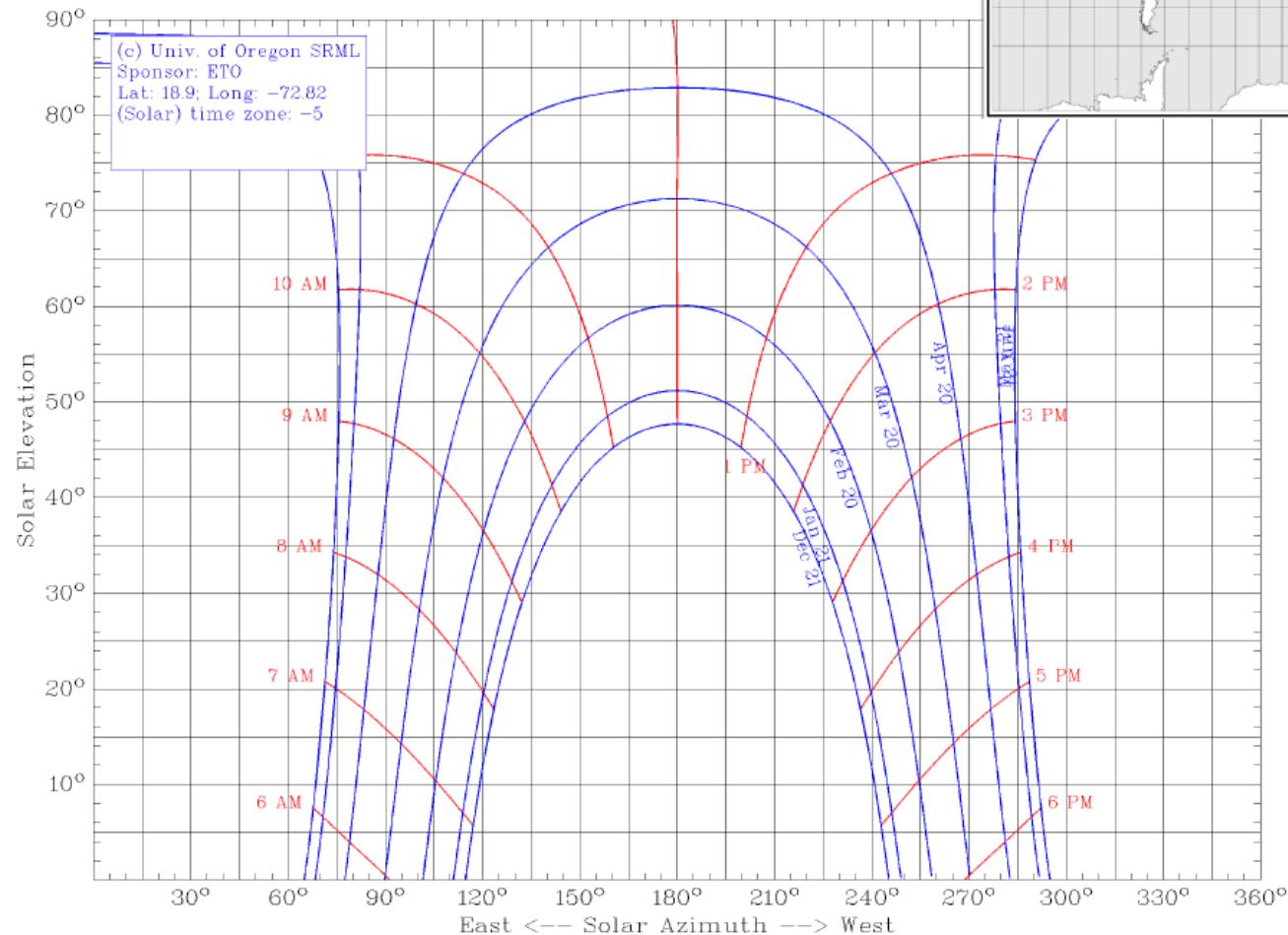


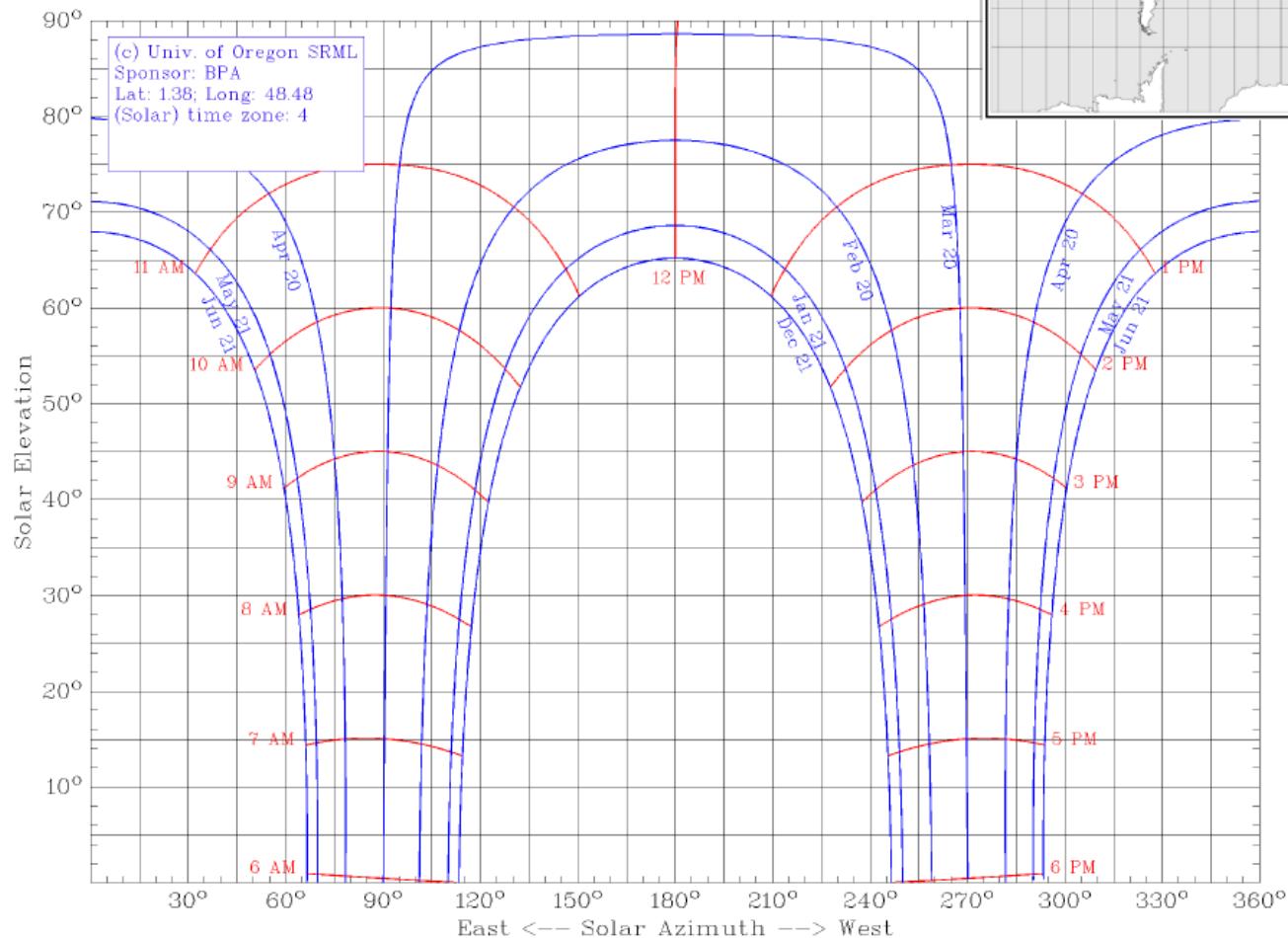


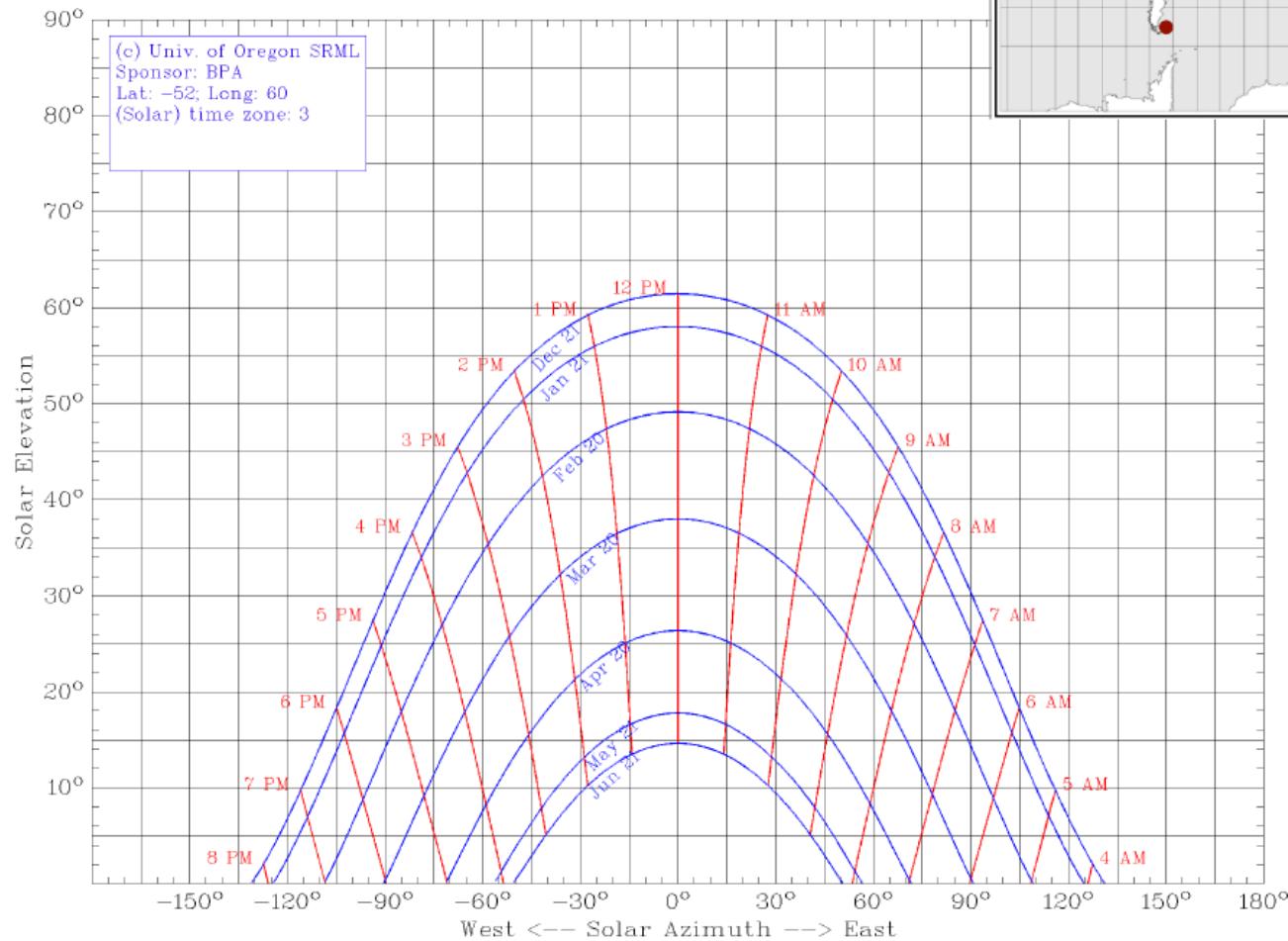












Autodesk Ecotect - Sun-Path Diagram

File Graph Shading Display Table Help

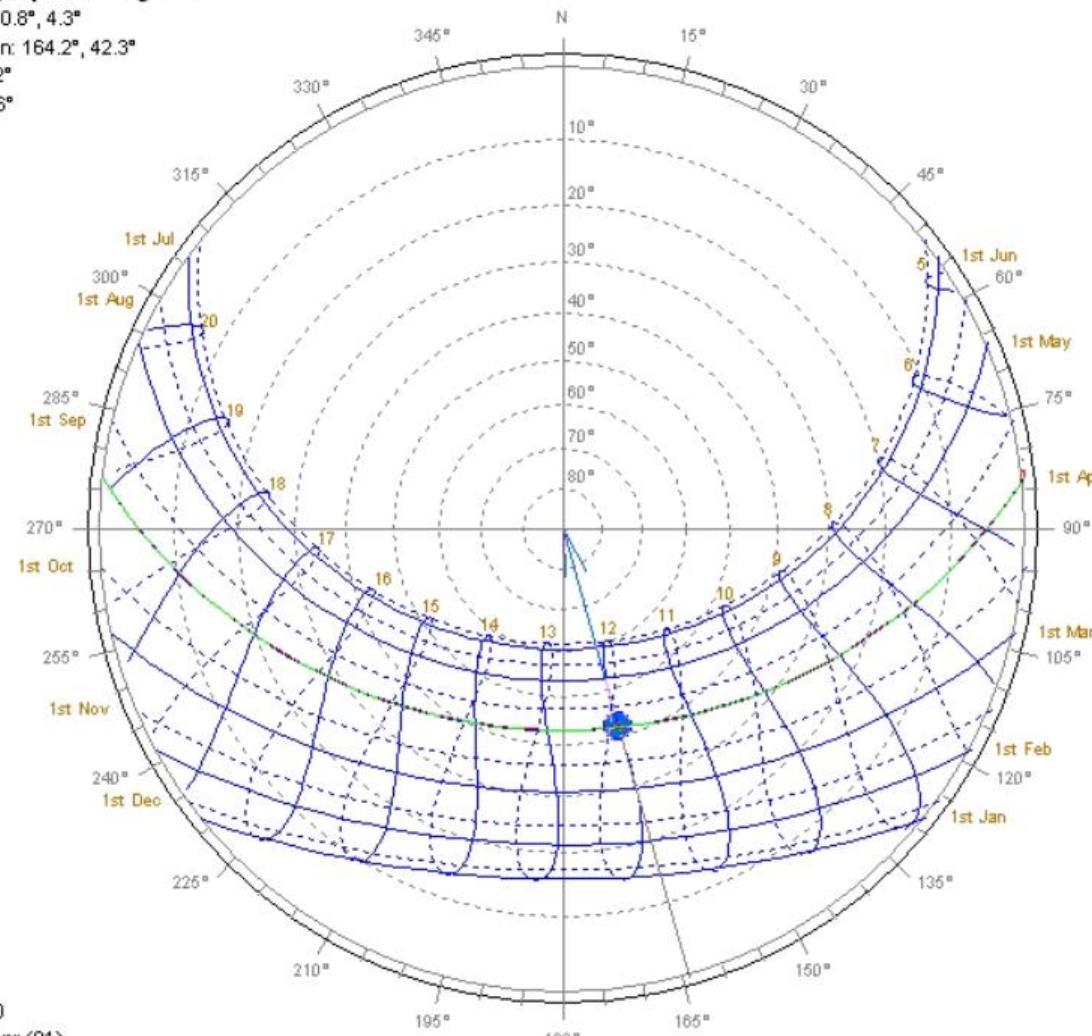
Stereographic Diagram

Location: 50.8°, 4.3°

Sun Position: 164.2°, 42.3°

HSA: 164.2°

VSA: 136.6°



Time: 12:00

Date: 1st Apr (91)

Dotted lines: July-December.

Spherical / Equidistant / Stereographic / BRE Sun-Path / Orthographic / Waldram / Tabular

Sun-Path Settings

DISPLAY SETTINGS

Stereographic

Hourly Sun Position

Show in Visualise Page

Overlay Percentages

OBJECT SHADING

Object: None

Mask: None

Calculate Shading...

Display ▶ Shading ▶

Show Shading Coefficients

» VIEW SHADING FILE

» EDIT SHADING

» OVERLAYED DATA

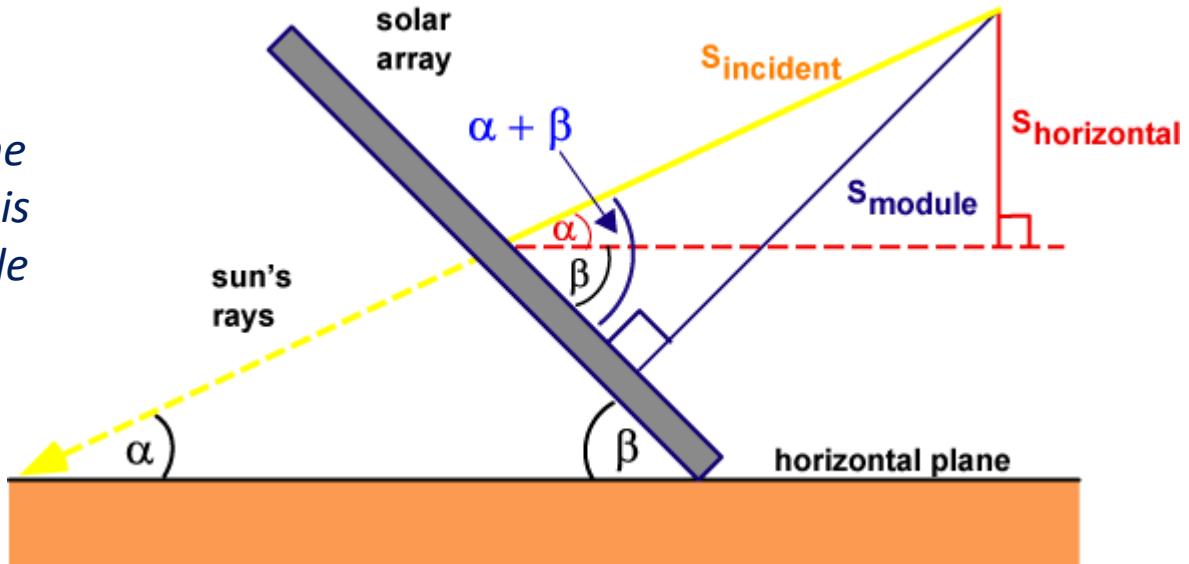
- +

Solar radiation on a tilted surface facing the sun

The considered surface is first supposed to be oriented towards the sun (i.e. facing the sun) with an arbitrary tilt angle β from the horizontal.

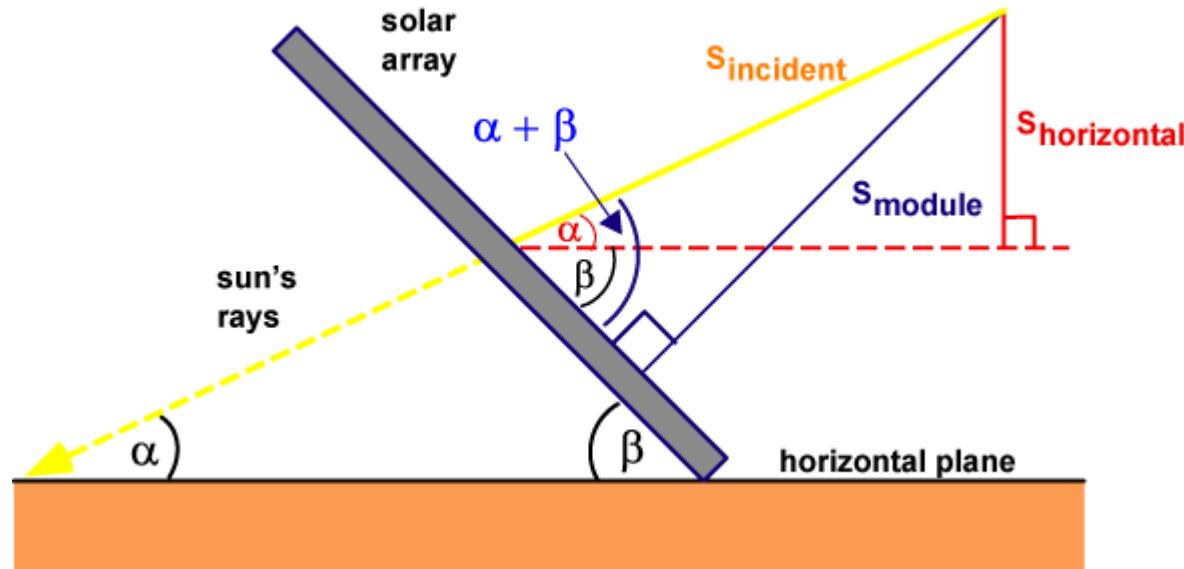
The amount of solar radiation incident on a tilted PV module surface is the component of the incident solar radiation which is perpendicular to the PV module surface.

$$\begin{aligned}\alpha &= \text{elevation angle} \\ \beta &= \text{surface tilt angle}\end{aligned}$$



- The power incident on a PV module depends not only on the power contained in the sunlight, but also on the angle between the module and the sun.
- When the absorbing surface and the sunlight are perpendicular to each other, the power density on the surface is equal to that of the sunlight. In other words, the power density will always be at its maximum when the PV module is perpendicular to the sun.
- As the angle between the sun and a fixed surface is continually changing, the power density on a fixed PV module is less than that of the incident sunlight.

Solar radiation on a tilted surface facing the sun



$$S_{\text{module}} = S_{\text{incident}} \sin(\alpha + \beta)$$

$$S_{\text{horizontal}} = S_{\text{incident}} \sin \alpha$$

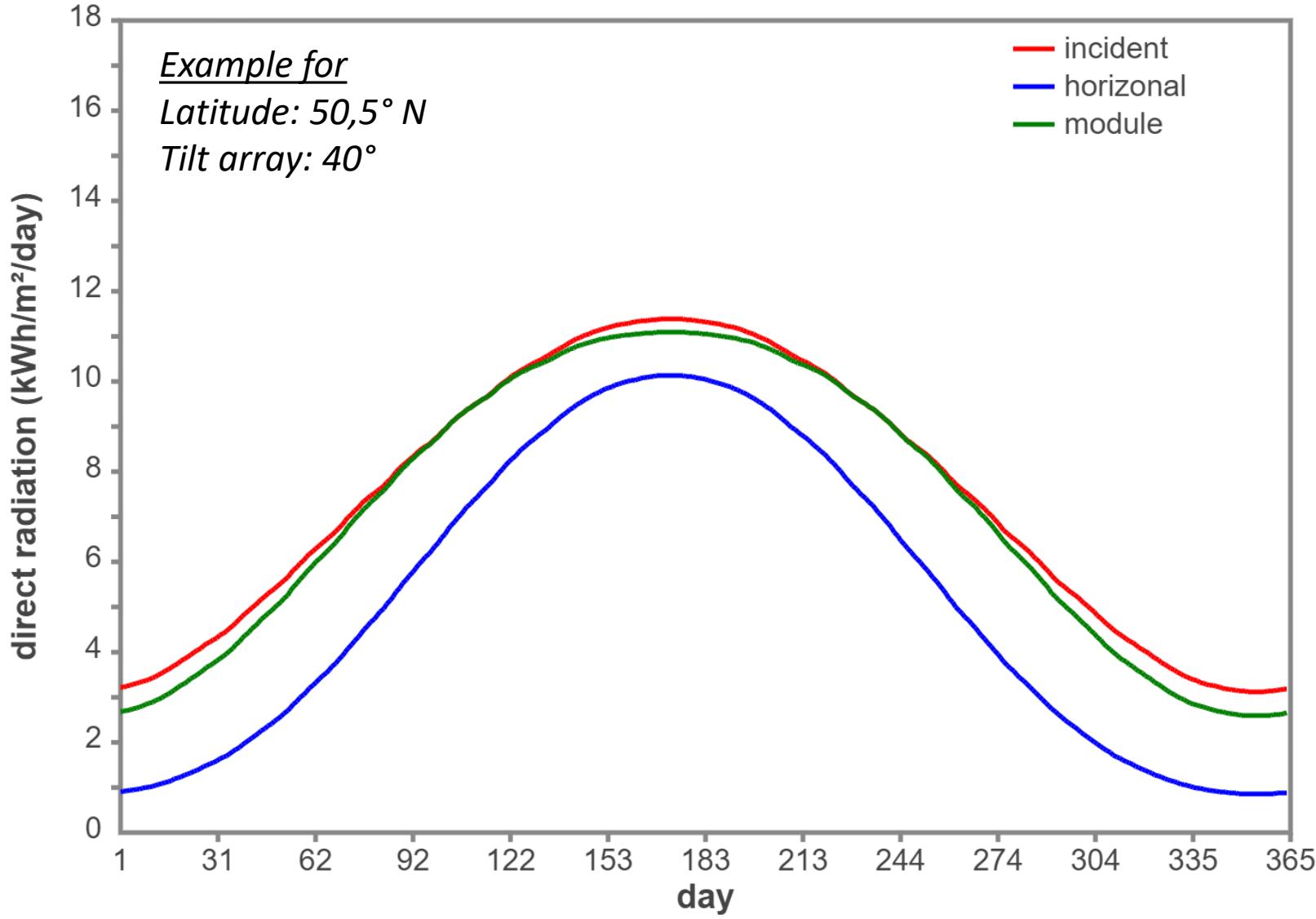
$$S_{\text{module}} = \frac{S_{\text{horizontal}} \sin(\alpha + \beta)}{\sin \alpha}$$

where

α is the *elevation angle*

β is the *tilt angle of the module* measured from the horizontal.

Simulator: <https://www.pveducation.org/pvcdrom/properties-of-sunlight/solar-radiation-on-a-tilted-surface>



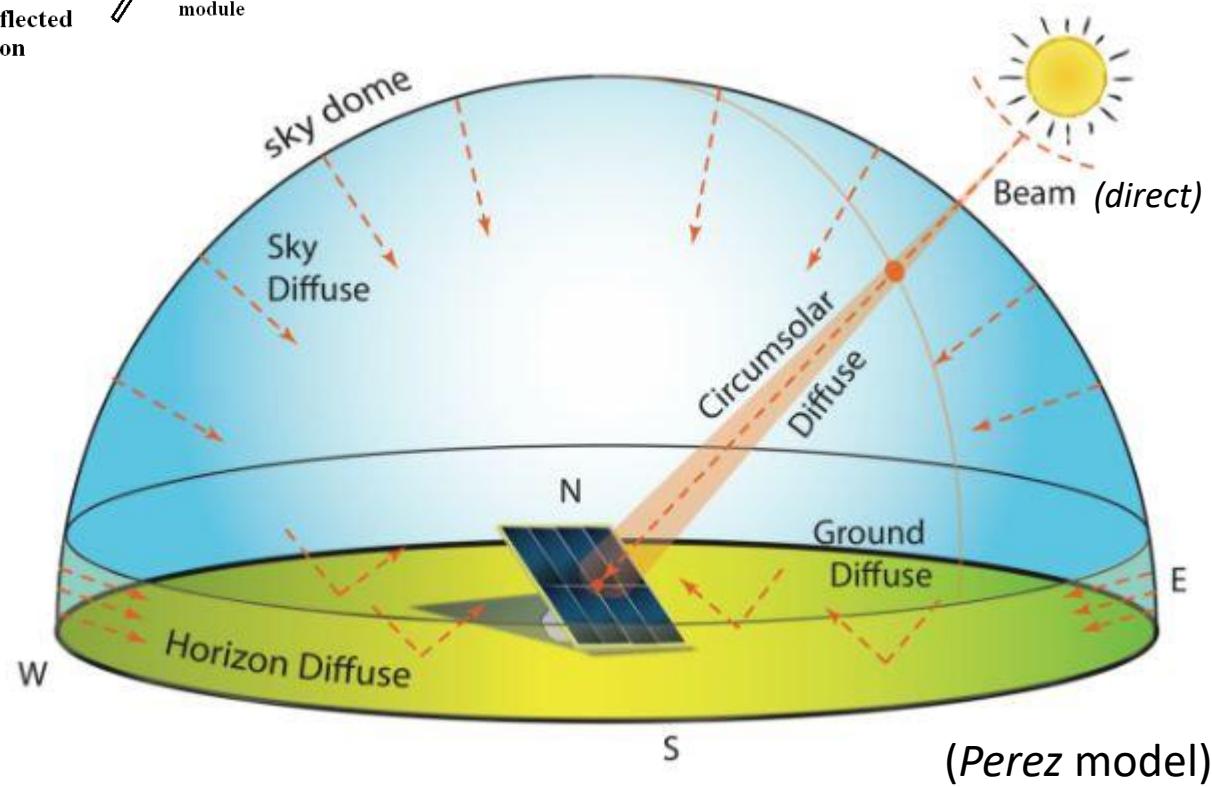
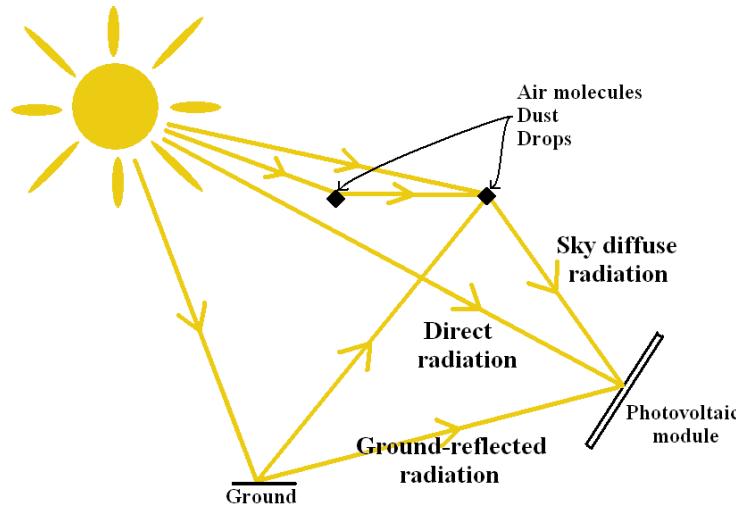
The Module Power is the solar radiation striking a tilted module. The module tilt angle is measured from the horizontal. The Incident Power is the solar radiation perpendicular to the sun's rays and is what would be received by a module that perfectly tracks the sun. Power on Horizontal is the solar radiation striking the ground and is what would be received for a module lying flat on the ground. These values should be regarded as maximum possible values at the particular location as they do not include the effects of cloud cover.

Solar radiation on a surface with arbitrary orientation and tilt

$$S_{\text{module}} = S_{\text{incident}} [\cos(\alpha)\sin(\beta)\cos(\Psi - \Theta) + \sin(\alpha)\cos(\beta)]$$

- S_{module} and S_{incident} are respectively the light intensities on the module and of the incoming light in W/m^2 , the S_{incident} being a direct only component.
- α is the *sun elevation angle*
- Θ is the *sun azimuth angle*
- β is the *module tilt angle*.
(A module lying flat on the ground has $\beta=0^\circ$, and a vertical module has a $\beta=90^\circ$)
- Ψ is the *azimuth angle that the module faces*.
Common practice leads to modules aligned to face towards the equator. A module in the northern hemisphere will be facing south with $\Psi=180^\circ$ and a module in the southern hemisphere will typically face directly north with $\Psi=0^\circ$. This is changing nowadays...
- A module that directly faces the sun so that the incoming rays are perpendicular to the module surface has the module tilt equal to the sun's zenith angle ($\beta = 90^\circ - \alpha$), and the module azimuth angle equal to the sun's azimuth angle ($\Psi = \Theta$), such that
 $S_{\text{module}} = S_{\text{incident}}$

Solar radiation: direct vs diffuse radiation



Usually available data in meteorological databases

Typical Meteorological Year Data

Direct Normal Irradiance (DNI), or *beam radiation*, is measured *at the surface of the Earth* at a given location with a surface element perpendicular to the sun.

(Particular case for slide 66 with $\alpha + \beta = 90^\circ$: $S_{\text{module}} = S_{\text{incident}}$)

It excludes diffuse solar radiation (radiation that is scattered or reflected by atmospheric components).

- Direct irradiance is equal to the extraterrestrial irradiance above the atmosphere minus the atmospheric losses due to absorption and scattering.
Losses depend on time of day (length of light's path through the atmosphere depending on the solar elevation angle), cloud cover, moisture content and other contents.
- The irradiance above the atmosphere also varies with time of year (because the distance to the sun varies), although this effect is generally less significant compared to the effect of losses on DNI.

Usually available data in meteorological databases (Typical Meteorological Year Data)

Diffuse Horizontal Irradiance (DHI), or *Diffuse Sky Radiation* is the radiation at the Earth's surface from light scattered by the atmosphere.

It is measured on a horizontal surface with radiation coming from all points in the sky excluding *circumsolar radiation* (radiation coming from the sun disk).

N.B. There would be almost no DHI in the absence of atmosphere.

Global Horizontal Irradiance (GHI) is the total irradiance from the sun on a horizontal surface on Earth.

It is the sum of direct normal irradiance, after accounting for the *sun elevation angle* α , and diffuse horizontal irradiance:

$$\text{GHI} = \text{DHI} + \text{DNI} \sin \alpha$$

Those data are ideally available in average values per hour (sometimes per 10 or 15 minutes)