Intensity and power

We restrict ourselves to the case of electromagnetic plane waves, described as $\vec{E}(\vec{r},t) = \vec{E_0}e^{j(\omega t-\vec{k}(\omega)\cdot\vec{r})}$, in which $\vec{E_0}$ is a complex vector and $\vec{k}(\omega)$ is the complex wavevector which depends on the angular frequency ω through a dispersion relationship. The intensity of such waves is:

$$I(\omega) = \frac{cn(\omega)\epsilon_0}{2}|\vec{E_0}|^2 \ , \tag{1}$$

in which ϵ_0 is the vacuum permittivity, c the speed of light in vacuum and $n(\omega)$ the real part of the index of refraction in the propagation medium. The intensity is the average power transported by the wave across a unit surface oriented perpendicularly to its direction of propagation ; it is expressed in W/m^2. The power P (Watts) transported by a wave across a surface of area $S_\perp$ oriented perpendicularly to the propagation direction is thus:

$$P(\omega) = S_\perp I(\omega) = S_\perp \frac{cn(\omega)\epsilon_0}{2}|\vec{E_0}|^2 \ . \tag{2}$$

Now, consider a (possibly multilayered) interface separating two media having $n_1(\omega)$ and $n_2(\omega)$ as real components of the indices of refraction of the media, respectively (Figure 1). A wave is incident from the first medium at angle ϕ_1, covering the surface S of the interface. It is reflected in the medium of incidence, and transmitted to the second medium. Meanwhile, part of the wave may also be absorbed by the interface.

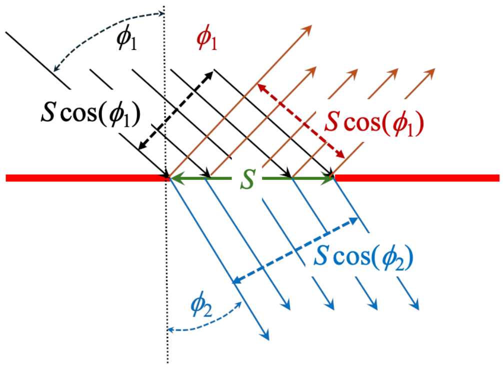

Figure 1: Illustration of a plane wave impinging on a surface of area S at oblique incidence.

Considering this illustration, we note that:

1. The Fresnel coefficients of reflection and transmission of that interface are noted r and t with a subscript indicating the orientation of the field either in the s or p direction which are, respectively, perpendicular and parallel to the plane of incidence (which is the plane containing the normal to the interface and the direction of propagation of the wave, itself identical to the direction of $\vec{k}$). The Fresnel coefficients are the ratios between the electric fields (reflected by the interface or transmitted through the interface) and the electric field in the medium of incidence, at the location of the interface. These coefficients are complex numbers because the fields generally differ in amplitude and phase.

2. The reflectivity R and transmissivity T of the interface (also called reflectance and transmittance) are defined as the ratios between the average power (reflected by the interface or transmitted through) and the incoming power of the incident wave.

   (a) **Case of reflection:** since the reflected field is in the same medium of index of refraction $n_1(\omega)$ as the incident field, and since the area of the reflected beam is identical to the one of the incident beam, the reflectivity or reflectance R is:

$$R = \frac{P_{r1}}{P_1} = \frac{|\vec{E_{r1}}|^2}{|\vec{E_1}|^2} = |r|^2 \ , \tag{3}$$

in which P_{r1} is the power reflected by S and P_1 is the power incident on S. The reflectance is thus simply the square of the magnitude of the Fresnel coefficient of reflection.

(b) **Case of transmission:** in contrast, the indices of refraction of the media of incidence and transmission are different, as well as the areas of the transmitted and incident beams (Figure 1). Therefore, the transmissivity or transmittance T is:

$$T = \frac{P_2}{P_1} = \frac{n_2}{n_1}\frac{\cos\phi_2}{\cos\phi_1}\frac{|\vec{E_2}|^2}{|\vec{E_1}|^2} = \frac{n_2\cos\phi_2}{n_1\cos\phi_1}|t|^2 \ , \tag{4}$$

in which P_2 is the transmitted power. The transmittance is thus not equal to the square of the magnitude of the Fresnel coefficient of transmission, which needs to be corrected for geometrical effects and the difference of the speed of light in both media. This formula does not hold when total external reflection occurs, since the wave in medium 2 is then not plane anymore.

3. The absorbance A, which is the power absorbed by the interface relative to the incident power, is obtained from energy conservation as:

$$A = 1 - R - T \ . \tag{5}$$

Note that a similar law does not hold for the coefficients of reflection and transmission.