Francisco Cuevas
Carlos Schwarzenberg
DMAT - UTFSM

Ayudantes
Diego Astaburuaga
Gabriel Riffo

MAT041 - Probabilidad y Estadística

Variables Aleatorias

$\boxed{X \text{ es discreta}}$

$$f_X(x) \geq 0$$

$$\sum_{x=-\infty}^{\infty} f_X(x) = 1$$

$$\mathbb{P}[X = x] = f_X(x)$$

$$F_X(x) = \mathbb{P}[X \leq x] = \sum_{u=-\infty}^{x} f_X(u)$$

$\boxed{X \text{ es continua.}}$

$$f_X(x) \geq 0$$

$$\int_{-\infty}^{\infty} f_X(x)\,dx = 1$$

$$\mathbb{P}[X = x] = 0$$

$$F_X(x) = \mathbb{P}[X \leq x] = \int_{-\infty}^{x} f_X(u)du$$

$$x_p \text{ es el percentil } p \Leftarrow F_X(x_p) = p$$

$$E[X] = \begin{cases} \displaystyle\sum_{x=-\infty}^{\infty} x f_X(x) & \text{(Caso discreto)} \\ \displaystyle\int_{-\infty}^{\infty} x f_X(x)\mathrm{d}x & \text{(Caso continuo)} \end{cases}$$

Sean a y b números reales. Entonces:
$E[a] = a$
$E[aX] = aE[X]$
$E[aX + b] = aE[X] + b$

$$V[X] = E[X^2] - (E[X])^2$$

$$E[X^2] = \begin{cases} \displaystyle\sum_{x=-\infty}^{\infty} x^2 f_X(x) & \text{(Caso discreto)} \\ \displaystyle\int_{-\infty}^{\infty} x^2 f_X(x)\,dx & \text{(Caso continuo)} \end{cases}$$

$$V[X] = E\left[(X - E[X])^2\right]$$

Sean a y b números reales. Entonces:
$V[a] = 0$
$V[aX] = a^2V[X]$
$V[aX + b] = a^2V[X]$

Sean X e Y variables aleatorias continuas. Sea $g : \mathbb{R} \to \mathbb{R}$ continua e inyectiva tal que $Y = g(X)$

$$f_Y(y) = f_X(g^{-1}(y)) \left|\frac{dg^{-1}(y)}{dy}\right| \mathbb{I}_{g(\mathrm{Rec}(x))}(y)$$

En general:

$$E[g(X)] = \begin{cases} \displaystyle\sum_{x=-\infty}^{\infty} g(x) f_X(x) & \text{(Caso discreto)} \\ \displaystyle\int_{-\infty}^{\infty} g(x) f_X(x)\mathrm{d}x & \text{(Caso continuo)} \end{cases}$$