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MAT041 - Probabilidad y Estadística

Vectores Aleatorios

$\boxed{(X,Y) \text{ discretos}}$

$$f_{XY}(x,y) \geq 0$$

$$\sum_{y=-\infty}^{\infty} \sum_{x=-\infty}^{\infty} f_{XY}(x,y) = 1$$

$$\mathbb{P}[X=x,\, Y=y] = f_{XY}(x,y)$$

$\boxed{(X,Y) \text{ continuos.}}$

$$f_{XY}(x,y) \geq 0$$

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{XY}(x,y)\,dx\,dy = 1$$

$$\mathbb{P}[X=x,\, Y=y] = 0$$

$\boxed{\text{Marginal de } X}$

$$F_X(x) = \lim_{y\to\infty} F_{XY}(x,y)$$

$$f_X(x) = \begin{cases} \displaystyle\sum_{y=-\infty}^{\infty} f_{XY}(x,y) & \text{, Discreto} \\ \displaystyle\int_{-\infty}^{\infty} f_{XY}(x,y)\,dy & \text{, Continuo} \end{cases}$$

$$E[X^r] = \begin{cases} \displaystyle\sum_{x=-\infty}^{\infty} \sum_{y=-\infty}^{\infty} x^r f_{XY}(x,y) & \text{, Disc.} \\ \displaystyle\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x^r f_{XY}(x,y)\,dx\,dy & \text{, Cont.} \end{cases}$$

$$V[X] = E[X^2] - E[X]^2$$

$\boxed{\text{Marginal de } Y}$

$$F_Y(y) = \lim_{x\to\infty} F_{XY}(x,y)$$

$$f_Y(Y) = \begin{cases} \displaystyle\sum_{x=-\infty}^{\infty} f_{XY}(x,y) & \text{, Discreto} \\ \displaystyle\int_{-\infty}^{\infty} f_{XY}(x,y)\,dx\,dy & \text{, Continuo} \end{cases}$$

$$E[Y^r] = \begin{cases} \displaystyle\sum_{x=-\infty}^{\infty} \sum_{y=-\infty}^{\infty} y^r f_{XY}(x,y) & \text{, Disc.} \\ \displaystyle\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} y^r f_{XY}(x,y)\,dx\,dy & \text{, Cont.} \end{cases}$$

$$V[Y] = E[Y^2] - E[Y]^2$$

Sea $g : \mathbb{R} \to \mathbb{R}$, entonces:

$$E[g(X,Y)] = \begin{cases} \displaystyle\sum_{x=-\infty}^{\infty} \sum_{y=-\infty}^{\infty} g(x,y)\cdot f_{XY}(x,y) & \text{,Disc.} \\ \displaystyle\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x,y)\cdot f_{XY}(x,y)\,dx\,dy & \text{,Cont.} \end{cases}$$

$$\mathrm{Cov}(X,Y) = E[XY] - E[X]E[Y]$$

$$\mathrm{Cov}(X,X) = V[X]$$

$$X \perp Y \Leftrightarrow \begin{cases} f_{XY}(x,y) & = f_X(x) f_Y(y) \\ F_{XY}(x,y) & = F_X(x) F_Y(y) \end{cases}$$

$$X \perp Y \Rightarrow \mathrm{Cov}(X,Y) = 0$$

$$\rho_{X,Y} = \frac{\mathrm{Cov}(X,Y)}{\sqrt{V[X]V[Y]}}$$

$$|\rho_{X,Y}| \leq 1$$

$\boxed{\text{Condicional } Y|_{X=x}}$

$$f_{Y|X=x}(y) = \frac{f_{XY}(x,y)}{f_X(x)}$$

$$E[Y^r|X=x] = \begin{cases} \displaystyle\sum_{y=-\infty}^{\infty} y^r f_{Y|X=x}(y) & \text{, Disc.} \\ \displaystyle\int_{\infty}^{\infty} y^r f_{Y|X=x}(y)\,dy & \text{, Cont.} \end{cases}$$

$$V[Y|x=x] = E[Y^2|x=x] - E[Y|x=x]^2$$

Análogo para el caso $X|_{Y=y}$

Sea $Y = g(X)$ con $Y_i = g_i(X_1, \ldots, X_p)$

$$f_Y(y_1, \ldots, y_p) = f_X(g^{-1}(Y)) \left| J\left(\frac{x_1, \ldots, x_p}{y_1, \ldots, y_p}\right) \right|,$$

donde

$$J\left(\frac{x_1, \ldots, x_p}{y_1, \cdots, y_p}\right) = \begin{bmatrix} \frac{\partial x_1}{\partial y1} & \cdots & \frac{\partial x_1}{\partial y_p} \\ \vdots & \ddots & \vdots \\ \frac{\partial x_p}{\partial y1} & \cdots & \frac{\partial x_p}{\partial y_p} \end{bmatrix}$$