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# MAT041 - Probabilidad y Estadística

## Normal Multivariada

### Distribución Normal Bivariada

$\boxed{\text{Notación}}$

$$E[X] = \mu_x$$
$$E[Y] = \mu_Y$$
$$V[X] = \sigma_X^2$$
$$V[Y] = \sigma_Y^2$$
$$\text{Cov}(X,Y) = \sigma_{XY}$$
$$\text{Corr}(X,Y) = \rho$$

$$\mu = \begin{pmatrix} \mu_x \\ \mu_y \end{pmatrix}$$
$$\Sigma = \begin{pmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{yx} & \sigma_y^2 \end{pmatrix}$$
$$\begin{pmatrix} X \\ Y \end{pmatrix} =\sim \mathcal{N}(\mu, \Sigma)$$
$$X \sim \mathcal{N}(\mu_X, \sigma_X^2)$$
$$Y \sim \mathcal{N}(\mu_Y, \sigma_Y^2)$$
$$\sigma_{XY} = 0 \Leftrightarrow X \perp Y$$

$$Y|_{X=x} \sim \mathcal{N}\left(\mu_{Y|X=x}, \sigma_{Y|X=x}^2\right)$$
$$\mu_{Y|X=x} = \mu_Y + \frac{\sigma_{XY}}{\sigma_X^2}(x - \mu_X)$$
$$\sigma_{Y|X=x}^2 = \sigma_Y^2(1-\rho^2)$$
$$X|_{Y=y} \sim \mathcal{N}\left(\mu_{X|Y=y}, \sigma_{X|Y=y}^2\right)$$
$$\mu_{X|Y=y} = \mu_X + \frac{\sigma_{XY}}{\sigma_Y^2}(y - \mu_Y)$$
$$\sigma_{X|Y=y}^2 = \sigma_X^2(1-\rho^2)$$

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### Distribución Normal Multivariada

$\boxed{\text{Notación}}$

$$X = (X_1, \ldots, X_p)^t$$
$$E[X_i] = \mu_i$$
$$V[X_i] = \sigma_i^2$$
$$\text{Cov}(X_i, X_i) = \sigma_{XY}$$
$$\text{Corr}(X_i, X_i) = \rho_{ij}$$

$$X \sim \mathcal{N}_p(\mu, \Sigma_{p\times p})$$
$$\mu = \begin{pmatrix} \mu_1 \\ \cdots \\ \mu_p \end{pmatrix}$$
$$X_i \sim \mathcal{N}(\mu_i, \sigma_i^2)$$

$$E[X] = \mu$$
$$V[X] = \Sigma$$
$$\Sigma_{p\times p} = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1p} \\ \sigma_{21} & \ddots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ \sigma_{p1} & \sigma_{p2} & \cdots & \sigma_p^2 \end{pmatrix}$$