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min $J_0(u) = \frac{1}{2}x^T(T)\, S^f x(T) + \frac{1}{2}\int_0^T (x^T Q x + u^T R u)\, dt$

$L_0 = \frac{1}{2} x^T Q x + u^T R u$

$\dot{x} = f(x(t), u(t))$

$= A(t)x(t) + Bu$ 系(t)

$H = L_0 + \lambda^T f$ $\boxed{\text{一阶必要条件}}$

$= \frac{1}{2}(x^T Q x + u^T R u) + \lambda^T (Ax + Bu)$

$\frac{\partial H}{\partial \lambda} = f = Ax + Bu = \frac{\partial H}{\partial x}$

$-\frac{\partial H}{\partial x} = \frac{\partial \lambda}{\partial t}$ ① 协态方程

☆ 横截条件

$\lambda(T)$ ☆

极值条件

$\frac{\partial H}{\partial u} = 0 \Rightarrow$

$\dot{\lambda} = -Qx - A^T\lambda(t)$ 协态方程

$\lambda^T = S^f x(T)$

$Ru + B^T\lambda = 0 \Rightarrow \boxed{u = -R^{-1}B^T\lambda}$

Assume that 猜想

☆ $\lambda(t) = S(t)x(t)$ $\lambda(t)$ $x(t)$

$\lambda(t)$ 协态 $\boxed{\text{过渡矩阵}}$ $A(t)$

② $\lambda(t) = S(t)x(t)$

$\frac{d\lambda(t)}{dt}$ ① ②

② $\frac{d\lambda}{dt} = \frac{dS}{dt}x(t) + S\frac{dx}{dt}$

$S f$

$= \frac{dS}{dt}x + S(Ax + Bu)$

$u \rightarrow -R^{-1}B^T\lambda \rightarrow Sx$

$\frac{d\lambda}{dt} = \frac{dS}{dt}x + S(Ax - BR^{-1}B^TSx)$

② $= \frac{dS}{dt}x + SAx - SBR^{-1}B^TSx$

① $= -Qx + A^T\lambda$, $\lambda \rightarrow Sx$

$= -Qx - A^TSx$

$\left(\frac{dS}{dt} + SA - SBR^{-1}B^TS + Q + A^TS\right)x = 0$

黎卡提 Riccati 方程

$$\boxed{\frac{dS}{dt} = -SA - A^TS - Q + SBR^{-1}B^TS}$$

S 矩阵 S(t)

非线性方程

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A(t) 时变矩阵

R(t)

找规律

R^{+} R 对称性

问题本身

最优控制问题

u(t)

控制参数化

参数设计 新问题

最优参数选择问题

反馈

① u(t)=

② $u(t)=-Kx(t)$

$u(t)=\varphi(y(t),t)$

线性

理论

$u(t)=-Kx(t)$

可观测性

可控性

稳定性

完备性

可镇定

全状态

非线性

最优控制问题

MISER3

matlab 编程 控制

设计

本科

我

wang 老师

日期:　　/

有限　　k　　$\boxed{K(k)}$

$x(k+1) = A(k)x(k) + B(k)u(k)$

$\lambda(k+1)$

$x(0) = x_0$　　连续

$J_0 = \Phi(x(T)) + \int_0^T L_0\, dt$　　二次型

$J_0 = \frac{1}{2}x^T(N)P(N)x(N) + \frac{1}{2}\sum_{k=0}^{N-1}[x^T(k)Q(k)x(k) + u^T(k)R(k)u(k)]$

横截条件　$\lambda(T) = \frac{\frac{1}{2}x^T(N)P(N)x(N)}{\partial x(N)} = 0$　　$\boxed{\lambda(N)}$　　N $\boxed{\text{fixed}}$

$H = L_0 + \lambda^T f = \frac{1}{2}x^T(k)Q(k)x(k) + \frac{1}{2}u^T(k)R(k)u(k)$

$+ \lambda^T(k+1)[A(k)x(k) + B(k)u(k)]$

一阶必要性条件

状态　$\frac{\partial H}{\partial \lambda} = \frac{\partial x}{\partial t} = f = A(k)x(k) + B(k)u(k)$

协态　$\boxed{-\frac{\partial H}{\partial x} = \frac{\partial \lambda}{\partial t}}$　连续

维数相符

离散　$\lambda(k) = \frac{\partial H(k)}{\partial x(k)} = Q(k)x(k) + A^T(k)\lambda(k+1)$

$\lambda(N) = 0$

维数

最优性条件　$\frac{\partial H}{\partial u} = 0$　　$R(k)u(k) + B^T(k)\lambda(k+1) = 0$

$u(k) = -R^{-1}(k)B^T(k)\lambda(k+1)$

连　$\lambda(t) = S(t)x(t)$

离　$\boxed{\lambda(k) = K(k)x(k)}$

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$\lambda(k) = Q(k)x(k) + A^T(k)\lambda(k+1)$ 协态方程

$\boxed{K(k)x(k)} = Q(k)x(k) + A^T(k)K(k+1)x(k+1)$ ✓

$x(k+1) = A(k)x(k) + B(k)u(k)$ 状态

$x(k+1) = A(k)x(k) - B(k)R^{-1}(k)B^T(k)\boxed{K(k+1)x(k+1)}$

$x(k+1) + BR^{-1}B^T K(k+1)x(k+1) = A(k)x(k)$

$\underbrace{[I + BR^{-1}B^T K(k+1)]}_{M} x(k+1) = A(k)x(k)$

$x(k+1) = M^{-1}A(k)x(k)$ ✓

$K(k)x(k) = Q(k)x(k) + A^T(k)K(k+1)[I + BR^{-1}B^T K(k+1)]A(k)x(k)$

$[K(k) - Q(k) - A^T(k)K(k+1)[I + BR^{-1}B^T K(k+1)]^{-1}]\,x(k) = 0$

$x(k) \neq 0$

$K(k) = Q(k) + A^T(k)K(k+1)[I + BR^{-1}B^T K(k+1)]^{-1}$

Ricatti 差分方程 $\boxed{u = -Kx(k)}$ ✓ ?

$(A+BC)^{-1} = A^{-1} - A^{-1}B(I + CA^{-1}B)^{-1}CA^{-1}$

应用 $A = I$. $B = B(k)$ $\boxed{C = R^{-1}B^T K(k+1)}$

$K(k) = Q(k) + A^T(k)X(k+1)A(k) - A^T(k)K(k+1)B(k)$ 倒向

$[R(k) + B^T(k)K(k+1)B(k)]^{-1}B^T(k)K(k+1)A(k)$.

$K(N) = P(N)$ 终端条件 $K(k)$ 已知

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$$\boxed{u(k) = -R^{-1}B^{T}\lambda(k+1)}$$

$\frac{\partial H}{\partial u}=0$

$\lambda(k) = K(k)X(k)$

$B(t)$　$B(t)$

$K(k+1)X(k+1)$

$u(k) = -R^{-1}B^{T}\lambda(k+1) = -R^{-1}B^{T}K(k+1)X(k+1)$

连

$B(k)$

离散

$B(k)$

A B 已知

K 要求

$[I+BR^{-1}B^{T}K(k+1)]X(k+1) = A(k)X(k)$

$K(k+1) = M^{-1}A(k)X(k)$

$\boxed{A(k)}$

$X = K(k)X(k)$

过渡阵

k

$u(k) = -R^{-1}B^{T}K(k+1)X(k+1)$

$u(k) = -R^{-1}B^{T}K(k+1)[I+BR^{-1}B^{T}K(k+1)]^{-1}A(k)X(k)$

$-R^{-1}B^{T}K(k+1)[I+BR^{-1}B^{T}K(k+1)]^{-1}\cdot$

$[(I+BR^{-1}B^{T}K(k+1))-(BR^{-1}B^{T}K(k+1))]\cdot A(k)X(k)$

$= -R^{-1}B^{T}K(k+1)\{I-[I+BR^{-1}B^{T}K(k+1)]^{-1}\cdot$

$B(k)R^{-1}(k)B^{T}K(k+1)\}A(k)X(k).$

$I-(I+BR^{-1}B^{T}K(k+1))^{-1}BR^{-1}B^{T}K(k+1)$

ABC

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$u(k)$

$$= -\{R^{-1}(k) - R^{-1}(k) B^T(k) K(k+1) [I + B(k) R^{-1}(k) \cdot B^T(k) K(k+1)]^{-1} B(k) R^{-1}(k)\} B^T(k) K(k+1) A X$$

$$-R^{-1} B^T K(k+1) + R^{-1} B^T K(k+1) [I + B(k) R^{-1} k \cdot B^T K(k+1)]^{-1} \cdot B(k) R^{-1}(k) B^T K(k+1)\} A(k) X(k)$$

$A(k) \cdot X(k)$

$$-\{R^{-1} - R^{-1}(k) B^T(k) K(k+1)\} B^T K(k+1) A X$$

$A = R(k) \quad B = B^T(k) K(k+1) \quad C = B(k)$

$$u(k) = -[R(k) + B^T(k) K(k+1) B(k)]^{-1} B^T(k) K(k+1) \cdot A(k) X(h) = -L(k) X(k)$$

$$L(k) = [R(k) + B^T(k) K(k+1) B(k)]^{-1} B^T(k) \cdot K(k+1) A(k)$$

$$u(k) = -L(k) X(k)$$

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$$x(k+1) = Ax(k) + Bu(k)$$

$$J = \sum_{k=0}^{\infty} [x^T(k) Q x(k) + u^T(k) R u(k)]$$ ✓

$$u(k) = -L x(k)$$

$$L = (R + B^T K B)^{-1} B^T K A$$

$$-K + Q + A^T K A - A^T K B (R + B^T K B)^{-1} B^T K A = 0$$ K

$$x(k+1) = [A - BL] x(k)$$ ✓

K = P 求P

$$P(k) = K(k)$$

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