Asymptote 范例教程

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本文是基于 Asymptote 1.88 的一组教程。
http://asy4cn.googlecode.com/

目录

第一章　赵爽的弦图

赵爽博士是位老知识分子，研究兴趣是天文历法与算学，一生精研《周髀算经》。不过赵老爷子年轻的时候书籍都是手写，最近才紧跟时代潮流用上了电脑。现在他要修订他研究《周髀算经》的札记，决定使用 LaTeX 来排版。

现在他遇到一个难题，就是他要画出笔记中讲解勾股定理的一幅弦图。听人介绍，几经比较之后，他决定使用现在炒得火热的 Asymptote。

赵博士的原图是手画的，线框多有不直不准的，图 1.1 就是旧年据手稿做的雕版图，赵博士并不满意。赵爽博士理想中的图，线条要平整美观，文字要清楚整齐，图形还要上色：朱实自然得用红色，黄实也该用黄色，以与注文一致——就是图 1.2 的样子。

计议已定，赵博士要开始正式的绘图了。

1.1　绘图环境

Asymptote 的安装并不复杂，在 Windows 下面就是下载运行那个安装包，在 Linux 下面一般也只需要下载对应的压缩包，解压就可以使用了。哦，赵博士用的就是 Windows。

点图标运行 Asymptote，就出现了交互式的命令行，提示符是一个 >。输入命令：

```
draw((0,0) -- (3cm,4cm));   // 一条直线
```

赵博士装的 GSView 立即弹了出来，里面已经画出一条倾斜的直线。再输入 quit，程序退出，并留下了一个叫做 out.eps 的图形文件，小菜一碟。

这里稍稍解释一下上面的一句代码。draw 是画线的命令，更准确地说，是 Asymptote 中的函数：它带有一个参数 (0,0) -- (3cm,4cm)，参数外面是圆括号，整个命令以分号结束。里面的 (0,0) -- (3cm,4cm) 是由两个

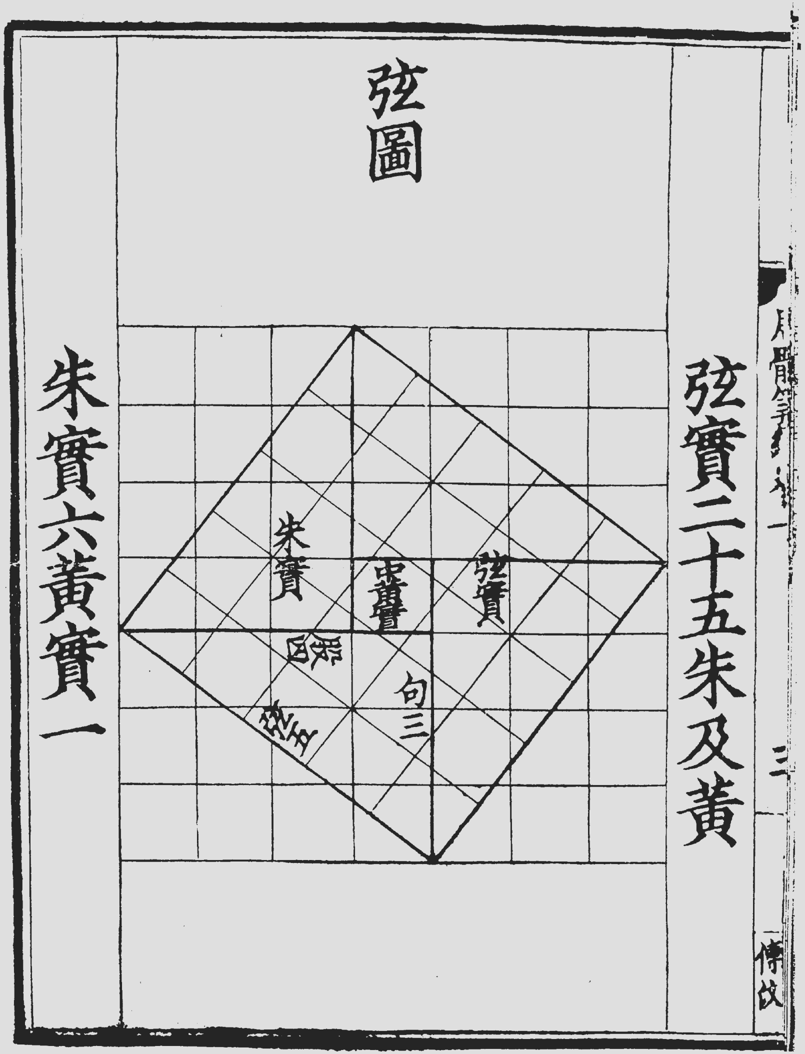


图 1.1: 旧年做的雕版

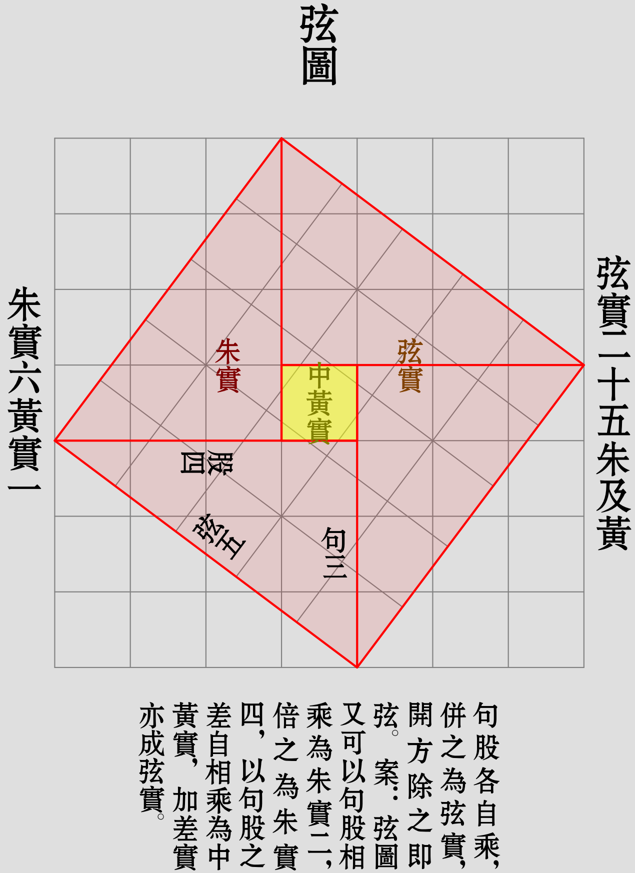

弦圖
朱實六黃實一
弦實二十五朱及黃
朱實
中黃實
弦實
股四
弦五
句三
句股各自乘，併之為弦實，開方除之即弦。案：弦圖又可以句股相乘為朱實二，倍之為朱實四，以句股之差自相乘為中黃實，加差實亦成弦實。

图 1.2: 理想中的新版本

坐标连接而成的直线，坐标是在直角坐标系下的，可以带单位 mm, cm, pt, bp, inch, inches，其意义与在TEX 中的一样，如果不带单位，则默认为 bp。行末以 // 开头的是注释，另有一种在 /* */ 之间的注释，与 C 语言相同。

不过赵博士写书讲求胸有成竹方才下笔，因此他更愿意使用更一种方式：打开一个文本编辑器，把上面的绘图命令都录入完毕，保存为一个名为 line.asy 的文件，最后把这个文件拖动到 Asymptote 的图标上面，就完成了整个作图。

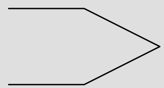
赵博士的小侄子不屑于这种快捷图标的编译方式，他直接进入命令行，输入 asy，就进入了 Asymptote 的交互环境；要是输入 asy line，就画出了刚才的保存的直线。

1.2 直线与绘图命令

弦图的图形其实很简单，都是直线、方块、三角形这些，而且为了计算的简便，所有长度也都是整数值。那么，首先就是在 Asymptote 中来画直线。

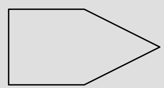
正如前面试验的时候做的那样，一条直线就是用 -- 把坐标连起来，再使用 draw 命令，直线就画出来了。事实上，可以用 -- 把坐标点连成折线：

```
draw( (0,0) -- (1cm,0) -- (2cm,0.5cm) -- (1cm,1cm) -- (0,1cm) );
```



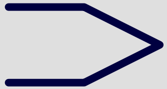
像这样把坐标用 -- 连结起来的，就成为一条路径。把直线而稍做修改 ，在后面连上一个特殊的坐标 **cycle**，就可以得到一条首尾相接的闭路径。如：

```
draw( (0,0) -- (1cm,0) -- (2cm,0.5cm) -- (1cm,1cm) -- (0,1cm) -- cycle );
```



画一条路径可以使用不同的颜色、粗细的笔，这只要给 draw 命令多加一个画笔 参数（多个参数用逗号分开）：

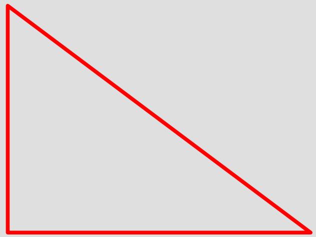
```
draw((0,0) -- (1cm,0) -- (2cm,0.5cm) -- (1cm,1cm) -- (0,1cm), darkblue+1mm);
```



这里 `darkblue` 是颜色，`1mm` 是线的粗细。`darkblue+1mm` 即指一毫米宽的深蓝色粗线。（可用的颜色名称可以参考 [1]）

赵博士画的是“勾三股四弦五”的红色三角形，这很容易：

```
draw( (0,0) -- (4cm,0) -- (0,3cm) -- cycle, red+0.5mm );
```

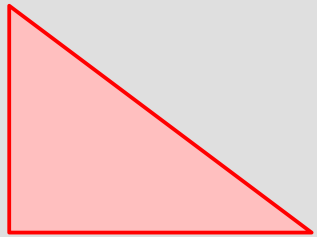


不过现在还需要的是实心的三角形，因此就需要一个新的绘图命令 `fill`，即填充。于是，红色三角形就成为

```
fill( (0,0) -- (4cm,0) -- (0,3cm) -- cycle, red );
```

但这样一来三角形的颜色就太重了，而且边界也不清楚。因此似乎应该先用浅红色填充一遍，然后再用深红色勾边。好在可以使用一个命令 filldraw 同时完成这两件事情，这样就不需要把一条路径写两遍了。即有：

```
filldraw((0,0) -- (4cm,0) -- (0,3cm) -- cycle,
    fillpen=palered, drawpen=red+0.5mm);
```



这里可以简单地直接写两个参数 palered, red+0.5mm，不过为了清晰起见还是使用 “键 = 值” 的写法，明确表示出填充的画笔和描线的画笔。

现在，赵博士的整个弦图的框架就呼之欲出了，就是画出四个三角形（图 1.3）：

```
filldraw( (4cm,0) -- (4cm,3cm) -- (0,3cm) -- cycle,
    fillpen=palered, drawpen=red+0.5mm);
filldraw( (7cm,4cm) -- (4cm,4cm) -- (4cm,0) -- cycle,
    fillpen=palered, drawpen=red+0.5mm);
filldraw( (3cm,7cm) -- (3cm,4cm) -- (7cm,4cm) -- cycle,
    fillpen=palered, drawpen=red+0.5mm);
filldraw( (0,3cm) -- (3cm,3cm) -- (3cm,7cm) -- cycle,
    fillpen=palered, drawpen=red+0.5mm);
```

还应该画出弦图的中间的“黄实”，用黄色填充。这部分是一个正方形，可以使用现成的 box(角点, 角点) 命令来产生矩形的路径，因而填充正中间的正方形就可以用：

```
fill( box((3cm,3cm), (4cm,4cm)), yellow );
```

这个填充的命令应该放在画线之前（以免覆盖描的红线）。

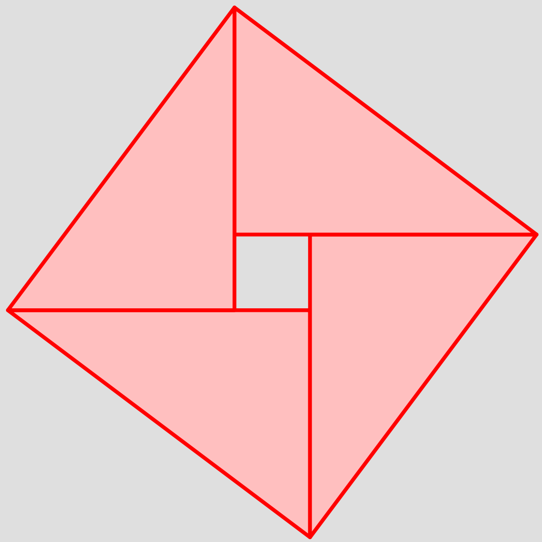


图 1.3: 弦图的初步框架

回顾前面的代码，赵博士觉得连续地写四个 filldraw 命令太重复了。他读了 Asymptote 的文档 [1]，才知道多条路径可以用符号 ^^ 连起来，一起使用，于是立即着手改进原来的代码。

而且，由于还打算在图的后面画出参考网格，图形的颜色还应该设置为半透明的。好在这并不难实现，只要稍稍改动一下填充的画笔，使用 opacity(数值) 来设定有一定不透明度（取值为 0 ∼ 1）的画笔，并把它加在原来的画笔上。于是赵博士最后写出了这样的代码：

```
fill( box((3cm,3cm), (4cm,4cm)), opacity(0.5)+yellow );
filldraw( (4cm,0) -- (4cm,3cm) -- (0,3cm) -- cycle
    ^^ (7cm,4cm) -- (4cm,4cm) -- (4cm,0) -- cycle
    ^^ (3cm,7cm) -- (3cm,4cm) -- (7cm,4cm) -- cycle
    ^^ (0,3cm) -- (3cm,3cm) -- (3cm,7cm) -- cycle,
    fillpen=opacity(0.1)+red, drawpen=red+0.5mm );
```

至此，弦图的主要框架（图 1.4）就此完成。

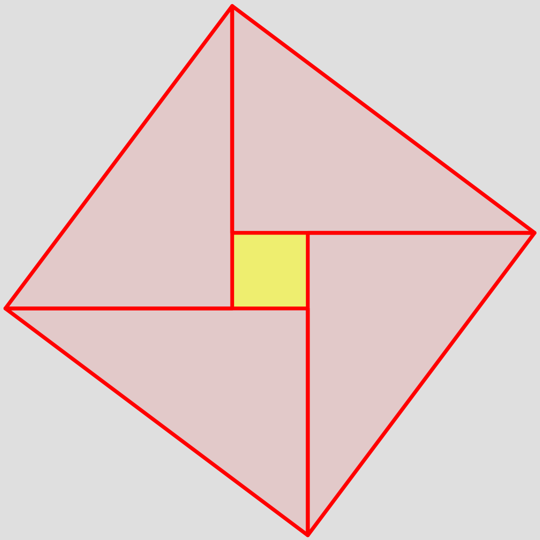


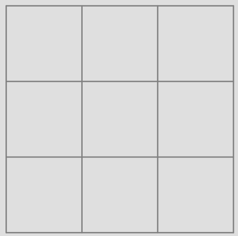
图 1.4: 弦图的进一步优化的框架

1.3 图形变换与功能模块的使用

然后是画作为长度参考的网格。其实网格一开始就该在图中画出来，这样后面画图准确与否才能看得清楚。不过现在赵博士是重作旧图，图样已定，网格就搁在弦图的主要图形后面才画了。

按说这个网格是十分简单的，无非就是画一些灰色的纵横细线。比如这个 3×3 的网格：

```
draw( (0,0) -- (3cm,0)
    ^^ (0,1cm) -- (3cm,1cm)
    ^^ (0,2cm) -- (3cm,2cm)
    ^^ (0,3cm) -- (3cm,3cm),
    gray );
draw( (0,0) -- (0,3cm)
    ^^ (1cm,0) -- (1cm,3cm)
    ^^ (2cm,0) -- (2cm,3cm)
    ^^ (3cm,0) -- (3cm,3cm),
    gray );
```



可无疑这个办法显得太麻烦了，赵博士要画的是 7×7 的网格，就要分别画出 16 条直线。这样的代码不仅不好写，而且容易出错，修改一下也很麻烦。

赵博士看到了手册 [1] 中讲循环语句的用法，似乎可以完成这件事。可是以赵博士的年龄，再去看什么编程什么变量的，命令不能一条一条执行下来，很不习惯，头脑就往往转不清楚。

于是赵博士就去论坛上咨询，一些人劝他去用几行循环语句，甚至有人已经把完整的函数做好了。但有一个结

果特别引人注目，有人指出，在 math 模块中已经定义好了一个 grid 函数，只要拿来用就可以了。赵博士立即精神大振，来看这个 grid 函数：

```
picture grid(int Nx, int Ny, pen p=currentpen)
```

这个是在 math 模块中 grid 函数的原型。它说明 grid 函数有 Nx, Ny 两个整数类型的必需参数，一个可选的画笔，并且返回一个 **picture**（图）类型的对象。

要使用模块的功能，需要在绘图之前导入这个模块，这只要使用

```
import 模块名;
```

因此，要使用 math 模块中的 grid 函数，只要在代码中写

```
import math;
```

就可以了。

grid 函数的行为看起来很奇怪，调用它会在一个单独的图上画出一个 Nx × Ny 的网格，网格的左下角在原点，间距为 1。要使用 grid 函数画的图形，要使用 add(图) 命令，把这个图形加在当前的图上：

```
import math;
add(grid(10,10,gray));
```

不过直接这样做的结果是只能得到一个小得已经看不清的网格。因此，必须对图形进行放缩。

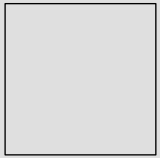
Asymptote 提供了平移、旋转、放缩、倾斜、反射等各种的仿射变换，来对坐标、路径、图形等元素进行变换（严格的函数原型参考 [1]）：

```
shift(坐标)          // 按坐标平移
shift(x, y)          // 按 (x, y) 平移
scale(倍数)          // 按倍数放缩
xscale(倍数)         // x 轴方向按倍数放缩
yscale(倍数)         // y 轴方向按倍数放缩
scale(x, y)          // 在 x 轴、y 轴方向分别按倍数 x, y 放缩
```

```
rotate(角度, z=(0,0)) // 按角度绕中心 z（默认为原点，逆时针）旋转
slant(因子)           // 按一定因子向右倾斜
reflect(a, b)         // 相对直线 a--b 反射
```

使用一个变换就是把这个变换乘在被变换对象的左边。例如一个放缩一个单位正方形：

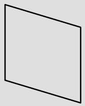
```
draw( scale(2cm) * box((0,0), (1,1)) );
```



这样的变换可以连续地做下去，例如

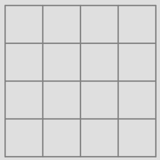
```
draw( rotate(90) * slant(0.3) * scale(1cm) * box((0,0), (1,1)) );
```

就是把一个单位正方形先放大，再倾斜，再旋转 $90°$：



有了这些变换，画出一个合适大小的网格就不再是什么难事了：

```
import math;
add( scale(5mm) * grid(4, 4, gray) );
```



赵博士的弦图有两个网格，不仅要大小合适，而且其中一个需要进行旋转和平移。平移的位置很明显，但旋转仍然需要一些计算。当然这难不倒精研天文算学多年的赵博士，这里弦实（5×5 的正方形）可以看作是顺时针旋转得到的，从朱实的三角形容易看出旋转的角度正好是 $\arctan(3/4)$。Asymptote 中也可以方便地调用返回角度的反三角函数 aTan 来计算这个角度。更详细的数学函数列表，参看 [1]。

于是，赵博士弦图中的网格，就可以这样方便地画出来了（图 1.5）：

```
// 网格
import math;
add( scale(1cm) * grid(7, 7, gray) );
add( shift(0,3cm) * rotate(-aTan(3/4)) * scale(1cm) * grid(5, 5, gray) );
// 弦图主体
fill( box((3cm,3cm), (4cm,4cm)), opacity(0.5)+yellow );
filldraw( (4cm,0) -- (4cm,3cm) -- (0,3cm) -- cycle
    ^^ (7cm,4cm) -- (4cm,4cm) -- (4cm,0) -- cycle
    ^^ (3cm,7cm) -- (3cm,4cm) -- (7cm,4cm) -- cycle
    ^^ (0,3cm) -- (3cm,3cm) -- (3cm,7cm) -- cycle,
    fillpen=opacity(0.1)+red, drawpen=red+0.5mm );
```

1.4 标注文字

现在要进行的是文字的标注。按照勾股定理的约定，赵博士打算在一个红色三角形内标注“朱实”，在黄色矩形处标注“黄实”，并为拼得的整个大矩形标注“弦实”；在另一红色三角形的三边标注“勾三”、“股四”、“弦五”的尺寸；最后在图形两侧加上说明的文字。

在标注文字之前，对于中文标签，应该先定义好中文环境和字体。Asymptote 会调用 LaTeX 来进行标签的处理，因而需要设置的就是 LaTeX 的编译引擎与一般的中文 LaTeX 文件导言区。在这里，赵博士决定使用 XeTeX 引擎与 xeCJK 宏包来处理中文。为此，在 Asymptote 源文件中，他使用了下面的设置代码：

```
settings.tex = "xelatex";
usepackage("xeCJK");
```

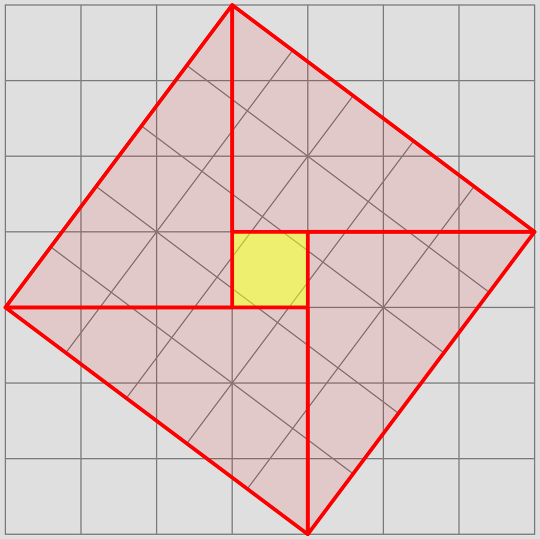


图 1.5: 带网格的弦图草图

```
texpreamble("\setCJKmainfont{SimSun}");
```

这里第一行是设置编译时所用的 TeX 引擎。后面 usepackage 命令就是 LaTeX 中的 **\usepackage** 命令的一个包装形式，里面的字符串参数就是宏包名；而 texpreamble 命令则把接收的参数直接放进 LaTeX 的导言区。

进行上述设置后，就可以正确使用中文标签了。标注的命令很简单，就是 label，参数正是标签文字和标签的位置。例如：

```
draw( (0,0) -- (1cm,1cm) -- (2cm,0) );
label( "中间", (1cm,0cm) );
```

就得到

可以在标签中使用任意的 LaTeX 代码，包括数学公式，例如：

```
label("$x = \sin\alpha$", (0,0));
```

就会正确地得到 $x = \sin\alpha$ 的标签。

现在，我们可以给弦图加上“朱实”、“黄实”和“弦实”的标签了。在前面的框架代码后面加上

```
label("朱实", (2cm,4cm));
label("黄实", (3.5cm,3.5cm));
label("弦实", (5cm,4cm));
```

以及设置字体的代码，就得到图 1.6 的结果。

下面则是要给三角形的三边进行标注。

与前面在一个点处标注不同，这里实际是给一条路径（直线）标注标签。因此，想要得到的是距离这条路径的中点一定方向距离加一个标签，而不是简单地取路径上的一点作为标签的位置。好在 Asymptote 确实也提供了这样的功能，仍然使用 label 命令，基本语法是：

```
label(标签, 路径)
```

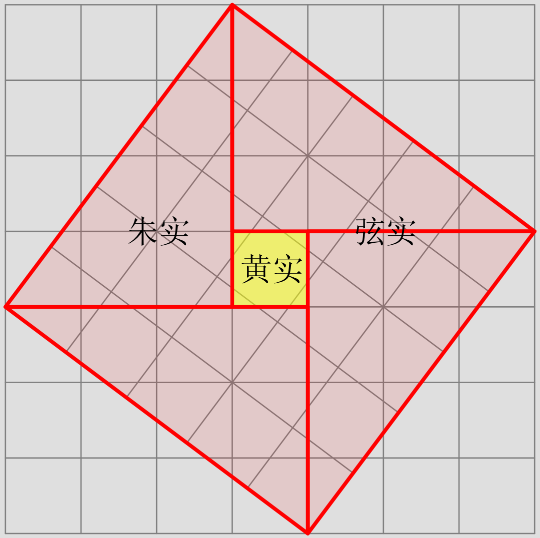
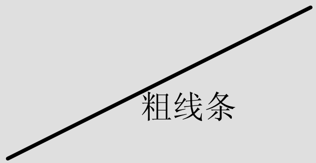


图 1.6: 带部分标注的弦图

这里默认会在路径中间的右侧（沿着路径行进方向）加标签。例如：

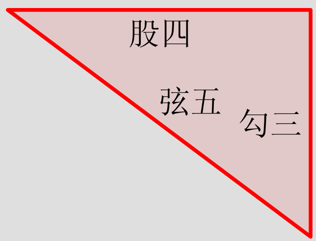
```
draw( (0,0) -- (4cm,2cm), linewidth(0.5mm) );
label("粗线条", (0,0) -- (4cm,2cm));
```

（其中的 linewidth 函数用来表示具有一定线宽的画笔）这段代码将得到



现在，赵博士就可以给三角形的三边加上“勾三”、“股四”、“弦五”的标签了，只要稍稍注意一下标签摆放的默认方向：

```
filldraw( (4cm,0) -- (4cm,3cm) -- (0,3cm) -- cycle,
    fillpen=opacity(0.1)+red, drawpen=red+0.5mm );
label( "勾三", (4cm,3cm) -- (4cm,0) );
label( "股四", (0,3cm) -- (4cm,3cm) );
label( "弦五", (4cm,0) -- (0,3cm) );
```

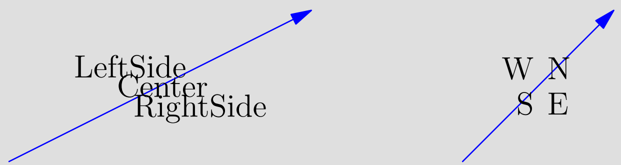


不过，为了得到正确的标签位置，不得不把原来逆时针画的线用顺时针方向重写，这多少让赵博士有些恼火：为什么不能在路径的左边标注标签呢？确实可以，很简单，只要给 label 命令再加上 align=LeftSide 选项（或

者简单地只用 LeftSide）就指定了在左边放置对齐。同理，还有向右对齐的 RightSide，在中间对齐的 Center 以及一般意义的相对方向 Relative(方向)。例如：

```
draw( (0,0) -- (4cm,2cm), blue, Arrow );
label( "LeftSide", (0,0) -- (4cm,2cm), align=LeftSide );
label( "RightSide", (0,0) -- (4cm,2cm), align=RightSide );
label( "Center", (0,0) -- (4cm,2cm), align=Center );

draw( (6cm,0)--(8cm,2cm), blue, Arrow );
label( "E", (6cm,0)--(8cm,2cm), Relative(E) );
label( "S", (6cm,0)--(8cm,2cm), Relative(S) );
label( "W", (6cm,0)--(8cm,2cm), Relative(W) );
label( "N", (6cm,0)--(8cm,2cm), Relative(N) );
```



E, S, W, N 分别是东南西北四个罗盘方向，用在 Relative 函数里面就表示相对于路径方向的四个方向。为明确，这里用 Arrow 选项在画线时加了箭头。

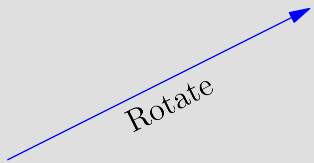
不过，这样的标签还是不能令赵博士满意。赵博士给三角形加标签，还希望标签随着三角形的边作旋转，使标签沿着边排列。为此，赵博士不得不又仔细查看了 [1]，在讲标注文字一节（label）他找到了一个构造标签的更高级的办法，即不仅仅是使用一个简单的字符串，而是使用

```
Label(标签)
```

函数进行构造。里面的“标签”参数仍然可以是原来的字符串，或是通过这个函数构造出来的高级的标签。这个函数可以带许多可选的其他参数，如 **position=位置** 的参数就可以指定标签放在路径中点之外的其他地方；而 embed=嵌入变换方式 的参数则可以解决标签自动旋转的问题。

这里暂且放下 **position** 参数。只来看 embed 参数的一个特例：Rotate(方向)。这个参数会让标签向着给定的方向旋转，如：

```
draw( (0,0)--(4cm,2cm), blue, Arrow );
label( Label("Rotate", Rotate((4,2))),
    (0,0)--(4cm,2cm) );
```



这里坐标 $(4,2)$ 正是这条直线的绘制方向。

终于，使用了上面的全部功能，赵博士完成了全部的图形标注工作（图 1.7）：

```
settings.tex = "xelatex";
usepackage("xeCJK");
texpreamble("\setCJKmainfont{SimSun}");

import math;
add( scale(1cm) * grid(7, 7, gray) );
add( shift(0,3cm) * rotate(-aTan(3/4)) * scale(1cm) * grid(5, 5, gray) );

fill( box((3cm,3cm), (4cm,4cm)), opacity(0.5)+yellow );
filldraw( (4cm,0) -- (4cm,3cm) -- (0,3cm) -- cycle
    ^^ (7cm,4cm) -- (4cm,4cm) -- (4cm,0) -- cycle
    ^^ (3cm,7cm) -- (3cm,4cm) -- (7cm,4cm) -- cycle
    ^^ (0,3cm) -- (3cm,3cm) -- (3cm,7cm) -- cycle,
    fillpen=opacity(0.1)+red, drawpen=red+0.5mm );
```

```
label("朱实", (2cm,4cm));
label("黄实", (3.5cm,3.5cm));
label("弦实", (5cm,4cm));
label( Label("勾三",Rotate(S)), (4cm,0)--(4cm,3cm), LeftSide );
label( Label("股四",Rotate(E)), (4cm,3cm)--(0,3cm), LeftSide );
label( Label("弦五",Rotate((4,-3))), (0,3cm)--(4cm,0), LeftSide );
```

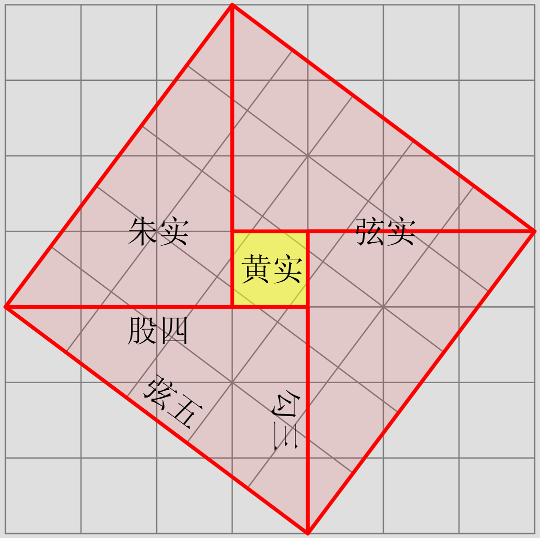


图 1.7: 带标注的完整弦图

习题

1. 查阅参考手册 [1]，看看都有哪些颜色可用。看看你的系统中安装了哪些中文字体。然后修改图 1.7，使用另一种你喜欢的字体进行标注；并且将“朱实”、“黄实”和“弦实”分别用红、黄、橙色标注。

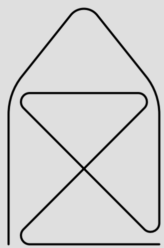
2. 查阅参考手册 [1]，看看 `draw`、`fill`、`label` 等命令都可以带哪些可选的参数，并自己举例子试验一下效果。
3. 代码

```
guide zhushi = (4cm,0) -- (4cm,3cm) -- (0,3cm) -- cycle;
```

   可以把一个三角形的路径保存在变量 `zhushi` 中，以后就可以使用 `draw(zhushi)` 这样的命令对此路径进行操作了[1]。

   考虑如何利用平移和旋转变换，只定义一个三角形的路径，就把弦图中的四个红色三角形都画出来。

4. 查阅手册 [1] 和 Asymptote 自带的例子，研究模块 roundedpath 的用法。并利用它尝试画出下面的图形：



   看看 Asymptote 中还有什么用法简单而又有趣的模块。

5. （较难）研究在 LaTeX 里中文直排的方法，尽量精确地复现出赵博士理想中的弦图效果（图 1.2）。

[1]这里定义了一个 **guide** 类型的变量，我们译之为“路向”，以区别于“路径”（**path**）。两个概念在 Asymptote 中同源而有别，但本章中二者可以互换并无区别，因此为方便我们也不区别而通称为路径。有关 Asymptote 语言中路径与路向的区别，以及关于变量定义和使用的详细内容，请参考后续的章节及手册 [1]。

第二章　André Deledicq 的铺砌插画

André 是一名兴趣广泛的法国数学教师，在他的新著《Le monde des pavages》(《铺砌世界》) 中，打算画一幅有关羊的铺砌插画：

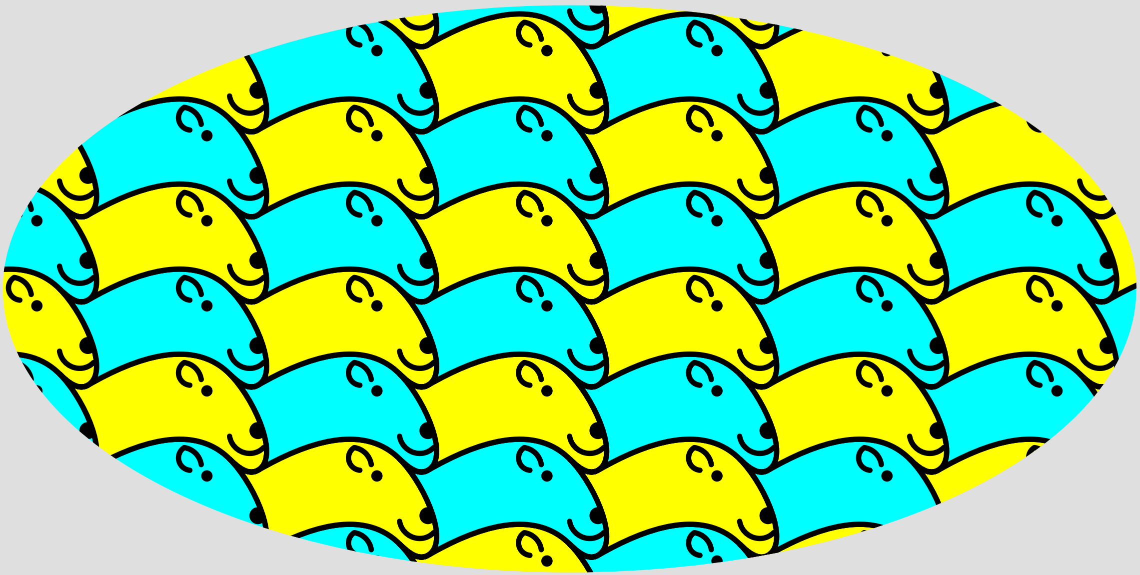
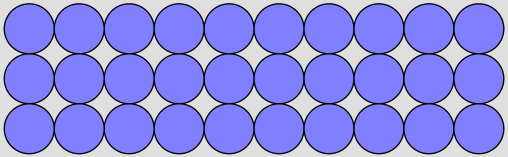


图 2.1: André 理想中的铺砌图

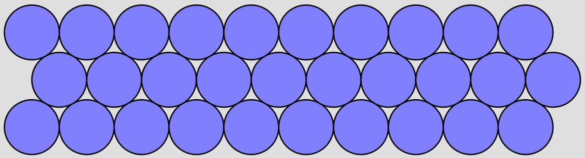
André 很清楚他要画的图形的数学理论，但 André 的朋友 Timothy 告诉他要画这样的图形多少是需要一些编程的知识的，对于他这样一位往日对计算机并不通晓的人来说可能会有困难。不过 André 并不以为意：这世上还有什么比数学更难的呢？于是他兴致勃勃的开始了。

2.1 从矩形到铺砌

铺砌图，顾名思义，就是像铺地板砖一样，把许多相同样式的图形平铺开来。不过，并不是什么图形都可以平铺填满整个平面的——比如圆形就不行。把许多圆形一个挨一个排列起来，也只能得到



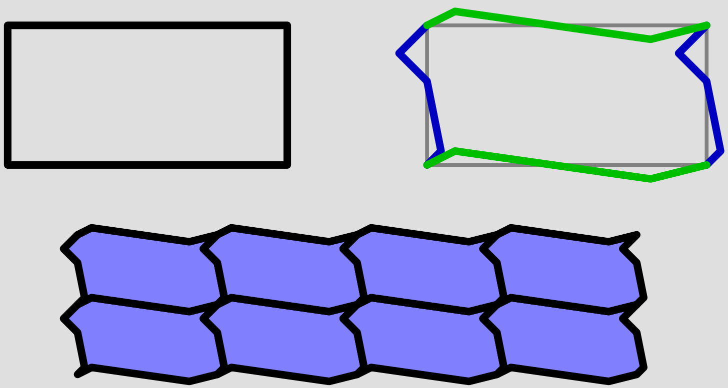
或者是



都会留下许多空隙。而矩形、平行四边形、六边形等等都可以不留空隙地把平面铺满。

但问题是，如何设计出 André 理想中的那种看起来形状不规则的铺砌图案呢？

身为数学教师的 André 当然有办法。其实不规则铺砌图案还是规则图案的变形。 André 要画的羊形铺砌图，其实就是从矩形铺砌变化而来的。只要把一个矩形图案的上下两边、左右两边分别变形，使得变形后的上边与下边、左边与右边还对应重合，就依然可以完美地拼合起来。这正是铺砌图案最基本的构成方式：



有了这个方法，对复杂的铺砌图，也只要从一个基本形状（比如矩形、正六边形）开始变形，就等到铺砌所需要的一块“砖”。

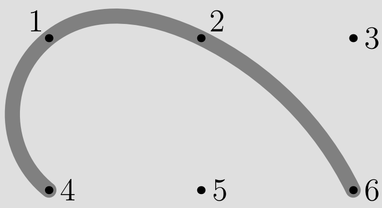
因此，要画出羊头形状铺砌图，只要把一个矩形按照上面的要求变形为一个羊头形状，在不同的位置重复画出就可以了。

2.2 变量与曲线

下面的问题就是，怎么画一个羊头呢？更具体地说，怎么画出羊头的曲线呢？

那么，首先要了解如何在 Asymptote 中描述曲线。1.2 节中提到 -- 连结一组坐标就成为直（折）线段；类似地，用 .. 连结坐标就得到经过这些坐标点的曲线：

```
size(5cm,0);
pair z1 = (0,1), z2 = (1,1), z3 = (2,1),
     z4 = (0,0), z5 = (1,0), z6 = (2,0);
guide p = z4 .. z1 .. z2 .. z6;
draw(p, gray+2mm);
```



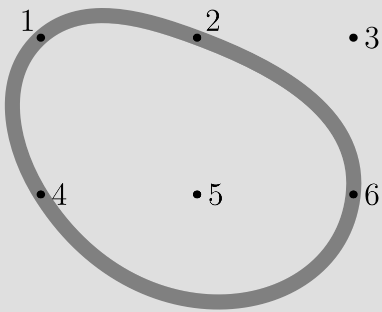
在这里，我们定义了一些变量以使代码清晰（这里略去了画点和标签的代码）。**pair** 类型的变量 z1, ⋯, z6 保存六个坐标，**guide** 类型的变量 p 保存一条曲线的路向。因而上面 size 之后的绘图代码就相当于

```
draw( (0,0) .. (0,1) .. (1,1) .. (2,0), gray+2mm );
```

其中前面的一句 size(5cm,0) 表示代码中的坐标只是相对位置，最后将整个图形按比例放缩为 5 cm 宽[1]。类似地，也可以使用 size(0,4cm) 把图形放缩到 4 cm 高。

最重要的当然还是曲线的表示。以 .. 连结的坐标会以一种尽量接近圆弧的方式连为经过这些点的光滑曲线。与画直线类似，**cycle** 可以作为一个特殊的坐标产生闭合曲线，即一条闭路向：

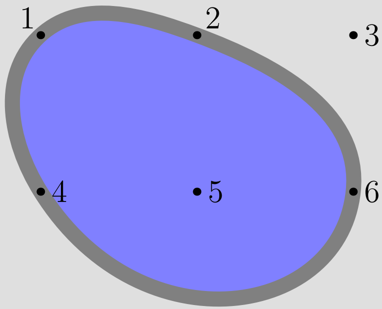
```
guide q = z4 .. z1 .. z2 .. z6 .. cycle;
draw(q, gray+2mm);
```



变量不仅仅是给了坐标、路向等对象一个简洁的名字，它也使得对同一个对象重复使用并进行不同的操作变得十分方便：

[1]注意坐标、图形会被放缩，但画笔的宽度不会放缩。

```
fill(q, lightblue);
draw(q, gray+2mm);
```

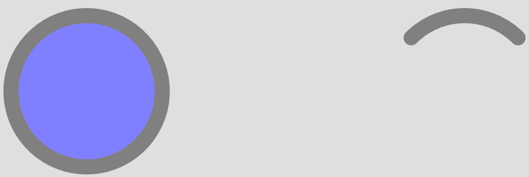


就像使用 box 可以直接得到矩形一样，最常用的曲线：圆、椭圆和圆弧，也可以使用现成的命令得到：

circle(c, r)	圆心 c，半径 r 的圆，这是逆时针方向的闭曲线；
ellipse(c, a, b)	中心为 c，长半轴 a，短半轴 b 的椭圆，这也是逆时针方向的闭曲线；
arc(c, r, angle1, angle2)	圆心 c，半径 r，角度从 angle1 到 angle2 的圆弧。

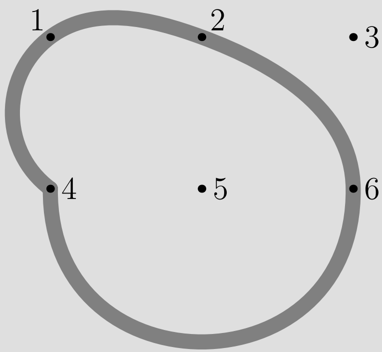
例如：

```
filldraw( circle((0,0), 1cm), lightblue, gray+2mm );
draw( arc((5cm,0), 1cm, 45, 135), gray+2mm );
```



一条用 **cycle** 产生的闭路向和简单地把首尾结点重合的路向是非常不同的。首先，只有闭路向可以填充颜色；其次，使用 **cycle** 连结的曲线在起点处是光滑连接的，而如果只是首尾结点重合则不会光滑连接。试将下面的曲线 q2 与上面的曲线 q 比较：

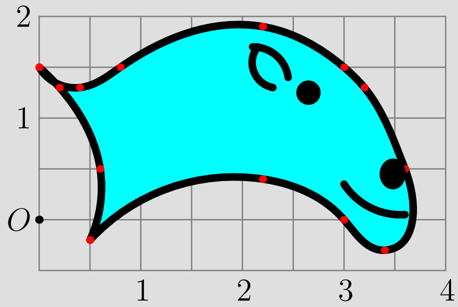
```
guide q2 = z4 .. z1 .. z2 .. z6 .. z4;
draw(q2, gray+2mm);
```



现在有了绘制曲线的方法，画出一个羊头就只是把草稿上的坐标连接起来而已。André 有一个纸上的草图，于是在描出几个点以后，他很快得到这样的结果（这里给图形增加了辅助网格）：

```
size(0,4cm);
pen outline = black+1mm;
// 头
guide head = (0.5,-0.2) .. (0.6,0.5) .. (0.2,1.3) .. (0,1.5) .. (0,1.5)
    .. (0.4,1.3) .. (0.8,1.5) .. (2.2,1.9) .. (3,1.5) .. (3.2,1.3)
    .. (3.6,0.5) .. (3.4,-0.3) .. (3,0) .. (2.2,0.4) .. (0.5,-0.2) .. cycle;
filldraw(head, cyan, outline);
dot(head, red+1mm); // 画出羊头曲线上的结点
// 五官
fill( circle((2.65,1.25), 0.12), outline );
```

```
fill( (3.5,0.3) .. (3.35,0.45) .. (3.5,0.6) .. (3.6,0.4) .. cycle, outline );
draw( (3,0.35) .. (3.3,0.1) .. (3.6,0.05), outline );
draw( (2.3,1.3) .. (2.1, 1.5) .. (2.15,1.7), outline );
draw( (2.1,1.7) .. (2.35,1.6) .. (2.45,1.4), outline );
```



在一开始，André 使用

```
pen outline = black+1mm;
```

定义一个 **pen** 类型的变量 `outline` 表示用来画羊头轮廓的画笔，以备使用。

然后，André 直接用 `..` 连结一组坐标来定义羊的头部轮廓：

```
guide head = (0.5,-0.2) .. (0.6,0.5) .. (0.2,1.3) .. (0,1.5) .. (0,1.5)
    .. (0.4,1.3) .. (0.8,1.5) .. (2.2,1.9) .. (3,1.5) .. (3.2,1.3)
    .. (3.6,0.5) .. (3.4,-0.3) .. (3,0) .. (2.2,0.4) .. (0.5,-0.2) .. cycle;
```

需要尖角的时候，就使用重复的相同点（如这里的起点）；曲线变化大的地方，取的点也比较密集。

最后五官的绘制。眼睛是填充的小黑圆，鼻子是黑色的卵形，耳朵和嘴都是简单的曲线。

于是，只要把这样一个图形一个挨一个地重复画许多遍，就可以得到 André 想要的铺砌效果了。设计羊头形状的工作无疑是最关键也最复杂的，因此 André 的任务现在就已经完成了一半。

不过继承了法国完美主义风气的 André 老师，很快挑出了毛病：这只羊头部的轮廓，并不完全是按照 2.1 节对矩形变形得到的——他的手稿基本上是这样设计的，但在使用 Asymptote 上绘图时则只是在手稿上相当随意地取了

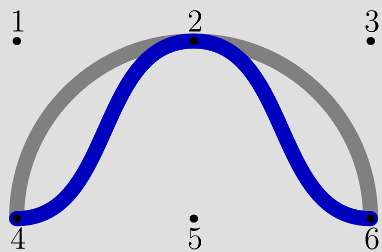
一些结点连结得到曲线，这个轮廓想必也并不能严丝合缝地一个个拼起来。还有一件很令他恼火的事情则是：要画出羊头的轮廓，他要画的点太多了，一个尖角用两个结点表示，也太不符合他的简洁美学了。因此，这个看上去相当不错的羊头一号，就被 André 老师无情地否决掉了。他决定发扬数学教师严谨简洁的作风，再做出更完美的羊头二号来。

2.3 细致的曲线调整与曲线操作

在 Asymptote 中，除了简单地使用 `--` 和 `..` 连接坐标来定义直线与曲线以外，也提供了更丰富的手段来对曲线进行更为细致的调整与操作。于是，就有了方向、张力、卷曲值这些东西。

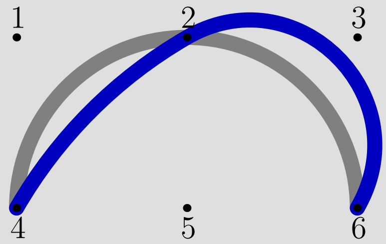
在曲线的结点处，可以使用以花括号括起的一个向量坐标来限定曲线在此处的切线方向 。例如：

```
draw(z4 .. z2 .. z6, gray+2mm);
draw(z4{right} .. z2{right} .. z6{right}, heavyblue+2mm);
```



在这里，常量 `right` 就相当于罗盘方向 `E`，也就是 `(0,1)`。事实上，在 Asymptote 中已经预定义好了上下左右四个方向：up, down, `left`, `right`，作为四个罗盘方向的同义词。此外，也可以使用预置的函数 `dir(角度)` 来表示指向此方向的单位向量，例如：

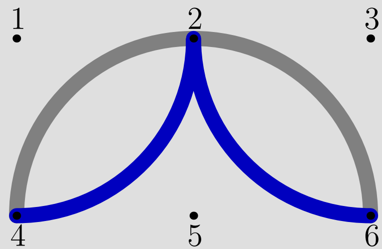
```
draw(z4 .. z2 .. z6, gray+2mm);
draw(z4{dir(60)} .. z2{dir(30)} .. z6{dir(-120)}, heavyblue+2mm);
```



当然，这里并不要求使用单位向量来限定曲线方向，使用任何长度的向量都得得到相同的效果。

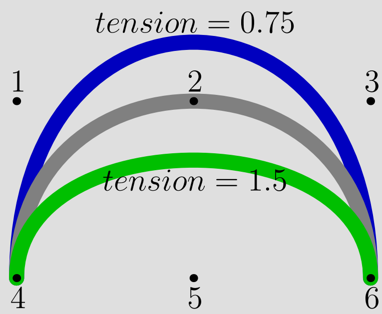
就像上面的例子，限定曲线的切线方向无疑使得定义一条曲线更加直观和方便。不仅如此，我们甚至还可以在曲线同一结点的两侧设置截然不同的切线方向，此时曲线就会在这个结点的位置得到一个尖角。如：

```
draw(z4 .. z2 .. z6, gray+2mm);
draw(z4 .. {up} z2 {down} .. z6, heavyblue+2mm);
```



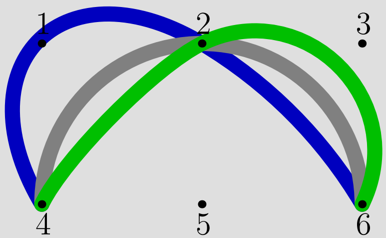
了解了曲线的方向，再来看曲线的张力。大体上讲，这是一个相对模糊的概念，因为它并不像曲线切线方向那样具有明确的几何特征。如果把结点看做是钉子，而把 Asymptote 的曲线看做是用若干钉子固定起来的一根弹簧或是硬橡皮绳，或许有助于理解所谓张力的概念。在不同的钉子之间，弹簧的张力越大，弹簧就越趋于直线的紧绷状态；反之，张力小时，弹簧就会因为松弛而自动张成圆弧状。张力用 **tension 张力值** 来设定，张力值是不小于 0.75 的实数，默认值 1 使得曲线最接近圆弧形状。例如：

```
draw(z4 {up} .. tension 0.75 .. {down} z6, heavyblue+2mm);
draw(z4 {up} .. tension 1 .. {down} z6, gray+2mm);
draw(z4 {up} .. tension 1.5 .. {down} z6, heavygreen+2mm);
```



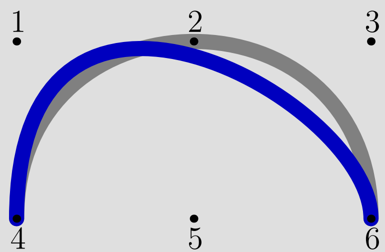
注意张力不仅影响两个结点之间的曲线形状，临近结点间的曲线形状也会受到影响。如：

```
draw(z4 .. tension 0.75 .. z2 .. z6, heavyblue+2mm);
draw(z4 .. tension 1 .. z2 .. z6, gray+2mm);
draw(z4 .. tension 1.5 .. z2 .. z6, heavygreen+2mm);
```



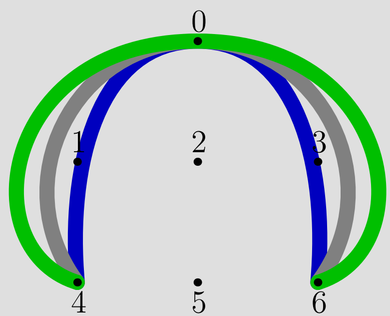
比弹簧或橡皮绳更强的是，Asymptote 曲线在结点两边的张力事实上也可以使用 **tension 起 and 末** 来分开设定：

```
draw(z4 {up} .. tension 1 .. {down} z6, gray+2mm);
draw(z4 {up} .. tension 0.75 and 2 .. {down} z6, heavyblue+2mm);
```



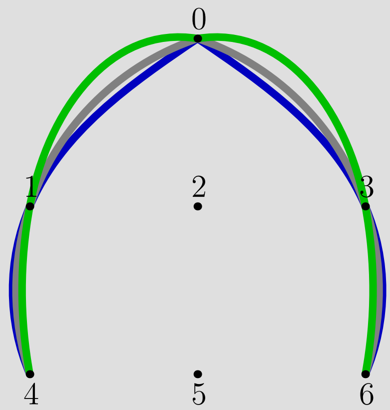
最后来看卷曲度。卷曲度影响曲线端点的弯曲程度，它的语法和设定曲线方向类似：**{curl 数值}**。卷曲度是一个非负值，其数值越大则曲线在端点的曲率越大，数值为 0 时则曲率趋于 0。在曲线的端点处，有默认的卷曲度 1，使得曲线接近圆弧。例如：

```
draw(z4 {curl 0} .. z0 .. {curl 0} z6, heavyblue+2mm);
draw(z4 {curl 1} .. z0 .. {curl 1} z6, gray+2mm);
draw(z4 {curl 5} .. z0 .. {curl 5} z6, heavygreen+2mm);
```



必须注意的是，卷曲度影响的是曲线的首尾端点的弯曲情况；而如果某个结点原本不是曲线的端点，那么曲线就会在此处折成尖角——特别地，如果相临端点都设置了卷曲度值，那么这两个结点之间就会是一段直线。例如：

```
draw(z4 .. z1 .. z0{curl 0} .. z3 .. z6, heavyblue+1mm);
draw(z4 .. z1 .. z0{curl 1} .. z3 .. z6, gray+1mm);
draw(z4 .. z1 .. z0{curl 5} .. z3 .. z6, heavygreen+1mm);
```

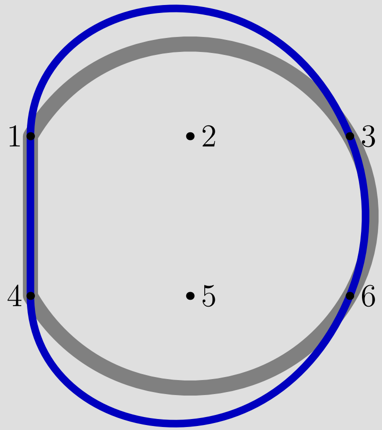


除了直线连接的符号 `--` 和曲线连接的符号 `..`，还有几个连接点坐标以得到曲线的连接符，它们分别是特殊的直线连接符 `---`、特殊的曲线连接符 `::` 以及将曲线首尾相接的 `&`。再加上连接几条不同曲线的 `^^`，就是全部连接曲线的符号了。

连接符 `---` 与 `--` 类似，都得到直线，单独使用 `---` 画出的直线和折线与使用 `--` 得到的结果并不能看出区别。但事实上，用 `---` 得到的并不是严格的直线折线，而只是非常接近直线的光滑曲线[2]。因而，如果把 `---` 与 `..` 配合使用，就能使得到的直线与曲线光滑地连接，而不像使用 `--` 那样得到不需要的尖角。例如：

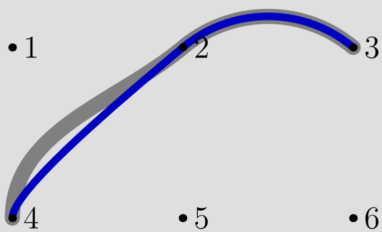
```
draw(z6 .. z4 -- z1 .. z3 .. cycle, gray+2mm);
draw(z6 .. z4 --- z1 .. z3 .. cycle, heavyblue+1mm);
```

[2]在 Asymptote 内部，连接符 `---` 是 `.. tension atleast infinity ..` 的缩写。**atleast** 关键字一般并不直接使用，故本文不作讨论，可参考 [2]。



连接符 :: 则与 .. 类似，都得到曲线[3]。不过连接符 :: 有一个特性，就是它将尽可能使两个坐标之间的曲线没有拐点（即尽可能避免两坐标间出现 S 形曲线，效果是在 .. 的基础上自动增加张力的结果）。例如：

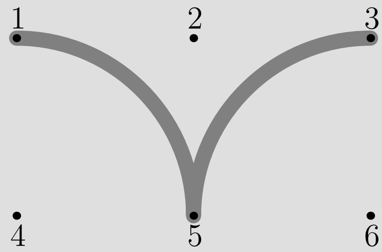
```
draw(z4{up} .. z2{dir(40)} .. z3, gray+2mm);
draw(z4{up} :: z2{dir(40)} .. z3, heavyblue+1mm);
```



& 连接符用于把几段首尾点依次重合的曲线直接相连，而不改变原来连接前曲线的形状。这事实上与使用重复点直接用 .. 生成尖角的效果相同，但在 Asymptote 中，连接符 & 会在生成的曲线中去除第一条曲线的尾结点（它应该是与第二条曲线的首结点重合的），因而不会出现像之前直接使用 .. 生成的曲线那样会有重复的结点。例如：

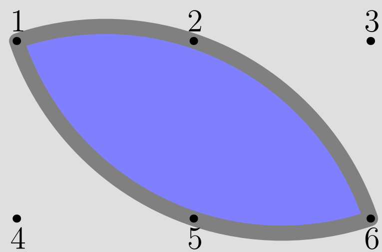
[3]在 Asymptote 内部，连接符 :: 是 .. **tension atleast** 1 .. 的缩写。Asymptote 的曲线功能和语法大多源自 METAPOST，不过在 METAPOST 中与 :: 功能相同的连接符是 ...。

```
draw(z1{right} .. {down} z5 & z5 {up} .. {right}z3, gray+2mm);
```



另一方面，相比直接单独绘制几条不相关的曲线，使用 & 连接符的一个重要作用是可以由此生成合适的闭曲线。例如：

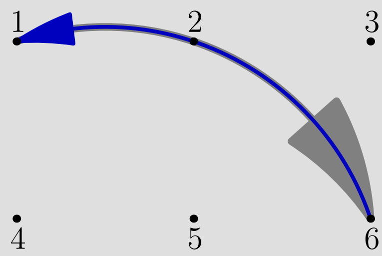
```
filldraw(z1 .. z2 .. z6 & z6 .. z5 .. z1 & cycle, lightblue, gray+2mm);
```



连接符 ^^ 在前面 section 1.2 中已经介绍过了。这里则指出它的一个特殊应用：在一个区域中“挖洞”填充。在数学中，平面上的一条简单闭曲线有的方向可以分为正负两种，逆时针方向为正，正时针方向为负。因此，在某个方向曲线围成的区域中用连接符 ^^ 加入一条反方向的闭曲线，就相当于在这个封闭区域中挖了一个洞。

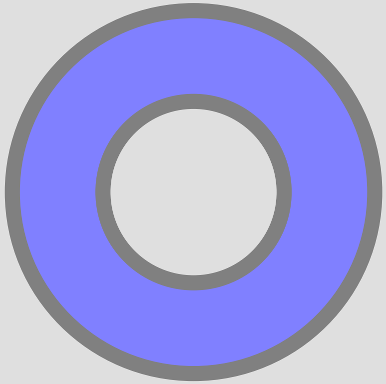
在给出相关的例子之前，我们首先给出 Asymptote 中把曲线方向逆转的函数 reverse(曲线)，它接受一条曲线（可以是路径或路向）作为参数，并返回此曲线的逆方向曲线。绘制时给曲线加上箭头可以明显看出其方向：

```
draw(z1 .. z2 .. z6, gray+1mm, Arrow);
draw(reverse(z1 .. z2 .. z6), heavyblue+0.5mm, Arrow);
```



于是，利用这种机制，在大圆内加上一个反方向的圆，把这两条曲线用 ^^ 连接，就可以用来填充得到环形：

```
size(5cm, 0);
filldraw(circle((0,0), 2) ^^ reverse(circle((0,0), 1)), lightblue, gray+2mm);
```

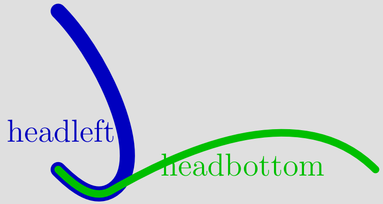


需要说明的是，使用 & 连接两条曲线，得到的结果是一条连续的曲线；而使用 ^^ 连接两条曲线，得到的结果并不是一条曲线，而只是把两条曲线合并在一起的曲线数组（数组在稍后的章节介绍）。因而它们在使用时是不同的。

上面介绍的内容，差不多就是 Asymptote 中全部基本的曲线控制方式了。有了这些， André 老师就可以按照 section 2.1 中的方式，精确地设计羊头的轮廓了。他分别定义羊头左边和下面的两根线：

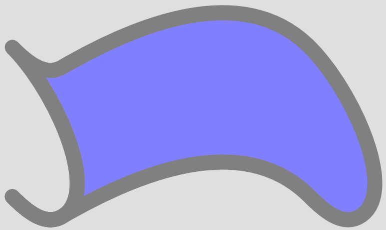
```
guide headleft = (0,1.5){SE} .. tension 1.4 .. (0.5,-0.2){dir(-150)} ..
      {NW}(0,0);
```

```
guide headbottom = (0,0){SE} .. {dir(30)}(0.5,-0.2) ..{SE}(3,0);
```



这里的两条曲线虽然看起来定义比较复杂，但它们都只是由三个结点连接而成的，定义更为直观，且在数学上更加简单。此外，明确限定了两条曲线在结点处的切线方向，可以保证它们可以精确拼接。于是，通过平移变换、使用 `reverse` 函数逆向并使用连接符 & 的拼接，就可以得到完整的羊头轮廓了：

```
guide head = headleft & headbottom &
    shift(3,0)*reverse(headleft) & shift(0,1.5)*reverse(headbottom) & cycle;
```

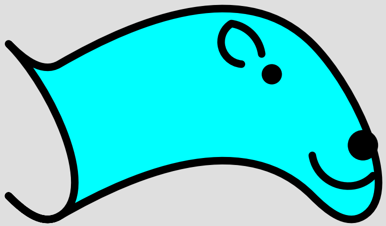


下面，羊的眼睛（eye）、耳朵（ear）、鼻子（muzzle）和嘴（mouth）就比较容易得到了：

```
guide eye = circle((2.6,1.2), 0.1);
guide ear = (2.3,1.3) .. (2.1,1.5) .. (2.2,1.7)
    & (2.2,1.7) .. (2.4,1.6) .. (2.5,1.4);
guide muzzle = circle((3.5,0.5), 0.15);
guide mouth = (3,0.4) .. (3.4,0.1) .. (3.6,0.2);
```

于是，André 就这样画出了一个完整的青色羊头：

```
size(5cm, 0);
filldraw(head, cyan, linewidth(1mm));
fill(eye ^^ muzzle);
draw(ear ^^ mouth, linewidth(1mm));
```



挑剔的 André 对这个羊头的曲线感到满意，因为作为一个铺砌图形，它绘制得完全精确。一个羊头已经可以很好地画出来了，剩下的问题就是重复地把这个羊头铺满平面，开始“铺砖”了。

2.4 循环与条件判断

要在平面上铺满相同的图形，就需要反复地在不同的位置进行绘制。Asymptote 中可以使用循环语句来完成这种重复的工作。

最常用的循环语句是 **for** 循环，其基本语法是：

```
for (初始化; 执行条件; 循环增量)
    循环语句体
```

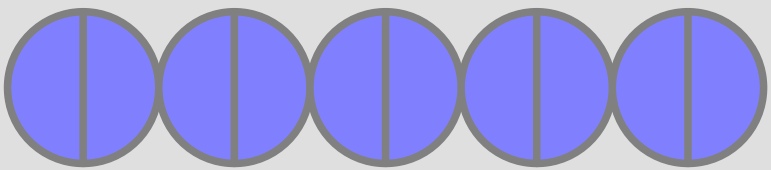
在一个循环语句中，Asymptote 首先运行初始化语句，随后开始不断运行循环语句体，每次运行语句体前检查执行条件以判断是否开始，而在运行语句体后运行循环增量的语句。例如下面的语句会画出起始角度为 0°, 30°, 60°, 90° 的四条曲线：

```
for (real angle = 0; angle <= 90; angle += 30)
    draw( (0,0){dir(angle)} .. {right}(6cm,0), gray+1mm );
```

在初始化部分定义了一个实数（**real**）类型的变量 angle，初始值为 0；进入循环的执行条件是角度变量 angle 不大于直角；而在每次循环体画完曲线后，变量 angle 的值就增加 30°。这里复合赋值运算符 += 表示在变量原有的基础上累加， var += x 就等价于 var = var + x，类似地减法、乘除法、取余数和乘方也有 -=、*=、 /=、%=、^= 这几种对应的赋值形式。对于整数 （**int**）类型的变量，还可以使用 ++var 和 --var 分别表示 var += 1 和 var -= 1。

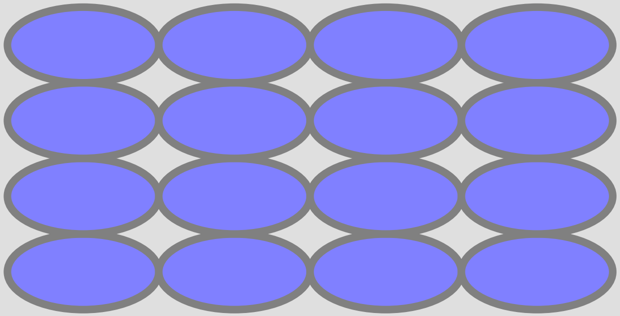
如果循环体的语句不止一个，可以用花括号把这些语句括起来。如：

```
for (int i = 0; i < 5; ++i) {
    filldraw(circle((2i*cm,0), 1cm), lightblue, gray+1mm);
    draw( (2i*cm,-1cm) -- (2i*cm,1cm), gray+1mm);
}
```



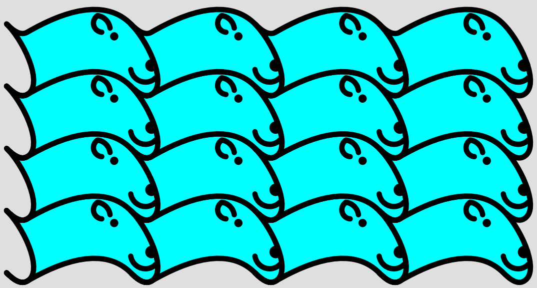
平面铺砌图形需要在横向和纵向两个方向重复延伸，因而可以使用两层嵌套的循环语句来完成这种图形：

```
for (real x = 0; x < 8cm; x += 2cm)
    for (real y = 0; y < 4cm; y += 1cm)
        filldraw(ellipse((x,y), 1cm, 0.5cm), lightblue, gray+1mm);
```



因此，有了前面的准备工作，André 就很容易通过一个双重循环来做出羊头的铺砌形状来：

```
// ……相关曲线的定义同前
size(7cm, 0);
for (int x = 0; x < 4; ++x) {
    for (int y = 0; y < 4; ++y) {
        transform pos = shift(3x, 1.5y);
        filldraw( pos * head, cyan, linewidth(2bp));
        fill( pos * (eye ^^ muzzle) );
        draw( pos * (ear ^^ mouth), linewidth(2bp));
    }
}
```



在这里，André 使用关于整数变量 x 和 y 的二重循环来控制整个铺砌图。在循环体中使用了变换类型 **transform** 的变量 pos 来保存从原点到每一块铺砌图形元的平移变换。在绘图时，给每条曲线都使用 pos 进行变换，就可以在正确的位置绘图，而不用直接修改原来的曲线。

现在，André 几乎已经得到了和图 2.1 相同的图形了。不过在 图 2.1 中，羊头的颜色是随位置交错变化的。一个笨办法是分别用两组循环来画出黄色和青色的羊头，但这会带来很多重复的代码。因此 André 决定在循环体内做一点手脚：出于数学教师的敏感，他发现羊头颜色的规律可以由整数 $x+y$ 的奇偶性来判定，于是只要在循环体内增加一点判断……

Asymptote 的条件判断语句正好可以完成这项工作。条件判断语句的基本语法是：

```
// 只有一个分支
if (条件)
    条件为真执行的语句块
// 有 else 块，有两个分支
if (条件)
    条件为真执行的语句块
else
    条件为假执行的语句块
```

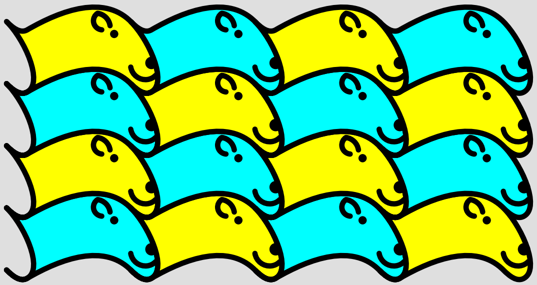
其中，可以用花括号括起多行的语句块。因而，可以使用如下的语句来通过判断 $x+y$ 的奇偶性来决定画笔的颜色：

```
pen color;
if ( (x+y) % 2 == 0 )   // 判断 x+y 是否为偶数
    color = cyan;
else
    color = yellow;
```

把上面的控制颜色的判断语句用到前面的循环体中，就可以得到交错色彩的铺砌图了：

```
// ……相关曲线的定义同前
size(7cm, 0);
for (int x = 0; x < 4; ++x) {
    for (int y = 0; y < 4; ++y) {
```

```
        transform pos = shift(3x, 1.5y);
        pen color;
        if ( (x+y) % 2 == 0 )
            color = cyan;
        else
            color = yellow;
        filldraw( pos * head, color, linewidth(2bp));
        fill( pos * (eye ^^ muzzle) );
        draw( pos * (ear ^^ mouth), linewidth(2bp));
    }
}
```



至此，André 大松一口气，他的铺砌图形基本上就绘制完成了。不过在他结束这一切的时候，他的同事，Asymptote 的专家 Philippe 告诉他，其实有关条件判断的语句还可以使用一个条件表达式来简化。在这里，要判断的条件比较简单，而判断的结果也只是青、黄两种颜色的值，因此，可以使用条件运算符 ? : 来代替。条件运算符是一个特殊的三元运算符，它的语法是：

```
条件 ? 条件为真的值 : 条件为假的值
```

因此前面的颜色判断和赋值语句就可以简化为：

```
pen color = (i+j)%2==0 ? cyan : yellow;
```

现在，André 的铺砌图形就完成了。下面只要把他的图形剪裁一下，就大工告成了。

2.5 图形的剪裁

André 最后的问题是把整个铺砌图形剪裁为椭圆形。为此，我们就需要引入在画线（draw）、填充（fill）、标注（label）之外的最后一种基本绘图命令，即剪裁（clip）。剪裁命令的基本语法是：

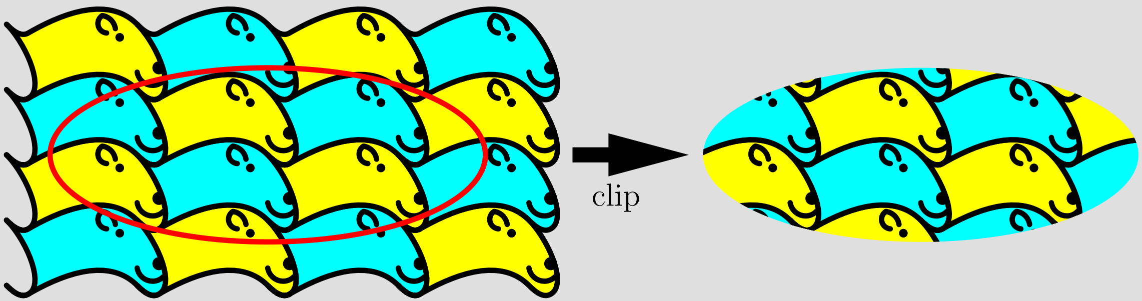
```
clip(图, 曲线);     // 图的默认值是当前图 currentpicture
```

如果省略 **picture** 类型的图参数，则剪裁命令会把当前正在绘制的图用给定的曲线裁剪开来；显示指定图参数则只剪裁给定的图中的内容。

剪裁命令的效果正是 André 需要的，为了用椭圆形剪裁图形，André 只需要增加一句

```
clip(ellipse((6,3), 5, 2));
```

就得到下图右侧的剪裁效果：



最后，André 得到了完整画出图 2.1 的全部代码：

```
// André 的铺砌图案

// 全局图形缩放
size(15cm, 0);

// 定义羊头曲线
```

```
guide headleft = (0,1.5){SE} .. tension 1.4 .. (0.5,-0.2){dir(-150)} ..
     {NW}(0,0);
guide headbottom = (0,0){SE} .. {dir(30)}(0.5,-0.2) ..{SE}(3,0);
guide head = headleft & headbottom &
    shift(3,0)*reverse(headleft) & shift(0,1.5)*reverse(headbottom) & cycle;
guide eye = circle((2.6,1.2), 0.1);
guide ear = (2.3,1.3) .. (2.1,1.5) .. (2.2,1.7)
    & (2.2,1.7) .. (2.4,1.6) .. (2.5,1.4);
guide muzzle = circle((3.5,0.5), 0.15);
guide mouth = (3,0.4) .. (3.4,0.1) .. (3.6,0.2);

// 绘制铺砌图形
for (int x = 0; x < 8; ++x) {
    for (int y = 0; y < 8; ++y) {
        transform pos = shift(3x, 1.5y);
        pen color;
        if ( (x+y) % 2 == 0 )
            color = cyan;
        else
            color = yellow;
        filldraw(pos * head, color, linewidth(2bp));
        fill(pos * (eye ^^ muzzle) );
        draw(pos * (ear ^^ mouth), linewidth(2bp));
    }
}

// 剪裁为椭圆形
clip(ellipse((12,6), 10, 5));
```

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Version 3, 29 June 2007

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```
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索引

参考文献

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