

# Imitation Learning by State-Only Distribution Matching

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## Abstract

Imitation Learning from observation describes policy learning in a similar way to human learning. An agent’s policy is trained by observing an expert performing a task. While many state-only imitation learning approaches are based on adversarial imitation learning, one main drawback is that adversarial training is often unstable and lacks a reliable convergence estimator. If the true environment reward is unknown and cannot be used to select the best-performing model, this can result in bad real-world policy performance. We propose a non-adversarial learning-from-observations approach, together with an interpretable convergence and performance metric.

Our training objective minimizes the Kulback-Leibler divergence (KLD) between the policy and expert state transition trajectories which can be optimized in a non-adversarial fashion. Such methods demonstrate improved robustness when learned density models guide the optimization. We further improve the sample efficiency by rewriting the KLD minimization as the Soft Actor Critic objective based on a modified reward using additional density models that estimate the environment’s forward and backward dynamics. Finally, we evaluate the effectiveness of our approach on well-known continuous control environments and show state-of-the-art performance while having a reliable performance estimator compared to several recent learning-from-observation methods.

## 1 INTRODUCTION

Imitation learning (IL) describes methods that learn optimal behavior that is represented by a collection of expert demonstrations. While in standard reinforcement learning (RL), the agent is trained on environment feedback using a reward signal, IL can alleviate the problem of designing effective reward functions. This is particularly useful for tasks where demonstrations are more accessible than designing a reward function. One popular example is to train traffic agents in

a simulation to mimic real-world road users [Kuefler *et al.*, 2017].

Learning-from-demonstrations (LfD) describes IL approaches that require state-action pairs from expert demonstrations [Ho and Ermon, 2016]. While actions can guide policy learning, it might be very costly or even impossible to collect actions alongside state demonstrations in many real-world setups. For example, when expert demonstrations are available as video recordings without additional sensor signals. One example of such a setup is training traffic agents in a simulation, where the expert data contains recordings of traffic in bird’s eye view [Kuefler *et al.*, 2017]. No direct information on the vehicle physics, throttle, and steering angle is available. Another example is teaching a robot to pick, move, and place objects based on human demonstrations [Osa *et al.*, 2018]. In such scenarios, actions have to be estimated based on sometimes incomplete information to train an agent to imitate the observed behavior.

Alternatively, learning-from-observations (LfO) performs state-only IL and trains an agent without actions being available in the expert dataset [Torabi *et al.*, 2019]. While LfO is a more challenging task than LfD, it can be more practical in case of incomplete data sources. Learning an additional environment model may help to infer actions based on expert observations and the learned environment dynamics [Torabi *et al.*, 2018a]. Like LfD, distribution matching based on an adversarial setup is commonly used in LfO [Torabi *et al.*, 2018b]. In adversarial imitation learning (AIL), a policy is trained using an adversarial discriminator, which is used to estimate a reward that guides policy training. While AIL methods obtain better performing agents than supervised methods like behavioral cloning (BC) using less data, adversarial training often has stability issues [Miyato *et al.*, 2018] and under some conditions is not guaranteed to converge [Jin *et al.*, 2020]. Additionally, estimating the performance of a trained policy without access to the environment reward can be very challenging. While the duality gap [Grnarova *et al.*, 2019; Sidheekh *et al.*, 2021] is a convergence metric suited for GAN based methods, it is difficult to use in the AIL setup since it relies on the gradient of the generator for an optimization process. In the AIL setup, the generator consists of the policy and the environment and therefore the gradient is difficult to estimate with black box environments. As an alternative for AIL setups the predicted reward (discriminator

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output) or the policy loss can be used to estimate the performance.

To address the limitations of AIL, we propose a state-only distribution matching method that learns a policy in a non-adversarial way. We optimize the Kulback-Leibler divergence (KLD) between the actionless policy and expert trajectories by minimizing the KLD of the conditional state transition distribution of the policy and the expert for all time steps. We estimate the expert state transition distribution using normalizing flows, which can be trained offline using the expert dataset. Thus, stability issues arising from the min-max adversarial optimization in AIL methods can be avoided. This objective is similar to FORM [Jaegle *et al.*, 2021], which was shown to be more stable in the presence of task-irrelevant features.

While maximum entropy RL methods [Ziebart, 2010] can improve policy training by increasing the exploration of the agent, they also add a bias if being used to minimize the proposed KLD. To match the transition distributions of the policy and the expert exactly, the state-next-state distribution of the policy is expanded into policy entropy, forward dynamics and inverse action model of the environment. It has been shown that such dynamic models can improve the convergence [Zhu *et al.*, 2020] and are well suited to infer actions not available in the dataset [Torabi *et al.*, 2018a]. We model these distributions using normalizing flow models which have been demonstrated to perform very well on learning complex probability distributions [Papamakarios *et al.*, 2019]. Combining all estimates results in an interpretable reward that can be used together with standard maximum entropy RL methods [Haarnoja *et al.*, 2018a]. The optimization based on the KLD provides a reliable convergence metric of the training and a good estimator for policy performance.

As contributions we derive SOIL-TDM (State Only Imitation Learning by Trajectory Distribution Matching), a non-adversarial LfO method which minimizes the KLD between the conditional state transition distributions of the policy and the expert using maximum entropy RL. We show the convergence of the proposed method using off-policy samples from a replay buffer. We develop a practical algorithm based on the SOIL-TDM objective and demonstrate its effectiveness to measure its convergence compared to several other state-of-the-art methods. Empirically we compared our method to the recent state-of-the-art IL approaches OPOLO [Zhu *et al.*, 2020], f-IRL [Ni *et al.*, 2020], and FORM [Jaegle *et al.*, 2021] in complex continuous control environments. We demonstrate that our method is superior especially if the selection of the best policy cannot be based on the true environment reward signal. This is a setting which more closely resembles real-world applications in autonomous driving or robotics where it is difficult to define a reward function [Osa *et al.*, 2018].

## 2 Background

In this work, we want to train a stochastic policy function  $\pi_\theta(a_t|s_t)$  in continuous action spaces with parameters  $\theta$  in a sequential decision making task considering finite-

horizon environments<sup>1</sup>. The problem is modeled as a Markov Decision Process (MDP), which is described by the tuple  $(S, A, p, r)$  with the continuous state spaces  $S$  and action spaces  $A$ . The transition probability is described by  $p(s_{t+1}|s_t, a_t)$  and the bounded reward function by  $r(s_t, a_t)$ . At every time step  $t$  the agent interacts with its environment by observing a state  $s_t$  and taking an action  $a_t$ . This results in a new state  $s_{t+1}$  and a reward signal  $r_{t+1}$  based on the transition probability and reward function. We will use  $\mu^{\pi_\theta}(s_t, a_t)$  to denote the state-action marginals at time step  $t$  of the trajectory distribution induced by the policy  $\pi_\theta(a_t|s_t)$ .

### 2.1 Maximum Entropy Reinforcement Learning and Soft Actor Critic

The standard objective in RL is the expected sum of undiscounted rewards  $\sum_{t=0}^T \mathbb{E}_{(s_t, a_t) \sim \mu^{\pi_\theta}} [r(s_t, a_t)]$ . The goal of the agent is to learn a policy  $\pi_\theta(a_t|s_t)$  which maximises this objective. The maximum entropy objective

$$J(\pi_\theta) = \sum_{t=0}^T \mathbb{E}_{(s_t, a_t) \sim \mu^{\pi_\theta}} [r(s_t, a_t) + \alpha \mathcal{H}(\pi_\theta(\cdot|s_t))] \quad (1)$$

introduces a modified goal for the RL agent, where the agent has to maximise the sum of the reward signal and its output entropy  $\mathcal{H}(\pi_\theta(\cdot|s_t))$  [Ziebart, 2010]. The parameter  $\alpha$  controls the stochasticity of the optimal policy by determining the relative importance of the entropy term versus the reward.

Soft Actor-Critic (SAC) [Haarnoja *et al.*, 2018a; Haarnoja *et al.*, 2018b] combines off-policy Q-Learning with a stable stochastic actor-critic formulation. The soft Q-function parameters  $\Psi$  can be trained with:

$$J_Q = \mathbb{E}_{(s_t, a_t) \sim D_{RB}} \left[ \frac{1}{2} (Q_\Psi(s_t, a_t) - (r(s_t, a_t) + \gamma \mathbb{E}_{s_{t+1}} [V_\Psi(s_{t+1})]))^2 \right] \quad (2)$$

The soft state value function is defined by:

$$V^\pi(s_t) := \mathbb{E}_{a_t \sim \pi} [Q(s_t, a_t) - \alpha \log \pi_\theta(a_t|s_t)] \quad (3)$$

Lastly, the policy is optimized by minimizing the following objective:

$$J_\pi = \mathbb{E}_{(s_t) \sim D_{RB}} [\mathbb{E}_{(a_t) \sim \pi_\theta} [\alpha \log \pi_\theta(a_t|s_t) - Q_\Psi(s_t, a_t)]] \quad (4)$$

### 2.2 Imitation Learning

In the IL setup, the agent does not have access to the true environment reward function  $r(s_t, a_t)$  and instead has to imitate expert trajectories performed by an expert policy  $\pi_E$  collected in a dataset  $\mathcal{D}_E$ .

In the typical **learning-from-demonstration** setup the expert demonstrations  $\mathcal{D}_E^{LfD} := \{s_t^k, a_t^k, s_{t+1}^k\}_{k=1}^N$  are given by action-state-next-state transitions. Distribution matching has been a popular choice among different LfD approaches. The policy  $\pi_\theta$  is learned by minimizing the discrepancy between the stationary state-action distribution induced by the expert  $\mu^E(s, a)$  and the policy  $\mu^{\pi_\theta}(s, a)$ . An overview and comparison of different LfD objectives resulting from this discrepancy minimization was done by Ghasemipour *et al.* [2019].

<sup>1</sup>An extension to infinite-horizons is given in section 3

Often the backward KLD is used to measure this discrepancy [Fu *et al.*, 2017]:

$$\min J_{LFD}(\pi_\theta) := \min \mathbb{D}_{KL}(\mu^{\pi_\theta}(s, a) || \mu^E(s, a)) \quad (5)$$

**Learning-from-observation (LfO)** considers a more challenging task where expert actions are not available. Hence, the demonstrations  $\mathcal{D}_E^{LfO} := \{s_t^k, s_{t+1}^k\}_{k=1}^N$  consist of state-next-state transitions. The policy learns which actions to take based on interactions with the environment and the expert state transitions. Distribution matching based on state-transition distributions is a popular choice for state-only IL [Torabi *et al.*, 2018b; Zhu *et al.*, 2020]:

$$\min J_{LfO}(\pi_\theta) := \min \mathbb{D}_{KL}(\mu^{\pi_\theta}(s, s') || \mu^E(s, s')) \quad (6)$$

### 3 Method

In a finite horizon MDP setting, the joint state-only trajectory distributions are defined by the start state distribution  $p(s_0)$  and the product of the conditional state transition distributions  $p(s_{i+1}|s_i)$ . For the policy distribution  $\mu^{\pi_\theta}$  and the expert distribution  $\mu^E$  this becomes:

$$\begin{aligned} \mu^{\pi_\theta}(s_T, \dots, s_0) &= p(s_0) \prod_{i=0..T-1} \mu^{\pi_\theta}(s_{i+1}|s_i), \\ \mu^E(s_T, \dots, s_0) &= p(s_0) \prod_{i=0..T-1} \mu^E(s_{i+1}|s_i) \end{aligned}$$

Our goal is to match the state-only trajectory distribution  $\mu^{\pi_\theta}$  induced by the policy with the state-only expert trajectory distribution  $\mu^E$  by minimizing the Kulback-Leibler divergence (KLD) between them:

$$\begin{aligned} J_{SOIL-TDM} &= \mathbb{D}_{KL}(\mu^{\pi_\theta} || \mu^E) \\ &= \mathbb{E}_{(s_T, \dots, s_0) \sim \mu^{\pi_\theta}} [\log \mu^{\pi_\theta} - \log \mu^E] \\ &= \sum_{i=0..T-1} \mathbb{E}_{(s_{i+1}, s_i) \sim \mu^{\pi_\theta}} [\log \mu^{\pi_\theta}(s_{i+1}|s_i) \\ &\quad - \log \mu^E(s_{i+1}|s_i)] \end{aligned} \quad (7)$$

The conditional expert state transition distribution  $\mu^E(s_{i+1}|s_i)$  can be learned offline from the demonstrations for example by training a conditional normalizing flow on the given state/next-state pairs. The policy induced conditional state transition distribution can be rewritten with the Bayes theorem using the environment model  $p(s_{i+1}|a_i, s_i)$  and the inverse action distribution density  $\pi'_\theta(a_i|s_{i+1}, s_i)$ :

$$\mu^{\pi_\theta}(s_{i+1}|s_i) = \frac{p(s_{i+1}|a_i, s_i) \pi_\theta(a_i|s_i)}{\pi'_\theta(a_i|s_{i+1}, s_i)} \quad (8)$$

It holds for any  $a_i$  where  $\pi' > 0$ . Thus, one can extend the expectation over  $(s_{i+1}, s_i)$  by the action  $a_i$  and the KLD minimization  $\min \mathbb{D}_{KL}(\mu^{\pi_\theta} || \mu^E)$  can be rewritten as

$$\begin{aligned} \min \sum_{i=0..T-1} \mathbb{E}_{(s_i, a_i, s_{i+1}) \sim \pi_\theta} [\log p(s_{i+1}|a_i, s_i) + \log \pi_\theta(a_i|s_i) \\ - \log \pi'_\theta(a_i|s_{i+1}, s_i) - \log \mu^E(s_{i+1}|s_i)] \end{aligned} \quad (9)$$

Now, by defining a reward function (also see A.2)

$$\begin{aligned} r(a_i, s_i) &:= \mathbb{E}_{s_{i+1} \sim p(s_{i+1}|a_i, s_i)} [-\log p(s_{i+1}|a_i, s_i) \\ &\quad + \log \pi'_\theta(a_i|s_{i+1}, s_i) + \log \mu^E(s_{i+1}|s_i)] \end{aligned} \quad (10)$$

that depends on the expert state transition likelihood  $\mu^E(s_{i+1}|s_i)$ , on the environment model  $p(s_{i+1}|a_i, s_i)$  and on the inverse action distribution density  $\pi'_\theta(a_i|s_{i+1}, s_i)$ . The state-only trajectory distribution matching problem can be transformed to a max-entropy RL task:

$$\begin{aligned} \min \mathbb{D}_{KL}(\mu^{\pi_\theta} || \mu^E) \\ &= \max \sum_{i=0..T-1} \mathbb{E}_{(a_i, s_i) \sim \pi_\theta} [-\log \pi_\theta(a_i|s_i) + r(a_i, s_i)] \\ &= \max \sum_{i=0..T-1} \mathbb{E}_{(a_i, s_i) \sim \pi_\theta} [r(a_i, s_i) + \mathcal{H}(\pi_\theta(\cdot|s))] \end{aligned} \quad (11)$$

In practice the reward function  $r(a_i, s_i)$  can be computed using monte carlo integration with a single sample from  $p(s_{i+1}|a_i, s_i)$  using the replay buffer.

This max-entropy RL task can be optimized with standard max-entropy RL algorithms. In this work, we applied the SAC algorithm [Haarnoja *et al.*, 2018a] as it is outlined in Section 2.1.

The extension to infinite horizon tasks can be done by introducing a discount factor  $\gamma$  as in the work by Haarnoja *et al.* [2018b]. In combination with our reward definition one obtains the following infinite horizon maximum entropy objective:

$$\begin{aligned} J_{ME-iH} &= \sum_{i=0.. \inf} \mathbb{E}_{(a_i, s_i) \sim \pi_\theta} \left[ \sum_{j=i.. \inf} \gamma^{j-i} \right. \\ &\quad \left. \mathbb{E}_{(a_j, s_j) \sim \pi_\theta} [r(a_j, s_j) + \mathcal{H}(\pi_\theta(\cdot|s_j)|s_i, a_i)] \right] \end{aligned} \quad (12)$$

#### 3.1 Algorithm

To evaluate the reward function, the environment model  $p(s_{i+1}|a_i, s_i)$  and the inverse action distribution function  $\pi'_\theta(a_i|s_{i+1}, s_i)$  have to be estimated. We model both distributions using conditional normalizing flows and train with maximum likelihood based on expert demonstrations and rollout data from a replay buffer. The environment model  $p(s_{i+1}|a_i, s_i)$  is modeled by  $\mu_\phi(s_{i+1}|a_i, s_i)$  with parameter  $\phi$  and the inverse action distribution function  $\pi'_\theta(a_i|s_{i+1}, s_i)$  is modeled by  $\mu_\eta(a_i|s_{i+1}, s_i)$  with parameter  $\eta$ .

The whole training process according to Algorithm 1 is described in the following<sup>2</sup>. The expert state transition model  $\mu^E(s_{t+1}|s_t)$  is trained offline using the expert dataset  $D_E$  which contains  $K$  expert state trajectories. We assume the expert distribution is correctly represented by the dataset. Density modeling of the expert state transitions can still result in overfitting when only few expert demonstrations are available (in the few sample limit). We improved the expert training process by adding Gaussian noise to the state values. The standard deviation of the noise is reduced during training so that the model has a correct estimate of the density without

<sup>2</sup>Code will be available at [https://github.com/\\*](https://github.com/*)

overfitting to the explicit demonstrations. With this approach we are able to successfully train expert state transition models on as few as one expert trajectory. The influence of this improved routine is studied in Appendix A.6. After this initial step the following process is repeated until convergence in each episode. The policy interacts with the environment for  $T$  steps to collect state-action-next-state information, which is saved in the replay buffer  $D_{RB}$ . The conditional normalizing flows for the environment model  $\mu_\phi(s_{i+1}|a_i, s_i)$  (policy independent) and the inverse action distribution model  $\mu_\eta(a_i|s_{i+1}, s_i)$  (policy dependent) are optimized using samples from the replay buffer  $D_{RB}$  for  $N$  steps. In Appendix A.3 we show that this reduces the KLD (Equation 9) in each step. Afterwards we use the learned models together with the samples from the replay buffer to compute a one-sample monte carlo approximation of the reward to train the Q-function. The policy  $\pi_\theta(a_t|s_t)$  is updated using SAC. The SAC-based Q-function training and policy optimization also minimize Equation 9 (see Equation 11) in each step. Together with the inverse action policy learning they lead to a converging algorithm since all steps reduce the KLD, which is bounded from below by 0. It is worth noting that the overall algorithm is non-adversarial, the inverse action policy optimization and the policy optimization using SAC both reduce the overall objective - the KLD. Contrary to AIL algorithms like OPOLO, it is not based on an adversarial nested min-max optimization. Additionally, we can estimate the similarity of state transitions from our policy to the expert during each optimization step, since we model all densities in the rewritten KLD from Equation 9. As a result we have a reliable performance estimate enabling us to select the best performing policy based on the lowest KLD between policy and expert state transition trajectories.

### 3.2 Relation to learning from observations

The LfO objective of previous approaches like OPOLO minimizes the divergence between the joint policy state transition distribution and the joint expert state transition distribution:

$$J_{LfO} = \mathbb{D}_{KL}(\mu^{\pi_\theta}(s', s) || \mu^E(s', s)) \quad (13)$$

which can be rewritten as (see A.1)

$$J_{LfO} = \mathbb{D}_{KL}(\mu^{\pi_\theta}(s_T, \dots, s_0) || \mu^E(s_T, \dots, s_0)) + \sum_{i=1..T-1} \mathbb{D}_{KL}(\mu^{\pi_\theta}(s_i) || \mu^E(s_i)) \quad (14)$$

Thus, this LfO objective minimizes the sum of the KLD between the joint distributions and the KLDs of the marginal distributions. The SOIL-TDM objective in comparison minimizes purely the KLD of the joint distributions. In case of a perfect distribution matching - a zero KLD between the joint distributions - the KLDs of the marginals also vanish so both objectives have the same optimum.

## 4 Related Work

Many recent IL approaches are based on inverse RL (IRL) [Ng and Russell, 2000]. In IRL, the goal is to learn a reward signal for which the expert policy is optimal. AIL algorithms

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### Algorithm 1 State-Only Imitation Learning by Trajectory Distribution Matching (SOIL-TDM)

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1: procedure SOIL-TDM( $D_E$ )
2:   Train  $\mu^E(s_{t+1}|s_t)$  with  $D_E : \{s_0, s_1, \dots, s_T\}_{k=0}^K$ 
3:   for episodes do
4:     for range( $T$ ) do ▷ generate data
5:        $\hat{a}_t \leftarrow \text{sample}(\pi_\theta(\hat{a}_t|s_t))$ 
6:        $s_{t+1} \leftarrow p_{sim}(s_{t+1}|s_t, \hat{a}_t)$  ▷ apply action
7:       Store  $(s_t, \hat{a}_t, s_{t+1})$  in  $D_{RB}$ 
8:     end for
9:     for range( $N$ ) do ▷ update dynamics models
10:       $\{(s_t, \hat{a}_t, s_{t+1})\}_{i=1}^B \sim D_{RB}$  ▷ sample batch
11:      train  $\mu_\eta(\hat{a}_t|s_{t+1}, s_t)$  and  $\mu_\phi(s_{t+1}|\hat{a}_t, s_t)$ 
12:    end for
13:    for range( $N$ ) do ▷ SAC Optimization
14:       $\{(s_t, \hat{a}_t, s_{t+1})\}_{i=1}^B \sim D_{RB}$  ▷ sample batch
15:       $a_t \leftarrow \text{sample}(\pi_\theta(a_t|s_t))$ 
16:      optimize  $\pi_\theta(a_t|s_t)$  with  $J_\pi$  from eq. 4
17:      ▷ estimate reward
18:       $r(s_t, \hat{a}_t) \leftarrow -\log \mu_\phi(s_{t+1}|\hat{a}_t, s_t) +$   

        $\log \mu_\eta(\hat{a}_t|s_{t+1}, s_t) + \log \mu^E(s_{t+1}|s_t)$ 
19:      optimize  $Q_\psi(\hat{a}_t, s_t)$  with  $J_Q$  from eq. 2
20:    end for
21:  end for
22: end procedure

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are popular methods to perform IL in a RL setup [Ho and Ermon, 2016; Fu *et al.*, 2017; Kostrikov *et al.*, 2020]. In AIL, a discriminator gets trained to distinguish between expert states and states coming from policy rollouts. The goal of the policy is to fool the discriminator. The policy gets optimized to match the state action distribution of the expert, using this two-player game. Based on this idea more general approaches have been derived based on f-divergences. Ni *et al.*[2020] derive an analytic gradient of any f-divergence between the agent and expert state distribution w.r.t. reward parameters. Based on this gradient they present the algorithm f-IRL that recovers a stationary reward function from the expert density by gradient descent. Ghasemipour *et al.*[2019] identify that IRL’s state-marginal matching objective contributes most to its superior performance and apply this understanding to teach agents a diverse range of behaviours using simply hand-specified state distributions.

A key problem with AIL for LfD and LfO is optimization instability [Miyato *et al.*, 2018]. Wang *et al.*[2019] avoid the instabilities resulting from adversarial optimization by estimating the support of the expert policy to compute a fixed reward function. Similarly, Brantley *et al.*[2020] use a fixed reward function by estimating the variance of an ensemble of policies. Both methods rely on additional behavioral cloning steps to reach expert-level performance. Liu *et al.*[2020] propose Energy-Based Imitation Learning (EBIL) which recovers fixed and interpretative reward signals by directly estimating the expert’s energy. Neural Density Imitation (NDI) [Kim *et al.*, 2021] uses density models to perform distribution matching. Deterministic and Discriminative Imitation (D2-Imitation) [Sun *et al.*, 2021] requires no adver-

serial training by partitioning samples into two replay buffers and then learning a deterministic policy via off-policy reinforcement learning. Inverse soft-Q learning (IQ-Learn) [Garg *et al.*, 2021] avoids adversarial training by learning a single Q-function to implicitly representing both reward and policy. The implicitly learned rewards from IQ-Learn show a high positive correlation with the ground-truth rewards.

LfO can be divided into model-free and model-based approaches. GAILfO [Torabi *et al.*, 2018b] is a model-free approach which uses the GAIL principle with the discriminator input being state-only. Yang *et al.* [2019] analyzed the gap between the LfD and LfO objectives and proved that it lies in the disagreement of inverse dynamics models between the imitator and expert. Their proposed method IDDM is based on an upper bound of this gap in a model-free way. OPOLO [Zhu *et al.*, 2020] is a sample-efficient LfO approach also based on AIL, which enables off-policy optimization. The policy update is also regulated with an inverse action model that assists distribution matching in a mode-covering perspective.

Other model-based approaches either apply forward dynamics models or inverse action models. Sun *et al.* [2019] proposed a solution based on forward dynamics models to learn time dependant policies. While being provably efficient, it is not suited for infinite horizon tasks. Alternatively, behavior cloning from observations (BCO) [Torabi *et al.*, 2018a] learns an inverse action model based on simulator interactions to infer actions based on the expert state demonstrations. GPRIL [Schroecker *et al.*, 2019] uses normalizing flows as generative models to learn backward dynamics models to estimate predecessor transitions and augmenting the expert data set with further trajectories, which lead to expert states. Jiang *et al.* [2020] investigated IL using few expert demonstrations and a simulator with misspecified dynamics. A detailed overview of LfO was done by Torabi *et al.* [2019].

#### 4.1 Method Discussion and Relation to FORM

While our proposed method SOIL-TDM was independently developed it is most similar to the state-only approach FORM [Jaegle *et al.*, 2021]. In FORM the policy training is guided by a conditional density estimation of the expert’s observed state transitions. In addition a state transition model  $\mu_{\Phi}^{\pi_{\theta}}(s_{i+1}|s_i)$  of the current policy is learned. The policy reward is estimated by:  $r_i = \log \mu^E(s_{i+1}|s_i) - \log \mu_{\Phi}^{\pi_{\theta}}(s_{i+1}|s_i)$ . The approach matches conditional state transition probabilities of expert and policy in comparison to the joint state-action (like GAIL) or joint state-next-state (like OPOLO or GAILfO) densities. The authors argue that this has consequences in the robustness of the different approaches. Namely, methods based on conditional state probabilities are less sensitive to erroneously penalizing features that may not be in the demonstrator data but lead to correct transitions. Hence, such methods may be less prone to overfit to irrelevant differences. Jaegle *et al.* [2021] demonstrate the benefit of such a conditional density matching approach.

In contrast to FORM we show in Equation 8 that the policies next-state conditional density  $\mu^{\pi_{\theta}}(s_{i+1}|s_i)$  can be separated into the policies action density and the forward- and the backward-dynamics densities. Using this decomposition

we show that the KLD minimization is equivalent to a maximum entropy RL objective (see Equation 9) with a special reward (see Equation 10). Here the entropy of the policy stemming from the decomposition of the conditional state-next-state density leads to the maximum entropy RL objective. Jaegle *et al.* [2021] mention that the second term in their reward objective can be viewed as an entropy-like expression. Hence, if this reward is optimized using a RL algorithm which includes some form of policy entropy regularization this entropy is basically weighted twice. In the experiments we show that this double accounting of the policy entropy negatively affects the sample efficiency of the algorithm in comparison to our method.

## 5 Experiments

We evaluate our proposed method described in Section 3 in a variety of different IL tasks and compare it against the baseline methods OPOLO, F-IRL and FORM. We evaluate and compare all methods in complex and high dimensional continuous control environments using the Pybullet physics simulation [Coumans and Bai, 2016 2019]. To evaluate the performance of all methods, the episode rewards of the trained policies are compared to reward from the expert policy. The expert data generation as well as the used baseline implementations are described in Appendix A.5.

Since we assume no environment reward is available as an early stopping criterion, we use other convergence estimates available during training to select the best policy for each method. In adversarial training the duality gap [Grnarova *et al.*, 2019; Sidheekh *et al.*, 2021] is an established method to estimate the convergence of the training process. In the IL setup the duality gap can be very difficult to estimate since it requires the gradient of the policy and the environment (i.e. the gradient of the generator) for the optimization process it relies on. We therefore use two alternatives for model selection for OPOLO. The first approach selects the model with the lowest policy loss and the second approach selects the model based on the highest estimated reward over ten consecutive epochs. For F-IRL we selected the model with the lowest estimated Jensen-Shannon divergence over ten epochs. To estimate the convergence of SOIL-TDM the policy loss based on the KLD from equation 9 is used. It can be estimated using the same models used for training the policy. Similarly, we used the effect models of FORM to estimate the convergence based on the reward.

The evaluation is done by running 3 training runs with ten test episodes (in total 30 rollouts) for each trained policy and calculating the respective mean and confidence interval for all runs. We plot the rewards normalized so that 1 corresponds to expert performance. Values above 1 mean that the agent has achieved a higher reward than the mean of the expert (episode rewards of the expert are reported in Appendix A.5). Implementation details of our method are described in Appendix A.4.

The evaluation results of the discussed methods on a suite of continuous control tasks with unknown true environment reward as a selection criterion are shown in Figure 1. The achieved rewards are plotted with respect to the number of

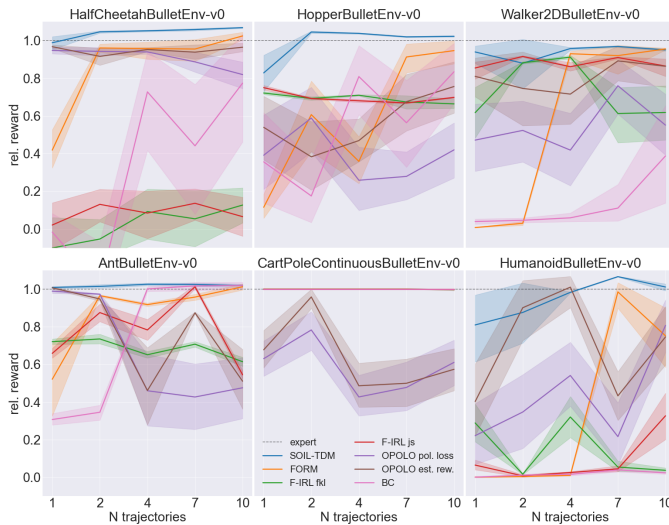


Figure 1: Unknown true environment reward selection criteria: Relative reward for different amount of expert trajectories on continuous control environments. The best policies based on estimated convergence values were selected. The value 1 corresponds to expert policy performance.

expert trajectories provided for training the agent. The confidence intervals are plotted using lighter colors.

If the true environment reward is unknown the results show that SOIL-TDM achieves or surpasses the performance of the baseline methods on all tested environments (with the exception of two and four expert trajectories in the HumanoidBulletEnv-v0 environment and two trajectories in the Walker2DBulletEnv-v0 environment). In general the adversarial based methods OPOLO and F-IRL exhibit a high variance of the achieved rewards using the proposed selection criteria. While loss and reward are well suited for selecting the best model in usual setups, the results demonstrate that they might be less expressive for estimating the convergence in adversarial training due to the min-max game of the discriminator and the policy. The stability of the SOIL-TDM training method is evident from the small confidence band of the results which gets smaller for more expert demonstrations. While the selection of FORM is also more stable compared to the adversarial methods it generally achieves lower rewards in the sample efficient regime of one and two expert trajectories.

Figure 2 shows the benchmark results of OPOLO, F-IRL, FORM, and SOIL-TDM if the true environment reward is used as an early stopping criterion. In this setup, our method still achieves competitive performance or surpasses OPOLO, F-IRL, and FORM. Additional figures for a comparison of the training performance and efficiency can be found in the Appendix A.7. An ablation study for our method is in the Appendix A.6.

## 6 Conclusion

In this work we propose a non-adversarial state-only imitation learning approach based on the minimization of the

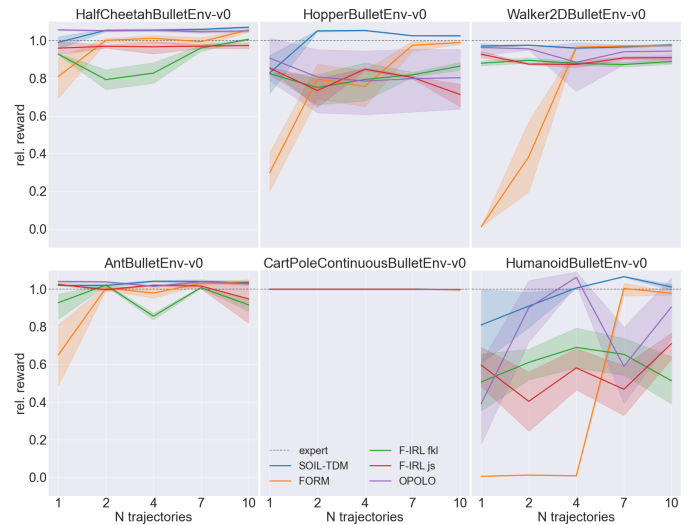


Figure 2: Best true environment reward selection criterion: Relative reward for different amount of expert trajectories on continuous control environments. The value 1 corresponds to expert policy performance.

Kulback-Leibler divergence between the policy and the expert state trajectory distribution. This objective leads to a maximum entropy reinforcement learning problem with a reward function depending on the expert state transition distribution and the forward and backward dynamics of the environment which can be modeled using conditional normalizing flows. The proposed approach is compared to several state-of-the-art learning from observations methods in a scenario with unknown environment rewards and achieves state-of-the-art performance.

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